Polynomial Regression

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🧠 1. What is Polynomial Regression?

Polynomial Regression is an extension of **Linear Regression** that models the relationship between the independent variable x and the dependent variable y as an **n-th degree polynomial**.

Equation:

$$y = \beta_0 + \beta_1 x + \beta_2 x^2 + \beta_3 x^3 + \ldots + \beta_n x^n + \epsilon$$

Here:

- $\beta_0, \beta_1, \ldots, \beta_n$ are coefficients
- ϵ is the error term

👉 It's still a **linear model in parameters** (since coefficients are linear), but **non-linear in features**.

📈 2. Why Polynomial Regression?

When your data shows a **curved or non-linear relationship**, a simple linear regression line cannot capture the trend.

Example:

- Growth of bacteria over time
- Relationship between experience and income
- Temperature vs. electricity consumption

x_1	y	x_2 (x_1^2)	x_3 (x_1^3)
1	2	1	1
3	3	9	27
4	5	16	64
6	7	36	216

🧮 3. Mathematical Intuition

Suppose you have input x and output y.

We can create new features:

$$x_1 = x, \quad x_2 = x^2, \quad x_3 = x^3, \ldots, x_n = x^n$$

x_1	x_2	x_3	y
1	1	1	2
3	9	27	3
4	16	64	5
6	36	216	7

Then perform **Linear Regression** on these **transformed features**.

So, we convert:

$$y = \beta_0 + \beta_1 x + \beta_2 x^2$$

into:

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

and apply ordinary least squares.

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_1^2 + \beta_3 x_1^3$$

$$y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3$$