Topic 13 Oscillations

Summary

Simple harmonic motion

- The period of an oscillation is the time taken to complete one oscillation.
- Frequency is the number of oscillations per unit time.
- Frequency f is related to period T by the expression $f = 1/T$
- The displacement of a particle is its distance from the equilibrium position.
- Amplitude is the maximum displacement.
- Simple harmonic motion (s.h.m.) is defined as the motion of a particle about a fixed point such that its acceleration a is proportional to its displacement x from the fixed point, and is directed towards the fixed point, $a \propto -x$ or $a = -\omega^2 x$
- The constant ω in the defining equation for simple harmonic motion is known as the angular frequency.
- For a particle oscillating in s.h.m. with frequency f, then $\omega = 2\pi f$
- Simple harmonic motion is described in terms of displacement x, amplitude x_0, frequency f, angular frequency ω by the following relations.

displacement:	$x = x_0 \sin \omega t$	or	$x = x_0 \cos \omega t$
velocity:	$v = x_0 \omega \cos \omega t$	or	$v = -x_0 \omega \sin \omega t$
acceleration:	$a = -x_0 \omega^2 \sin \omega t$	or	$a = -x_0 \omega^2 \cos \omega t$

- Remember that $\omega = 2\pi f$, and equations may appear in either form.

Energy of oscillations

- The kinetic energy E_k of a particle of mass m oscillating in simple harmonic motion with angular frequency ω and amplitude x_0 is $E_k = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$ where x is the displacement.
- The potential energy E_p of a particle of mass m oscillating in simple harmonic motion with angular frequency ω is $E_p = \frac{1}{2} m\omega^2 x^2$ where x is the displacement.
- The total energy E_{tot} of a particle of mass m oscillating in simple harmonic motion with angular frequency ω and amplitude x_0 is $E_{tot} = \frac{1}{2} m\omega^2 x_0^2$
- For a particle oscillating in simple harmonic motion $E_{tot} = E_k + E_p$ and this expresses the law of conservation of energy.

Free and damped oscillations

- Free oscillations are oscillations where there are no resistive forces acting on the oscillating system.
- Damping is produced by resistive forces which dissipate the energy of the vibrating system.
- Light damping causes the amplitude of vibration of the oscillation to decrease gradually. Critical damping causes the displacement to be reduced to zero in the shortest time possible, without any oscillation of the object. Overdamping also causes an exponential reduction in displacement, but over a greater time than for critical damping.
- The natural frequency of vibration of an object is the frequency at which the object will vibrate when allowed to do so freely.
- Forced oscillations occur when a periodic driving force is applied to a system which is capable of vibration.
- Resonance occurs when the driving frequency on the system is equal to its natural frequency of vibration. The amplitude of vibration is a maximum at the resonant frequency.

Definitions and formulae

- Simple harmonic motion (s.h.m.): motion where the acceleration of a particle is proportional to the displacement from a fixed point and is directed towards the fixed point (equilibrium position).
- $a = -(2\pi f)^2 x = -\omega^2 x$, where ω is the angular frequency = $2\pi f$
- Hence $x = x_o \sin 2\pi ft$ (or $x = x_o \cos 2\pi ft$)
- $T = 1/f = 2\pi/\omega$
- $v = v_o \cos \omega t$ and $v = \pm (x_o^2 + x^2)^{1/2}$
- $E_k = \frac{1}{2} m\omega^2 (x_o^2 - x^2)$
- $E_p = \frac{1}{2} m\omega^2 x^2$
- $E_{total} = E_p + E_k = \frac{1}{2} m\omega^2 x_o^2$
- Resonance: maximum amplitude occurs when driving frequency is equal to the natural frequency.