SOLUTIONS

Name:
Department:

QUIZ 2

In how many ways can three x's, three y's and three z's be arranged so that no consecutive triple of the same letter appears.

c_1. condition on an arrangement that 3 x's appear consecutively.

Similarly define c_2, c_3 for y and z.

$S_0 =$ the number of all arrangements

$$= \frac{9!}{3!\,3!\,3!}$$

$$N(c_1) = \frac{7!}{3!\,3!} = N(c_2) = N(c_3) \Rightarrow S_1 = 3 \cdot \frac{7!}{3!\,3!}$$

$$N(c_1, c_2) = \frac{5!}{3!} = N(c_1, c_3) = N(c_2, c_3)$$

$$\Rightarrow S_2 = 3\,\frac{5!}{3!}$$

$$N(c_1, c_2, c_3) = 3! = S_3$$

From the principle of inclusion/exclusion

$$N(\bar{c}_1, \bar{c}_2, \bar{c}_3) = S_0 - S_1 + S_2 - S_3$$

$$= \frac{9!}{3!\,3!\,3!} - 3\,\frac{7!}{3!\,3!} + \frac{3 \cdot 5!}{3!} - 3!$$

Solutions

Name:
Department:

Quiz 2

In how many ways can one distribute ten distinct prizes among four students with exactly two students getting nothing?

$c_1 =$ condition that 1st student gets no prize.

Similarly ~~we~~ define conditions c_2, c_3, c_4 for 2nd, 3rd, 4th students getting no prize.

We compute

$E_2 = S_2 - 3S_3 + 6S_4$ (from the formula that generalizes inclusion/exclusion)

$N(c_1, c_2) = 2^{10} \Rightarrow S_2 = 6 \cdot 2^{10}$

$N(c_1, c_2, c_3) = 1 \Rightarrow S_3 = 4 \cdot 1$

$N(c_1, c_2, c_3, c_4) = 0 \Rightarrow S_4 = 0$

$\Rightarrow E_2 = 6 2^{10} - 3 \cdot 4$