Have you ever shouted "Eureka!" in the bath? That fun word comes from a brilliant man named **Archimedes**, who lived over 2,000 years ago in a city called Syracuse. He was one of the greatest scientists and mathematicians ever!

Archimedes loved solving puzzles. One day, the king asked him if his new crown was made of pure gold. Archimedes thought and thought. While getting into a full bathtub, he saw the water spill over the edge. He realized something amazing: the water that spilled out was exactly the same volume as his body. **He could use water to measure the crown!** He was so excited he jumped out and ran through the streets, shouting "Eureka!" (which means "I found it!"). His idea is called **buoyancy**, and it explains why huge metal ships can float.

Archimedes also had a super math brain. He worked out a very close number for **Pi** (π), the special number we use for circles. He loved the sphere and cylinder so much he wanted them carved on his tombstone.

To protect his city from enemy ships, people say he built a giant metal claw that could lift a ship right out of the sea!

Archimedes was so focused on his work that, according to legend, when a soldier disturbed him while he was drawing a math problem in the sand, he said, "Don't disturb my circles!" He shows us how powerful a curious mind can be, and how much fun it is to discover how the world works.

True or False

1. Archimedes shouted "Eureka!" when he solved the problem of the king's crown.
2. Archimedes discovered Pi (π), which is a number we use for triangles.
3. A story says Archimedes was drawing in the sand when a soldier came to him.

"Wh" Questions

1. **What** did Archimedes shout when he had his big idea in the bath?
2. **Why** did the king ask Archimedes to look at his new crown?
3. **What** two shapes did Archimedes love so much he wanted them on his tombstone?

Find the Misspelled Word

Read each sentence and find the word that is spelled incorrectly.

1. Archimedes was a brilliant **mathematishun** from long ago.
2. He used water to figure out the **volum** of the king's crown.
3. People say he built a giant **mashine** to lift ships out of the water.

Fill in the Gaps

Fill in the gaps with the following words:

Shapes, water, bath, thinker, Eureka, crown, math, circles

Archimedes was a very clever **......1..........** from Syracuse. His most famous story is about him taking a **......2..........** . He noticed the **......3..........** rising and had a brilliant idea. He was so excited he yelled "**......4..........** !" This idea helped him solve the mystery of the king's **......5..........** . He also loved **......6..........** and figured out a lot about **......7..........** and **......8..........** .

Grammar

Simple present

Exercise 1: Matching Exercise (Simple Present Verbs)

Instructions: Match the subject on the left with the correct form of the verb on the right to make a logical sentence.

Subjects:

1. I
2. She

3. The dog
4. We
5. My brother and I
6. The bus
7. Maria
8. Cats

Verbs:

A. plays the piano beautifully.

B. drink milk every morning.

C. starts its route at 7 a.m.

D. barks at the mail carrier.

E. enjoy playing video games on weekends.

F. live in a big blue house.

G. studies for her tests at the library.

H. sleep for many hours during the day.

Exercise 2: Fill-in-the-Blank (Simple Present Tense)

Instructions: Fill in each blank with the correct **Simple Present** form of the verb given in parentheses ( ).

The Park on Saturday

Every Saturday, my family and I (1. **go**) _________ to the city park. My mom (2. **pack**) _________ a big picnic basket. My little sister, Lily, (3. **love**) _________ to feed the ducks. She (4. **throw**) _________ small pieces of bread into the pond. My dad (5. **read**) _________ his book under a tall tree. I (6. **fly**) _________ my red kite when the wind is strong.

Our dog, Max, always (7. **come**) _________ with us. He (8. **chase** - negative) _________ the birds, but he (9. **watch**) _________ them very closely. We all (10. **have**) _________ a wonderful time together.

Cardinal Numbers:

0	zero, nought
1	one
2	two
3	three
4	four
5	five
6	six
7	seven
8	eight
9	nine
10	ten

11	eleven
12	twelve
13	thirteen
14	fourteen
15	fifteen
16	sixteen
17	seventeen
18	eighteen
19	nineteen
20	twenty

21	twenty-one
22	twenty-two
23	twenty-three
24	twenty-four
30	thirty
31	thirty-one
40	forty
50	fifty
60	sixty
70	seventy
80	eighty
90	ninety

100	one hundred
101	one hundred and one
152	one hundred and fifty-two
200	two hundred
1,000	one thousand
1,000,000	one million
1,000,000,000	one billion
1,000,000,000,000	one trillion

⚠ In English, when we write cardinal numbers, we separate thousands with a comma (,)

For numbers in the hundreds:
British English: 120 = *one hundred **and** twenty*
American English: 120 = *one hundred twenty*

For numbers in the thousands
British English: 3,486 = *three **thousand**, four hundred and eighty-six*
American English: 3,486 = *thirty-four **hundred**, eighty-six*

NOTE: British English takes "and" following "hundred". American English omits "and". [8]

How to Say Numbers in the Hundreds:
Say numbers in the hundreds by beginning with numerals one through nine followed by "hundred". Finish by saying the last two digits:
- 350 – three hundred fifty
- 425 – four hundred twenty-five
- 873 - eight hundred seventy-three
- 112 - one hundred twelve

How to Say Numbers in the Thousands:
Say a number up to 999 followed by "thousand." Finish by reading the hundreds when applicable:
- 15,560 – **fifteen thousand** *five hundred sixty*
- 786,450 – **seven hundred eighty-six thousand** *four hundred fifty*
- 342,713 - **three hundred forty-two thousand** *seven hundred thirteen*
- 569,045 - **five hundred sixty-nine thousand** *forty-five*

How to Say Numbers in the Millions
For millions, say a number up to 999 followed by "million." Finish by saying first the thousands and then the hundreds when applicable:
- 2,450,000 – **two million** four hundred fifty thousand
- 27,805,234 - **twenty-seven million eight** hundred five thousand two hundred thirty-four
- 934,700,000 – **nine hundred thirty-four million** seven hundred thousand
- 589,432,420 - **five hundred eighty-nine million** four hundred thirty-two thousand four hundred twenty [9]

Part 1: Hundreds

Write the number in words.

1. 218 → ______________________________
2. 645 → ______________________________
3. 999 → ______________________________
4. 103 → ______________________________

Part 2: Thousands

Write the number in words.

1. 12,430 → ______________________________
2. 305,210 → ______________________________
3. 478,009 → ______________________________
4. 891,765 → ______________________________

Part 3: Millions

Write the number in words.

1. 3,000,000 → ______________________________
2. 45,320,100 → ______________________________
3. 789,654,321 → ______________________________
4. 120,005,000 → ______________________________

Ordinal numbers:

We use **ordinal numbers** to talk about the "order" of things or to define a thing's position in a series.

first	1st	**eleventh**	11th	**twenty-first**	21st	**eighthieth**	80th
second	2nd	**twelfth**	12th	**twenty-second**	22nd	**ninetieth**	90th
third	3rd	**thirteenth**	13th	**twenty-third**	23rd	**hundredth**	100th
fourth	4th	**fourteenth**	14th	**twenty-fourth**	24th	**hundred and first**	101st
fifth	5th	**fifteenth**	15th	**thirtieth**	30th	**hundred and fifty-second**	152nd
sixth	6th	**sixteenth**	16th	**thirty-first**	31st	**two hundredth**	200th
seventh	7th	**seventeenth**	17th	**fortieth**	40th	**thousandth**	1,000th
eighth	8th	**eighteenth**	18th	**fiftieth**	50th	**millionth**	1,000,000th
ninth	9th	**nineteenth**	19th	**sixtieth**	60th	**billionth**	1,000,000,000th
tenth	10th	**twentieth**	20th	**seventieth**	70th	**trillionth**	1,000,000,000,000th

Fractions:

In the fraction $\frac{a}{b}$ (a over b):

- a is called the **numerator**
- b is called the **denominator**.

A **proper fraction** has its numerator less than its denominator, e.g. $\frac{3}{4}$

An **improper fraction** has its numerator more than its denominator, e.g. $\frac{9}{2}$

How to Talk About Fractions:

Say the top number as a cardinal number, followed by the ordinal number + "s:"

- 3/8 - three-eighth**s**
- 5/16 - five-sixteenth**s**
- 2/32 - two thirty-second**s**

Exceptions to this rule are:

- 1/4, 3/4 - one-quarter, three quarters
- 1/3, 2/3 - one third, two-thirds

Read numbers together with fractions by first stating the number followed by "and" and then the fraction:

- 4 7/8 - four and seven-eighths

½	**a half OR one half**
⅓	**a third OR one third**
¼	**a quarter OR one quarter**
⅕	**a fifth OR one fifth**
¾	**three quarters**
⅛	**an eighth OR one eighth**
⅔	**two thirds**
⅗	**three fifths**
⅝	**five eighths**
1½	**one and a half**
5¾	**five and three quarters**

Before you read

Discuss these questions.

1. What is the difference between visual and symbolic reasoning in mathematics?
2. What is Euclid's contribution to mathematics?
3. What is most interesting about Euclid's geometry to modern mathematicians?

A Key terms

Match these terms with their definitions.

1. notation	a) not representing any specific value
2. arbitrary	b) having all sides of equal length
3. equilateral	c) any series of signs or symbols used to represent quantities or elements in a specialized system
4. a solid	d) a solid figure having four plane faces
5. tetrahedron	e) a solid figure having eight plane faces
6. octahedron	f) a closed surface in three-dimensional space
7. vertex (pl.vertices)	g) the point of intersection of two sides of a plane figure or angle
8. an obtuse angle	h) (of a triangle) having two sides of equal length
9 .an isosceles triangle	j) (of an angle) lying between 90° and 180°

Introduction

Euclid of Alexandria is the most famous Greek geometer, known not for original discovery but for brilliantly synthesizing existing knowledge. His greatest work, the "Elements", became a timeless bestseller. While he wrote many texts, only five survive: the Elements, Division of Figures, Data, Phenomena, and Optics.

The Elements is his masterpiece, systematically explaining two- and three-dimensional geometry. The other works are supplements: the Phenomena applies geometry to astronomy using spheres, and the Optics explores perspective—how the eye transforms a three-dimensional scene into a two-dimensional image.

Euclid's true genius was his method. He moved from merely stating truths to proving them through logic. He begins by listing some assumptions and then stating their logical consequences, a statement we call a theorem. The Elements itself breaks up into 13 logically ordered books, culminating in the proof that only five regular (Platonic) solids exist.

For modern thinkers, the key is not its content, but its logical structure. Unlike earlier mathematicians, Euclid does not merely assert that some theorem is true. He provides a proof, where each step is a logical consequence of some of the previous steps. This deductive framework, building inevitable conclusions from clear starting points, is his enduring legacy, defining rigorous mathematical reasoning for centuries.

True or False

Read the statements and write T if the statement is true or F if it is false.

1. Euclid is primarily celebrated for discovering entirely new mathematical theorems.
2. His book Optics investigates how we perceive three-dimensional space in two dimensions.
3. The Elements is a single, short book about shapes in a plane.
4. Euclid's major contribution was creating a logical structure for proving geometric facts.
5. The climax of the Elements is the proof that there are exactly five regular solids.

WH- Questions

Answer the following questions based on the text.

1. What is the main subject of Euclid's Phenomena?
2. How did Euclid's approach differ from that of earlier mathematicians?
3. Why is Euclid's Elements so significant to modern mathematics?

Fill-in-the-Blanks Passage

Complete the summary by filling in the blanks with words or short phrases from the text.

Consequences, perspective, method, assert, logical, consequence , synthesizer, proof,

Euclid, the famous Greek geometer, was a great (1) ………….. His most important work, the Elements, is not a collection of new discoveries but a brilliant (2) ………….. system. He begins with clear assumptions and then shows the (3) logical ………….. , which are called theorems. Unlike his predecessors, he did not just (4) ………….. that a theorem was true; he provided a step-by-step (5) ………….. . In this proof, every new statement must be a (6) logical ………….. of a previous step. This rigorous (7) ………….. or structure is considered his greatest legacy. His other works, like the Optics, explored topics such as visual (8) ………….. , explaining how the eye interprets a 3D world.

Grammar

Simple present

Here are three activities focused on the **Simple Present Tense**, covering affirmative, negative, and interrogative forms.

Activity 1: Fill in the Blanks

Complete each sentence using the correct form of the verb in parentheses (affirmative or negative).

1. She ________ coffee every morning. (drink – affirmative)
2. They ________ to school on Sundays. (go – negative)
3. My brother ________ television in the afternoon. (watch – negative)
4. I ________ my homework after dinner. (do – affirmative)
5. The train ________ at 8 PM. (arrive – affirmative)
6. We ________ in a big city. (live – negative)
7. He ________ English very well. (speak – affirmative)
8. Cats ________ vegetables. (eat – negative)

Activity 2: Make the Questions

Form questions in the Simple Present tense using the words given.

1. (you / like / pizza) → ______________________________?
2. (she / work / in a hospital) → ______________________________?
3. (they / play / football / on Saturdays) → ______________________________?
4. (the store / open / at 9 AM) → ______________________________?
5. (he / have / a brother) → ______________________________?
6. (we / need / milk) → ______________________________?
7. (your parents / speak / French) → ______________________________?
8. (it / rain / a lot here) → ______________________________?

Activity 3: Correct the Sentences

Each sentence has one mistake. Find it and rewrite the sentence correctly. The mistake could be in the verb form (affirmative/negative) or in question structure.

1. **Incorrect:** She don't like cold weather.

   Correct: ________________________________

2. **Incorrect:** Do he lives nearby?

   Correct: ________________________________

3. **Incorrect:** They plays basketball every week.

   Correct: ________________________________

4. **Incorrect:** We doesn't understand the lesson.

   Correct: ________________________________

5. **Incorrect:** You walks to the park?

   Correct: ________________________________

6. **Incorrect:** The bus not stop here.

   Correct: ________________________________

7. **Incorrect:** Does they have a car?

   Correct: ________________________________

8. **Incorrect:** I not eat meat.

   Correct: ________________________________

maths

+ : plus
addition : addition
additionner : to add
la somme : the sum

Examples :

- 10 + 2 = 12 Ten **plus** two **equals** twelve *or* Ten **plus** two **is** twelve.
- 23 + 5 + 6 = 34

Twenty-three **plus** five **plus** six **equals** thirty-four.
Or Twenty-three **plus** five **plus** six **is** thirty-four.

- 405 + 67 + 12 = 484

Four hundred and five **plus** sixty-seven **plus** twelve **equals** four hundred and eighty-four.
Or Four hundred **and** five **plus** sixty-seven **plus** twelve **is** four hundred and eighty-four

Note that we usually say **equals** NOT **equal.**

– : minus

soustraction : **subtraction**

soustraire : **to subtract / to take away** (enlever)

la différence : **the difference**

Examples :

•16 - 13 = 3 : Thirteen **from** sixteen **leaves** three
Or Sixteen **minus** thirteen **equals** three.
Or Sixteen **minus** thirteen **is** three.
•47 - 11 = 36
Eleven **from** forty-seven **leaves** thirty-six.
Or Forty-seven **minus** eleven **equals** thirty-six.
Or Forty-seven **minus** eleven **is** thirty-six.

× : multiplied by, times

multiplication : **multiplication**

multiplier : **to multiply**

fois (multiplié par) : **times**

le produit : **the product**

Examples :

- 8 x 5 = 40

Eight **times** five **equals** forty *or* Eight five**s** **is** forty.

- 35 x 11 = 385

Thirty-five **times** eleven **equals** three hundred and eighty-five.
or
Thirty-five eleven**s** **is** three hundred and eighty-five.

÷ or / : divided by

division : **division**

diviser : **to divide**

divisé par : **divided by**

le quotient : **the quotient**

Examples :

- 6 x 30 + 5 = 185

Six **times** thirty **plus** five **equals** one hundred and eighty-five *or*
Six **times** thirty **plus** five **is** one hundred eighty-five.

- 23 - 7 + 14 x 11 = 170

Twenty three **minus** seven **plus** fourteen **times** eleven **equals** one hundred and seventy *or*
Twenty three **minus** seven **plus** fourteen **times** eleven **is** one hundred and seventy.

Examples :

- 62 : 2 = 31

Sixty-two **divided by** two **equals** thirty-one *or* Two **into** sixty-two **is** thirty-one *or* **If you divide** sixty-two **by** two **you get** thirty-one

- 140 : 5 = 28

One hundred and forty **divided by** five **equals** twenty-eight.
or
Five **into** one hundred and forty **is** twenty-eight.

Logical Proof and Formal Reasoning (Beginner Level)

Logical proof and formal reasoning help us understand **why** something is true. Instead of relying on feelings or guesses, we use clear rules and steps to reach correct conclusions. These ideas are especially important in mathematics, but they are also useful in everyday thinking.

A **logical proof** is a clear, step-by-step explanation that shows how a conclusion follows from known facts. Each step must have a reason.
Example (Logical Proof):

1. All even numbers are divisible by 2.
2. The number 8 is an even number.
3. Therefore, 8 is divisible by 2.
   This is a logical proof because the conclusion follows directly from the given facts.

Formal reasoning means thinking in an organized and precise way. It usually starts with basic statements called **assumptions** or **axioms**, which we accept as true. From these, we use rules of logic to reach new conclusions.
Example (Formal Reasoning):

- Assumption: All birds have wings.
- Assumption: A sparrow is a bird.
- Conclusion: A sparrow has wings.
  This reasoning is formal because it follows a clear logical structure.

Another common type of reasoning is **deductive reasoning**, where we move from a general rule to a specific case.
Example (Deduction):

- All squares have four sides.
- This shape is a square.
- Therefore, this shape has four sides.

There is also **inductive reasoning**, where we observe several examples and make a general conclusion.
Example (Induction):

- The sun rose yesterday.
- The sun rose today.
- Therefore, the sun will probably rise tomorrow.

Learning logical proof and formal reasoning helps beginners think clearly and carefully. These skills improve problem-solving, help us explain our ideas, and allow us to make better decisions in mathematics and in everyday life.

Here are **four True/False statements** related to logical proof and formal reasoning:

1. A logical proof shows **why** something is true, step by step.
2. Formal reasoning allows conclusions to be drawn without any assumptions or rules.
3. Deductive reasoning moves from a general rule to a specific case.

4. Inductive reasoning guarantees that the conclusion is always correct.

Match the items in **Column A** with the correct items in **Column B**.

Column A

1. Logical Proof
2. Formal Reasoning
3. Deductive Reasoning
4. Inductive Reasoning

Column B
A. Thinking step by step from a general rule to a specific case.
B. A clear explanation showing why something is true using known facts.
C. Observing several examples and making a general conclusion.
D. Using structured, precise thinking based on assumptions or rules.

WH-Questions

1. **What** is a logical proof?
2. **Why** do we use logical proofs instead of guessing?
3. **What** does formal reasoning start with?
4. **Who** is an example of a mathematician known for formal reasoning?
5. **How** does deductive reasoning work?
6. **How** is inductive reasoning different from deductive reasoning?
7. **What** is the purpose of each step in a logical proof?
8. **Where** can we use logical proof and formal reasoning outside of mathematics?
9. **Which** type of reasoning moves from observations to general conclusions?
10. **Which** type of reasoning moves from general rules to specific cases?

Grammar

Simple past

1. What is the Simple Past?

The Simple Past tense is used to talk about actions or events that **happened at a specific time in the past**.
Example:

- I visited my friend yesterday.
- She watched a movie last night.

---

2. Forming the Simple Past

A. Regular verbs

- Add **-ed** to the base form of the verb.
  Examples:
- walk → walked
- play → played
- cook → cooked

Note: If the verb ends in **-e**, just add **-d**.

- like → liked
- live → lived

Spelling changes for regular verbs:

- For verbs ending in **consonant + y**, change **y** → **i + ed**:
  - study → studied
  - try → tried
- For short verbs with one syllable ending in **consonant-vowel-consonant**, double the final consonant:
  - stop → stopped
  - plan → planned

B. Irregular verbs

- Irregular verbs do **not** follow a simple pattern.
- You must **memorize their past forms**.
  Examples:
- go → went
- eat → ate
- see → saw

---

3. Forming Negative Sentences

- Use **did not (didn't) + base verb**.
  Examples:
- I did not go to school yesterday.
- She didn't watch TV last night.

---

4. Forming Questions

- Use **Did + subject + base verb**.
  Examples:
- Did you visit your friend yesterday?
- Did they watch the movie?

---

5. Signal Words for Simple Past

These words often indicate the Simple Past tense:

- yesterday, last night/week/year, ago, in 2010, when I was a child

Example:

- I played football **yesterday**.
- She moved to London **last year**.

Part 1: Fill in the blanks (Regular and Irregular Verbs)

Complete the sentences with the correct form of the verb in parentheses.

1. Yesterday, I _______ (play) football with my friends.
2. She _______ (go) to the market last Saturday.
3. We _______ (watch) a movie last night.
4. He _______ (eat) an apple for breakfast.
5. They _______ (study) English yesterday.

Part 2: Change to Negative Sentences

Rewrite the sentences in the negative form.

1. I visited my grandmother yesterday. → ___________________________________
2. He watched TV last night. → ___________________________________
3. She went to school yesterday. → ___________________________________

Part 3: Make Questions (Interrogative)

Change the sentences into questions.

1. You played football yesterday. → ___________________________________
2. They saw a movie last weekend. → ___________________________________
3. She cooked dinner last night. → ___________________________________

Part 4: Mixed Practice (Affirmative, Negative, Interrogative)

Decide which form is correct for each sentence. Write **A** for affirmative, **N** for negative, **Q** for question.

1. Did you visit your friend yesterday? ________
2. I didn't go to school yesterday. ________
3. She played the piano yesterday. ________
4. We didn't watch TV last night. ________
5. They went to the park yesterday. ________

Maths

Read and calculate the following operations:

- **$30 - 5 + 12 \times 10 =$**
- **$42 - 8 + 9 \times 15 =$**
- **$25 - 3 + 16 \times 9 =$**
- **$50 - 14 + 8 \times 20 =$**
- **$40 - 6 + 11 \times 13 =$**
- **$28 - 4 + 15 \times 12 =$**
- **$35 - 7 + 14 \times 11 =$**
- **$60 - 18 + 10 \times 15 =$**
- **$22 - 5 + 17 \times 9 =$**
- **$45 - 9 + 12 \times 14 =$**

$=$ is equal to
$\neq$ is not equal to
$\approx$ is approximately equal to
$<$ is less than
$\leq$ is less than or equal to
$>$ is greater than
$\geq$ is greater than or equal to
$\therefore$ therefore
$\sqrt{\ }$ the square root of

Rewrite each of these statements using mathematical symbols.

(a) 19 is less than 45
(b) 12 plus 18 is equal to 30
(c) 0.5 is equal to 1 2

(d) 0.8 is not equal to 8.0
(e) -34 is less than 2 times -16
(f) therefore the number *x* equals the square root of 72
(g) a number (*x*) is less than or equal to negative 45
(h) π is approximately equal to 3.14
(i) 5.1 is greater than 5.01
(j) the sum of 3 and 4 is not equal to the product of 3 and 4
(k) the difference between 12 and -12 is greater than 12
(l) the sum of -12 and -24 is less than 0

(m) 12 times (*x*) is approximately -40

Say whether these mathematical statements are true or false.

(a) $0.599 > 6.0$
(b) $5 \times 1999 \approx 10\,000$
(c) $8.1 = 8\frac{1}{10}$
(d) $6.2 + 4.3 = 4.3 + 6.2$
(e) $20 \times 9 \geq 21 \times 8$
(f) $6.0 = 6$
(g) $-12 > -4$
(h) $19.9 \leq 20$
(i) $1000 > 199 \times 5$
(j) $\sqrt{16} = 4$
(k) $35 \times 5 \times 2 \neq 350$
(l) $20 \div 4 = 5 \div 20$
(m) $20 - 4 \neq 4 - 20$
(n) $20 \times 4 \neq 4 \times 20$

Different types of numbers

Make sure you know the correct mathematical words for the types of numbers in the table.

Number	Definition	Example
Natural number	Any whole number from 1 to infinity, sometimes called 'counting numbers'. 0 is not included.	1, 2, 3, 4, 5, . . .
Odd number	A whole number that cannot be divided exactly by 2.	1, 3, 5, 7, . . .
Even number	A whole number that can be divided exactly by 2.	2, 4, 6, 8, . . .
Integer	Any of the negative and positive whole numbers, including zero.	. . . −3, −2, −1, 0, 1, 2, 3, . . .
Prime number	A whole number greater than 1 which has only two factors: the number itself and 1.	2, 3, 5, 7, 11, . . .
Square number	The product obtained when an integer is multiplied by itself.	1, 4, 9, 16, . . .
Fraction	A number representing parts of a whole number, can be written as a common (vulgar) fraction in the form of $\frac{a}{b}$ or as a decimal using the decimal point.	$\frac{1}{2}, \frac{1}{4}, \frac{1}{3}, \frac{1}{8}, \frac{13}{3}, 2\frac{1}{2}$ 0.5, 0.2, 0.08, 1.7

1 Here is a set of numbers: $\{-4, -1, 0, \frac{1}{2}, 0.75, 3, 4, 6, 11, 16, 19, 25\}$

List the numbers from this set that are:

(a) natural numbers
(b) even numbers
(c) odd numbers
(d) integers
(e) negative integers
(f) fractions
(g) square numbers
(h) prime numbers
(i) neither square nor prime.

2 List:

(a) the next four odd numbers after 107
(b) four consecutive even numbers between 2008 and 2030
(c) all odd numbers between 993 and 1007
(d) the first five square numbers
(e) four decimal fractions that are smaller than 0.5
(f) four vulgar fractions that are greater than $\frac{1}{2}$ but smaller than $\frac{3}{4}$.

3 State whether the following will be odd or even:

(a) the sum of two odd numbers
(b) the sum of two even numbers
(c) the sum of an odd and an even number
(d) the square of an odd number
(e) the square of an even number
(f) an odd number multiplied by an even number.

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A **logical proof** is a clear, step-by-step explanation that shows how a conclusion follows from known facts. Each step must have a reason.
Example (Logical Proof):

1. All even numbers are divisible by 2.
2. The number 8 is an even number.
3. Therefore, 8 is divisible by 2.
   This is a logical proof because the conclusion follows directly from the given facts.

Formal reasoning means thinking in an organized and precise way. It usually starts with basic statements called **assumptions** or **axioms**, which we accept as true. From these, we use rules of logic to reach new conclusions.
Example (Formal Reasoning):

- Assumption: All birds have wings.
- Assumption: A sparrow is a bird.
- Conclusion: A sparrow has wings.
  This reasoning is formal because it follows a clear logical structure.

Another common type of reasoning is **deductive reasoning**, where we move from a general rule to a specific case.
Example (Deduction):

- All squares have four sides.
- This shape is a square.
- Therefore, this shape has four sides.

There is also **inductive reasoning**, where we observe several examples and make a general conclusion.
Example (Induction):

- The sun rose yesterday.
- The sun rose today.
- Therefore, the sun will probably rise tomorrow.

Learning logical proof and formal reasoning helps beginners think clearly and carefully. These skills improve problem-solving, help us explain our ideas, and allow us to make better decisions in mathematics and in everyday life.

Here are **four True/False statements** related to logical proof and formal reasoning:

1. A logical proof shows **why** something is true, step by step.
2. Formal reasoning allows conclusions to be drawn without any assumptions or rules.
3. Deductive reasoning moves from a general rule to a specific case.

4. Inductive reasoning guarantees that the conclusion is always correct.

Match the items in **Column A** with the correct items in **Column B**.

Column A

1. Logical Proof
2. Formal Reasoning
3. Deductive Reasoning
4. Inductive Reasoning

Column B

A. Thinking step by step from a general rule to a specific case.
B. A clear explanation showing why something is true using known facts.
C. Observing several examples and making a general conclusion.
D. Using structured, precise thinking based on assumptions or rules.

WH-Questions

1. **What** is a logical proof?
2. **Why** do we use logical proofs instead of guessing?
3. **What** does formal reasoning start with?
4. **Who** is an example of a mathematician known for formal reasoning?
5. **How** does deductive reasoning work?
6. **How** is inductive reasoning different from deductive reasoning?
7. **What** is the purpose of each step in a logical proof?
8. **Where** can we use logical proof and formal reasoning outside of mathematics?
9. **Which** type of reasoning moves from observations to general conclusions?
10. **Which** type of reasoning moves from general rules to specific cases?

Grammar

Simple past- revision

Instructions: Conjugate the verb in brackets into the correct simple past tense form for the sentence.

Affirmative Sentences

1. Last night, I (to watch) ______ a science documentary.
2. She (to bake) ______ a chocolate cake for the party.

3. They (to travel) ______ to Spain for their vacation.
4. The mail (to arrive) ______ very late this morning.
5. We (to study) ______ together at the library.
6. He (to fix) ______ his bicycle over the weekend.
7. The movie (to start) ______ at 8 o'clock sharp.
8. I (to finish) ______ reading that novel.
9. My parents (to call) ______ me yesterday.
10. The team (to win) ______ the championship game.

Negative Sentences

1. I (to understand / not) ______ ______ the instructions.
2. She (to enjoy / not) ______ ______ the cold weather.
3. They (to finish / not) ______ ______ the project on time.
4. He (to remember / not) ______ ______ to buy milk.
5. We (to go / not) ______ ______ to the meeting.
6. The computer (to work / not) ______ ______ properly.
7. You (to tell / not) ______ ______ me about your plan.
8. I (to have / not) ______ ______ enough money for the ticket.
9. The children (to eat / not) ______ ______ their vegetables.
10. My phone (to ring / not) ______ ______ all afternoon.

10 Interrogative Sentences

1. (you / to see) ______ ______ the solar eclipse?
2. (she / to pass) ______ ______ her final exam?
3. (they / to move) ______ ______ to a new apartment?
4. (he / to find) ______ ______ his missing wallet?
5. (we / to receive) ______ ______ the package yet?
6. (the dog / to bark) ______ ______ at the mailman?
7. (I / to lock) ______ ______ the car door?
8. (your friends / to arrive) ______ ______ safely?
9. (it / to snow) ______ ______ in the mountains?

10. (the store / to open) _____ _____ on Sunday?

Mixed Sentences (Affirmative, Negative, Interrogative)

1. **Affirmative:** We finally (to decide) _____ on a name for the puppy.
2. **Interrogative:** (you / to decide) _____ _____ what to order yet?
3. **Negative:** No, I (to decide / not) _____ _____. The menu is too big.
4. **Affirmative:** She (to graduate) _____ from university in June.
5. **Interrogative:** (her family / to celebrate) _____ _____ with a party?
6. **Negative:** Her sister (to attend / not) _____ _____ because she was sick.
7. **Affirmative:** They (to plant) _____ new flowers in the garden.
8. **Interrogative:** (the flowers / to grow) _____ _____ quickly?
9. **Negative:** Unfortunately, they (to grow / not) _____ _____ well in the shade.
10. **Affirmative:** I (to hear) _____ a strange noise outside my window.

Maths

Multiples and factors

You can think of the multiples of a number as the 'times table' for that number. For example, the multiples of 3 are $3 \times 1 = \mathbf{3}$, $3 \times 2 = \mathbf{6}$, $3 \times 3 = \mathbf{9}$ and so on.

Multiples

A **multiple** of a number is found when you multiply that number by a positive integer. The first multiple of any number is the number itself (the number multiplied by 1).

Worked example 1

(a) What are the first three multiples of 12?
(b) Is 300 a multiple of 12?

(a)	12, 24, 36	To find these multiply 12 by 1, 2 and then 3. $12 \times 1 = 12$ $12 \times 2 = 24$ $12 \times 3 = 36$
(b)	Yes, 300 is a multiple of 12.	To find out, divide 300 by 12. If it goes exactly, then 300 is a multiple of 12. $300 \div 12 = 25$

Exercise 1.3

1 List the first five multiples of:

(a) 2 (b) 3 (c) 5 (d) 8
(e) 9 (f) 10 (g) 12 (h) 100

2 Use a calculator to find and list the first ten multiples of:

(a) 29 (b) 44 (c) 75 (d) 114
(e) 299 (f) 350 (g) 1012 (h) 9123

3 List:

(a) the multiples of 4 between 29 and 53
(b) the multiples of 50 less than 400
(c) the multiples of 100 between 4000 and 5000.

4 Here are five numbers: 576, 396, 354, 792, 1164. Which of these are multiples of 12?

5 Which of the following numbers are not multiples of 27?

(a) 324 (b) 783 (c) 816 (d) 837 (e) 1116

The lowest common multiple (LCM)

The lowest common multiple of two or more numbers is the smallest number that is a multiple of all the given numbers.

Worked example 2

Find the lowest common multiple of 4 and 7.

M_4 = 4, 8, 12, 16, 20, 24, **28**, 32	List several multiples of 4. (Note: M_4 means multiples of 4.)
M_7 = 7, 14, 21, **28**, 35, 42	List several multiples of 7.
LCM = 28	Find the lowest number that appears in both sets. This is the LCM.

Exercise 1.4

FAST FORWARD

Later in this chapter you will see how prime factors can be used to find LCMs. ▶

1 Find the LCM of:

(a) 2 and 5 (b) 8 and 10 (c) 6 and 4
(d) 3 and 9 (e) 35 and 55 (f) 6 and 11
(g) 2, 4 and 8 (h) 4, 5 and 6 (i) 6, 8 and 9
(j) 1, 3 and 7 (k) 4, 5 and 8 (l) 3, 4 and 18

A **factor** is a number that divides exactly into another number with no remainder. For example, 2 is a factor of 16 because it goes into 16 exactly 8 times. 1 is a factor of every number. The largest factor of any number is the number itself.

F_{12} means the factors of 12.

To list the factors in numerical order go down the left side and then up the right side of the factor pairs. Remember not to repeat factors.

Worked example 3

Find the factors of:

(a) 12 **(b)** 25 **(c)** 110

(a)	$F_{12} = 1, 2, 3, 4, 6, 12$	Find pairs of numbers that multiply to give 12: 1×12 2×6 3×4 Write the factors in numerical order.
(b)	$F_{25} = 1, 5, 25$	1×25 5×5 Do not repeat the 5.
(c)	$F_{110} = 1, 2, 5, 10, 11, 22, 55, 110$	1×110 2×55 5×22 10×11

Exercise 1.5

1 List all the factors of:

(a) 4 **(b)** 5 **(c)** 8 **(d)** 11 **(e)** 18
(f) 12 **(g)** 35 **(h)** 40 **(i)** 57 **(j)** 90
(k) 100 **(l)** 132 **(m)** 160 **(n)** 153 **(o)** 360

2 Which number in each set is not a factor of the given number?

(a) 14 {1, 2, 4, 7, 14}
(b) 15 {1, 3, 5, 15, 45}
(c) 21 {1, 3, 7, 14, 21}
(d) 33 {1, 3, 11, 22, 33}
(e) 42 {3, 6, 7, 8, 14}

FAST FORWARD

Later in this chapter you will learn more about divisibility tests and how to use these to decide whether or not one number is a factor of another.

3 State true or false in each case.

(a) 3 is a factor of 313 **(b)** 9 is a factor of 99
(c) 3 is a factor of 300 **(d)** 2 is a factor of 300
(e) 2 is a factor of 122 488 **(f)** 12 is a factor of 60
(g) 210 is a factor of 210 **(h)** 8 is a factor of 420

4 What is the smallest factor and the largest factor of any number?

The highest common factor (HCF)

The highest common factor of two or more numbers is the highest number that is a factor of all the given numbers.

Worked example 4

Find the HCF of 8 and 24.

F_8 = <u>1</u>, <u>2</u>, <u>4</u>, <u>8</u>	List the factors of each number.
F_{24} = <u>1</u>, <u>2</u>, 3, <u>4</u>, 6, <u>8</u>, 12, 24	Underline factors that appear in both sets.
HCF = 8	Pick out the highest underlined factor (HCF).

Exercise 1.6

FAST FORWARD

You will learn how to find HCFs by using prime factors later in the chapter.

1 Find the HCF of each pair of numbers.

(a) 3 and 6 **(b)** 24 and 16 **(c)** 15 and 40 **(d)** 42 and 70

(e) 32 and 36 **(f)** 26 and 36 **(g)** 22 and 44 **(h)** 42 and 48

2 Find the HCF of each group of numbers.

(a) 3, 9 and 15 **(b)** 36, 63 and 84 **(c)** 22, 33 and 121

3 Not including the factor provided, find two numbers that have:

(a) an HCF of 2 **(b)** an HCF of 6

4 What is the HCF of two different prime numbers? Give a reason for your answer.

Living maths

Word problems involving HCF usually involve splitting things into smaller pieces or arranging things in equal groups or rows.

5 Simeon has two lengths of rope. One piece is 72 metres long and the other is 90 metres long. He wants to cut both lengths of rope into the longest pieces of equal length possible. How long should the pieces be?

6 Ms Sanchez has 40 canvases and 100 tubes of paint to give to the students in her art group. What is the largest number of students she can have if she gives each student an equal number of canvasses and an equal number of tubes of paint?

7 Indira has 300 blue beads, 750 red beads and 900 silver beads. She threads these beads to make wire bracelets. Each bracelet must have the same number and colour of beads. What is the maximum number of bracelets she can make with these beads?