⇒ Jean Baptiste Joseph fourier [21 march 1768 - 16 may 1930]

⇒ Peter Gustav Lejune Dirichlet [13 feb 1805- 5 may 1859]

\# Fourier Series → periodic signals only } Analysis

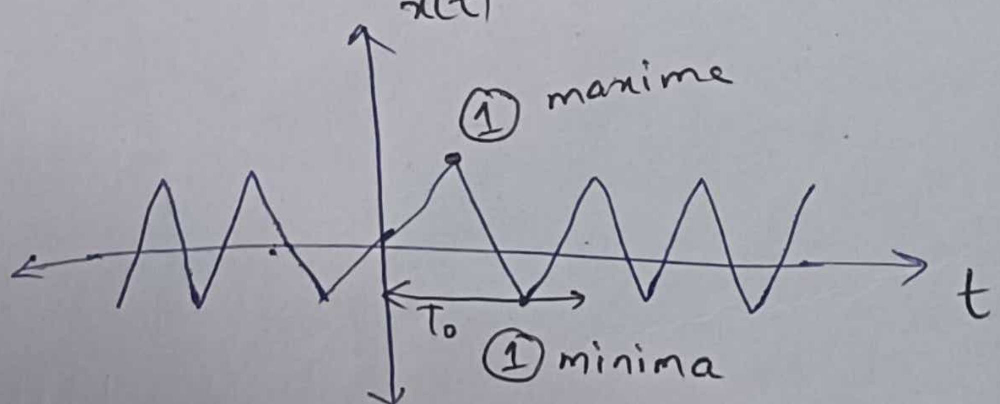
\# Fourier transforms → Non-periodic } Analysis

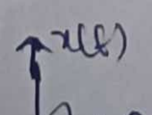
\# Laplace transforms ⟶ Designing

* Fourier series:

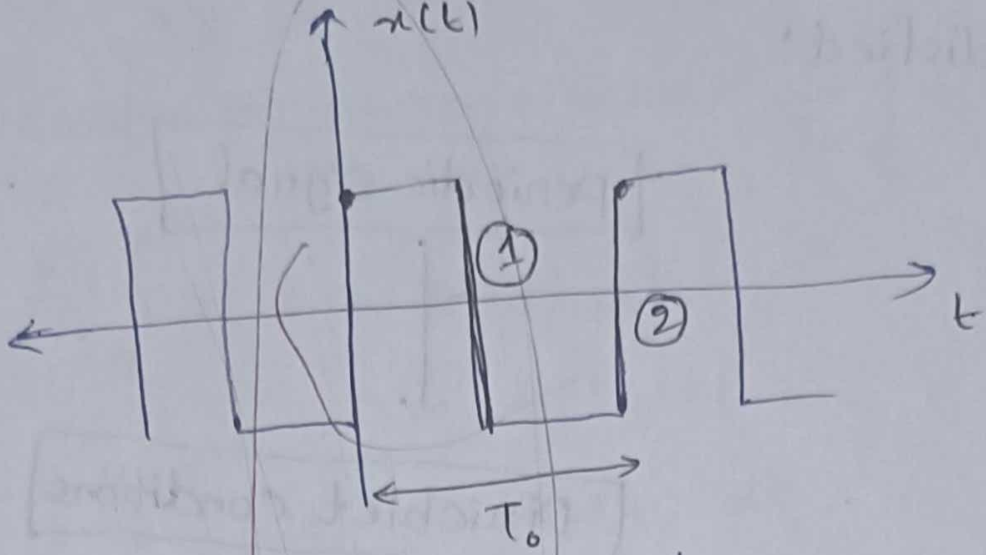
\# <u>Dirichlet</u> <u>Conditions</u>:

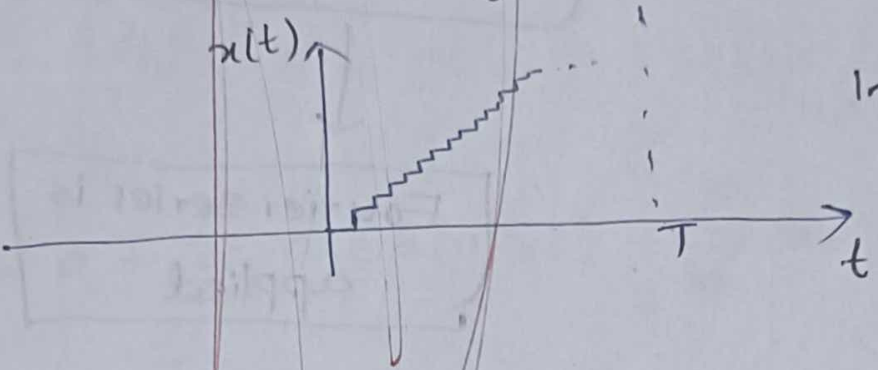
① Signal should have finite number of maxima and minima over the range of time period.





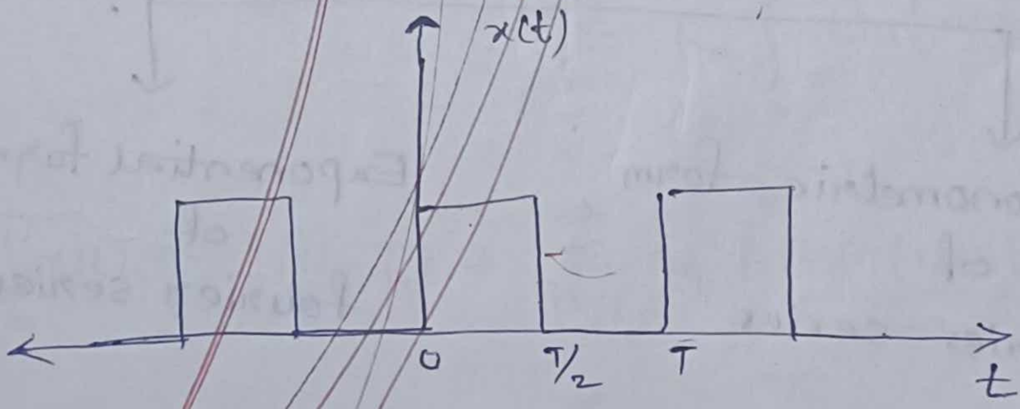
discontinuities over the range of time period

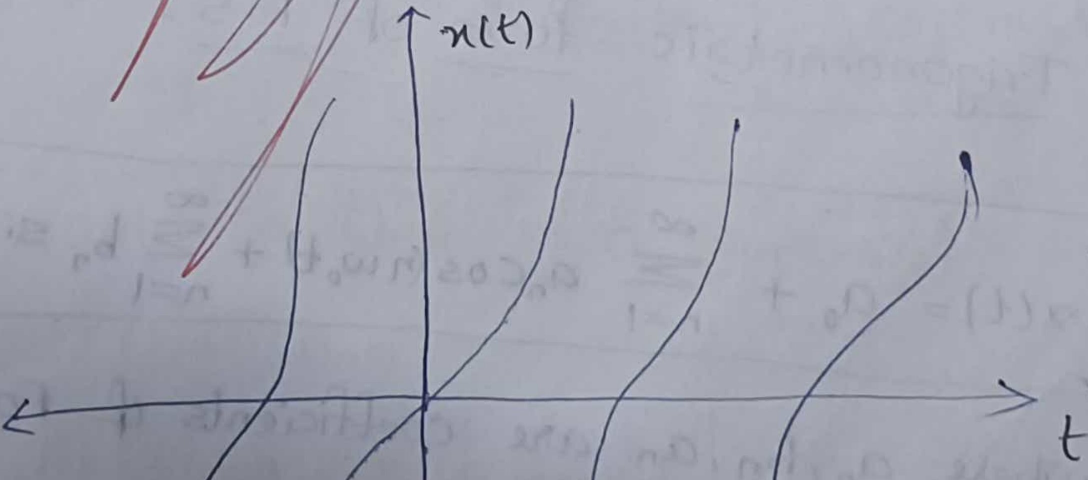




Infinite discontinuities

③ signal should be absolutely integrable over the time period.





the following conditions

satisfied:

Periodic signal

↓

Dirichlet conditions

↓

Fourier series is applied

Fourier Series

↓ Trigonometric form of fourier series

↓ Exponential form of fourier series.

① Trigonometric form of F.S:

$$\boxed{x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)}$$

Fourier

$2\omega_0, 3\omega_0, 4\omega_0 \longrightarrow$ harmonics.

$a_0 =$ DC value (or) Average value $= \frac{1}{T_0} \int_{T_0} x(t)\, dt$

$$a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos(n\omega_0 t)\, dt$$

$$b_n = \frac{2}{T_0} \int_{T_0} x(t) \cdot \sin(n\omega_0 t)\, dt$$

(i)

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$$\Rightarrow \int_0^{T_0} x(t)\, dt = \int_0^{T_0} a_0\, dt + \sum_{n=1}^{\infty} a_n \int_0^{T_0} \cos(n\omega_0 t)\, dt$$

$$+ \sum_{n=1}^{\infty} b_n \int_0^{T_0} \sin(\underline{n\omega_0 t})\, dt$$

$$\Rightarrow \int_0^{T_0} x(t)\, dt = (a_0 \cdot T_0) + \sum_{n=1}^{\infty} a_n \left| \frac{\sin(n\omega_0 t)}{n\omega_0} \right|_0^{T_0}$$

$$+ \sum_{n=1}^{\infty} b_n \left| -\frac{\cos(n\omega_0 t)}{n\omega_0} \right|_0^{T_0}$$

$$\Rightarrow \int_0^{T_0} x(t)\, dt = (a_0 \cdot T_0) + \sum_{n=1}^{\infty} a_n \frac{\sin(n\omega_0 T_0)}{n\omega_0}$$

$$+ \sum^{\infty} b_n \left[\frac{1}{} - \cos(n\omega_0 t_0) \right]$$

$$+ \sum_{n=1}^{\infty} b_n \left[\frac{1}{n\omega_0} - \frac{\cos(2\pi n)}{n\omega_0} \right]$$

$$\sin(2\pi n) = 0 \quad \forall n$$

$$\cos(2\pi n) = 1 \quad \forall n$$

$$\Rightarrow \int_0^{T_0} x(t)\,dt = a_0 T_0 + 0 + 0$$

$$\boxed{a_0 = \frac{1}{T_0} \int_0^{T_0} x(t)\,dt} \quad (i)$$

(ii) <u>proof:</u>

$$\therefore x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$$\Rightarrow x(t) = a_0 + a_1 \cos\omega_0 t + a_2 \cos 2\omega_0 t + \dots\dots + a_n \cos n\omega_0 t + \dots\dots$$

$$+ b_1 \sin\omega_0 t + b_2 \sin 2\omega_0 t + \dots\dots + b_n \sin n\omega_0 t + \dots\dots$$

$$\Rightarrow x(t).\cos(n\omega_0 t) = a_0 \cos(n\omega_0 t) + a_1 \cos\omega_0 t.\cos(n\omega_0 t) +$$

$$a_2 \cos(2\omega_0 t).\cos(n\omega_0 t) + \dots\dots + a_n \cos^2(n\omega_0 t) + \dots\dots$$

$$+ a_2 \int_0^{T_0} \cancelto{0}{\cos(2\omega_0 t).\cos(n\omega_0 t)\,dt} + \cdots\cdots + a_n \int_0^{T_0} \cos^2(n\omega_0 t)\,dt +$$

$$\cdots + b_1 \int_0^{T_0} \cancelto{0}{\sin(\omega_0 t)\cos(n\omega_0 t)\,dt} + b_2 \int_0^{T_0} \cancelto{0}{\sin(2\omega_0 t).\cos(n\omega_0 t)\,dt}$$

$$+ \cdots\cdots + b_n \int_0^{T_0} \cancelto{0}{\sin(n\omega_0 t).\cos(n\omega_0 t)\,dt} + \cdots\cdots$$

$$\Rightarrow \int_0^{T_0} x(t).\cos(n\omega_0 t)\,dt = a_n \int_0^{T_0} \left[\frac{1+\cos(2n\omega_0 t)}{2}\right] dt$$

$$\Rightarrow \int_0^{T_0} x(t).\cos(n\omega_0 t)\,dt = a_n \cdot \frac{T_0}{2}$$

$$\boxed{\therefore a_n = \frac{2}{T_0} \int_0^{T_0} x(t).\cos n\omega_0 t \; dt}$$

(iii) <u>proof:</u>

$$\therefore x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

$$\Rightarrow x(t) = a_0 + a_1 \cos\omega_0 t + a_2 \cos 2\omega_0 t + \cdots + a_n \cos n\omega_0 t + \cdots$$

$$+ \; b_1 \sin\omega_0 t + b_2 \sin 2\omega_0 t + \cdots + b_n \sin n\omega_0 t + \cdots$$

$$+ a_2 \cos(2\omega_0 t).\sin(n\omega_0 t) + \cdots\cdots + a_n \cos(n\omega_0 t).\sin(n\omega_0 t) + \cdots$$

$$+ b_1 \sin(\omega_0 t)\sin(n\omega_0 t) + b_2 \sin(2\omega_0 t)\sin(n\omega_0 t) + \cdots\cdots$$

$$+ b_n \sin^2(n\omega_0 t) + \cdots\cdots$$

$$\Rightarrow \int_0^{T_0} x(t).\sin(n\omega_0 t)\,dt = \cancelto{0}{\int_0^{T_0} a_0 \sin(n\omega_0 t)\,dt} + a_1 \cancelto{0}{\int_0^{T_0} \cos(\omega_0 t).\sin(n\omega_0 t)\,dt}$$

$$+ a_2 \cancelto{0}{\int_0^{T_0} \cos(2\omega_0 t).\sin(n\omega_0 t)\,dt} + \cdots\cdots + a_n \cancelto{0}{\int_0^{T_0} \cos(n\omega_0 t).\sin(n\omega_0 t)\,dt} + \cdots$$

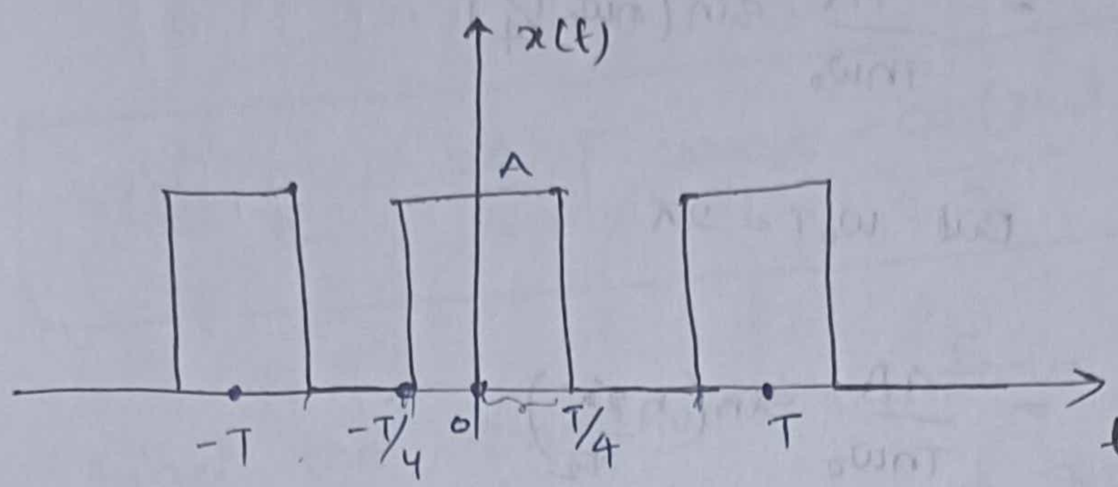
$$+ b_1 \cancelto{0}{\int_0^{T_0} \sin(\omega_0 t).\sin(n\omega_0 t)\,dt} + b_2 \cancelto{0}{\int_0^{T_0} \sin(2\omega_0 t).\sin(n\omega_0 t)\,dt}$$

$$+ \cdots\cdots + b_n \int_0^{T_0} \sin^2(n\omega_0 t)\,dt + \cdots\cdots$$

$$\Rightarrow \int_0^{T_0} x(t).\sin(n\omega_0 t)\,dt = b_n \int_0^{T_0} \left[\frac{1-\cos(2n\omega_0 t)}{2}\right] dt$$

$$\Rightarrow \int_0^{T_0} x(t).\sin(n\omega_0 t)\,dt = b_n . \frac{T_0}{2}$$

fourier series of the following wave form.



$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$$

(i) $a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt$

$= \frac{1}{T}\int_{-T/4}^{T/4} A.dt = \frac{1}{T} \cdot A\left[\frac{T}{4}+\frac{T}{4}\right]$

$= \frac{A}{\cancel{T}} \cdot \frac{\cancel{T}}{2}$

$\boxed{a_0 = \frac{A}{2}}$

(ii) $a_n = \frac{2}{T_0}\int_0^{T_0} x(t)\cos(n\omega_0 t)\,dt$

$= \frac{2}{T}\int_{-T/4}^{T/4} A\cos(n\omega_0 t)\,dt$

$$= \frac{4A}{Tn\omega_0} \cdot \sin(n\omega_0 T/4)$$

But $\omega_0 T = 2\pi$

$$= \frac{4A}{Tn\omega_0} \sin\left(n \frac{2\pi}{4_2}\right)$$

$$= \frac{4A}{Tn\omega_0} \sin\left(\frac{n\pi}{2}\right)$$

$$= \frac{4A}{2\pi n} \sin\left(\frac{n\pi}{2}\right)$$

$$\boxed{a_n = \frac{2A}{n\pi} \sin\left(\frac{n\pi}{2}\right)}$$

$$a_n = \begin{cases} \frac{2A}{n\pi}(-1)^n \text{ ; odd} \\ 0 \text{ ; even} \end{cases}$$

(iii) $$b_n = \frac{2}{T} \int_0^T A \sin(n\omega_0 t)\, dt$$

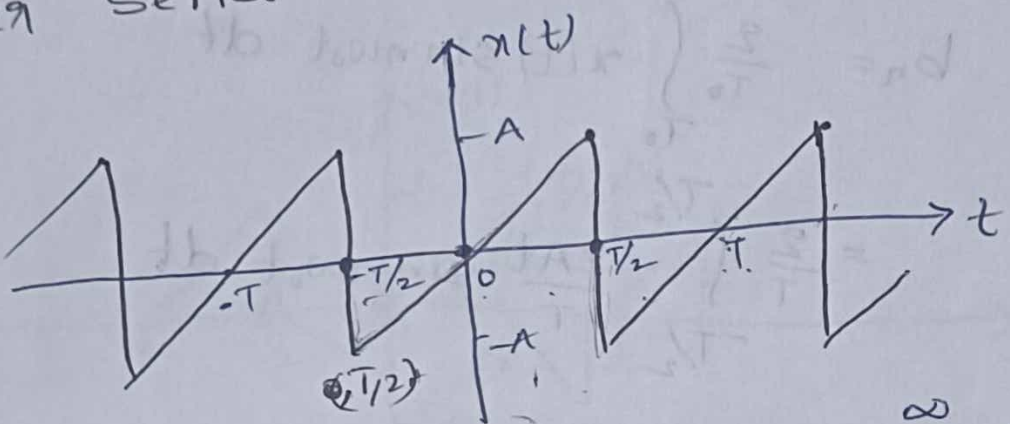
$$= \frac{2A}{T} \Big|_0^T \frac{-\cos(n\omega_0 t)}{n\omega_0}$$

$$b_n = \frac{-2A}{\quad} [0] = 0$$

$$2 \qquad n=1$$

$$\boxed{x(t) = \frac{A}{2} + \frac{2A}{\pi}\left[\cos\omega_0 t - \frac{\cos(3\omega_0 t)}{3} + \cdots\right]}$$

* Find the trigonometric form of fourier series for the signal shown below.



Sol:- $x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)$

(i) $a_0 = \frac{1}{T_0}\int_0^{T_0} x(t)\,dt$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \frac{2A}{T_0} t\,dt$$

$$= \frac{2A}{T_0^2}\left[\frac{T_0^2}{8} - \frac{T_0^2}{8}\right]$$

$$\boxed{a_0 = 0}$$

$$= \frac{4A}{T^2}\left[\frac{t\sin(n\omega_0 t)}{n\omega_0} + \frac{\cos(n\omega_0 t)}{(n\omega_0)^2}\right]_{-T/2}$$

$$= \frac{4A}{T^2}(0)$$

$$\boxed{a_n = 0}$$

(iii) $b_n = \frac{2}{T_0}\int_{T_0} x(t)\sin n\omega_0 t \, dt$

$$= \frac{2}{T}\int_{-T/2}^{T/2} \frac{2At}{T}\sin n\omega_0 t \, dt$$

$$= \frac{4A}{T^2}\left[\frac{-t\cos n\omega_0 t}{n\omega_0} + \cancel{\frac{\sin n\omega_0 t}{(n\omega_0)^2}}\right]_{-T/2}^{T/2}$$

$$= \frac{\overset{2}{\cancel{4}}A}{T^2}\left[\frac{-T/\cancel{2}\cos(n\pi)}{n\omega_0} - \frac{T/\cancel{2}\cos n\pi}{n\omega_0}\right]$$

$$= -\frac{2A}{T^2}\cdot\frac{\cancel{2}T\cos n\pi}{n\frac{\cancel{2}\pi}{T}}$$

$$= \frac{-2A}{\cancel{T^2}}\cdot\frac{\cancel{T^2}\cos n\pi}{\pi n}$$

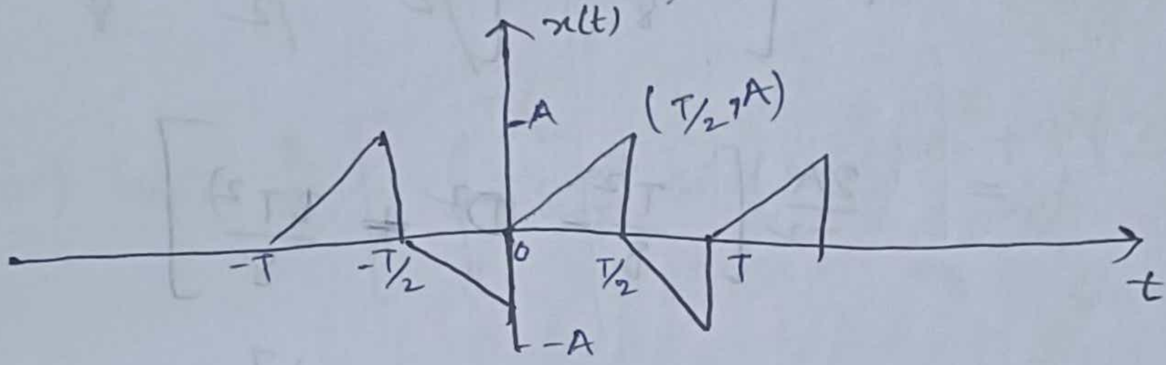
$$b_n = \frac{-2A\cos n\pi}{\pi n}$$

$$n=1 \qquad n\pi$$

$$= \sum_{n=1}^{\infty} \frac{2A}{n\pi} (-1)^{n+1} \sin(n\omega_0 t)$$

$$x(t) = \left\{ \sum_{n=1}^{\infty} \frac{2A}{n\pi} (-1)^{n+1} \sin(n\omega_0 t) \right.$$

(3)



Sol:

$$a_0 = \frac{1}{T} \left[\int_0^{T/2} x_1(t)\,dt + \int_{T/2}^{T} x_2(t)\,dt \right]$$

for $x_1(t)$:

$$y = \frac{2A}{T} x$$

$$\Rightarrow \boxed{x = \frac{2A}{T} t}$$

for $x_2(t)$: $(T/2, 0)$ and $(T, -A)$

$$\rightarrow m = \frac{-A}{T/2} = -\frac{2A}{T}$$

(i) $a_0 = \frac{1}{T}\left[\int_0^{T/2} \frac{2At}{T}\,dt + \int_{T/2}^{T} -\frac{2At}{T} + A\,dt\right]$

$= \frac{2A}{T^2}\left[\Big|_0^{T/2} \frac{t^2}{2} - \Big|_{T/2}^{T} t^2/2 + T/2\,t\right]$

$= \frac{2A}{T^2}\left[\frac{T^2}{8} - \left[\cancel{\frac{T^2}{2}} + \cancel{\frac{T^2}{2}} - \frac{T^2}{8} + \frac{T^2}{4}\right]\right]$

$= \frac{2A}{T^2}\left[\frac{T^2}{8} - 0^2 + \frac{3T^2}{8}\right]$

$= \frac{2A}{T^2}\left[\frac{T^2 - 0^2 + (3T^2)}{8}\right]$

$= \frac{\cancel{2}A}{\cancel{T^2}}\left[-\frac{4}{8}\right]$

$\boxed{a_0 = -A}$

(ii) $a_n = \frac{2}{T}\left[\int_0^{T/2} \frac{2At}{T}\cos(n\omega_0 t)\,dt + \int_{T/2}^{T}\left(-\frac{2At}{T} + A\right)\cos(n\omega_0 t)\,dt\right]$

$= \frac{2}{T}\left[\frac{2A}{T}\Big|^{T/2} \frac{t\sin(n\omega_0 t)}{\quad} + \cos(n\omega_0 t)\right.$

$$- \frac{2A}{T(n\omega_0)^2}\left(1-\cos n\pi\right)\Bigg]$$

$$= \frac{2}{T}\left[\frac{2A\cos n\pi}{(n\omega_0)^2 T} - \frac{2A}{(n\omega_0)^2 T} - \frac{2A}{(n\omega_0)^2 T} + \frac{2A}{(n\omega_0)^2 T}\cos n\pi \right]$$

$$a_n = \frac{2\times 2A}{(n\omega_0 T)^2}\left[2\cos n\pi - 2 \right]$$

$$\text{(iii) } b_n = \frac{2}{T}\left[\int_0^{T/2} \frac{2At}{T}\sin(n\omega_0 t)\,dt + \int_{T/2}^{T} \left(-\frac{2At}{T}+A\right)\sin n\omega_0 t\,dt \right]$$

$$= \frac{2}{T}\left[\frac{2A}{T}\Big|_0^{T/2} \frac{-t\cos(n\omega_0 t)}{n\omega_0} - \frac{2A}{T}\Big|_{T/2}^{T} \frac{-t\cos(n\omega_0 t)}{n\omega_0} + A\Big|_{T/2}^{T} \frac{\cos(n\omega_0 t)}{n\omega_0} \right]$$

$$= \frac{2}{T}\left[\frac{2A}{T}\left(\frac{-\frac{T}{2}\cos n\pi}{n\omega_0}\right) - \frac{2A}{T}\Big|_{T/2}^{T} \frac{-t\cos(n\omega_0 t)}{n\omega_0} + A\Big|_{T/2}^{T} \frac{\cos n\omega_0 t}{n\omega_0} \right]$$

$$\boxed{b_n = \frac{A}{\pi n}\left(1-\cos n\pi\right)}$$

$$\infty \qquad \sum^{\infty} \frac{A}{}\left(1-\cos n\pi\right)$$

(1) t-axis symmetry $a_0 = 0$

(2) even symmetry

$x(-t) = x(t)$? ? $b_n = 0$

(3) odd symmetry ? $a_n = 0$?

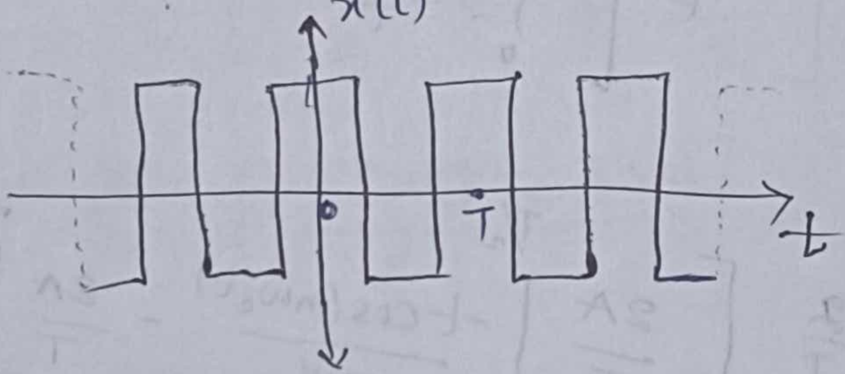
$x(-t) = -x(t)$

(4) Half-wave Symmetry

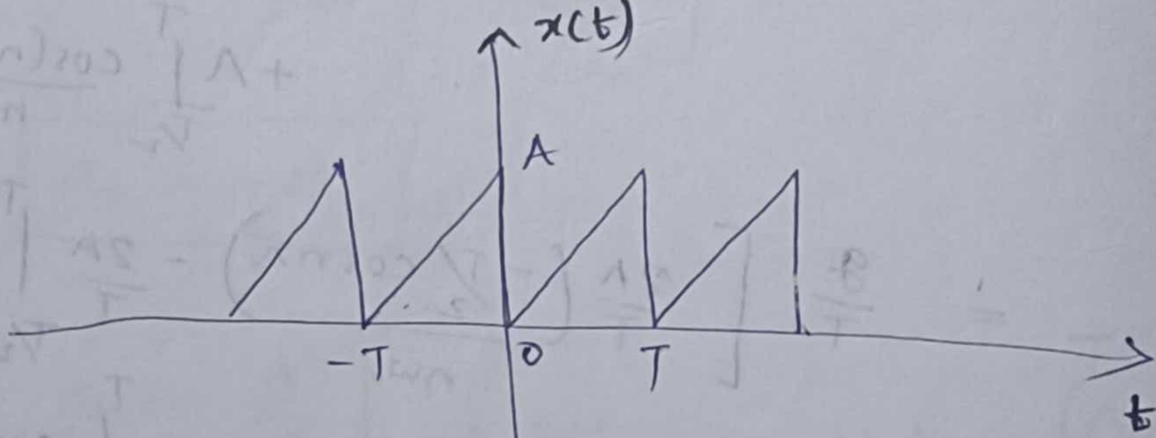
$n = 1, 3, 5, 7 \cdots$ odd harmonics

$x(t) = -x(t \pm \frac{T}{2})$

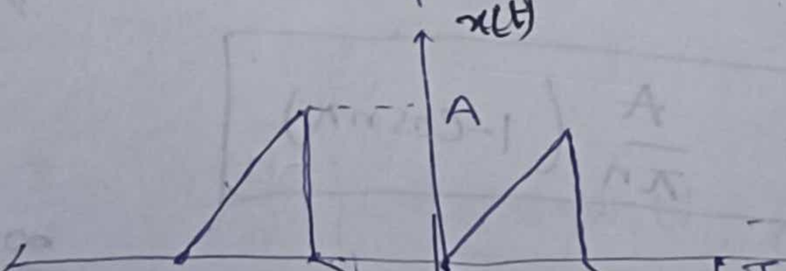
Eg:



Eg:

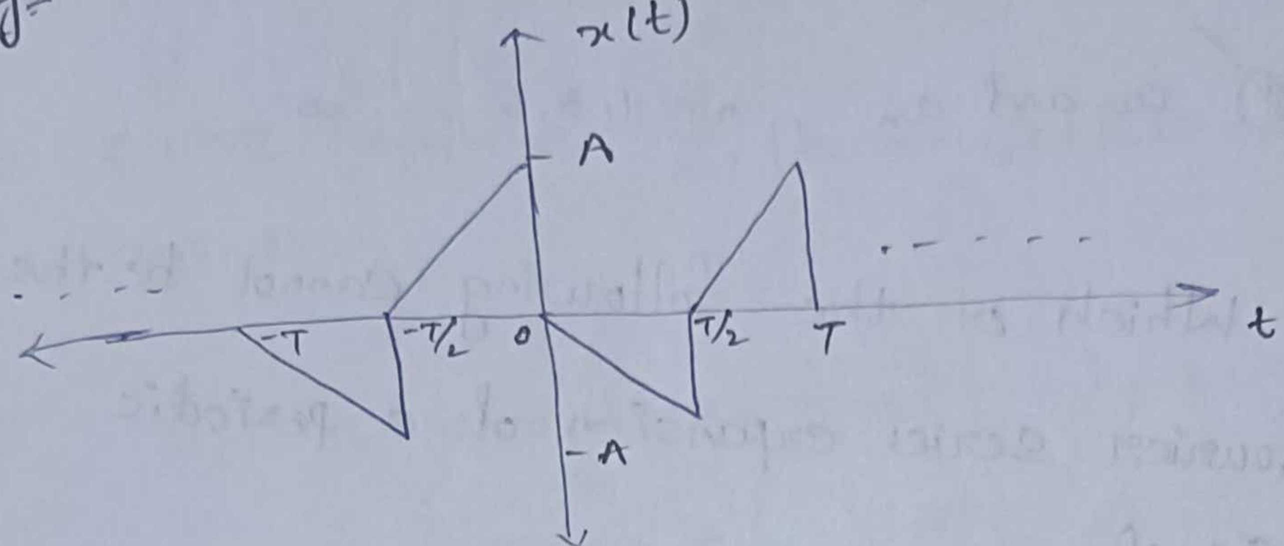


Eg:



$$+\frac{2A}{\pi}\left[\sin\omega_0 t+\frac{\sin 3\omega_0 t}{3}+\frac{\sin 5\omega_0 t}{5}+\dots\right]$$

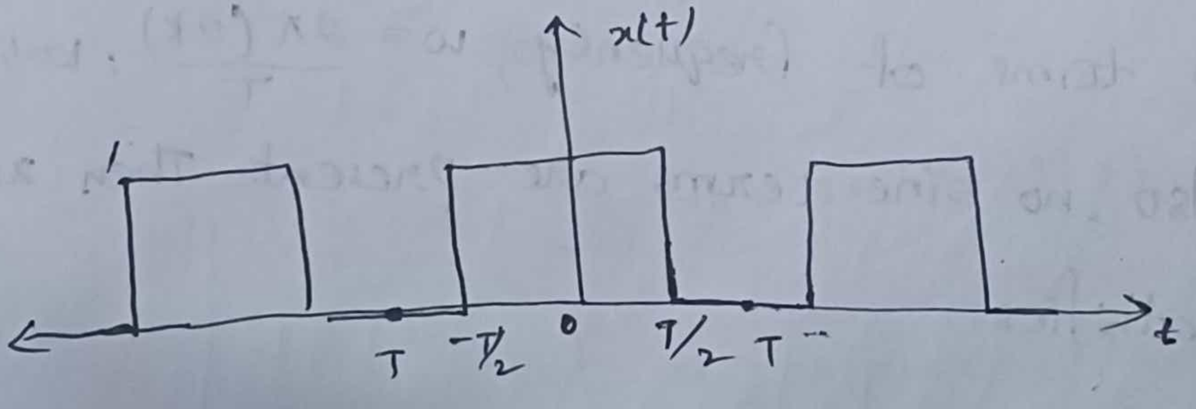
Eg:-



$$x(t)=\frac{4A}{\pi^2}\left[\cos\omega_0 t+\frac{\cos 3\omega_0 t}{3^2}+\frac{\cos 5\omega_0 t}{5^2}+\dots\right]$$

$$-\frac{2A}{\pi}\left[\sin\omega_0 t+\frac{\sin 3\omega_0 t}{3}+\frac{\sin 5\omega_0 t}{5}+\dots\right]$$

* The fourier expansion $f(t)=a_0+\sum_{n=1}^{\infty}(a_n\cos n\omega_0 t + b_n\sin n\omega_0 t)$ of the periodic signal shown below will contain the following non-zero terms.



(b) a_0 and a_n, $n = 1, 2, 3, \dots \infty$

(c) a_0, a_n and b_n, $n = 1, 2, 3, \dots \infty$

(d) a_0 and a_n, $n = 1, 3, 5 \dots \infty$

* Which of the following cannot be the fourier series expansion of a periodic signal.

(a) $x(t) = 2\cos t + 3\cos 3t$

(b) $x(t) = 2\cos \pi t + 7\cos t$

(c) $x(t) = \cos t + 0.5$

(d) $x(t) = 2\cos(1.5\pi t) + \sin(3.5\pi t)$

* $x(t)$ is a real valued function of a real variable with period 'T'. Its trigonometric fourier series expansion contains no terms of frequency $\omega = \frac{2\pi(2k)}{T}$; $k = 1, 2, 3 \dots$ Also, no sine terms are present. Then $x(t)$ satisfies:

(a) $x(t) = -x(t-T)$

(c) $x(t) = x(T-t) = -x(t-T/2)$

(d) $x(t) = x(t-T) = x(t-T/2)$

* prove $x(t) = a_0 + \sum_{n=1}^{\infty} [a_n \cos n\omega_0 t + b_n \sin n\omega_0 t]$ is periodic.

Sol:- $x(t) = a_0 + (a_1 \cos \omega_0 t + a_2 \cos 2\omega_0 t + \ldots) + b_1 \sin \omega_0 t + b_2 \sin 2\omega_0 t + \ldots$

$\Rightarrow$ Let $\frac{2\pi}{\omega_0} = T_0$

$\Rightarrow$ The fundamental time periods of each individual function is as follows:

$T_0, \; T_0/2, \; T_0/3, \; T_0/4 \ldots$

$\frac{T_0}{T_0/2} = 1/2$ and $\frac{T_0}{T_0/3} = 1/3$ and $\frac{T_0}{T_0/4} = 1/4 \ldots$

All are rational. so, it is periodic

(or)

$\therefore x(t+T) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0(t+T) + b_n \sin n\omega_0(t+T)$

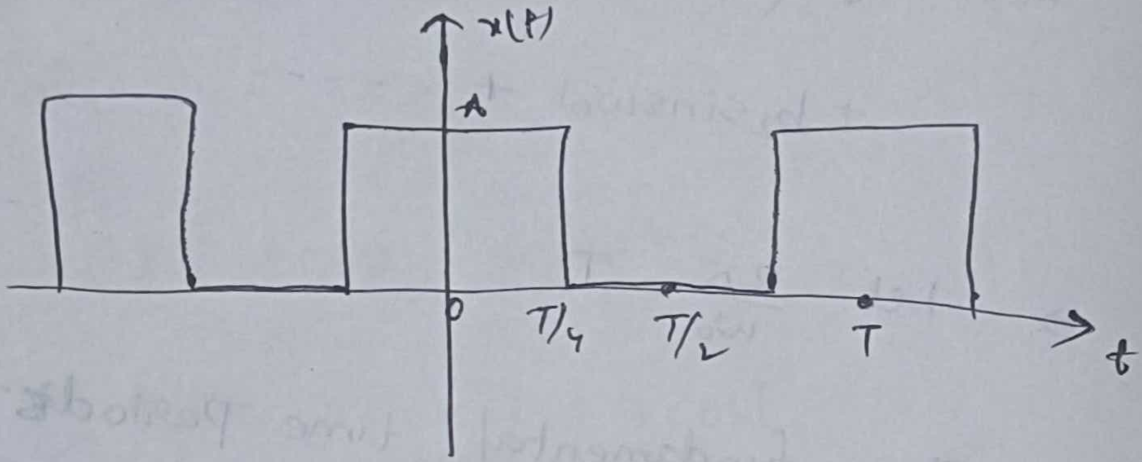
$= a_0 + \sum_{n=1}^{\infty} a_n \cos(2n\pi + n\omega_0 t) + b_n \sin(2n\pi + n\omega_0 t)$

$$\boxed{x(t) = \sum_{n=-\infty}^{+\infty} C_n e^{jn\omega_0 t}}$$

$C_n \rightarrow$ Fourier Series Coefficient

where, $C_n = \frac{1}{T_0} \int_{T_0} x(t)\, e^{-jn\omega_0 t}\, dt$

<u>Eg</u>:-



<u>Sol</u>:-
$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$\Rightarrow C_n = \frac{1}{T_0} \int_{T_0} x(t)\, e^{-jn\omega_0 t}\, dt$$

$$x(t) = \begin{cases} 0 & ; -T/2 < t < -T/4 \\ A & ; -T/4 < t < T/4 \\ 0 & ; T/4 < t < T/2 \end{cases}$$

$$\therefore C_n = \frac{1}{T} \left[\int_{-T/4}^{T/4} A e^{-jn\omega_0 t}\, dt \right]$$

$-T/4$

$$\Rightarrow C_n = \frac{-A}{T\left(jn\frac{2\pi}{T}\right)}\left[e^{-jn\frac{2\pi}{T}\cdot\frac{T}{4}} - e^{jn\frac{2\pi}{T}\cdot\frac{T}{4}}\right]$$

$$\Rightarrow C_n = \frac{-A}{jn2\pi}\left[e^{-jn\pi/2} - e^{+jn\pi/2}\right]$$

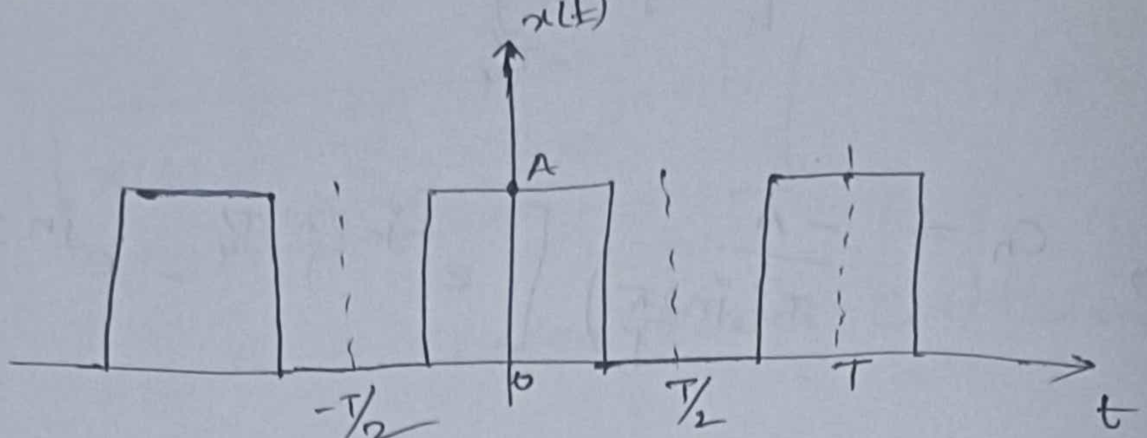
$$\Rightarrow C_n = \frac{A}{n\pi}\left[\frac{e^{jn\pi/2} - e^{-jn\pi/2}}{2j}\right]$$

$$\boxed{C_n = \frac{A}{n\pi}\sin\left(\frac{n\pi}{2}\right)}$$

$$\therefore x(t) = \sum_{n=-\infty}^{\infty} \frac{A}{n\pi}\sin\left(\frac{n\pi}{2}\right) e^{jn\omega_0 t}$$

$$= \frac{A}{2} + \frac{2A}{\pi}\left[\cos\omega_0 t - \frac{\cos 3\omega_0 t}{3} + \cdots\right]$$

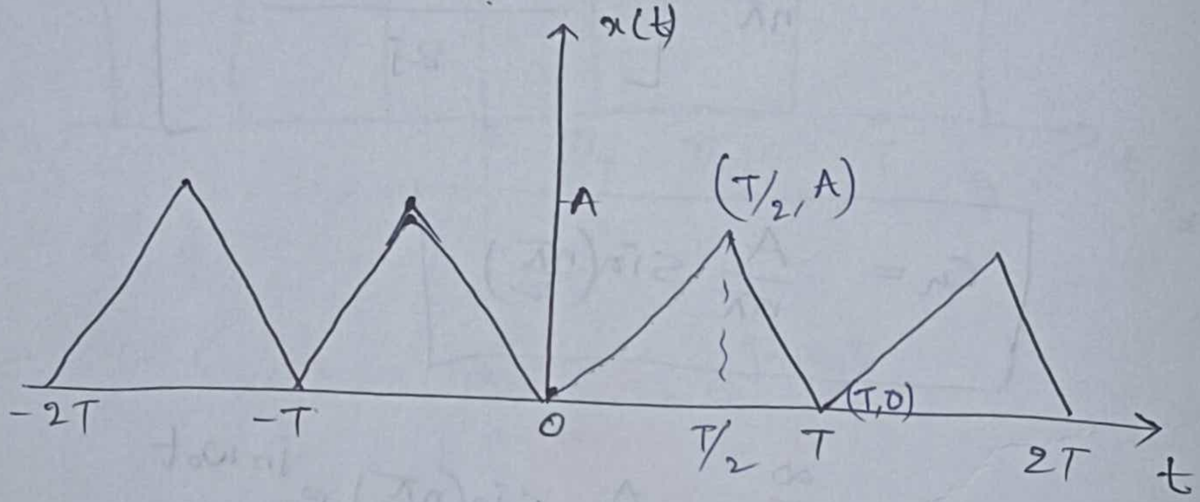
① ✓



Sol: $$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

$$a_0 = \frac{A}{2} \; ; \; a_n = \frac{2A}{n\pi} \sin\left(\frac{n\pi}{2}\right) \; ; \; b_n = 0$$

②



Sol:

$$a_0 = \frac{1}{T}\int_0^T x(t)\,dt$$

$$\Rightarrow a_0 = \frac{1}{T}\left[\int_0^{T/2} \frac{2A}{T} t\,dt + \int_{T/2}^{T} -\frac{2A}{T}(t-T)\,dt\right]$$

$$= \frac{1}{T}\left[\frac{2At^2}{2T}\Bigg|_0^{T/2} - \frac{2A}{T}\frac{(t-T)^2}{2}\Bigg|_{T/2}^{T}\right]$$

$$= \frac{1}{T}\left[\frac{AT^2}{4T} + \frac{AT^2}{4T}\right]$$

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$$\boxed{a_0 = \frac{A}{2}}$$

$$a_n = \frac{2}{T}\left[\int_0^{T/2} \frac{2At}{T}\cos n\omega_0 t \, dt + \int_{T/2}^{T} \frac{-2A}{T}(t-T)\cos n\omega_0 t \, dt\right]$$

$$= \frac{2}{T}\left[\frac{2At}{T}\frac{\sin n\omega_0 t}{n\omega_0} + \frac{2A(1)}{T}\frac{\cos n\omega_0 t}{(n\omega_0)^2}\Bigg|_0^{T/2} - \Bigg|_{T/2}^{T} \frac{2A}{T}(t-T)\frac{\sin n\omega_0 t}{n\omega_0} + \frac{2A}{T}\frac{\cos n\omega_0 t}{(n\omega_0)^2}\right]$$

$$= \frac{4A}{T^2}\left[\left(\frac{T}{2}\frac{\sin\left(n\frac{2\pi}{T}\cdot\frac{T}{2}\right)}{n\omega_0} + \frac{\cos\left(n\frac{2\pi}{T}\cdot\frac{T}{2}\right)}{(n\omega_0)^2} - \frac{1}{(n\omega_0)^2}\right) - \right.$$

$$\left.\left(\frac{\cos\left(n\frac{2\pi}{T}\cdot T\right)}{(n\omega_0)^2} + \frac{T}{2}\frac{\sin\left(n\frac{2\pi}{T}\cdot\frac{T}{2}\right)}{(n\omega_0)} - \frac{\cos\left(n\frac{2\pi}{T}\cdot\frac{T}{2}\right)}{(n\omega_0)^2}\right)\right]$$

$$= \frac{4A}{T^2}\left[0 + \frac{\cos n\pi}{(n\omega_0)^2} - \frac{1}{(n\omega_0)^2} - \frac{1}{(n\omega_0)^2} + 0 + \frac{\cos n\pi}{(n\omega_0)^2}\right]$$

$$= \frac{4A}{\cancel{T^2}}\left(\frac{+2}{n^2 4\pi^2}\cdot\cancel{T^2}\right)(\cos n\pi - 1)$$

$$\boxed{a_n = \frac{+2A}{n^2\pi^2}\left((-1)^n - 1\right)}$$

$$a_n = \begin{cases} \dfrac{-4A}{n^2\pi^2} & ; n \to \text{odd} \\ & n \to \text{even} \end{cases}$$

$$T/2$$

$$= \frac{2}{T}\left[\frac{2At}{T}\frac{\cos n\omega_0 t}{-n\omega_0} + \frac{2A}{T}\frac{\sin n\omega_0 t}{(n\omega_0)^2}\right]_0^{T/2} - \left|\frac{2A}{T}(t-T)\frac{\cos n\omega_0 t}{-n\omega_0}\right._{T/2}^{T}$$

$$+ \frac{2A}{T}\frac{\sin n\omega_0 t}{(n\omega_0)^2}\Bigg]$$

$$= \frac{4A}{T^2}\left[\frac{T/2 \cos\left(n\frac{2\pi}{T}\cdot\frac{T}{2}\right)}{-n\omega_0} + \cancel{\frac{\sin\left(n\frac{2\pi}{T}\cdot\frac{T}{2}\right)}{(n\omega_0)^2}} + 0 - 0\right.$$

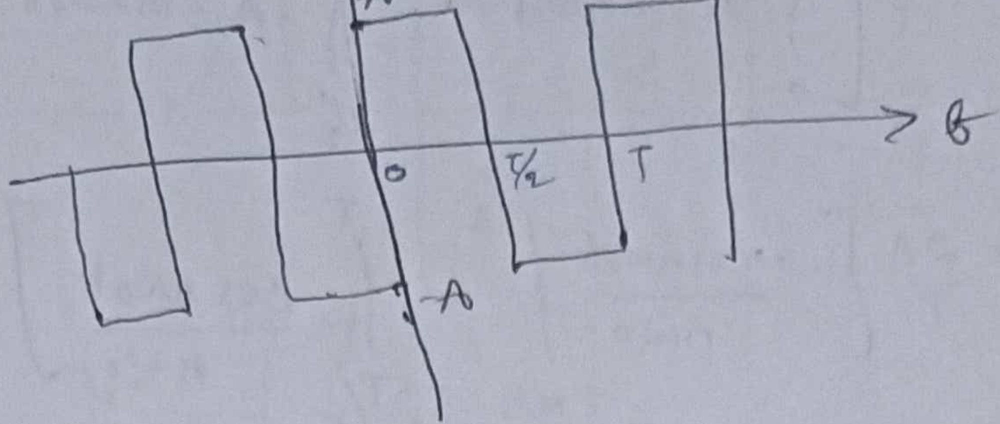
$$- \left(\cancel{\frac{\sin\left(n\frac{2\pi}{T}\cdot T\right)}{(n\omega_0)^2}} - T/2\,\frac{\cos\left(n\frac{2\pi}{T}\cdot\frac{T}{2}\right)}{n\omega_0} - \cancel{\frac{\sin\left(n\frac{2\pi}{T}\cdot\frac{T}{2}\right)}{(n\omega_0)^2}}\right)\Bigg]$$

$$= \frac{4A}{T^2}\left[\frac{T/2\cos(n\pi)}{-n\omega_0} + T/2\,\frac{\cos(n\pi)}{n\omega_0}\right]$$

$$\boxed{b_n = 0}$$

$$x(t) = \frac{A}{2} + \sum_{n=1,3,\ldots}^{\infty} \frac{-4A}{n^2\pi^2}\cos n\omega_0 t$$

$$x(t) = \frac{A}{2} - \frac{4A}{\pi^2}\left(\frac{\cos\omega_0 t}{1^2} + \frac{\cos 3\omega_0 t}{3^2} + \cdots\right]$$



$$a_0 = \frac{1}{T}\left[\int_0^{T/2} A\,dt + \int_{T/2}^{T} -A\,dt\right]$$

$$= \frac{1}{T}\left[A(T/2) - (AT - AT/2)\right]$$

$$= \frac{1}{T}\left[AT/2 - AT/2\right]$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{2}{T}\left[\int_0^{T/2} A\cos n\omega_0 t\,dt + \int_{T/2}^{T} -A\cos n\omega_0 t\,dt\right]$$

$$= \frac{2A}{T}\left[\frac{\sin n\omega_0 t}{n\omega_0}\Big|_0^{T/2} - \frac{\sin n\omega_0 t}{n\omega_0}\Big|_{T/2}^{T}\right]$$

$$= \frac{2A}{T}\left[\frac{\sin(n\frac{2\pi}{\cancel{T}}\cdot \cancel{T}/2)}{n\omega_0} - 0 - \left[\frac{\sin(n\frac{2\pi}{\cancel{T}}\cdot\cancel{T})}{n\omega_0} - \frac{\sin(n\frac{2\pi}{\cancel{T}}\cdot\cancel{T}/2)}{n\omega_0}\right]\right]$$

$$= \frac{2\pi}{T}(0)$$

$$\boxed{a_n = 0}$$

$$b_n = \frac{2}{T}\left[\int_0^{T/2} A \sin n\omega_0 t\, dt + \int_{T/2}^{T} -A \sin n\omega_0 t\, dt\right]$$

$$= \frac{2A}{T}\left[\left.\frac{-\cos n\omega_0 t}{n\omega_0}\right|_0^{T/2} + \left.\frac{\cos n\omega_0 t}{n\omega_0}\right|_{T/2}^{T}\right]$$

$$= \frac{2A}{Tn\omega_0}\left[\frac{-\cos\left(n\frac{2\pi}{T}\cdot\frac{T}{2}\right)}{1} + 1 + \cos\left(n\frac{2\pi}{T}\cdot T\right) - \cos\left(n\frac{2\pi}{T}\cdot\frac{T}{2}\right)\right]$$

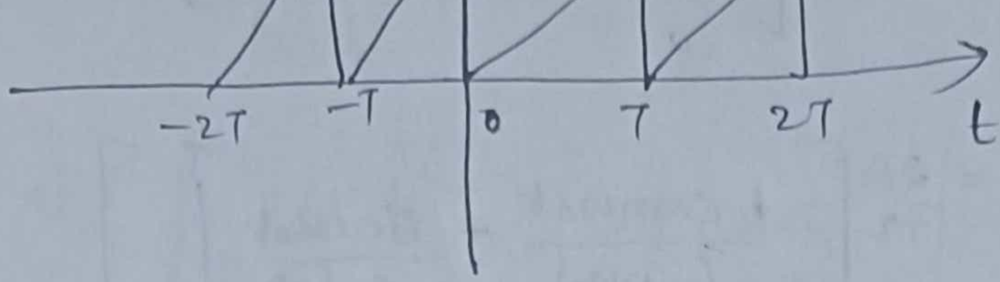
$$= \frac{2A}{T\cdot n\cdot\frac{2\pi}{T}}\left[1 - \cos n\pi + 1 - \cos n\pi\right]$$

$$= \frac{2A}{n\pi}\left[1 - \cos n\pi\right]$$

$$b_n = \begin{cases} \frac{4A}{n\pi} & ; \; n = 1, 3, 5 \ldots \\ 0 & ; \; n = 2, 4, 6 \ldots \end{cases}$$

$$x(t) = \sum_{n=1,3,5}^{\infty} \frac{4A}{n\pi} \sin n\omega_0 t$$

$$x(t) = \frac{4A}{\pi}\left[\sin\omega_0 t + \frac{\sin 3\omega_0 t}{3} + \frac{\sin 5\omega_0 t}{5} + \cdots\right]$$



Sol:

$$a_0 = \frac{1}{T}\left[\int_0^T \frac{A}{T}t\, dt\right]$$

$$= \frac{1}{T}\left(\frac{At^2}{2T}\Big|_0^T\right)$$

$$= \frac{1}{T}\left(\frac{AT^2}{2T}\right)$$

$$\boxed{a_0 = \frac{A}{2}}$$

$$a_n = \frac{2}{T}\left[\int_0^T \frac{A}{T}t \cos n\omega_0 t\, dt\right]$$

$$= \frac{2A}{T^2}\left[\frac{t \sin n\omega_0 t}{n\omega_0} + \frac{\cos n\omega_0 t}{(n\omega_0)^2}\Big|_0^T\right]$$

$$= \frac{2A}{T^2}\left[\frac{T \sin\left(n\frac{2\pi}{\cancel{T}}\cdot\cancel{T}\right)}{n\omega_0} + \frac{\cos\left(n\frac{2\pi}{\cancel{T}}\cdot\cancel{T}\right)}{(n\omega_0)^2} - 0\ \frac{1}{(n\omega_0)^2}\right]$$

$$= \frac{2A}{T^2}\left[\frac{1}{(n\omega_0)^2} - \frac{1}{(n\omega_0)^2}\right]$$

$$\boxed{a_n = 0}$$

$$= \frac{2A}{T^2}\left[\frac{t\cos n\omega_0 t}{(-n\omega_0)} + \frac{\sin n\omega_0 t}{(n\omega_0)^2} \Big|_0^T \right]$$

$$= \frac{2A}{T^2}\left[\frac{T\cos(n\frac{2\pi}{\cancel{T}}\cdot\cancel{T})}{(-n\omega_0)} + \frac{\sin(n\frac{2\pi}{\cancel{T}}\cdot\cancel{T})}{(n\omega_0)^2} \right]$$

$$= \frac{2A}{T^{\cancel{2}}}\left(\frac{\cancel{T}}{-n\omega_0} \right)$$

$$= \frac{-2A}{\cancel{T}\cdot n\frac{2\pi}{\cancel{T}}}$$

$$\boxed{b_n = \frac{-A}{n\pi}}$$

$$x(t) = \frac{A}{2} + \sum_{n=1}^{\infty} \frac{-A}{n\pi}\sin(n\omega_0 t)$$

$$x(t) = \frac{A}{2} - \frac{A}{\pi}\left(\frac{\sin\omega_0 t}{1} + \frac{\sin 2\omega_0 t}{2} + \frac{\sin 3\omega_0 t}{3} + \cdots \right)$$

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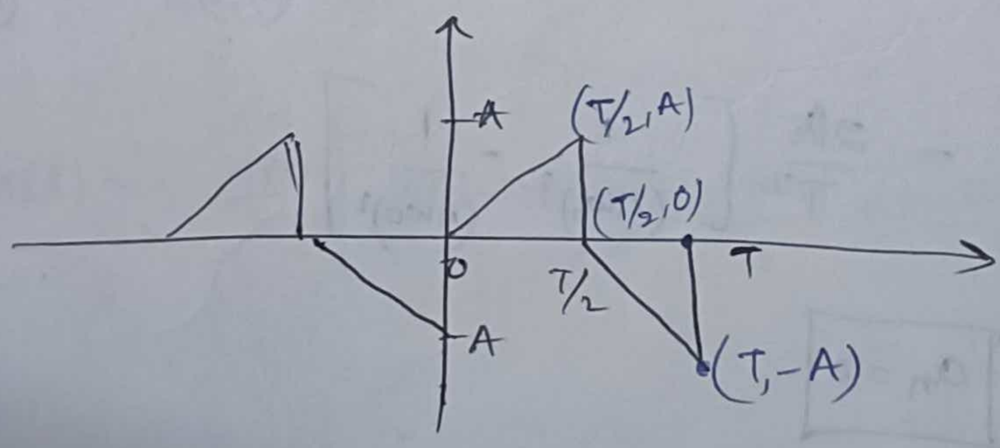

$$= \frac{1}{T}\left[\frac{2At^2}{2T}\Big|_0^{T/2} - \Big|_{T/2}^{T} \frac{2A}{T}\frac{(t-T/2)^2}{2} \right]$$

$$= \frac{1}{T}\left[\frac{AT^2}{4T} + \frac{AT^2}{4T} \right]$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{2}{T}\left[\int_0^{T/2} \frac{2A}{T} t \cos n\omega_0 t \, dt + \int_{T/2}^{T} -\frac{2A}{T}(t-T/2)\cos n\omega_0 t \, dt \right]$$

$$= \frac{2}{T}\left[\frac{2A}{T} t \frac{\sin n\omega_0 t}{n\omega_0} + \frac{2A}{T}\frac{\cos n\omega_0 t}{(n\omega_0)^2} \Big|_0^{T/2} - \Big|_{T/2}^{T} \frac{2A}{T}(t-T/2)\frac{\sin n\omega_0 t}{n\omega_0} + \frac{2A}{T}\frac{\cos n\omega_0 t}{(n\omega_0)^2} \right]$$

$$= \frac{4A}{T^2}\left[\left(\cancel{\frac{T/2 \sin(n\frac{2\pi}{T}.\frac{T}{2})}{n\omega_0}} + \frac{\cos(n\frac{2\pi}{T}.\frac{T}{2})}{(n\omega_0)^2} - \frac{1}{(n\omega_0)^2} \right) - \left(\cancel{\frac{T/2 \sin(n\frac{2\pi}{T}.T)}{n\omega_0}} + \frac{\cos(n\frac{2\pi}{T}.T)}{(n\omega_0)^2} - \frac{\cos(n\frac{2\pi}{T}.\frac{T}{2})}{(n\omega_0)^2} \right) \right]$$

$$= \frac{4A}{T^2}\left[\frac{\cos n\pi}{(n\omega_0)^2} - \frac{1}{(n\omega_0)^2} - \frac{1}{(n\omega_0)^2} + \frac{\cos n\pi}{(n\omega_0)^2} \right]$$

$$\frac{}{T^2 \cdot \frac{n^2 4\pi^2}{T^2}}\left[\, 2(\cos n\pi - 1)\right]$$

$$= \frac{2A}{n^2\pi^2}\left((-1)^n - 1\right)$$

$$a_n = \begin{cases} \dfrac{-4A}{n^2\pi^2} ; & n = 1,3,5\ldots \\ 0 ; & n = \text{even} \end{cases}$$

$$b_n = \frac{2}{T}\left[\int_0^{T/2} \frac{2A}{T} t \sin(n\omega_0 t)\,dt + \int_{T/2}^{T} -\frac{2A}{T}(t - T/2) \sin n\omega_0 t\,dt\right]$$

$$= \frac{2}{T}\left[\frac{2A}{T} t \frac{\cos n\omega_0 t}{-n\omega_0} + \frac{2A}{T}\frac{\sin(n\omega_0 t)}{(n\omega_0)^2}\Bigg|_0^{T/2} - \Bigg|_{T/2}^{T} \frac{2A}{T}(t - T/2)\frac{\cos(n\omega_0 t)}{-n\omega_0} + \frac{2A}{T}\frac{\sin(n\omega_0 t)}{(n\omega_0)^2}\right]$$

$$= \frac{4A}{T^2}\left[\left(\frac{T}{2}\frac{\cos\left(n\left(\frac{2\pi}{T}\cdot\frac{T}{2}\right)\right)}{-n\omega_0} + \cancel{\frac{\sin\left(n\frac{2\pi}{T}\frac{T}{2}\right)}{(n\omega_0)^2}}^{\,0} - 0\right)\right.$$

$$- \left(\frac{T}{2}\frac{\cos\left(n\frac{2\pi}{T}\cdot T\right)}{(-n\omega_0)} + \cancel{\frac{\sin\left(n\frac{2\pi}{T}\cdot T\right)}{(n\omega_0)^2}}_{\,0} - \cancel{\frac{\sin\left(n\frac{2\pi}{T}\frac{T}{2}\right)}{(n\omega_0)^2}}\right.$$

$$T^2\left(\quad \frac{\quad}{(-n\omega_0)} \quad \frac{2}{-n\omega_0} \right)$$

$$= \frac{4AT}{2\pi^2 \cdot n \frac{2\pi}{T}} \left(-\cos n\pi + 1 \right)$$

$$= \frac{A}{n\pi} \left(-(-1)^n + 1 \right)$$

$$b_n = \begin{cases} \frac{2A}{n\pi} ; & n \to \text{odd} \\ 0 ; & n \to \text{even} \end{cases}$$

$$x(t) = \sum_{n=1,3,5\ldots}^{\infty} \frac{-4A}{n^2\pi^2} \cos n\omega_0 t + \sum_{n=1,3,5\ldots}^{\infty} \frac{2A}{n\pi} \sin n\omega_0 t$$

$$x(t) = \frac{-4A}{\pi^2} \left[\frac{\cos \omega_0 t}{1^2} + \frac{\cos 3\omega_0 t}{3^2} + \frac{\cos 5\omega_0 t}{5^2} + \cdots \right]$$

$$+ \frac{2A}{\pi} \left(\frac{\sin \omega_0 t}{1} + \frac{\sin 3\omega_0 t}{3} + \frac{\sin 5\omega_0 t}{5} + \cdots \right]$$

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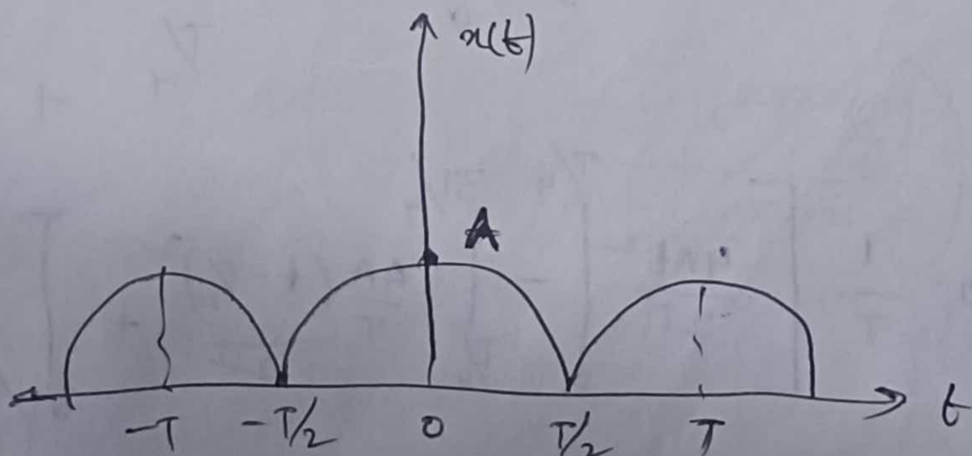


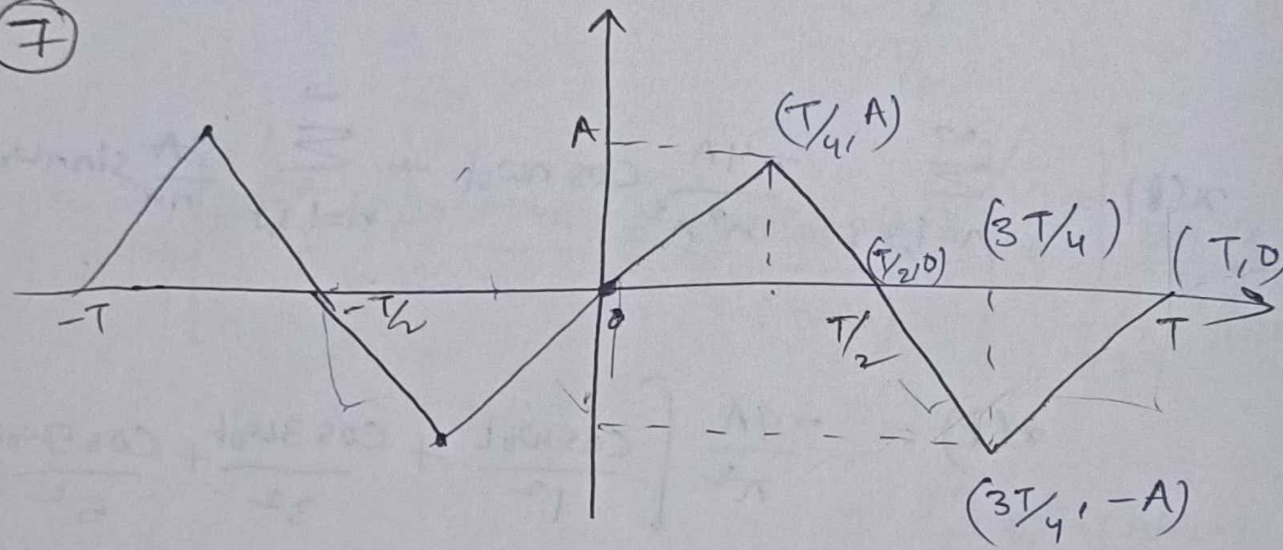
$$a_0 = \frac{1}{T}\left[\int_{-T/2} A\cos\left(\frac{\omega_0 t}{2}\right)dt\right]$$

$$= \frac{1}{T}\left[2\int_0^T A\cos\left(\frac{\omega_0 t}{2}\right)dt\right]$$

$$= \frac{2A}{T}\left[\frac{\sin\frac{\omega_0 t}{2}}{\omega_0/2}\Bigg|_0^T\right]$$

$$= \frac{2A}{\cancel{T}\cdot\frac{2\pi}{2\cancel{T}}}\left[\sin\left(\frac{2\pi}{2T}\cdot T\right)\right]$$

(7)



Sol:

$$a_0 = \frac{1}{T}\left[\int_0^{T/4}\frac{4A}{T}t\,dt + \int_{T/4}^{3T/4} -\frac{4A}{T}(t - T/2)\,dt + \int_{3T/4}^{T}\frac{4A}{T}(t-T)\,dt\right]$$

$$\Rightarrow a_0 = \frac{1}{T}\left[\frac{4At^2}{2T}\Bigg|_0^{T/4} - \Bigg|_{T/4}^{3T/4}\frac{4A}{T}\frac{(t-T/2)^2}{2} + \Bigg|_{3T/4}^{T}\frac{4A}{T}\frac{(t-T)^2}{2}\right]$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{2}{T}\left(\frac{4A}{T}\right)\left[\int_0^{T/4} t\cos n\omega_0 t\,dt + \int_{T/4}^{3T/4} -(t-T/2)\cos n\omega_0 t\,dt + \int_{3T/4}^{T} (t-T)\cos n\omega_0 t\,dt\right]$$

$$= \frac{8A}{T^2}\left[\left.\frac{t\sin n\omega_0 t}{n\omega_0} + \frac{\cos n\omega_0 t}{(n\omega_0)^2}\right|_0^{T/4} - \left|_{T/4}^{3T/4} (t-T/2)\frac{\sin n\omega_0 t}{n\omega_0} + \frac{\cos n\omega_0 t}{(n\omega_0)^2} + \right|_{3T/4}^{T} (t-T)\frac{\sin n\omega_0 t}{n\omega_0} + \frac{\cos n\omega_0 t}{(n\omega_0)^2}\right]$$

$$= \frac{8A}{T^2}\left[\left(\frac{T}{4}\cancel{\frac{\sin\left(n\frac{2\pi}{T}\cdot\frac{T/4}{2}\right)}{n\omega_0}} + \cancel{\frac{\cos\left(n\frac{2\pi}{T}\cdot\frac{T/4}{2}\right)}{(n\omega_0)^2}} - \cancel{\frac{1}{(n\omega_0)^2}}\right)\right.$$

$$- \left(\cancel{\frac{\frac{T}{4}\sin\left(n\frac{2\pi}{T}\cdot\frac{3T}{4}\right)}{n\omega_0}} + \cancel{\frac{\cos\left(n\frac{2\pi}{T}\frac{3T}{4}\right)}{(n\omega_0)^2}} + \frac{T}{4}\cancel{\frac{\sin\left(n\frac{2\pi}{T}\frac{T/4}{2}\right)}{n\omega_0}} - \cancel{\frac{\cos\left(n\frac{2\pi}{T}\frac{T/4}{2}\right)}{(n\omega_0)^2}}\right)$$

$$\left. + \left(\cancel{\frac{\cos\left(n\frac{2\pi}{T}\cdot T\right)}{(n\omega_0)^2}} + \frac{T}{4}\cancel{\frac{\sin\left(n\frac{2\pi}{T}\cdot\frac{3T}{4}\right)}{n\omega_0}} - \cancel{\frac{\cos\left(n\frac{2\pi}{T}\frac{3T}{4}\right)}{(n\omega_0)^2}}\right)\right.$$

$$b_n = \frac{2}{T}\left(\frac{4A}{T}\right)\left[\int_0 t\sin n\omega_0 t\,dt + \int_{T/4} -(t-T/2)\sin n\omega_0 t\,dt + \int_{3T/4}^{T} (t-T)\sin(n\omega_0 t)\,dt\right]$$

$$= \frac{8A}{T^2}\left[\left.\frac{-t\cos n\omega_0 t}{n\omega_0} + \frac{\sin n\omega_0 t}{(n\omega_0)^2}\right|_0^{T/4} - \left|(t-T/2)\frac{\cos n\omega_0 t}{-n\omega_0} + \frac{\sin n\omega_0 t}{(n\omega_0)^2}\right|_{T/4}^{3T/4} + \left|(t-T)\frac{\cos(n\omega_0 t)}{-n\omega_0} + \frac{\sin n\omega_0 t}{(n\omega_0)^2}\right|_{3T/4}^{T}\right]$$

$$= \frac{8A}{T^2}\left[\left(\cancel{\frac{-\frac{T}{4}\cos\left(n\frac{2\pi}{T}\cdot\frac{T}{4}\right)}{n\omega_0}} + \frac{\sin\left(n\frac{2\pi}{T}\cdot\frac{T}{4}\right)}{(n\omega_0)^2} - 0\right)\right.$$

$$- \left(\cancel{\frac{\frac{T}{4}\cos\left(n\frac{2\pi}{T}\cdot\frac{3T}{4}\right)}{-n\omega_0}} + \frac{\sin\left(n\frac{2\pi}{T}\cdot\frac{3T}{4}\right)}{(n\omega_0)^2} - \cancel{\frac{\frac{T}{4}\cos\left(n\frac{2\pi}{T}\cdot\frac{T}{4}\right)}{n\omega_0}} - \frac{\sin\left(n\frac{2\pi}{T}\cdot\frac{T}{4}\right)}{(n\omega_0)^2}\right)$$

$$+ \left(\frac{\sin\left(n\frac{2\pi}{T}\cdot T\right)}{(n\omega_0)^2} - \cancel{\frac{\frac{T}{4}\cos\left(n\frac{2\pi}{T}\cdot\frac{3T}{4}\right)}{n\omega_0}} - \frac{\sin\left(n\frac{2\pi}{T}\cdot\frac{3T}{4}\right)}{}\right.$$

$$\Big[\cancel{\frac{\quad}{n\omega_0}} \quad \frac{\quad}{(n\omega_0)^2} - \frac{\quad}{(n\omega_0)^2} \Big]$$

$$= \frac{\overset{4}{\cancel{8}}A}{\cancel{T^2} n^2 \frac{\cancel{4}\pi^2}{\cancel{T^2}}} \cancel{2} \left(\sin\left(\frac{n\pi}{2}\right) - \sin\left(\frac{3n\pi}{2}\right) \right)$$

$$= \frac{4A}{n^2\pi^2} \left(\sin\left(\frac{n\pi}{2}\right) - \sin\left(2n\pi - \frac{n\pi}{2}\right) \right)$$

$$= \frac{4A}{n^2\pi^2} \left(\sin\left(\frac{n\pi}{2}\right) + \sin\left(\frac{n\pi}{2}\right) \right)$$

$$= \frac{8A}{n^2\pi^2} \sin \frac{n\pi}{2}$$

$$b_n = \begin{cases} \frac{8A}{n^2\pi^2} (\pm 1) \sin\left(\frac{n\pi}{2}\right); & n \rightarrow \text{odd} \\ 0 \quad ; & n \rightarrow \text{even} \end{cases}$$

$$x(t) = \sum_{n=1}^{\infty} \frac{8A}{n^2\pi^2} \sin\left(\frac{n\pi}{2}\right) \sin(n\omega_0 t)$$

$$x(t) = \frac{8A}{\pi^2} \left[\sin \omega_0 t - \frac{\sin(3\omega_0 t)}{3^2} + \frac{\sin(5\omega_0 t)}{5^2} + \cdots \right]$$

$\cos(2\omega_0 t + \pi/4)$

Sol:

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$\therefore x(t) = 3 + \cancel{2}\left[\frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{+\cancel{2}j}\right] + \frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}$$

$$+ \frac{e^{j(2\omega_0 t + \frac{\pi}{4})} + e^{-j(2\omega_0 t + \frac{\pi}{4})}}{2}$$

$$\Rightarrow x(t) = 3 - je^{j\omega_0 t} + je^{-j\omega_0 t} + \frac{e^{j\omega_0 t}}{2} + \frac{e^{-j\omega_0 t}}{2}$$

$$+ \frac{e^{j(2\omega_0 t + \pi/4)}}{2} + \frac{e^{-j(2\omega_0 t + \frac{\pi}{4})}}{2}$$

$$\Rightarrow x(t) = 3 + \left(\frac{1}{2} - j\right] e^{j\omega_0 t} + \left[\frac{1}{2} + j\right] e^{-j\omega_0 t}$$

$$+ \left(\frac{1+j}{\sqrt{2}}\right)\frac{e^{j2\omega_0 t}}{2} + \left(\frac{1-j}{\sqrt{2}}\right)\frac{e^{-j2\omega_0 t}}{2}$$

$$\therefore C_0 = 3, \quad C_1 = \frac{1-2j}{2}, \quad C_2 = \left(\frac{1+j}{2\sqrt{2}}\right)$$

$$C_{-1} = \frac{1+2j}{2}, \quad C_{-2} = \frac{1-j}{2\sqrt{2}}$$

$$C_n = \begin{cases} \left(-\frac{1}{3}\right)^n & n \geq 0 \\ 0 & n < 0 \end{cases}$$

what is $x(t)$?

<u>sol</u> ÷

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\frac{2\pi}{T} \cdot t}$$

$$= \sum_{n=0}^{\infty} \left(\frac{-1}{3}\right)^n e^{j2n\pi t}$$

$$= \sum_{n=0}^{\infty} \left(\frac{-1}{3}\right)^n e^{j2n\pi t}$$

$$= 1 + \frac{-1}{3} e^{j2\pi t} + \left(\frac{-1}{3}\right)^2 e^{j4\pi t} + \left(-\frac{1}{3}\right)^3 e^{j6\pi t} \cdots$$

$$\boxed{x(t) = \frac{1}{1 + \frac{1}{3} e^{j2\pi t}}}$$

*

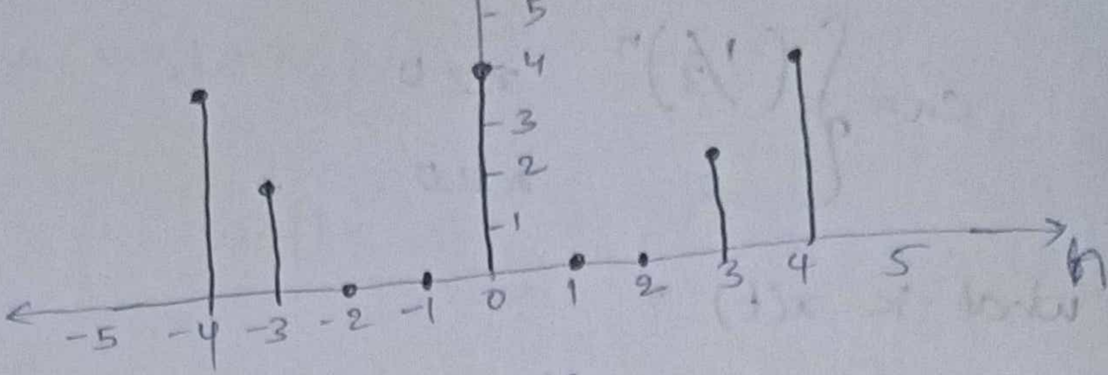
$$C_n = \begin{cases} \left[\frac{1}{2}\right]^n, & n \geq 0 \\ 0, & n < 0 \end{cases} \quad ; \text{ Period} = 2$$

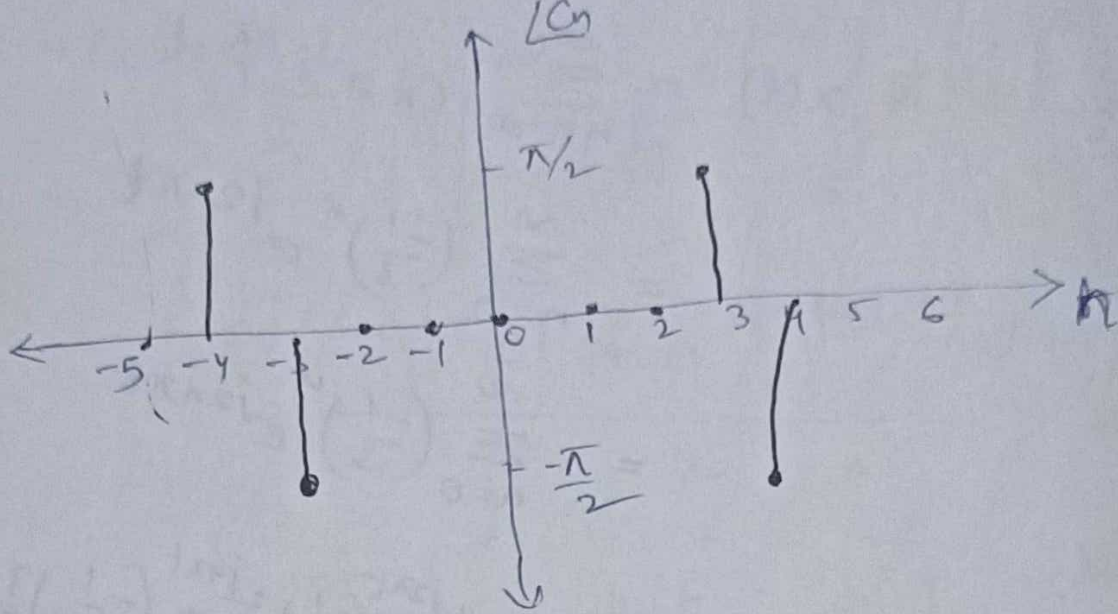
$x(0) = ?$

<u>sol</u> ÷

$$x(t) = \sum_{n=0}^{\infty} \left(\frac{1}{2}\right)^n e^{jn\pi t}$$

$$= 1 + \frac{1}{2} e^{j\pi t} + \left(\frac{1}{2}\right)^2 e^{j2\pi t} + \cdots\cdots$$





$C_0 = 4\angle 0 = 4e^{j0}$

$C_3 = 2\angle \pi/2 = 2e^{j\pi/2}$

$C_{-3} = 2\angle -\pi/2 = 2e^{-j\pi/2}$

$C_{+4} = 4\angle -\pi/2 = 4e^{-j\pi/2}$

$C_{-4} = 4\angle \pi/2 = 4e^{j\pi/2}$

$$x(t) = C_4 e^{-j4\omega_0 t} + C_{-3} e^{-j3\omega_0 t} + C_0 e^{0t} + C_3 e^{j3\omega_0 t} + C_4 e^{j4\omega_0 t}$$

$$= 4e^{j\pi/2} e^{-j4\omega_0 t} + 2e^{-j\pi/2} e^{-j3\omega_0 t} + 4 + 2e^{j\pi/2} e^{j3\omega_0 t}$$
$$+ 4e^{-j\pi/2} . e^{j4\omega_0 t}$$

$-j(4\omega_0 t - \pi/2)$ $\quad -j(3\omega_0 t + \pi/2)$ $\quad j(3\omega_0 t + \pi$

$$\Rightarrow x(t) = \frac{8\left(e^{j(4\omega_0 t - \frac{\pi}{2})} + e^{-j(4\omega_0 t - \frac{\pi}{2})}\right)}{2} + 4 + \frac{4\left(e^{j(3\omega_0 t + \frac{\pi}{2})} + e^{-j(3\omega_0 t + \frac{\pi}{2})}\right)}{2}$$

$$x(t) = \underline{\underline{8\cos\left(4\omega_0 t - \frac{\pi}{2}\right) + 4 + 4\cos\left(3\omega_0 t + \frac{\pi}{2}\right)}}$$

* Relation between fourier coefficients (a_0, a_n, b_n with c_n):

$$x(t) = \sum_{n=-\infty}^{+\infty} c_n e^{jn\omega_0 t} \quad \text{and}$$

$$\Rightarrow x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$\Rightarrow x(t) = a_0 + \sum_{n=1}^{\infty} a_n \left[\frac{e^{jn\omega_0 t} + e^{-jn\omega_0 t}}{2}\right] + b_n \left[\frac{e^{jn\omega_0 t} - e^{-jn\omega_0 t}}{2j}\right]$$

$$\Rightarrow x(t) = a_0 + \sum_{n=1}^{\infty} \frac{a_n}{2} e^{jn\omega_0 t} + \frac{a_n}{2} e^{-jn\omega_0 t}$$

$$- \frac{jb_n e^{jn\omega_0 t}}{} \quad \frac{jb_n e^{-jn\omega_0 t}}{}$$

$$= a_0 + \sum_{n=1}^{\infty} \underbrace{\left[\frac{a_n - jb_n}{2}\right]}_{C_n} e^{jn\omega_0 t} + \sum_{n=1}^{\infty} \underbrace{\left[\frac{a_n + jb_n}{2}\right]}_{C_{-n}} e^{-jn\omega_0 t}$$

$$= a_0 + \sum_{n=1}^{\infty} C_n e^{jn\omega_0 t} + \sum_{n=1}^{\infty} C_{-n} e^{-jn\omega_0 t}$$

$$= a_0 + \sum_{n=1}^{\infty} C_n e^{jn\omega_0 t} + \sum_{n=-1}^{-\infty} C_n e^{jn\omega_0 t}$$

$$x(t) = C_0 + \sum_{n=-1}^{-\infty} C_n e^{jn\omega_0 t} + \sum_{n=1}^{\infty} C_n e^{jn\omega_0 t}$$

$$\boxed{\therefore x(t) = \sum_{n=-\infty}^{+\infty} C_n e^{jn\omega_0 t}}$$

$$C_n = \left[\frac{a_n - jb_n}{2}\right] \text{ and } C_{-n} = \left[\frac{a_n + jb_n}{2}\right]$$

Now,

$$C_n = \frac{a_n - jb_n}{2} \quad \text{—— (1)}$$

$$C_{-n} = \frac{a_n + jb_n}{2} \quad \text{—— (2)}$$

eqn (1) + eqn (2) $\Rightarrow$ $\boxed{C_n + C_{-n} = a_n}$

eqn (1) - eqn (2) $\Rightarrow$ $C_n - C_{-n} = -jb_n$

$$\therefore \operatorname{Re}(C_n) + j\operatorname{Im}[C_n] = \frac{a_n}{2} - j\frac{b_n}{2}$$

$$\Rightarrow \operatorname{Re}[C_n] = \frac{a_n}{2}$$

$$\boxed{a_n = 2\operatorname{Re}[C_n]}$$

$$\operatorname{Im}[C_n] = \frac{-b_n}{2}$$

$$\boxed{b_n = -2\operatorname{Im}[C_n]}$$

* Fourier series representation of a signal $x(t)$ is $x(t) = \sum_{n=-\infty}^{+\infty} \frac{3}{4+(3n\pi)^2} e^{jn\pi t}$

(a) Find F.T.P. (b) Find the average value value of the signal $x(t)$ (c) One term in the expansion is $A_0 \cos 6\pi t$, then calculate the value of A_0. (d) One term in the expansion is $B_0 \sin 6\pi t$, then $B_0 = ?$

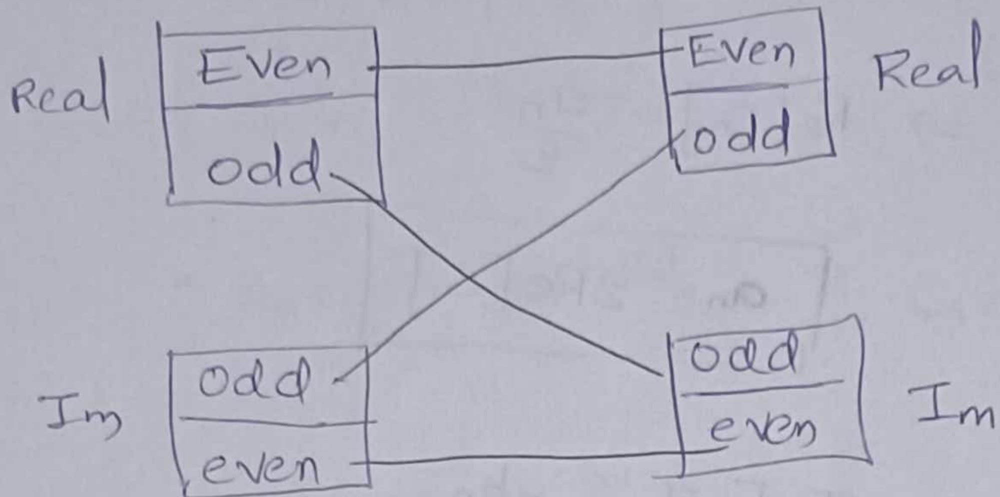
sol:

(a) $T_0 = 2$

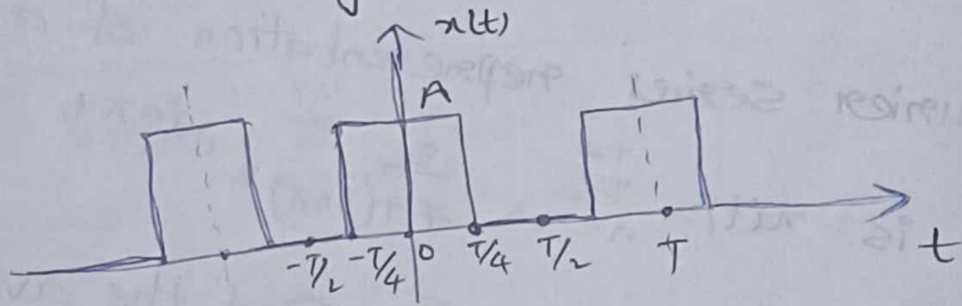
(b) Avg value $= \frac{3}{4}$

4+324π

shortcut:



* Determine the fourier series coefficient for the following periodic signal



(a) $\frac{A}{j\pi k}\sin\left[\frac{\pi}{2}k\right]$ (b) $\frac{A}{\pi j k}\cos\left[\frac{\pi}{2}k\right]$

(c) $\frac{A}{\pi k}\sin\left[\frac{\pi}{2}k\right]$ (d) $\frac{A}{\pi k}\cos\left[\frac{\pi}{2}k\right]$

Sol:

$$\boxed{x(t) = \text{Real} + \text{Even}}$$

Since,

$$x(t) = \begin{cases} 0 & -T/2 < t < -T/4 \\ A & -T/4 < t < T/4 \\ 0 & T/4 < t < T/2 \end{cases}$$

If $x(t)$ is Real and Even, then

C_n should be Even and Real,

so, option c is correct.

* $x(t) \longrightarrow C_n = \begin{cases} 2 & ; n=0 \\ j\left[\frac{1}{2}\right]^{|n|} & ; \text{otherwise.} \end{cases}$

which of the following is true:

(a) $x(t)$ is real valued signal

(b) $x(t)$ is an even signal

(c) $\frac{d\,x(t)}{dt}$ is an even signal

(d) Both (b) and (c)

1. Linearity
2. property of Conjugation &
3. Time Reversal
4. Time Scaling
5. Time shifting
6. Frequency shifting
7. Convolution in time
8. Multiplication in time
9. Differentiation in time
10. Integration in time.
11. Parseval's Power theorem.

(1) Linearity:

$$x_1(t) \longrightarrow C_{1n} \qquad C_n = \alpha C_{1n} + \beta C_{2n}$$

$$x_2(t) \longrightarrow C_{2n}$$

If $y(t) = \alpha x_1(t) + \beta x_2(t)$, then:

$$C_n = \frac{1}{T_0} \int_0^{T_0} y(t)\, e^{-jn\omega_0 t}\, dt$$

$$\boxed{C_n = \alpha C_{1n} + \beta C_{2n}}$$

Note: This holds only when the $x_1(t)$ and $x_2(t)$ have same time period.

(2) property of conjugation:

$$x(t) \longrightarrow C_n$$

$$\therefore C_n = \frac{1}{T_0} \int_0^{T_0} x(t)\, e^{-jn\omega_0 t}\, dt$$

$$\Rightarrow C_n^* = \frac{1}{T_0} \int_0^{T_0} x^*(t)\, e^{jn\omega_0 t}\, dt$$

$$\Rightarrow \boxed{C_{-n}^* = \frac{1}{T_0} \int_0^{T_0} x^*(t)\, e^{-jn\omega_0 t}\, dt}$$

so, $\boxed{x^*(t) \longrightarrow C_{-n}^*}$

(3) Time Reversal:

$$x(t) \longrightarrow C_n.$$

$$x(-t) \longrightarrow C_n'$$

$$T_0 \int_0$$

Let $-t = \tau \Rightarrow -dt = d\tau$

when $t = 0$, $\tau = 0$

$t = T_0$, $\tau = -T_0$

$$\Rightarrow C_n' = \frac{1}{T_0}\int_0^{-T_0} x(\tau)\, e^{jn\omega_0\tau}(-d\tau)$$

$$\Rightarrow C_n' = \frac{1}{T_0}\int_{-T_0}^{0} x(\tau)\, e^{jn\omega_0\tau}\, d\tau$$

$$\boxed{C_n' = C_{-n}}$$

(4) Time scaling:

$$x(t) \longrightarrow C_n \quad (\omega_0)$$

$$x(at) \longrightarrow C_n' \quad (\omega_0 a) \quad (a > 1)$$

$$\therefore C_n' = \frac{1}{T_0/a}\int_0^{T_0/a} x(at)\, e^{-jn\underline{\omega_0}at}\, dt$$

$$\Rightarrow \text{Let } at = \tau \Rightarrow a\,dt = d\tau$$

$$c_n' = \frac{1}{T_0} \int_0^{T_0} x(\tau)\, e^{-jn\omega_0 \tau}\, d\tau$$

$$\boxed{\therefore\ c_n' = c_n}$$

[Since; during time scaling, F.T.P becomes $\frac{T_0}{a}$]

(5) Time shifting:

$$x(t) \longrightarrow c_n \;(\omega_0)$$

$$x(t+a) \longrightarrow c_n' \;(\omega_0)$$

$$\therefore\ c_n' = \frac{1}{T_0} \int_0^{T_0} x(t+a)\, e^{-jn\omega_0 t}\, dt$$

Let $t + a = \tau \Rightarrow dt = d\tau$

when $t = 0,\ \tau = a$

$t = T_0,\ \tau = a + T_0$

$$\Rightarrow c_n' = \frac{1}{\,} \int^{a+T_0} x(\tau)\, e^{-jn\omega_0(\tau - a)}\, d\tau$$

$$\Rightarrow c_n' = e^{jn\omega_0 a}\left[\frac{1}{T_0}\int_{T_0} x(\tau)\, e^{-jn\omega_0\tau}\, d\tau\right]$$

$$\boxed{c_n' = e^{jn\omega_0 a}\, c_n}$$

Similarly,

if $x(t) \longrightarrow c_n$.

then $x(t-t_0) \longrightarrow e^{-jn\omega_0 t_0}\, c_n$

(6) Frequency shifting:

$x(t) \longrightarrow c_n$.

then, $e^{jm\omega_0 t}\, x(t) \longrightarrow c_{n-m}$

$e^{-jm\omega_0 t}\, x(t) \longrightarrow c_{n+m}$

$$\therefore c_n = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-jn\omega_0 t}\, dt$$

$$\Rightarrow c_{n-m} = \frac{1}{T_0}\int_0^{T_0} x(t)\, e^{-j(n-m)\omega_0 t}\, dt$$

$$\Rightarrow c_{n-m} = \frac{1}{T_0}\, e^{jm\omega_0 t}\int^{T_0} x(t)\, e^{-jn\omega_0 t}\, dt$$

(7) Convolution in time :

$$x_1(t) \longrightarrow C_{1n}$$

$$x_2(t) \longrightarrow C_{2n}$$

$$x_1(t) * x_2(t) = y(t) \longrightarrow C_n$$

$$\therefore \quad C_n = \frac{1}{T_0}\int_0^{T_0} y(t)\, e^{-jn\omega_0 t}\, dt$$

$$\Rightarrow C_n = \frac{1}{T_0}\int_0^{T_0} \left[x_1(t) * x_2(t)\right] e^{-jn\omega_0 t}\, dt$$

$$\Rightarrow C_n = \frac{1}{T_0}\left[\int_0^{T_0}\left[\int_0^{T_0} x_1(\tau)\, x_2(t-\tau)\, d\tau\right] e^{-jn\omega_0 t}\, dt\right]$$

$$\Rightarrow C_n = \frac{1}{T_0}\left[\int_0^{T_0}\int_0^{T_0} x_1(\tau)\, x_2(t-\tau)\, \frac{e^{jn\omega_0\tau}}{e^{jn\omega_0\tau}}\, d\tau\, e^{-jn\omega_0 t}\, dt\right]$$

$$\Rightarrow C_n = \frac{1}{T_0}\int_0^{T_0} x_1(\tau)\cdot e^{-jn\omega_0\tau}\, d\tau \cdot \int_0^{T_0} x_2(t-\tau)\, e^{-jn\omega_0(t-\tau)}\, dt$$

$$\Rightarrow C_n = C_{1n}\cdot T_0 \cdot \frac{1}{T_0}\int_0^{T_0} x_2(t-\tau)\, e^{-jn\omega_0(t-\tau)}\, dt$$

let $t-\tau = \alpha \Rightarrow dt = d\alpha$

T_0
$-T$

$\rightarrow C_n = C_{1n} \cdot T_0 \cdot C_{2n}$

$$\boxed{C_n = T_0 (C_{1n} \cdot C_{2n})}$$

(8) Multiplication in Time:

$x(t) \longrightarrow C_n$

$x_2(t) \longrightarrow B_n$

$x_1(t) \cdot x_2(t) \longrightarrow C_n * B_n$

$$x_1(t) = \sum_{k=-\infty}^{\infty} C_k e^{jkw_0 t} \quad ; \quad x_2(t) = \sum_{p=-\infty}^{\infty} B_p e^{jpw_0 t}$$

$$\therefore x_1(t) \cdot x_2(t) = \sum_{k=-\infty}^{\infty} C_k e^{jkw_0 t} \cdot \sum_{p=-\infty}^{\infty} B_p e^{jpw_0 t}$$

$$\Rightarrow x_1(t) \cdot x_2(t) = \sum_{p=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} C_k B_p e^{jkw_0 t} e^{jpw_0 t}$$

$$= \sum_{p=-\infty}^{\infty} \sum_{k=-\infty}^{\infty} C_k B_p e^{j(k+p)w_0 t}$$

$k+p = n \Rightarrow p = n-k$

when $p = -\infty$, $n = -\infty$

$$\boxed{x_1(t)\cdot x_2(t) = \sum_{n=-\infty}^{\infty} (C_n * B_n)\, e^{jn\omega_0 t}}$$

where

$$C_n * B_n = \sum_{k=-\infty}^{\infty} C_k B_{n-k}.$$

(9) Differentiation in Time:

$$x(t) \longrightarrow C_n$$

$$\frac{d}{dt} x(t) \longrightarrow jn\omega_0 C_n$$

$$\therefore x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$\frac{d}{dt} x(t) = \sum_{n=-\infty}^{\infty} [C_n \cdot jn\omega_0] e^{jn\omega_0 t}$$

similarly,

$$\frac{d^k}{dt^k} x(t) \longrightarrow (jn\omega_0)^k C_n$$

(10) Integration in time:

$$x(t) \longrightarrow C_n$$

$$\int^{t} x(\tau)\, d\tau \longrightarrow \frac{C_n}{jn\omega_0}$$

$$\rightarrow \int_{-\infty}^{t} x(\tau)\, d\tau = \sum_{n=-\infty}^{\infty} C_n \left. \frac{e^{jn\omega_0 \tau}}{jn\omega_0} \right|_{-\infty}$$

$$\therefore \int_{-\infty}^{t} x(\tau)\, d\tau = \sum_{n=-\infty}^{\infty} \left[\frac{1}{jn\omega_0} C_n \right] e^{jn\omega_0 t}$$

(11) Parseval's power theorem:

$$x(t) \longrightarrow C_n$$

$$\therefore \quad P_{avg} = \frac{1}{T_0} \int_{0}^{T_0} |x(t)|^2\, dt$$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$x^*(t) = \sum_{n=-\infty}^{\infty} C_n^* e^{-jn\omega_0 t}$$

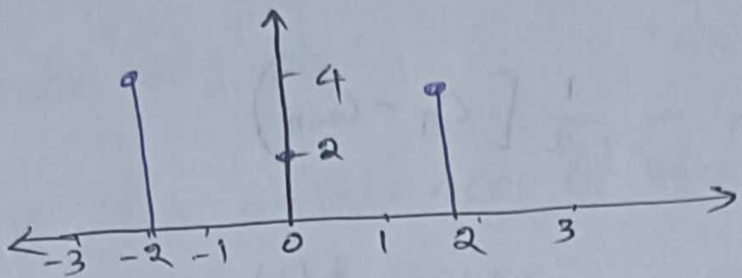
$$\Rightarrow \quad P_{avg} = \frac{1}{T_0} \int_{0}^{T_0} x(t) \cdot x^*(t)\, dt$$

$$= \frac{1}{T_0} \int_{0}^{T_0} x(t) \cdot \sum_{n=-\infty}^{\infty} C_n^* e^{-jn\omega_0 t}\, dt$$

$$= \sum_{n=-\infty}^{\infty} C_n^* \frac{1}{T_0} \int_{0}^{T_0} x(t) e^{-jn\omega_0 t}\, dt$$

$$P_{avg} = \sum_{n=-\infty} |C_n|^2$$

* Find the average power of signal $x(t)$:



$$P_{avg} = \sum_{n=-\infty}^{\infty} |C_n|^2 = 4^2 + 2^2 + 4^2 = 36\,W$$

* Find C_n' interms of C_n where

$$x(t) \longrightarrow C_n$$

$$y(t) \longrightarrow C_n'$$

1. $y(t) = x(t+1) - x(t-1)$

2. $y(t) = e^{-j2\omega_0 t} x(t)$

3. $y(t) = \frac{d^2}{dt^2} x(t)$

4. $y(t) = \text{odd } x(t)$

5. $y(t) = \text{Real } x(t)$

Sol:-

1. $C_n' = e^{jn\omega_0} C_n - e^{-jn\omega_0} C_n$

$$= C_n \left(\frac{e^{jn\omega_0} - e^{-jn\omega_0}}{2j} \right) 2j$$

③ $c_n' = (jn\omega_0)^2 c_n = -n^2\omega_0^2 c_n$

④ $y(t) = \dfrac{x(t) - x(-t)}{2}$

$$c_n' = \frac{1}{2}\left[c_n - c_{-n}\right)$$

⑤ $y(t) = \dfrac{x(t) + x^*(t)}{2}$

$$c_n' = \frac{1}{2}\left[c_n + c_{-n}^*\right]$$