General Physics 1

for Sciences and Engineering Faculties

Chapter 4 - Static Equilibrium and Elasticity

Static Equilibrium

Equilibrium implies that the object moves with both constant velocity and constant angular velocity relative to an observer in an inertial reference frame.

Will deal now with the special case in which both of these velocities are equal to zero

- This is called *static equilibrium.*

Static equilibrium is a common situation in engineering.

The principles involved are of particular interest to civil engineers, architects, and mechanical engineers.

Rigid Object in Equilibrium

In *the particle in equilibrium model* a particle moves with constant velocity because the net force acting on it is zero.

- The objects often cannot be modeled as particles.

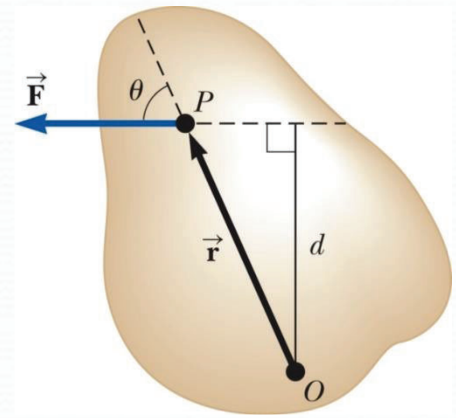
For an extended object to be in equilibrium, a second condition of equilibrium must be satisfied.

- This second condition involves the rotational motion of the extended object.

Torque

$$\vec{\tau} = \vec{\mathbf{F}} \times \vec{\mathbf{r}}$$

- The tendency of the force to cause a rotation about O depends on F and the moment arm *d.* The net torque on a rigid object causes it to undergo an angular acceleration.



The net external force on the object must equal zero.

- $\sum \vec{\mathbf{F}}_{ext} = 0$
- If the object is modeled as a particle, then this is the only condition that must be satisfied.

The net external torque on the object about any axis must be zero.

- $\sum \vec{\tau}_{ext} = 0$
- This is needed if the object cannot be modeled as a particle.

These conditions describe the **rigid object in equilibrium analysis model.**

We will restrict the applications to situations in which all the forces lie in the xy plane. There are three resulting equations:

- $\Sigma F_x = 0$, $\Sigma F_y = 0$
- $\Sigma \tau_z = 0$

Center of Mass

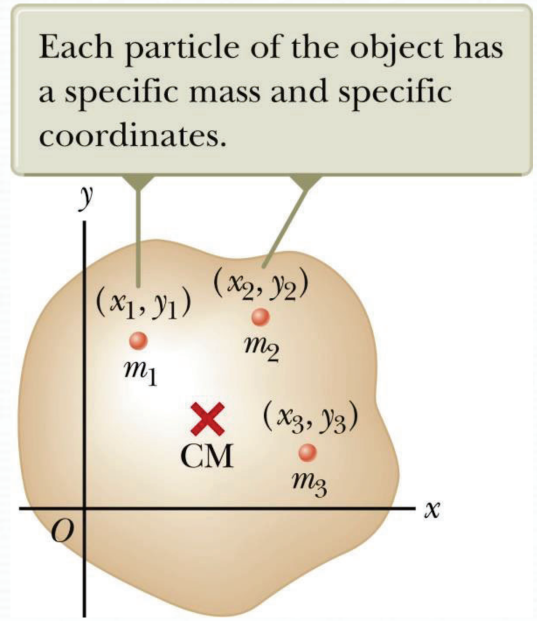
An object can be divided into many small particles.

- Each particle will have a specific mass and specific coordinates.

The x coordinate of the center of mass will be

$$x_{CM} = \frac{\sum_i m_i x_i}{\sum_i m_i}$$

Similar expressions can be found for the y and z coordinates.

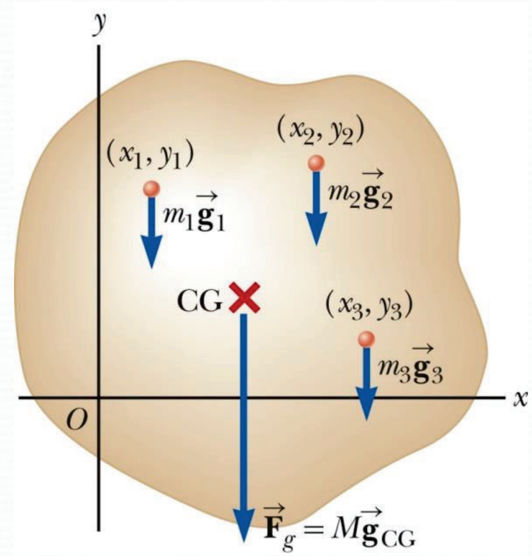


Center of Gravity

All the various gravitational forces acting on all the various mass elements are equivalent to a single gravitational force acting through a single point called the center of gravity (CG).

Each particle contributes a torque about an axis through the origin equal in magnitude to the particle's weight multiplied by its moment arm.

The center of gravity of the object coincides with its center of mass.



Problem-Solving Strategy – Equilibrium Problems

Conceptualize

- Identify all the forces acting on the object.
- Image the effect of each force on the rotation of the object if it were the only force acting on the object.

Categorize

- Confirm the object is a rigid object in equilibrium.
- The object must have zero translational acceleration and zero angular acceleration.

Analyze

- Draw a diagram.
- Show and label all external forces acting on the object.
- Particle under a net force model: he object on which the forces act can be represented in a free body diagram as a dot because it does not matter where on the object the forces are applied.
- Rigid object in equilibrium model: Cannot use a dot to represent the object because the location where the forces act is important in the calculations.

Problem-Solving Strategy – Equilibrium Problems, 2

Analyze, cont

- Establish a convenient coordinate system.
- Find the components of the forces along the two axes.
- Apply the first condition for equilibrium ($\Sigma F=0$). Be careful of signs.
- Choose a convenient axis for calculating the net torque on the rigid object.
- Choose an axis that simplifies the calculations as much as possible.
- Apply the second condition for equilibrium.
- The two conditions of equilibrium will give a system of equations.
- Solve the equations simultaneously.

Finalize

- Make sure your results are consistent with your diagram.
- If the solution gives a negative for a force, it is in the opposite direction to what you drew in the diagram.
- Check your results to confirm $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma \tau = 0$.

Center of Gravity of Humans

Another technique used to determine the center of gravity of humans is described in the figure below.

A board of length l is supported at its ends resting on scales adjusted to read zero with the board alone.

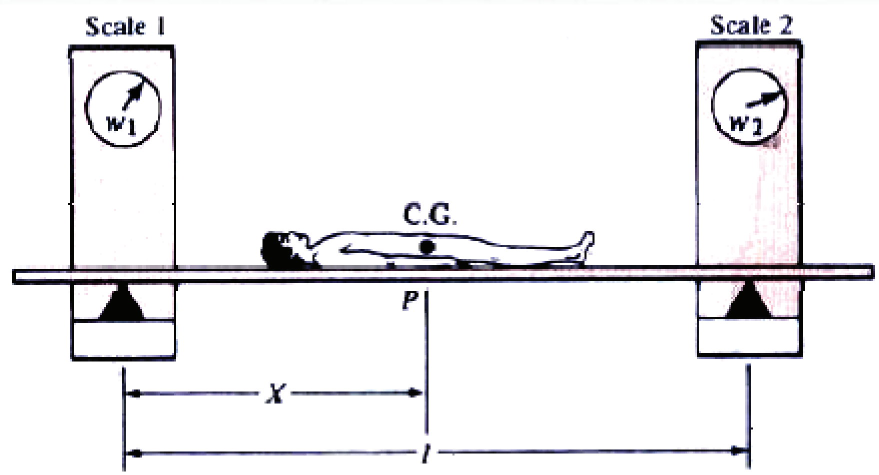
When a person lies on the board the scales read w1 and w2.

The condition for the torque $\Sigma\tau = 0$ can be used to Find X.

□ The torque about point P is

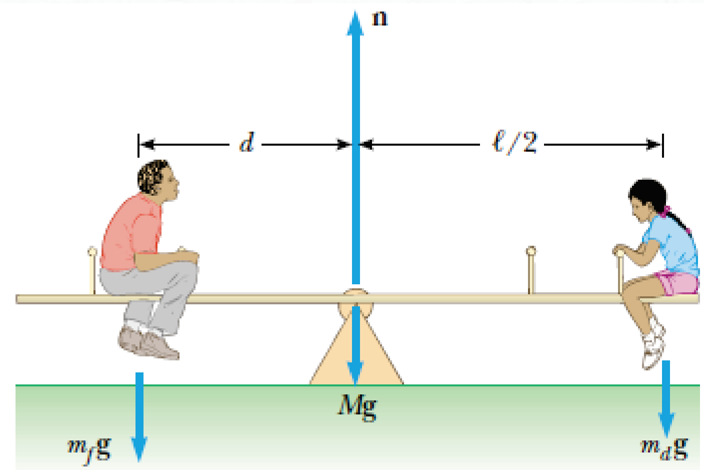
$$\boxed{Xw_1 - (l - X)w_2 = 0}$$

$$\boxed{X = \frac{lw_2}{w_1 + w_2}}$$



The Seesaw Revisited Example

A seesaw consisting of a uniform board of mass ***M*** and length **L** supports a father and daughter with masses m_f and m_d, respectively, as shown in Figure. The support (called the *fulcrum*) is under the center of gravity of the board, the father is a distance ***d*** from the center, and the daughter is a distance **L/2** from the center.



(A) Determine the magnitude of the upward force **n** exerted by the support on the board.

- $\Sigma F_y = 0$

$$n - m_f g - m_d g - Mg = 0$$

$$n = \boxed{m_f g + m_d g + Mg}$$

- $\Sigma F_x = 0$ The equation also applies, but we do not need to consider it because no forces act horizontally on the board.)

(B) Determine where the father should sit to balance the system.

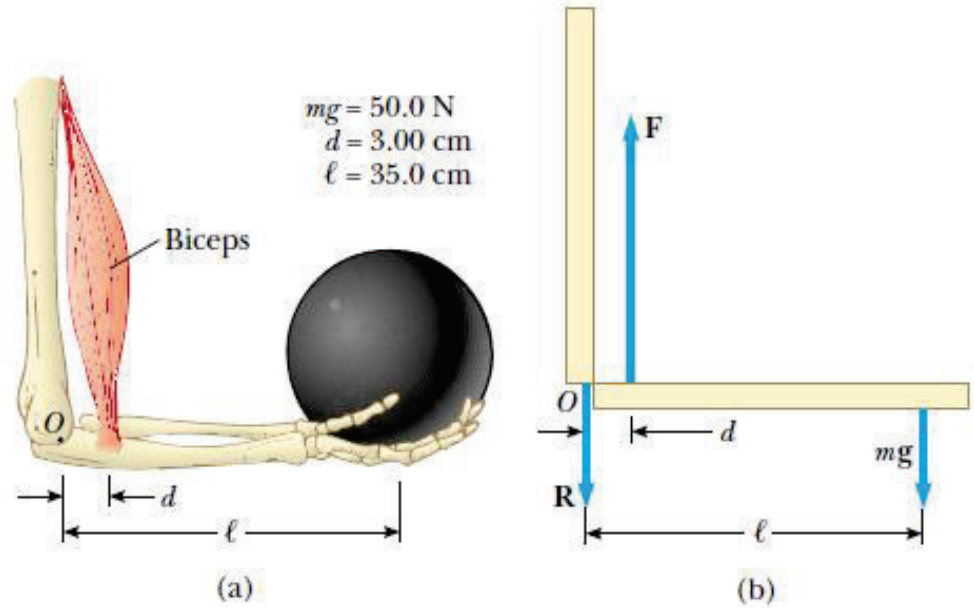
- $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma \tau_z = 0$ that

$$(m_f g)(d) - (m_d g)\frac{\ell}{2} = 0$$

$$d = \boxed{\left(\frac{m_d}{m_f}\right)\frac{1}{2}\ell}$$

Example **A Weighted Hand**

F is the upward force exerted by the biceps and R is the downward forc exerted by the upper arm at the joint.



$$\sum F_y = F - R - 50.0\ \text{N} = 0$$

$$\sum \tau = Fd - mg\ell = 0$$

$$F(3.00\ \text{cm}) - (50.0\ \text{N})(35.0\ \text{cm}) = 0$$

$$F = \boxed{583\ \text{N}}$$

This value for *F* can be substituted into to give *R* =533 N. As this example shows, the forces at joints and in muscles can be extremely large.

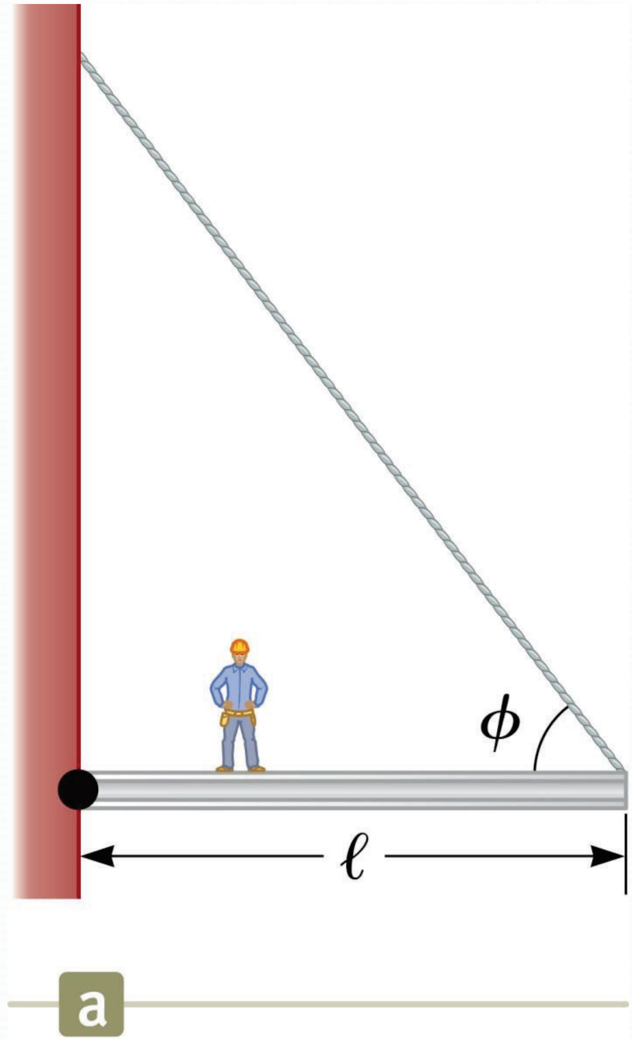
Horizontal Beam Example

Conceptualize

- The beam is uniform.
- So the center of gravity is at the geometric center of the beam.
- The person is standing on the beam.
- What are the tension in the cable and the force exerted by the wall on the beam?

Categorize

- The system is at rest, categorize as a rigid object in equilibrium.



Horizontal Beam Example, 2

Analyze

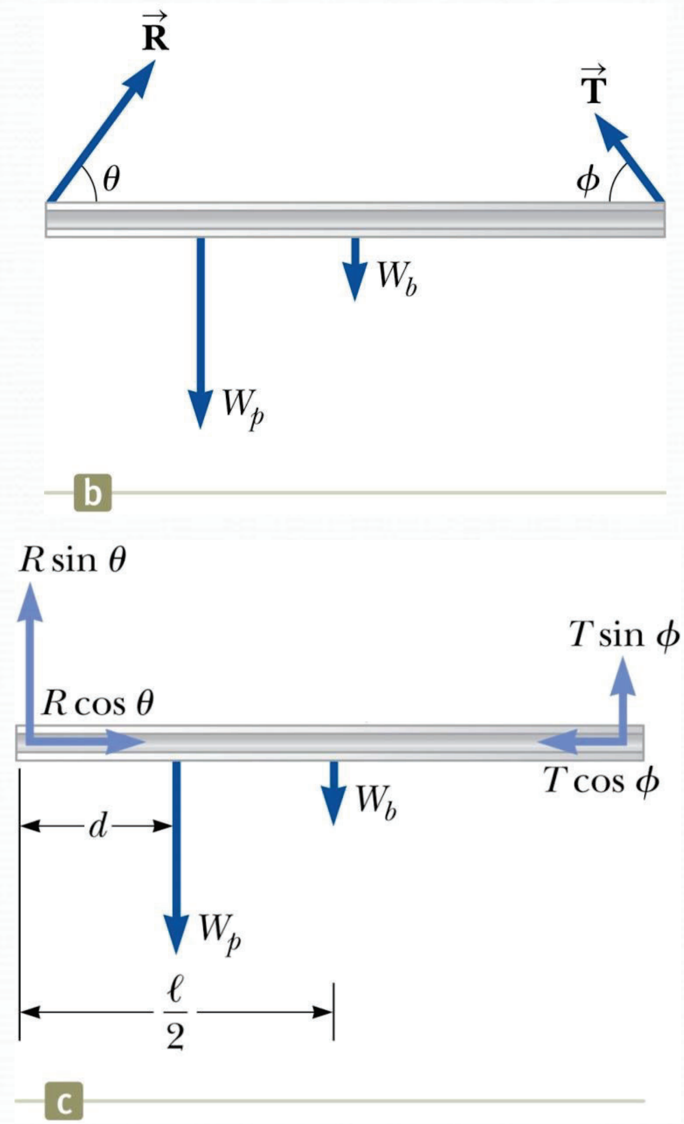
- Draw a force diagram.
- Use the pivot in the problem (at the wall) as the pivot.
    - This will generally be easiest.
- Note there are three unknowns (T, R, θ).

Analyze, cont.

- The forces can be resolved into components.
- Apply the two conditions of equilibrium to obtain three equations.
- Solve for the unknowns.

Finalize

- The positive value for θ indicates the direction of R was correct in the diagram.



Ladder Example

Conceptualize

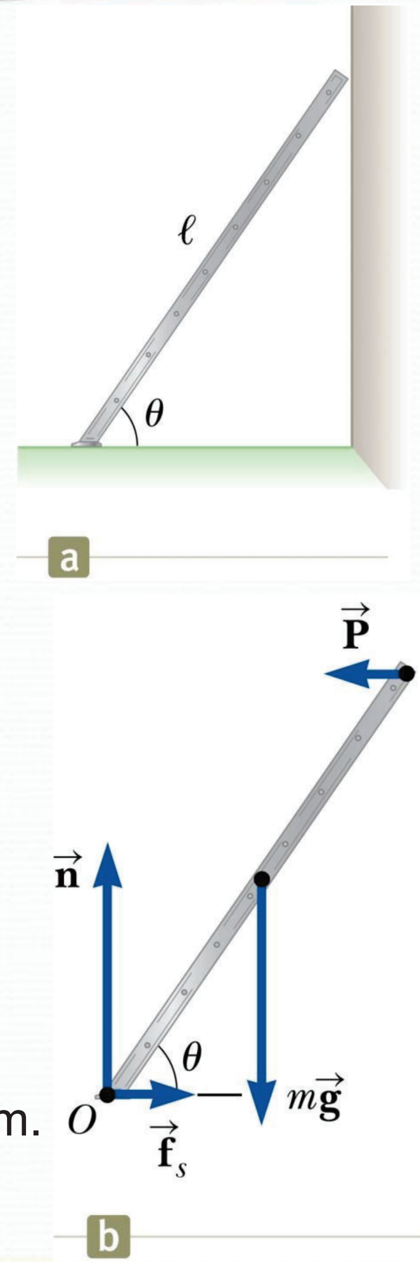
- The ladder is uniform.
- So the weight of the ladder acts through its geometric center (its center of gravity).
- There is static friction between the ladder and the ground.

Categorize

- Model the object as a rigid object in equilibrium.

Analyze

- Draw a diagram showing all the forces acting on the ladder.
- The frictional force is $f_s = \mu_s n$.
- Let O be the axis of rotation.
- Apply the equations for the two conditions of equilibrium.
- Solve the equations.



Elasticity

So far we have assumed that objects remain rigid when external forces act on them.

- Except springs

Actually, all objects are deformable to some extent.

- It is possible to change the size and/or shape of the object by applying external forces.

Internal forces resist the deformation.

Stress

- Is proportional to the force causing the deformation
- It is the external force acting on the object per unit cross-sectional area.

Strain

- Is the result of a stress
- Is a measure of the degree of deformation

Elastic Modulus

The elastic modulus is the constant of proportionality between the stress and the strain.

- For sufficiently small stresses, the stress is directly proportional to the stress.
- It depends on the material being deformed.
- It also depends on the nature of the deformation.

The elastic modulus, in general, relates what is done to a solid object to how that object responds.

$$elastic\ \mathrm{mod}ulus \equiv \frac{stress}{strain}$$

Various types of deformation have unique elastic moduli.

Young's Modulus: Measures the resistance of a solid to a change in its length

Shear Modulus: Measures the resistance of motion of the planes within a solid parallel to each other

Bulk Modulus: Measures the resistance of solids or liquids to changes in their volume

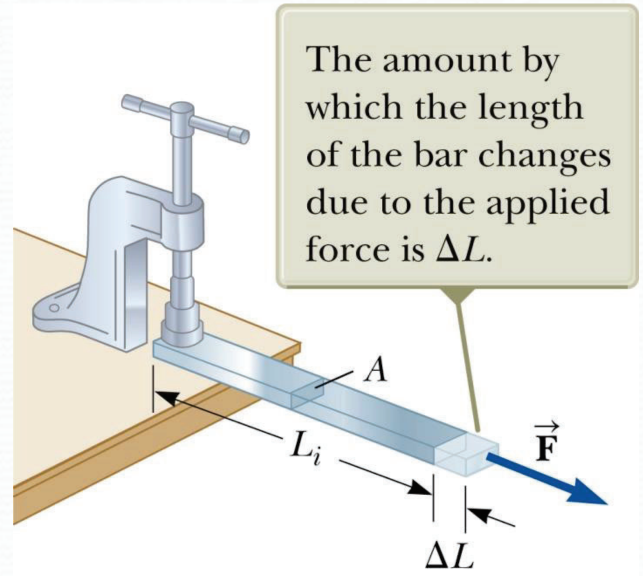
Young's Modulus

The bar is stretched by an amount ΔL under the action of the force F.

The **tensile stress** is the ratio of the magnitude of the external force to the cross-sectional area A.

The **tension strain** is the ratio of the change in length to the original length.

Young's modulus, Y, is the ratio of those two ratios:



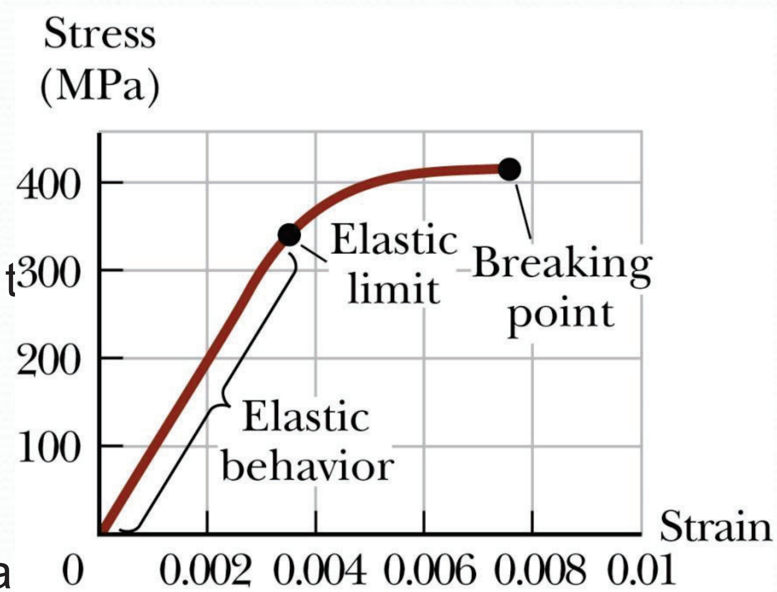
$$Y \equiv \frac{\text{tensile stress}}{\text{tensile strain}} = \frac{F/A}{\Delta L/L_i}$$

Units are N / m^2

Experiments show that for certain stresses, the stress is directly proportional to the strain. This is t elastic behavior part of the curve.

When the stress exceeds **the elastic limit**, the substance will be permanently deformed.

With additional stress, the material ultimately brea



Shear Modulus

Another type of deformation occurs when a force acts parallel to one of its faces while the opposite face is held fixed by another force.

This is called a ***shear stress.***

For small deformations, no change in volume occurs with this deformation.

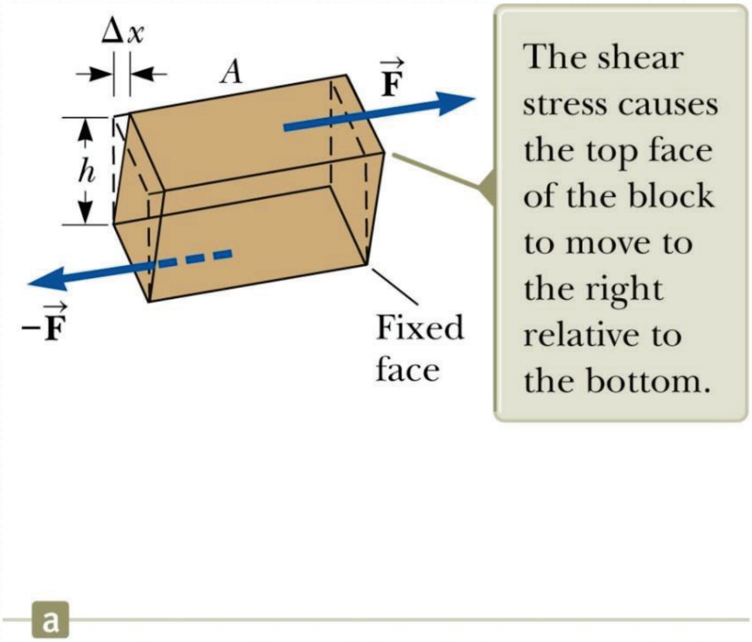
- A good first approximation

The shear strain is Δx / h.

- Δx is the horizontal distance the sheared face moves.
- h is the height of the object.

The shear stress is F / A.

- F is the tangential force.
- A is the area of the face being sheared.



The shear modulus is the ratio of the shear stress to the shear strain.

$$S = \frac{\text{shear stress}}{\text{shear strain}} = \frac{F/A}{\Delta x/h}$$

Units are N / m^2

Example

A 200-kg load is hung on a wire having a length of 4.00 m, cross-sectional area $0.200 \times 10^{-4} m^2$, and Young's modulus $8.00 \times 10^{10} N/m^2$. What is its increase in length?

Example

Assume that Young's modulus for bone is 1.5x10^10 N/m2 and that a bone will fracture if more than 1.5x10^8 N/m2 is exerted. (a) What is the maximum force that can be exerted on the femur bone in the leg if it has a minimum effective diameter of 2.50 cm? (b) If a force of this magnitude is applied compressively, by how much does the 25.0-cm-long bone shorten?

Example

A man leg can be thought of as a shaft of bone 1.2 m long. If the strain is 1.3×10^{-4} when the leg supports his weight, by how much is his leg shortened?

Example

What is the spring constant of human femur under compression of average cross-sectional area 10-3 m2 and length 0.4 m?

Young's modulus for the bone is 1.5 × 1010 N⁄m2

$$\boxed{k = \frac{YA}{l_o}}$$

$$\boxed{k = \frac{YA}{l_o} = 3.75\times10^{6}\,N/m}$$

Example

The average cross-sectional area of a woman femur is $10^{-3}m^{2}$ and it is 0.4 m long.

The woman weights 750 N (a) what is the length change of this bone when it supports half of the weight of the woman? (b) Assuming the stress-strain relationship is linear until fracture, what is the change in length just prior to fracture?

Example: Shear stress on the spine

Between each pair of vertebrae of the spine is a disc of cartilage of thickness 0.5 cm. Assume the disc has a radius of 0.04 m. The shear modulus of cartilage is 1 10^7 N=m^2. A shear force of 10 N is applied to one end of the disc while the other end is held fixed. (a) What is the resulting shear strain? (b) How far has one end of the disc moved with respect to the other end?

Solution: (a) The shear strain is caused by the shear force,

$$\begin{aligned}
\text{strain} &= \frac{F}{AS} \\
\text{strain} &= \frac{10\ \mathrm{N}}{\pi(0.04\ \mathrm{m})^2(1\times 10^7\ \mathrm{N/m^2})} \\
\text{strain} &= 1.99\times 10^{-4}.
\end{aligned}$$

(b) A shear strain is dened as the displacement over the height,

$$\begin{aligned}
\text{strain} &= \frac{\Delta x}{h} \\
\Delta x &= h \times \text{strain} \\
\Delta x &= (0.5\ \mathrm{cm})(1.99\times 10^{-4}) \\
\Delta x &= 0.99\ \mu\mathrm{m}.
\end{aligned}$$

Bulk Modulus

Another type of deformation occurs when a force of uniform magnitude is applied perpendicularly over the entire surface of the object.

The object will undergo a change in volume, but not in shape.

The volume stress is defined as the ratio of the magnitude of the total force, F, exerted on the surface to the area, A, of the surface.

- This is also called the ***pressure.***

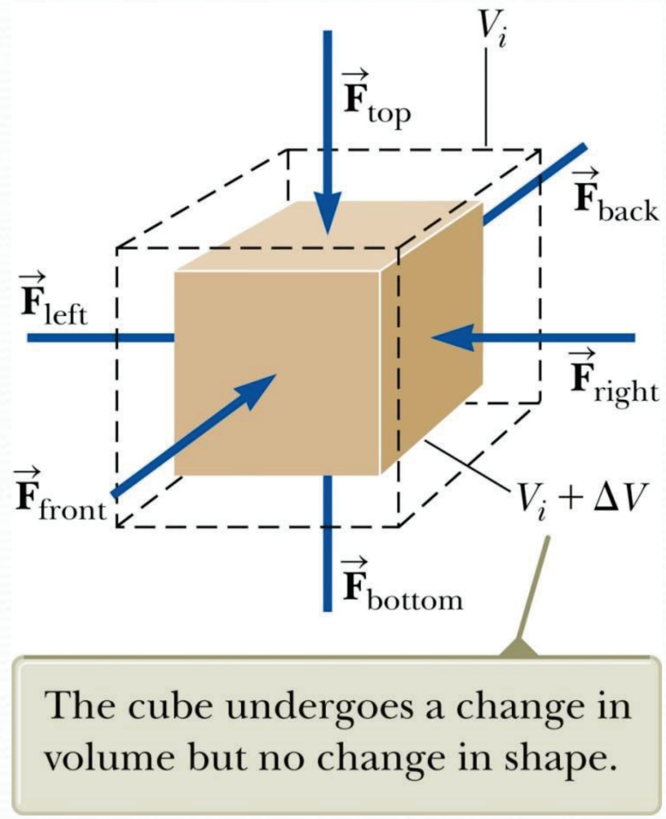
The volume strain is the ratio of the change in volume to the original volume.

The bulk modulus is the ratio of the volume stress to the volume strain.

$$B = \frac{\text{volume stress}}{\text{volume strain}} = -\frac{\Delta F / A}{\Delta V / V_i} = -\frac{\Delta P}{\Delta V / V_i}$$

The negative indicates that an increase in pressure will result in a decrease in volume.

The compressibility is the inverse of the bulk modulus.



Moduli and Types of Materials

Both solids and liquids have a bulk modulus.

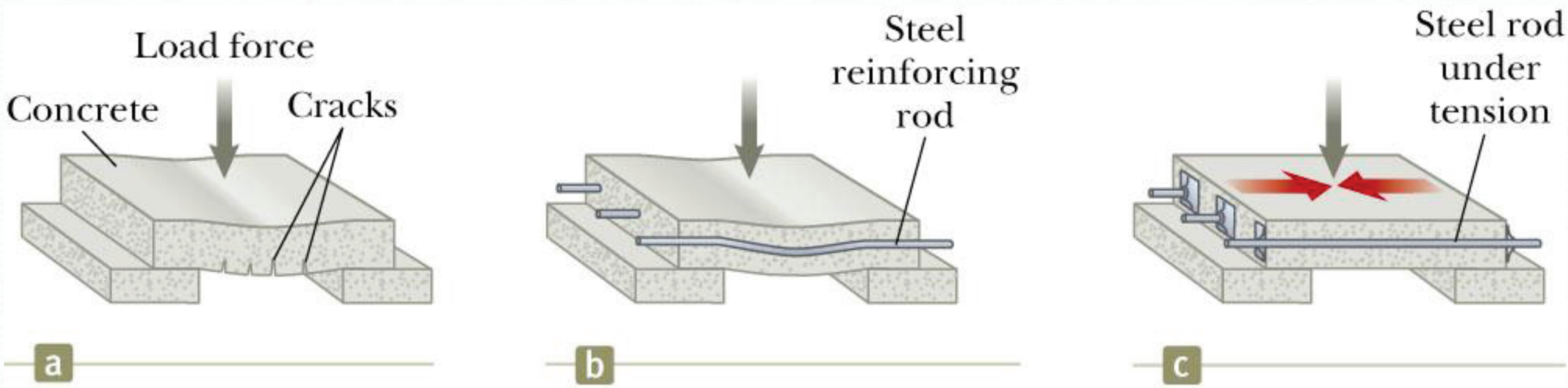
Liquids cannot sustain a shearing stress or a tensile stress.

- If a shearing force or a tensile force is applied to a liquid, the liquid will flow in response.

TABLE 12.1 *Typical Values for Elastic Moduli*

Substance	Young's Modulus (N/m^2)	Shear Modulus (N/m^2)	Bulk Modulus (N/m^2)
Tungsten	35×10^{10}	14×10^{10}	20×10^{10}
Steel	20×10^{10}	8.4×10^{10}	6×10^{10}
Copper	11×10^{10}	4.2×10^{10}	14×10^{10}
Brass	9.1×10^{10}	3.5×10^{10}	6.1×10^{10}
Aluminum	7.0×10^{10}	2.5×10^{10}	7.0×10^{10}
Glass	$6.5–7.8 \times 10^{10}$	$2.6–3.2 \times 10^{10}$	$5.0–5.5 \times 10^{10}$
Quartz	5.6×10^{10}	2.6×10^{10}	2.7×10^{10}
Water	—	—	0.21×10^{10}
Mercury	—	—	2.8×10^{10}

Prestressed Concrete



If the stress on a solid object exceeds a certain value, the object fractures.

Concrete is normally very brittle when it is cast in thin sections.

- The slab tends to sag and crack at unsupported areas.

The slab can be strengthened by the use of steel rods to reinforce the concrete.

The concrete is stronger under compression than under tension.

Pre-stressed Concrete, cont.

A significant increase in shear strength is achieved if the reinforced concrete is pre-stressed.

As the concrete is being poured, the steel rods are held under tension by external forces.

These external forces are released after the concrete cures.

This results in a permanent tension in the steel and hence a compressive stress on the concrete.

This permits the concrete to support a much heavier load.