

Assignment 1
Munerah Alfayez

1. Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a longterm contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of \$32. Each Mini requires 40 minutes of labor and generates a unit profit of \$24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week. a. Clearly define the decision variables b. What is the objective function? c. What are the constraints? d. Write down the full mathematical formulation for this LP problem.

a. Clearly define the decision variables

In this scenario, there are two decision variables

X_1 = Number of the Collegiate bags to produce each week

X_2 = Number of the Mini bags to produce each week

b. What is the objective function?

The objective is to maximize the profit by identifying how many units to be produced each week

The objective function is $Z = 32 X_1 + 24 X_2$, while Z is the profit in \$ / unit

c. What are the constraints?

Supply constraint $3(X_1) + 2(X_2) \leq 5000$

Labor constraint $45(X_1) + 40(X_2) \leq 84000 \text{ min /week } (35 \times 40 \times 60)$

Demand constraint $X_1 \leq 1000, X_2 \leq 1200$

Where $X_1 \geq 0, X_2 \geq 0$

d. Write down the full mathematical formulation for this LP problem.

Let X_1 = Number of the Collegiate bags to produce each week

Let X_2 = Number of the Mini bags to produce each week

Let Z = the profit

Maximum profit function is $Z = 32 X_1 + 24 X_2$

$$3(X_1) + 2(X_2) \leq 5000$$

$$45(X_1) + 40(X_2) \leq 84000 \text{ min /week (} 35 \cdot 40 \cdot 60 \text{)}$$

$$X_1 \leq 1000, X_2 \leq 1200$$

Where $X_1 \geq 0, X_2 \geq 0$

The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of \$420, \$360, and \$300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved. The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively. Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day. At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product. Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

a. Define the decision variables

In this scenario, there are six decision variables

L_1 = number of large size products to produce in plant 1 each day

L_2 = number of large size products to produce in plant 2 each day

L_3 = number of large size products to produce in plant 3 each day

M_1 = number of medium size products to produce in plant 1 each day

M_2 = number of medium size products to produce in plant 2 each day

M3 = number of medium size products to produce in plant 3 each day

S1 = number of small size products to produce in plant 1 each day

S2 = number of small size products to produce in plant 2 each day

S3 = number of small size products to produce in plant 3 each day

b. Formulate a linear programming model for this problem

The objective is to maximize the profit by identifying how many units to be produced each day

Maximum profit function is

$Z = 420 L1 + 420 L2 + 420 L3 + 360 M1 + 360 M2 + 360 M3 + 300S1 + 300S2 + 300 S3$, while Z is the profit in \$ / unit

Production capacity constraints

$$L1 + M1 + S1 \leq 750$$

$$L2 + M2 + S2 \leq 900$$

$$L3 + M3 + S3 \leq 450$$

Storage capacity constraints

$$20 L1 + 15 M1 + 12 S1 \leq 13000$$

$$20 L2 + 15 M2 + 12 S2 \leq 12000$$

$$20 L3 + 15 M3 + 12 S3 \leq 5000$$

Sales forecast constraints

$$L1 + L2 + L3 \leq 900$$

$$M1 + M2 + M3 \leq 1200$$

$$S1 + S2 + S3 \leq 750$$

The plant's excess production capacity can be used to produce the new product

$$\frac{L1+M1+S1}{750} = \frac{L2+M2+S2}{900} = \frac{L3+M3+S3}{450}$$

To simplify it

$$900 (L1 + M1 + S1) - 750 (L2 + M2 + S2) = 0$$

$$450 (L2 + M2 + S2) - 900 (L3 + M3 + S3) = 0$$

$$450 (L1 + M1 + S1) - 750 (L3 + M3 + S3) = 0$$

Where $L1, M1, S1, L2, M2, S2, L3, M3, S3 \geq 0$