1. Counting

Basic Principles of Counting ÷ There are two principles

i) Addition Principle

ii) Multiplication principle

i) Addition Principle ÷ (or)

- If there are m ways to do one thing and n ways to do another thing. But both the things cannot be done at once. Then the number of ways to do any one of the thing is given by m+n
- Whenever we see 'or' in a question then we use ~~multip~~ Addition principle.

ii) Multiplication principle ÷ (and)

- If there are m ways to do one thing and n ways to do another thing. Both the things can be done at once. The number of ways to do both the things is given by "m×n".
- Whenever we see "and" in a question then we use multiplication principle

Note ÷

Addion (OR)

Multiplication (And)

Q:- A icecream shop has 3 different flavours of cone, 4 different flavours of cup icecreams. ~~and~~ In howmany ways
i) One can choose any one of the icecream
ii) One can choose combination of both the icecreams.

soln:- i) Any one of the icecream
here we can use cone or cup icecreams
cone - 3 cup - 4
From addition principle - 3+4=7 ways

ii) Combination of both icecreams
means we must choose cone and cup icecream
From multiplication principle - 3×4=12 ways

Permutations:- Arrangement of a set of objects in a definite order.

Ex:- Arrange 2 3 4 in all possible orders.

2 3 4
2 4 3
3 2 4
3 4 2
4 2 3
4 3 2

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$$\boxed{{}^{n}P_{r} = \frac{n!}{(n-r)!}}$$

n - no. of objects
r - no. of digits or objects to be taken

Q:- Write the no. of permutations of Yuvraj

Soln:- no. of letters = 6
no. of permutations or digits r = 6

$${}^{6}P_{6} = \frac{6!}{6-6!} = \frac{6!}{0!} = 6!$$

$$= 6 \times 5 \times 4 \times 3 \times 2 \times 1$$

$$= 720$$

Q:- Find the no. of 3 digit arrangements of Yuvraj

Soln:- $n = 6$
$r = 3$

$${}^{6}P_{3} = \frac{6!}{(6-3)!} = \frac{6!}{3!} = \frac{6 \times 5 \times 4 \times \cancel{3!}}{\cancel{3!}}$$

$$= 120$$

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Case-1: If any objects are repeating.

- In this case we divide the repeating object out of the total permutations.

Ex: In howmany ways we can arrange MATHEMATICS

Soln: MATHEMATICS

No. of letters = 11

here a is repeating two times

t is repeating two times

m is repeating two times

No of permutations $= \frac{11!}{2!2!2!}$

In general, if we want to calculate the no of arrangements where the

P_1 objects are repeating P_1 times

P_2 objects are repeating P_2 times

⋮

P_n objects are repeating P_n times

Then no of permutations can be given by

$$= \frac{n!}{P_1 P_2 P_3 \cdots P_n} \quad (or) \quad \frac{nP_r}{P_1 P_2 P_3 \cdots P_n}$$

Case 2: Some objects want to stay together.

In this case we assume them as a single unit and find the arrangements for them.

Ex: find the no. of arrangements in DAUGHTER such that all vowels occur together

Soln: DAUGHTER

$n = 8$.

Vowels in DAUGHTER are $\boxed{AUE}$

After assuming $\boxed{AUE}$ as a single unit we get $\boxed{AUE}$ D G H T R

now $n = 6$

The no. of arrangements $= 3! \times 6!$

since $\boxed{AUE}$ can have internal arrangement in $3!$ ways so by multiplication principle we have $3! \times 6!$ ways.

Case 3: Some objects want to stay in a fixed position:

Ex: Find no. of permutations of Yuvraj such that Y is always in the first place and r is in the fourth place

$$Y \ \_ \ \_ \ r \ \_ \ \_$$

$$\times 2 \times 3 \times 4$$

here out of 6, 2 are fixed. Remaining 4 letters

No. of arrangements $= 4 \times 3 \times 2 \times 1$

Combination :- Selection of no of objects.

- Here order is not important. Only selection from the given set of object

$$\boxed{{}^{n}C_r = \frac{n!}{(n-r)!\,r!}}$$

It can also derived from permutations

$$\boxed{{}^{n}C_r = \frac{{}^{n}P_r}{r!}}$$

$$\boxed{{}^{n}C_r = \frac{n!}{(n-r)!\,r!}}$$

Ex :- Find the no. of combinations of Yuvraj

$n = 7$

$r = 7$

$$\boxed{{}^{7}C_7 = 1}$$

Since here order is not important the no of combinations are 1

Ex :- ~~Find~~ Select a combination of 3 letters of Yuvraj

$n = 7$

$r = 3$

$${}^{7}C_3 = \frac{7!}{(7-3)!\,3!} = \frac{7!}{4! \times 3!}$$

Problems solve

1) LOGORITHM

(i) Repetition is not allowed

here $n=9$, $r=3$

$$9P_3 = \frac{9!}{(9-3)!} = \frac{9!}{6!}$$

(ii) Repetition is allowed

$$\square \quad \square \quad \square$$

$$9 \quad 9 \times 9$$

$$= 9 \times 9 \times 9 = 9^3$$

2) LEADER

$n=6$, $r=6$, E is repeating two times

$$6P_6 = \frac{6!}{2!} = \frac{6 \times 5 \times 4 \times 3 \times \cancel{2!}}{\cancel{2!}}$$

$$= 360$$

3) $n = 10$

$r = 4$

no of arrangements $= 10P_4$

$$= \frac{10!}{(10-4)!} = \frac{10!}{6!}$$

4) HOLIDAY

$n = 7, r = 7$

$^nP_r = {}^7P_7 = 7!$ ways we can arrange holiday

Our condition is I ~~and L always~~ always come to left of L.

As there are equal possibilities that I always come to Left of L and I always come to right of L.

So no. of permutations $= \frac{7!}{2}$

5) CLIMATE

$\underline{I}$ _ $\underline{A}$ _ E _ _

3! → vowels

4! → consonants

$= 3! \times 4!$

6) MATHEMATICS

$\boxed{AAEI}$ M T H M T C S

1 2 3 4 5 6 7 8

$$= \frac{4!}{2!} \times \frac{8!}{2! \times 2!}$$

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8) $n=5$
$r=3$

$nP_r = {}^5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!}$

9) no of boys $=5$, no of girls $=4$
no of boys reqd $r=3!$, no of girls reqd $=3$
$= {}^5C_3 \times {}^4C_3$

10) (i) Four cards of same suit
$= {}^{12}C_4 + {}^{12}C_4 + {}^{12}C_4 + {}^{12}C_4$

(ii) Four cards belonging to four diff suits
$= {}^{12}C_1 \times {}^{12}C_1 \times {}^{12}C_1 \times {}^{12}C_1$

(iii) Face cards
$= {}^{12}C_4$

(iv) Two red and two black
$= {}^{26}C_2 \times {}^{26}C_2$

(v) Four of same colour
$= {}^{26}C_4 + {}^{26}C_4$

11) 7 ~~total~~ commity
9 boys, 4 girls
a) Exactly three girls
= GGG BBBB
$= {}^4C_3 \times {}^9C_4$

ii) Atleast 3 girls

= GGG BBB GGGG BBB

$^4C_3 \times {}^9C_4 + {}^4C_3 \times {}^9C_3$

iii) Atmost 3 girls

$= {}^9C_7 + {}^4C_1 \times {}^9C_6 + {}^4C_2 \times {}^9C_5 + {}^4C_3 \times {}^9C_4$

---

12) $^nC_2 = 21$

$$\frac{n!}{(n-2)!\,2!} = 21$$

$$\frac{(n) \times (n-1) \times \cancel{(n-2)!}}{2! \times \cancel{(n-2)!}} = 21$$

$(n)(n-1) = 21 \times 2$

$n^2 - n = 42$

$n^2 - n - 42 = 0$

$n = 7,\ n = -6$ (since n cant be negative)

$\boxed{n = 7}$

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Binomial theorem

$(a+b)^0 = 1$

$(a+b)^1 = a+b$

$(a+b)^2 = a^2+2ab+b^2$

$(a+b)^3 = a^3+3a^2b+3ab^2+b^3$

$(a+b)^4 = a^4+4a^3b+6a^2b^2+4ab^3+b^4$

Observations from above

- We can observe in a binomial expension the power of first variable is decreasing by 1
- Also the power of second variable is increasing by 1.
- No of terms in a binomial expansion $= n+1$ where $n =$ power.

Pascal Triangle :- It can be expressed by the coefficients of binomial expansion

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

$(a+b)^n = {}^nC_0 a^n b^0 + {}^nC_1 a^{n-1} b^1 + {}^nC_2 a^{n-2} b^2 +$

$\cdots\cdots + {}^nC_{n-1} a^1 b^{n-1} + {}^nC_n a^0 b^n$

In general the binomial expression can be given by

$$\boxed{= \sum_{k=0}^{n} {}^nC_k \, a^{n-k} b^k}$$

<u>Middle term:-</u>

<u>Case 1:-</u> n is even

→ there are odd number of terms

→ middle term is only one

→ middle term $= \left(\frac{n}{2}+1\right)^{th}$ term

<u>Case 2:-</u> n is odd

→ Total terms in expansion : Even

→ There are two middle terms

→ $\left(\frac{n+1}{2}\right)^{th}$ and $\left(\frac{n+1}{2}+1\right)^{th}$ term

To find the r^{th} term

$$\boxed{T_r = {}^nC_{r-1}\, a^{n-r+1}\, b^{r-1}}$$

$$\boxed{T_{r+1} = {}^nC_r\, a^{n-r}\, b^r}$$

Q:- Solve

1) $(x+5)^4$

ii) Solve middle term of $(x+5)^4$

iii) Find the 5^{th} term of $(x+5)^4$

soln:-

(i) $(x+5)^4$

$n = 4$

$$= {}^4C_0 (x)^4 (5)^0 + {}^4C_1 (x)^3 (5) + {}^4C_2 (x)^2 (5)^2 + {}^4C_3 (x)(5)^3 + {}^4C_4 (x)^0 (5)^4$$

(ii) middle term $(x+5)^4$

$n=4 \Rightarrow$ there are odd no of terms

middle term $= \left(\frac{4}{2}+1\right) = 3^{rd}$ term

$$T_3 = T_{2+1} = {}^4C_2 (x)^2 (5)^2$$

(iii) 5^{th} term

$$T_{4+1} = {}^4C_4 (x)^0 (5)^4$$

Catalon Numbers

$$\boxed{C_n = \frac{1}{n+1}\left({}^{2n}C_n\right)}$$

$$C_0 = \frac{1}{0+1}\left({}^{0}C_0\right) = 1$$

$$C_1 = \frac{1}{1+1}\left({}^{2}C_1\right) = \frac{2}{2} = 1$$

$$C_2 = \frac{1}{3}\left({}^{4}C_2\right) = 2$$

$$C_3 = \frac{1}{4}\left({}^{6}C_3\right) = 5$$

$$C_4 = \frac{1}{5}\left({}^{8}C_4\right) = 14$$

$$C_5 = \frac{1}{6}\left({}^{12}C_6\right) = 42$$

$$\vdots$$

$$\boxed{C_n = 1, 1, 2, 5, 14, 42 - - -}$$

Applications

Parantheris ÷

n=0 ⟶ 1 way

n=1 ( ) ⟶ 1 way

n=2 ()() (()) ⟶ 2 ways

n=3 ()()() (())() ()(()) (()()) ((())) ⟶ 5 ways

n=4 ⟶ 14 ways

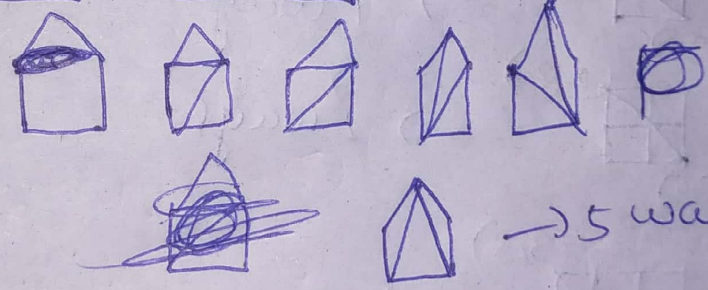
1, 1, 2, 5, 14, 42, - - -

Polygon Triangulation

n=2 ⟶ 1 way

n=3  ⟶ 1 way

n=4  ⟶ 2 ways

n=5  ⟶ 5 ways

n=6  ⟶ 14 ways

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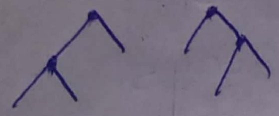
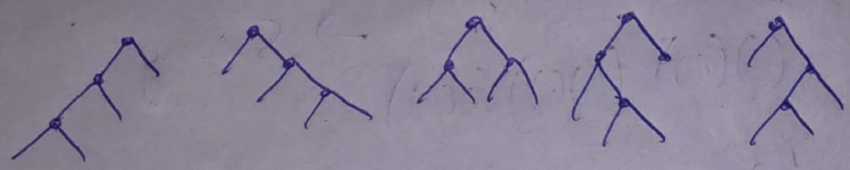
Binary trees

n=0 → 1 way

n=1  → 1 way

n=2  → 2 ways

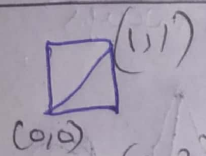
n=3  → 5 wa

n=4 — 14 ways

n=5 — 42 ways

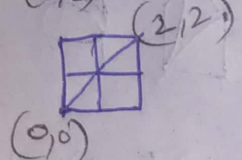
1, 1, 2, 5, 14, 42, — — —

Reach (0,0), (n,n) without crossing diagonal

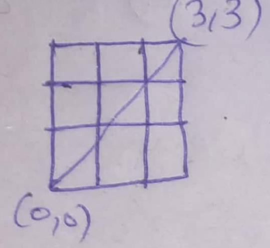


→ 1 way

→ 1 way



→ 2 ways



→ 5 ways

1, 1, 2, 5, — — —