Assignment_3

2022-10-16

Transportation problem

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Heart Start produces automated external defibrillators (AEDs) in each of two different plants (A and B). The unit production costs and monthly production capacity of the two plants are indicated in the table below. The AEDs are sold through three wholesalers. The shipping cost from each plant to the warehouse of each wholesaler along with the monthly demand from each wholesaler are also indicated in the table. How many AEDs should be produced in each plant, and how should they be distributed to each of the three wholesaler warehouses so as to minimize the combined cost of production and shipping?

```
knitr::opts_chunk$set(echo = TRUE)
```

The unit production costs and monthly production capacity of the two plants are indicated in the table below. The AEDs are sold through three wholesalers. The shipping cost from each plant to the warehouse of each wholesaler along with the monthly demand from each wholesaler are also indicated in the table

```
tab <- matrix(c(22,14,30,600,100,
                16,20,24,625,120,
                80,60,70,"-","-") , ncol=5 , byrow=TRUE)

colnames(tab) <- c("Warehouse1","Warehouse2","Warehouse3","Prod cost","Prod Capacity")
row.names(tab) <- c("Plant A","Plant B","Demand")
tab <- as.table(tab)
tab
```

```
##         Warehouse1 Warehouse2 Warehouse3 Prod cost Prod Capacity
## Plant A 22         14         30         600       100
## Plant B 16         20         24         625       120
## Demand  80         60         70         -         -
```

The above transportation problem can be formoulated as below

$$\text{Min } \; TC = 22X_{11} + 14X_{12} + 30X_{13}$$

$$+16X_{21} + 20X_{22} + 24X_{23}$$

subject to

#Production Capacity consnstraints Production plant A :

$$X_{11} + X_{12} + X_{13} \leq 100$$

Production Plant B :

$$X_{21} + X_{22} + X_{23} \leq 120$$

#Demand Consntraints

Demand Warehouse 1 :

$$X_{11} + X_{21} \geq 80$$

Demand Warehouse 2 :

$$X_{12} + X_{22} \geq 60$$

Demand Warehouse 3 :

$$X_{13} + X_{23} \geq 70$$

Non-negativity of the variables

$$X_{ij} \geq 0$$

Where

$$i = 1, 2, 3$$

And

$$j = 1, 2, 3$$

1. Formulate and solve this transportation problem using R

#Solving the above transportation cost minimization problem using R . This Transportation problem is unbalanced one . Demand is less than supply by 10 , So I create a dummy variable in column 4 with transportation cost zero and demand 10 .

```
library(lpSolveAPI)
library(lpSolve)
# Set up cost matrix
costs <- matrix(c(622,614,630,0,
                641,645,649,0) , ncol=4 , byrow=TRUE)

# Set up cost table for each warehouse

colnames(costs) <- c("Warehouse1","Warehouse2","Warehouse3","Dummy")
row.names(costs) <- c("Plant A","Plant B")
costs <- as.table(costs)
costs
```

```
##         Warehouse1 Warehouse2 Warehouse3 Dummy
## Plant A        622        614        630     0
## Plant B        641        645        649     0
```

```
# Set up constraints signs and right-hand sides (production side )
row.signs <- rep("<=",2)
row.rhs <- c(100,120)

# Demand consntraints
col.signs <- rep(">=",4)
col.rhs <- c(80,60,70,10)

#Run

lptrans <- lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)

#Values of all 6 variables
lptrans$solution
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   60   40    0
## [2,]   80    0   30   10
```

```
# Value of the objective function
lptrans$objval
```

```
## [1] 132790
```

When solved the transportation problem, I got the values of the variables as

$$x_{12} = 60$$

$$x_{13} = 40$$

$$x_{21} = 80$$

$$x_{23} = 30$$

$$x_{24} = 10$$

132,790 is the minimum combined cost of production and shipping founded for the optimal solution . In order to minimize the combined cost of production and shipping Plant A should produce 100 units , 60 units for the warehouse 2 and 40 units for warehouse 3 . Plant B should produce 110 units , 80 units for the warehouse 1 and 30 units for warehouse 3 .

2. Formulate the dual of this transportation problem

$$P^d \text{ :price at destination}$$

$$P^o \text{ :price at the plant (operation)}$$

$$i \text{ : warehouse number}$$

$$\text{Max } VA = (80P_1^d + 60P_2^d + 70P_3^d) - (100P_1^o + 120P_2^o)$$

$$= 80P_1^d + 60P_2^d + 70P_3^d - 100P_1^o - 120P_2^o$$

#Subject to

#Plant A consntraints :

$$P_1^d - P_1^o \geq 622$$

$$P_2^d - P_1^o \geq 614$$

$$P_3^d - P_1^o \geq 630$$

#Plant B consntraints :

$$P_1^d - P_2^o \geq 641$$

$$P_2^d - P_2^o \geq 645$$

$$P_3^d - P_2^o \geq 649$$

Non-negativity of the variables

$$P_i^j \geq 0$$

Where

$$i = 1, 2, 3$$

These constraints can be modified as :

#Plant A consntraints :

$$P_1^d \geq 622 + P_1^o$$

$$P_2^d \geq 614 + P_1^o$$

$$P_3^d \geq 630 + P_1^o$$

#Plant B consntraints :

$$P_1^d \geq 641 + P_2^o$$

$$P_2^d \geq 645 + P_2^o$$

$$P_3^d \geq 649 + P_2^o$$

$$P_i^j \geq 0$$

Where

$$i = 1, 2, 3$$

3. Make an economic interpretation of the dual

While the objective function of the Primal is to minimize the cost at the operation side (plants), the objective function of the Dual is to maximize the revenue at the destination side (warehouses).

The price at the destination should be equal to or more than the price at the plant (production)in addition to the cost to make a profit. Which is already represented by the Dual constraints.

$$P_i^d >= costs + P_i^o$$

To translate that into an economic term

$$MR_i >= MC_i$$

Marginal revenue (MR): refers to the marginal revenue which is the increase in revenue that results from a one-unit increase in production. Marginal cost (MC): refers to the cost added by producing one additional unit

When marginal revenue equals marginal cost, profit has reached its maximum, so production should be stopped at that point as increasing production more would not make any more profits and eventually lose money. So, in this problem the aim is to reach the point where

$$MR = MC$$

```
lptrans$duals
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
```

As it is known , the solutions of the Dual is nothing , but the shadow prices of the Primal . The dual solution of this problem indicate that all of the shadow prices for the primal problem is equal to zero , which means there is no possibility to increase profit or decrease cost by reallocation resources (The result we have conclude is the feasible solution) .in this case marginal revenue is equal to marginal cost .

$$MR = MC$$