Multiple Linear Regression

26 June 2025 17:07

🧠 What is Multiple Linear Regression?

Multiple Linear Regression (MLR) is a supervised learning algorithm that models the linear relationship between one **dependent variable** (target) and **two or more independent variables** (features).

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_n x_n + \varepsilon$$

Intercept

$y = B_0 + B_1 x$

Intercept

slope

Where:

- y: Target variable (e.g., **House Price**)
- $x_1, x_2, \ldots, x_n$: Feature variables (e.g., size, bedrooms, age, etc.)
- β_0: Intercept (bias term)
- $\beta_1, \beta_2, \ldots, \beta_n$: Coefficients (weights) (slope)
- ε: Error term (residuals)

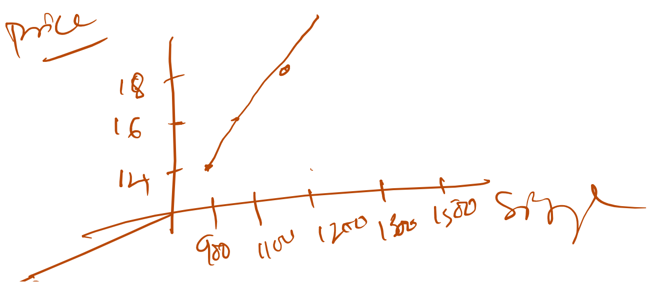
Problem: House price prediction

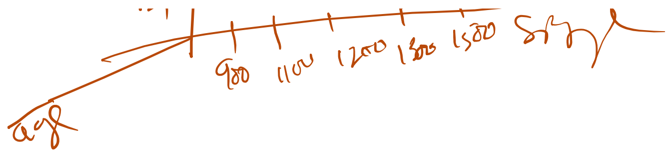
Sample dataset:

Size (sqft) (x_1)	Bedrooms (x_2)	Age (years) (x_3)	Price (₹ Lakhs) (y)
1200	2	10	18.0
1500	3	5	25.0
900	2	20	14.0
2000	4	2	36.0
1800	3	7	30.0
1100	2	15	16.0
2200	4	3	39.0
1300	2	12	20.0
1700	3	6	28.0
1000	2	18	15.0

$\hat{y} = 96, \; E_3 = 114.961 = 82$

$\hat{y} = 202.5 \quad E_3 = 166.6$





$$Y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$

Initialize with Random values

$$\beta_0 = 0, \ \beta_1 = 0.1, \ \beta_2 = 0.5, \ \beta_3 = 0.25$$

$$\hat{y} = 0 + 0.1 \times x_1 + 0.5 \times x_2 + 0.25 \times x_3$$

$(1200, 2, 10)$

$x_1 \quad x_2 \quad x_3$

$y = 18$

$$\hat{y} = 0 + 0.1 \times 1200 + 0.5 \times 2 + 0.25 \times 10$$

$$\hat{y} = 120 + 1 + 2.5$$

$$\hat{y} = 123.5$$

Loss $\rightarrow$ MAE | MSE | RMSE

$$MAE = \frac{1}{n} \sum_{i=1}^{n} | y_i - \hat{y}_i |$$

$n = 10$

$$= |18 - 123.5|$$

$$E_1 = 105.5$$

$x_1 \quad x_2 \quad x_3$

$(1500, 3, 5)$

$y = 25$

$$\beta_0 = 0, \ \beta_1 = 0.1, \ \beta_2 = 0.5, \ \beta_3 = .25$$

$$\hat{y} = B_0 + B_1 x_1 + B_2 x_2 + B_3 x_3$$

$$\hat{y} = 152.75$$

$$E_2 = |y - \hat{y}|$$

$$E_2 = |25 - 152.75| = 127.75$$

$$MAE = \frac{1}{n} \sum_{i=1}^{n} | y_i - \hat{y}_i |$$

$$E = MAE = \frac{1}{10} [(105.5) + (127.75) + 82 + 166.5 \ldots]$$

$$B_0 = B_0 - \alpha \frac{\partial E}{\partial B_0} \qquad E = |y - \hat{y}|$$

$$B_1 = B_1 - \alpha \frac{\partial E}{\partial B_1} \qquad \frac{\partial E}{\partial B_0} |(y - (B_0 + B_1 x_1 + B_2 x_2 + B_3 x_3)|$$

$$B_2 = B_2 - \alpha \frac{\partial E}{\partial B_2} \qquad \frac{\partial E}{\partial B_0} = 1$$

$$B_3 = B_3 - \alpha \frac{\partial E}{\partial B_3}$$

$$\frac{\partial E}{\partial B_1} = \frac{\partial}{\partial B_1} | y - (B_0 + B_1 x_1 + B_2 x_2 + B_3 x_3) |$$

$$\frac{\partial E}{\partial B_1} = x_1$$

$$\frac{\partial E}{\partial B_2} = x_2, \quad \frac{\partial E}{\partial B_3} = x_3$$

$$\frac{\partial E}{\partial B_2} = x_2, \quad \frac{}{\partial B_3} \text{---}$$

$\alpha \rightarrow$ Learning Rate $\rightarrow$ 0.01

$$B_0 = B_0 - \alpha * \frac{\partial E}{\partial B_0}$$

$$B_0 = 0 - 0.01 * 1$$

$$B_0 = -0.01$$

$$B_1 = B_1 - \alpha * \frac{\partial E}{\partial B_1}$$

$$= 0.1 - 0.01 \times \left[\frac{1200 + 1500 + 900 - -}{10} \right]$$

$$= 0.1 - 0.01 \times \left[\frac{14700}{10} \right]$$

$$= 0.1 - 14.7$$

$$= \underline{-14.6}$$

$$B_2 = B_2 - \alpha * \frac{\partial E}{\partial B_2}$$

$$B_2 = 0.5 - 0.01 * \sum_{i=1}^{n} \frac{x_{2i}}{n}$$

$$= 0.5 - 0.01 * \frac{27}{10}$$

$$= 0.5 - 0.027$$

$= 0.5 \qquad\qquad 10$

$= 0.5 - 0.027$

$B_2 = \underline{0.473}$

$$B_3 = B_3 - \alpha * \frac{\partial E}{\partial B_3}$$

$$B_3 = 0.25 - 0.01 * \sum_{i=1}^{n} \frac{x_{3i}}{n}$$

$= 0.25 - 0.01 * 9.8$

$= 0.152$

Iteration 2

$$\hat{y} = -0.01 + (-14.6)x_1 + 0.473x_2 + 0.152x_3$$