

# Evaluation Metrics Assignment

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## Classification Model Evaluation – Numerical Problem

### Problem Statement

A medical diagnostic system is tested on a dataset of 12 patients to predict whether they have a certain disease.

- 1 indicates **Disease Positive**
- 0 indicates **Disease Negative**

Here are the **true labels** and the **predicted probabilities** given by the model:

Patient	True Label (Actual $y$ )	Predicted Probability ( $\hat{y}$ )
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1	1	0.95
2	0	0.85
3	1	0.90
4	0	0.10
5	1	0.80
6	0	0.05
7	1	0.60
8	1	0.30
9	0	0.70
10	0	0.40
11	1	0.20
12	0	0.25

### Tasks

Assume the classification threshold is 0.5, i.e.,

- $\hat{y} \geq 0.5 \rightarrow$  predict 1 (disease)
- $\hat{y} < 0.5 \rightarrow$  predict 0 (no disease)

### Task 1: Compute Predicted Labels

Based on a threshold of 0.5, calculate the predicted class labels ( $y_{pred}$ ).

### Task 2: Confusion Matrix

From the true and predicted labels, compute:

- True Positives (TP)
- False Positives (FP)
- False Negatives (FN)
- True Negatives (TN)

## Task 3: Compute Evaluation Metrics

### 1. Accuracy

$$\text{Accuracy} = \frac{TP + TN}{TP + FP + FN + TN}$$

### 2. Precision

$$\text{Precision} = \frac{TP}{TP + FP}$$

### 3. Recall (Sensitivity)

$$\text{Recall} = \frac{TP}{TP + FN}$$

### 4. Specificity

$$\text{Specificity} = \frac{TN}{TN + FP}$$

### 5. F1 Score

$$\text{F1 Score} = 2 \cdot \frac{\text{Precision} \cdot \text{Recall}}{\text{Precision} + \text{Recall}}$$

### 6. False Positive Rate (FPR)

$$\text{FPR} = \frac{FP}{FP + TN}$$

## Task 4: ROC Curve Table

Generate a table of **TPR** and **FPR** at different thresholds (e.g., 0.9, 0.8, ..., 0.1), and create a rough sketch of the ROC curve (or compute AUC numerically if you prefer).

## Task 5: Compute AUC-ROC

Using the TPR and FPR values from Task 4:

- Plot the **ROC curve** (TPR vs FPR).
- Use the **trapezoidal rule** to compute **AUC**:

$$\text{AUC} = \int_0^1 \text{TPR}(FPR) dFPR \approx \sum \left( \frac{\text{TPR}_i + \text{TPR}_{i+1}}{2} \cdot (\text{FPR}_{i+1} - \text{FPR}_i) \right)$$

## Deliverables Expected

- Final confusion matrix
- All metric values (rounded to 2 decimals)
- ROC points table (optional bonus)