Internship Defense

Control of electron temperature profile in H-mode tokamak plasmas

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Outline of the presentation

- Context and motivation
- Previous works and objectives
- Control-oriented model
- Stability of the open loop system
- Distributed control and stability of the closed loop system
- Control implementation and results
- Conclusion and future work

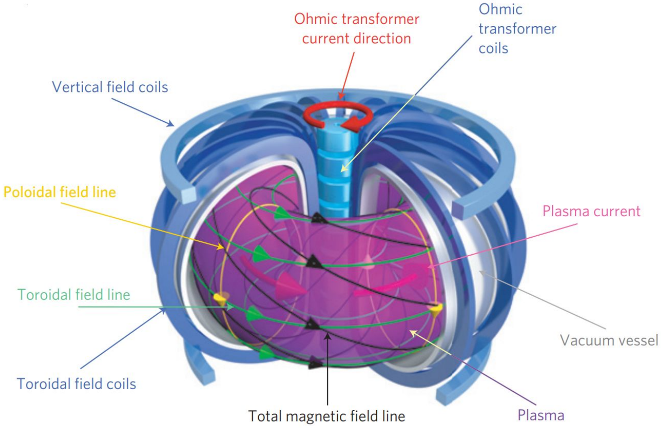
Context and motivation

➔ Thermonuclear fusion

$$ {}^{2}_{1}\mathrm{D} + {}^{3}_{1}\mathrm{T} \longrightarrow {}^{4}_{2}\mathrm{He}(\,3.5\,\mathrm{Mev}) + {}^{1}_{0}\mathrm{n}(\,14.1\,\mathrm{Mev}) $$

➔ Magnetic confinement and tokamaks

$$ n_e \tau_E \geq \frac{2T}{\langle \sigma v \rangle E_\alpha} $$



➔ Advanced scenarios and plasma profile control

Previous works and objectives

- Data driven model (Moreau et al., 2008)
- Discretized model for control (Kim and Lister, 2012)
- The coupling (Mavkov et al., 2017)

First-principle model (Felici, 2011)

Distributed control (Bribiesca Argomedo et al., 2013)

The nonlinearity

➔ Control of the electron temperature profile in H-mode plasmas

Control-oriented model

Control-oriented model

Heat transport dynamics

$$\frac{3}{2}\frac{\partial(n_e T_e)}{\partial t} = \frac{1}{a^2}\frac{1}{x}\frac{\partial}{\partial x}\left(x n_e \chi_e \frac{\partial T_e}{\partial x}\right) - P_{sinks} + P_{sources}$$

$$P_{sources} = \cancel{P_{OH}} + P_{aux}$$

$$\cancel{P_{sinks} = P_{ei} + P_{e,rad}}$$

$$\frac{\partial T_e}{\partial x}(0,t) = 0, \forall t \geq 0$$

$$T_e(1,t) = T_{e,edge}(t), \forall t \geq 0$$

$$T_e(x,0) = T_0(x), \forall x \in [0,1]$$

- I_p is computed offline and defines the discharge phases.
- T_i is not modeled.

Control-oriented model

Heat diffusivity model

Bohm/gyro-Bohm empirical model (Erba et al., 1997) $\chi_e = \chi_{Be} + \chi_{gBe}$

$$\chi_{Be} = \alpha_B \frac{T_e}{eB_{\phi 0}} {L^*_{p_e}}^{-1} \langle L^*_{T_e} \rangle^{-1} q^2, \quad {L^*_{p_e}}^{-1} = \frac{a|\nabla p_e|}{p_e}$$

$$\chi_{gBe} = \alpha_{gB} \frac{T_e}{eB_{\phi 0}} {L^*_{T_e}}^{-1} \rho^*, \quad \rho^* = \frac{m_e^{1/2} T_e^{1/2}}{eB_{\phi 0}}, \quad {L^*_{T_e}}^{-1} = \frac{a|\nabla T_e|}{T_e}$$

$$\langle L^*_{T_e} \rangle^{-1} = \frac{T_e(x=0.8) - T_e(x=1)}{T_e(x=1)}, \quad p_e = n_e T_e$$

➔ q is the safety factor, $B_{\phi 0}$ is toroidal magnetic field at the center and ρ^* is the electron gyroradius.

Control-oriented model

Heat diffusivity model (Edge model)

The pedestal in represented by introducing a suppression function to the electron heat diffusivity (Pianroj and Onjun, 2012)

$$\chi_{e_s} = \chi_e \times f_s$$

$$f_s = \frac{1}{1 + C\left(\frac{\omega_{E\times B}}{\gamma_{ITG}}\right)^2} \times \frac{1}{max(1, (s - s_{thres})^2)}$$

Suppression of electron heat diffusivity due to :

1) $\omega_{E\times B}$ flow shearing rate together with reduction of turbulence growth rate γ_{ITG} .
2) the magnetic shear s exceeds a threshold s_{thres}.

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Control-oriented model

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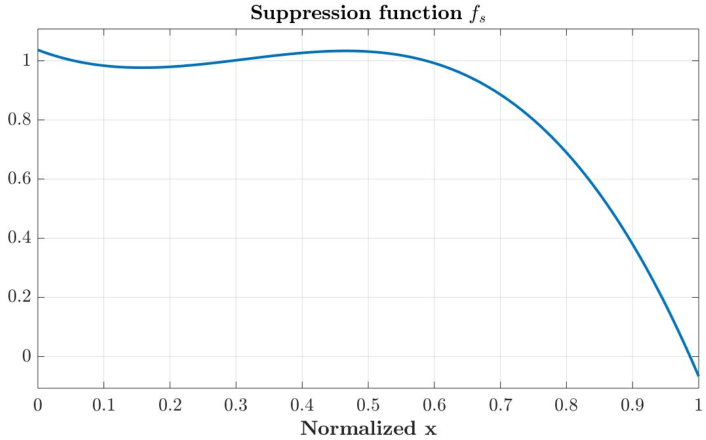


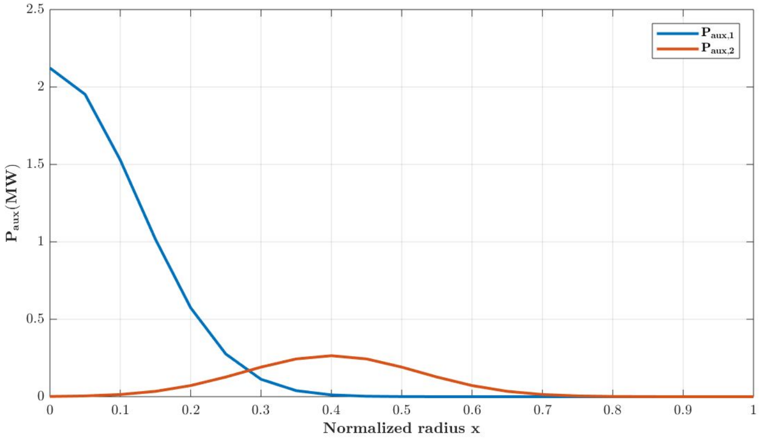
Control-oriented model

Model inputs

Electron Cyclotron Heating and Current Drive antennas (ECH/ECCD) approximated by weighted Gaussian distributions

$$P_{aux,i}(x,t) = P_i(t) \exp\left\{\frac{-4(x - x_{dep,i})^2}{w_{dep,i}^2}\right\} / \int_0^a \exp\left\{\frac{-4(x - x_{dep,i})^2}{w_{dep,i}^2}\right\}$$

$$P_{aux}(x,t) = \sum_i P_{aux,i}(x,t)$$



Stability of the open loop system

Stability of the open loop system

Integral inequalities of polynomial integrands

The aim is to verify
$$\int_0^1 f(x, D^\alpha u(x))\ dx \geq 0$$
where **u** verify the boundary condition:
$$Q\begin{pmatrix} D^{\alpha-1}u(1) \\ D^{\alpha-1}u(0) \end{pmatrix} = 0$$
and the integrand is polynomial on its second argument, so we can write **f** in quadratic-like form and we add terms due to the uniqueness of the integral expression:
$$\int_0^1 \left[\left(\xi^{\lceil \frac{k}{2} \rceil}(D^\alpha u(x))\right)^T F(x)\ \xi^{\lceil \frac{k}{2} \rceil}(D^\alpha u(x)) + \frac{d}{dx}\left(\left(\xi^{\lceil \frac{k}{2} \rceil}(D^{\alpha-1} u(x))\right)^T H(x)\ \xi^{\lceil \frac{k}{2} \rceil}(D^{\alpha-1} u(x)) \right) \right] dx$$
$$+ \left[\left(\xi^{\lceil \frac{k}{2} \rceil}(D^{\alpha-1} u(x))\right)^T H(x)\ \xi^{\lceil \frac{k}{2} \rceil}(D^{\alpha-1} u(x)) \right]_1^0 \geq 0$$

Stability of the open loop system

Integral inequalities of polynomial integrands

After manipulation,

$$\int_0^1 \left(\xi^{\lceil \frac{k}{2} \rceil}(D^\alpha u(x))\right)^T (F(x) + \bar{H}(x))\, \xi^{\lceil \frac{k}{2} \rceil}(D^\alpha u(x))\, dx$$

we get:

$$+ \boxed{\begin{bmatrix} \xi^{\lceil \frac{k}{2} \rceil}(D^{\alpha-1}u(0)) \\ \xi^{\lceil \frac{k}{2} \rceil}(D^{\alpha-1}u(1)) \end{bmatrix}^T \begin{bmatrix} H(0) & 0 \\ 0 & -H(1) \end{bmatrix} \begin{bmatrix} \xi^{\lceil \frac{k}{2} \rceil}(D^{\alpha-1}u(0)) \\ \xi^{\lceil \frac{k}{2} \rceil}(D^{\alpha-1}u(1)) \end{bmatrix} \geq 0}$$

Need to prove $F(x) + \bar{H}(x) \geq 0, \quad \forall x \in [0,1]$ (**F** and **H** are polynomial on **x**)

Positivstellensatz : Find $N(x) \in \Sigma^{n_F \times n_F}[x]$, such that :

$$F(x) + \bar{H}(x) - N(x)w(x) \in \Sigma^{n_F \times n_F}[x]$$

solved using **SOSTOOLS** toolbox.

$$k = deg(f(.,z))$$

$$n = \dim(u)$$

$$F : [0,1] \to \mathbb{S}^{\sigma(n\alpha, \lceil \frac{k}{2} \rceil)}$$

$$H : [0,1] \to \mathbb{S}^{\sigma(n(\alpha-1), \lceil \frac{k}{2} \rceil)}$$

$$n_F = \sigma(n\alpha, \lceil \tfrac{k}{2} \rceil)$$

$$w(x) := x(x-1)$$

Stability of the open loop system

Stability analysis

Lyapunov candidate function:

$$V(T_e) = \frac{1}{2}||T_e||^2_{2,P(x)} = \frac{1}{2}\int_0^1 x^2 P(x) T_e^2 \, dx, \qquad P(x) > 0 \quad \forall x \in [0,1]$$

and the system dynamics :

$$\frac{\partial T_e}{\partial t} = \frac{A}{x}\frac{\partial}{\partial x}\left(x\left(B(x)\frac{\partial T_e}{\partial x} + C(x)\underline{\sqrt{T_e}}\frac{\partial T_e}{\partial x}\right)\frac{\partial T_e}{\partial x}\right), \quad \forall x \in [0,1]$$

Stability of the open loop system

Stability analysis

Lyapunov candidate function:

$$V(T_e) = \frac{1}{2}||T_e||^2_{2,P(x)} = \frac{1}{2}\int_0^1 x^2 P(x) T_e^2 \, dx, \qquad P(x) > 0 \quad \forall x \in [0,1]$$

and the system dynamics :

$$\frac{\partial \mathcal{T}_e}{\partial t} = \frac{A}{x}\frac{\partial}{\partial x}\left(x \left(B(x)\frac{\partial \mathcal{T}_e}{\partial x} + C(x)\underline{\mathcal{T}_e}\frac{\partial \mathcal{T}_e}{\partial x}\right)\frac{\partial \mathcal{T}_e}{\partial x}\right), \qquad \forall x \in [0,1]$$

For asymptotic stability we must verify :

$$\dot{V}(T_e) = \boxed{\dot{V}(\mathcal{T}_e)} + \int_0^1 x^2 P(x) T_e \left(\frac{\partial T_e}{\partial x} - \frac{\partial \mathcal{T}_e}{\partial x}\right) dx < 0$$

Stability of the open loop system

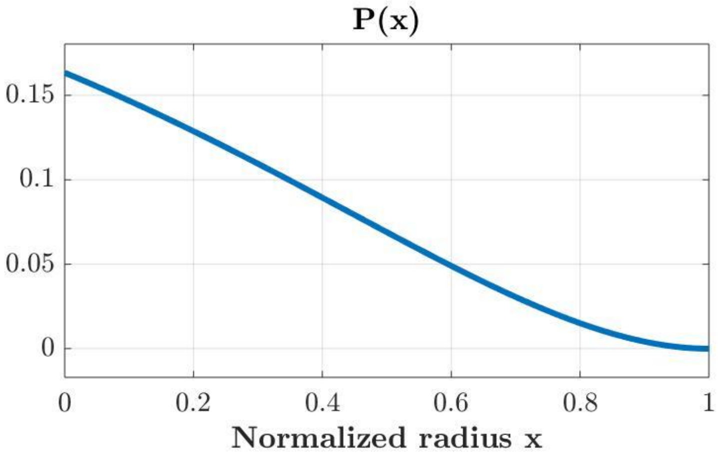
Stability analysis

In quadratic-like form :

$$\dot{V}(\mathcal{T}_e) = \int_0^1 \begin{bmatrix} \mathcal{T}_e \\ \frac{\partial \mathcal{T}_e}{\partial x} \\ \mathcal{T}_e \frac{\partial \mathcal{T}_e}{\partial x} \\ (\frac{\partial \mathcal{T}_e}{\partial x})^2 \end{bmatrix}^T \begin{bmatrix} 0 & 0 & 0 & \frac{\alpha_4}{4} \\ 0 & 0 & \frac{\alpha_4}{4} & \frac{\alpha_1}{2} \\ 0 & \frac{\alpha_4}{4} & \alpha_3 & \frac{\alpha_2}{2} \\ \frac{\alpha_4}{4} & \frac{\alpha_1}{2} & \frac{\alpha_2}{2} & 0 \end{bmatrix} \begin{bmatrix} \mathcal{T}_e \\ \frac{\partial \mathcal{T}_e}{\partial x} \\ \mathcal{T}_e \frac{\partial \mathcal{T}_e}{\partial x} \\ (\frac{\partial \mathcal{T}_e}{\partial x})^2 \end{bmatrix} dx$$

Using **SOSTOOLS**, we compute **P(x)**

to make $\dot{V}(\mathcal{T}_e) < 0$



Distributed control and stability of the closed loop system

Distributed control

We apply a **PI** control action to the error mode :

$$\begin{cases} \dfrac{\partial \mathcal{T}_e}{\partial t} = \mathcal{D}(t, x, \mathcal{T}_e) + u \\ \mathcal{E}(t, x) = \mathcal{T}_e - r(x) \end{cases} \longrightarrow \frac{\partial \mathcal{E}}{\partial t} = \mathcal{D}(t, x, \mathcal{E} + r) + u \longrightarrow \frac{\partial \mathcal{E}}{\partial t} = \mathcal{D}(t, x, \mathcal{E}) + \mathcal{R}_{est}(t, x, \mathcal{E}, r) + u$$

The feedback control input and the closed loop system :

$$u_{des} = -\mathcal{R}_{est}(t, x, \mathcal{E}, r) - \alpha_P \mathcal{E} - \int_{t_0}^{t} \alpha_I \mathcal{E} dt \longrightarrow \begin{cases} \dfrac{\partial \mathcal{E}}{\partial t} & = \mathcal{D}(t, x, \mathcal{E}) - \alpha_P \mathcal{E} + I \\ \dfrac{\partial I}{\partial t} & = -\alpha_I \mathcal{E} \end{cases}$$

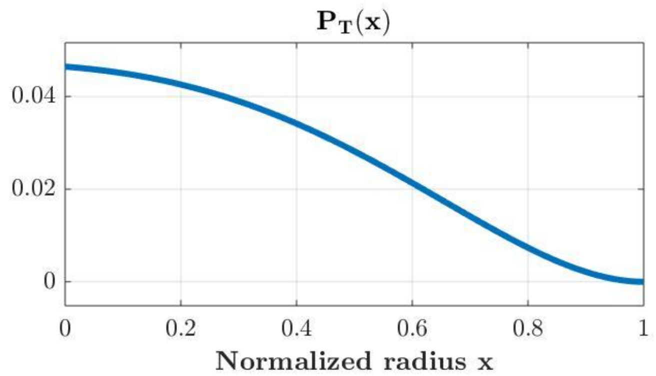
stability of the closed loop system

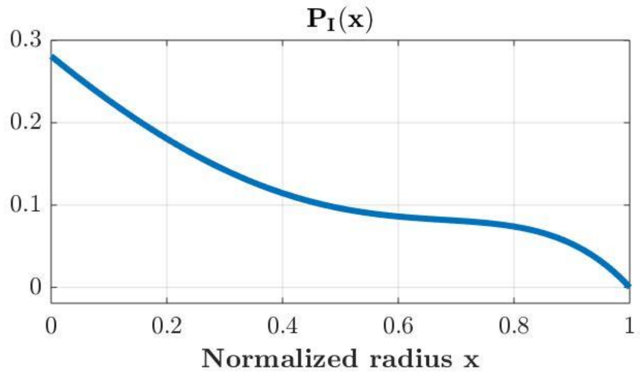
Lyapunov candidate function and its time derivative in quadratic-like form :

$$V(\mathcal{T}_e, I) = \frac{1}{2}\int_0^1 x^2 P_T(x)\mathcal{T}_e^2 \, dx + \frac{1}{2}\int_0^1 x^2 P_I(x) I^2 \, dx,$$
$$P_T(x) > 0 \ and \ P_I(x) > 0 \quad \forall x \in [0,1]$$

$$\dot{V}(\mathcal{T}_e, I) = \int_0^1 \begin{bmatrix} \mathcal{T}_e \\ \frac{\partial \mathcal{T}_e}{\partial x} \\ \mathcal{T}_e \frac{\partial \mathcal{T}_e}{\partial x} \\ (\frac{\partial \mathcal{T}_e}{\partial x})^2 \\ I \end{bmatrix}^T \begin{bmatrix} \alpha_5 & 0 & 0 & \frac{\alpha_4}{4} & \frac{\alpha_6}{2} \\ 0 & 0 & \frac{\alpha_4}{4} & \frac{\alpha_1}{2} & 0 \\ 0 & \frac{\alpha_4}{4} & \alpha_3 & \frac{\alpha_2}{2} & 0 \\ \frac{\alpha_4}{4} & \frac{\alpha_1}{2} & \frac{\alpha_2}{2} & 0 & 0 \\ \frac{\alpha_6}{2} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \mathcal{T}_e \\ \frac{\partial \mathcal{T}_e}{\partial x} \\ \mathcal{T}_e \frac{\partial \mathcal{T}_e}{\partial x} \\ (\frac{\partial \mathcal{T}_e}{\partial x})^2 \\ I \end{bmatrix} dx$$

and the weighting functions ;



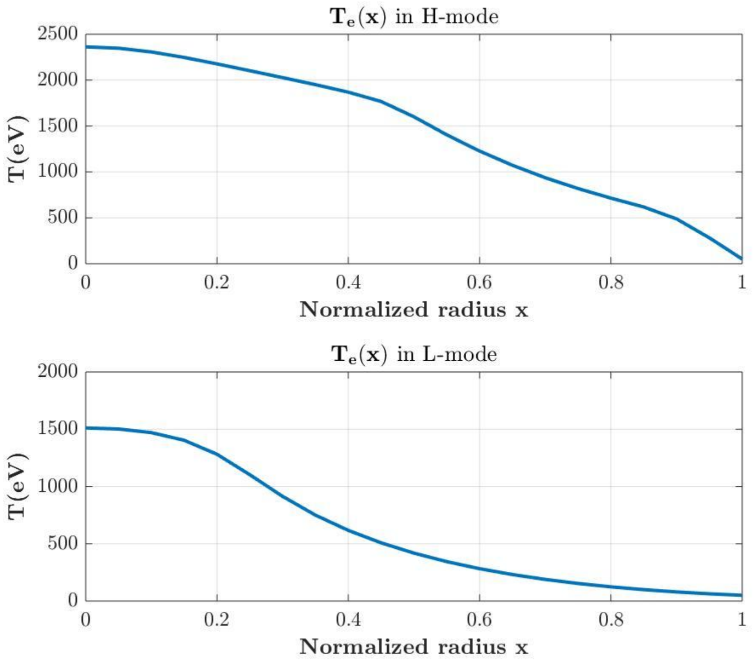


Control implementation and results

Control implementation

Use RAPTOR with TCV settings :

➔ H-mode heat diffusivity model



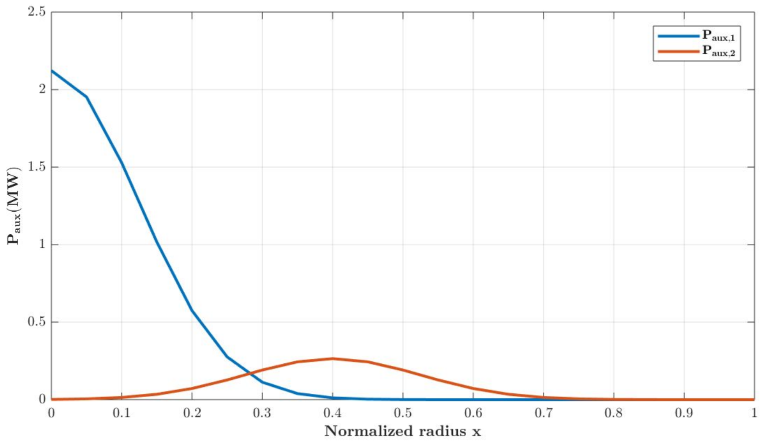
Control implementation

Use **RAPTOR** with **TCV** settings :

➔ H-mode heat diffusivity model

➔ Power inputs : **ECH** source at **xdep=0** and **wdep=0.35**, and **ECCD** source at **xdep=0.4** and **wdep=0.35**.

➔ The actual system inputs $u_{ac} = [P_1, P_2]^T$ solution to the optimization problem

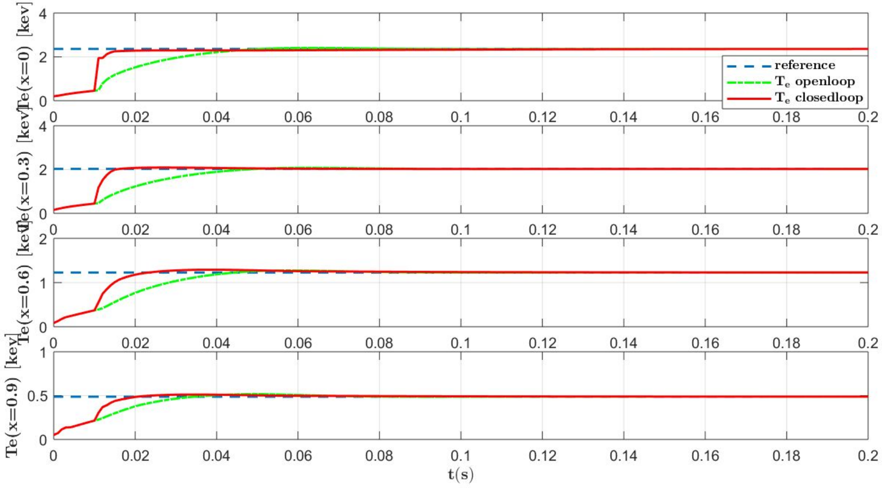


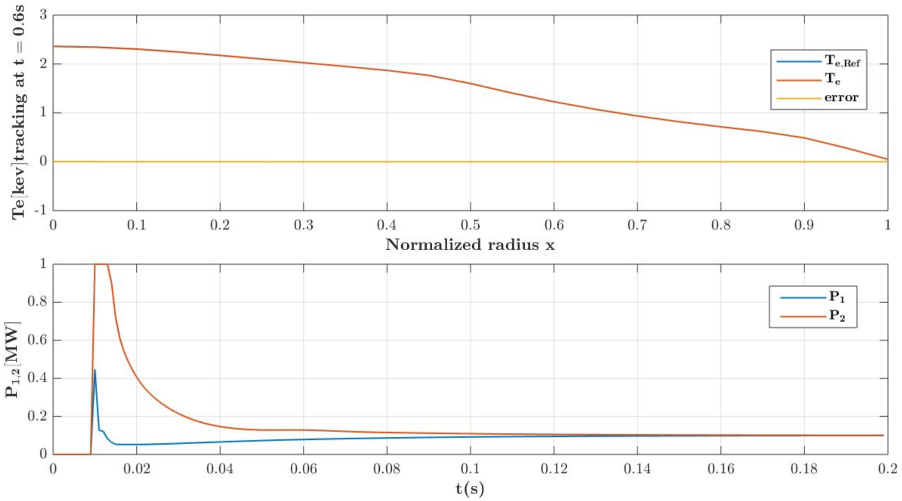
$$u_{ac}^* = arg\,\min_{u_{ac}} \int_0^1 \left(u_{des} - P_{aux}(u_{ac})\right)^2 dx$$ subject to : $$0 \leq P_1 \leq 1\text{MW}$$ $$0 \leq P_2 \leq 1\text{MW}$$

Results

The reference profile with $P_1 = 0.1$ MW and $P_2 = 0.1$ MW

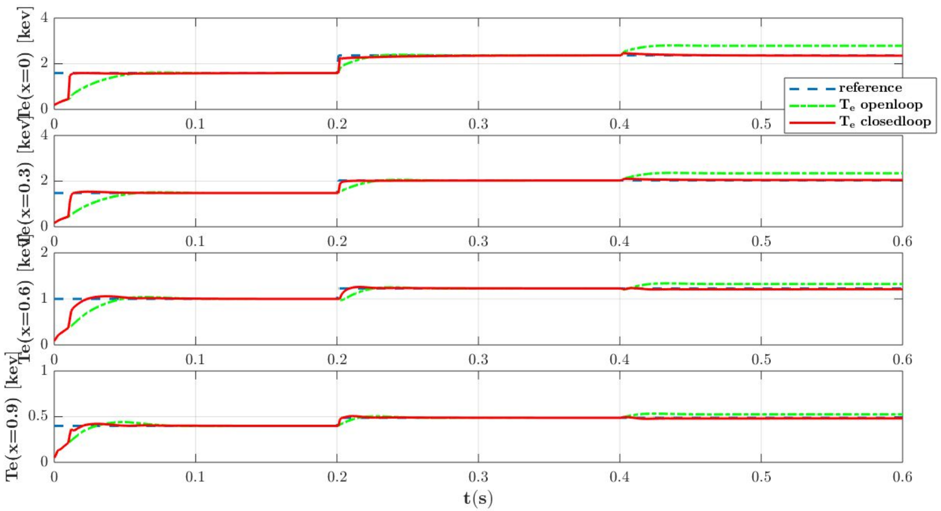
and the **PI** parameters are : $\alpha_I = 4 \times 10^3$ and $\alpha_P = 2 \times 10^5$

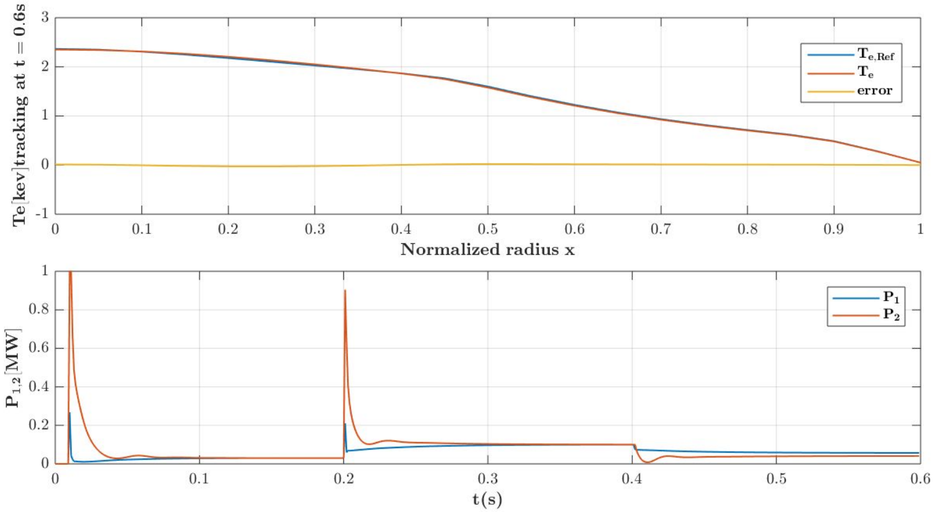




Results

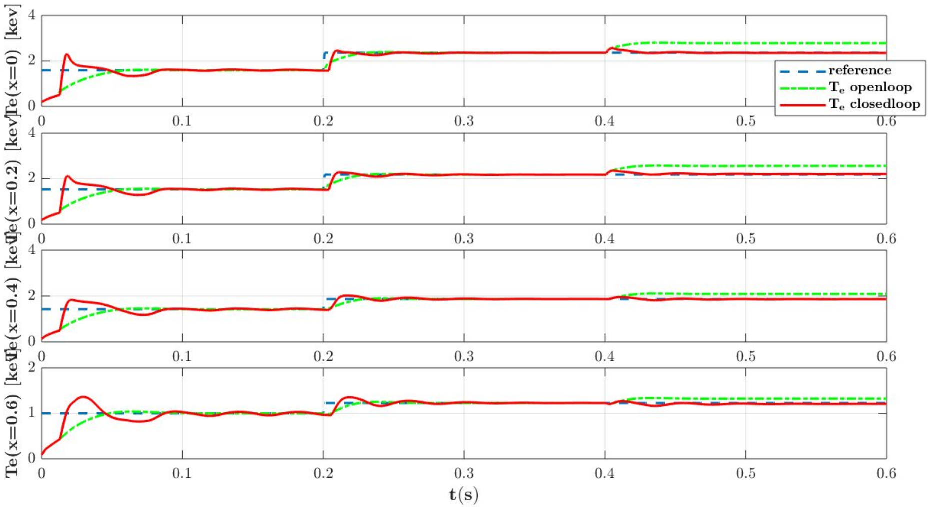
Change the reference profile at **t = 0.2 s** + add disturbance at **t = 0.4 s**, a third **ECCD** source at **xdep = 0.2**, **wdep = 0.35** and **P3 = 0.1 MW**.

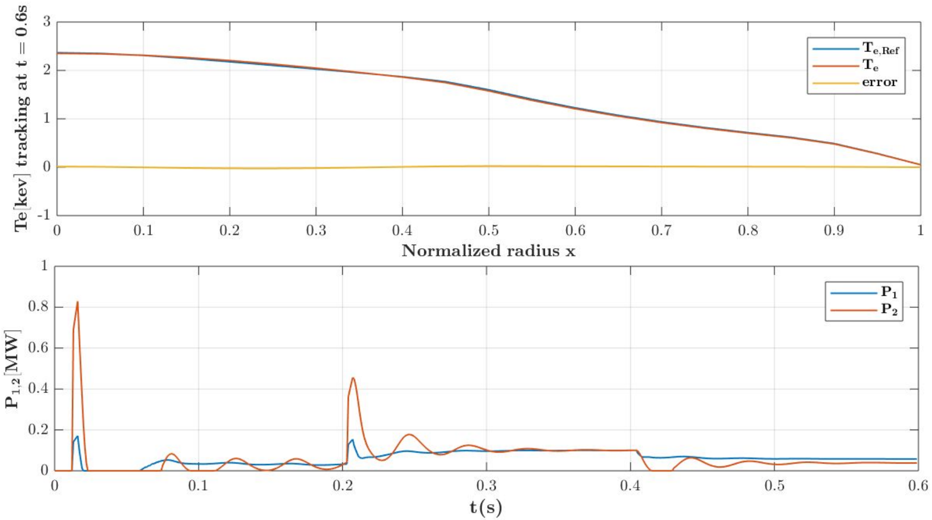




Results

Add time delays of **3 ms** to the control loop





Conclusions

- Use a control-oriented model for the electron temperature profile in H-mode.
- Analyze the stability of the nonlinear PDE describing the dynamics.
- Apply a PI control action and analyze the stability of the closed loop system
- Evaluate the control strategy on **RAPTOR** considering the input constraints.

Future work

- Considering the coupling between the plasma profiles as well as the two timescales dynamics.
- Investigating transport models that describe internal transport barrier (ITB) scenarios.
- The use of more elaborate control strategies, boundary control, and the consideration of experimental validation on tokamaks.

References

Argomedo, F. B., Witrant, E., Prieur, C., Brémond, S., Nouailletas, R., & Artaud, J. F. “Lyapunov-based distributed control of the safety-factor profile in a tokamak plasma”. Nuclear Fusion, 53(3) (2013): 033005.

Erba, M., et al. “Development of a non-local model for tokamak heat transport in L-mode, H-mode and transient regimes.” Plasma Physics and Controlled Fusion 39.2 (1997): 261.

Felici, F. “Real-time control of tokamak plasmas: from control of physics to physics-based control.” (2011).

Kim, S. H., and J. B. Lister. “A potentially robust plasma profile control approach for ITER using real-time estimation of linearized profile response models.” Nuclear Fusion 52.7 (2012): 074002.

Mavkov, B., Witrant, E., & Prieur, C. “Distributed control of coupled inhomogeneous diffusion in tokamak plasmas (full version).”(2017).

Moreau, D., et al. “A two-time-scale dynamic-model approach for magnetic and kinetic profile control in advanced tokamak scenarios on JET.” Nuclear Fusion 48.10 (2008): 106001.

Pianroj, Y., & Onjun, T. “Simulations of H-mode plasmas in tokamak using a complete core-edge modeling in the BALDUR code.” Plasma Science and Technology 14.9 (2012): 778.