Assignment_3

2022-10-16

Transportation problem

Munerah Al-Fayez

Heart Start produces automated external defibrillators (AEDs) in each of two different plants (A and B). The unit production costs and monthly production capacity of the two plants are indicated in the table below. The AEDs are sold through three wholesalers. The shipping cost from each plant to the warehouse of each wholesaler along with the monthly demand from each wholesaler are also indicated in the table. How many AEDs should be produced in each plant, and how should they be distributed to each of the three wholesaler warehouses so as to minimize the combined cost of production and shipping?

```
knitr::opts_chunk$set(echo = TRUE)
```

The unit production costs and monthly production capacity of the two plants are indicated in the table below. The AEDs are sold through three wholesalers. The shipping cost from each plant to the warehouse of each wholesaler along with the monthly demand from each wholesaler are also indicated in the table

```
tab <- matrix(c(22,14,30,600,100,
                16,20,24,625,120,
                80,60,70,"-","-") , ncol=5 , byrow=TRUE)

colnames(tab) <- c("Warehouse1","Warehouse2","Warehouse3","Prod cost","Prod Capacity")
row.names(tab) <- c("Plant A","Plant B","Demand")
tab <- as.table(tab)
tab
```

```
##         Warehouse1 Warehouse2 Warehouse3 Prod cost Prod Capacity
## Plant A 22         14         30         600       100
## Plant B 16         20         24         625       120
## Demand  80         60         70         -         -
```

The above transportation problem can be formoulated as below

$$\text{Min } TC = 22X_{11} + 14X_{12} + 30X_{13}$$

$$+16X_{21} + 20X_{22} + 24X_{23}$$

/text{subject to}

#Production Capacity consnstraints Production plant A :

$$X_{11} + X_{12} + X_{13} \leq 100$$

Production Plant B :

$$X_{21} + X_{22} + X_{23} \leq 120$$

#Demand Consntraints

Demand Warehouse 1 :

$$X_{11} + X_{21} \geq 80$$

Demand Warehouse 2 :

$$X_{12} + X_{22} \geq 60$$

Demand Warehouse 3 :

$$X_{13} + X_{23} \geq 70$$

Non-negativity of the variables

$$X_{ij} \geq 0$$

Where

$$i = 1, 2, 3$$

And

$$j = 1, 2, 3$$

1. Formulate and solve this transportation problem using R

#Solving the above transportation cost minimization problem using R . This Transportation problem is unbalanced one . Demand is less than supply by 10 , So I create a dummy variable in column 4 with transportation cost zero and demand 10 .

```
library(lpSolveAPI)
library(lpSolve)
# Set up cost matrix
costs <- matrix(c(622,614,630,0,
                641,645,649,0) , ncol=4 , byrow=TRUE)

# Set up cost table for each warehouse

colnames(costs) <- c("Warehouse1","Warehouse2","Warehouse3","Dummy")
row.names(costs) <- c("Plant A","Plant B")
costs <- as.table(costs)
costs
```

```
##         Warehouse1 Warehouse2 Warehouse3 Dummy
## Plant A        622        614        630     0
## Plant B        641        645        649     0
```

```
# Set up consntraints signs and right-hand sides (production side )
row.signs <- rep("<=",2)
row.rhs <- c(100,120)

# Demand consntraints
col.signs <- rep(">=",4)
col.rhs <- c(80,60,70,10)

#Run

lptrans <- lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)

#Values of all 6 variables
lptrans$solution
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   60   40    0
## [2,]   80    0   30   10
```

```
# Value of the objective function
lptrans$objval
```

```
## [1] 132790
```

When solved the transportation problem, I got the values of the variables as

$$x_{12} = 60$$

$$x_{13} = 40$$

$$x_{21} = 80$$

$$x_{23} = 30$$

$$x_{24} = 10$$

132,790 is the minimum combined cost of production and shipping founded for the optimal solution . In order to minimize the combined cost of production and shipping Plant A should produce 100 units , 60 units for the warehouse 2 and 40 units for warehouse 3 . Plant B should produce 110 units , 80 units for the warehouse 1 and 30 units for warehouse 3 .

2. Formulate the dual of this transportation problem

$$\text{Max } VA = (80P_1^d + 60P_2^d + 70P_3^d) - (100P_1^o - 120P_2^o)$$

Subject to

#Plant A consntraints :

$$P_1^d - P_1^o \geq 22$$

$$P_2^d - P_1^o \geq 14$$

$$P_3^d - P_1^o \geq 30$$

#Plant B consntraints :

$$P_1^d - P_2^o \geq 16$$

$$P_2^d - P_2^o \geq 20$$

$$P_3^d - P_2^o \geq 24$$

Non-negativity of the variables

$$P_i^j \geq 0$$

Where

$$i = 1, 2, 3$$

3. Make an economic interpretation of the dual

```
lptrans$duals
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
```

As it is known , the solutions of the Dual is nothing , but the shadow prices of the Primal . The dual solution of this problem indicate that all of the shadow prices for the primal problem is equal to zero , which means there is no possibility to increase profit or decrease cost by reallocation resources (The result we have conclude is the feasible solution) .in this case marginal revenue is equal to marginal cost .

$$MR = MC$$