

# MATHEMATICS

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Paper 9709/11

Paper 11

## Key messages

The efficient use of a variety of algebraic techniques was central to the solution of most questions. Candidates who displayed good algebraic skills coped well with paper. Some questions were left unanswered by some candidates. Using the available wealth of past papers, both for this variant and 9709/12 and 9709/13, should ensure that candidates are familiar with the many ways in which knowledge of the syllabus can be tested. Centres should be aware that sketch graphs are not expected to be presented on graph paper.

## General comments

The use of clearly stated methods with good annotation of the stages within them was a feature of the best answers. In questions where a diagram was given or could be used clear references to the diagram were most helpful for examiners seeking to award method marks

## Comments on specific questions

### **Question 1**

Good use was made of the given formula for the binomial expansion. Those who selected the appropriate term reached the solution quickly whilst some insisted on presenting the whole expansion before selection. The expansion of the negative term proved problematic for some.

Answer:  $-67.5$

### **Question 2**

The use of the trigonometrical identity to obtain a quadratic equation in  $\cos\theta$  was the only successful route used in the solution of this equation. Those who solved the quadratic usually realised one solution must be discarded and the other led, in most cases, to two solutions for  $\theta$ .

Answer:  $\theta = 48.2^\circ, 311.8^\circ$

### **Question 3**

The volume of revolution of an area between the curve and the y-axis proved unfamiliar to some but the use of  $x = f(y)$  for the curve equation was a useful hint to those who adopted the correct method. Although squaring  $x$  did not always produce 3 terms and the integration of negative powers of  $y$  was inconsistent there were many good examples of complete solutions.

Answer:  $22\pi$

#### Question 4

In part (i) the use of the chain rule:  $\frac{dy}{dx}_{x=0} \times 0.3$  was not widely recognised but some found  $\frac{dy}{dt}$  in terms of  $x$ .

Finding the equation in part (ii) was attempted well with many candidates finding the constant of integration correctly. This type of integral continues to be dealt with correctly although the presence of the '2' caused problems for some.

Answers: (i) -0.6 (ii)  $y = 2x - \frac{16}{3}\sqrt{3x+4} + 12$

#### Question 5

In part (i) many candidates realised that two equations in  $x$  and  $y$  for area and perimeter could be derived from the information given in the question and used these to derive the required expression for A. The algebraic manipulation used in the elimination of  $y$  suggested by the given result was often seen. Those who set the perimeter to equal  $8x + 4y$  would have been well advised to re-check their assumption when the correct result failed to materialise.

In part (ii) most successful solutions came from differentiation and setting  $\frac{dA}{dx}$  to zero. Finding  $x$  from this was completed more successfully than finding  $y$ . Some candidates chose to use the maximum of quadratic functions at  $x = \frac{-b}{2a}$  and some used a graphical approach. Both methods were used successfully to find  $x$ .

In spite of there being no need, some candidates insisted on showing A to be a maximum.

Answer: (ii)  $x = 20, y = 24$

#### Question 6

In part (a) the easiest route to the solutions for  $m$  via the discriminant was not the most favoured. Some found the gradient of the curve and substituted this for  $m$  but only those who then equated their new linear equation with the quadratic made any headway.

Part (b) was very well answered. In part (i) the values of  $a$  and  $b$  were found successfully either by using the factors  $(x-1)(x-9)$  or by forming a pair of simultaneous equations.

In part (ii) a variety of successful methods included calculus, completing the square and using the symmetry of the function. Those who chose to use  $x = \frac{-b}{2a}$  had to be careful to distinguish between the values they had found and the coefficients of the quadratic.

Answer: (a)  $m = -12, m = 4$  (b)(i)  $a = -10, b = 9$  (ii)  $(5, -16)$

#### Question 7

The formulae booklet was used effectively to quote the arc length and sector area formulae.

In part (i) the successful candidates used the sine and cosine of angle OCD or angle COD to find CD and OD. Arc CB was found directly or as arc AB – arc AC. Although the use of radians was strongly suggested in the question the use of degrees or a mixture of radians and degrees was not uncommon and difficult to credit.

In part (ii) the quickest route to the solution using the area of sector  $OCB$  as a starting point was not always seen. Those who used the area of the quarter circle  $AOB$  as a starting point were also able to reach the correct solution. Maintaining a suitable degree of accuracy in working out the side lengths and areas ensured the final answer was within the required range.

Answers: (i)  $r\cos\theta + r - r\sin\theta + r\left(\frac{\pi}{2} - \theta\right)$  (or equivalent) (ii) 6.31

### Question 8

In an unstructured question such as this, a diagram and clear explanation enables examiners to give most credit for partially correct solutions. Those who found the gradient of  $AB$  and equated this to  $\frac{dy}{dx}$  were able to find the coordinates of  $C$  and  $D$ . Most candidates who reached this stage realised the mid-point and gradient of  $CD$  were required. The product of perpendicular gradients was well understood but some went back to the gradient of  $AB$  instead of  $CD$ . Some successful solutions came from the locus of points equidistant from  $C$  and  $D$ .

Answer:  $y = \frac{-1}{2}x$

### Question 9

In part (a) the  $n^{\text{th}}$  term and  $S_{\infty}$  formulae were used well producing many completely correct solutions. Candidates who were able to link the properties of arithmetic progressions with basic trigonometry produced good solutions in part (b).

In part (i) the use of two expressions for  $d$ , one expression for  $d$  and an expression for the third term or two expressions for the sum of three terms were the favoured routes to the given result. Some candidates chose to evaluate all the terms from the given result and demonstrate how they formed an arithmetic progression.

Although most candidates could quote the formula for  $S_{20}$  in part (ii), finding numerical values for  $a$  and  $d$  proved to be troublesome. Good answers were produced by those who found the exact values of  $\sin x$  and  $\cos x$  and from those who found  $\tan^{-1}x$  in degrees or radians and then the values of  $\sin x$  and  $\cos x$ .

Answers: (a) 250 (b)(ii) 70

### Question 10

In part (i) the use of the scalar product on  $\overrightarrow{OA}$  and  $\overrightarrow{OB}$  produced many successful solutions. Those who chose to find the magnitude of each vector made the task of finding  $k$  somewhat harder for themselves. Part (ii) would have produced many more correct answers but for sign errors. The method of finding and equating the magnitudes of  $\overrightarrow{OC}$  and  $\overrightarrow{AB}$  was well understood.

In part (iii) those candidates who were able to find the unit vectors of  $\overrightarrow{OA}$  and  $\overrightarrow{OC}$  and to scale these accordingly usually went on to find the magnitude of  $\overrightarrow{DE}$  successfully. The best answers were often accompanied by a sketch showing the vectors.

Answers: (i)  $k = 4.5$  (ii)  $k = 4, k = -8$  (iii)  $\sqrt{85}$

### Question 11

In part (i) the range of  $f(x)$  presented in the form given for the domain was expected. Many other forms were presented and if they had the correct end points full credit was given. Candidates who appreciated the full range of values of the sine function are given by the domain were able to deduce these end points.

The use of  $x = 0$  and  $f(x) = 0$  in part (ii) was well understood by most candidates. The use of degrees for the intersection with the  $x$  axis was not uncommon but some credit was given for the method. Although perhaps obvious, it was important to state the coordinates of the point of intersection with the  $y$  axis.

To gain both marks in part (iii) the sketch needed to show a recognisable section of the sine curve in the given domain with the range from part (i) and the axis intercepts from part (ii). Most successful candidates realised there was a link between the three parts although some plotted a series of points calculated within the domain.

Candidates who had found the  $x$  coordinate in part (ii) were usually able to repeat the change of subject process to find the inverse successfully in part (iv). The use of the relationship between the range and domain of functions and their inverses was seen in all correct solutions

Answers: (i)  $-5 \leq x \leq 3$  (ii)  $(0, -1)$ ,  $(0.253, 0)$  (iv)  $f^{-1}(x) = \sin^{-1} \frac{x+1}{4}$ ,  $-5 \leq x \leq 3$ ,  $\frac{-\pi}{2} \leq f^{-1}(x) \leq \frac{\pi}{2}$

# MATHEMATICS

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Paper 9709/12

Paper 12

## Key messages

Candidates should try not to be thrown off course by questions set in context. The mathematics involved in such questions is often quite straightforward providing candidates do not panic but concentrate on reading the question very carefully when they are extracting the relevant information. This comment was made in last year's report but unfortunately many candidates did not seem to have followed this advice and the question set in context, again, was the one which they were least successful with.

When definite integrals are evaluated it is important that both of the limits can be clearly seen to have been substituted into the integral. Many candidates missed out a necessary line of working which meant that if the integration was incorrect, no credit could be given.

## General comments

The paper seemed to be well received by the candidates, many good and excellent scripts were seen. The paper seemed to work well; with a number of questions being reasonably straightforward, particularly near the beginning of the paper, giving all candidates the opportunity to show what they had learned and understood. However, some questions provided more of a challenge, even for candidates of good ability. The vast majority of candidates appeared to have sufficient time to complete the paper. Questions 9, in particular, but also 5(ii) and 11(ii) proved to be a difficult challenge for many. The standard of presentation was generally good with candidates setting their work out in a clear readable fashion with very few candidates dividing the page into two vertically.

## Comments on specific questions

### **Question 1**

The question proved to be a very accessible start to the paper with a great many candidates demonstrating a good knowledge of composite functions and scoring full marks. A few candidates made numerical errors when substituting 2 into  $gf(x)$  but it was very pleasing that virtually no candidates attempted to multiply the functions together.

*Answer:* 2

### **Question 2**

The vast majority of candidates realised the need to integrate and were generally able to do this successfully although a good number did forget to divide by  $-2$  or divided by 2 instead. Many weaker candidates, but also some quite good ones, used the equation of a straight line rather than integrating and received no credit.

*Answer:*  $y = \frac{4}{(5-2x)} + 3$

### Question 3

Very many candidates found this vector question to be straightforward and scored full marks. The majority finding  $\mathbf{AB}$  and then successfully adding this onto  $\mathbf{OB}$ . A good number of candidates forgot to find the unit vector and a few did not understand what was required for this operation.

Answer: 
$$\frac{(6\mathbf{i} - 3\mathbf{j} + 6\mathbf{k})}{9}$$

### Question 4

Two different approaches were generally adopted for both parts of this question. Many good candidates worked out which term or terms were required whilst others worked out the full expansions. Both methods were generally successful although a number of candidates didn't fully account for the minus sign and others did not seem to understand the phrase 'is independent of  $x$ ' and gave answers such as  $-160x$ . Weaker candidates sometimes missed the second part of this question out or gave answers coming from just one term rather than a combination of two.

Answer:  $-160, -140$

### Question 5

This question was very well done by stronger candidates but weaker ones struggled and either missed out the second part completely or failed to justify the given answer. In part (i) successful candidates realised the need to find  $AC$  first, using triangle  $ABC$  before using Pythagoras theorem with  $MC$ . This was usually successfully done although some candidates either failed to evaluate  $\tan \frac{\pi}{3}$  or left the final answer as the sum of two terms. In part (ii) many candidates seemed to confuse themselves by referring to angle  $\theta$  when they actually meant angle  $MAC$ . Others failed to justify the  $\frac{\pi}{6}$  or make clear the link between that and the two angles  $BAM$  and  $MAC$ .

Answer: (i)  $\sqrt{13} x$

### Question 6

The vast majority of candidates attempted this question although many made mistakes, particularly in part (i). Candidates could usually find  $PQ$  and  $PT$  but  $QT$  proved much more of a challenge. Most candidates realised that the answer would involve  $\cos \alpha$  but some thought that  $OT$  was  $\frac{\cos \alpha}{r}$  rather than the reciprocal of this and a good number failed to subtract  $r$  from  $OT$  in order to obtain  $QT$  or did  $r$  minus  $OT$  instead. Part (ii) was more successfully answered with full marks often seen but a significant number did fail to give the final answer to 2 significant figures as instructed in the question. Some candidates rounded earlier in their working and this often led to an incorrect final answer.

Answer: (i)  $\frac{r}{\cos \alpha} - r$  (ii) 34

### Question 7

The vast majority of candidates worked with the left hand side of the identity in part (i) and realised the need to combine the fractions. Many were successful and earned full marks but a significant minority were unable to do so or made errors particularly with the numerator. Extra practice with this technique will hopefully improve future performance. Some candidates, when they were unable to prove the identity, did not attempt part (ii) even though the answer in part (i) was given. Some weaker candidates failed to use the identity or misread the question and ignored the  $\sin \theta$  and so the relatively straightforward task of isolating  $\tan \theta$

became much more difficult. A number of candidates who were able to isolate  $\tan \theta$  obtained  $\frac{3}{4}$  rather than  $\frac{4}{3}$  and others gave the answers to 3 significant figures rather than one decimal place as required for angles.

Answer: (ii) 53.1, 233.1

### Question 8

Many fully correct answers were seen for this question, with candidates successfully finding the gradient in the first part and the perpendicular gradient and the midpoint in the second part. A small number of candidates seemed to be confused with the question and tried to use the perpendicular gradient in the first part rather than equating two gradients or, more commonly, using the equation of  $AB$  and substituting for point  $C$  along with point  $A$  or  $B$ . Similarly in part (ii) by far the most common approach was to use the equation of the perpendicular bisector.

Answer: (i) 2.8 (ii) 0.6

### Question 9

This question was, by far, the worst one done by candidates. Many seemed to struggle to understand the situation described or were confused by what type of series was needed. In part (a) many seemed to be trying to find the amount left in the tank, or the sum of the amount lost over 30 days, rather than the amount lost on the 30<sup>th</sup> day. Hence many candidates tried to use formula for the sum an arithmetic series rather than the  $n$ th term. Conversely in parts (b) and (ii) many used the  $n$ th term formula when the sum formula was required. Some candidates who were able to correctly form and solve the quadratic equation in part (b) were then unable to interpret the answer of 40.4 as meaning that the tank would become empty during the 41<sup>st</sup> day. A few weak candidates tried to use the arithmetic series formula in part (ii) but most realised that it was now a geometric series. Generally it was only the strongest candidates who managed to find the amount lost in this final part and they were then usually able to find the percentage of the amount left, but a few did find the percentage lost.

Answer: (i)(a) 68, (b) 41 (ii) 17.75%

### Question 10

This question was well done by many candidates and full marks were seen quite often. Some weaker candidates struggled with the  $\frac{8}{x}$  term and were unable to successfully differentiate, or more commonly,

square and integrate it. Quite often  $y^2$  became  $\frac{64}{x^2} + 4x^2$ . Candidates were familiar with the required

techniques in part (ii) but many failed to read the question carefully enough and only obtained one stationary point,  $M$ , ignoring the requirement that  $x < 0$ . In part (iii) many candidates failed to clearly show both of the limits substituted and because the integral was often incorrect it was not possible to award any method marks. It is important with limits that the calculations which are put into calculators are also written down.

Answer: (i)  $-8x^{-2} + 2,16x^{-3}, -64x^{-1} + 32x + \frac{4x^3}{3} + c$  (ii)  $M(2,8), (-2,-8)$ , maximum (iii)  $\frac{220\pi}{3}$

**Question 11**

This final question was well attempted by most candidates with only the weakest of candidates missing it out completely or failing to score any marks. In part (i) the vast majority of candidates formed and attempted to solve a quadratic equation although a few did fail to incorporate the 3. Many were able to find the correct two critical values although some either did not attempt to solve the inequality or wrote down the incorrect region. Part (ii) was found to be the most challenging part of the question for candidates but there were many who were able to equate, re-arrange and then use the discriminant to obtain the required equation. Others successfully differentiated and then equated the gradients. Both of the minus signs in part (iii) caused problems for some candidates but there were also many correct answers. Some candidates failed to see the correct link between parts (iii) and (iv) and answers of 4 or -3 were quite common. In part (v) weaker candidates did not use the completed square form in order to find the inverse function and were therefore unable to make any progress.

**Answer:** (i)  $x \leq 2, x \geq 4$ , (iii)  $4 - (x - 3)^2$  (iv) 3 (v)  $\sqrt{4 - x} + 3$

# MATHEMATICS

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Paper 9709/13  
Paper 13

## Key messages

- 1 Last November it was reported that a significant minority of candidates attempted to compress their work so that it occupied only two or three sheets of paper, a practice which can lead to the omission of essential working and, potentially, the loss of marks. It is pleasing to report that this practice was far less widespread in this examination session. Indeed, the introduction of standard answer booklets with at least 12 pages available together with supplementary booklets if required, induced quite a number of candidates to be extremely extravagant with space by ignoring the printed lines and often leaving several printed lines between each line of their working. It should be noted, however, that it is expected that from May 2017 these Answer booklets will be replaced by Question/Answer booklets and, for each question or part question there will be a set amount of working space. This amount of working space will be more than adequate providing candidates use the printed lines as a guide.
- 2 Notwithstanding the comment in 1 above, there are still candidates who are not showing sufficient working. This is particularly true in the case of questions involving direct integration. Candidates who do not show the actual result of the integration process will not, in general, score any marks.
- 3 In questions which require an *exact* answer it is necessary for candidates to employ an exact method. An inexact method which perhaps fortuitously rounds to the required answer will therefore not gain full marks. (See **Question 11**.)
- 4 Since scripts are now scanned and then marked on-line, Examiners no longer have access to the paper script when marking but only the scanned image on their computer screen. This can occasionally cause difficulties for Examiners when the candidate places a minus sign directly on a printed ruled line. Candidates should be advised to try to place minus signs between the printed ruled lines.

## General comments

The paper was generally well received by candidates and many very good scripts were seen. Most candidates seemed to have sufficient time to finish the paper. In trigonometric questions, candidates should be aware that sometimes answers are required in degrees and sometimes in radians and it would be advisable before embarking on the solution of the question for candidates to check the required range and set their calculators to degrees or radians as appropriate.

## Comments on specific questions

### **Question 1**

This was a straightforward question and candidates responded very well – a large majority scoring full marks. Candidates generally chose one of three approaches: using the general term, expanding out the full series or simple recognition of the required coefficient.

*Answer:* 90.

## Question 2

Strong candidates almost always scored full marks in this question but there were various errors to which candidates of modest ability succumbed. Most candidates appeared to know that they had to square  $y$  but a surprising number failed to reach  $x^3 + 1$  before attempting to integrate and hence the simplicity of the integration was lost on them. The correct limits were usually applied but a few used an incorrect upper limit. Rarely were the limits interchanged and rarely was  $\pi$  omitted.

Answer:  $6\pi$ .

## Question 3

Almost all candidates completed part (i) correctly and almost as many were successful with part (ii). There were some errors made with the integration of the  $x^{-3}$ . term and occasionally the constant of integration was forgotten and hence the last 2 marks for the substitution were lost.

Answers: (i)  $k = -4$ ; (ii)  $y = 2x^3 + 2x^{-2} + 5$ .

## Question 4

This was a question that rarely posed significant problems for candidates. The great majority found all 4 possibilities for  $r$  and  $d$  before rejecting  $r = 1$  and  $d = 0$ , although failure to reject these values was not penalised.

Answer:  $r = 5, d = 6$ .

## Question 5

It was pleasing to see this question answered so well. The most common error was the failure to apply the chain rule when finding the first and/or second derivative and thereby losing a factor of 2. Also occasionally only one root was found ( $\frac{3}{4}$ ) from the solution of the quadratic equation, usually omitting a  $\pm$  on the way.

Almost all candidates determined the nature of the stationary points using the second derivative and this was done confidently and accurately. Candidates who attempted the first derivative method were usually unsuccessful, often because they did not reference specific  $x$  values.

Answer:  $x = \frac{1}{4}$  maximum,  $x = \frac{3}{4}$  minimum.

## Question 6

On the whole this question was well done with most candidates achieving at least 6 marks out of 7. The majority of candidates realised that the required area was that of the triangle minus the sum of the three sectors. Although most candidates realised that angle  $ACB$  was a right angle, a significant number did not. Those who had recognised the right angled triangle usually achieved either full marks or just lost the final mark through not working to a sufficient degree of accuracy for the three sectors. This loss of accuracy was often caused by the use of degrees rather than radians. The correct formula for the area of a sector was used by most candidates. A number of candidates who had noted that angle  $C$  was  $90^\circ$  then thought that  $A$  must be  $30^\circ$  and  $B$  must be  $60^\circ$ , presumably because this was the same proportion as the radii of the three circles.

Answer: 0.464.

### Question 7

This question provided a challenge even for some candidates of high ability. Candidates of more modest ability very often scored only the first mark. It was given that the rate of change of  $y$  was constant. Candidates would have found the question much easier if they had let  $dy/dt = k$ , for example, and then it follows that  $dx/dt = \frac{1}{2}k$  and  $dy/dx = 2$ . However, relatively few candidates followed this route and very many got confused with the notation and application of the chain rule and foundered. Some candidates got lucky and guessed that  $dy/dx = 2$  and correctly equated their derivative with 2 and simplified to a 3-term quadratic in  $\sqrt{x}$ . Many candidates who reached this point were then able correctly to solve firstly for  $\sqrt{x}$  and finally for  $x$ . However, some candidates who had arrived at a correct equation involving  $\sqrt{x}$  proceeded to attempt to remove the square root simply by squaring every term.

Answer:  $x = 9/4, 1$ .

### Question 8

Most candidates managed to transform the given equation into a quadratic equation in  $\cos x$  but many lost marks by giving the solutions in degrees rather than radians. (See **General Comments** above.) Surprisingly, a significant number of candidates seemed to miss the second request under part (i) – that of solving the equation, many going on to solve the first equation but labelling their answer (ii), ignoring the second equation in which  $x$  was replaced by  $2x$ . Part (ii) was not well done. Of those who realised the connection with part (i) many just halved their solutions from part (i), not appreciating that what was required was to first list solutions for  $2x$  in the interval  $0 \rightarrow 2\pi$  before dividing by 2. It was common to see candidates start the whole process from the beginning again so making a lot of extra work for themselves.

Answers: (i) 2.42, 0; (ii) 1.21, 1.93, 0, 3.14.

### Question 9

This question was generally answered very well and provided a good source of marks – even for the less strong candidates. In part (i) the need to find expressions for the vectors  $AB$  and  $CB$  was recognised by most and most were able to find these by correctly subtracting the given position vectors of  $A$ ,  $B$  and  $C$ . There were the occasional slips in the subtraction, usually in the  $\mathbf{k}$  coefficient of vector  $AB$  with  $p - 4$  appearing instead of  $p + 4$ . Most knew they then needed to find expressions for the magnitudes of these two vectors and to then equate and solve for  $p$ . Slips in squaring the brackets or even in combining the constant terms or terms in  $p$  meant a few candidates lost the final mark. In part (ii) most candidates knew the processes required to find an angle between two vectors by using the scalar product. A very few substituted the value of  $p$  found in part (i) instead of the required value of  $p = 1$  given in part (ii). Some candidates reversed both of their vectors found in part (i) which still gives the required angle, but a few reversed just one of their vectors resulting in the incorrect obtuse angle between the vectors.

Answers: (i)  $p = 2$ ; (ii)  $75.3^\circ$ .

### Question 10

Candidates generally did not seem comfortable with this question and, after **Question 7**, was the least well answered. This does not apply to part (i) which, apart from occasional slips in finding  $b$ , was answered successfully by most candidates. Part (ii) proved to be quite challenging for most candidates. Some reached 2 as their answer but without really demonstrating they were confident in what they were doing. Relatively few correctly chose the negative value to be the maximum value of  $q$ . Part (iii) was more accessible but, surprisingly, a wide range of false methods led to incorrect answers. In part (iv) most candidates used a correct method for the inverse function but at the end forgot the  $\pm$  or chose the positive sign. In previous examinations candidates have shown generally that they realised that the domain of the inverse function was the same set of values as the range of the original function but in this paper there were a surprising number of errors in candidates' answers.

Answers: (i)  $a = 3, b = -12$ ; (ii) maximum  $q = -2$ ; (iii)  $y \geq 33$ ; (iv)  $(fg)^{-1}(x) = -\sqrt{\frac{x+21}{6}}$ ,  $x \geq 33$ .

**Question 11**

It was very surprising and pleasing to see the variety of methods and areas of Pure Mathematics used in answering this question. In part (i) a significant number of candidates did not take note of the need to find the exact area and so lost marks by, for example, using a calculator to find angles and so introducing rounding errors. Candidates' attempts for part (ii) were generally successful with many scoring full marks. The most common method was to find equations of  $AB$  and  $CD$  and to solve simultaneously. Most coped well with fractions and negative numbers, although the last mark was sometimes lost due to an arithmetic error.

Answers: (i) 34; (ii) 2/5.

# MATHEMATICS

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Paper 9709/21  
Paper 21

## Key messages

Candidates are reminded to read the rubric on the front of the paper and take note especially of the level of accuracy required. Many candidates lost marks unnecessarily due to answers given to the wrong degree of accuracy. Note should also be made of the importance of the word 'Hence' which implies that work done in a previous part of the question is to be used.

## General comments

Well prepared candidates were able to produce scripts of a high standard showing a good understanding of the syllabus objectives and how to apply techniques learned both appropriately and correctly. There appeared to be no issues with time.

## Comments on specific questions

### **Question 1**

Many candidates were able to differentiate  $e^{4x}$  correctly, but fewer were able to differentiate  $6 \ln(2x + 3)$  with the same level of success. The most common error was failing to multiply by 2 to give  $\frac{12}{2x + 3}$ , with  $\frac{6}{2x + 3}$  being the most common incorrect answer. Most candidates realised the necessity of then substituting  $x = 0$  to find the gradient. Some candidates merely substituted  $x = 0$  into the equation for  $y$ , not realising that they first needed to differentiate the given equation in order to find the gradient.

*Answer:* 8

### **Question 2**

Many candidates attempted this question by trying to express the given equation in terms of either  $\sin\theta, \cos\theta$  or a mixture of both, thus making the problem more cumbersome and time consuming. Consequently those that used these methods were less successful than those candidates who worked with  $\tan\theta$ .

Frequently, candidates who used  $\cot\theta = \frac{1}{\tan\theta}$  then mistakenly expressed  $\tan 2\theta \times \tan\theta$  as  $\tan^2\theta$  or  $\tan^2 2\theta$ , not realising the need to use the double angle formula.

Candidates who did solve the equation using a correct method often only gave the acute angle solution rather than both solutions within the required range.

A few candidates tend to miss off the angle, in this case  $\theta$ , when dealing with trigonometric functions which in this question caused confusion due to the combination of single and double angles. Centres are asked to encourage candidates to write out any trigonometric ratios/functions without omissions.

*Answer:*  $28.1^\circ, 151.9^\circ$

### Question 3

Few candidates were able to rearrange the given equation to obtain a correct quadratic equation in  $e^x$ . Those that did usually went on to produce a correct solution, although some candidates did stop once they had found the two solutions for  $e^x$ , rather than continue and use natural logarithms to obtain the solutions for  $x$ . Many candidates lost an accuracy mark by not giving the solutions to the required 3 significant figures level of accuracy.

The most common error was to attempt to take logarithms immediately; candidates who used this approach had no success.

Answer:  $-0.405, 1.39$

### Question 4

- (i) Almost all candidates were able to obtain full marks in this part of the question, either by algebraic long division, comparing coefficients or by synthetic division. A few candidates failed to translate their synthetic division result into a quotient and remainder, thus showing that there is a need for candidates to ensure that they have actually answered the question.
- (ii) The word ‘Hence’ was used to show that this part of the question required the use of the result from the first part of the question. Some candidates failed to appreciate this and often repeated the first part of the question again. Some candidates also confused the request for factorisation with solving and thus gave only numerical values for  $x$  rather than a product of 3 factors. This highlights again the need to ensure that question has actually been answered.
- (iii) Very few correct solutions were seen. Most candidates simply wrote down the 3 solutions using their factors from part (ii) completely ignoring the modulus sign.

Answer: (i)  $8x^2 + 14x - 15$  (ii)  $(x + 2)(2x + 5)(4x - 3)$  (iii)  $\pm \frac{3}{4}$

### Question 5

- (i) Most candidates attempted to differentiate the parametric equations with respect to  $\theta$ . A common error was to obtain  $\frac{dy}{d\theta} = 3\cos 2\theta$ , rather than  $\frac{dy}{d\theta} = 6\cos 2\theta$ . There was often confusion with notation and the correct use of the variables. Correct results for  $\frac{dy}{dx}$  were seen but all too frequently, use of the appropriate double angle formula was not made, meaning that the given expression could not be obtained.
- (ii) Equating the given result from part (i) to zero and attempting to solve, was often attempted with varying degrees of success. Many candidates found  $\theta = \frac{\pi}{4}$  but did not then find the corresponding Cartesian coordinates. Centres should remind candidates that when dealing with questions involving trigonometric functions and calculus, angles are in radians unless specifically stated otherwise.
- (iii) Few candidates recognised the necessity of finding  $\theta$  by equating  $2\sqrt{3}$  or  $\frac{3}{2}\sqrt{3}$  with the given parametric forms. Again the need to ensure that the question is answered completely was highlighted in this part as candidates often then failed to use their value of  $\theta$  to find the gradient as required.

Answer: (ii)  $\frac{\pi}{4}, (2, 3)$  (iii)  $-\frac{3}{8}$

**Question 6**

- (i) This part was usually done well by those candidates who recognised the need to differentiate the given function as either a quotient or an appropriate product. Equating to  $\frac{1}{2}$  and re-arranging to obtain the given form usually followed correctly from a correct differentiation.
- (ii) This type of question always appears to cause candidates problems. One approach was to make use of the equation from part (i) and write it in the form  $f(p) = p - \sqrt{\frac{48p-16}{p^2+8}}$  and substitute in for  $p$  thus obtaining a change of sign between  $p = 2$  and  $p = 3$ . Some conclusion mentioning the change of sign implying that a solution lay between the two given values was then expected. Often no conclusion was drawn. The more common approach was to substitute  $p = 2$  and  $p = 3$  into the given formula from part (i). This was where most candidates stopped. It was then necessary to make a comparison with the value used and value obtained in both cases.
- (iii) Usually done well by most candidates, the main problem was failure to give the final result to the correct level of accuracy.

Answer: (iii) 2.728

**Question 7**

- (a) Very few candidates recognised that they needed to write the integrand as  $\frac{1}{\cos^2 2x} + \cos^2 2x$  to start with. Even fewer realised that they needed to write  $\frac{1}{\cos^2 2x}$  as  $\sec^2 2x$  and use the double angle formula to re-write  $\cos^2 2x$  before integration could be attempted. Very few correct solutions were seen.
- (b) Most candidates that attempted this question were able to obtain the form  $2x + k \ln(3x - 2)$ , but quite a few were unable to obtain the correct value for  $k$  with 6 being the most common value seen. Most were able to make a correct use of limits and apply appropriate laws of logarithms, but few were able to express 20 as  $\ln e^{20}$  in order to obtain their answer in the required form.

Answer: (a)  $\frac{1}{2} \tan 2x + \frac{1}{2} x + \frac{1}{8} \sin 4x + c$  (b)  $\ln 16e^{20}$

# MATHEMATICS

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Paper 9709/22  
Paper 22

## Key messages

Candidates are reminded to read the rubric on the front of the paper and take note especially of the level of accuracy required. Many candidates lost marks unnecessarily due to answers given to the wrong degree of accuracy. Note should also be made of the importance of the word 'Hence' which implies that work done in a previous part of the question is to be used

## General comments

Well prepared candidates were able to produce scripts of a high standard showing a good understanding of the syllabus objectives and how to apply techniques learned both appropriately and correctly. There appeared to be no issues with time.

## Comments on specific questions

### **Question 1**

Many candidates were able to get as far as  $3x \log 5 = 4y \log 7$ , using either base 10 logarithms or natural logarithms. However candidates were often unable to manipulate the different terms in order to isolate  $\frac{x}{y}$ . Of those that did, often answers were given to more than the required 4 significant figure level of accuracy.

Answer: 1.612

### **Question 2**

- (i) Most candidates were able to use algebraic long division and obtain a correct quotient, however many candidates assumed that all remainders have to be numeric and thus often many did not give their remainder as a linear factor in  $x$ .
- (ii) Some candidates chose to start the problem again by failing to recognise the implications of the word 'Hence'. Some candidates attempted to 'spot roots' by substitution.

Answer: (i) Quotient  $2x - 3$ , remainder  $-25x + 18$  (ii)  $p = 16$ ,  $q = -15$

### **Question 3**

- (i) Usually done well by the majority of candidates, there were the occasional sign errors.
- (ii) Again candidates failed to realise the significance of the word 'Hence' with many starting to solve the problem from the start. Solutions in degrees were common even though the required range had been given in radians. There were also a lot of misconceptions about how equations in terms of  $\cot x$  were solved. This highlights a problem that could be addressed by centres.

Answer: (i)  $-6, \frac{4}{5}$  (ii) 0.896

#### Question 4

- (i) Many candidates were able to use the compound angle formulae correctly to show the required result.
- (i) (a) Again the word ‘Hence’ failed to guide many candidates into using the result from part (i). Too many candidates do not understand the meaning of ‘exact value and so chose to use their calculators to evaluate  $\sin 105^\circ + \sin 165^\circ$ . It was hoped that candidates would make use of part (i) and relate  $\theta$  to  $45^\circ$  and realise that  $\sin 105^\circ + \sin 165^\circ = \sqrt{3} \cos 45^\circ$  leading to an exact answer.
- (b) Most candidates were able to reach  $\cos^2 \theta = \frac{1}{\sqrt{3}}$ , but some were unable to deal with the square root of a square root. Of those that did solve the resulting equation correctly, often only the acute angle solution was given.

Answer: (ii)(a)  $\sqrt{\frac{3}{2}}$  or  $\frac{1}{2}\sqrt{6}$  (b)  $40.6^\circ, 139.4^\circ$

#### Question 5

- (i) Candidates needed to start with the differentiation of a product and then make a correct use of logarithms. Unfortunately there was a lot of incorrect working with logarithms and contrived manipulation to get the given result.
- (ii) This type of question always appears to cause candidates problems. One approach was to make use of the equation from part (i) and write it in the form  $f(p) = p - 3 \ln\left(\frac{20}{p+3}\right)$  and substitute in for  $p$  thus obtaining a change of sign between  $p = 3.3$  and  $p = 3.5$ . Some conclusion mentioning the change of sign implying that a solution lay between the two given values was then expected. Often no conclusion was drawn. The more common approach was to substitute  $p = 3.3$  and  $p = 3.5$  into the given formula from part (i). This was where most candidates stopped. It was then necessary to make a comparison with the value used and value obtained in both cases.
- (iii) Unfortunately 12 or 13 iterations were needed if starting from an end point. Candidates should realise that fewer iterations are needed if the midpoint of the given interval is used. Having said that, there were many completely correct solutions. There did not appear to be any problems with the accuracy of the final answer although some candidates chose not to write down all their iterations or show enough to lead to the final correct result.

Answer: (iii) 3.412

#### Question 6

- (a) Very few correct solutions were seen. Candidates were expected to write the integrand in the form  $2e^{-2x} + \frac{1}{2}e^{-x}$ , having divided each term in the numerator by  $2e^{2x}$ . A common error was to write the integrand as  $2e^{-2x}(4 + e^x)$ .
- (b) Most candidates realised that  $\ln(2x+5)$  was involved but many failed to obtain  $\frac{1}{2}\ln(2x+5)$ . Although most were able to make correct use of the laws of logarithms, some did not give their final answer as a single logarithm as required.
- (c) It is important that candidates read the question properly as many used more than the two strips asked for in the application of the trapezium rule. Errors in the evaluation of the  $y$  values were also common.

Answer: (a)  $-e^{-2x} - \frac{1}{2}e^{-x} + c$  (b)  $\ln\frac{5}{3}$  (c) 3.9

**Question 7**

- (i) A good attempt to differentiate the given parametric equations with respect to  $t$  was made by most candidates. There were the occasional errors with signs, but many were able to obtain the given result correctly.
- (ii) Correct methods of solution were common with candidates equating the result from part (i) to zero and attempting to solve. It should be noted that as candidates are using calculus, angles are in radians unless specifically stated otherwise. Having solved the equation correctly, some mistakenly thought that  $x = \frac{\pi}{2}$  and not  $t$ .
- (iii) Very few candidates continued on to part (iii). Of those that did, many correctly attempted to solve  $\cos 2t = -\frac{1}{3}$  but premature approximation often lead to inaccurate final answers.

*Answer:* (ii) (2,-2) (iii)  $-\frac{12}{\sqrt{3}}$  or  $-4\sqrt{3}$  or  $-6.93$  and  $\frac{12}{\sqrt{3}}$  or  $4\sqrt{3}$  or  $6.93$

# MATHEMATICS

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Paper 9709/23

Paper 23

## Key messages

Candidates are reminded to read the rubric on the front of the paper and take note especially of the level of accuracy required. Many candidates lost marks unnecessarily due to answers given to the wrong degree of accuracy. Note should also be made of the importance of the word ‘Hence’ which implies that work done in a previous part of the question is to be used.

## General comments

Well prepared candidates were able to produce scripts of a high standard showing a good understanding of the syllabus objectives and how to apply techniques learned both appropriately and correctly. There appeared to be no issues with time.

## Comments on specific questions

### **Question 1**

Many candidates were able to get as far as  $3x\log 5 = 4y\log 7$ , using either base 10 logarithms or natural logarithms. However candidates were often unable to manipulate the different terms in order to isolate  $\frac{x}{y}$ . Of those that did, often answers were given to more than the required 4 significant figure level of accuracy.

*Answer:* 1.612

### **Question 2**

- (i) Most candidates were able to use algebraic long division and obtain a correct quotient, however many candidates assumed that all remainders have to be numeric and thus often many did not give their remainder as a linear factor in  $x$ .
- (ii) Some candidates chose to start the problem again by failing to recognise the implications of the word ‘Hence’. Some candidates attempted to ‘spot roots’ by substitution.

*Answer:* (i) Quotient  $2x - 3$ , remainder  $-25x + 18$  (ii)  $p = 16$ ,  $q = -15$

### **Question 3**

- (i) Usually done well by the majority of candidates, there were the occasional sign errors.
- (ii) Again candidates failed to realise the significance of the word ‘Hence’ with many starting to solve the problem from the start. Solutions in degrees were common even though the required range had been given in radians. There were also a lot of misconceptions about how equations in terms of  $\cot x$  were solved. This highlights a problem that could be addressed by centres.

*Answer:* (i)  $-6, \frac{4}{5}$  (ii) 0.896

#### Question 4

- (i) Many candidates were able to use the compound angle formulae correctly to show the required result.
- (i) (a) Again the word ‘Hence’ failed to guide many candidates into using the result from part (i). Too many candidates do not understand the meaning of ‘exact value and so chose to use their calculators to evaluate  $\sin 105^\circ + \sin 165^\circ$ . It was hoped that candidates would make use of part (i) and relate  $\theta$  to  $45^\circ$  and realise that  $\sin 105^\circ + \sin 165^\circ = \sqrt{3} \cos 45^\circ$  leading to an exact answer.
- (b) Most candidates were able to reach  $\cos^2 \theta = \frac{1}{\sqrt{3}}$ , but some were unable to deal with the square root of a square root. Of those that did solve the resulting equation correctly, often only the acute angle solution was given.

Answer: (ii)(a)  $\sqrt{\frac{3}{2}}$  or  $\frac{1}{2}\sqrt{6}$  (b)  $40.6^\circ, 139.4^\circ$

#### Question 5

- (i) Candidates needed to start with the differentiation of a product and then make a correct use of logarithms. Unfortunately there was a lot of incorrect working with logarithms and contrived manipulation to get the given result.
- (ii) This type of question always appears to cause candidates problems. One approach was to make use of the equation from part (i) and write it in the form  $f(p) = p - 3 \ln\left(\frac{20}{p+3}\right)$  and substitute in for  $p$  thus obtaining a change of sign between  $p = 3.3$  and  $p = 3.5$ . Some conclusion mentioning the change of sign implying that a solution lay between the two given values was then expected. Often no conclusion was drawn. The more common approach was to substitute  $p = 3.3$  and  $p = 3.5$  into the given formula from part (i). This was where most candidates stopped. It was then necessary to make a comparison with the value used and value obtained in both cases.
- (iii) Unfortunately 12 or 13 iterations were needed if starting from an end point. Candidates should realise that fewer iterations are needed if the midpoint of the given interval is used. Having said that, there were many completely correct solutions. There did not appear to be any problems with the accuracy of the final answer although some candidates chose not to write down all their iterations or show enough to lead to the final correct result.

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#### Question 6

- (a) Very few correct solutions were seen. Candidates were expected to write the integrand in the form  $2e^{-2x} + \frac{1}{2}e^{-x}$ , having divided each term in the numerator by  $2e^{2x}$ . A common error was to write the integrand as  $2e^{-2x}(4 + e^x)$ .
- (b) Most candidates realised that  $\ln(2x+5)$  was involved but many failed to obtain  $\frac{1}{2}\ln(2x+5)$ . Although most were able to make correct use of the laws of logarithms, some did not give their final answer as a single logarithm as required.
- (c) It is important that candidates read the question properly as many used more than the two strips asked for in the application of the trapezium rule. Errors in the evaluation of the  $y$  values were also common.

Answer: (a)  $-e^{-2x} - \frac{1}{2}e^{-x} + c$  (b)  $\ln\frac{5}{3}$  (c) 3.9

**Question 7**

- (i) A good attempt to differentiate the given parametric equations with respect to  $t$  was made by most candidates. There were the occasional errors with signs, but many were able to obtain the given result correctly.
- (ii) Correct methods of solution were common with candidates equating the result from part (i) to zero and attempting to solve. It should be noted that as candidates are using calculus, angles are in radians unless specifically stated otherwise. Having solved the equation correctly, some mistakenly thought that  $x = \frac{\pi}{2}$  and not  $t$ .
- (iii) Very few candidates continued on to part (iii). Of those that did, many correctly attempted to solve  $\cos 2t = -\frac{1}{3}$  but premature approximation often lead to inaccurate final answers.

*Answer:* (ii) (2,-2) (iii)  $-\frac{12}{\sqrt{3}}$  or  $-4\sqrt{3}$  or  $-6.93$  and  $\frac{12}{\sqrt{3}}$  or  $4\sqrt{3}$  or  $6.93$

# MATHEMATICS

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Paper 9709/31  
Paper 31

## General comments

The standard of work on this paper varied considerably and resulted in a wide spread of scores from zero to full marks. Some excellent scripts were seen but there were also scripts from candidates who appeared poorly equipped for the paper and made very little progress.

Most candidates made good progress with solving the modular equation (**Question 1(i)**), solving the trig. equation (**Question 3**), using the iterative formula (**Question 6(iii)**) and the use of partial fractions (**Question 8(i)**). Those questions that were less well done were solving the variable separable equation (**Question 4**), vector geometry (**Question 9**) and complex numbers (**Question 10**). Candidates from several centres offered little or no attempt at all at the last two questions (vectors and complex numbers), suggesting that perhaps they had not studied these topics in sufficient depth. There were many candidates who were familiar with the basic methods of calculus, but then differentiated for the integration by parts (**Question 2**) and integrated to locate the stationary point (**Question 5**).

Much of the work was clearly set out and easy to follow. Candidates who split their work so that the solution to a single question appears here and there, mixed up with other work, should be aware that they are making their work more difficult to mark and they are making it more difficult for themselves to check through.

When a question asks for the answer to be given as an exact value (as in **Question 2**) then candidates should be aware that calculator methods will not gain credit. Similarly, when a result is given (as in **Question 6(ii)** and **Question 7(i)**) then a full explanation is expected.

Candidates are reminded of the need to read the questions carefully and to ensure that their response matches the request. In **Question 6(ii)** many candidates were clearly prepared to demonstrate that the equation has a root between 1.4 and 1.5 (as has been asked in similar questions in the past) but did not show that the iterative formula provided was equivalent to the original equation.

Where numerical and other answers are given after the comments on individual questions that follow it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only ‘correct answer’.

## Comments on specific questions

### Question 1

- (i) Some candidates found one root by inspection (usually  $x = -2$ ) and did not attempt to find the second root. Those candidates who started by forming a quadratic equation were usually successful in reaching both roots, provided they started with  $4(x - 1)^2 = 9x^2$  and not the popular incorrect form  $2(x - 1)^2 = 3x^2$ .
- (ii) Only a small number of candidates made use of the link between the two parts of this question and went directly to a solution of  $5^x = \frac{2}{5}$ . The majority of candidates started again from the beginning, sometimes starting with incorrect equations in logarithms. Some candidates attempted to solve  $5^x = -2$  by first taking out the negative and then replacing it in their final answer.

Answers: (i)  $-2$ ,  $\frac{2}{5}$  (ii)  $-0.569$

### Question 2

The majority of candidates identified the need to use integration by parts and showed clear working, however some candidates appeared to differentiate rather than integrate. Errors in the integration usually involved an incorrect sign in the formula for integration by parts, or not dealing correctly with division by  $-2$ . Many candidates dealt correctly with the limits, although some did not appear to have used the lower limit. This question asked for an exact value for the integral, so decimal alternatives were not accepted. The correct decimal equivalent answer sometimes followed incorrect work – where the two methods disagree, a candidate should see this as a signal to go back through their work to identify the error.

Answer:  $\frac{1}{4} - \frac{1}{2}e^{-1}$

### Question 3

The great majority of candidates substituted correctly and went on to form a correct quadratic equation in  $\cos\theta$  but there was evidence of some confusion over the meaning of  $\operatorname{cosec}\theta$  and  $\cot\theta$ . The correct quadratic was invariably followed by the correct angle. The additional incorrect solution  $\theta = 48.2^\circ$  caused some candidates to lose the final mark.

Answer:  $131.8^\circ$

### Question 4

Those candidates who recognised this as a variable separable equation usually completed the first stage correctly and integrated to obtain  $\ln y$ . The integration of  $\frac{1-2x^2}{x}$  with respect to  $x$  proved to be more difficult, with many candidates not taking the straight forward step of splitting the fraction to obtain  $\frac{1}{x} - 2x$ . There were some correct attempts to apply integration by parts, which was a more complicated approach than that intended. Most candidates who completed the integration went on to use the limits correctly to find the constant of integration. Several candidates who converted to a form free of logarithms before evaluating the constant often retained the constant in the incorrect form “+C”.

Answer:  $y = 2xe^{(1-x^2)}$

### Question 5

There are two key steps at the start of this problem – differentiation, and use of the double angle formula. Candidates who differentiated first usually used the product formula correctly, although there were some slips in the differentiation of  $\cos 2x$ , sometimes involving both the sign and the 2. These candidates then needed to deal with both  $\sin 2x$  and  $\cos 2x$ . Those candidates who chose to start by expressing  $\cos 2x$  in terms of  $\sin x$  had to differentiate  $-2\sin^3 x$  as a function of a function. This was often successful, and this alternative method often gave tidier final solutions. The question specifies that  $0 < x < \frac{1}{2}\pi$ , and candidates should be aware that the differentiation requires the angles to be in radians. A final answer in degrees was common, but not accepted.

Answer: 0.421

### Question 6

- (i) Many candidates produced good sketches showing the functions  $5e^{-x}$  and  $\sqrt{x}$ . Very few of these sketches had any marking on them or accompanying comment to explain how the sketch indicated the presence of one root.
- (ii) This part of the question asked candidates to demonstrate the equivalence of the two equations  $5e^{-x} = \sqrt{x}$  and  $x = \frac{1}{2} \ln\left(\frac{25}{x}\right)$ . A small number of candidates gave clear and concise solutions. Many candidates did not appear to understand what they needed to do, or did not give sufficient explanation to support the given answer. Some candidates had clearly spent time practicing on similar questions from past papers and set about finding an interval containing the root of the equation, which was not relevant here.
- (iii) This part of the question can be answered independently of the previous parts, and many candidates worked through correctly, with the required accuracy, to deduce the correct answer. Those candidates who simply write down the correct answer should be aware that they will score no marks if they do not give sufficient working to demonstrate use of the required method.

*Answer:* (iii) 1.43

### Question 7

- (i) The majority of candidates recognised the need to use implicit differentiation and applied this correctly. There were some arithmetic and algebraic errors in moving from  $3x^2 - 6xy - 3x^2 \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 0$  to the given answer, and some candidates had  $\frac{dy}{dx}$  rather than 0 on the right hand side of the initial equation.
- (ii) This part of the question proved to be challenging for many candidates. It required an understanding that for the tangent to be parallel to the x-axis, they needed  $\frac{dy}{dx} = 0$  and therefore  $x^2 - 2xy = 0$ . Candidates who got as far as  $x^2 - 2xy = 0$  often overlooked the possibility that  $x = 0$  and focused on  $x = 2y$ . Most substituted this correctly into the initial equation and reached the correct solution provided they avoided arithmetic and algebraic errors.

*Answer:* (ii)  $(-2, -1)$ ,  $(0, \sqrt[3]{3})$

### Question 8

- (i) The majority of candidates split the fraction correctly into the form  $\frac{A}{x+1} + \frac{B}{x-3} + \frac{C}{(x-3)^2}$  but the alternative form  $\frac{A}{x+1} + \frac{Bx+C}{(x-3)^2}$  was also accepted. Many candidates are very proficient in completing this process, with some showing very little working to support their answers. Candidates should understand that in showing no working, if the process goes wrong they will earn no credit for having a correct method.
- (ii) Many candidates gave a correct expansion of  $(1+x)^{-1}$ . The other two terms were more demanding, requiring the steps  $(x-3)^{-1} = (-3)^{-1}\left(1-\frac{x}{3}\right)^{-1}$  and  $(x-3)^{-2} = (-3)^{-2}\left(1-\frac{x}{3}\right)^{-2}$ . Only a small number of candidates dealt correctly with both  $(-3)^{-1}$  and  $(-3)^{-2}$  and went on to reach the correct final answer.

*Answers:* (i)  $\frac{1}{x+1} + \frac{3}{x-3} + \frac{12}{(x-3)^2}$ , (ii)  $\frac{4}{3} - \frac{4}{9}x + \frac{4}{3}x^2$

### Question 9

- (i) Those candidates who had prepared to answer a question on vectors had several options in tackling the equation of the plane. The most common approach was to find two vectors parallel to the plane and then use the vector product to find the normal vector. The alternative approach of substituting the position vectors of  $A$ ,  $B$  and  $C$  into the equation to obtain three equations in four unknowns and then find the ratio of the coefficients was also popular. The most common error was to include the position vector of  $D$  in the working, when it is not given that  $D$  lies on the plane.
- (ii) Candidates with a good understanding of vectors had no difficulty in forming the equation of the line through  $D$  parallel to  $OA$  and finding  $P$ , the point of intersection of this line with the plane. Having found the position vector of  $P$ , it was relatively straight forward to find the vector  $\overrightarrow{DP}$  and hence the given length. Sufficient working (e.g.  $\sqrt{2^2 + 6^2 + 4^2}$ ) needed to be seen to demonstrate that the candidate had indeed reached the given answer correctly.

Answer: (i)  $5x - 2y + 3z = 5$

### Question 10

- (a) There was some confusion between the square and the square root of  $7 - (6\sqrt{2})i$  but the majority of candidates who had prepared to answer a question on complex numbers showed a good understanding of the correct method to find the roots. Provided they reached the correct quadratic in  $x^2$  or  $y^2$  and remembered that they were looking for two roots, candidates usually reached the correct answer. A few candidates attempted to find the square roots by first writing  $7 - (6\sqrt{2})i$  in the form  $r(\cos\theta + i\sin\theta)$ . This is a valid method for finding the square root of a complex number but in this case it is not appropriate because it does not yield the exact answer required by the question.
- (b) Many candidates produced an Argand diagram showing the correct circle for the locus  $|w - 1 - 2i| = 1$ . The half line for  $\arg(z - 1) = \frac{\pi}{4}$  proved to be more challenging. The final two marks of the paper were scored by the small number of candidates who correctly identified the point for which the value of  $|w - z|$  was least.

Answers: (a)  $\pm(3 - i\sqrt{2})$ , (b)(ii)  $\sqrt{2} - 1$

# MATHEMATICS

Paper 9709/32

Paper 32

## Key messages

The standard of work on this paper varied considerably and resulted in a wide spread of marks from zero to full marks. Nearly all questions discriminated successfully and yet were accessible to well prepared candidates. Some excellent scripts were seen but there were also scripts from candidates who appeared poorly equipped for the paper and made very little progress. The questions or parts of questions that were generally done well were **Q.2** (binomial expansion), **Q.5** (trigonometry) and **Q.8** (differentiation and iteration). Those that were done least well were **Q.1** (indicial equation), **Q.7 (i)** (partial fractions), **Q.9** (vector geometry) and **Q.10** (complex numbers).

In general the presentation of work was good, though there were some very untidy scripts. Candidates who present highly disordered work make it difficult for examiners to assess and find the best mark for the work. More importantly, untidiness and fragmentation must make it difficult for candidates to review and improve their attempts at each question.

There were questions, e.g. **Q.1**, **Q.3**, **Q.5** and **Q.7** where it was common to find candidates losing marks because of failure to use brackets correctly. Candidates need to check carefully for misuse of brackets and the algebraic errors that can ensue.

Some candidates lost marks by using a calculator to provide answers in cases where this was inappropriate. For example, in **Q.3** a correct decimal value for the integral was often seen after incorrect working, gaining no credit. In **Q.10 (a)** the correct complex roots were sometimes written down without sufficient previous working, again gaining no credit. It may be helpful to say that on this paper the five places where the use of a calculator was intrinsic to the answering of a question or part of a question were **Q.1**, **Q.5 (ii)**, **Q.8 (iii)** and **(iv)**, and **Q.10 (b) (ii)**.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

## Comments on specific questions

### **Question 1**

Most candidates began by attempting to equate the logarithms of both sides. On the left the omission of brackets in  $(3x - 1)\ln 4$  was a common error. Some changed the term on the left to  $4^{3x-1} / 3$  and followed it incorrectly with  $(3x - 1)\ln(4 / 3)$ . On the right the question gives the term as  $3(5^x)$  yet many seemed unaware of the significance of the brackets and incorrectly logged the term as  $x\ln 15$ ,  $3x\ln 5$  or as  $(\ln 3)x\ln(5)$ . There were some interesting variants. Some worked, quite suitably, with logs to base 10 or to base 4. Sometimes an attempt was made to convert the term on the right to a power of 4 and then equate the index to  $3x - 1$ .

To 6 decimal places the correct answer is 0.974685. This is closer to 0.974 than 0.975 to 3 decimal places. Thus in calculating the final answer, premature approximation risked losing the accuracy needed for the final mark. Those who delayed using calculators until they had prepared an exact answer such as  $(\ln 12 / \ln(64 / 5))$  were better placed for full marks.

Answer: 0.975

## Question 2

This was well answered. The majority converted the given expression to the form  $(1-2x)^{-\frac{1}{2}}$  and substituted correctly in the formula for  $(1+x)^n$ . A few substituted  $2x$  rather than  $-2x$  and some forgot to replace  $x$  in the formula by  $-2x$  for all the terms. The coefficients were usually simplified correctly though some left the  $x^3$  coefficient in the incomplete form  $\frac{15}{6}$ .

Answer:  $1 + x + \frac{3}{2}x^2 + \frac{5}{2}x^3$

## Question 3

This question discriminated well. Most candidates completed a first application of integration by parts with an integral of the form  $ax^2 \cos 2x + b \int x \cos 2x \, dx$  but errors in the values or signs of the constants  $a$  and  $b$  were not uncommon. Success in completing the integration needed good control of signs and brackets. For example, the sign of the final term  $\frac{1}{4}\cos 2x$  in the complete integral is the product of four minuses and was quite often incorrect. Limits were usually substituted correctly but some either failed to show the insertion of the lower limit or assumed the value there was zero without any evidence. The question asked for an exact answer but this was often given as a decimal.

Answer:  $\frac{1}{8}(\pi^2 - 4)$

## Question 4

A good number of candidates differentiated correctly using the quotient or product rule and a correct derivative of  $(\ln x)^2$ , but some interpreted  $(\ln x)^2$  as  $\ln(x^2)$  and worked with  $2\ln x$ . Those with a correct derivative usually obtained two values for  $\ln x$  though some overlooked the solution  $\ln x = 0$ . Many who obtained the solution  $x = e^2$  lost a mark by giving the corresponding  $y$ -coordinate as a decimal rather than as an exact value as requested.

Answer:  $(1, 0); (e^2, 4e^{-2})$

## Question 5

Though a significant minority failed to progress far, usually having failed to express  $\cos 4\theta$  correctly in terms of  $2\theta$ , there were a good number of correct proofs of the identity in part (i) by various routes. It was surprising that many of the candidates who began work on the left side chose to express it in terms of  $\cos 2\theta$  and  $\cos \theta$  when the target expression on the right was in terms of  $\sin \theta$ . Shorter solutions were obtained by working in terms of  $\sin 2\theta$  and  $\sin \theta$ . Attempts at proofs beginning on the right side were less frequent but some were completed successfully. In part (ii) most candidates used the result from part (i) to obtain an equation in  $\sin^4 \theta$  and many found correct solutions in the first and second quadrants, but only a minority found the solutions in the third and fourth quadrants from the negative root of the equation in  $\sin \theta$ . Premature approximation was quite common and caused loss of accuracy in the first decimal place of the values for the angles.

Answer: (ii)  $68.5^\circ, 111.5^\circ, 248.4^\circ, 291.5^\circ$

### Question 6

In part (i) most candidates separated variables correctly and integrated to obtain the term  $\ln x$ . A good number went on to find a term of the form  $k \ln(3 + \cos 2\theta)$ . The commonest incorrect values of  $k$  were  $-2$ ,

$-1$ ,  $\frac{1}{2}$ ,  $1$  and  $2$ . Most then went on to use a correct method to evaluate a constant of integration. There were some imaginative and successful approaches to the trig integral in which the integrand was rearranged as  $\frac{2\sin\theta\cos\theta}{4 - 2\sin^2\theta}$ ,  $\frac{2\sin\theta\cos\theta}{2 + 2\cos^2\theta}$ , or their simplified equivalents. In part (ii), of those with a correct expression for  $x$ , relatively few gave  $\cos 2\theta$  its greatest value in order to find the least value of  $x$ .

Answers: (i)  $x = \sqrt{\left(\frac{27}{3 + \cos 2\theta}\right)}$  (ii)  $\frac{3\sqrt{3}}{2}$

### Question 7

Part (i) was not well answered. The majority of candidates worked on an incorrect form of fractions, e.g.  $\frac{A}{2x+1} + \frac{B}{x+2}$ , and seemed unconcerned by false identities such as  $4x^2 + 7x + 4 = A(x+2) + B(2x+1)$ , which cannot be satisfied by any values of  $A$  and  $B$ . A substantial minority worked on an appropriate form of partial fractions, e.g.  $A + \frac{B}{2x+1} + \frac{C}{x+2}$ , either stating it at the beginning or reaching it after dividing the numerator of  $f(x)$  by the denominator, and were often successful in determining the constants. A few candidates worked on one of the incomplete forms  $\frac{B}{2x+1} + \frac{Dx+E}{x+2}$  and  $\frac{Fx+G}{2x+1} + \frac{C}{x+2}$ . They too were usually successful in determining the constants but invariably went no further, scoring three marks out of five for the section.

Before integrating in part (ii), some candidates made costly errors by failing to transfer their constants to the right fractions. Nevertheless, whatever the outcome of part (i), most candidates integrated their fractions successfully. Those who had correctly determined one of the incomplete forms in part (i), e.g.

$\frac{1}{2x+1} + \frac{2x+2}{x+2}$ , could have integrated and scored full marks for this section, either by first rewriting the second fraction as  $2 - \frac{2}{x+2}$  or using integration by parts, but hardly anyone did this.

Answer: (i)  $2 + \frac{1}{2x+1} - \frac{2}{x+2}$

### Question 8

This was well answered throughout. In part (iii) most candidates evaluated a relevant expression such as  $e^a - \sin a \tan a$  at  $a = 1$  and  $a = 1.5$ , but not all completed the proof by referring to the difference in the signs of the calculated values. Some believed that calculating the values of the right hand side of the given equation was sufficient when in fact comparison of the values of both sides was essential.

Answers: (iv) 1.317

### Question 9

- (i) Some attempts failed from the beginning by taking  $\overline{OB}$  to be (4, 0, 1) in component form, or by using a diagram with cyclic form  $ABDC$  rather than  $ABCD$  as stated in the question. Those that succeeded in finding the position vector of  $D$  used either vector addition, e.g.  $\overline{OD} = \overline{OC} + \overline{CD}$ , or the fact that  $AC$  and  $BD$  have the same midpoint. A slower method was to set up and solve vector equations for  $AD$  and  $CD$ . A common error was to equate  $\overline{CD}$  to  $\overline{AB}$  rather than  $\overline{BA}$ . To show  $ABCD$  to be a rhombus candidates only needed to compare the lengths of a pair of adjacent sides such as  $AB$  and  $AC$  or show that the diagonals  $AC$  and  $BD$  were perpendicular. A common error was to think that the equality of a pair of parallel sides was sufficient.
- (ii) This was answered fairly well. A normal vector  $(a, b, c)$  for the plane was usually found by calculating the vector product of  $\overline{OA}$  and  $\overline{BC}$  or solving a pair of relevant equations such as  $a + 2b + 3c = 0$  and  $2a + b - 2c = 0$ . The fourth constant  $d$  was found by inserting the coordinates of  $B$  or  $C$  in the equation. Some candidates lacked a sound method for finding the normal, sometimes taking it to be  $\overline{OA}$  or  $\overline{BC}$ . Some tried to use  $A$  to determine  $d$ .

Answers: (i)  $3\mathbf{i} + 3\mathbf{j} + \mathbf{k}$  (ii)  $-7x + 8y - 3z = 29$

### Question 10

- (a) Most candidates used the quadratic formula to find the roots. Errors in the initial application were common. In particular the simplification of the discriminant was often incorrect. Using this method the roots first emerged with a complex denominator,  $2i$  or  $i$ , but then some candidates found it difficult to convert them to the required form. The preliminary step of multiplying the equation by  $i$  to give  $z^2 - 2iz - 3 = 0$  avoided this problem. Some substituted  $x + iy$  for  $z$  in the given equation and equated real and imaginary parts to zero, but only a few found the correct roots this way. Correct roots without the necessary preliminary working were seen and gained no credit.
- (b) (i) The majority plotted the point representing  $4 + 3i$  and attempted to draw the perpendicular bisector of the line joining it to the origin. Those with unequal scales on the axes found this difficult. An alternative approach was to find and plot a Cartesian equation for the locus, e.g.  $8x + 6y = 25$ . A common error was to take the locus to be a circle with centre at the above point
- (ii) This was well answered by those with a correct diagram.

Answers: (a)  $\sqrt{2} + i, -\sqrt{2} + i$  (b)(ii)  $2+1.5i; 2.5, 0.64$

# MATHEMATICS

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Paper 9709/33

Paper 33

## General comments

The standard of work on this paper varied considerably and resulted in a wide spread of marks from zero to full marks. All questions discriminated successfully and yet were accessible to well-prepared candidates. The questions or parts of questions that were generally done well were **Question 1** (inequalities), **Question 3** (trigonometry), **Question 6** (numerical roots of an equation), **Question 8(i)** (intersection of lines), **Question 9(i)/(ii)** (Argand diagram and division of complex numbers), **Question 10(i)** (partial fractions). Those that were done least well were **Question 2** (logarithms), **Question 5** (differential equation), **Question 7(ii)** (integration), **Question 8(ii)** (perpendicular distance from point to line) and **Question 10(ii)** (binomial expansions).

It was pleasing to see that both teachers and candidates had taken on board many of the comments stressed in recent reports, namely that when attempting a question candidates need to be aware that it is essential that sufficient working is shown to indicate how they arrive at their answers, whether they are working towards a given answer or an answer that is not given. This was strongly emphasised in relation to the details required when solving a quadratic equation and when substituting limits into an integral.

However, there are two extremely important points that teachers and candidates need to address. The first arises from the very weak understanding that candidates have of the word '**exact**' (see **Question 2(i)/(ii)**, **Question 7(ii)**, **Question 8(ii)** and **Question 9(iii)**). Candidates need to choose a **method** that leads to an **exact answer**, as in **Question 9(iii)**, (see specific comments on this question), and to continue working with exact arithmetic as in **Questions 2(i)/(ii)**, **7(ii)** and **8(ii)**. If candidates opt for a method that is only possible with the use of the calculator then no marks are possible. Likewise if candidates have a suitable approach but then introduce decimals, as in **Question 2** (logarithms), then again from that point onwards no further marks are possible. The other point relates to accuracy, see **Question 4(ii)** and the final part of **Question 5**, where to acquire answers accurate to 3 significant figures required candidates to work to a much higher degree of accuracy.

Where numerical and other answers are given after the comments on individual questions it should be understood that alternative forms are often acceptable and that the form given is not necessarily the only 'correct answer'.

## Comments on specific questions

### **Question 1**

Most candidates opted to square and then to factorise. Although some omitted to square the 2, hence producing an incorrect quadratic equation. However, provided they showed the required working they were still able to obtain a method mark. Factorisation was usually correctly undertaken, as was the presentation of the final answer. The odd candidate had the symbol  $\leqslant$  instead of the symbol  $<$ .

$$\text{Answer: } -5 < x < \frac{3}{5}$$

## Question 2

- (i) Although told to take logarithms and to obtain an exact value of the gradient of the line many didn't and went straight to decimal values. Answers to all bases were accepted in this part of the question, however using bases other than 10 or e when solving (ii) meant that simplification of the final answer was difficult. Rare to see any candidate compare their expression with the form  $ay = bx + c$  and then state hence a straight line.
- (ii) Most candidates knew what was required but since all their coefficients were expressed as decimals they avoided the necessary work of combining logarithms. Common answers were  $x = 0.773$  or  $0.774$  for which no credit was given.
- (iii) NB: Notation  $\log_3 4$  is not  $\log \frac{4}{3}$

**Answer:** (i)  $y\ln 3 = 2\ln 4 - x\ln 4$ ,  $- \frac{\ln 4}{\ln 3}$ ; (ii)  $x = \frac{\ln 4}{\ln 6}$  or  $\log_{36} 16$ , etc.

## Question 3

- (i) Most candidates obtained  $R = 3$  and  $\alpha = 41.81^\circ$ . Although a few candidates had  $\alpha = 48.19^\circ$ .
- (ii) Evaluation of  $\cos^{-1}(0.4)$  was usually correct, however finding  $x$  resulted in numerous errors. Often  $\frac{1}{2}x$  was never seen, only  $x$ . Sometimes only  $\cos^{-1}(0.4)$  was multiplied by 2, other times final result was divided by 2 instead of being multiplied by 2. Occasionally candidates found another solution in the interval  $0^\circ < x < 360^\circ$ .

**Answer:** (i)  $R = 3$   $\alpha = 41.81^\circ$ ; (ii)  $216.5^\circ$

## Question 4

- (i) Candidates found this part relatively straight forward, although  $\frac{dx}{dt} = -\sin t$  and  $\frac{dy}{dt} = \frac{1}{1+\sin t}$  were commonly seen. Most candidates showed sufficient working to justify the given answer.
- (ii) Most reached as far as a  $t$  value, and some managed  $x = 1.56$ , however few obtained  $-0.898$ . As mentioned in general comments too often inaccurate answers, such as  $-0.897$ ,  $-0.896$  or  $-0.9$ .

**Answer:** (ii)  $1.56$ ,  $-0.898$

## Question 5

This question was usually done very well or very badly. Most candidates could separate variables, but then had trouble reaching  $e^{2y}$ . Too many tried to integrate  $\frac{1}{e^{-2y}}$  directly and incorrectly finished with  $\ln$  of some exponential expression. The  $x$  term also caused problems with little progress if candidates failed to spot that Pythagoras was required. Another common error in the  $x$  term was a muddle of signs. Those candidates who were able to make a positive attempt at both integrals, apart from odd slips, were able to score the method mark for the evaluation of the constant term, although a few candidates failed to introduce any constant when performing their integration.

The final part needed the substitution of  $x = \frac{\pi}{4}$  into their expression, provided that it had a term containing  $e^{\pm 2y}$  in order to require the necessary taking of logarithms work. Since candidates had often acquired errors in their expressions many failed to clearly show what they were doing and without seeing the substitution of  $x = \frac{\pi}{4}$  and a statement ' $y = 2\ln(\dots\dots)$ ' this mark was unavailable. Simply stating an incorrect value  $y$ , although candidates may have performed the work on their calculator, cannot be rewarded.

Again, as in **Question 4**, failure to work to the accuracy required resulted in incorrect answers of 0.18 or 0.178 instead of the correct answer of 0.179.

Answer:  $\frac{1}{2}e^{2y} = \tan x - x + \frac{1}{2}; 0.179$

### Question 6

- (i) Candidates found this part relatively straight forward. Although some had to fudge their answer since incorrectly differentiated  $\cos \frac{1}{2}x$ . Most candidates showed sufficient working to gain the mark for the given answer.
- (ii) Candidates correctly used a variety of expressions to establish this result. However, the question clearly states 'by calculation' hence it is necessary to state the function that one is using and the resulting numerical values, not just the function is  $< 0$  and  $> 0$ .
- (iii) Very well done, the only error usually seen was that 2.1551 followed by 2.1532 shows convergence to 2.15.

Answer: (iii) 2.15

### Question 7

- (i) Most candidates successfully found  $\frac{du}{dx}$  and substituted for  $x$  and  $dx$ . However, with the answer being given many failed to show sufficient working in regard to the indices work, also in relation to the change of limits. Just changing limits when the given integral was in terms of  $x$  to those in the given integral when it was in terms of  $u$  was not sufficient without some simple calculation to show that this operation had actually been performed. Some candidates encountered errors in their indices.
- (ii) The obvious approach was to split  $I$  into three fractions, from which it was easy to convert to three terms, each simple to integrate. Unfortunately many candidates who followed this approach converted  $\frac{1}{2u^3}$  to  $2u^{-3}$ . The same error was also regularly seen for those who integrated by parts. Occasionally candidates inappropriately decided to try to integrate the original integral instead of using the hint given in (i). The substitution of limits was usually shown in sufficient detail to acquire the method mark, however, many then decided to use their calculator giving their exact answer as 0.0341, so failed to score the final accuracy mark.

Answer:  $\frac{1}{2}\ln 2 - \frac{5}{16}$

### Question 8

- (i) The vector **OB** caused the weaker candidates to use  $2\mathbf{i} + 3\mathbf{j}$  instead of  $2\mathbf{i} + 3\mathbf{k}$ . Many candidates gave the direction vector **AB** as the equation of the line through points A and B. Far too many candidates simply equated the given line with point A or point B. It was insufficient simply to solve two of the three equations and then state that the lines did not intersect, since the question states that the lines don't intersect. A full checking that the parameters failed to satisfy the third equation was necessary.

When solving equations it needs to be systematically, that is, solve the first two equations for **both parameters** and then check whether these parameters satisfy the third equation; or having solved the first two equations for one parameter then solve another pair of equations and establish whether the new parameter values agree.

- (ii) Another question that was usually very well done or very badly done. Obviously there are numerous methods for solving this type of problem, see Mark Scheme for a sample of these. However, most of these require one to establish a vector from A to a point C on the given line prior to taking the scalar or vector product with the direction vector of the line. Too many candidates opted incorrectly to use either vector **AB** or vector **OA** instead of vector **AC** in this operation. Unfortunately, those candidates who decided to use the scalar product approach then needed to use Pythagoras. This is perfectly possible to undertake without a calculator, but most candidates decided that with square roots/etc. to use the calculator so preventing them establishing the exact given answer.

**NB**  $0.70710$  is not  $\frac{1}{\sqrt{2}}$

### Question 9

- (i) The values of  $u$  and  $v$  and the diagram were almost always correct. Most candidates stated that OB and AC were parallel but many failed to state that their lengths were equal.
- (ii) This part was well done with very few errors. The odd error that did occur was when the denominator was evaluated as  $2^2 - i^2 = 3$ .
- (iii) This part was done very badly. Some of the most frequent errors were

$$\text{Angle } AOB = \frac{\pi}{2} + \tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{3}\right) = \frac{3}{4}\pi$$

$$\text{Angle } AOB = \pi - \tan^{-1}(3) + \tan^{-1}\left(\frac{1}{2}\right) = \frac{3}{4}\pi$$

These expressions were often produced without any effort being made to evaluate them and to justify them being equal to the given exact answer. Even if such calculations had been undertaken the use of a calculator would have been required, see question rubric. However, even had the question rubric not prevented the use of a calculator, the exact answer given would have made such a method completely inappropriate in this situation.

A few candidates calculated the lengths of OA, OB, OC, AC, etc., all possible using simple arithmetic, and then went on to successfully obtain some of the angles in the diagram, for example

and angle COB =  $\frac{\pi}{2}$  and angle AOC =  $\frac{\pi}{4}$ . Likewise using the scalar product or the cosine rule to

establish that the angle AOB =  $\cos^{-1}\left(-\frac{1}{\sqrt{2}}\right)$  was appropriate.

There were many incorrect statements seen involving  $\arg u$  and  $\arg v$  as wrongly evaluated positive acute angles. Only a few candidates obtained the most simple and direct solution, which equated the required angle AOB to the argument of the complex number found in (ii), namely angle AOB =

$$\arg\left(\frac{u}{v}\right) = \tan^{-1}(-1) = \frac{3}{4}\pi. \text{ Some attempts at this form of solution were incorrect due to } \tan^{-1}(-1)$$

being evaluated as  $-\frac{\pi}{4}$

**Question 10**

- (i) Most candidates showed a good understanding of the methods for finding partial fractions and the vast majority selected the correct forms. However, the odd candidate did introduce four unknown constants into their three partial fractions and hence were unable to acquire a unique answer. Fortunately those who included an un-necessary unknown whole number constant, which they then found to be zero, were able to recover the partial fraction mark.
- (ii) Many candidates miscopied when commencing this section, so  $(x - 3)^{-1}$ ,  $(x + 1)^{-1}$  and  $(x + 1)^{-2}$  all seen too often. Most errors occurred when attempting to rearrange the denominators of the partial fractions to the form  $(1 \pm y)^n$ , where  $n = -1$  or  $-2$ . Very common to see  $(x - 1)^{-2}$  become  $-(1 - x)^{-2}$ .  
The term  $-3(x + 3)^{-1}$  often incorrectly became  $-9(1 + \frac{x}{3})^{-1}$

Answer: (i)  $\frac{-3}{x+3} + \frac{1}{x-1} + \frac{2}{(x-1)^2}$  or  $\frac{-3}{x+3} + \frac{x+1}{(x-1)^2}$ ; (ii)  $\frac{10}{3}x + \frac{44}{9}x^2$

# MATHEMATICS

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Paper 9709/41  
Paper 41

## General comments

There were some excellent candidates who produced very good answers on this paper. However, a significant number of candidates were quite badly prepared for the examination.

Some candidates lost marks due to not giving answers to 3 sf as requested and also due to prematurely approximating within their calculations leading to the final answer. Candidates should be reminded that if an answer is required to 3 sf then their working should be performed to at least 4 sf. In **Questions 4 and 5** the tangent of an angle was given in the question. In these questions it was not necessary to determine the actual angle to 1 dp as this often leads to premature approximation and frequently also to loss of accuracy marks.

One of the rubrics on this paper is to take  $g = 10$  and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve the correct answer unless this value is used.

## Comments on specific questions

### Question 1

- (i) Most candidates produced good diagrams for this part in the shape of a trapezium. Candidates should be reminded that they should correctly and clearly annotate their diagrams in order to obtain full marks for the question. An error that was seen was to use 0.9 as the constant speed reached on the  $v$ -axis

Answer: Trapezium with 0, 3, 9 and 13 shown on the  $t$ -axis. 2.7 shown on the  $v$ -axis

- (ii) Again most candidates attempted to evaluate the area enclosed by the trapezium in order to calculate the distance travelled by the lift.

Answer: Total distance travelled = 25.7 m

### Question 2

- (i) Many candidates found the correct work done against friction by multiplying the friction force of 40 N by the distance travelled of 36 m. Some wrongly introduced the angle of 20 degrees which was not relevant in this calculation.

Answers: Work done against friction = 1440 J

- (ii) In this part of the question candidates needed to evaluate the change in potential energy as  $PE = mgh$  where  $h = 36 \sin 20$ . Some forgot to include the  $\sin 20$  component in their answer.

Answer: Change in GPE = 3080 J

- (iii) The quickest method for this part was simply to add together the answers to parts (i) and (ii). However many candidates worked out the pulling force as the component of the weight plus the frictional force and then multiplied this expression by the distance 36 m. This was longer but a perfectly correct method.

Answer: Work done by the pulling force is 4520 J

### Question 3

- (i) Most candidates approached this part correctly. As the car was travelling at constant speed, the required driving force is equal to the resistance force of 300 N. The rate at which the engine is working, is in fact the power and this is found by multiplying the driving force by the speed of  $40\text{ ms}^{-1}$ . Some did not give the answer in the required units of kW

Answer: Rate at which the engine is working is 12 kW

- (ii) Most candidates found 90% of the power found in part (i) and used this to determine the driving force but some forgot to include the resistance when using Newton's law to find the required acceleration.

Answer: The acceleration of the car is  $0.132\text{ ms}^{-2}$

### Question 4

The best method of approach here was to resolve forces vertically and horizontally. However, even though it was clear that this was being attempted, many did not show their working in a clear manner, often setting up a table of components but not combining them correctly. For those who found values for  $P \cos \theta$  and  $P \sin \theta$  the next step was either to divide and set up an equation for  $\tan \theta$  or to use Pythagoras to eliminate  $\theta$ . Most candidates attempted this and some excellent solutions were seen but often incorrect components had been found and so these candidates were only able to score method marks.

Answers:  $P = 48.1$     $\theta = 28.7$

### Question 5

Many candidates found this to be a difficult question. It was necessary to resolve forces acting on both particles. Often it was unclear as to which direction the candidate was assuming the motion was taking place. It is necessary to be consistent with the directions of travel of the two particles. When the two equations of motion were stated they were found to be simultaneous equations and elimination methods could be used to find the required values of the acceleration and the tension.

Answers:  $a = \frac{10}{3} = 3.33\text{ ms}^{-2}$     $T = \frac{200}{3} = 66.7\text{ N}$

### Question 6

- (i) Most candidates knew that they needed to differentiate the given expression for  $v$  to find the acceleration. Many candidates found the critical value of 2.5 seconds but some were confused when attempting to set up the inequality.

Answer:  $t < 2.5$

- (ii) It was first necessary to determine the times at which the particle was at instantaneous rest and most found these as  $t = 1$  and  $t = 4$ . Most candidates knew that they had to integrate the given expression for  $v$  between limits of 1 and 4 to determine the displacement. This answer was negative and some had problems justifying the change in sign to find the distance travelled between the two times.

Answer: Distance between the two positions = 27 m

- (iii) Most candidates who attempted this part knew that they needed to set the expression found in part (ii) for displacement to zero. This gave a cubic equation in  $t$  but a factor of  $t$  could be taken out reducing it to a quadratic. It was necessary to use the formula to solve the problem.

Answer: The two values of  $t$  at which the particle passes through O are  $t = 2.31$  and  $t = 5.19$

### Question 7

There were some excellent answers seen for this question but many candidates did not score many marks on this part.

- (i) (a)** This part required candidates to apply Newton's second law along the direction of the plane with the forces acting being the given 200 N force up the plane and the weight component of the particle acting down the plane. Some candidates forgot to include the weight component.

Answer: Acceleration of the particle =  $3.25 \text{ ms}^{-2}$

- (i) (b)** Once the acceleration of the particle had been found in part **(i)** the velocity could be found using the constant acceleration formula  $v^2 = 2as$  where  $s = 12$  giving  $v^2 = 78$ . An alternative method was to use the work/energy principle where KE gain = WD by driving force – PE gain although very few used this method.

Answer: Change in kinetic energy = 1170 J

- (ii) (a)** This part was not generally well done. Candidates first needed to find the normal reaction and use this in the equation  $F = \mu R$ . The equation of motion along the plane now involves three forces, the 200 N applied force, the weight component of the particle and the friction force. Equating these forces to mass times acceleration gave the required acceleration.

Answer: The acceleration of the particle is  $2.12 \text{ ms}^{-2}$

- (ii) (b)** This part was found to be difficult by many candidates. As the driving force now makes an angle with the plane, the normal reaction now involves a component of this force in addition to the weight component. Again once the normal reaction is found it can be used in the equation  $F = \mu R$ , giving a different friction force to that in part **(ii) (a)**. The equation of motion along the plane now involves three forces, the  $200 \cos 10^\circ$  N applied force, the weight component of the particle and the new friction force. Again applying Newton's second law gives the required acceleration.

Answer: The acceleration of the particle is  $2.16 \text{ ms}^{-2}$

# MATHEMATICS

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Paper 9709/42  
Paper 42

## General comments

The paper was generally well done by many candidates although as usual a wide range of marks was seen.

The presentation of the work was good in most cases and as the papers are now scanned, it is important to write answers clearly using black pen.

Some candidates lost marks due to not giving answers to 3 sf as requested and also due to prematurely approximating within their calculations leading to the final answer, particularly in **Questions 1, 2, 3, 5, 6 and 7**. Candidates should be reminded that if an answer is required to 3 sf then their working should be performed to at least 4 sf.

In **Questions 1 and 3** the sine of a required angle was given. In both cases it was not necessary to calculate the angle itself as the sines and cosines required could be evaluated exactly. However, many candidates often proceeded to find the relevant angles to 1 decimal place and immediately lost accuracy and in some cases marks.

One of the rubrics on this paper is to take  $g = 10$  and it has been noted that virtually all candidates are now following this instruction. In fact in some cases it is impossible to achieve the correct given answer unless this value is used.

## Comments on specific questions

### **Question 1**

Many candidates immediately found the angle  $\alpha$  as 36.9 degrees and subsequently lost accuracy marks as it is possible to solve the problem using the given value of  $\sin \alpha$  along with  $\cos \alpha = 4/5$ . Most candidates resolved forces horizontally and vertically although some resolved along the directions of the 6 N and 8 N forces which was also perfectly acceptable. This question was fairly well attempted by most candidates, but full marks were only scored by those who used exact values of  $\sin \alpha$  and  $\cos \alpha$ .

*Answer:* Magnitude of the resultant is 3N. The direction of the resultant is in the negative x-direction

### **Question 2**

- (i) Almost all candidates performed well on this part of the question, by setting the given expression for  $v$  to zero and solving the resulting quadratic equation.

*Answers:*  $t = 0.5$  and  $t = 1.5$

- (ii) Most candidates integrated the given expression with respect to  $t$  between the limits of 0.5 and 1.5 but many failed to realise that this integral gave the displacement  $s$ , and this resulted in a negative value of  $-2/3$ . The final mark for this part was only awarded to those who justified the change of sign, by stating that the distance travelled was  $2/3$

*Answer:* The distance travelled between the two times was  $2/3$  m

### Question 3

- (i) This part of the question was generally well done by candidates who used the definition that  $PE = mgh$ , where  $h = x \sin \alpha$  and the value of  $\sin \alpha$  given in the question was used. Again some found the angle  $\alpha$  which lead to some inaccuracy. Some candidates wrongly used the distance  $x$  up the plane rather than  $h$ .

Answer: Change in gravitational potential energy of the particle =  $400 x/13 = 30.8x$

- (ii) The question asked for an energy approach and most candidates followed this method obtaining expressions for the work done against the constant resistance as  $15x$  and the loss in kinetic energy of the particle as  $\frac{1}{2} \times 8 \times 5^2 = 100$ . The GPE had been found in part (i) and the method was to use the work/energy equation in the form  $KE \text{ loss} = GPE \text{ gain} + WD$  against resistance. Some good solutions were seen. Some candidates forgot to include the effect of the resistance and others included extra terms in their energy equation.

Answer:  $x = 2.18$

### Question 4

- (i) Most candidates correctly found the required distance by using the area property under the  $v-t$  graph. Very few incorrect answers to this part were seen.

Answer: The distance covered in the first 42 seconds = 319.8 m

- (ii) In this part it was first necessary to determine the remaining distance to be run over the final 10 seconds. Even though the 3sf answer to part (i) was 320 m for which full marks were scored, it was necessary to use the exact answer of 319.8 m to part (i) in order to find the remaining distance as 80.2 in order to obtain the correct value of  $V$ . The most straightforward method was to equate this remaining distance to the area under the curve between  $t = 42$  and  $t = 52$ . Some candidates found the deceleration and then used the constant acceleration equations to find the distance. This was a longer method but perfectly correct. It should be noted that when an answer is given as in this case, full detail should be shown in the solution in order to justify the answer.

Answer: Answer given as  $V = 7.84$

- (iii) Candidates again scored well in this part. Once the answer to part (ii) had been found, either use of the gradient of the graph or the constant acceleration equations could be used. Some candidates were confused over the sign of the answer. The question asked for the deceleration and this answer is positive, whereas the acceleration over this stage is negative.

Answer: Deceleration =  $0.036 \text{ ms}^{-2}$

### Question 5

Candidates found this question to be quite challenging. An error which occurred frequently was to assume that the normal reaction at the block was  $25 \cos 30$  whereas in fact the reaction also involved a component of  $T$ . Most candidates resolved forces along and perpendicular to the plane. Some also wrongly included an acceleration even though it was stated that the block was in equilibrium. Some good answers were seen and those who successfully found the correct answer showed impressive accuracy in their calculations.

Answer:  $T = 17.5$

### Question 6

- (i) (a) Most candidates successfully realised that the required driving force, for constant speed, was equal to the resistive force and then used the formula  $P = Fv$  to find the power. Some failed to give their final answer in kW

Answer:  $P = 62 \text{ kW}$

- (i) (b) In this part it was first necessary to determine the new driving force acting. The new power is 400 000 W and the driving force is  $40\ 000/40$ . Many forgot that the speed was still  $40\ \text{ms}^{-1}$  and found a new speed. Some also forgot to include the resistive force when finding the deceleration.

Answer: Instantaneous deceleration of the car is  $0.5\ \text{ms}^{-2}$

- (ii) For this part of the question it was necessary to find the driving force required to maintain a constant speed. Two effects had to be included, namely the resistance force and the component of the weight of the car down the slope. Some candidates forgot to include one or other of these effects. Once these two forces had been combined, the constant speed could be found using  $v = P/F$

Answer: Constant speed =  $26.0\ \text{ms}^{-1}$

### Question 7

- (i) Almost all candidates approached this question by applying Newton's second law to each of the particles. Many good solutions were seen to this problem. An error that was often seen was when applying Newton's law to particle A along the direction of the string, the weight was included as one of the forces when in fact it was at right angles to this direction. Most correctly found the acceleration and then used the constant acceleration equations to find the required time. Although it was possible to use energy methods to solve the problem, very few adopted this approach.

Answer: Time taken to reach the ground is  $\frac{\sqrt{6}}{6} = 0.408$  seconds

- (ii) Candidates found this part to be quite difficult. Again the most common approach to the first stage of the motion was to apply Newton's second law to both particles and similar errors with regard to the weight component were seen. Most successfully found the friction force acting on particle A and then, using Newton's second law applied to both particles, attempted to find the acceleration. A common error in part (ii) was to use the values of acceleration or time found in part (i). In this case it was necessary to determine the velocity of the particles as B hit the ground and then to use this value as the initial velocity of A as it decelerated to rest.

Answer: Total distance travelled by A is 1.1 m

# MATHEMATICS

Paper 9709/43

Paper 43

## Key messages

- Non-exact numerical answers are required correct to three significant figures as stated on the question paper. Candidates are also reminded to maintain sufficient accuracy in their working to achieve this level of accuracy in their final answers.
- Candidates are reminded of the importance of a clear and complete force diagram in problem solving and a consideration of all forces when resolving or when writing down equations of motion.

## General comments

This examination proved to be straightforward for many candidates. There was much work of a very high standard including many excellent, well presented scripts. **Question 3** was found to be the easiest question whilst **Question 2(ii)**, **Question 5(iii)** and **Question 7(ii)** provided the most challenge.

## Comments on specific questions

### Question 1

- (i) This question was straightforward for the majority of candidates who frequently gained full marks. Errors were usually due to the use of  $h = 20$  instead of  $h = 20 \sin 30^\circ$  leading to 1600 J, or the use of  $\cos 30^\circ$  instead of  $\sin 30^\circ$  leading to 1386 J.
- (ii) Candidates needed to distinguish between force and work done. Some candidates calculated the difference between the total work done and the gain in potential energy (346 J), omitting to divide by the distance travelled in order to find the frictional force. Others found the total force (57.3 N) but omitted to subtract the component of weight down the plane.

Answer: 800 J 17.3 N

### Question 2

- (i) Nearly all candidates applied constant acceleration formulae accurately although a few found only one of the two quantities.
- (ii) Part (ii) proved to be more challenging. Candidates attempted to form an equation in  $t$  but the equation was often incorrect due to the different starting times for Alan and Ben. It was common to see equations such as  $4t = 15 + 6(t - 5)$  using a 5 second time difference for the constant speeds rather than a 10 second time difference.

Answer: 15 m 6 ms<sup>-1</sup> 90 m

### Question 3

This question was answered very well by most candidates and fully correct solutions were common. Candidates knew how to resolve forces in equilibrium to form simultaneous equations in  $P$  and  $\theta$ . The few errors that occurred were usually due to a trigonometric slip when resolving, or difficulty in solving the simultaneous equations. Accuracy was not usually an issue, but candidates should beware of approximating early e.g.  $\sin \theta = 12.9/13.7$  leading to  $\theta = 70.3$  instead of 70.8. A significant number of candidates found horizontal and vertical components rather than forming simultaneous equations and were then able to obtain  $P$  and  $\theta$  successfully using Pythagoras' Theorem and trigonometry.

Answer: 13.7 70.8

### Question 4

Most candidates attempted to resolve the forces parallel and perpendicular to the plane as expected and often found both values of  $X$  accurately and concisely.  $X$  was sometimes shown acting **down** the plane, and occasionally shown in a horizontal direction instead of parallel to the plane, leading to a more complex situation. Those who started with  $X$  acting **down** the plane either stated  $X = -23.1$ , usually with no explanation for the negative value, or solved their equation making a sign error and concluding  $X = 23.1$  as given in the question. Some candidates found only one answer rather than a least **and** a greatest value for  $X$ . A few stated  $X = 15g \sin 20^\circ$ , assuming no friction despite the rough plane. Those who attempted to form inequalities rather than equations often experienced difficulty with the direction of the inequality, concluding e.g.  $X \leq 23.1$ .

Answer: 79.5

### Question 5

This question differentiated well between candidates with most able to answer part (i) and a variety of errors seen in parts (ii) and (iii).

- (i)  $P = DF \times v$  was usually applied accurately with a driving force of 650 N for the situation of constant speed. A few candidates erroneously included the weight e.g.  $v = 20000/14000$  or  $v = 20000/(14000 + 650)$ .
- (ii) Some candidates had difficulty in transferring from the horizontal road to the hill, continuing with  $DF = 650 \text{ N}$  or using  $DF = 1400g \sin \theta$ . Common incorrect answers were  $P = 20000 \text{ W}$  from  $P/v = 1400g \sin \theta$  (no resisting force), and  $P = 135000 \text{ W}$  from  $P/v = 1400g \sin \theta - 650$  (resisting force acting up the hill). Other errors included a misuse of  $\sin \theta = 1/7$  e.g.  $\sin 1/7$  or  $\cos(\sin^{-1} 1/7)$  used instead of  $\sin \theta$ .
- (iii) Whilst nearly all candidates calculated 80% of the power found in part (ii) as required, many oversimplified the new situation of acceleration **down** the hill, omitting either the weight component or the resisting force or both. Thus, common incorrect answers were  $a = 0.293 \text{ ms}^{-2}$  from  $21200/20 - 650 = 1400 a$ , and  $a = 0.757 \text{ ms}^{-2}$  from  $21200/20 = 1400 a$ . Some candidates found a deceleration, ( $a = -1.14 \text{ ms}^{-2}$ ), following a sign error in  $21200/20 - 650 - 1400g \sin \theta = 1400 a$  suggesting uphill motion.

Answer:  $30.8 \text{ ms}^{-1}$   $26500 \text{ W}$   $1.72 \text{ ms}^{-2}$

### Question 6

- (i) Many candidates found this to be a straightforward pulley question. The most successful method of solution was the application of Newton's Second law to each particle to form simultaneous equations in  $T$  and  $a$ , often leading to full marks. Some candidates calculated the time correct to two instead of three significant figures and a few used  $g$  as the acceleration of the system. Those who attempted to form a single equation in either  $T$  or  $a$  made more errors, usually either with signs or oversimplifications such as  $1.3g - T = T - 0.7g$ .

- (ii) Candidates were expected to consider the two stages of motion. Errors occurred when the acceleration for the connected particles was taken as  $g$ , and also when the acceleration from part (i) was used after one particle reached the plane. The most frequently seen incorrect answer was 2.6 m, found by adding 2 m instead of 4 m to the 0.6 m distance travelled after the 1.3 kg particle reached the plane. Although the vast majority of candidates attempted solutions using constant acceleration formulae as expected, there were also some attempts at using a work/energy method which depended on considering the work done by the tension as well as PE and KE for the first stage of motion.

Answer: 9.1 N 1.15 s 4.6 m

### Question 7

- (i) The majority of candidates recognised that deceleration required the solution of  $6t - 2 < 0$ . Incorrect answers included  $t < 3$ ,  $t \leq 1/3$ ,  $t = 1/3$  and  $t > 1/3$ .
- (ii) This part proved to be more difficult. Most candidates recognised that integration was needed to find  $v(t)$  and  $s(t)$ , but were often unable to find the two constants of integration. Having obtained  $v = 3t^2 - 2t + C$ , many attempted to find  $C$  either by assuming  $t = 0$ ,  $v = 0$  leading to  $C = 0$  or by assuming constant speed  $t = 1$ ,  $v = 7/1 = 7$  leading to  $C = 6$ . Some attempted to use constant acceleration formulae to find  $C$  despite the varying acceleration. Those who correctly attempted to integrate  $3t^2 - 2t + C$  sometimes obtained  $3t^2 - 2t + Ct + C$  rather than involving a second constant of integration and then continued to find **two** expressions for  $s$ .
- (iii) Candidates usually attempted to solve  $v(t) = 10$  as required but frequently used an erroneous  $v(t)$  from part (ii).  $t = 2.19$  was often found as the solution to  $3t^2 - 2t = 10$  following a zero or omitted constant of integration.

Answer:  $t < 1/3$   $s = t^3 - t^2 + 2t + 5$  2s

# MATHEMATICS

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Paper 9709/51

Paper 51

## General comments

Fewer candidates than in the past lost marks because they gave answers to 2 significant figures instead of 3 as requested on the question paper.

A small number of candidates lost marks due to premature approximating within their calculations leading to an incorrect answer.

Candidates should be reminded that if an answer is required to 3 significant figures then their working should be performed to at least 4 significant figures.

Very few candidates now use  $g = 9.8/81$ . The instructions on the question paper ask for  $g = 10$  to be used.

Occasionally candidates use a wrong formula. A formula booklet is provided so it is good to keep a check on it.

The easier questions proved to be **1** and **3**.

The harder questions proved to be **2**, **4(ii)** and **7**.

## Comments on specific questions

### **Question 1**

This question was quite well done. It could be solved in numerous different ways. The most frequently seen method was to find  $v$ , the vertical velocity, at the point where the speed is  $12 \text{ m s}^{-1}$ . The use of Pythagoras's theorem gives  $v^2 = 12^2 - (16\cos 45)^2$  leading to  $v = 4$ . Then  $v = u + at$  is used to give  $-4 = 4 - gt$ . Hence  $t = 0.8 \text{ s}$ .

*Answer:*  $0.8 \text{ s}$

### **Question 2**

This question caused quite a lot of problems. It involved finding distances using trigonometry.

- (i) Some candidates used the formula for a semi-circular lamina and not a wire. A clear diagram with the required measurements marked would have been very helpful. The required distance is  $OG\cos 70 + 0.4\sin 70$  where  $OG$  is the centre of mass of the wire from the mid-point  $O$  of  $AB$ .
- (ii) To find  $W$  it is necessary to take moments about  $A$  so that the forces on the hinge are not involved.

*Answers:* (i)  $0.463 \text{ m}$  (ii) Weight =  $21.2 \text{ N}$

### Question 3

This question was quite well done.

- (i) Most candidates arrive at the given answer.
- (ii) A number of candidates were unable to integrate  $e^{-x}$  correctly. Occasionally the limits were incorrectly used or  $c = 0$  was just assumed.

Answers: (i)  $v dv/dx = 5 - 2e^{-x}$  (ii)  $v = 2.05$

### Question 4

- (i) This part of the question was generally well done. Candidates realised they needed to use  $F = \mu R$ , where  $F$  = friction force and  $R$  = normal reaction.
- (ii) This part proved to be rather difficult. Let  $A$  be the point of contact of the cylinder at the point nearest the bottom of the plane. The weight would then act through  $A$ . The centre of mass of the whole solid can be found in terms of  $x$ , where  $x$  is the required height of the cylinder. By taking moments about  $O$ , the centre of the base of the cylinder, would give  $Wx/2 + W(1.1 + x) = 2W \times OG$ , where  $OG = 0.4\tan 70^\circ$ .  $OG$  being the distance of the centre of mass of the solid.  $x$  can now be calculated.

Answers: (i) Least value of  $\mu = 0.364$  (ii) Greatest height of cylinder = 0.372 m

### Question 5

This question was quite well done.

- (i) By using horizontal and vertical motion, the 2 velocities can be found. Then by Pythagoras, the magnitude of the resultant velocity can be calculated. The required angle can be found by using the trigonometry of a right angled triangle.
- (ii) Vertical motion gives the vertical velocity as the particle hits the ground. The horizontal velocity remains unchanged. The required angle can now be found.

Answers: (i) Speed of projection = 5 m s<sup>-1</sup> and  $\theta = 36.9^\circ$  (ii) 80.1° with the horizontal

### Question 6

Most candidates knew in this question to resolve vertically and to use Newton's Second Law horizontally. This question was usually well done.

- (i) (a) This part of the question required Newton's Second Law horizontally with acceleration =  $r\omega^2$  used. This was generally well done.
- (b) By resolving vertically the contact force could be calculated.
- (ii) In this part of the question it was necessary to put the normal contact force equal to zero. By resolving vertically  $T$  could be found. Then by using Newton's Second Law, the speed could be found.

Answers: (i)(a) Angular speed = 4.67 rad s<sup>-1</sup> (i)(b) Contact force = 0.986 N (ii) Speed = 1.61 m s<sup>-1</sup>

### Question 7

This type of question on the elastic string always proves to be very difficult for candidates. It proved to be the hardest question on the paper. A good diagram would certainly be helpful.

- (i) This part of the question required the use of the formula  $T = \lambda x/l$ . This would give the equation  $12(1.6 - 1.2)/1.2 = mgsin30$ . This leads to the correct answer.
- (ii) A 4 term energy equation is now required. The equation should contain a KE term, a PE term and 2 EE terms.
- (iii) The new equilibrium position must now be found.  $T = \lambda x/l$  is used in order to do this. With this new extension, another 4 term energy equation is required. This equation will contain a KE term, a PE term and 2 EE terms.

Answers (i) mass = 0.8 kg (ii) Initial KE = 1 J (iii) Greatest speed =  $1.5 \text{ m s}^{-1}$

# MATHEMATICS

Paper 9709/52

Paper 52

## General comments

Fewer candidates than in the past lost marks because they gave answers to 2 significant figures instead of 3 as requested on the question paper.

A small number of candidates lost marks due to premature approximating within their calculations leading to an incorrect answer.

Candidates should be reminded that if an answer is required to 3 significant figures then their working should be performed to at least 4 significant figures.

Very few candidates now use  $g = 9.8/9.81$ . The instructions on the question paper ask for  $g = 10$  to be used.

Occasionally candidates use a wrong formula. A formula booklet is provided so it is a good idea to keep a check on it.

The easier questions proved to be **2(i)**, **3(ii)**, **4(ii)** and **5**.

The harder questions proved to be **2(ii)**, **2(ii)** and **6(ii)**.

## Comments on specific questions

### **Question 1**

This question was generally well done.

- (i) It was necessary to use  $s = ut + 1/2at^2$  vertically to find the height of the particle at time 0.8 seconds. The result is arrived at by subtracting 0.5 from the answer found.
- (ii) The time to the greatest height can be calculated and by subtracting this from 0.8 the answer is found.

*Answers:* (i) 1.1 m (ii) 0.2 s

### **Question 2**

- (i) This part of the question was well done by most of the candidates.
- (ii) Many candidates found this part of the question rather difficult. The solution required the setting up of a 3 term energy equation involving PE, KE and EE.
- (iii) Here again an energy equation is required. If  $e$  is the extension then the energy equation required would be  $10e^2 / (2 \times 0.4) = 5(e + 1)$ .

*Answers:* (i) Modulus of elasticity = 10 (ii) Speed of projection =  $2 \text{ m s}^{-1}$  (iii) Greatest extension = 0.483 m

### Question 3

- (i) Quite a number of candidates used  $y = 0$  instead of  $y = 8$ .
- (ii) This part of the question was quite well done.
- (iii) Quite number of candidates stopped short of the final answer by only calculating the vertical component of the velocity,

Answers: (i)  $x = 4$  (ii) Angle =  $63.4^\circ$  and speed =  $5 \text{ m s}^{-1}$  (iii) Speed =  $13.6 \text{ m s}^{-1}$

### Question 4

- (i) Too many candidates thought that the radius of the hole was  $0.1 \text{ m}$  and found its area to be  $\pi \times 0.1^2$  and used this value. The question tells them that the cross-section is  $0.03 \text{ m}^2$ . Note that the edge of the hole does not touch  $AB$ .
- (ii) This part of the question was usually well done. A number of candidates took the force to be horizontal instead of vertical.
- (iii) Too many candidates simply said  $\tan\theta = 0.212/0.368$  instead of  $\tan\theta = (0.4 - 0.212)/0.368$ .

Answers: (i)  $0.212 \text{ m}$ ,  $0.368 \text{ m}$  (ii)  $F = 37.1$  (iii)  $\theta = 27.1$

### Question 5

This question was generally well answered and proved to be a good source of marks.

- (i) Some candidates found it difficult to explain that the particle moved when  $t = 2$ . The required equation was usual set up without too much difficulty.
- (ii) Most candidates knew that they were required to integrate. A number of candidates used  $t = 0$  instead of  $t = 2$ .
- (iii) Again candidates knew to integrate.

Answers: (i)  $t = 2$ ,  $a = 1.5t - 3$  (ii)  $v = 0.75t^2 - 3t + 3$  (iii)  $x = 0.25t^3 - 1.5t^2 + 3t - 2$

### Question 6

Most candidates knew that they had to resolve vertically and also to use Newton's Second Law horizontally in order to set up 2 equations. These could then be solved simultaneously to find the required tensions. Some sign errors were seen. Also some trig functions were incorrect.

- (i) This part was quite well done. Usual the mistake occurred when the candidate tried to solve the equations.
- (ii) The resolving vertically equation was usually correct. Not many candidates managed to find the new radius so sadly not many correct solutions were seen.

Answers: (i)  $T_A = 0.753 \text{ N}$ ,  $T_B = 2.74 \text{ N}$  (ii)  $T_A = 0$

# MATHEMATICS

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Paper 9709/53

Paper 53

## General comments

Fewer candidates than in the past lost marks because they gave answers to 2 significant figures instead of 3 as requested on the question paper.

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## Comments on specific questions

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*Answer:*  $0.8 \text{ s}$

### **Question 2**

This question caused quite a lot of problems. It involved finding distances using trigonometry.

- (i) Some candidates used the formula for a semi-circular lamina and not a wire. A clear diagram with the required measurements marked would have been very helpful. The required distance is  $OG\cos 70 + 0.4\sin 70$  where  $OG$  is the centre of mass of the wire from the mid-point  $O$  of  $AB$ .
- (ii) To find  $W$  it is necessary to take moments about  $A$  so that the forces on the hinge are not involved.

*Answers:* (i)  $0.463 \text{ m}$  (ii) Weight =  $21.2 \text{ N}$

### Question 3

This question was quite well done.

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Answers: (i)  $v dv/dx = 5 - 2e^{-x}$  (ii)  $v = 2.05$

### Question 4

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- (ii) This part proved to be rather difficult. Let  $A$  be the point of contact of the cylinder at the point nearest the bottom of the plane. The weight would then act through  $A$ . The centre of mass of the whole solid can be found in terms of  $x$ , where  $x$  is the required height of the cylinder. By taking moments about  $O$ , the centre of the base of the cylinder, would give  $Wx/2 + W(1.1 + x) = 2W \times OG$ , where  $OG = 0.4\tan 70^\circ$ .  $OG$  being the distance of the centre of mass of the solid.  $x$  can now be calculated.

Answers: (i) Least value of  $\mu = 0.364$  (ii) Greatest height of cylinder = 0.372 m

### Question 5

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- (i) By using horizontal and vertical motion, the 2 velocities can be found. Then by Pythagoras, the magnitude of the resultant velocity can be calculated. The required angle can be found by using the trigonometry of a right angled triangle.
- (ii) Vertical motion gives the vertical velocity as the particle hits the ground. The horizontal velocity remains unchanged. The required angle can now be found.

Answers: (i) Speed of projection = 5 m s<sup>-1</sup> and  $\theta = 36.9^\circ$  (ii) 80.1° with the horizontal

### Question 6

Most candidates knew in this question to resolve vertically and to use Newton's Second Law horizontally. This question was usually well done.

- (i) (a) This part of the question required Newton's Second Law horizontally with acceleration =  $r\omega^2$  used. This was generally well done.
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### Question 7

This type of question on the elastic string always proves to be very difficult for candidates. It proved to be the hardest question on the paper. A good diagram would certainly be helpful.

- (i) This part of the question required the use of the formula  $T = \lambda x/l$ . This would give the equation  $12(1.6 - 1.2)/1.2 = mgsin30$ . This leads to the correct answer.
- (ii) A 4 term energy equation is now required. The equation should contain a KE term, a PE term and 2 EE terms.
- (iii) The new equilibrium position must now be found.  $T = \lambda x/l$  is used in order to do this. With this new extension, another 4 term energy equation is required. This equation will contain a KE term, a PE term and 2 EE terms.

Answers (i) mass = 0.8 kg (ii) Initial KE = 1 J (iii) Greatest speed =  $1.5 \text{ m s}^{-1}$

# MATHEMATICS

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Paper 9709/61  
Paper 61

## Key messages

To do well in this paper, candidates must work with 4 significant figures or more in order to achieve the accuracy required. Candidates should also show all working, so that in the event of a mistake being made, credit can be given for method; a wrong answer with no working shown, scores no marks. Candidates should label graphs and axes including units, and choose sensible scales.

## General comments

It was pleasing to see that at least some of the candidates who took this paper had a good knowledge of the syllabus. However, a large proportion of candidates who entered this exam would benefit from further practice with past papers.

## Comments on specific questions

### **Question 1**

This question was a straightforward start to the paper involving a standard normal distribution and those candidates who knew what a normal distribution was, scored well on it.

*Answer:* 0.174

### **Question 2**

The first stage of this question was well attempted by almost everybody. However, the second stage involved recognising a binomial distribution situation. Of those who did not recognise the binomial situation, some used common sense and multiplied the probabilities by 3, which is the equivalent of  ${}^3C_1$ , and hence gained full marks.

*Answer:* 0.939

### **Question 3**

- (i) A good question involving tree diagrams, which was well done by almost everybody. There are alas still some candidates who do not know when to add probabilities and when to multiply.
- (ii) This was an easy conditional probability question and well done by many. There are many examples of such exercises in text books.

*Answer:* (i) 0.66 (ii) 0.824

#### Question 4

This question proved difficult for many candidates to make a start on. Candidates were asked to find the mean and variance of the number of green sweets when 2 sweets were taken from a bag of 2 green and 5 blue sweets. They were not told how to do it. The bright candidates, and the ones who had been taught and were familiar with this type of question, had no problems. Some candidates did not spot ‘without replacement’ and used a binomial situation, and thus lost some marks but still gained 3 method marks.

Answers: 4/7 (0.571), 50/147 (0.340)

#### Question 5

- (i) Many good attempts were made on this question, with normal tables being used correctly by those who had been taught the subject.
- (ii) The normal approximation to the binomial is a subject that occurs almost every year so it was disappointing that approximately half of those who succeeded in part (i) were unable to do this part of the question.
- (iii) Candidates should be prepared to quote the theory for this type of question.

Answers: (i) 0.945 (ii) 0.0875 (iii)  $np > 5$ ,  $nq > 5$

#### Question 6

- (a) (i) This was a different type of question to the usual perms and combs. Good candidates scored well in this, but also the weak candidates who could not do anything else on the paper, were happy to spend time on this question listing all possible numbers, developing a pattern and evaluating the answer, which was indeed often correct. Some candidates listed the numbers with different digits, some listed the numbers with repeated digits and subtracted from 900.
- (ii) This question was best solved by listing numbers starting with 7 which were odd and had no repetitions, then starting with 8 and then starting with 9. Credit was given for appreciating that the last digit could only be one of 1, 3, 5, 7 or 9 and indeed for numbers starting with 7 or 9 could only be one of four digits.
- (b) This type of question has occurred many times before and was well done by many candidates. Credit was given in the form of Method marks for recognising the options and summing them, providing the options were three-factor outcomes either multiplied or added.

Answers: (i) 648 (ii) 104 (iii) 2700

**Question 7**

- (i) This question provided some easy marks for candidates who had been well taught to label axes, choose sensible scales (NOT going up in 15s or even 16s) and knew what the drawing of a cumulative frequency curve meant. There were many candidates who drew a histogram which scored 0 marks. For those candidates who did obtain the cumulative frequencies (1 mark) and managed to have linear scales and to put labels (cf going up, money spent in \$s going across) (1 mark) they still had to plot the upper end points and join them up from (0, 0) in order to gain the final 2 marks.
- (ii) When asked to estimate the median, because the question did not specify which method to use, it was acceptable to use linear interpolation or to read it from the graph by drawing a line across from 80 to the graph and then down. Similarly the inter-quartile range could use linear interpolation or, preferable, draw a line across from 120 and another from 40 and read both quartiles from the graph, then subtract them. There was some lee-way in the readings from the graph but candidates who used an inappropriate scale did not usually gain full marks here.
- (iii) Again candidates could either use linear interpolation to estimate the answer, or read from the graph by drawing a line up from 115 to the graph and then across, and subtracting this number from 160.
- (iv) This was a pleasing end to the paper with the majority of candidates gaining full marks.

**Answers:** (ii) 60, 40 (iii) 9 (iv) 64.1

# MATHEMATICS

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Paper 9709/62

Paper 62

## Key messages

Candidates should be encouraged to show all necessary workings. If a simple slip is made, then credit can be awarded for the processes seen. A significant number of candidates did not show sufficient working to make their approach clear, and were unable to gain full credit.

Candidates should be aware that they need to work to a greater degree of accuracy than the 3 significant figures required in the answer to ensure that final answers are correct.

Candidates should be aware that a stem-and-leaf diagram is a representation of the data, and as with other graphical plots needs to be constructed accurately.

## General comments

It was pleasing that many candidates seem to have prepared well for the examination, and many good solutions were seen. There did not appear to be any issue with the accessibility of the questions, nor the time available for the paper.

Some candidates made more than one attempt at a question and then failed to indicate which their final solution was. This is often to the disadvantage of the candidate. Candidates should be reminded that corrections should be made clearly, not overwriting previous answers, as it is often not possible to determine their intention.

Answers to **Questions 1, 3 and 6** were generally stronger than other questions.

Candidates should be aware that the mark allocated to each part of a question is an indication of the work that can be expected to be undertaken.

## Comments on specific questions

### **Question 1**

- (i) There was a clear understanding of the requirements for a tree diagram. The best solutions ensured that the outcomes were in line, as this enabled the diagram to be read more easily. Some candidates did not include the branch for Milk with Hot Chocolate, which indicates that this is not a possible outcome. It was unfortunate that where candidates did include the Hot Chocolate and No Milk, the zero probability was often not stated on the final branch.
- (ii) Candidates who had failed to gain credit in (i) often completed this part well. Good solutions calculated  $P(\text{coffee} \cap \text{milk})$  and  $P(\text{milk})$  separately before combining in the conditional probability formula. It was disappointing to see the correct unsimplified solution inaccurately evaluated on a surprising number of occasions. Many candidates stated an exact value as they were able to simplify the accurate 2 decimal place answers for the numerator and denominator. Some candidates failed to follow the general instruction that answers should be to 3 significant figures and rounded to 2 decimal places.

**Answer:** (ii) 0.192

## Question 2

- (i) Although the allocation of 1 mark should have prompted candidates to realise that this part required a simple approach, many long and complex calculations were seen. Good solutions identified that the probability of waiting between 5 and 10 minutes was the difference of the stated probabilities of waiting for 10 minutes and waiting for 5 minutes.
- (ii) The majority of candidates correctly attempted to use the normal distribution approximation. The best solutions provided clear workings for deriving the mean and variance using their value from (i), and included the continuity correction in the standardisation formula. The inclusion of a sketch of the normal distribution curve often assisted in clarifying the required area. A few candidates did show that the normal approximation was appropriate, which is to be commended. A number of candidates did not use 4 significant figures within their solutions, so their final result was inaccurate and penalised. A number of candidates appear to have considered that as the third significant figure in their decimal answer was zero, it could be omitted. This changes the implied accuracy of their answer and is unacceptable.

Answers: (i) 0.72 (ii) 0.990

## Question 3

- (i) Most candidates included the correct values for  $x$  in the probability distribution table. A number of candidates did not use their knowledge of Pure Mathematics 1 to determine the probability for each value of eggs from the general term stated. Weaker solutions often did not equate the sum of their probabilities to 1, and were therefore unable to solve for  $k$ .
- (ii) Candidates who had calculated a multiple of  $k$  in (i) often substituted into their probability distribution table correctly and accurately evaluated the mean and variance. Good solutions included a clear statement of the terms being calculated in both expressions. A few candidates either omitted to subtract the  $mean^2$  or squared the wrong terms in the variance formula.

Answers: (i)  $k = 1/10$  (ii) mean = 3, variance = 1

## Question 4

- (i) Almost all candidates recognised that the binomial distribution was the appropriate approach to take. Candidates need to be encouraged to recognise key words such as 'at least', as many interpreted incorrectly and did not include 14 people in their solution set. Some candidates performed complex calculations to identify the probability for making a purchase. Good solutions often determined this using  $1-P(\text{not buying})$ . The best solutions used their calculators efficiently, stating the calculations required, recording calculation stages to 4 or more significant figures and then stating a final solution derived from the values retained on the calculator. Weaker solutions rounded the term values to 3 or 4 significant figures, which then produced an inaccurate answer. Some candidates seem to have mistaken decimal places as significant figures and worked to 3 or 4 decimal places throughout.
- (ii) Most candidates constructed an inequality of the correct format. Poor solutions used the answer from (i) or other values that had been calculated previously. Good solutions provided clear logarithmic manipulations to solve the inequality. However, many simply stated a single value, which may have been achieved by trial and improvement, but had misinterpreted the inequality and therefore could gain little credit.

Answers: (i) 0.0171 (ii) 24

## Question 5

- (i) Although almost all solutions included an attempt at a stem-and-leaf diagram, relatively few full credit answers were seen. Good solutions often included a draft diagram, which group the data into blocks initially, with a final diagram sorting these groups into the appropriate order. This allowed candidates to use a consistent spacing between terms, as expected with a visual presentation of statistical data. A number of candidates used 2 single-sided diagrams, which could gain little credit. Many keys were incomplete, with the most common errors being the omission of units or towns.

Candidates should be reminded that a single key is required for all stem-and-leaf diagrams, as many gave separate keys for each town.

- (ii) Good solutions used the stem-and-leaf diagram efficiently, matching answers with positions within the diagram. If candidates had read the question again after completing their answer, many would have realised that they had calculated the interquartile range for the wrong town. Some candidates used only the single digit from the stem-and-leaf diagram as their final answer. The best solutions often stated all the quartiles for both towns and then identified the required values with clear statements and calculations seen.
- (iii) Good solutions interpreted the information as a real life situation, making statements such as 'Bronlea is more windy than Rogate'. At this level, candidates are expected to go beyond making statements like 'the median of Bronlea is higher than Rogate'.

Answers: (ii) median = 23 kph, IQR = 16 kph

### Question 6

Candidates of all abilities attempted this question with some success.

- (i) Good solutions applied the normal standardisation formula with accuracy using the data provided. This included not using a continuity correction as suggested by the wording of the data. Many candidates sketched the curve to help identify the required area value. Weaker solutions often failed to reach the final answer.
- (ii) Most candidates were able to quote the required normal standardisation formula and use the tables accurately to convert the stated probability. It was unfortunate that a number of solutions were seen where the z-value had been rounded to 3 significant figures at the start of the solution. Weaker candidates failed to use the tables appropriately. Many candidates do not appear to be aware of the critical values for the normal distribution are listed on the tables provided.
- (iii) Few candidates recognised the symmetrical relationship between this part and (i), and recalculated the probability rather than just stating their previous answer. It was noticeable that a number of candidates reached a different value for the probability. Almost all candidates realised that they needed to multiply the number of days in a year by their probability, although some chose to ignore the information within the question and used 360 days. It was encouraging that most candidates provided an integer value in response to the real-life context.

Answers: (i) 0.295 (ii) 7.83 (iii) 107 or 108 days.

### Question 7

- (a) (i) The standard nature of this part enabled most candidates to present their understanding of permutations. Good solutions removed the effect of the repeated letters. Candidates should be aware that when an exact value is calculated, then it should be presented as their final solution.
- (ii) Many candidates found this question challenging. There were a variety of approaches that could be taken to fulfil the conditions within the question. Good solutions included explanation of the process that was being undertaken at each stage. Weaker solutions failed to account for the different places that the W's could be placed. A number of candidates who answered (i) correctly then failed to utilise the same approach to the repeated letters here. Without clear justification, presenting the correct final value did not guarantee any credit, as a number of partial solutions provided the same answer.
- (b) It was pleasing that many candidates who struggled with (a)(ii) did successfully attempt this question. Good solutions initially identified the 4 possible outcomes that fulfilled the criteria as a 'table' and then linked all the option calculations with the appropriate outcome. Weaker solutions had a more haphazard approach to the outcomes, resulting in either incorrect outcomes being included, or correct outcomes omitted. Almost all solutions used combinations correctly within their calculations.

Answers: (i) 302400 (ii) 20160 (iii) 3990

# MATHEMATICS

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Paper 9709/63  
Paper 63

## Key messages

Candidates should be encouraged to show all necessary workings. If a simple slip is made, then credit can be awarded for the processes seen. A si number of candidates did not show sufficient working to make their approach clear, and were unable to gain full credit.

Candidates should be aware that they need to work to a greater degree of accuracy than the 3 significant figures required in the answer to ensure that final answers are correct.

## General comments

The paper proved to be accessible to most candidates and they generally answered the questions in numerical order with each question starting on a new page. Many excellent candidates only lost a mark for not labelling their box plots in **Question 2** and/or premature approximation in **Question 7iii**. The combinations question was split into short parts and this made it more accessible than in other papers. All of the questions were well answered by a large number of candidates.

## Comments on specific questions

### **Question 1**

- (i) Most candidates completed the table correctly. A few entered probabilities instead of the required numbers and they received no marks.
- (ii) Many candidates correctly tried to show that the product of the probabilities was equal to the probability of the intersection but a number of them did not follow up their ideas with evaluated answers and the correct conclusion. Some incorrectly tried to show that the product equalled 1. Some attempted the more challenging conditional probability method and a few correctly showed that the  $P(X|Y)$  was not equal to the  $P(X)$  or  $P(Y|X)$  was not equal to the  $P(Y)$ .

*Answer:* (i) 6 19 25 (ii) Not independent  
2 3 5  
8 22

### **Question 2**

- (i) Few complete solutions were seen, as most candidates did not label their scaled line with both time and seconds. The best solutions used scales which enabled the boxes to be plotted accurately, using a single scale so that appropriate comparisons between the groups could be made. It was regrettable that a number of answers had the ‘whiskers’ passing through the ‘box’. Candidates are expected to be aware of the purpose of box plots, and little credit can be gained where the plots either do not line up or are superimposed as this prevents comparisons.
- (ii) The strongest candidates produced two statements comparing girls and boys from two of the categories Location, Spread and Distribution and used statistical vocabulary e.g. range or interquartile range and median. If they mentioned skewness they explained that it was positive skew.

### Question 3

- (i) The majority of candidates answered this question successfully. A few thought that the two-way table was a ‘probability distribution’ and a significant number produced probabilities that were half the correct value for outcomes 1 to 5, producing values that summed to 21/36.
- (ii) Almost all candidates were awarded the method mark.

**Answer:** (i)

x	0	1	2	3	4	5
$P(X = x)$	1/6	10/36	8/36	6/36	4/36	2/36

(ii) 35/18 or 1.94

### Question 4

Able candidates often scored full marks on this question. Part ii caused the most difficulties while part iii proved to be the most accessible.

- (i) Those candidates who began by finding the coded mean were usually successful in finding c. Candidates who opted to find  $\sum x$  first often correctly subtracted 1 845 but then omitted to divide by 9. The most common incorrect answer was 18 000.
- (ii) As with part (i), there were two possible approaches with those opting to use the coded values to find the variance being the most successful. Those who used uncoded values needed to expand  $\sum(x - c)^2$  which often caused difficulties. Some weaker candidates incorrectly combined coded and uncoded values and a significant number presented the standard deviation as the variance.
- (iii) Most of those candidates who correctly found the new total rental price went on to correctly find the price of the additional apartment. The most common error was for candidates to multiply the new mean by 9 rather than 10.

**Answers:** (i) 2 000 (ii) 11 025 (iii) 1 360

### Question 5

- (i) There were many fully correct solutions seen. A surprisingly common error was to see a z-value of 1.15 rather than 1.015. Other incorrect z-values were seen from inaccurate use of the tables and some candidates put their standardised expression equal to a probability rather than a z-value.
- (ii) It was pleasing to see a large number of fully correct solutions. Many candidates were able to translate the height of a comfortable desk for Jodu into a margin around the mean and then use it in the calculation of a z-value. Only a few candidates incorrectly used a continuity correction. Most candidates correctly gave a whole number as their final answer.

**Answers:** (i) 0.985 (ii) 293 or 294

### Question 6

- (i) Most candidates answered this question correctly. Only a small number ignored the repeated letters and gave 9! as their final answer.
- (ii) Many candidates gained one of the two available marks. The most frequent error was to multiply the correct answer by 2.
- (iii) As in (ii), many candidates gained just one of the two available marks. A common error was to multiply the correct answer by 4! while others gave an answer of 6! without dividing by 2.
- (iv) This question was less well done with a common error being an answer of 6 rather than 3. Candidates who listed the correct solutions tended to be more successful.
- (v) Many candidates who had incorrectly answered (iv) recovered in this part of the question. A commonly seen incorrect answer was 10. In some cases this resulted from candidates correctly

giving the number of selections with no Rs and two Rs and adding this to an incorrect number of selections for one R from **(iv)**. In other cases, it resulted from the use of 5C3 where candidates did not consider separately the different number of Rs. Some others incorrectly considered different numbers of Es. It was pleasing to see many candidate setting out their solutions clearly with headings for no Rs, one R and 2 Rs.

Answers: **(i)** 7 560 **(ii)** 210 **(iii)** 360 **(iv)** 3 **(v)** 7

### Question 7

Many candidates lost marks in this question through premature approximation and giving answers to 2 significant figures.

- (i)** It was apparent that many candidates were well prepared for this style of question and earned full marks. However, a significant number incorrectly tried to use a normal distribution.
- (ii)** This question proved more challenging and many candidates produced a binomial term. Those who correctly calculated  $0.35 \times 0.35 \times 0.35 \times 0.65$  often gave the answer to 2 significant figures – confusing 3 significant figures with 3 decimal places.
- (iii)** This question was well answered by many candidates but the majority of those lost the last answer mark by prematurely approximating the P(7) and/or the mean and variance. A number of candidates miscalculated the mean and variance using 0.65 instead of P(7).

Answer: **(i)** 0.541 **(ii)** 0.0279 **(iii)** 0.290

# MATHEMATICS

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Paper 9709/71  
Paper 71

## General comments

This paper generated a complete spread of marks; there were some good marks awarded, but also there were many cases where candidates appeared unprepared for the demands of the paper. Questions that were, in general, well attempted were **Questions 2(ii)**, and **6(ii)** and **(iii)** whilst **Questions 3** and **7** proved to be particularly demanding. Questions requiring an explanation of statistical terms were not well attempted.

Presentation was not always good; it is important that all necessary working is shown with solutions presented clearly and logically. In questions where it is required to show that a given answer is correct, it is of particular importance that all necessary steps in the calculation are shown; marks can be withheld for lack of essential working.

Timing did not appear to be a problem for candidates.

## Comments on specific questions

### **Question 1**

A variety of different approaches were tried in answering this question. Candidates using  $N(100/3, 250/9)$  were usually successful, though errors involving an incorrect continuity correction or an invalid comparison were seen. It was important that a clear and valid comparison was made (1.486 with 1.282 or alternatively 0.0686 with 0.1). This comparison can be shown as an inequality statement or on a clearly labelled diagram. The final conclusion is best written in context and as a non-definite statement (as below).

*Answer:* There is some evidence that  $p < 1/6$

### **Question 2**

In part **(i)** some candidates used an acceptable definition of ‘random’ and put it into the context given. However, many candidates struggled to convey the meaning of ‘random’. Part **(ii)** was usually well attempted, but in part **(iii)** the definition of ‘population’ in the given context was not well answered; many candidates thought that the population was all the employees, rather than the distances travelled by all the employees. Weaker candidates did not appreciate the difference between a sample and a population.

*Answer:* Each employee has an equal chance of being chosen  
4      4.36  
Distances travelled by all employees at the firm

### **Question 3**

Candidates were not all successful in finding the value of  $p$  (0.61). In part **(ii)**, whilst some candidates were able to find the value for  $z$  (2.321), very few then went on to find the correct confidence level.

*Answer:* 0.61  
98%

#### Question 4

Candidates were required to state their Null and Alternative Hypotheses; this was not always done. A two-tail test should have been set-up as the test was to determine if the mean time had changed. The comparison between 1.684 and 1.96 (or an equivalent, valid comparison) should have been clearly shown either as an inequality statement or on a clearly labelled diagram in order to validate the conclusion drawn. The final conclusion is best written in context and as a non-definite statement (as below). There were candidates who were able to state the probability of a Type I error in part (ii).

Answer: No evidence that the mean time has changed.

0.05

#### Question 5

Some candidates successfully found the probabilities required in this question. Common errors, on both parts of the question, included incorrect calculation of the variance and calculating the wrong tail probability. Questions on this topic are usually well attempted; this was, in general, the case here but there were some candidates who seemed unaware of how to approach this question.

Answer: 0.0147

0.705

#### Question 6

In part (i) some candidates were able to interpret the given scenario and state the greatest distance travelled, and in part (ii) many candidates attempted to integrate  $f(x)$  with correct limits and equate their answer to 1 in order to show that  $k$  was  $\frac{3}{4}$ . In part (iii) the integral of  $xf(x)$  was required; this was also quite well attempted, though incorrect limits were seen on occasions. Part (iv) was not quite so well attempted, and even those who attempted to integrate  $f(x)$  between 0 and 1 often left their answer as a probability (0.3125) whereas the question required the expected number of turns ( $400 \times 0.3125$ ). Candidates usually find questions on probability density functions reasonably accessible; this was, in general, the case here, with some candidates answering the question with confidence. However, there were also some candidates who were unable to make an attempt at the question.

Answer: 2 m

1.2 m.

125

#### Question 7

Part (i) of this question was not well attempted. Candidates were required to find the values of  $\lambda$  for both the spoons and the forks (0.8 and 0.9 respectively) and then use the Poisson distribution to find the probability of at least one defective spoon and multiply it by the probability of at least one defective fork. Many candidates did not realise what was required, and often did not multiply separate probabilities. In some cases candidates failed to use Poisson despite the instructions given in the question. Candidates attempted part (ii) with slightly more confidence, though not always with the correct value of  $\lambda$ . In part (iii) candidates with good knowledge of the Poisson distribution were generally able to form two correct equations to find  $\lambda$  and  $p$ .

Answer: 0.327

0.757

3      0.149

# MATHEMATICS

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Paper 9709/72  
Paper 72

## General comments

Candidates, in general, answered this paper with confidence and were able to demonstrate the skills required. Questions answered well were **Questions 1 and 5**, whilst **Question 6** caused problems for some candidates.

In general, candidates adhered to the required level of accuracy; there were a few cases (particularly on **Question 2**) where there was confusion between 3 decimal places and the required 3 significant figures. Presentation was generally good, though it is important that candidates write all figures clearly and unambiguously. Essential working was shown in most cases; this being of particular importance in questions requiring a given answer to be shown.

No evidence was noted that candidates were unable to finish in the allocated time.

## Comments on specific questions

### **Question 1**

This question was well attempted giving many candidates a successful start to the paper. Common errors included omission of  $\sqrt{50}$  when standardising, calculation errors, and finding the wrong tail probability. Weaker candidates sometimes confused two different methods of finding the required probability.

*Answer:* 0.989

### **Question 2**

It is important when asked to explain ‘in the context of the question’ that more than just a text book definition is given. The explanation must make reference to the situation presented; this reference was not always clearly made by candidates in part (i) of this question. When explaining both Type I and Type II errors, it is important that the conclusion being drawn and the reality of the situation, thus illustrating the error, are both made clear in the context given.

Part (ii) required the probability of a Type I error to be calculated. Some candidates correctly used  $B(15, 0.9)$  and found the required probability, though as the required answer was 0.0556 (or 0.0555) there were cases where the answer was given as 0.056 (indicating a confusion between the 3.s.f required and, in this case, a reduced level of accuracy of 3.d.p). As many answers on this paper are probabilities (and therefore decimals smaller than 1), it is important that candidates appreciate and adhere to the level of accuracy required. A common error was to omit the  $X = 12$  term in the Binomial calculation, and there were some candidates who used incorrect distributions.

*Answer:* Conclude that less than 90% are satisfied when this is not true  
0.0556

### Question 3

The reason for using a sample rather than the whole population was generally well answered in part (i), with most candidates giving a sensible answer relating to time or cost. However, very general comments like 'it is easier' were not specific enough.

Whilst many candidates were able to recall the correct formula for a 95% confidence interval, errors were made in its application; many candidates used the value of  $x$  as 64.7 instead of 65.7, and incorrect values of  $z$  were also used.

It was important in part (ii) (b) that a clear statement saying that 64.7 was not in the confidence interval was made before a conclusion was drawn.

Answer: Population too big

64.9 to 66.5

CI does not include 64.7 therefore probably has affected the mean bounce height

### Question 4

This was a reasonably well attempted question, though confusion between different methods was evident on occasions. Not all candidates stated the required Null and Alternative Hypothesis, and for those following the most standard method using  $N(42, 42)$ , a common error was to omit or to use an incorrect continuity correction. It was important that a clear and valid comparison was shown in order to reach the required conclusion; this could be given as an inequality or on a clearly labelled diagram. As has been mentioned in the past, the conclusion is best given in context, and as a non-definite statement (see below).

Answer: No evidence that the mean has changed

### Question 5

This was a particularly well attempted question. Many candidates correctly used  $N(520, 70)$ , or equivalent, in part (i) and  $N(-10, 386.2)$  in part (ii). Whilst many fully correct answers were seen, some candidates made errors in calculating the variance in both part (i) and part (ii); confusion between standard deviation and variance being evident. Other common errors included incorrect inclusion or exclusion of the box as appropriate to the method used, and finding the wrong tail probability.

Answer: 0.884

0.305

### Question 6

Part (i) of this question was usually well attempted, however, parts (ii) and (iii) were not. In part (i) the probability of exactly 5 hair dryers was required (with  $\lambda = 6.8$ ). However, in part (ii) the question required candidates to find the probability of less than 4 hair dryers and less than 5 hair dryers respectively (with  $\lambda = 3.4$ ). A large number of candidates did not appreciate this, and merely calculated the probability of exactly 4 hair dryers, and exactly 5. It is important that candidates always read the given scenario carefully in order to interpret the situation correctly.

In part (iii) candidates were required to find  $P(\leq n)$  for different values of  $n$ , until 0.9 was reached ( $n=6$  giving 0.942). Many candidates, incorrectly, used  $P(n)$  and attempted to solve an incorrect equation.

Answer: 0.135

0.744

0.870

6 hair dryers

### Question 7

There were parts of this question that were well attempted, and parts that were poorly attempted. Many candidates were unable to use the symmetry of the given diagram to find the required probability in (a). Parts **b(i)** and **(ii)** were generally well attempted, though the final mark on **(ii)** was often not gained as candidates left their answer 0.5625 or 0.563, omitting the final stage required i.e. to multiply by 100 (since the model was given in hundreds of hours). Similarly, there was confusion with ‘hundreds of hours’ in part **b(iii)**, where either 150 should have been compared with 290, or if working in hundreds of hours 1.5 with 2.9. Also, in part **b(iv)** many candidates, incorrectly, gave their answer as 290 instead of 2.9. Many candidates attempted incorrect calculations to find a value for  $a$ , despite the question clearly requesting candidates to “state a suitable value”.

Answer: 0.8

56.3

Max  $x$  is 1.5, less than 2.9

Any  $a$  such that  $2.9 \leq a < 5$

# MATHEMATICS

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Paper 9709/73  
Paper 73

## Key messages

Candidates should consider whether a combination of normal variables is a sum or a multiple of a single normal variable.

Candidates need to be familiar with the meaning of the Central Limit theorem, and how it is applied, particularly the distribution of the sample mean.

## General comments

The majority of scripts were well presented, and solutions were clear, with calculations carried out using sufficient figures to produce accurate results. It is clear a significant number of candidates are using calculators with the normal distribution as a built in function. It is important that calculator displays are accurately copied onto the answer booklet and all working is shown. **Questions 1, 2, 5(i) and (ii) and 7(i)** were a good source of marks, **Questions 4, 5(iii) 6(iii) and 6(iv)** as well as the last part of **7(ii)** proved the most demanding

## Comments on specific questions

### **Question 1**

The majority of candidates scored full marks obtaining the 98% confidence interval for the mean. A small number used the wrong value for  $z$ , most often 2.054, and a small number tried to make an adjustment to the unbiased variance given.

*Answer:* 191 to 194

### **Question 2**

Most candidates provided full solutions to this question on hypothesis testing in the context of a normal distribution. A few candidates failed to correctly state the hypotheses, or used an incorrect parameter or some had no parameter at all. The majority of candidates calculated the correct test statistic, although a number of candidates used 1.2 in the standardisation rather than  $1.2/\sqrt{16}$ . Having obtained the test statistic most candidates then performed a clear test either comparing with the  $z$  value 1.645 or an area comparison with 0.05. Almost all conclusions correctly stated that there was evidence of an increase in the mean.

*Answer:* Reject  $H_0$ , there is evidence that the mean yield has increased

### **Question 3**

Many candidates scored full marks on both parts of this question. The majority of candidates identified that the Poisson was the suitable approximating distribution in part (i), and having calculated the correct mean, then used this to calculate  $P(X > 2)$ . The most common error was the incorrect inclusion of  $P(X = 2)$  calculating  $1 - P(X < 1)$ , rather than  $1 - P(X < 2)$ . A minority of candidates incorrectly tried to use the normal distribution, which was inappropriate with the given values of  $n$  and  $p$ .

In part (ii) Many candidates were able to identify the correct justification for the use of the Poisson approximation, indicating the values in the situation of  $n$  and either of  $np$  or simply  $p$ .

*Answers:* (i) 0.430 (ii)  $n > 50$ ,  $np < 5$  (or  $p < 0.1$ ).

#### Question 4

This question assessed the candidates' ability to significance test in the context of a discrete distribution. This is a topic that many candidates continue to find difficult to score highly on. A number of candidates scored the first mark for the hypotheses and no more, often as they incorrectly attempted to work with the normal distribution. The majority of marks in part (i) were awarded for correctly obtaining the critical region ( $X < 2$ ). As a minimum candidates were required to find  $P(X < 2)$  and  $P(X < 3)$ , deducing the correct critical region. A number of candidates lost a mark as they made rounding errors in one or both probabilities. It was then expected that as the number of sightings (2) was in the critical region that there was evidence of fewer sightings of Golden Eagles. Some candidates with the correct critical region did not use this, but performed the test by comparing with  $P(X) < 0.05$ . This was accepted. There were three common reasons why a number of candidates failed to obtain full marks in this part. The first was having found the critical region, no test was performed. The second was stating the critical region as  $P(X < 2)$  rather than just  $X < 2$ . The third was by trying to perform the test by simply comparing  $P(X = 2)$  with 0.05.

In part (ii) candidates were required to state the Type I error as the size of the critical region. Those candidates who had calculated a critical region in part (i) usually recognised this. In part (iii) most candidates correctly stated that a Type II error could not be made as  $H_0$  had been rejected. A number of candidates incorrectly stated that the reason was that no new mean was given.

Answers: (i)  $X < 2$ , reject  $H_0$ . Evidence of fewer sightings. (ii) 0.0203 (iii) As  $H_0$  rejected

#### Question 5

Questions testing continuous random variables are often well answered and parts (i) and (ii) continued this pattern. In part (i) the majority of candidates were able to produce sufficiently clear and full solutions to demonstrate the given value of  $k$ . A very small number of candidates made errors, either by not applying  $k$  to the evaluation with both limits, or by using decimals rather than exact fractions. In part (ii) Almost all candidates found the correct value for  $E(T)$ .

Part (iii) proved much more challenging. Candidates were expected to find  $P(X < E(T))$  and subtract 0.5 from this. Many candidates used an alternative method trying initially to find the value of the median. Many succeeded in finding the value despite the need to solve a cubic equation. They then found the required value by integrating the pdf between these two values. Some candidates lost the accuracy mark as their values for the median had insufficient figures. In part (iv) many candidates correctly used the definition of the range of the pdf to find the maximum time needed to complete the test. A number of candidates incorrectly thought a calculation with the pdf was required.

Answers: (i)  $k = 3/250$  (ii) 6.875 (iii) 0.0361 (iv) 10 (minutes)

#### Question 6

In part (i) many candidates correctly obtained the required probability by combining the Poisson means and calculating the  $P(X + Y = 4 | \lambda = 3.9)$ . A significant number of candidates used a combination method finding the 5 different ways that  $X + Y = 5$ , and summing these. Candidates are advised to use the most efficient route, as combinations are easily missed.

Part (ii) was not as well answered as part (i). Candidates were required to state the sampling distribution of the mean of 75 values. Many stated the sampling distribution of the sum of 75 values. Most candidates who recognised what was required identified the normal distribution and the mean, but stated the variance as 1.6 not  $1.6/75$ .)

Part (iii) was then answered correctly by many candidates who had not scored in part (ii). Many fully correct solutions were seen, although some candidates incorrectly interpreted more than 1.7 as greater than 1.75.

The final part(iv) proved most challenging of all. The required answer was that the Central Limit theorem (CLT) was needed as  $X$  was not normally distributed. Many indicated that the CLT was not needed as we could assume the mean was normally distributed, or that the distribution of  $X$  was unknown, even though it was stated to be Poisson.

Answers: (i) 0.195 (ii)  $N(1.6, 1.6/75)$  (iii) 0.247 (iv) Required as  $X$  not normally distributed.

### Question 7

Many candidates obtained full marks on the first part of the question. The main source of error was in the calculation of the variance of 20 bags of sugar and one box, when candidates treated the 20 bags as a multiple rather than a sum of independent normal variables. A small number of candidates incorrectly worked with standard deviations, rather than variances. Nearly all candidates recognised that standardisation and the use of tables was required. In part (ii) only a minority of candidates obtained all the available marks. The common error was to fail to recognise that the question required the probability that the two full boxes differed in mass by less than 0.02 kg. A significant number of candidates gave an answer of 0.5418, which was the probability that the difference in weight between the first box and the second was less than 0.02 kg (i.e. the probability that  $X_1 - X_2 < 0.02$ ), whereas, in fact, the probability of  $|X_1 - X_2| < 0.02$  was required. It is important that candidates read the question carefully to fully interpret what is needed.

Answers: (i) 0.0684 (ii) 0.0836