Unit-3
Logic

1.Conjunction statement: A conjunction statement is a logical statement that connects two other statements using the word "and." It is true only if both of the individual statements are true. The symbol used for conjunction is $\wedge$.
Truth table:

p	**q**	**p $\wedge$ q**
0	0	0
0	1	0
1	0	0
1	1	1

Examples:

Statement 1: It is raining outside.
Statement 2: I am wearing a raincoat.

Conjunction statement: It is raining outside and I am wearing a raincoat.

Statement 1: The number is even.
Statement 2: The number is divisible by 3.

Conjunction statement: The number is even and it is divisible by 3.

2.Disjunction statement:
A disjunction statement is a logical statement that connects two other statements using the word "or." It is true if at least one of the individual statements is true. The symbol used for disjunction is ∨.

Truth table:

p	q	p ∨ q
0	0	0
0	1	1
1	0	1
1	1	1

Examples:

Statement 1: The class is cancelled.
Statement 2: The teacher is absent.

Disjunction statement: The class is canceled or the teacher is absent.

Statement 1: The price of gold increases.
Statement 2: The stock market crashes.

Disjunction statement: The price of gold increases or the stock market crashes.

3.Conditional statement:
A conditional statement is a logical statement that connects two other statements using the phrase "if-then." It is false only if the hypothesis (the "if" part) is true and the conclusion (the "then" part) is false. The symbol used for conditional is →.
Truth table:

p	q	p → q
0	0	1
0	1	1
1	0	0

1 1   1

Examples:

Statement 1: If it rains, then the ground gets wet.
Statement 2: It is raining.

Conditional statement**:** If it rains, then the ground gets wet.

Statement 1: If I study hard, then I will pass the exam.
Statement 2: I studied hard.

Conditional statement: If I study hard, then I will pass the exam.

4.Biconditional statement: A biconditional statement is a logical statement that connects two other statements using the phrase "if and only if." It is true if both statements have the same truth value. The symbol used for biconditional is ↔.

Truth table:

p q p ↔ q
0 0   1
0 1   0
1 0   0
1 1   1

Examples:

Statement 1: A figure is a square if and only if it has four equal sides and four right angles.
Statement 2: A figure has four equal sides and four right angles.

Biconditional statement: A figure is a square if and only if it has four equal sides and four right angles.

Statement 1: I will go to the beach if and only if it is sunny.
Statement 2: It is sunny.

Biconditional statement: I will go to the beach if and only if it is sunny.

5.Negation

In the truth table, the "not" operator takes a single input value p, and produces a single output value ¬p, which is the negation of p. If p is true (1), then ¬p is false (0). If p is false (0), then ¬p is true (1).

Truth Table

p	¬p
0	1
1	0

For example, if we have the statement "It is sunny today," we can represent it as p = 1. The negation of this statement would be "It is not sunny today," which we can represent as ¬p = 0.

6.Inverse: The inverse of a conditional statement switches the hypothesis and the conclusion of the original statement. It is false only if the original statement is true and the inverse is false.
Truth table:

p	q	¬p	¬q	¬p → ¬q
0	0	1	1	1
0	1	1	0	0
1	0	0	1	1
1	1	0.	0.	1

Examples:

Statement: If an animal is a cat, then it has fur.
Inverse: If an animal does not have fur, then it is not a cat.

Statement: If a number is even, then it is divisible by 2.
Inverse: If a number is not divisible by 2, then it is not even.

7.Converse: The converse of a conditional statement switches the hypothesis and the conclusion of the original statement. It is false only if the original statement is false and the converse is true.
Truth table:

p	**q**	**p → q**	**q → p**
0	0	1	1
0	1	1	0
1	0	0	1
1	1.	1	1

Examples:

Statement: If it rains, then the ground gets wet.
Converse: If the ground gets wet, then it rains.

Statement: If a number is positive, then it is greater than 0.
Converse: If a number is greater than 0, then it is positive.

The contrapositive of a conditional statement switches the hypothesis and the conclusion of the original statement and negates both. It is true only if the original statement is true.

Truth table:

p	**q**	**¬q**	**¬p**	**¬q → ¬p**
0	0	1	1	1
0	1	0	1	1
1	0	1	0	0
1	1	0	0	1

Examples:

Statement: If a person is a student, then they are young.
Contrapositive: If a person is not young, then they are not a student.

Statement: If a function is continuous, then it is differentiable.
Contrapositive: If a function is not differentiable, then it is not continuous.

Algebra of Statements

Algebra of Statements is a set of rules for manipulating logical statements using logical operators. It is a way of reasoning and deriving new statements from given ones. The algebra of statements provides a systematic way of working with statements and determining their truth values.

The following are some of the rules of Algebra of Statements:

1.Commutative laws:

- $p \land q \equiv q \land p$
- $p \lor q \equiv q \lor p$

The commutative laws state that the order of the operands can be changed without changing the truth value of the statement. In other words, "and" and "or" are commutative operations.

2.Associative laws:

- $(p \land q) \land r \equiv p \land (q \land r)$
- $(p \lor q) \lor r \equiv p \lor (q \lor r)$

The associative laws state that the grouping of the operands can be changed without changing the truth value of the statement. In other words, "and" and "or" are associative operations.

3.Distributive laws:

- $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$
- $p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$

The distributive laws state that "and" and "or" can be distributed over each other. In other words, they allow us to break down complex statements into simpler ones.

4.Identity laws:

- $p \land T \equiv p$
- $p \lor F \equiv p$

The identity laws state that the truth value of a statement remains unchanged when combined with T (true) or F (false) using "and" or "or" operations, respectively.

5.Negation laws:

- $p \land \neg p \equiv F$
- $p \lor \neg p \equiv T$

The negation laws state that the negation of a statement combined with the statement itself always evaluates to F (false) for "and" operations and T (true) for "or" operations.

Using these rules of algebra of statements, we can simplify complex logical expressions and determine their truth values. These rules provide a formal system for manipulating logical statements, which is used extensively in computer science, mathematics, and philosophy.

6.Demorgans law

- $\neg(p \land q) \equiv \neg p \lor \neg q$
- $\neg(p \lor q) \equiv \neg p \land \neg q$

Problems

1.~(p ∨ (~q ∧ p))

to simplify the expression ~(p ∨ (~q ∧ p)), we can use de morgan's laws:

~(p ∨ (~q ∧ p))
= ~(p) ∧ ~(~q ∧ p) [de morgan's law]
= (~p) ∧ (~~q ∨ ~p) [de morgan's law]
= (~p) ∧ (q ∨ ~p) [double negation]
= (q ∨ ~p) ∧ (~p) [commutative property]

so the simplified expression is (q ∨ ~p) ∧ (~p).

2.Simplify (P ∨ Q) ∧ (~P ∨ Q) using algebraic laws of statements.

We can simplify (P ∨ Q) ∧ (~P ∨ Q) using the distributive law of conjunction over disjunction:

(P ∨ Q) ∧ (~P ∨ Q) = [(P ∨ Q) ∧ ~P] ∨ [(P ∨ Q) ∧ Q] (distributive law)

Now, we can simplify each of the two terms on the right using the distributive law in the opposite direction:

[(P ∨ Q) ∧ ~P] ∨ [(P ∨ Q) ∧ Q] = [(P ∧ ~P) ∨ (Q ∧ ~P)] ∨ [(P ∧ Q) ∨ (Q ∧ Q)] (distributive law)

Since P ∧ ~P is always false and Q ∧ Q is equivalent to Q, we can simplify further:

[(P ∧ ~P) ∨ (Q ∧ ~P)] ∨ [(P ∧ Q) ∨ (Q ∧ Q)] = (Q ∧ ~P) ∨ (P ∧ Q) (identity law)

Finally, we can use the commutative law of conjunction to rearrange the terms:

(Q ∧ ~P) ∨ (P ∧ Q) = Q ∧ (P ∨ ~P) (commutative law)

Since P ∨ ~P is always true, we can simplify further to:

Q ∧ (P ∨ ~P) = Q

Therefore, (P ∨ Q) ∧ (~P ∨ Q) simplifies to Q.

Rules of Inferences

The Rules of Inference are the fundamental logical principles that allow us to make valid arguments in deductive reasoning. They provide a systematic method for drawing conclusions based on a set of premises, or statements that are assumed to be true.

There are several commonly used rules of inference, including modus ponens, modus tollens, hypothetical syllogism, disjunctive syllogism, addition, simplification, conjunction, resolution, contraposition, and transitive inference. Each of these rules follows a specific pattern of logical reasoning that allows us to draw valid conclusions from given premises.

Modus ponens and modus tollens are the two basic conditional inference rules. Modus ponens states that if P implies Q and P is true, then Q must be true. Modus tollens states that if P implies Q and Q is false, then P must be false.

Hypothetical syllogism and disjunctive syllogism are two other important rules of inference. Hypothetical syllogism allows us to chain together conditional statements, while disjunctive syllogism allows us to eliminate one of two possible options in a disjunction.

Addition, simplification, and conjunction are three rules of inference that are used for manipulating conjunctions. Addition allows us to add a statement to a conjunction, simplification allows us to extract one component of a conjunction, and conjunction allows us to combine two statements into a conjunction.

Resolution is a rule of inference that is used to simplify logical statements by combining disjunctions. Contraposition is a rule of inference that allows us to swap the terms in a conditional statement, while maintaining the same logical structure. Finally, transitive inference allows us to chain together multiple conditional statements to form a longer, more complex argument.

Problem: 1
Use the modus ponens inference rule to prove that if P then Q, and P is true, then Q is true.
Solution:

Given:
$P \rightarrow Q$ (If P then Q)
P (P is true)

To prove:
Q (Q is true)

Using modus ponens:
$P \rightarrow Q$ (Premise)
P (Premise)
Therefore, Q (Conclusion)

Problem: 2
Use the modus tollens inference rule to prove that if P then Q, and Q is false, then P is false.
Solution:

Given:
$P \rightarrow Q$ (If P then Q)
~Q (Q is false)

To prove:
~P (P is false)

Using modus tollens:
$P \rightarrow Q$ (Premise)
~Q (Premise)
Therefore, ~P (Conclusion)

Problem: 3
Use the hypothetical syllogism inference rule to prove that if P then Q, and Q then R, then if P then R.
Solution:

Given:
$P \rightarrow Q$ (If P then Q)
$Q \rightarrow R$ (If Q then R)

To prove:
$P \rightarrow R$ (If P then R)

Using hypothetical syllogism:
$P \rightarrow Q$ (Premise)
$Q \rightarrow R$ (Premise)
Therefore, $P \rightarrow R$ (Conclusion)

Problem:4

Use the disjunctive syllogism inference rule to prove that either P or Q is true, and P is false, then Q is true.
Solution:

Given:
$P \vee Q$ (P or Q is true)
$\sim P$ (P is false)

To prove:
Q (Q is true)

Using disjunctive syllogism:
$P \vee Q$ (Premise)
$\sim P$ (Premise)
Therefore, Q (Conclusion)

Problem:5
Use the addition inference rule to prove that P is true.
Solution:

Given:
None

To prove:
P is true

Using addition:
P (Premise)
Therefore, P (Conclusion)

Problem: 6
Use the simplification inference rule to prove that if P and Q are both true, then P is true.
Solution:

Given:
$P \wedge Q$ (P and Q are both true)

To prove:
P is true

Using simplification:
$P \wedge Q$ (Premise)
Therefore, P (Conclusion)

Problem: 7
Use the conjunction inference rule to prove that if P is true and Q is true, then P and Q are both true.
Solution:

Given:
P is true
Q is true

To prove:
$P \wedge Q$ is true

Using conjunction:
P (Premise)
Q (Premise)
Therefore, $P \wedge Q$ (Conclusion)

Problem:8
Use the resolution inference rule to prove that if P or Q is true, and not P or R is true, then Q or R is true.
Solution:

Given:
$P \vee Q$ (P or Q is true)
$\sim P \vee R$ (Not P or R is true)

To prove:
$Q \vee R$ is true

Using resolution:
$P \vee Q$ (Premise)
$\sim P \vee R$ (Premise)
Therefore, $Q \vee R$ (Conclusion)

Problem: 9
Use the contraposition inference rule to prove that if P then Q, then if not Q then not P.
Solution:

Given:
$P \rightarrow Q$ (If P then Q)

To prove:
$\sim Q \rightarrow \sim P$ (If not Q then not P)

Using contraposition:
$P \rightarrow Q$ (Premise)
$\sim Q \rightarrow \sim P$ (Conclusion)

Problem: 10
Use the transitive inference rule to prove that if P then Q, and Q then R, then if P then R.
Solution:

Given:
$P \rightarrow Q$ (If P then Q)
$Q \rightarrow R$ (If Q then R)

To prove:
$P \rightarrow R$ (If P then R)

Using transitive inference:
$P \rightarrow Q$ (Premise)
$Q \rightarrow R$ (Premise)
Therefore, $P \rightarrow R$ (Conclusion)