Nonlinear PDE-Based control of the electron temperature in H-mode tokamak plasmas

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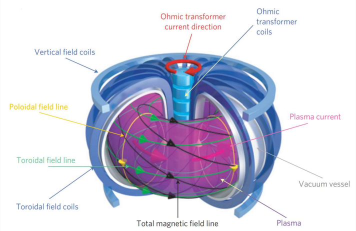
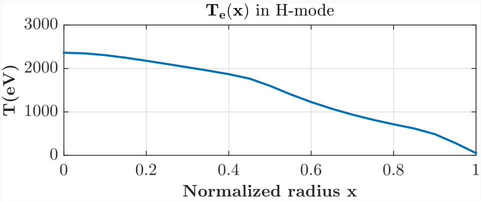
Table of contents

INTRODUCTION

Introduction

- Fixed flux surface geometry, 1D transport.
- Plasma profile control using auxiliary heating sources.
- The High confinement mode (H-mode) is the target (increase τ_E).





Introduction

- Data driven model ([Moreau et al., 2008])
- Discretized model for control ([Ou et al., 2008])
- The coupling ([Mavkov et al., 2017])
- First-principle model ([Witrant et al., 2007])
- Distributed control ([Bribiesca Argomedo et al., 2013])
- The nonlinearity of the electron temperature dynamics T_e

Control of the electron temperature profile in TCV[1] H-mode plasmas.

- Infinite-dimensional model: Nonlinear parabolic PDE.
- Lyapunov control function approach.
- Sum of squares technique.

[1] Tokamak à Configuration Variable at EPFL in Lausanne, Switzerland.

SYSTEM DESCRIPTION AND CONTROL PROBLEM

Electron temperature dynamics

- Electron temperature dynamics T_e

$$\frac{3}{2}\frac{\partial(n_e T_e)}{\partial t} = \frac{1}{a^2}\frac{1}{x}\frac{\partial}{\partial x}(x n_e \chi_e \frac{\partial T_e}{\partial x}) + P, \tag{1}$$

with boundary conditions:

$$\frac{\partial T_e}{\partial x}(0, t) = 0, \;\; T_e(1, t) = T_{e,edge}(t), \forall t \geq t_0 \tag{2}$$

and the initial condition:

$$T_e(x, t_0) = T_0(x), \forall x \in [0, 1] \tag{3}$$

$\chi_e(x, t)$; the electron heat diffusivity,
$P(x, t) = P_{ohm}(x, t) + P_{aux}(x, t) - P_{rad}(x, t)$

Electron heat diffusivity model

- Semi-empirical model: the modified bohm/gyrobohm model [Pianroj and Onjun, 2012]:

$$\begin{cases} \chi_e = \chi_{e_c} \times f_s, \quad \chi_{e_c} = (2\chi_{Be} + \chi_{gBe}), \\ \chi_{Be} = 4 \times 10^{-5} R \left| \frac{\nabla (n_e T_e)}{n_e B_{\phi_0}} \right| q^2 \left(\frac{T_{e,0.8} - T_{e,1.0}}{T_{e,1.0}} \right), \\ \chi_{gBe} = 5 \times 10^{-6} \sqrt{T_e} \left| \frac{\nabla T_e}{B_{\phi_0}^2} \right|, \\ f_s = \frac{1}{1+k\left(\frac{\omega_{E\times B}}{\gamma_{ITG}}\right)^2} \times \frac{1}{max(1,(s-s_{thres})^2)} \end{cases} \tag{4}$$

f_s; taken as polynomial approximation $f_s(x)$ of the experimental results obtained in the same paper.

Control problem

To formulate the control system model, we use the assumptions:

1. $q(x,t)$ is constant in time and the density dynamics can be neglected.
2. $\frac{\partial T_e}{\partial x} \leq 0$ always and everywhere and $T_{e,edge} \ll T_{e,center}$ and to be neglected.

We get:

$$\frac{\partial T_e}{\partial t} = \frac{A}{x}\frac{\partial}{\partial x}\left(x\left(B(x)+C(x)\sqrt{T_e}\right)\left(\frac{\partial T_e}{\partial x}\right)^2\right) + u, \quad \forall x \in [0,1] \quad (5)$$

with boundary conditions,

$$\frac{\partial T_e}{\partial x}(0,t) = 0, \;\; T_e(1,t) = 0, \forall t \geq t_0 \qquad (6)$$

where: $A = \frac{2}{3a^2}$, $B(x) = \frac{-8\times10^{-5}RL_{T_e}}{B_{\phi_0}}q^2(x)f_s(x)$, $C(x) = \frac{-5\times10^{-6}}{B_{\phi_0}^2}f_s(x)$.

STABILITY ANALYSIS AND DISTRIBUTED CONTROL

Stability of the open-loop system

Nominal stability of $(\bar{u}(x), \bar{T}_e(x))$ with Lyapunov function candidate:

$$V(T_e) = \frac{1}{2}\int_0^1 xP_{T_e}(x)(T_e - \bar{T}_e)^2\,dx, \quad P_{T_e}(x) > 0 \tag{7}$$

Theorem

Suppose that for a given $\alpha_1 > 0$, there exist a polynomial $P_{T_e}(x) > 0$ and a 5×5 symmetric polynomial matrix $H(x)$ with $H(0) \geq 0$, $H_{1,1}(1) \leq 0$ and:

$$F(x) + \bar{H}(x) \geq 0, \quad \forall x \in [0, 1] \tag{8}$$

where (8) is a differential matrix inequality.

Then the time derivative $\dot{V}$ of V along the solutions of (5)-(6) verifies:

$$\dot{V}(T_e) \leq -\alpha_1 V(T_e) + \int_0^1 xP_{T_e}(x)(T_e - \bar{T}_e)\tilde{u}\,dx \tag{9}$$

where $\tilde{u}$ is defined as $\tilde{u} = u - \bar{u}$.

Stability of the open-loop system

proof: along [Valmorbida et al., 2015]

Perform the time derivative of the Lyapunov function, integration by part:

$$\dot{V}(T_e)+\alpha_1 V(T_e) = -\int_0^1 G(x, \sqrt{T_e}, D^1 T_e)\, dx + \int_0^1 x P_{T_e}(x)(T_e - \bar{T}_e)\tilde{u}\, dx \tag{10}$$

Change of variable $\tau = \sqrt{T_e}$, we write G in terms of τ as:

$$G(x, D^1\tau) = \xi(D^1\tau)^T F(x) \xi(D^1\tau)$$

$$\xi(D^1\tau) := \left[1, \tau, \frac{\partial \tau}{\partial x}, \tau^2, \tau\frac{\partial \tau}{\partial x}, \left(\frac{\partial \tau}{\partial x}\right)^2, \tau^3, \tau^2\frac{\partial \tau}{\partial x}, \ldots \right.$$
$$\left. \tau\left(\frac{\partial \tau}{\partial x}\right)^2, \left(\frac{\partial \tau}{\partial x}\right)^3, \tau^4, \tau^3\frac{\partial \tau}{\partial x}, \tau^2\left(\frac{\partial \tau}{\partial x}\right)^2, \tau\left(\frac{\partial \tau}{\partial x}\right)^3\right]^T$$

Stability of the open-loop system

We also have:

$$\int_0^1 \xi(D^1\tau)^T F(x)\xi(D^1\tau)\ dx = \int_0^1 \Big[\xi(D^1\tau)^T F(x)\xi(D^1\tau) + \frac{d}{dx}\left(\mu(\tau)^T H(x)\mu(\tau)\right)\Big] dx - \left(\mu(\tau)^T H(x)\mu(\tau)\right)\Big|_0^1 \tag{11}$$

With $\mu(\tau) := \left[1, \tau, \tau^2, \tau^3, \tau^4\right]^T$, and finally:

$$\dot{V}(T_e) + \alpha_1 V(T_e) = -\int_0^1 \xi(D^1\tau)^T \underbrace{(F(x) + \bar{H}(x))}_{\geq 0} \xi(D^1\tau) + \underbrace{H_{1,1}(1)}_{\leq 0} - \mu(\tau(0))^T \underbrace{H(0)}_{\geq 0} \mu(\tau(0)) + \int_0^1 x P_{T_e}(x)(T_e - \bar{T}_e)\tilde{u}\ dx$$

■

Calculation of the weighting Function

To solve the differential matrix inequality $F(x) + \bar{H}(x) \geq 0$ over $\{x|x(1-x) \geq 0\}$ for $P_{T_e}(x)$, we cast the problem into a Sum Of Squares problem (SOSP):

Find $P_{T_e}(x),\ H(x),\ N(x)$ subject to:

- $F(x) + \bar{H}(x) - N(x)x(1-x) \in \Sigma^{14\times 14}[x]$,
- $N(x) \in \Sigma^{14\times 14}[x]$,
- $H(0) \geq 0, H_{1,1}(1) \leq 0$.

The problem is then formulated as an SDP and solved using Yalmip and the SOS module. The resulting $P_{T_e}(x)$ is a decreasing polynomial strictly positive on $[0, 1]$.

Distributed control

Corollary

If the conditions of Theorem 1 are verified, we choose the control input $u_{ctrl} = \bar{u} + \tilde{u}$, where $\tilde{u}$ is calculated to verify the equality:

$$\int_0^1 x P_{T_e}(x)(T_e - \bar{T}_e)\tilde{u}\, dx = -\alpha_2 V(T_e) \tag{12}$$

- Exponential stability of the closed-loop system with convergence rate $\alpha_1 + \alpha_2$
- An explicit control law from (12) is the proportional controller:

$$u_{ctrl} = \bar{u} - \frac{\alpha_2}{2}(T_e - \bar{T}_e) \tag{13}$$

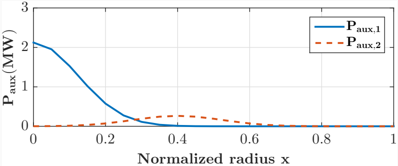
CONTROL IMPLEMENTATION AND RESULTS

RAPTOR simulator and Input constraints

- RAPTOR[2] (EPFL) is a lightweight, simplified transport physics code used as a nonlinear plasma simulator and as a real time state observer.
- Radio-Frequency heating: ECRH[3] and ECCD[4] are gaussian distributed.

$$P_{aux,i}(x,t) = P_i(t)\frac{e^z}{\int_0^1 e^z V' dx},$$

$$z = \frac{-4(x - x_{dep,i})^2}{w_{dep,i}^2}$$



[2] Rapid Plasma Transport simulatOR.
[3] Electron Cyclotron Resonance Heating
[4] Electron Cyclotron Current Drive

Tracking control

- Implementation with the engineering input $u_{ac} = [P_1(t), P_2(t)]^T$

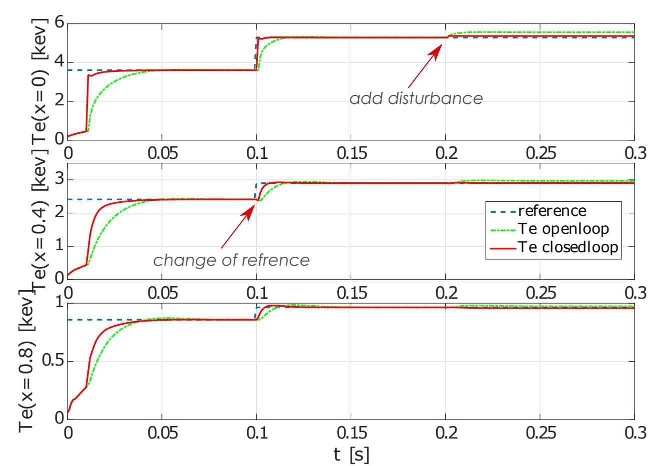
$$u_{ac}^* = arg\ \underset{u_{ac}}{min} \int_0^1 \Big[\underbrace{xP_{T_e}(x)\Big(\big(P_{aux}(u_{ac}) - u_{ref}\big) + \frac{\alpha_2}{2}\big(T_e - T_{e,ref}\big)\Big)}_{Cf\ Corollary} \Big]^2 dx \tag{14}$$

subject to: $\quad 0 \leq P_1 \leq 1\mathrm{MW},\ 0 \leq P_2 \leq 1\mathrm{MW}.$
where $(T_{e,ref},\ u_{ref})$ are the reference temperature profile and its corresponding input.

- In RAPTOR, we use TCV configuration with H-mode transport model [Kim et al., 2016], with ECRH at $x_{dep} = 0$ and ECCD at $x_{dep} = 0.4$.

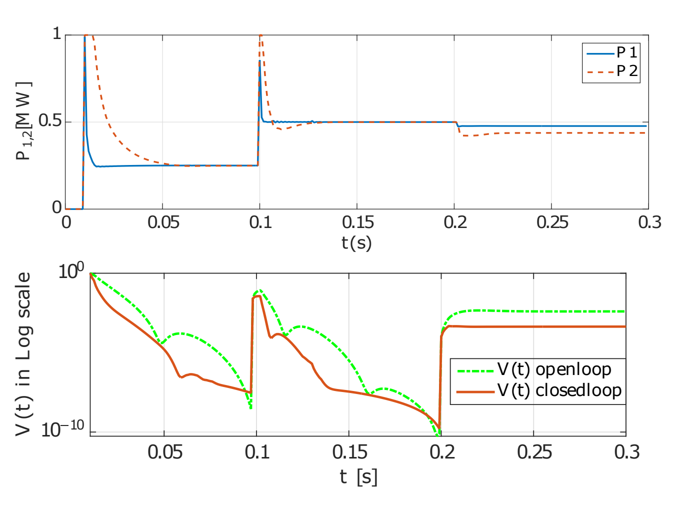
Simulation results

- Changing the reference profile at $t = 0.1\ s$ and adding a disturbance at $t = 0.2\ s$ (third ECCD source at $x_{dep} = 0.2$ with $w_{dep} = 0.35$ and $P_3 = 0.1$ MW).



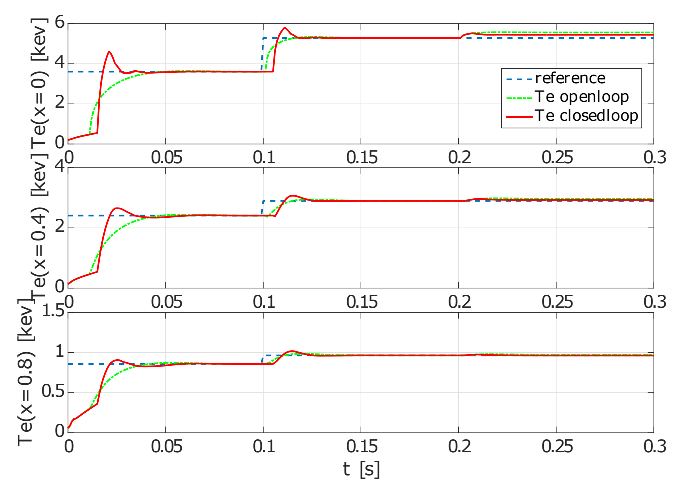
Simulation results

- Time-evolution of $P_{1,2}$ and Lyapunov function evaluation:



Simulation results

- Considering the computation and the transportation of the control signal as a time-delay of 5 *ms*.



CONCLUSION AND FUTURE WORK

Conclusion

- Stability analysis and control of the electron temperature profile in H-mode tokamak plasmas.
- Using Lyapunov approach and sum of squares techniques.
- RAPTOR plasma simulator was used in H-mode and the control input constraints were taken into account.
- The robustness of the controller was investigated with respect to changing operating point, input disturbances and time-delays.

Future work

- Include the ohmic heating power density $P_{ohm}(x, t)$ and the radiated power density $P_{rad}(x, t)$ dynamics to the system.
- Explore central-core-edge diffusivity models that enables internal transport barrier (ITB) [Kim et al., 2016].
- Add the integral action to the analysis to handle the static error.

THANK YOU!

Stability of the open-loop system

$$F(x) + \bar{H}(x) =$$

$$\begin{bmatrix}
f_9 + h_1 & h_2 & h_3 & f_7 + h_4 & h_5 & 0 & h_6 & h_7 & 0 & 0 & f_8 + h_8 & h_9 & f_5 & 0 \\
\bullet & f_7 + h_{10} & h_{11} & h_{12} & h_{13} & 0 & f_8 + h_{14} & h_{15} & f_5 & 0 & h_{16} & h_{17} & f_6 & 0 \\
\bullet & \bullet & 0 & h_{18} & 0 & 0 & h_{19} & f_5 & 0 & 0 & h_{20} & f_6 & 0 & 0 \\
\bullet & \bullet & \bullet & f_8 + h_{21} & h_{22} & f_5 & h_{23} & h_{24} & f_6 & 0 & h_{25} & 2h_{27} & f_1 & f_3 \\
\bullet & \bullet & \bullet & \bullet & f_5 & 0 & h_{26} & f_6 & 0 & 0 & h_{27} & f_1 & f_3 & 0 \\
\bullet & \bullet & \bullet & \bullet & \bullet & 0 & f_6 & 0 & 0 & 0 & f_1 & f_3 & 0 & 0 \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & h_{28} & h_{29} & f_1 & f_3 & h_{30} & h_{31} & f_2 & f_4 \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & f_1 & f_3 & 0 & \frac{3}{4} h_{31} & f_2 & f_4 & 0 \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & 0 & 0 & f_2 & f_4 & 0 & 0 \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & 0 & f_4 & 0 & 0 & 0 \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & h_{32} & h_{33} & 0 & 0 \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & 0 & 0 & 0 \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & 0 & 0 \\
\bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & 0
\end{bmatrix} \tag{15}$$

Stability of the open-loop system

$$f_1 = \tfrac{4}{9}(ABxP'_{T_e}),\ f_2 = \tfrac{2}{3}(ACxP'_{T_e}),\ f_3 = \tfrac{4}{5}(ABxP_{T_e}),\ f_4 = (ACxP_{T_e}),$$
$$f_5 = -\frac{4}{9}(ABx(P'_{T_e}\bar{T}_e + P_T\bar{T}_e{}')),\ f_6 = -\tfrac{2}{5}(ACx(P'_{T_e}\bar{T}_e + P_T\bar{T}_e{}')),$$
$$f_7 = \frac{1}{3}(\alpha_1 xP_{T_e}\bar{T}_e - xP_{T_e}\bar{u}),\ f_8 = -\frac{1}{10}(\alpha_1 xP_{T_e}),$$
$$f_9 = (xP_{T_e}\bar{T}_e\bar{u} - \frac{1}{2}\alpha_1 xP_{T_e}\bar{T}_e{}^2).\ h_1 = \tfrac{dH_{1,1}}{dx},\ h_2 = \tfrac{dH_{1,2}}{dx},\ h_3 = H_{1,2},\ h_4 = \tfrac{2}{3}\tfrac{dH_{1,3}}{dx},$$
$$h_5 = H_{1,3},\ h_6 = \frac{1}{2}\frac{dH_{1,4}}{dx},\ h_7 = H_{1,4},\ h_8 = \frac{2}{5}\frac{dH_{1,5}}{dx},\ h_9 = H_{1,5},\ h_{10} = \tfrac{2}{3}\tfrac{dH_{1,3}}{dx} + \tfrac{dH_{2,2}}{dx},$$
$$h_{11} = H_{1,3} + H_{2,2},\ h_{12} = \frac{1}{2}\frac{dH_{1,4}}{dx} + \frac{dH_{2,3}}{dx},\ h_{13} = H_{1,4} + 2H_{2,3},$$
$$h_{14} = \frac{2}{5}\frac{dH_{1,5}}{dx} + \frac{dH_{2,4}}{dx},\ h_{15} = H_{1,5} + 3H_{2,4},\ h_{16} = \tfrac{dH_{2,5}}{dx},\ h_{17} = 4H_{2,5},$$
$$h_{18} = H_{1,4} + H_{2,3},\ h_{19} = H_{1,5} + H_{2,4},\ h_{20} = H_{2,5},\ h_{21} = \frac{2}{5}\frac{dH_{1,5}}{dx} + \frac{dH_{3,3}}{dx},$$
$$h_{22} = H_{1,5} + 2H_{3,3},\ h_{23} = \tfrac{dH_{3,4}}{dx},\ h_{24} = 3H_{3,4},\ h_{25} = \tfrac{dH_{3,5}}{dx},\ h_{26} = 2H_{3,4},\ h_{27} = 2H_{3,5},$$
$$h_{28} = \frac{dH_{4,4}}{dx}, h_{29} = 3H_{4,4},\ h_{30} = \tfrac{dH_{4,5}}{dx},\ h_{31} = 4H_{4,5},\ h_{32} = \tfrac{dH_{5,5}}{dx},\ h_{33} = 4H_{5,5}. \tag{16}$$

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