Theorem of Total Probability

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Theorem: Law of Total Probability

Let $A_1, A_2, \ldots, A_k$ be a complete system of events, i.e.,

$$A_i \cap A_j = \varnothing \quad \text{for } i \neq j, \qquad \bigcup_{i=1}^{k} A_i = \Omega.$$

Then, for any event $B \subseteq \Omega$, we have:

$$P(B) = \sum_{j=1}^{k} P(B \cap A_j) = \sum_{j=1}^{k} P(A_j)\, P(B \mid A_j).$$

This is the **total probability formula**.

Example 1

Let us assume that a student lives in Ouisslan and wants to attend a Master's class. He can use three types of transportation: A, B, and C, where:

- A means coming on foot,
- B means coming by bus,
- C means using an electric scooter.

The probabilities of choosing each type of transportation are:

$$P(A) = 0.3, \quad P(B) = 0.4, \quad P(C) = 0.3$$

We want to find the probability that the student arrives late to the Master's class. We denote this event by E: "Arrives late".
Giving that the probability that he arrives late when using each mode is:

$$P(E \mid A) = 0.15, \quad P(E \mid B) = 0.10, \quad P(E \mid C) = 0.05$$

Based on the **law of total probability**, we have:

$$\begin{aligned} P(E) &= P(A)\, P(E \mid A) + P(B)\, P(E \mid B) + P(C)\, P(E \mid C) \\ &= 0.3 \times 0.15 + 0.4 \times 0.10 + 0.3 \times 0.05 \\ &= 0.10 \end{aligned}$$

I would like to propose an additional example in order to enhance our understanding of the theoreme.

Example 2

When playing an electronic game, the machine chooses among the levels N1,N2,N3 with respective probabilities 0.6, 0.3 and 0.1 we consider the event G:"Winning a game". The probability of winning at levels N1,N2,N3 is 0.9, 0.7 and 0.2 respectively. Determine the probability of winnig a game.

Good luck!