Solution

Name:
Department:

QUIZ 3

Define the integer sequence $a_0, a_1, a_2 \ldots$, recursively by

(1) $a_0 = 1, a_1 = 1, a_2 = 1$; and

(2) For $n \geq 3$, $a_n = a_{n-1} + a_{n-3}$.

Prove that $a_n \geq (\sqrt{2})^{n-2}$ for all $n \geq 2$.

$S(n)$ $\quad a_n \geq (\sqrt{2})^{n-2}$

Note that $S(2)$ is correct.

$a_3 = 2$, $a_4 = 3$, so $S(3)$ and $S(4)$ are correct.

Assume $S(2), S(3), \ldots, S(k)$ are correct with $k \geq 4$. Then

$$a_{k+1} = a_k + a_{k-2} \underset{\text{by induction}}{\geq} (\sqrt{2})^{k-2} + (\sqrt{2})^{k-4} = 3(\sqrt{2})^{k-4} \geq 2\sqrt{2}(\sqrt{2})^{k-4} = (\sqrt{2})^{k-1}$$

Hence $a_{k+1} \geq (\sqrt{2})^{k-1}$.

So $S(k+1)$ is true

Solutions

Name:
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QUIZ 3

For $n \geq 0$, let F_n denote the nth Fibonacci number. Prove that

$$F_0 + F_1 + F_2 + \cdots + F_n = F_{n+2} - 1.$$

for all $n \geq 0$.

(Recall that Fibonacci numbers are defined recursively as $F_0 = 0$, $F_1 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 2$.)

Let $S(n)$: $F_0 + F_1 + \cdots + F_n = F_{n+2} - 1$

$S(1)$ is true because $F_0 + F_1 = F_3 - 1$

$F_3 = 3$

Assume $S(k)$ is true

$$F_0 + F_1 + \cdots + F_k + F_{k+1} \underset{\text{by induction}}{=} F_{k+2} - 1 + F_{k+1} \underset{\text{by definition of Fibonacci numbers}}{=} F_{k+3} - 1$$

So $S(k+1)$ is also true.