# Helmholtz equation with different PoU functions

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## 1 Problem definition

Consider the one-dimensional Helmholtz equation with Dirichlet boundary condition over $\Omega = [0, 8]$

$$\begin{cases} \dfrac{\mathrm{d}^2 u(x)}{\mathrm{d}x^2} - \lambda u(x) = f(x) \quad x \in \Omega, \\ u(0) = c_1, \quad u(8) = c_2. \end{cases} \tag{1}$$

Once an explicit form of $u$ is given, $c_1$, $c_2$, and $f$ can be computed.

And we use the following explicit solution to (1) in our code

$$u(x) = \sin(3\pi x + \frac{3\pi}{20}) \cos(2\pi x + \frac{\pi}{10}) + 2. \tag{2}$$

For this one-dimensional problem, the commonly used PoU functions are

$$\psi_n^a(x) = \mathbb{I}_{-1 \le \tilde{x} < 1}, \tag{3}$$

and

$$\psi_n^b(x) = \begin{cases} \dfrac{1 + \sin(2\pi\tilde{x})}{2} & -\dfrac{5}{4} \le \tilde{x} < -\dfrac{3}{4}, \\ 1 & -\dfrac{3}{4} \le \tilde{x} < \dfrac{3}{4}, \\ \dfrac{1 - \sin(2\pi\tilde{x})}{2} & \dfrac{3}{4} \le \tilde{x} < \dfrac{5}{4}, \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

See Figure 1 for the visualization of $\psi^a$ and $\psi^b$.

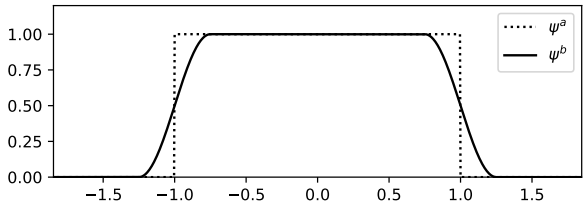


Figure 1: Visualization of $\psi^a(x)$ in (3) and $\psi^b(x)$ in (4).

## 2 Sample code

In our current implementation, we use Python to coding, two kinds of PoU functions are implemented in code *RFM_helm1d_PoU_psi_a_pytorch.py* and *RFM_helm1d_PoU_psi_b_pytorch.py* respectively.

For more specific comments, see the annotation in the codes.