Tautology

(i) Tautology : A statement is said to be a tautology if it is true for all logical possibilities i.e. its truth value always T. it is denoted by t.

Prove statement $p \vee \sim (p \wedge q)$ is a tautology

p	q	$p \wedge q$	$\sim (p \wedge q)$	$p \vee \sim (p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Contradiction / Fallacy

(ii) Contradiction : A statement is a contradiction if it is false for all logical possibilities. i.e. its truth value always F. It is denoted by c.

Prove the statement $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction

p	q	$\sim p$	$\sim q$	$p \vee q$	$(\sim p \wedge \sim q)$	$(p \vee q) \wedge (\sim p \wedge \sim q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	F

(iv) Negation of biconditional : $\sim (p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$ we know that

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$\therefore \sim (\mathbf{p} \leftrightarrow \mathbf{q}) \equiv \sim [(\mathbf{p} \rightarrow \mathbf{q}) \wedge (\mathbf{q} \rightarrow \mathbf{p})]$

$\equiv \sim (\mathbf{p} \rightarrow \mathbf{q}) \vee \sim (\mathbf{q} \rightarrow \mathbf{p})$

$\equiv (\mathbf{p} \wedge \sim \mathbf{q}) \vee (\mathbf{q} \wedge \sim \mathrm{p})$

$p \longleftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$\equiv (\sim p \vee q) \wedge (\sim q \vee p)$

$\therefore \sim (p \longleftrightarrow q) \equiv \sim [(\sim p \vee q) \wedge (\sim q \vee p)]$

$\equiv \sim (\sim p \vee q) \vee \sim (\sim q \vee p)$

$\equiv (\sim(\sim p) \wedge \sim q) \vee (\sim(\sim q) \wedge \sim p)$

$\equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

Q. The negation of the Boolean expression $p \vee (\sim p \wedge q)$ is equivalent to: [JEE Mains Online-2020]

(A) $\sim p \vee \sim q$

(B) $\sim p \vee q$

(C) $\ldots \wedge \sim q$

(D) $\ldots \sim q$

$$\sim(p \vee (\sim p \wedge q))$$
$$\sim p \wedge \sim(\sim p \wedge q)$$
$$\sim p \wedge (\sim(\sim p) \vee \sim q)$$
$$\sim p \wedge (p \vee \sim q).$$
$$(\sim p \wedge p) \vee (\sim p \wedge \sim q)$$
$$c \vee (\sim p \wedge \sim q)$$
$$\underline{\sim p \wedge \sim q.}$$

$\sim(p \vee q) \equiv \sim p \wedge \sim q$

$\sim(\sim p) \equiv p$

$\sim p \wedge p \equiv c.$

$c \vee p \equiv p$

Q. The negative of the statement $\sim p \wedge (p \vee q)$ is [JEE Mains (Feb)2021]

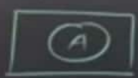
(A) $\sim p \vee q$

(B) $p \vee \sim q$

(C) $\sim p \wedge q$

(D) $p \wedge \sim q$

U



$U \cap A = A$

$\sim(\sim p \wedge (p \vee q))$

$\sim(\sim p) \vee \sim(p \vee q)$

$p \vee (\sim p \wedge \sim q)$

$\underbrace{(p \vee \sim p)} \wedge (p \vee \sim q)$

$t \wedge (p \vee \sim q)$

$p \vee \sim q$.

Associativity

$p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$

$p \vee (q \vee r)$ … $\vee r$

$p \vee (q \wedge$ … $(p \vee r)$

$p \wedge$ …

Q. The negation of the Boolean expression $x \leftrightarrow \sim y$ is equivalent to :

[JEE Mains Online-2020]

(A) $(\sim x \wedge y) \vee (\sim x \wedge \sim y)$

$x \wedge \sim y) \vee (\sim x \wedge y)$

$x \wedge y) \wedge (\sim x \vee \sim y)$

$(x \wedge y) \vee (\sim x \wedge \sim y)$

$x \leftrightarrow y \equiv (x \to y) \wedge (y \to x)$

$x \to y \equiv \sim x \vee y$

$$x \leftrightarrow \sim y \equiv (x \to \sim y) \wedge (\sim y \to x)$$
$$\equiv (\sim x \vee \sim y) \wedge (\sim(\sim y) \vee x).$$
$$\equiv (\sim x \vee \sim y) \wedge (y \vee x).$$

Negation : $\sim(x \leftrightarrow \sim y) \equiv \sim[(\sim x \vee \sim y) \wedge (y \vee x)]$
$$\equiv [\sim(\sim x \vee \sim y) \vee \sim(y \vee x)]$$
$$\equiv (x \wedge y) \vee (\sim y \wedge \sim x).$$

Q. The statement $(p \to (q \to p)) \to (p \to (p \vee q))$ is : [JEE Mains Online-2020]

(A) a tautology

uivalent to $(p \wedge q) \vee (\sim q)$

valent to $(p \vee q) \wedge (\sim p)$

radiction

$$(p \to (q \to p)) \longrightarrow (p \to (p \vee q))$$

$$(\sim p \vee (q \to p)) \longrightarrow (\sim p \vee (p \vee q))$$

$$(\sim p \vee (\sim q \vee p)) \longrightarrow (\underbrace{(\sim p \vee p)}_{=t} \vee q)$$

$$(t \vee q)$$

$$\sim(\sim p \vee (\sim q \vee p)) \vee t \equiv t$$

$\sim p \vee p \equiv t$

$t \vee q \equiv t$

Q. The statement $(p \wedge (p \to q) \wedge (q \to r)) \to r$ is : [JEE Mains (August)2021]

(A) a tautology

(B) equivalent to $p \to \sim r$

(C) a fallacy

(D) equivalent to $q \to \sim r$

$$(p \wedge (p \to q) \wedge (\sim q \vee r)) \to r$$

$$\equiv \sim(p \wedge (\sim p \vee q) \wedge (\sim q \vee r)) \vee r$$

$$\equiv \sim p \vee (\sim(\sim p) \wedge \sim q) \vee (\sim(\sim q) \wedge \sim r) \vee r$$

$$\equiv (\sim p \vee (p \wedge \sim q)) \vee (q \wedge \sim r)) \vee r$$

$$\equiv ((\sim p \vee p) \wedge (\sim p \vee \sim q)) \vee (q \wedge \sim r)) \vee r$$

$$\equiv ((\sim p \vee \sim q) \vee (q \wedge \sim r))\vee r$$

$$\equiv (((\sim p \vee \sim q) \vee q) \wedge ((\sim p \vee \sim q) \vee \sim r)) \vee r$$

$t \wedge p \equiv p$

$t \vee p \equiv t$

$\sim p \vee p \equiv t$

$p \vee (q \wedge r)$

$(p \vee q) \wedge (p \vee r)$

Distribut

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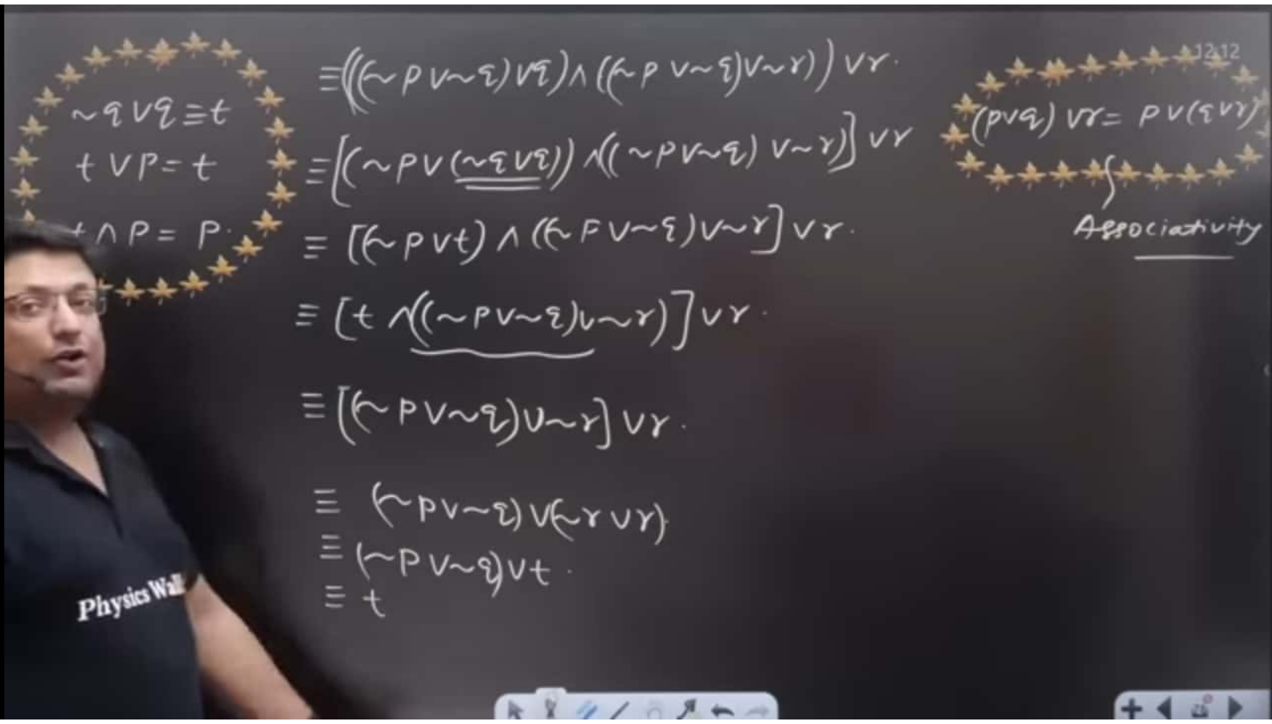
$\sim q \vee q \equiv t$
$t \vee p = t$
$t \wedge p = p$
$\equiv (((\sim p \vee \sim q) \vee q) \wedge ((\sim p \vee \sim q) \vee \sim r)) \vee r$
$\equiv [(\sim p \vee (\sim q \vee q)) \wedge ((\sim p \vee \sim q) \vee \sim r)] \vee r$
$\equiv [(\sim p \vee t) \wedge ((\sim p \vee \sim q) \vee \sim r)] \vee r$
$\equiv [t \wedge ((\sim p \vee \sim q) \vee \sim r)] \vee r$
$\equiv [(\sim p \vee \sim q) \vee \sim r] \vee r$
$\equiv (\sim p \vee \sim q) \vee (\sim r \vee r)$
$\equiv (\sim p \vee \sim q) \vee t$
$\equiv t$
$(p \vee q) \vee r = p \vee (q \vee r)$
Associativity