Bayes' formula:

So we take $A_1, \ldots A_K$ be a complet system of event.

(we mean a complet system it must meet two conditionnal.

$$\begin{cases} ① \; A_i \cap A_j = \emptyset \\ ② \; \bigcup_{i=1}^{K} A_i = \Omega \end{cases}$$
)

and let B be an event with $P(B) > 0$.

So according to the conditionnal Proba.

$$\boxed{P(A_j/B) = \frac{P(A_j \cap B)}{P(B)}} \quad (*)$$

$$P(B/A_j) = \frac{P(B \cap A_j)}{P(A_j)}$$

$$\Rightarrow P(B/A_j) \cdot P(A_j) = P(B \cap A_j) = P(A_j \cap B)$$

so we replace in $(*)$

$$\boxed{P(A_j/B) = \frac{P(B/A_j) \cdot P(A_j)}{P(B)}}$$

this one way of writing the formula.

according to law total probability.

$$P(A_j/B) = \frac{P(B/A_j) \cdot P(A_j)}{\sum_{i=1}^{k} P(B/A_i) \cdot P(A_i)}$$

generalization of Bayes' formula.

Exp :

* you have a couple and they got two different children and you're asserted that at least on of those children is going to be a girl.

What's the probability of two girls given at least one girl ??

* Each child can be
B : Boy
G : Girl.

So all possible combinations is are 4 cases.

1. G G ← 1/4
2. G B ← 1/4
3. B G ← 1/4
4. B B ← 1/4.

Total valid cases = 4

So Each case has a probability 1/4

So according to Bayes' formula.

$$P(2G/1G) = \frac{P(1G/2G) \cdot P(2G)}{P(1G)}$$

$$= \frac{1 \times 1/4}{3/4} = \boxed{\frac{1}{3}}$$