CAMBRIDGE
INTERNATIONAL EXAMINATIONS

NOVEMBER 2002

GCE Advanced Subsidiary Level

MARK SCHEME

MAXIMUM MARK : 50

SYLLABUS/COMPONENT :9709 /4

MATHEMATICS
(Mechanics 1)

UNIVERSITY of CAMBRIDGE
Local Examinations Syndicate

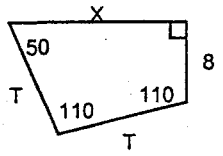
1		Driving force = 20 000/25	B1	
		For using Newton's 2nd law (3 terms needed) [20 000/25 – 600 = 1000a]	M1	
		Acceleration is 0.2ms^{-2}	A1	3

Notes: $\frac{20000-600}{25}=1000a$ scores B0 M1; $\frac{20000}{25}-\frac{600}{25}=1000a$ scores B1 M0

20000 = 25(1000a + 600) scores B1 M1 20 000/25 – 600 = 1000ga scores B1 M0

2	(i)	For 20×7 or 140 and ½ 4×7 or 14	B1	
		Valid argument that $s_1 + s_2 > 154$ (AG)	B1	2
		Alternatively: Approx distance is 20×7 + 4×7 k (where ½ < k < 1) M1 Whose value (shown) is (clearly) > 154 A1		
	(ii)	For using area property with correct signs [140 + 20 – ½ 10x8]	M1	
		Distance is 120m	A1	2

Note: 140 + 20 + 20 – 20 scores M0 in (ii)

3	(i)	For resolving forces on R vertically (3 terms needed)	M1	
		$T\sin50^\circ = T\sin20^\circ + 0.8g$	A1	
		Tension is 18.9 N (18.5 from $g = 9.81$ or $g = 9.8$)	A1	3
	(ii)	For resolving forces on R horizontally	M1	
		$X = T\cos50^\circ + T\cos20^\circ$	A1	
		$X = 29.9$ (8tan75°) (29.3 from g = 9.81 or g = 9.8)	A1ft	3
	Alternatively (by scale drawing): Correct quadrilateral drawn to scale M1  $18.4 \le T \le 19.4$ A2 $29.4 \le X \le 30.4$ A2 $T = 18.9$ and $X = 29.9$ A1			

Notes: F_y = Tsin50 – Tsin20 – 0.8g scores M0 in (i) and F_x = X – Tcos50 – Tcos20 scores M0 in (ii).

Note that sin/cos mix can score M1 A0 A0 M1 A0 A0 at best (this error leads to negative values for T and X).

None of the four A marks can be scored unless and until $T_1 = T_2$ is stated or implied, where T_1 and T_2 are the tensions in the two parts of the string.

Many candidates try to use Lami's theorem. In order to score any marks the candidate needs to reduce the system to one of 3 forces. Two examples of how this might be done, and how it should be marked, are shown below. [The general idea is that M1 is given for a **complete** method for X, A1 for a correct equation in X (only) and A1 for X = 29.9, and similarly for T.]

For example reducing the system to 3 forces of magnitudes 2Tcos35, X and 8, attempting to find the angles 105 and 165 and applying Lami M1
X/sin105 = 8/sin165 A1
X = 29.9 A1
Applying Lami to find T M1
2Tcos35/sin90 = 8/sin165 A1
T = 18.9 A1

Reducing the system to 3 forces of magnitudes T, T and $\sqrt{8^2 + X^2}$ and attempting to find the angles 70, 145 and 145 and applying Lami M1
$\sqrt{8^2 + X^2}$ sin 15 = 8 A1
X = 29.9 A1
Applying Lami to find T M1
T/sin 145 = $\sqrt{8^2 + 29.9^2}$ /sin 70 A1
T = 18.9 A1

4	(i)	For using $v = u - gt$, with $v = 0$, to find t [$5 - 10t = 0$]	M1	
		Time to maximum height of A is $5/g$	A1	
		For using $h = ut - \frac{1}{2} gt^2$ and evaluating $h_B(0.5) - h_A(0.5)$	M1	
		Difference in heights is 1.5m (1.53 from g = 9.81 or g = 9.8)	A1	4
	SR For difference in maximum heights (max 1 out of 4) 1.95 m (1.99 from g = 9.81 or 9.8) B1			
	(ii)	For attempting to solve $h_B - h_A = 0.9$ for t [$8t - 5t = 0.9$]	M1	
		$t = 0.3$	A1	
		For using $h = ut - \frac{1}{2} gt^2$ with the value of t found [$h = 5 \times 0.3 - \frac{1}{2} 10 \times 0.09$]	M1	
		Height of A is 1.05 m (1.06 from g = 9.81 or g = 9.8)	A1	4

Notes: Using a = +g in v = u + at scores M0 at the first stage in (i) and using a = +g in $s = ut + \frac{1}{2} at^2$ scores M0 at the second stage in (i) (notwithstanding the resultant 'correct' answer).

Allow error in sign of the terms $\frac{1}{2} gt^2$ in expressions for h_B and h_A for both the first M1 and the first A1 in (ii).

Using a = +g in $s = ut + \frac{1}{2} at^2$ scores M0 at the second stage in (ii).

Note that $5^2 = 8^2 - 2g(s + 0.9)$ leads entirely fortuitously to the 'correct' answer 1.05 in (ii) (but this doesn't apply when g is taken as 9.8). This solution scores 0 out of 4 in (ii).

5	(i)	$R = 15 \times 10 \times \cos 35^\circ = 123$ (AG)	B1	1
	(ii)	For resolving forces along the plane (either case)	M1	
		$150\sin 35^\circ = X + F$ and $150\sin 35^\circ = 5X - F$	A1	
		For eliminating F or X	M1	
		$X = 28.7$ (ft from wrong F or wrong positive μ) (28.1 from g = 9.81 or g = 9.8)	A1ft	
		F or $\mu R = 10g\sin 35^\circ$ or equivalent (may be implied) (57.36)	A1	
		For using $F = \mu R$ [$57.36 = \mu\, 122.9$ or $100 \sin 35^\circ = \mu\, 150 \cos 35^\circ$]	M1	
		Coefficient of friction is 0.467 (ft for positive value from wrong X) [(2/3)tan 35°]	A1ft	7
	SR for the case where a candidate does not use F explicitly and uses $F \le \mu R$ (and not $F = \mu R$) implicitly (max 4 out of 7) For resolving forces along the plane (either case) M1 $150\sin 35^\circ - X \le \mu R$ and $5X - 150\sin 35^\circ \le \mu R$ A1 For eliminating X (it is not possible to eliminate μR) M1 $\mu R \ge 100\sin 35^\circ$ or equivalent A1			

Notes: Do not allow answers from $g = 9.81$ or $g = 9.8$ in (i).
Accept any answer which rounds to 123 in (i).
Accept sin instead of cos for first M1 in (ii).
$F_1 = 150\sin 35^\circ - X - F$ and $F_2 = 5X - F - 150\sin 35^\circ$ scores M0 in (ii).
$150\sin 35^\circ - X - F = 15a$ and $5X - F - 150\sin 35^\circ = 15a$ → $300\sin 35^\circ - 6X = 0$ → $X = 28.7$ scores M1 M1 in (ii), but none of the three A marks unless and until a is set equal to zero.
If F is taken in the wrong direction the candidate can score M1 A0 M1 A1 (not fortuitous) A0 M1 A0 in (ii).
Allow $\mu = 0.466$ (however the inaccuracy arises) - this is because it would be harsh to regard 57.36/123, which equals 0.466, as p.a., since 123 is a printed answer.
The value of g should not affect the value of μ, but allow 0.457 or 0.458 from a mix (56.27/123 or 56.21/123) because 123 is a printed answer.

6	(i)(a)	For using $F = \mu mg \cos a$ $[0.25 \times 1.2g\cos 25^\circ]$	M1	
		Frictional component is 2.72 N (2.719) (2.67 from g = 9.81 or 2.66 from g = 9.8)	A1	2
	SR for the candidate who uses $F \leq \mu R$ instead of $F = \mu R$ (max 1 out of 2) $F \leq 2.72$ B1			
	(b)	For using Newton's 2nd law (3 terms needed) $[1.2g\sin 25^\circ - 2.719 = 1.2a]$	M1	
		Acceleration is 1.96 ms^{-2} (1.92 from $g = 9.81$ or 9.8) (ft for positive value of a from incorrect F).	A1ft	2
	(c)	For using $v^2 = 2as$ $[v^2 = 2x1.96x4]$	M1	
		Speed is 3.96ms^{-1} (3.92 from $g = 9.81$ or 9.8) (ft for 8.00 (accept 8.0 or 8) following a sin/cos mix)	A1ft	2
	(ii)(a)	PE Loss is 36J (35.3 from $g = 9.81$ or 9.8)	B1	1
	(b)	For using PE loss = KE gain from bottom of slope, or PE loss = KE gain WD against friction from top of slope, or $v^2 = v_{vert}^2 + v_{horiz}^2$ and $v_{vert}^2 = (-3.96\sin 35^\circ)^2 + 2g \times 3$	M1	
		$36 = \frac{1}{2}\, 1.2(v^2 - 3.96^2)$ or $1.2g(4\sin 25^\circ + 3) = \frac{1}{2}\, 1.2v^2 + 2.719 \times 4$ or $v^2 = [(-3.96\sin 35^\circ)^2 + 2g \times 3] + (3.96\cos 35^\circ)^2$	A1ft	
		Speed is 8.70 ms^{-1} (8.62 from $g = 9.81$ or 8.61 from $g = 9.8$)	A1	3
	SR (max 1 out of 3) $v^2 = 3.96^2 + 2g \times 3$ B1 ft			

Notes: Allow sin 25 instead of cos 25 for M1 in (i)(a).
Allow cos 25 instead of sin 25 for M1 in (i)(b).
$1.2g\sin 25^\circ - 2.719 = 1.2ga$ scores M0 in (i)(b).
Accept ± 36 in (ii)(a).
Allow M1 for $\frac{1}{2}\, 1.2v^2 = 36$ in (ii)(b).
Accept 8.7 (for 8.70) in (ii)(b).

7	(i)	$v(200) = 0.12\text{x}200 - 0.0006\text{x}40\,000 = 0$	B1	
		For using $a = \mathrm{d}v/\mathrm{d}t$ and evaluating $a(200)$ or $a(200 + \varepsilon)$ for suitably small ε $[a = 0.12 - 0.0012\text{x}200]$	M1	
		Acceleration is 0.12 ms^{-2} (accept $a = -0.12$) (must be from $\varepsilon = 0$)	A1	3
	(ii)	For attempting to solve $\mathrm{d}v/\mathrm{d}t = 0$ or using $t = ½\ 200$ (may be implied)	M1	
		$t = 100$ (ft incorrect 2-term $\mathrm{d}v/\mathrm{d}t$ in (i))	A1ft	
		Maximum speed is 6ms^{-1}	A1	3
	(iii)	For integrating v	M1	
		$s = 0.06t^2 - 0.0002t^3$ (+C)	A1	
		Displacement is 800m	A1	3
	(iv)	For attempting to solve $s = 0$	M1	
		$t = 300$	A1	2

Notes; The M mark in (ii) is not dependent on the M mark in (i), the $\mathrm{d}v/\mathrm{d}t$ used may be what the candidate thinks is $\mathrm{d}v/\mathrm{d}t$.
800 + C is not acceptable for seond A1 in (iii).
T = 0 or 300 is not acceptable for A1 in (iv).