Math 132: Discrete Mathematics
Problem session 3

1. How many ordered quadruples (x_1, x_2, x_3, x_4) of integers such that $-5 \leq x_i \leq 5$ sum to
   (a) 10 (b) 15 (c) 20 (d) 25?

2. How many 6-card hands from a standard deck of 52 cards (13 in each of the four suits) have at least one card from each suit?

3. How many 5-card hands from a standard deck of 52 cards (which has 4 Kings, 4 Queens, and 4 Jacks) have at least one of each face card?

4. How many permutations of the 26 letters in the English alphabet do *not* contain MATH, PATH, SUM, or THEORY?

5. How many numbers in $\{1, 2, \ldots, 10000\}$ are *not* multiples of 3, 4, or 5?

6. Let D_n be the number of derangements of n numbers. Recall that this is the number of permutations of $\{1, 2, \ldots, n\}$ such that for no $1 \leq i \leq n$ is i in the ith position of the permutation.

   Show that $n! = \sum_{i=0}^{n} \binom{n}{i} \cdot D_i$.

7. Show that $D_0 = 1$, $D_1 = 0$, and $D_n = (n-1) \cdot (D_{n-1} + D_{n-2})$ for $n \geq 2$.

8. Show that $D_n = n \cdot D_{n-1} + (-1)^n$ for $n \geq 1$.