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I Problem 3

Given- Cut off frequency (up) = 1000 Hz
Cut off frequency (down) = 350 Hz
Sampling frequency = 5000 Hz
Range = 100 Hz to 10000 Hz
$\alpha_s = 10$ $\alpha_p = 3$

Formulae

$\omega_p = 2\pi \times 1000$

$\omega_s = 2\pi \times 350$

$\Omega_p = \frac{2}{T} \tan\left(\frac{\omega_p T}{2}\right)$

$\Omega_s = \frac{2}{T} \tan\left(\frac{\omega_s T}{2}\right)$

$T = 1/f_s$

Solution

i) $T = 1/f_s = \frac{1}{5000} = 2 \times 10^{-4}$

ii) $\Omega_p = \frac{2}{2\times10^{-4}} \tan\left(\frac{2\pi \times 1000 \times 2 \times 10^{-4}}{2}\right)$

$\Omega_p = 7265.4$ rad/sec

iii) $\Omega_s = \frac{2}{2\times10^{-4}} \tan\left(\frac{2\pi \times 350 \times 2 \times 10^{-4}}{2}\right)$

$\Omega_s = 2235.26$ rad/sec

iv)

$$N = \frac{\log \sqrt{\frac{10^{(0.1\alpha_p)} - 1}{10^{(0.1\alpha_s)} - 1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$$

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Putting values we get

$$N = \frac{0.478}{0.511} \simeq 0.935$$

$$N \approx 1$$

v) $H(s) = \frac{1}{1+s}$ for $N=1$

$$s = \frac{\Omega_p}{s}$$

$$H(s) = \frac{1}{1 + \frac{7265}{s}}$$

$$H(s) = \frac{s}{s+7265}$$

vi) $H(z) = H(s)\Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$

$$H(z) = \frac{\frac{2}{2\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{\frac{2}{2\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 7265}$$

$$H(z) = \frac{y(n)}{x(n)}$$

$$\frac{y(n)}{x(n)} = \frac{10^4\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{10^4\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 7265}$$

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$$\frac{y(n)}{x(n)} = \frac{10^4 - 10^4 z^{-1}}{1 + z^{-1}}$$

$$\frac{10^4 - 10^4 z^{-1} + 7265\,(1 + z^{-1})}{1 + z^{-1}}$$

$$\frac{y(n)}{x(n)} = \frac{10^4 - 10^4 z^{-1}}{10^4 - 10^4 z^{-1} + 7265 + 7265 z^{-1}}$$

$$\frac{y(n)}{x(n)} = \frac{10^4 - 10^4 z^{-1}}{10^4 - 2735 z^{-1} + 7265}$$

$$\frac{y(n)}{x(n)} = \frac{10^4 - 10^4 z^{-1}}{17265 - 2735 z^{-1}}$$

~~$17265\,y(n) - 2735\,y(n-1) = 10^4 x(n) - 10^4 x(n-1)$~~

$$\frac{y(n)}{x(n)} = \frac{10000\,(1 - z^{-1})}{17265\left(1 - \frac{2735}{17265} z^{-1}\right)}$$

$$\frac{y(n)}{x(n)} = \frac{0.579\,(1 - z^{-1})}{1 - 0.158 z^{-1}}$$

$$y(n) - 0.158\,y(n-1) = 0.579\,x(n) - 0.579\,x(n-1)$$

$$\therefore y(n) = 0.158\,y(n-1) + 0.579\,x(n) - 0.579\,x(n-1)$$

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II Problem-3

For $F_s = 4000\,Hz$

i) $T = \frac{1}{F_s} = 2.5 \times 10^{-4}$

ii) $\Omega_p = \frac{2}{T}\tan\left(\frac{2\pi \times 1000 \times T}{2}\right)$

$\Omega_p = \frac{2}{2.5\times10^{-4}}\tan\left(\frac{2000\pi \times 2.5\times10^{-4}}{2}\right)$

$\Omega_p = 8000\,rad/s$

iii) $\Omega_s = \frac{2}{T}\tan\left(\frac{2\pi \times 350 \times 2.5\times10^{-4}}{2}\right)$

$\Omega_s = 2256\,rad/s$

iv) $N = \log\sqrt{\frac{10^{(0.1\alpha_p)}-1}{10^{(0.1\alpha_s)}-1}}$

$\log\left(\frac{\Omega_s}{\Omega_p}\right)$

$N = \frac{\pm 0.47}{\pm 0.54} = 0.87$

$N \approx 1$

v) $H(S) = \frac{1}{1+S} = \frac{1}{1+\frac{8000}{S}} = \frac{S}{S+8000}$

$H(S) = \frac{S}{S+8000}$

vi) $H(z) = H(s)\Big|_{s=\frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$

$$H(z) = \frac{\frac{2}{2.5\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{\frac{2}{2.5\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+8000}$$

$$H(z) = \frac{8000(1-z^{-1})}{8000(1-z^{-1})+8000(1+z^{-1})}$$

$$H(z) = \frac{8000(1-z^{-1})}{16000}$$

$$H(z) = \frac{y(n)}{x(n)} = 0.5 - 0.5z^{-1}$$

$$\boxed{y(n) = 0.5x(n) - 0.5x(n-1)}$$

III Problem-3

For $F_s = 2500Hz$

i) $T = \frac{1}{F_s} = \frac{1}{2500} = 4 \times 10^{-4}$

ii) $\Omega_p = \frac{2}{T} \tan\left(\frac{2\pi \times 1000 \times T}{2}\right)$

$\Omega_p = \frac{2}{4 \times 10^{-4}} \tan\left(\frac{2\pi \times 1000 \times 4 \times 10^{-4}}{2}\right)$

$\Omega_p = 15388.41 \text{ rad/s}$

iii) $\Omega_s = \frac{2}{T} \tan\left(\frac{2\pi \times 350 \times T}{2}\right)$

$\Omega_s = \frac{2}{4 \times 10^{-4}} \tan\left(\frac{2\pi \times 350 \times 4 \times 10^{-4}}{2}\right)$

$\Omega_s = 2352.82 \text{ rad/s}$

iv) $N = \frac{\log \sqrt{\frac{10^{0.1\alpha_p} - 1}{10^{0.1\alpha_s} - 1}}}{\log\left(\frac{\Omega_s}{\Omega_p}\right)}$

$N = \frac{-0.85}{-0.815}$ $\quad N \approx 1$

v) $H(s) = \frac{s}{s + 15388}$

vi) $H(z) = H(s)\Big|_{s = \frac{2}{T}\left(\frac{1 - z^{-1}}{1 + z^{-1}}\right)}$

$$H(z) = \frac{\frac{2}{4\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}{\frac{2}{4\times10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)+15388}$$

$$H(z) = \frac{5000 - 5000z^{-1}}{5000(1-z^{-1}) + 15388(1+z^{-1})}$$

$$= \frac{5000 - 5000z^{-1}}{20388 + 10388z^{-1}}$$

$$H(z) = \frac{0.24 - 0.24z^{-1}}{1 + 0.509z^{-1}}$$

$$\frac{y(n)}{x(n)} = \frac{0.24 - 0.24z^{-1}}{1 + 0.509z^{-1}}$$

$$y(n) + 0.509\,y(n-1) = 0.24\,x(n) - 0.24\,x(n-1)$$

$$\boxed{y(n) = 0.24\,x(n) - 0.24\,x(n-1) - 0.509\,y(n-1)}$$

IV Problem 3

For $F_s = 7500$ Hz

i) $T = \frac{1}{f_s} = 1.333 \times 10^{-4} = \frac{4}{3} \times 10^{-4}$

ii) $\Omega_p = \frac{2}{T} \tan\left(\frac{2\pi \times 1000 \times T}{2}\right)$

$= \frac{2}{1.33 \times 10^{-4}} \tan\left(\frac{2\pi \times 1000 \times 1.33 \times 10^{-4}}{2}\right)$

$\Omega_p = 6678.43$ rad/s

iii) $\Omega_s = \frac{2}{T} \tan\left(\frac{2\pi \times 350 \times T}{2}\right)$

$\Omega_s = 2211.79$ rad/sec

iv) $N = \frac{-0.47}{-0.979}$ $\quad N \simeq 1$

v) $H(s) = \frac{S}{S + 6678}$

vi) $H(z) = H(s)\Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$

$H(z) = \frac{2 \times 3}{4 \times 10^{-4}} \frac{(1-z^{-1})}{(1+z^{-1})}$

$\frac{3}{2 \times 10^{-4}} \frac{(1-z^{-1})}{(1+z^{-1})} + 6678$

$H(z) = \frac{1.5 \times 10^4 (1-z^{-1})}{1.5 \times 10^4 (1-z^{-1}) + 6678 (1+z^{-1})}$

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$$H(z) = \frac{15000 - 15000z^{-1}}{2159.61 - 8340.39z^{-1}}$$

$$\frac{y(n)}{x(n)} = \frac{0.69 - 0.69z^{-1}}{1 - 0.38z^{-1}}$$

$$y(n) = 0.69x(n) - 0.69x(n-1) + 0.38y(n-1)$$

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Problem 3

For $F_S = 10000$ Hz

i) $T = \frac{1}{F_S} = 1 \times 10^{-4}$

ii) $\Omega_p = \frac{2}{T} \tan\left(\frac{2\pi \times 1000 \times T}{2}\right)$

$\Omega_p = 6498.39$ rad/s

iii) $\Omega_s = \frac{2}{T} \tan\left(\frac{2\pi \times 350 \times T}{2}\right)$

$\Omega_s = 2208.02$ rad/s

iv) $N = \frac{-0.47}{-0.46} = 1.03 \approx 1$

$N = 1$

v) $H(s) = \frac{6}{s + 6498}$

vi) $H(z) = H(s)\Big|_{s = \frac{2}{T}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)}$

$H(z) = \frac{2}{10^{-4}\left(\frac{1-z^{-1}}{1+z^{-1}}\right)} \Big/ \frac{20}{10^{-4}}\left(\frac{1-z^{-1}}{1+z^{-1}}\right) + 6498$

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$$H(z) = \frac{20000\,(1-z^{-1})}{20000(1-z^{-1}) + 6498(1+z^{-1})}$$

$$H(z) = \frac{20000\,(1-z^{-1})}{26498 - 13502z^{-1}}$$

$$H(z) = \frac{0.754 - 0.754z^{-1}}{1 - 0.509z^{-1}}$$

$$\frac{y(n)}{x(n)} = \frac{0.754 - 0.754z^{-1}}{1 - 0.509z^{-1}}$$

$$y(n) = 0.754x(n) - 0.754x(n-1) + 0.509y(n-1)$$