Topic 10 Ideal gases

Summary

- The equation of state for an ideal gas relates the pressure p, volume V and thermodynamic temperature T of n moles of gas: $pV = nRT$ where $R = 8.3$ J K^{-1} mol^{-1}, the molar gas constant.
- For N molecules of gas: $pV = NkT$ where $k = 1.38 \times 10^{-23}$ J K^{-1}, the Boltzmann constant.
- The relation between R and k is $k = R/N_A$ where $N_A = 6.02 \times 10^{23}$ mol^{-1}, the Avogadro constant (the number of atoms in 0.012 kg of carbon-12).
- The assumptions of the kinetic theory of gases are

1 Molecules behave as identical, hard, perfectly elastic spheres.
2 Volume of the molecules is negligible compared with the volume of the containing vessel.
3 There are no forces of attraction or repulsion between atoms.
4 There are many molecules, all moving randomly.

- Kinetic theory equation: $pV = \frac{1}{3}Nm\langle c^2\rangle$, $p = \frac{1}{3}\rho\langle c^2\rangle$
- Average kinetic energy of a molecule: $\langle E_k\rangle = \frac{1}{2}m\langle c^2\rangle = \frac{3}{2}kT$
- Root-mean-square speed of molecules: $c_{rms} = \sqrt{(3kT/m)}$

Definitions and formulae

- The equation of state for an ideal gas is $pV = nRT$ (T in Kelvin, n is the number of moles).
- N_A is the number of atoms in 0.012 kg of carbon-12.
- One mole has 6.02×10^{23} particles
- Basic assumptions of the kinetic theory of gases:

1 all the molecules behave as identical, hard spheres that have elastic collisions
2 the volume of the molecules is negligible compared with the volume of the containing vessel
3 there are no forces of attraction or repulsion between molecules
4 there are a very large number of molecules all moving randomly.

- Deduce $pV = \frac{Nm < c^2 >}{3}$ and compare with $pV = NkT$ to deduce that the average translational kinetic energy of a molecule is proportional to T.
- $k = R/N_A$
- Ideal gas <k.e.> of a molecule $\propto T$ (absolute temperature)