Homework #1: Tensegrity structures

MAE 290A, Numerical Methods

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November 4, 2019

Abstract

In this homework assignment some tensegrity structures are analyzed using a given code to design and calculate its state which could be potentially inconsistent, undetermined, both potentially inconsistent and undetermined or none of the previous with only one exact solution called *static determinance*. In particular, a Fourth Order Michell Truss and a Non-Minimal Tensegrity Prism with four bars will be analyzed in some set up conditions using the given code developed by T. Bewley [1–3] and the previous analysis made in [4] of Tensegrity Structures. As an additional input to the existent work, a formulation of the Direct Stiffness Method [5] is given in concordance with the existent code. As a result, the code could be used to obtain the displacements and deformation of the structure when external loads are applied defining the Stiffness Matrix of each element and finally constructing a system in the form $A\mathbf{x} = \mathbf{b}$ for displacements. Some cases are analyzed to prove its validity.

1 Michell Truss Structure

According with the description made in [4], a code named `MitchelTruss4.m` is made to construc different configuration of a Michell Truss structure of order four.

A case represented in the figure 1, with only a vertical load in its point at the extreme has been computed. Based in the results, the structure is neither potentially inconsistent nor undetermined. In this state, there is not soft modes in the structure which correspond to the so called *static determinance* state. Thus it is not pretensionable, provided that it is possible to find a unique solution to the problem.

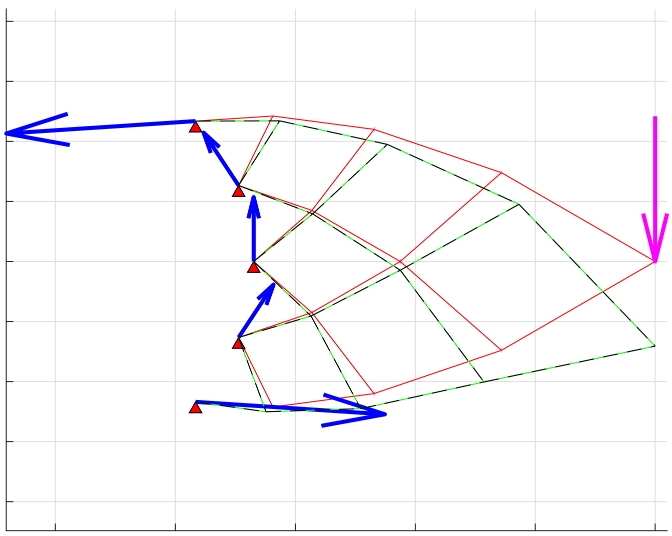


Figure 1: Michell Truss structure of order four. In black and green the result of the Direct Stiffness Method.

Once the problem is solved by the codes given by [3] it has been include a calculation for the displacements based on the Stiffness Matrix of the entire system. Every member has been characterized as a bar which allows rotation in its extremes[1]. Therefore, each matrix is constructed only by the Young Modulus (E) and the area of each bar. In this case, all the members have been selected to be equal.

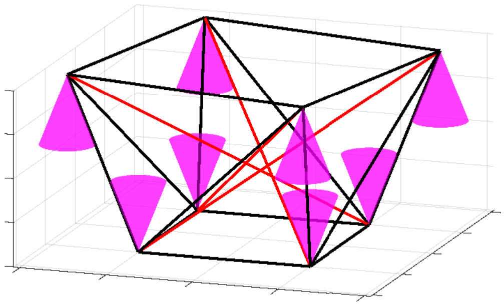


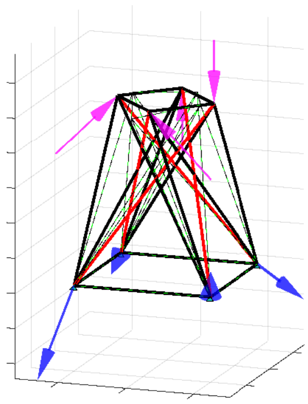
Figure 2: Non-Minimal Tensegrity Prism with four bars.

2 Non-Minimal Tensegrity Prism with four bars

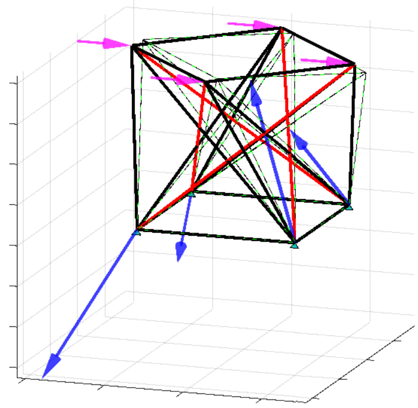
The second structure analyzed in this work is a Non-Minimal Tensegrity Prims with four bars. This configuration is represented in the figure ??. It turns out that this configuration is potentially inconsistent, implying the presence of soft modes. The rank in the system is 17, meaning that there is some degrees of freedom that gives us freedom to pretensionable the structure under zero load applied (in some cases it is not pretensionable under zero load applied so it is considered when the structure is under load).

After the linear program it is found that each string is at least above a certain value of tension so it is less likely that they go slack at some point when loads are applied.

Finally, some different configurations are presented in figures 3a and 3b computed with the added code for the Direct Stiffness Method.



(a)



(b)

Figure 3: Different cases to visualize the displacements experimented by the structure when loads are applied and the four points in base are fixed. Results of the codes `NonminimalPrism4Fixed1.m` and `NonminimalPrism4Fixed2.m`.

3 Conclusions

In the present work two different structures are analyzed. It is found that the first one, Michell Truss Structure of order four leads to a deterministic problem so that there is a unique solution for its static state, without any soft mode.

On the other hand, for a four bars Non-Minimal

[1] Note that for the stiffness matrix of a string the code should be modified in case the the string is not under traction

Prism it is found that it is potentionaly inconsistent and undetermined. For the configuration studied it was possible to pretensionable the structure and therefore find a solution with no slack string under no loads applied.

Finally, a code has been created to compute the Direct Stiffness Method that uses the previous information to solve for the displacements of the structure once the properties of the material are specified. With this code a more specific knowledge of the structure can be achieved, related with the physical nature of the elements.

References

[1] T. Bewley. Stabilization of low-altitude balloon systems, part 1: rigging with a single taut ground tether, with analysis as a variable-length pendulum. *Jet Propulsion Lab (JPL) and UC San Diego*, 2019.

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[4] Robert Skelton and Mauricio Oliveira. *Tensegrity Systems*. 01 2009.

[5] Robert J Melosh. Basis for derivation of matrices for the direct stiffness method. *AIAA Journal*, 1(7):1631–1637, 1963.