

Calculation of uncertainty in a result

Whenever we make a measurement, there is always some uncertainty in the result. This is simply because no measuring instrument has perfect precision.

Whenever we state a measurement, the number of quoted decimal places and the uncertainty should give clear evidence for the precision of the measurement. Note that some students refer to the 'error' in the measurement. An uncertainty is not an error.

In many situations, in order to obtain the value of a physical quantity, several other quantities are measured. Each of these measured quantities has an uncertainty and these uncertainties must be combined in order to determine the uncertainty in the value of the physical quantity. There are some basic rules for determining the uncertainty when measurements are combined. These are:

- Whenever two measurements are added together, the actual uncertainties are added.
- Whenever two measurements are subtracted, the actual uncertainties are added.
- Whenever two measurements are multiplied together, the percentage uncertainties are added.
- Whenever two measurements are divided, the percentage uncertainties are added.
- The actual uncertainty in the final answer should be quoted to one significant figure.
- The final answer should be quoted to the same decimal place as the actual uncertainty.

The rules are best illustrated by means of a sample calculation.

Suppose that we are determining a quantity Q . The quantity Q is found using the expression

$$Q = \frac{c(a+b)^3}{(e-f)}$$

where $a = 98 \pm 1$ (units)

$b = 15.2 \pm 0.2$ (units)

$c = 0.124 \pm 0.001$ (units)

$e = 14.65 \pm 0.05$ (units)

$f = 12.65 \pm 0.05$ (units)

The quantities a and b are added and so we add the actual uncertainties giving $(a + b) = 113.2 \pm 1.2$ (units).

Note that we have not reduced the uncertainty to one significant figure at this stage because we are at the start of the process, not the end.

$(a + b)$ is cubed which means 'multiply by itself three times'. So we take three times the percentage uncertainty in $(a + b)$ to find the uncertainty in $(a + b)^3$.

Percentage uncertainty in $(a + b)^3$ is $(1.2/113.2) \times 3 \times 100 = 3.2\%$

Percentage uncertainty in c is $(0.001/0.124) \times 100 = 0.81\%$

$(e - f) = 2.0$ and the uncertainty in $(e - f)$ is ± 0.1 (units)

So, percentage uncertainty in $(e - f)$ is $(0.1/2.0) \times 100 = 5.0\%$

Note that although the percentage uncertainty in either e or f is small, when we take the difference, then this increases significantly the percentage uncertainty in the result.

The quantities $(a + b)^3$, c and $(e - f)$ are all multiplied together or divided and so we add the percentage uncertainties in the quantities.

Total percentage uncertainty = 9.0%

Now, $Q = 89935$ (units)

Actual uncertainty in $Q = \pm(9.0 \times 89935)/100 = \pm 8094$ (units)

Therefore, we quote the uncertainty as ± 8000 and the value of Q as $90\,000$ (units). That is, in this case, Q is quoted to the nearest thousand because the uncertainty is to the nearest thousand.

So,

$$Q = (9.0 \pm 0.8) \times 10^4 \text{ (units)}$$

The uncertainty is written to one significant figure and, in this case, this is one decimal place. The value of Q is therefore written to the same number of decimal places.

Significant figures and decimal places

In Physics, numerical answers to calculations play an important role in our understanding not only of basic concepts, but also of the world in which we live. However, a numerical answer can be very misleading unless we take care to express that answer to an appropriate number of decimal places or significant figures, depending on whichever is appropriate.

Decimal places (d.p.)

The number of decimal places in an answer is the number of figures that appear to the right of the decimal point.

For example,

26.4 is expressed to one decimal place

26.43 is expressed to two decimal places

26.430 is expressed to three decimal places

26.4308 is expressed to four decimal places, etc.

The significance of the number of decimal places is that it indicates the least value to which the number is calculated or the quantity is measured. This is explained for the examples above in the table on page 10.

number	number of decimal places	significance
26.4	1	quoted to the nearest tenth
26.43	2	quoted to the nearest hundredth
26.430	3	quoted to the nearest thousandth
26.4308	4	quoted to the nearest ten-thousandth

Significant figures (sig. fig. or s.f.)

The number of significant figures is the number of characters shown in an answer or a quantity.

For example,

6 is quoted to one significant figure

63 is quoted to two significant figures

634 is quoted to three significant figures

6.345 is quoted to four significant figures.

Note that the number of significant figures does not take into account the position of the decimal point.

A problem arises when there are zeros at the end of a number. If the number is 600, then has this number been quoted to one, or two, or three significant figures? This problem is overcome by using index notation.

For example,

6×10^2 is quoted to one significant figure

6.0×10^2 is quoted to two significant figures

6.00×10^2 is quoted to three significant figures

Where a number of zeros are given before a number they do not count as significant figures. The number 0.0063 has two significant figures.

The number of significant figures quoted for the value of a quantity gives an indication of the degree of precision to which that quantity has been determined. The greater the number of significant figures, the greater the precision.

General rules for the use of decimal places and significant figures in examinations

Calculations

Where data is given so that a calculation may be carried out, then the data will be given to a certain number of significant figures. This is usually either two or three.

When giving the final answer, this should be quoted to the same number of significant figures as the data.

A warning! Calculations may involve more than one stage, so that the final answer is obtained from two or more initial calculations. For example, when calculating the speed of a car, then the distance travelled has to be found and also the time taken must to be determined before finally the distance is divided by the time to calculate the speed.

Suppose the data for the calculation is given to two significant figures.

On calculating the values of the distance and the time, they may well not work out to two significant figures and the calculator may give more. If the values for the distance and the time are rounded to two significant figures then it must be remembered that the values are less precise. So, when these less precise values are divided one by the other, the final answer for the speed may well be outside any acceptable limit of accuracy!

As a general rule:

Where a calculation involves a number of parts that lead up to a final answer, then each individual part should retain two significant figures more than is intended for the final answer.

Measurements

The number of significant figures to which a measurement is quoted should reflect the precision of the instrument that is used to make that measurement.

For example, measuring a length:

If the length is quoted as 60 cm, then it can be measured to the nearest 10 cm and we would expect the length to be between 55 cm and 64 cm.

Quoting the length as 64 cm would mean it can be measured to the nearest 1 cm and we would expect the length to be between 63.5 cm and 64.4 cm.

So, using a metre rule with which we can measure to the nearest millimetre, an answer of 64.2 cm would be appropriate.

For an ammeter with scale divisions of 0.2 A then an appropriate value of current would be 4.7 A.

Where data is expressed in a table, then there should be consistency in the number of decimal places for any group of measurements. For example, temperatures of 72.6 °C, 73.2 °C, 73 °C, 73.4 °C and 72.8 °C would not be acceptable because one value has lower precision than the rest.

Graphs

Graphs are an important feature of work in Physics and we frequently take values of coordinates. In general, we can read to the nearest half square of the graph grid. The number of decimal places to which the coordinates are quoted should reflect this precision.

For example, a graph grid has 2 mm squares and a scale is drawn to measure voltage where the number of 2 mm squares between the 2.0 V mark and the 4.0 V mark is 10. Then each 2 mm square represents 0.2 V. We can read to the nearest half square and that represents 0.1 V. So, it would be appropriate to give the coordinate as, for example, 3.7 V. Quoting it as 3.70 V would not be acceptable.

Final answers with their uncertainties

Physics is the science of measurement. Measurements are of little value unless it is possible to determine the reliability of the measurement. That implies the degree of uncertainty in that answer.

In AS Level [Topic 2](#), the way in which the uncertainty in an answer is determined was discussed. It should be remembered that an uncertainty is not an error.

Having calculated the quantity and its uncertainty, then a final answer should be quoted. This is in two stages.

Firstly, the uncertainty should be quoted to one significant figure. If, for example, the uncertainty is found to be ± 0.86 , then the uncertainty should be given as ± 0.9 . There is little point in quoting hundredths when the uncertainty is in tenths!

Secondly, the final answer should be quoted to the same number of decimal places as the uncertainty.

The final answer and its uncertainty should be quoted to the same number of decimal places.

For example, 65.7 ± 0.9 (with its unit) means that the answer is quoted to tenths because the uncertainty is in tenths. There would be no point in quoting the answer to hundredths or thousandths.

Proportionality and linearity

There is sometimes confusion between these two types of relationship. They are not the same and, at this level of your studies, you should be able to distinguish between them.

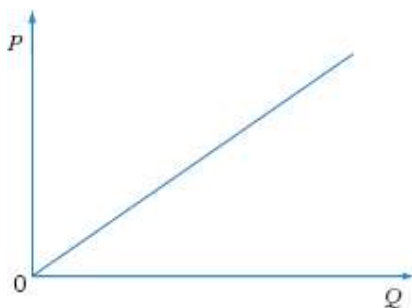
Proportionality

If two quantities P and Q are directly proportional to one another then, if one quantity is doubled, then the other will also be doubled. So, multiplying one quantity by a number, then the other quantity will also be multiplied by that same number.

*Two quantities P and Q are **directly proportional** to each other if they follow the relation $P = kQ$ where k is a constant.*

Hence $\frac{P}{Q} = k$ which is **constant**

Direct proportionality can be represented by the graph below.



We can see that the graph is a straight line. The gradient of the graph line is the constant k in the relation $P = kQ$.

Note that the line must pass through the origin.

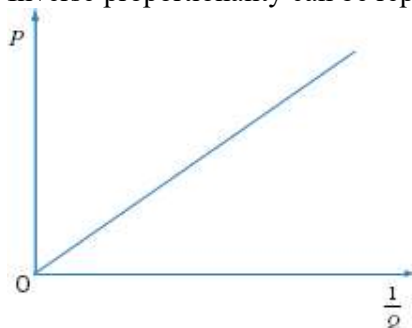
Two quantities may be proportional to each other, but not directly proportional. For example, P may be proportional to the square of Q . That is $P = cQ^2$ where c is a constant.

Proportionality can occur with any power of Q .

Two quantities may be **inversely** proportional to each other. This means, for example, that if doubling one quantity, then the other is halved.

*Two quantities P and Q are **inversely proportional** to each other if they follow the relation $P = \frac{k}{Q}$ where k is a constant.*

Inverse proportionality can be represented by the graph below, where P is plotted against $1/Q$.



The graph is a straight line. The gradient of the graph line is the constant k in the relation $P = \frac{k}{Q}$.

Note that the line must pass through the origin.

Inverse proportionality can occur with any power of Q .

Linearity

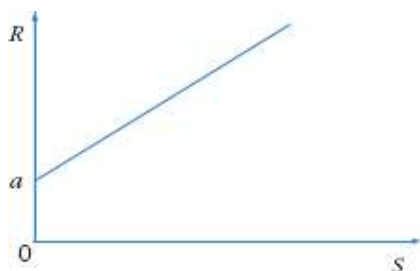
Linearity means that one quantity changes by the same factor as the change in another quantity.

Note the difference between proportionality and linearity.

For proportionality, the ratio of the two quantities that are proportional is always constant.

For linearity, the ratio of the changes in the two quantities is a constant.

Linearity can be represented by the graph below.



We say that R changes linearly with change in S . Note that the line of the graph does not pass through the origin.

The relationship can be represented by the expression

$R = a + bS$, where a and b are constants.

The constant a is the intercept on the R -axis.

If there is a change ΔR in R and the corresponding change in S is ΔS , then the constant b which is the gradient of the graph is given by

$$b = \frac{\Delta R}{\Delta S}$$

Relationships may also be based on powers. For example

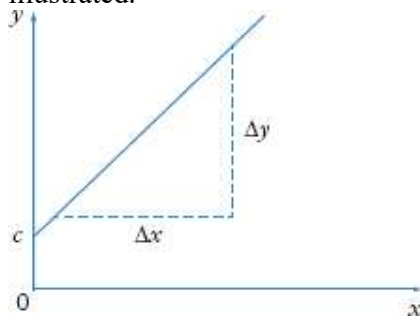
$R = a + bS^n$, where a , b and n are constants. A graph of R against S^n gives a straight line graph with gradient b and an intercept on the R axis of a .

Straight-line graphs

The representation of data in a graphical form is a very important means by which relationships between variables can be determined.

The plotting of data points provides an averaging technique which may well be superior to an arithmetical mean. Where an arithmetical mean is calculated, each set of data has an equal weighting. When using a best-fit line on a graph, the average is weighted towards those data points close to the line. A wayward point (anomalous point) can be detected and allowance made – perhaps taking a new set of measurements.

An important type of graph which is used frequently in AS/A -Level Physics is the straight-line graph, as illustrated.



The equation representing this graph is

$$y = mx + c$$

where m and c are constants.

The constant m is the gradient of the graph, $m = \frac{\Delta y}{\Delta x}$.

The constant c is the intercept on the y -axis.

If a variable y is thought to vary linearly with variable x , then plotting this graph will enable the following:

- the straight line with an intercept of c verifies a **linear** relationship between y and x
- determination of the values of the gradient m and the intercept c enables the exact form of the relationship to be established.

If the intercept is zero, the straight line passes through the origin. The relationship is $y = mx$. This special case with $c = 0$ means that y is **proportional** to x .

Relationships other than linear relationships can also be verified in this way. However, the suspected relationship must be changed into one of the form $y = mx + c$, then use the graph you plot to determine the constants. Some examples of this technique are shown in the table below.

suspected relationship	modified form of equation	y-axis plot	x-axis plot	gradient	intercept on y-axis
$P = aQ^n$	$\lg P = n \lg Q + \lg a$	$\lg P$	$\lg Q$	n	$\lg a$
$A = Be^{-kt}$	$\ln A = \ln B - kt$	$\ln A$	t	$-k$	$\ln B$
$R = eS^2 + fS$	$R/S = eS + f$	R/S	S	e	f