15. $\Delta S = \frac{}{T}$ 14. heat engine 20. hyp...

17. temperature 18. heat capacity 19. $J.\ kg^{-1}\ K^{-1}$ 24. Boyle's law

21. temperature 22. adiabatic 23. mercury thermometer 28. the gas molecule

25. temperature 26. mica 27. positive

possess momentum. 29. specific heat capacity 30. –40

EXERCISE "B"

WORKED PROBLEMS: APPLICATION

W.P. 11.1

i) The normal body temperature is 98.60F. What is the temperature on Celsius scale?

ii) At what temperature do the Fahrenheit and Celsius temperature readings coincide?

Solution

(i) $F = 98.6,\ C = ?$

$$\frac{C}{5} = \frac{F-32}{9}$$

$$\frac{C}{5} = \frac{98.6-32}{9} \Rightarrow 9C = (5 \times 98.6) - (5 \times 32)$$

$$C = \frac{493-160}{9} = \boxed{37^0C}$$

(ii) Let $C = F = x$

$$\frac{C}{5} = \frac{F-32}{9} \Rightarrow \frac{x}{5} = \frac{x-32}{9}$$

$$9x = 5x - (32 \times 5)$$

$$9x - 5x = -160$$

$$4x = -160$$

$$x = \frac{-160}{4} = \boxed{-40}$$

W.P. 11.2

A steel rod has a length of exactly 0.2 cm at 30°C. What will be its length at 60°C?

Solution

$L = 0.2\ cm = 0.02m,\ T_1 = 30^0C,\ T_2 = 60^0C,\ L' = ?$

For steel, $\alpha = 1.1 \times 10^{-5}\ C^{0-1}$

$L' = L(1 + \alpha\,\Delta T)$

$L' = 0.002\ [1 + 1.1 \times 10^{-5}\ (60 - 30)]$

$$L' = 0.002 + (0.002 \times 1.1 \times 10^{-5} \times 30)$$

$$L' = 2.0006 \times 10^{-3}\ m = 2.0006 \times 10^{-1}\ cm.$$

$$\boxed{L' = 0.20006\ cm}$$

W.P. 11.3

Find the change in volume of an aluminium sphere of 0.4 m radius when it is heated from 0° to 100°C.

Solution

$\Delta V = ?,\quad r = 0.4\ m,\quad T_1 = 0^0C,\quad T_2 = 100^0C$

$\beta = 7.2 \times 10^{-5}\ C^{0-1}$

$$\Delta V = \beta V \Delta T = \beta \times \frac{4}{3}\pi r^3 \times \Delta T$$

$$\Delta V = 7.2 \times 10\text{-}5 \times \frac{4}{3} \times 3.14 \times (0.4)^3 \times (100 - 0)$$

$$\Delta V = 3014 \times 0.064 \times 10^{-5} = 192.92 \times 10^{-5}$$

$$\boxed{\Delta V = 0.00192\ m^3}$$

W.P. 11.4

Calculate the root-mean-square speed of hydrogen molecule at 800 K. (Mass of proton = 1.67 x 10⁻²⁷ kg and k = 1.38 x 10⁻²³)

Solution

$T = 800\ k,\quad m = 2 \times 1.67 \times 10^{-27}\ kg,\quad V_{rms} = ?$

$$V_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 800}{2 \times 1.67 \times 10^{-27}}}$$

$$V_{rms} = \sqrt{\frac{3312}{3.34} \times 10^4} = \sqrt{9916167.6}$$

$$\boxed{V_{rms} = 3148.9\ m/s}$$

W.P. 11.5

(a) ***Determine the average value of the kinetic energy of the particles of an ideal gas at 0°C and at 50°C.***

(b) ***What is the kinetic energy per mole of an ideal gas at these temperature?***

Solution

(a) (i) $T = 0^0C = 273\ K,\quad \langle k.e. \rangle = ?$

$$\langle k.e. \rangle = \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 273 = 565.1 \times 10^{-23}$$

$$\boxed{\langle k.e. \rangle = 5.65 \times 10^{-23}\ J} \quad \text{.............................(1)}$$

(ii) $T = 50^0C = 273 + 50 = 323\ K$, $<k.e.> = ?$

$<k.e.> = \frac{3}{2}kT = \frac{3}{2} \times 1.38 \times 10^{-23} \times 323 = \boxed{6.69 \times 10^{-21}\ J}$ ....(2)

(b) K.e. per mole = E = ?

(i) $E = N_A \times <k.e.> = 6.02 \times 10^{23} \times 5.65 \times 10^{-21} = 34.013 \times 10^2$

$\boxed{E = 34013\ J/mole}$ ....(1)

(ii) $E = N_A \times <k.e.> = 6.02 \times 10^{23} \times 6.69 \times 10^{-21} = \boxed{4027.3\ J/mole}$ ....(2)

W.P. 11.6

A 2 kg iron block is taken from a furnace, where its temperature was 650°C and placed on a large block of ice at 0°C. Assuming that all the heat given up by the iron is used to melt the ice, how much ice is melted?

Solution

Mass of the iron block = m = 2 kg, Sp. heat of iron = 499.8 J $kg^{-1}K^{-1}$

Temp. of iron block = $T_1 = 650^0C$, Temp of the ice block = $T_2 = 0^0C$,

Sp. Latent heat of fusion of ice, $H_f = 3.36 \times 10^5\ Jkg^{-1}$,

Mass of ice melted = M = ?

Now, heat lost by the iron block = heat gained by the ice block :

$\therefore mc\,(T_2 - T_1) = MH_f$

$2 \times 499.8\ [650 - 0] = M \times 3.36 \times 10^5$

$M = \frac{649740}{3.36 \times 10^5} = \boxed{1.9\ Kg}$

W.P. 11.7

In a certain process, 400 J of heat is supplied to a system and at the same time 150 J of work is done by the system. What is the increase in internal energy of the system?

Solution

$Q = 400\ J$, $W = 150\ J$, $\Delta U = ?$

$\Delta U = Q - W$ $\Delta Q = \Delta U + \Delta W$

$\Delta U = 400 - 150$

$\boxed{\Delta U = 250\ J}$

W.P. 11.8

There is an increase of internal energy of 400 J, when 800 J of work is done by a system. What is the amount of heat supplied during this process?

Solution

$U = 400\ J$, $W = 800\ J$, $Q = ?$

$Q = \Delta U + W$

$$Q = 400 + 800$$
$$\boxed{Q = 1200\ J}$$

W.P. 11.9

A heat engine performs 200 J of work in each cycle and has efficiency of 20 %. For each cycle or operation (a) how mush heat is absorbed and (b) how much heat is expelled?

Solution

$$W = 200\ J, \qquad \eta = 20\ \% = \frac{20}{100} = 0.2, \qquad Q_1 = ? \qquad Q_2 = ?$$

$$\therefore \quad \eta = \frac{W}{Q_1}$$

Q_1 = absorb
Q_2 = released

$$0.2 = \frac{200}{Q_1}$$

$$Q_1 = \frac{200}{0.2} = \frac{2000}{2} = \boxed{1000\ J}$$

Now, $W = Q_1 - Q_2$

$$Q_2 = Q_1 - W = 1000 - 200$$

$$Q_2 = \boxed{800\ J}$$

W.P. 11.10

A heat engine operates between two reservoirs at temperatures of 25°C and 300°C. What is the maximum efficiency of this engine?

Solution

$$T_1 = 300^0C = 300 + 273 = 573K, \qquad T_2 = 25^0C = 25 + 273 = 298K, \qquad \eta = ?$$

$$\eta = 1 - \frac{T_2}{T_1}$$

$$\eta = 1 - \frac{298}{573} \qquad \eta = 1 - 0.52$$

$$\boxed{\eta = 0.48} \quad \text{or} \quad \eta = \frac{0.48}{100} = \boxed{48\ \%}$$

W.P. 11.11

The low temperature reservoir of a carnot engine is at 7°C and has an efficiency of 40 %. How much the temperature of the high – temperature reservoir be increased to increase the efficiency to 50 %?

Solution

$$\eta = 40\ \% = 0.40$$
$$T_2 = 7^0C = 280K$$

$T_1 = ?$

$\eta' = 50\ \% = 0.50$

$T'_2 = 7^0C = 280K$

$T'_1 = ?$

Required: $T_1' - T_1 = ?$

For the original condition:

$$\eta = 1 - \frac{T_2}{T_1}$$

$$0.40 = 1 - \frac{280}{T_1}$$

$$\frac{280}{T_1} = 0.60$$

$$T_1 = \frac{280}{0.60}$$

$T_1 = 466.66$ K..........................(1)

Now, for the new condition:

$$\eta' = 1 - \frac{T_2'}{T_1'}$$

$$0.50 = 1 - \frac{280}{T_1'}$$

$$\frac{280}{T_1'} = 1 - 0.50$$

$$\frac{280}{T_1} = 0.50$$

$$T_1' = \frac{280}{0.50} = 560K$$ ....(2)

Required temp. $= T'_1 - T_1 = 560 - 466.66 = \boxed{93.33\ \ K}$

W.P. 12.1 *Two unequal point charges repel each other with a force of 0.2 N when they are 10 cm apart. Find the force which each exerts on the other when they are 1cm apart.*

Solution

$$F = 0.2N, r = 10\text{ cm} = 0.1m$$

If $r' = 1cm = 0.01m,\ F' = ?$

$$F = K\frac{q_1 q_2}{r^2} \Rightarrow \frac{Fr^2}{K} = q_1 q_2 \quad \text{———} (1)$$

$$\text{Now,}\quad F' = K\frac{q_1 q_2}{r'^2} \quad \text{———} (2)$$

Putting eq. (1) in (2), we get

$$F' = \frac{K}{r'^2} \times \frac{Fr^2}{K} = \frac{Fr^2}{r'^2} = \frac{0.2 \times (0.1)^2}{(0.01)^2} = \frac{0.2 \times 0.01}{0.0001}$$

$$\boxed{F' = 20N}$$

W.P. 12.2 *Two point charges of $+1 \times 10^{-4}$C and -1×10^{-4} C are placed at a distance of 40cm from each other. A charge of $+6 \times 10^{-5}$ C is placed midway between them. What is the magnitude and direction of the force on it?*

Solution

$q_1 = +1 \times 10^{-4}C,\quad q_2 = -1 \times 10^{-4} C,\quad q_3 = +6 \times 10^{-5} C$

$AB = 40cm = 0.4m,\ F_3 = ?$

$AC = BC = 0.2m$

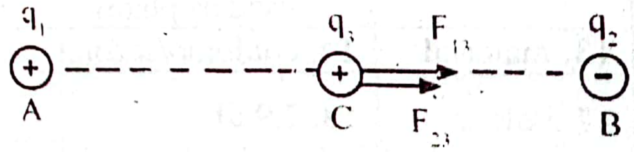


Fig. 12 - 28

Now, Force on q_3 due to q_1 is

$$F_{13} = K\frac{q_1 q_3}{(AC)^2} = \frac{9 \times 10^9 \times 1 \times 10^{-4} \times 6 \times 10^{-5}}{(0.2)^2}$$

$$F_{13} = 1350\ N$$

$$\text{Also,}\quad F_{23} = K\frac{q_2 q_3}{(BC)^2} = \frac{9 \times 10^9 \times 1 \times 10^{-4} \times 6 \times 10^{-5}}{(0.2)^2}$$

$$F_{23} = 1350\ N$$

Resultant force on charge q_3 is $F_3 = F_{13} + F_{23}$

$$F_3 = 1350 + 1350 = 2700$$

$$\boxed{F_3 = 2.7 \times 10^3\ N}$$

W.P. 12.3 ***Three point charges, each of 4μC, are placed at the three corners of a square of side 20cm. Find the magnitude of the force on each.***

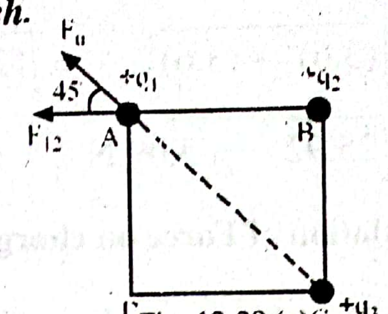


Fig. 12-29 (a)

Solution

$q_1 = q_2 = q_1 = 4 \times 10^{-6}C$

AB = BC = CD = DA = 20cm = 0.2m

$F_1 = ?,\ F_2 = ?,\ F_3 = ?$

Calculation of force on charge q1 (F_1)

Force on q_1 due to q_2 is $F_{12} = K \dfrac{q_1 q_2}{(AB)^2}$

$$F_{12} = 9 \times 10^9 \times \frac{4 \times 10^{-6} \times 4 \times 10^{-6}}{(0.2)^2} = 3.6\ N$$

To find AC, we have $AC^2 = AB^2 + BC^2$ [using Pythagorean theorem for Δ ABC]

$$AC^2 = (0.2)^2 + (0.2)^2 = 0.08m^2$$

Now, $$F_{13} = K\frac{q_1 q_3}{(AC)^2} = 9 \times 10^9 \times \frac{4 \times 10^{-4} \times 4 \times 10^{-6}}{0.08} = 1.8\ N$$

To calculate magnitude of the resultant (F_1) of F_{12} and F_{13}, we have from the 'cosine law':

$$F_1 = \sqrt{F_{12}^2 + F_{13}^2 + 2F_{12}^2F_{13}^2\cos\theta}$$

$$F_1 = \sqrt{(3.6)^2 + (1.8)^2 + 2x3.6x1.8x\cos45^0}$$

$$F_1 = \sqrt{12.96 + 3.24 + 12.96 \times 0.907} = \sqrt{25.36} = 5.036$$

$$\boxed{F_1 = 5.04\ N}$$

Calculation of force on charge q2 (F_2)

Force on q_2 due to q_1 is $F_{21} = K\dfrac{q_2 q_1}{(AB)^2}$

$$F_{21} = \frac{9 \times 10^9 \times 4 \times 10^{-6} \times 4 \times 10^{-6}}{(0.2)^2} = 3.6\ N$$

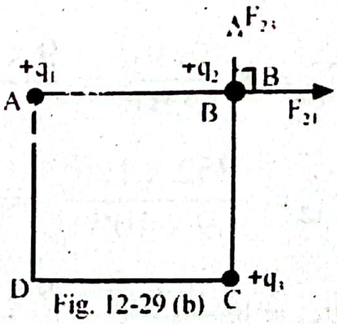


Fig. 12-29 (b)

Also, $$F_{23} = K\frac{q_2 q_3}{(BC)^2}$$

$$F_{23} = \frac{9 \times 10^9 \times 4 \times 10^{-6} \times 4 \times 10^{-6}}{(0.2)^2} = 3.6\ N$$

To calculate magnitude of the resultant (F_2) of F_{21} and F_{23} we have

$$F_1 = \sqrt{F_{21}^2 + F_{23}^2} \qquad [\because \theta = 90^\circ \text{ and } 90 = 0]$$

$$F_1 = \sqrt{(3.6)^2 + (3.6)^2} \qquad = \sqrt{12.96 + 12.96}$$

$$F_1 = \sqrt{25.92} \qquad = 5.09 \text{ N}$$

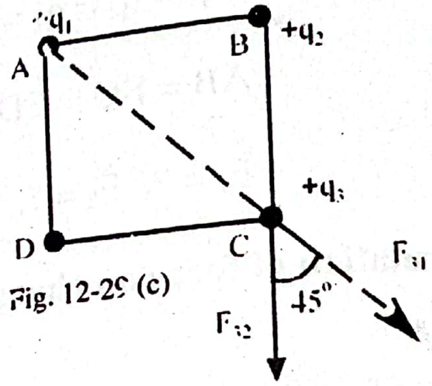


Fig. 12-29 (c)

Calculation of Force on charge q_3 (F_3)

Force on q_3 due to q_1 is $F_{31} = K \dfrac{q_3 q_1}{(AB)^2}$

$$F_{31} = \frac{9 \times 10^9 \times 4 \times 10^{-6} \times 4 \times 10^{-6}}{0.08} = 1.8 \text{ N}$$

Also, $F_{32} = K \dfrac{q_3 q_2}{(BC)^2} = \dfrac{9 \times 10^9 \times 4 \times 10^{-4} \times 4 \times 10^{-6}}{(1.2)^2} = 3.6 \text{ N}$

Now, $F_3 = \sqrt{F_{31}^2 + F_{32}^2 + F_{31}^2 F_{32}^2 \cos 45^\circ}$

$$F_3 = \sqrt{(1.8)^2 + (93.6)^2 + 2 \times 1.8 \times 3.6 \times 0.707} = \sqrt{25.36} \quad \boxed{= 5.04 \text{ N}}$$

W.P. 12-4 *Three charges $q_1 = + 7 \times 10^{6}$ C, $q_2 = + - 4 \times 10^{6}$ C and $q_3 + - 5 \times 10^{6}$C are placed at the vertices of a triangle as shown in the diagram. The sides of the triangle measure 3,4 and 5 cm. Determine the magnitude and direction of the force on the charge q_1.*

Solution

$q_1 = + 7 \times 10^{-6}$C, $q_2 = -4 \times 10^{-6}$C, $q_3 = 5 \times 10^{-6}$C

AB = 3cm = 0.03m, BC = 5cm = 0.05m, AC = 4cm = 0.04m

Force on q_1 is $F_1 = ?$

Now, Force on q_1 due to q_2 is

$$F_{12} = K \frac{q_1 q_2}{(AB)^2} = \frac{9 \times 10^9 \times 7 \times 10^{-6} \times 4 \times 10^{-6}}{(0.03)^2}$$

$$F_{12} = \frac{252 \times 10^{-3}}{9 \times 10^{-4}} = 28 \times 10 = 280 \text{ N}$$

$$F_{13} = K \frac{q_1 q_3}{(AC)^2} = \frac{9 \times 10^9 \times 7 \times 10^{-6} \times 5 \times 10^{-6}}{(0.04)^2}$$

$$F_{13} = \frac{315 \times 10^{-3}}{1.6 \times 10^{-3}} = 196.87 \text{ N}$$

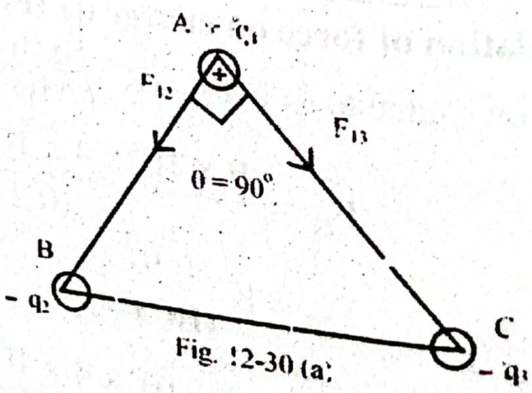


Fig. 12-30 (a)

The Pythagorean theorem is valid for the triangle Δ ABC, because $(BC)^2 = (5)^2 = 25cm^2$ and

$$BC^2 = BA^2 + AC^2 = (3)^2 + (4)^2 = 16 + 9 = 25cm^2$$

Hence the angle θ is 90°.

Now, the resultant force (F_1) is

$$F_1 = \sqrt{(F_{12}^2) + (F_{13}^2)} = \sqrt{(280)^2 + (196.87)^2} = \sqrt{78400 + 38757.8}$$

$$F_1 = \sqrt{117157.8} \quad \boxed{= 342.28\ N}$$

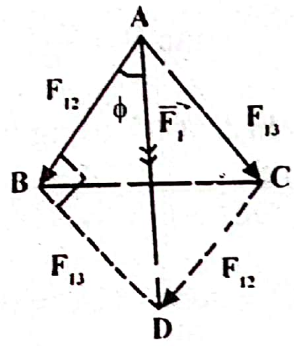


Fig 12-30 (b)

Using the law of sines for ΔABD,

$$\frac{\sin \varnothing}{F_{13}} = \frac{\sin < ABD}{F_1}$$

$$\frac{\sin \varnothing}{196.87} = \frac{\sin 90^\circ}{342.28} \Rightarrow \sin \varnothing = \frac{1}{342.28} \times 196.87$$

$$\sin \varnothing = 0.57517 \Rightarrow \varnothing = \sin^{-1}(0.57517) = \boxed{35.1^\circ}$$

Resultant force = 342.3 N, 35.1° with AB (force F_{12})

WP. 12.5 *Two small spheres, each having a mass of 0.1 gm, are suspended from the same point by silk threads each 20 cm long. The spheres are given equal charges and they are found to repel each other coming to rest 24 cm apart. Find the charge on each.*

Solution

Mass = $m = 0.1g = 0.0001$ kg, $q_1 = q_2 = q$ (say)

$$OA = OB = 20cm = 0.20m$$

$$AB - 24cm = 0.24m \implies AC = BC = \frac{24}{2} = 0.12m$$

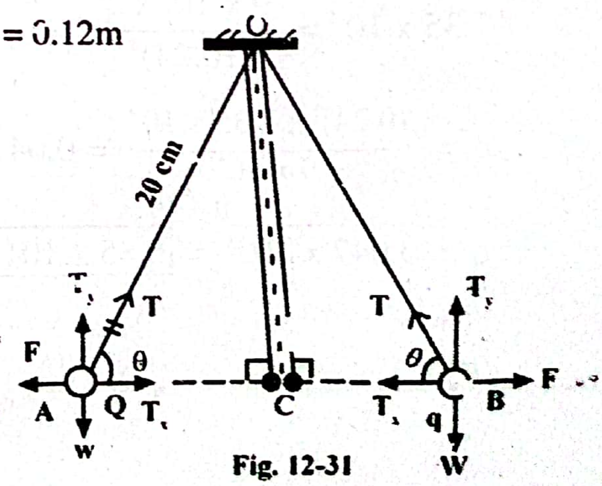


Fig. 12-31

Forces acting on spheres A and B are:

(i) weight (w = mg)

(ii) E.s. force of repulsion = F = ?

(iii) Tension (T)

Resolving Tension T, we have $T_x = T \cos \theta$

$T_y = T \sin \theta$

Applying the condition of equilibrium on charge A,

(i) $\Sigma F_x = 0 \implies T_x + (-F) = 0$

$$\therefore T\cos\theta - F = 0 \Rightarrow T\cos\theta = F \quad \text{——— (1)}$$

(ii) $\Sigma F_y = 0 \Rightarrow Ty + (-W) = 0$

$$T\sin\theta - mg = 0$$

$$T\sin\theta = mg \quad \text{——— (2)}$$

Dividing eq. (2) by eq. (1), we get

$$\frac{T\sin\theta}{T\cos\theta} = \frac{mg}{F}$$

$$\therefore \tan\theta = \frac{mg}{F} \Rightarrow F = \frac{mg}{\tan\theta} \quad \text{——— (3)}$$

In Δ OAC, using Pythagorean theorem

$$OC^2 + CA^2 = AO^2$$

$$OC^2 = AO^2 - CA^2 = (0.20)^2 - (0.12)^2 = 0.04 - 0.0144$$

$$OC = \sqrt{0.0256} = 0.16 \text{ m}$$

Thus, $\tan\theta = \frac{OC}{CA} = \frac{0.16}{0.12} = 1.333$ ——— (4)

Substituting eq. (4) in eq (3), we get

$$\therefore F = \frac{mg}{1.333} = \frac{0.0001 \times 9.8}{1.333}$$

$$F = 7.35 \times 10^{-4} \text{ N} \quad \text{——— (5)}$$

Using Coulomb's Law: $F = K\frac{q \times q}{(AB)^2}$

$$7.35 \times 10^{-4} = \frac{9 \times 10^9 \times q^2}{(0.24)^2}$$

$$q^2 = \frac{(0.24)^2 \times 7.35 \times 10^{-4}}{9 \times 10^9} = 0.047 \times 10^{-13}$$

$$q = \sqrt{0.047 \times 10^{-13}} = \boxed{6.85 \times 10^{-8}\text{ C}}$$

W.P. 12.6 ***Two charges of + 2 x 10^{-7} and – 5 x 10^{-7} are placed at a distance of 50 cm from each other. Find a point on the line joining the charges at which the electric field is zero.***

Solution

When P is taken in between A and B, the two intensities are in the same direction and cannot cancel. When it is taken to the right of B, the 'attractive' force due to q_2 (on the test +ve charge) is larger than the 'repulsive' force due to q_1. Thus resultant intensity is not zero. But in the region to the left of A, the intensities can cancel each other (because they are opposite).

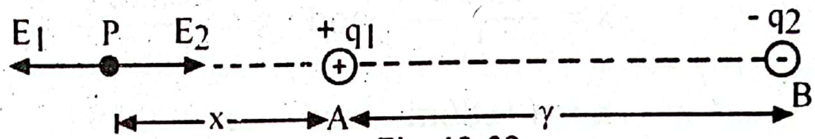


Fig. 12-32

Given: $q_1 = 2 \times 10^{-7}C$

$q_2 = -5 \times 10^{-7}C$

AB = r = 50 cm = 0.50m

Let P be a point where the electric field due to the charges q_1 and q_2 is zero.
Let AP = x.

Now, electric intensity at P due to q_1 = electric intensity at P due to q_2

$$E_1 = E_2$$

$$\frac{1}{4\pi\epsilon_o} \frac{q_1}{(AP)^2} = \frac{1}{4\pi\epsilon_o} \frac{q_2}{(BP)^2}$$

$$\frac{2 \times 10^{-7}}{x^2} = \frac{5 \times 10^{-7}}{(0.50 + x)^2}$$

$$5x^2 = 2\left(\frac{1}{2} + y\right)^2$$

$$5x^2 = 2\left[\frac{1}{4} + 2\left(\frac{1}{2}\right) x + x^2\right]$$

$$5x^2 = \frac{1}{2} + 2x + 2x^2$$

$$5x^2 - 2x^2 - 2x - \frac{1}{2} = 0$$

$$3x^2 - 2x - \frac{1}{2} = 0$$

$$6x^2 - 4x - 1 = 0$$

The standard quadratic formula is : $ax^2 + bx + c = 0$.
Here $a = 6$, $b = -4$ and $c = -1$.

$$\therefore\ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \Rightarrow x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 6 \times (-1)}}{2 \times 6}$$

$$x = \frac{4 \pm \sqrt{40}}{12} \Rightarrow x = \frac{4 \pm 6.324}{12}$$

$$x = \frac{10.324}{12} \quad \text{and} \quad x = \frac{-2.324}{12}$$

$x = +0.86\text{m}$ and $x = -0.194$ [Discarded]

$\therefore \boxed{x = 0.86\text{m} = 86\text{cm}}$ [It is away from 2×10^{-7} charge]

W.P. 12.7 ***What are the electric field and potential at the centre of a square whose diagonals are 60 cm each, when (a) charges each of +2 μC are placed at the four centres and (b) charges of +2 μC are placed on the adjacent corners and –4 μC on other corners?***

Solution

Part (a) (i) Calculation of electric intensity

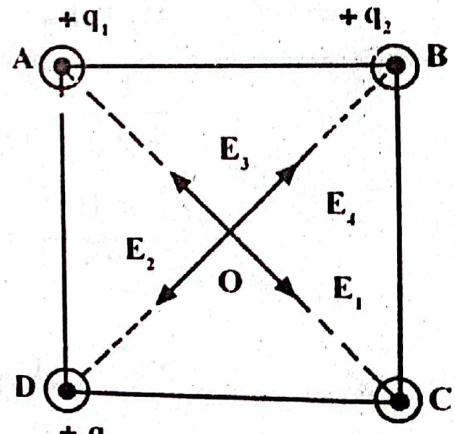


Fig. 12-33 (a)

$q_1 = q_2 = q_3 = q_4 = +2 \times 10^{-6}\text{C}$

$AC = BD = 60\text{cm}$

$$OA = OB = OC = OD = \frac{60}{2} = 30\text{cm} = 0.3\text{m}$$

Electric intensity at 'O' due to q_1 is $E_1 = K \dfrac{q_1}{(OA)^2}$

$$E_1 = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{(0.3)^2} = \frac{18}{0.09} \times 10^3$$

$$E_1 = 2 \times 10^5\ \text{N/C} \quad \text{———— (1)}$$

Similarly, $$E_2 = K\frac{q_2}{(OB)^2} = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{(0.3)^2} = 2 \times 10^5\ \text{NC} \quad \text{———— (2)}$$

$$E_3 = K\frac{q_3}{(OC)^2} = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{(0.3)^2} = 2 \times 10^5\ \text{NC} \quad \text{———— (3)}$$

$$E_4 = K\frac{q_4}{(OD)^2} = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{(0.3)^2} = 2 \times 10^5\ \text{NC} \quad \text{———— (4)}$$

Now, from the vector diagram, $E_1 + (-E_3) = 2 \times 10^5 - 2 \times 10^5 = 0$

$$E_2 + (-E_4) = 2 \times 10^5 - 2 \times 10^5 = 0$$

Hence, the net intensity at 'O' is zero $\Rightarrow$ $\boxed{E = 0}$

a (ii): Calculation of Potential

Potential at 'O' due to q_1 is $V_1 = K\dfrac{q_1}{OA} = \dfrac{9 \times 10^9 \times 2 \times 10^{-6}}{0.3} = 6 \times 10^4\ V$ — (1)

Similarly, $V_2 = K\dfrac{q_2}{OB} = \dfrac{9 \times 10^9 \times 2 \times 10^{-6}}{0.3} = 6 \times 10^4\ V$ — (2)

$$V_3 = K\frac{q_3}{OC} = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{0.3} = 6 \times 10^4\ V \text{ — (3)}$$

$$V_4 = K\frac{q_4}{OD} = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{0.3} = 6 \times 10^4\ V \text{ — (4)}$$

The total potential at 'O' is $V = V_1 + V_2 + V_3 + V_4$

$$V = (6 \times 10^4) + (6 \times 10^4) + (6 \times 10^4) + (6 + 10^4)$$

$$V = 10^4\,(6 + 6 + 6 + 6)$$

$$V = 24 \times 10^4 = \boxed{2.4 \times 10^5\ V}$$

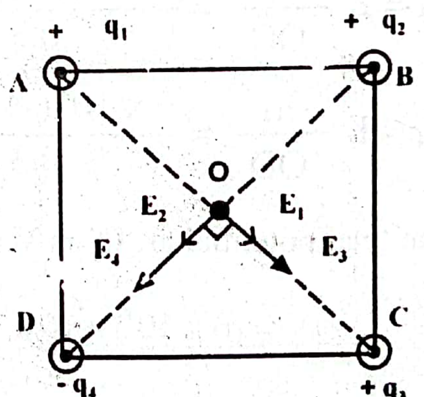


Fig. 12-33 (b)

Part (b) (I): Calculation of electric intensity

$q_1 = q_2 = +2 \times 10^{-6}\ C$ and $q_3 = q_4 = -4 \times 10^{-6} C$

Electric intensity at 0 due to q_1 is $E_1 = K\dfrac{q_1}{(OA)^2}$

$$E_1 = 9 \times 10^9 = \frac{2 \times 10^{-6}}{(0.3)^2} = 2 \times 10^5\ N/C \text{ ——— (1)}$$

$$E_2 = K\frac{q_2}{(OB)^2} = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{(0.3)^2} = 2 \times 10^5\ N/C \text{ ——— (2)}$$

$$E_3 = K\frac{q_3}{(OC)^2} = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{(0.3)^2} = 4 \times 10^5\ N/C \text{ ——— (3)}$$

$$E_4 = K\frac{q_4}{(OD)^2} = \frac{9 \times 10^9 \times 2 \times 10^{-6}}{(0.3)^2} = 4 \times 10^5\ N/C \text{ ——— (4)}$$

Now, from the vector diagram, $E' = E_1 + E_3 = 2 \times 10^5 + 4 \times 10^5 = 10^5 (2+4)$

$$E' = 6 \times 10^5 \text{ N/C} \quad \text{——— (5)}$$

Similarly,

$$E' = E_2 + E_4 = 2 \times 10^5 + 4 \times 10^5$$

$$E'' = 6 \times 10^5 \text{ N/C} \quad \text{——— (6)}$$

The included angle between E' and E'' is 90°. Hence, the resultant intensity is

$$E = \sqrt{E'^2 + E''^2}$$

$$E = \sqrt{(6 \times 10^5)^2 + (6 \times 10^5)^2} = \sqrt{36 \times 10^{10} + 36 \times 10^{10}}$$

$$E = \sqrt{10^{10}(36+36)} = 10^5 \times \sqrt{72} = 8.48 \times 10^5 \text{ N/C}$$

$$\boxed{E = 8.5 \times 10^5 \text{ N/C}}$$

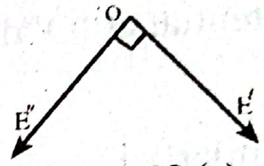


Fig. 12-33 (c)

D (ii): Calculation of potential

Potential at 'O' due to q_1 is $V_1 = K \dfrac{q_1}{OA} = \dfrac{9 \times 10^9 \times 2 \times 10^{-6}}{0.3} = 6 \times 10^4$ V ——— (

Similarly, $V_2 = K \dfrac{q_2}{OB} = \dfrac{9 \times 10^9 \times 2 \times 10^{-6}}{0.3} = 6 \times 10^4$ V ——— (2)

$$V_3 = K \frac{q_3}{OC} = \frac{9 \times 10^9 (-4) \times 10^{-6}}{0.3} = -12 \times 10^4 \text{ V} \quad \text{——— (3)}$$

$$V_4 = K \frac{q_4}{OD} = \frac{9 \times 10^9 (-4) \times 10^{-6}}{0.3} = -12 \times 10^4 \text{ V} \quad \text{——— (4)}$$

The total potential at 'O' is $V = V_1 + V_2 + V_3 + V_4$

$$V = (6 \times 10^4) + (6 \times 10^4) + (-12 \times 10^4) + (-12 \times 10^4)$$

$$V = 10^4 (6 + 6 - 12 - 12)$$

$$V = -12 \times 10^4 \text{ volt} \Longrightarrow \boxed{V = -1.2 \times 10^5 \text{ volt}}$$

W.P. 12.8 ***A particle carrying a charge of 10^{-5} C starts from rest in a uniform electric field of intensity 50 Vm^{-1}. Find the force on the particle and the kinetic energy it acquires when it has moved 1m.***

Solution

$q = 10^{-5}$c, $E = 50\ Vm^{-1}$, $v_i = 0$, k.e. = ?, s = 1m

Force on the particle, $F = qE = 10^{-5} \times 50 = 50 \times 10^{-5}$ N = $\boxed{5 \times 10^{-4} \text{ N}}$

Gain in k.e. = work done on the particle

k.e. = Fs

$k.e. = 5.0 \times 10^{-4} \times 1 = \boxed{5 \times 10^{-4} J}$

W.P. 12.9 ***A proton of mass 1.67×10^{-27}kg and charge 1.6×10^{-19}C is to be held motionless between two horizontal parallel plates 10cm apart. Find the voltage required to be applied between the plates.***

Solution

$M = 1.67 \times 10^{-27}kg, q = 1.6 \times 10^{-19}C, \quad d = 10cm = \dfrac{10}{100} = 0.1m, \quad V = ?$

When the particle is in equilibrium,

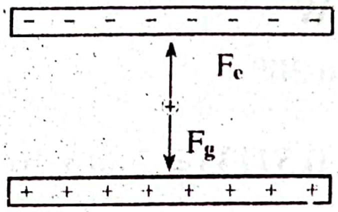


Fig. 12-34

gravitational force = electrostatic force

$Fg = Fe$

$Weight = qE$

$$mg = q\left(\frac{V}{d}\right) \;............[\, E = -\frac{\Delta V}{\Delta d} \text{ or } E = \frac{V}{d}\,]$$

$$\therefore V = \frac{mgd}{q}$$

$$V = \frac{1.67 \times 10^{-27} \times 9.8 \times 0.1}{1.6 \times 10^{-19}} = 1.0228 \times 10^{-27} \times 10^{19}$$

$$\boxed{V = 1.02 \times 10^{-8} \text{ volt}}$$

W.P. 12.10 ***A small sphere of weight 5×10^{-3} N is suspended by a silk thread 50mm long which is attached to a point on a large charged insulating plane. When a charge of 6×10^{-8}C is placed on the ball, the thread makes an angle of 30° with the vertical. What is the charge density of the plane?***

Solution

$W = 5 \times 10^{-3}N, AC = 50mm = \dfrac{50}{1000} = 0.05m$

$q = +6 \times 10^{-8} C, \theta = 30°, \quad$ charge density, $\sigma = ?$

The positively charged sphere is in equilibrium at C due to the positively charged sheet. Applying the condition of equilibrium on charge at C,

(i) $\Sigma Fx = 0 \Longrightarrow T \sin\theta + (-F) = 0$

$$\therefore T \sin\theta = F \quad \text{———— (1)}$$

(ii) $\Sigma Fy = 0 \Longrightarrow T\cos\theta + (-W) = 0$

$$T\cos\theta = W \quad \text{—————— (2)}$$

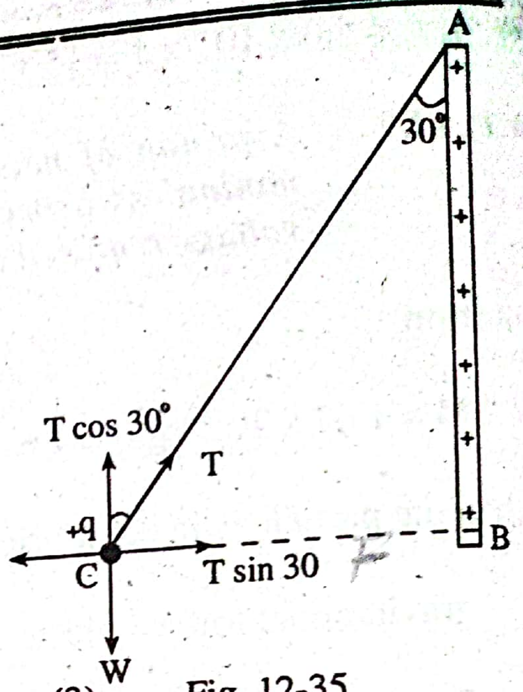


Fig. 12-35

Dividing eq (1) by (2), we get

$$\frac{T\sin\theta}{T\cos\theta} = \frac{F}{W}$$

$$\tan\theta = \frac{F}{W}$$

$$\therefore F = W \times \tan 30°$$

$$F = 5 \times 10^{-3} \times 0.57735 = 2.88 \times 10^{-3}\ N \quad \text{—————— (3)}$$

Now, electric intensity at a point C due to the charged sheet is

$$E = \frac{\sigma}{2\epsilon_o}$$

$$\therefore \sigma = 2\epsilon_o E$$

$$\sigma = \frac{F}{q} 2\epsilon_o$$

$$\sigma = 2 \times 8.85 \times 10^{-12} \times \frac{2.88 \times 10^{-3}}{6 \times 10^{-8}} = 8.496 \times 10^{-12+8-3}$$

$$\boxed{\sigma = 8.5 \times 10^{-7}\ C/m^2}$$

W.P. 12.11 ***How many electrons should be removed from each of the two similar spheres each of 10g so that the electrostatic repulsion is balanced by the gravitational force.***

Solution

Number of electrons n = ?, Mass of each sphere, $m_1 = m_2 = m = 10g\ {}^{10}/_{1000} = 0.01kg$,
Let charge on each sphere $= q_1 = q_2 = q$.

According to the condition: e.s. force = grav. force

$$F_e = F_g$$

$$K.\frac{q \times q}{r^2} = G\frac{m \times m}{r^2}$$

$$q^2 = \frac{G}{K} \times m^2$$

$$\therefore q^2 = \frac{6.67 \times 10^{-11}}{9 \times 10^9} \times (0.01)^2 = 7.41 \times 10^{-5} \times 10^{-11} \times 10^{-9} = 7.41 \times 10^{-25}$$

$$q = \sqrt{74.1 \times 10^{-26}} = 8.608 \times 10^{-13} C$$

$$\text{Number of electrons, } n = \frac{q}{e} = \frac{8.608 \times 10^{-13}\ C}{1.6 \times 10^{-19}\ C}$$

$$\boxed{\therefore n = 5.38 \times 10^6}$$

W.P. 12.12 ***There is a potential difference of 150v between two conductors of a power line. A charge of 600 C is carried from one conductor tc the other. What work is required? if the time necessary to transport the charge is 1.25 second, how much power is used?***

Solution

$\Delta V = 150V$, $q = 600$ C, $\Delta W = ?$ $\Delta t = 1.25$ s, $P = ?$

$$\Delta W = q\,\Delta V = 600 \times 150 = 90000 = \boxed{9.0 \times 10^4 J}$$

$$P = \frac{\Delta W}{\Delta t} = \frac{9 \times 10^4}{1.25} = \boxed{7.2 \times 10^4\ W}$$

W.P. 12.13 ***A metal sphere of 100 mm radius has a charge of 4.25 x 10^{-9}C. What is the potential (a) at its surface (b) at its centre. (c) What is the potential energy of charge 2.5 x 10^{-6}C at a point 150 mm from the centre of the sphere.***

Solution

$r = 100$ mm $= 0.1$m, $q = 4.25 \times 10^{-9}$C, $V = ?$

(c) P.e. = ? if $q' = 2.5 \times 10^{-6}$C, $r = 150$mm $= 0.15$m

(a) $V = K\frac{q}{r} = 9 \times 10^9 \times \frac{4.25 \times 10^{-9}}{0.1} = 382.5 = \boxed{3.825 \times 10^2\ V}$

(b) Potential at the centre of a sphere is the same as that its surface.

Hence potential at the centre = $\boxed{3.825 \times 10^2\ V}$

(c) $P.e. = \Delta W = \Delta V\ 'q' = K\frac{q}{r}q'$

$$P.e. = \frac{kqq'}{r} = \frac{9 \times 10^9 \times 4.25 \times 10^{-9} \times 2.5 \times 10^{-6}}{0.15} = 637.5 \times 10^{-6}\ J$$

$$\boxed{P.e. = 6.37 \times 10^{-4}\ J}$$

W.P. 12.14 ***An electron having an initial velocity of 10^3 m/s is directed from a distance of 1mm at another electron whose position is fixed. How close to the stationary electron will the other approach?***

Solution

$v_1 = 10^3$ m/s, $q_1 = q_2 = e = 1.6 \times 10^{-19}$C,

Distance between electrons, $r = 1mm = 10^{-3}$ m,

Closest distance of approach = x = ?

For electron, $m = 9.1 \times 10^{-31}$kg.

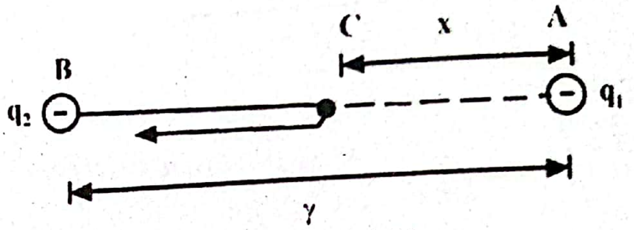


Fig. 12-36

Now, loss in k.e. = gain in p.e.

$$\frac{1}{2} mv^2 = (\text{p.e. at C}) - (\text{p.e. at B})$$

$$\frac{1}{2} mv^2 = \frac{K.e.e}{x} - \frac{K.e.e}{r}$$

$$\frac{1}{2} mv^2 = Ke^2\left(\frac{1}{x} - \frac{1}{x}\right)$$

$$\frac{1}{2} \times 9.1 \times 10^{-31} \times (10^3)^2 = 9 \times 10^9 \times (1.6 \times 10^{-19})^2 \left(\frac{1}{x} - \frac{1}{10^{-3}}\right)$$

$$4.55 \times 10^{-25} = 23.04 \times 10^{-38+9} = \left(\frac{1}{x} - 10^3\right)$$

$$\frac{4.55 \times 10^{-25}}{23.04 \times 10^{-29}} = \left(\frac{1}{x} - 10^3\right)$$

$$0.1975 \times 10^4 = \frac{1}{x} - 10^3$$

$$1975 + 10^3 = \frac{1}{x}$$

$$\frac{1}{x} = 1975 + 1000 \rightarrow \frac{1}{x} = 2975$$

$$x = \frac{1}{2975} = 3.36 \times 10^{-4} = 0.000336m$$

or $x = 0.000336 \times 1000 = \boxed{0.336mm}$

W.P. 12.15 ***Find the equivalent capacitance and charge on each of the capacitor shown in the diagram.***

$$C_1 = 2\mu F, \quad C_2 = 4\mu F \quad \text{and} \quad C_3 = 6\mu F$$

Solution

C_1 and C_2 are in parallel

$$\therefore C' = C_1 + C_2$$
$$C' = 2 + 4$$
$$C' = 6\mu F \text{ ———————— (1)}$$

Now, C' and C_3 are in series,

$$\therefore \frac{1}{Ce} = \frac{1}{C'} + \frac{1}{C_3}$$

$$\frac{1}{Ce} = \frac{1}{6} + \frac{1}{6} + \frac{1+1}{6} = \frac{2}{6} = \frac{1}{3}$$

$$\boxed{Ce = 3\mu F}$$

Total charge, $q = C_e V = 3 \times 10^{-6} \times 10 = \boxed{30 \times 10^{-6} C}$

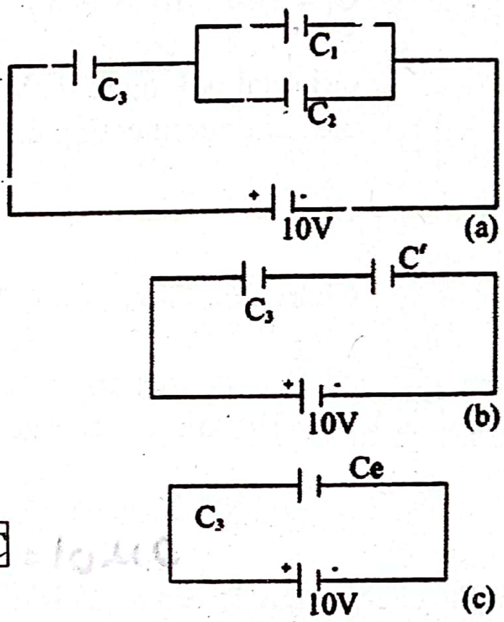


Fig. 12-37

The charge on C_3 is $q_3 = 30\mu C$ and on C' is $q' = 30\mu C$

P.d. across C' is $V' = \frac{q'}{C'} = \frac{30 \times 10^{-6}}{6 \times 10^{-6}} = 5V$

Charge on C_1 is $q_1 = C_1 V_1 = 2 \times 10^{-6} \times 5 = \boxed{10 \times 10^{-6} C}$

Thus, $q_1 = 10\mu C$, $q_2 = 20\ \mu C$ and $q_3 = \boxed{30\mu C}$

W.P. 12.16 ***Two capacitors of 2 μF and 8 μF are joined in series and a potential difference of 300 volts is applied. Find the charge and potential difference for each capacitor.***

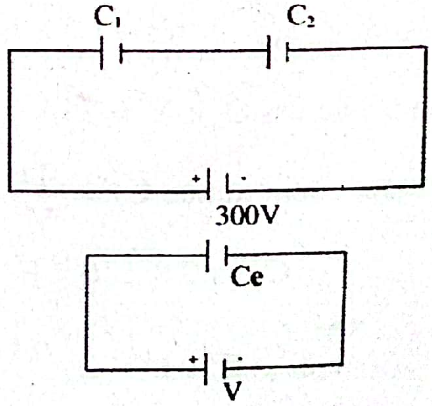


Fig. 12-38

Solution

$C_1 = 2 \times 10^{-6} F$, $C_2 = 8 \times 10^{-6} F$

C_1 and C_2 are in series

$$\therefore \frac{1}{Ce} = \frac{1}{2} + \frac{1}{8} = \frac{4+1}{8} = \frac{5}{8}$$

$$Ce = \frac{1}{2} = 1.6\mu F$$

Total charge, $q = C_e V = 1.6 \times 10^{-6} \times 300$

$$\boxed{q = 4.8 \times 10^{-4} C}$$

Now, C_1 and C_2 are in series, hence $q_1 = q_2 = q = 4.8 \times 10^{-4}$ C

P.d. across C_1 is $V_1 = \dfrac{q_1}{C_1} = \dfrac{4.8 \times 10^{-4}}{2 \times 10^{-6}} = 2.4 \times 10^{2} = \boxed{240 \text{ volts}}$

P.d. across C_2 is $V_2 = \dfrac{q_2}{C_2} = \dfrac{4.8 \times 10^{-4}}{8 \times 10^{-6}} = 0.6 \times 10^{2} = \boxed{60 \text{ volts}}$

W.P. 12.17 ***A Capacitor of 100 pF is charged to a potential difference of 50 V. If th plates are then connected in parallel to another capacitor and it is foun that the potential difference between its plates falls to 35 volts, what is th capacitance of the second capacitor?***

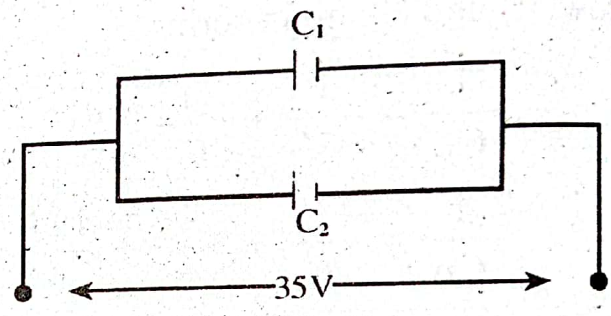


Fig. 12-39

Solution

$C_1 = 100 \times 10^{-12}$ F $= 1 \times 10^{-10}$F

Original p.d. on C_1 is V = 50V,
p.d. after connecting C_2 is V' = 35 V

C_2 = ?

Charge on C_1 is $q_1 = C_1V = 1 \times 10^{-10} \times 50 = 5.0 \times 10^{-9}$C

When charged C_1 and uncharged C_2 are connected in parallel (after removing the battery the p.d. is V' = 35V; and the charge on C1, now, is

$$q'_1 = C_1V' = 1 \times 10^{-10} \times 35 = 3.5 \times 10^{-9} \text{ C}$$

Let the charge transferred to C_2 is q'_2, then

$$q_1 = q'_1 + q'_2 \Longrightarrow q'_2 = q_1 - q'_1$$

$$q'_2 = (5.0 \times 10^{-9}) - (3.5 \times 10^{-9}) \; (5.0 - 3.5)$$

$$q'_2 = 1.5 \times 10^{-9} \text{ C}$$

The p.d across C_2 is $V' = 35$ V

Hence, capacitance $C_2 = \dfrac{q'_2}{V'} = \dfrac{1.5 \times 10^{-9}}{35} = 0.0428 \times 10^{-9}$ F

$C_2 = 42.8 \times 10^{-12}$ F

$\boxed{\therefore \; C_2 = 43 \text{ pF}}$

W.P. 12.18 ***Find the equivalent capacitance of the combination shown in the figure 12.40.***

$C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = C_7 = 3\mu F$

$C_8 = C_9 = 2\mu F$

Solution

C_3, C_4 and C_5 are in series, then $C_1, C_2, \& C_3$ are in Series then

$$\frac{1}{C'} = \frac{1}{C_3} + \frac{1}{C_4} + \frac{1}{C_5}$$

$$\frac{1}{C'} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{1+1+1}{3} = \frac{3}{3} = 1$$

$$C' = 1\mu F \text{ ——— (1)}$$

Now, C' and C_8 are in parallel, then

$$C'' = C' + C_8 = 1 + 2 = 3\mu F \text{ ——— (2)}$$

C_8, C'' and C_6 are in series, then

$$\frac{1}{C'''} = \frac{1}{C_2} + \frac{1}{C''} + \frac{1}{C_6} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

$$\frac{1}{C'''} = \frac{3}{3} = 1$$

$$C''' = 1\mu F \text{ ——— (3)}$$

C''' and C_9 are in parallel, then

$$C'''' = C''' + C_9 = 1 + 2 = 3\mu F \text{ ——— (4)}$$

Now, C'''', C_1 and C_7 are in series, then

$$\frac{1}{C_e} = \frac{1}{C''''} + \frac{1}{C_1} + \frac{1}{C_7} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$$

$$\frac{1}{C_e} = \frac{3}{3} = 1$$

$$\therefore \boxed{C_e = 1\mu F}$$

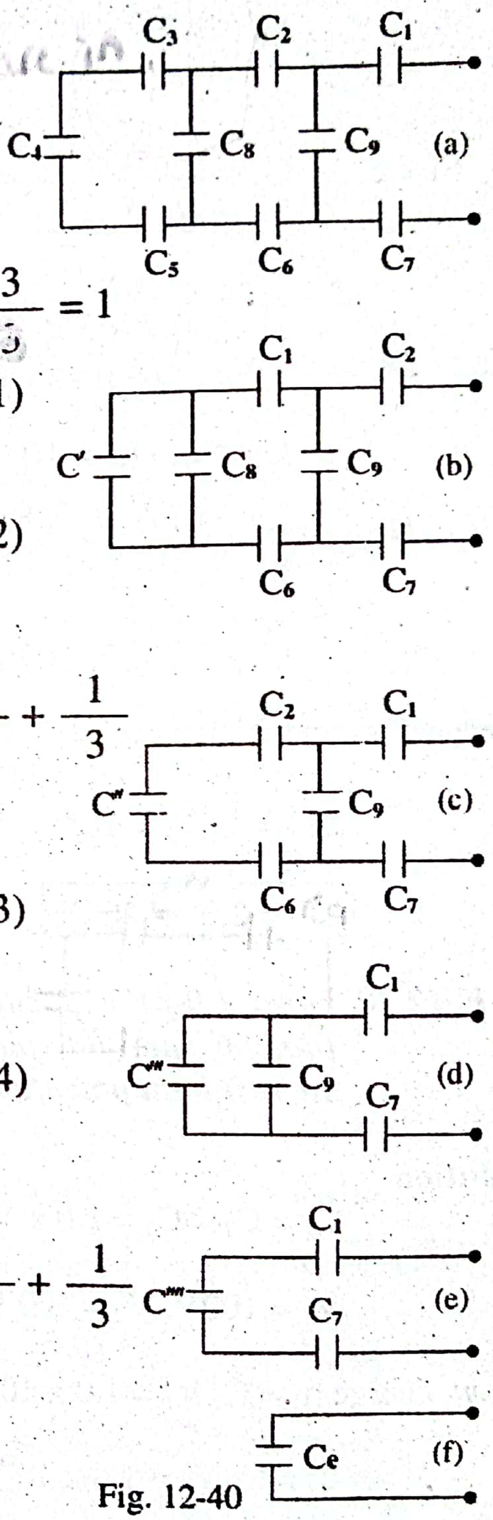


Fig. 12-40

W.P. 12.19 ***A parallel plate capacitor has plates 30cm x 30cm separated by a distance of 2 cm. By how much does the capacitance change if a dielectric slab of the same area but of thickness 1.5cm is slipped between the plates. The dielectric constant of the material is 2.***

Solution

$$A = 30cm \times 30cm = 900cm^2 = \frac{900}{100 \times 100}\ 0.09m^2$$

$$d = 2cm = \frac{2}{100} = 0.02m. \quad t = 1.5cm = \frac{1.5}{100} = 0.015m,$$

$$\epsilon_0 = 8.85 \times 10^{-12}\ C^2 N^{-1} m^{-2}$$

$$\epsilon_r = 2$$

Since,
$$C_d = \frac{\epsilon_0 A}{(d-t) + \frac{t}{\epsilon_r}} = \frac{8.85 \times 10^{-12} \times 0.09}{(0.02 - 0.015) + \left(\frac{0.015}{2}\right)}$$

$$C_d = \frac{0.7965 \times 10^{-12} \times 0.09}{(0.005 + 0.0075)} = \frac{0.7965}{(0.0125)} \times 10^{-12} = 63.72 \times 10^{-12}$$

$$C_d = 6.372 \times 10^{-11}F \quad \text{———} (1)$$

Now,
$$C = \frac{\epsilon . A}{d} = \frac{8.85 \times 10^{-12} \times 0.09}{0.02} = 39.825 \times 10^{-12}$$

$$C = 3.982 \times 10^{-11}F \quad \text{———} (2)$$

Increase is capacitance, $\Delta C = C_d - C$

$$\Delta C = 5.372 \times 10^{-11} - 3.982 \times 10^{-11} = 10^{-11}\ (6.372 - 3.982)$$

$$\boxed{\Delta C = 2.39 \times 10^{-11}F}$$

W.P. 12.20 ***Three 1.0-pF capacitors are charged separately to the potential difference of 100,200 and 300 volts. The Capacitors are then joined in parallel. What the resultant potential difference?***

Solution

$$C_1 = C_2 = C_3 = 1.0 \times 10^{-12} F$$

$V_1 = 100V$, $V_2 = 200$ V, $V_3 = 300$ V, then resultant p.d. = V = ?

Now, charge $q_1 = C_1 V_1 = 1.0 \times 10^{-12} \times 100 = 1 \times 10^{-10}$ C

$$q_2 = C_2 V_2 = 1.0 \times 10^{-12} \times 200 = 2 \times 10^{-10} C$$

$$q_3 = C_3 V_3 = 1.0 \times 10^{-10} \times 300 = 3 \times 10^{-10} C$$

The capacitors are in parallel, hence $q = q1 + q2 + q3$

$$q = 1 \times 10^{-10} + 2 \times 10^{-10} + 3 \times 10^{-10}\ (1 + 2 + 3)$$

$$q = 6 \times 10^{-10}\, C \text{ ———— } (1)$$

The equivalent, $C_e = C_1 + C_2 + C_3$

$$C_e = 1.0 \times 10^{-12} + 1.0 \times 10^{-12} + 1.0 \times 10^{-12} = 10^{-12}(1+1+1)$$

$$C_e = 3 \times 10^{-12}\, F \text{ ———— } (2)$$

The total p.d., $V = \dfrac{q}{C_e} = \dfrac{6 \times 10^{-10}}{3 \times 10^{-12}} = 2 \times 10^2$

$\therefore \boxed{V = 200 \text{ volts}}$

W.P. 12.21 ***Compare the capacitance of two identical capacitors with dielectrics inserted as shown in the diagrams. The dielectric constants are K_1 and K_2.***

Solution

Consider a capacitor of area of the plates 'A' and separation between the plates 'd' with two media of dielectric constants K_1 and K_2.

Case (a): Parallel Combination

Consider the capacitor as a combination of two capacitors C_1 and C_2 of half the area of the plates (A/2) for each, which are joined in parallel to form a single capacitor.

Now, $$C_1 = \frac{K_1 \epsilon_o A/2}{d} \text{ ———— } (1)$$

$$C_2 = \frac{K_2 \epsilon_o A/2}{d} \text{ ———— } (2)$$

Since C_1 and C_2 are in parallel, hence the equivalent capacitance,

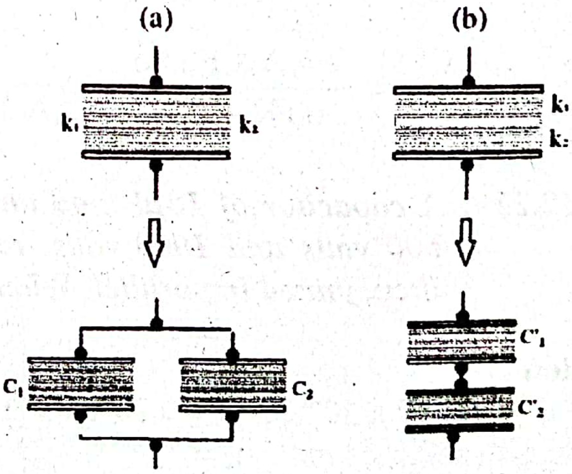
$$C_a = C_1 + C_2$$

$$C_a = \frac{K_1 \epsilon_o A/2}{d} + \frac{K_2 \epsilon_o A/2}{d}$$

$$C_a = \frac{K_1 \epsilon_o A}{2d} + \frac{K_2 \epsilon_o A}{2d}$$

$$C_a = \frac{K_1 \epsilon_o A + K_2 \epsilon_o A}{2d}$$

$$C_a = \frac{\epsilon_o A(K_1 + K_2)}{2d} \text{ ———— } (3)$$



(a) Parallel (b) Series

Fig. 12-41

Case (b): Series Combination

Consider the capacitor as a combination of two capacitors C'_1 and C'_2 of half the sepa-ration between the plates (d/2), which are joined in series to form a single capacitor.

Now, $C'_1 = \frac{K_1 \epsilon_0 A}{(d/2)} = \frac{2K_1 \epsilon_0 A}{d}$ ——————(4)

$$C'_2 = \frac{K_2 \epsilon_0 A}{(d/2)} = \frac{2K_2 \epsilon_0 A}{d} \text{ ——————(5)}$$

Since C'_1 and C'_2 are in series, hence the equivalent capacitance,(C_b) is given b.

$$\frac{1}{C_b} = \frac{1}{C'_1} + \frac{1}{C'_2} \Rightarrow Cb = \frac{C'_1 \times C'_2}{C'_1 + C'_2}$$

$$C_b = \frac{(2K_1\epsilon_0 A/d) \times (2K_2\epsilon_0 A/d)}{(\frac{2K_1\epsilon_0 A}{d}) + (\frac{2K_2\epsilon_0 A}{d})}$$

$$C_b = \frac{4K_1K_2\epsilon_0{}^2A^2/d^2}{(\frac{2K_1\epsilon_0 A + 2K_2\epsilon_0 A}{d})} = \frac{4K_1K_2\epsilon_0{}^2A^2}{d^2} \times \frac{d}{2\epsilon_0 A(K_1 + K_2)}$$

$$C_b = \frac{2K_1K_2\epsilon_0 A}{d} \times \frac{1}{(K_1 + K_2)}$$

$$C_b = \frac{2\epsilon_0 A\, K_1 K_2}{d\,(K_1 + K_2)} \text{ ——————(6)}$$

Comparison: Dividing equation (6) by (3), we get

$$\frac{C_b}{C_a} = [\frac{2\epsilon_0 A\, K_1 K_2}{d\,(K_1 + K_2)}] \div [\frac{\epsilon_0 A\,(K_1 + K_2)}{2d}]$$

$$\frac{C_b}{C_a} = \frac{2\epsilon_0 A \times K_1 K_2}{d\,(K_1 + K_2)} \times \frac{2d}{\epsilon_0 A\,(K_1 + K_2)} = \boxed{\frac{4\, K_1 K_2}{(K_1 + K_2)^2}}$$

W.P. 12.22 ***A capacitor of 10μF and one of 20 μF are connected across batteries of 600 volts and 1000 volts, respectively, and then disconnected. They are then, joined in parallel. What is the charge on each capacitor?***

Solution

$C_1 = 10 \times 10^{-6}$ F, $C_2 = 20 \times 10^{-6}$ F, $V_1 = 600$ V, $V_2 = 1000$ V

$q_1 = ?$ and $q_2 = ?$

The charge on the first capacitor is $q_1 = C_1 V_1 = 10 \times 10^{-6} \times 600$

$q_1 = 0.006$ C ——————(1)

The charge on the second capacitor is $q_2 = C_2 V_2 = 20 \times 10^{-6} \times 1000$

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$$q_2 = 0.02\ C \text{ ——— } (2)$$

In parallel combination, the total charge is

$$q = q_1 + q_2 = 0.006 + 0.02 = 0.026\ C \text{ ——— } (3)$$

The equivalent capacitance is $C_p = C_1 + C_2 = 10 + 20 = 30$

$$C_p = 30 \times 10^{-6}\ F \text{ ——— } (4)$$

The total p.d., $V = \dfrac{q}{C_p} = \dfrac{0.026}{30 \times 10^{-6}} = 866.6V$ ——— (5)

After parallel combination,
the charge on C_1 is $q'_1 = C_1 V = 10 \times 10^{-6} \times 866.6 = 8666 \times 10^{-6}$

$$\boxed{q'_1 = 8.67 \times 10^{-3}\ C}$$

The charge on C_2 is $q'_2 = C_2 V = 20 \times 10^{-6} \times 866.6 = 17.33 \times 10^{-3}$

$$\boxed{q'_2 = 17.33 \times 10^{-3}\ C}$$

W.P. 12.23 ***Attempt the problem 12.22 with the difference that the capacitors are joined in series after being charged, as before.***

Solution

The charge on the first capacitor, $q_1 = C_1 V_1 = 10 \times 10^{-6} \times 600 = 0.006C$

The charge on the second capacitor, $q_2 = C_2 V_2 = 20 \times 10^{-6} \times 1000 - 0.02C$

In series combination, $V = V_1 + V_2 = 600 + 1000 = 1600\ V$

$$\text{Now, } \frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{10} + \frac{1}{20}$$

$$C_s = \frac{20}{3}\mu F$$

The charge on each capacitor is the same. So

$$q_1 = q_2 = C_s V = \frac{20}{3} \times 10^{-6} \times 1600 = \boxed{1.067 \times 10^{-12} C}$$

W.P. 12.24 ***How many excess electrons must be placed on each of two small spheres spaced 3cm apart if the force of repulsion between the spheres is to be 10^{-19}N.***

Solution

Number of electrons, $n = \dfrac{q}{e}$?, $r = 3cm = \dfrac{q}{100} = 0.03m$, $F = 10^{-19}N$

EXERCISE – B

WORKED PROBLEMS: APPLICATIONS

W.P. 13.1 ***A certain battery is rated at 80 ampere hour. How many coulomb of charge can this battery supply?***

Solution

$I = 80A, \quad t = 1h = 60 \times 60 = 3600\ s, \quad q = ?$

$q = I \times t = 80 \times 3600 = \boxed{2.88 \times 10^5 C}$

$\frac{^{2}36_{4}}{144}$

W.P. 13.2

A silver wire 2-m long is to have a resistance of 0.5. Ω. What should its diameter be? ($\rho = 1.52 \times 10^{-8}\ \Omega.m.$)

Solution

$L = 2m, \quad R = 0.5\ \Omega, \quad d = ?$

$$R = \rho.\frac{L}{A} = \rho.\frac{L}{(\pi r^2)} = \rho.\frac{1}{\left(\frac{\pi d^2}{4}\right)} = \frac{4\rho L}{\pi d^2}$$

$$\therefore 0.5 = \frac{4 \times 1.52 \times 10^{-8} \times 2}{3.14 \times d^2} \Rightarrow d^2 = \frac{4 \times 1.52 \times 10^{-8} \times 2}{0.5 \times 3.14}$$

$$d^2 = \frac{12.16}{1.57} \times 10^{-8} = 7.745 \times 10^{-8} \qquad d = \sqrt{7.745 \times 10^{-8}}$$

$$\boxed{d = 2.78 \times 10^{-4}\ m}$$

W.P. 13.3 ***A current of 6A is drawn from a 120-V line. What power is being developed? How much energy in joule and KWh is expended if the current is drawn steadily for one week?***

Solution

$I = 6A;\ V = 120\ V;\ P = ?\ ;\ \text{energy},\ W = ?,\quad t = 1wk = (7 \times 24 \times 60 \times 60)\ s$

$P = VI = 120 \times 6 = \boxed{720\ W}$

$W = Pt = 720 \times (7 \times 24 \times 60 \times 60) = \boxed{4.35 \times 10^8\ J}$

$$W = \frac{4.35 \times 10^8 J}{(3.6 \times 10^6\ J/k\ Wh)} = 120.8\ kWh$$

W.P. 13.4 ***Currents of 3A (1.5 A) flow through two wires. One that has a potential difference of 60V across its ends and another that has a potential difference of 120 V across its end. Compare the rate at which energy passes through each wire.***

Solution

$I_1 = I_2 = 3A, \quad V_1 = 60V, \quad V_2 = 120\ V, \quad \frac{P_1}{P_2} = ?$

$I_2 = 1.5A$

For 1^{st} wire, $P_1 = V_1 I_1 = 60 \times 3 = 180$ W

For 2^{nd} wire, $P_2 = V_2 I_2 = 120 \times 3 = 360$ W $120 \times 1.5 = 180$ W

$$\frac{P_1}{P_2} = \frac{180}{360} = \frac{1}{2} = \boxed{1:2}$$ $$\frac{P_1}{P_2} = \frac{180}{180} = 1$$ Ans

W.P. 13.5 (a) ***A wire carries a current of 1A. How many electrons pass a point in the wire in each second?***

Solution

$I = 1A$, $t = 1$ s, Number of electrons, $N = ?$

$$\therefore N = \frac{q}{e} = \frac{1 \times t}{e} = \frac{1 \times 1}{1.6 \times 10^{-19}\ C} = \boxed{6.25 \times 10^{18} \text{ electrons}}$$

W.P. 13.5 (b) ***An electric drill rated at 400 W is connected to a 240 V power line. How much current does it draw?***

Solution

$P = 400w$, $V = 240$ V, $I = ?$

$VI = P$ $240 \times I = 400$ $$I = \frac{400}{240} = \boxed{1.666A}$$

W.P. 13.6 ***Resistance of 20, 40 & 50 Ω are connected in parallel across a 50-V power source. Find the equivalent resistance of the set and the current in each resistor.***

Solution

$R_1 = 20\Omega$, $R_2 = 40\Omega$, $R_3 = 50\Omega$, $V = 50V$, $R_e = ?$ $I_1 = ?, I_2 = ?, I_3 = ?$

For parallel combination, $$\frac{1}{R_e} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{20} + \frac{1}{40} + \frac{1}{50}$$

$$\frac{1}{R_e} = \frac{10+5+4}{200} = \frac{19}{200} \Rightarrow Re = \frac{200}{19} = \boxed{10.5\Omega}$$

In parallel combination, p.d. is the same across each resistor.

$$I_1 = \frac{V}{R_1} = \frac{50}{20} = 2.5\text{ A},$$

$$I_2 = \frac{V}{R_2} = \frac{50}{40} = 1.25\text{ A} \quad \text{and } I_3 = \frac{V}{R_3} = \frac{50}{50} = \boxed{1A.}$$

W.P. 13.7 ***(a) Find the equivalent resistance of the network shown in figure.***
(b) What is the current in 8-Ω resistor if the p.d. of 12V is applied to the network?

Solution

$R_1 = 5\Omega$, $R_2 = 8\Omega$. $R_3 = 6\Omega$,
$R_4 = 3\Omega$, $R_e = ?$ $V = 12$ volts
R_1 and R_2 are in parallel thus

$$\frac{1}{R'}=\frac{1}{R_1}+\frac{1}{R_2}=\frac{1}{5}+\frac{1}{8}=\frac{8+5}{40}=\frac{13}{40}\quad R'=\frac{40\Omega}{13}$$

R' and R_3 are in series, thus

$$R''=R'+R_3=\frac{40}{13}+6=\frac{40+78}{13}=\frac{118}{13}$$

R'' and R_4 are in parallel, thus

$$\frac{1}{R_e}=\frac{1}{R'}+\frac{1}{R_4}=\frac{13}{118}+\frac{1}{3}=\frac{39+118}{354}=\frac{157}{354}$$

$$R_e=\frac{354}{157}=\boxed{2.25\Omega}$$

Total current, $I=\frac{V}{R_e}=\frac{12}{2.25}=5.33$ A

p.d. across R'' is 12V

Current through R'' is

$$I_1=\frac{V}{R''}=\frac{12}{\left(\frac{118}{13}\right)}=\frac{12\times 13}{118}=1.32\text{A}$$

Current through R' is $I_1=1.32$ A

p.d. across R' is $V^1=I_1R^1=1.32\times\frac{40}{13}=4.07$v

p.d. across R_2 is $V^1=4.07$ V

Current through R_2 is $I_3=\frac{V'}{R_2}=\frac{4.07}{8}=0.508$ A

Required current = $\boxed{0.51\text{ A}}$

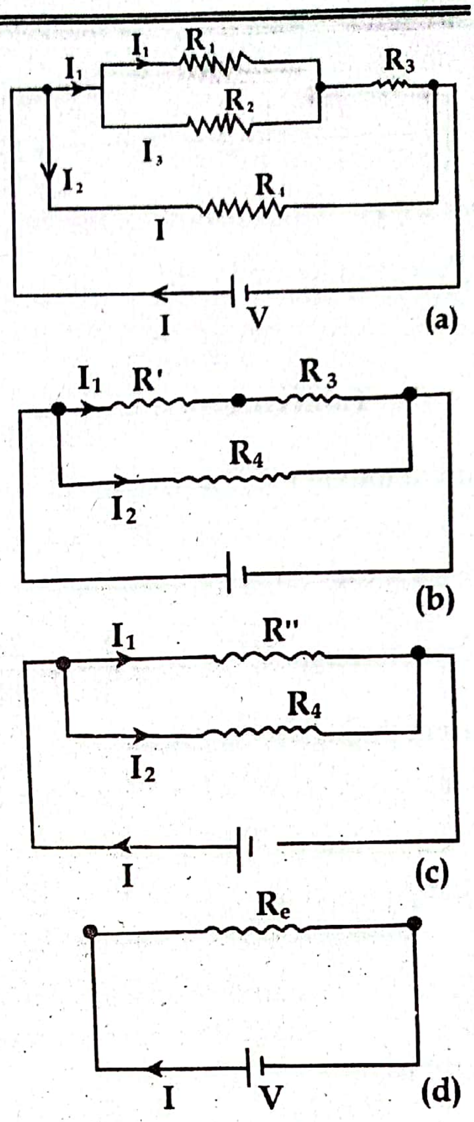
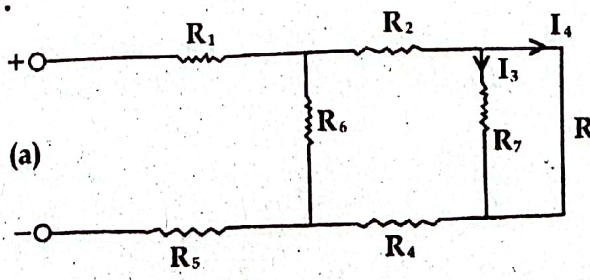


Fig 13-14

W.P. 13.8 ***A 60-V potential difference is applied to the circuit shown below. Find the current in the 10-Ω resistor.***



Solution

$R_1=5\Omega$, $R_2=4\Omega$,
$R_3=15\Omega$, $R_4=4\Omega$,
$R_5=3\Omega$, $R_6=14\Omega$,
$R_7=105$, $V=60$v.

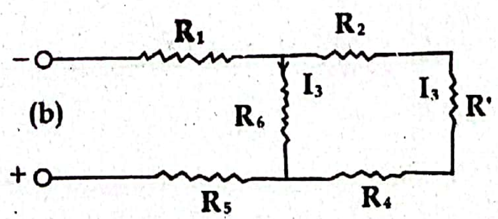
R_3 and R_7 are in parallel, thus

$$\frac{1}{R'}=\frac{1}{R_3}+\frac{1}{R_7}=\frac{1}{15}+\frac{1}{10}=\frac{2+3}{30}=\frac{5}{30}=\frac{1}{6}$$

$R^1=6\Omega$

$R_2\,R'$ and R_4 are in series, thus

$R''=R_2+R'+R_4=4+6+4=14\Omega$



R" and R_6 are in parallel, thus

$$\frac{1}{R'''} = \frac{1}{R''} + \frac{1}{R_6} = \frac{1}{14} + \frac{1}{14} = \frac{2+2}{14} = \frac{1}{7}$$

$R''' = 7\,\Omega$

R''', R_1 and R_5 are in series, hence

$R_e = R''' + R_1 + R_5 = 7 + 5 + 3 = \boxed{15\Omega}$

$$\textbf{Total current, } I = \frac{V}{Re} = \frac{60}{15} = 4A$$

Current through R''' = 4A

p.d. across R''' is V''' = IR''' = 4 x 7 = 28V

p.d. across R'' = 28V

$$\text{Current through R'' is } I_2 = \frac{V'''}{R''} = \frac{28}{14} = 2A$$

Current through R' is 2A.

p.d. across R' is V' = I_2 R' = 2 x 6 = 12V

p.d. across R_7 is 12V.

$$\text{Current through } R_7 \text{ is } I_3 = \frac{V^1}{R_7} = \frac{12}{10} = \boxed{1.2\ A}$$

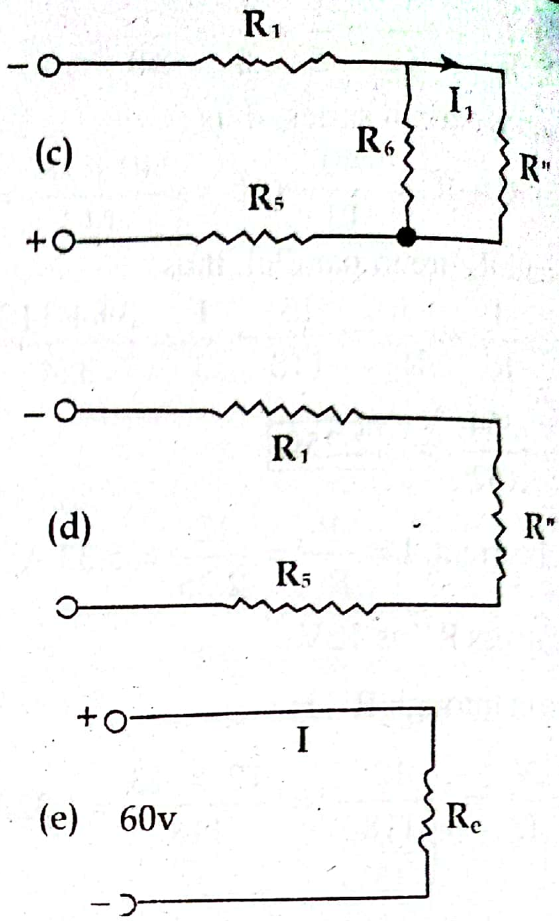


Fig. 13-15

W.P. 13.9 ***A source of what potential difference is needed to charge a battery of 24V e.m.f. and internal resistance of 0.1Ω at a rate of 70A?***

Solution

V = ? E = 24V, r = 0.1Ω, I = 70A

For charging a battery, required p.d., V = E + Ir = 24 + (70 x 0.1)

$V = 24 + 7 = \boxed{31\text{ volt}}$

W.P. 13.10 ***A battery of 20 V is connected to a 10Ω load and a current of 1.8 A flows. Find the internal resistance of the battery and its terminal voltage.***

Solution

E = 20 V, R = 10Ω, I = 1.8A, V = ?

$V = IR = 1.8 \times 10 = \boxed{18V}$

$\therefore V = E - Ir \Rightarrow V + Ir = E$

$$Ir = E - V \Rightarrow r = \frac{E - V}{1} = \frac{20 - 18}{1.8}$$

$$r = \frac{2}{1.8} = \boxed{1.1\Omega}$$

W.P. 13.11 ***A 40- Ω resistor is to be wound from platinum wire 0.1mm in diameter. How much wire is needed? (ρ = 11 x 10^{-8} Ω.m)***

Solution

$R = 40\Omega$, $d = 0.1\text{mm}$, $r = \frac{0.1}{2}$ $= 0.5\text{mm} = 0.5 \times 10^{-3}.\text{m}, L = ?$

$R = \frac{\rho L}{A}$ $R = \frac{\rho L}{\pi r^2}$ $40 = \frac{11 \times 10^{-8} \times L}{3.14 \times (0.05 \times 10^{3})}$

$L = \frac{40 \times 3.14 \times 2.5 \times 10^{-3} \times 10^{-6}}{11 \times 10^{-8}}$ $= 28.54 \times 10^{-1}$ $= \boxed{2.85\text{m}}$

W.P. 13.12 ***The battery of a pocket calculator supplies 0.35 A at a p.d. of 6V. What is the power rating of the calculator?***

Solution

$I = 0.35$ A, $V = 6V$, $P = ?$

$P = VI = 6 \times 0.35 = \boxed{2.1\text{ W}}$

W.P. 13.13 ***A current of 5A through a battery is maintained for 30 s and in this time 600J of chemical energy is transformed into electrical energy. (a)what is the e.m.f. of the battery? (b)How much electric power is available for joule heating and other uses?***

Solution

$I = 5A$; $t = 30$ s; energy, $W = 600$ J, e.m.f., $E = ?$; $P = ?$

Energy or work, $W = EIt$ $\Rightarrow$ $600 = E \times 5 \times 30$

$E = \frac{600}{5 \times 30} = \boxed{4V}$

$P = EI = 4 \times 5 = \boxed{20W}$

W.P. 13.14 ***A 12- Ω resistor is connected in series with a parallel combination of 10 resistors, each of 200 Ω. What is the net resistance of the circuit?***

Solution

$R_1 = 12\Omega$, R_2 to $R_{11} = 200\Omega$ each

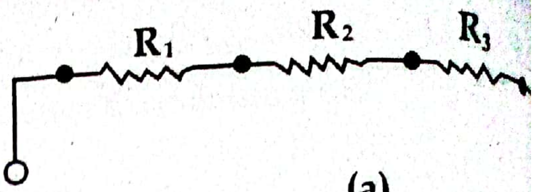
When 10 resistors, each of 200 Ω are in parallel, $R' = \frac{R}{n} = \frac{200}{10} = 20\Omega$

Net resistance, $R_e = R_1 + R' = 12 + 20 = \boxed{32\Omega}$

W.P. 13.15 ***Three equal resistors each of 12Ω can be connected in four different ways. What is the equivalent resistance of each combination?***

(i) First Arrangement

$$R_e = R_1 + R_2 + R_3 = 12 + 12 + 12 = \boxed{36\Omega}$$



(a)

(ii) Second Arrangement

$$\frac{1}{R_3} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{12} + \frac{1}{12} + \frac{1}{12}$$

$$\frac{1}{R_3} = \frac{1+1+1}{12} = \frac{3}{12} = \frac{1}{4}$$

$$\boxed{R_3 = 4\Omega}$$

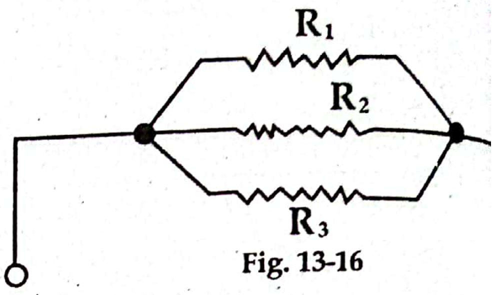


Fig. 13-16

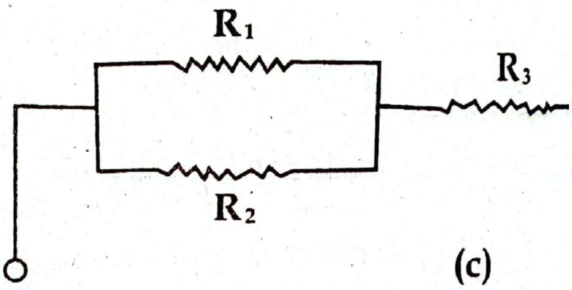
(iii) Third Arrangement

$R_1 = R_2$ are in parallel, then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{12} + \frac{1}{12} = \frac{2}{12} = \frac{1}{6}$$

$$R' = 6\Omega$$

$$R_e = R' + R_3 = 6 + 12 = \boxed{18\Omega}$$



(c)

(iv) Fourth Arrangement

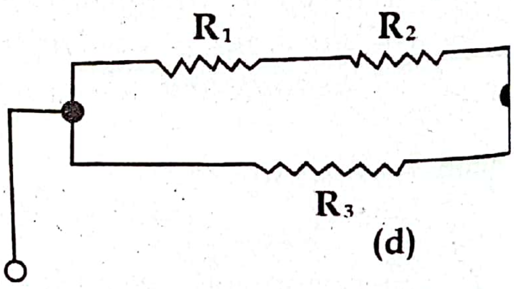
R_1 and R_2 are in series, then

$$R' = R_1 + R_2 = 12 + 12 = 24\Omega$$

$R_1 = R_3$ are in parallel, then

$$\frac{1}{R_e} = \frac{1}{R'} + \frac{1}{R_3} = \frac{1}{24} + \frac{1}{12} = \frac{1+2}{24} = \frac{3}{24}$$

$$R_3 = \frac{24}{3} = 8\Omega$$



(d)

W.P. 13.10 ***Find the resistance at*** $50°C$ ***of a copper wire 1mm in diameter and 3m long(*** $\rho = 1.60 \times 10^{-8}$ ***Ω.m,*** $\alpha = 0.0039°C^{-1}$ ***)***.

Solution

$$d = 2mm, r = \frac{2mm}{2} = 1mm = 1 \times 10^{-3}m, \quad L = 3m, \quad \Delta t = 50 - 0 = 50°C$$

$$R_o\ \rho = \frac{L}{A} = \rho\ \frac{L}{\pi r^2} = 1.60 \times 10^{-8} \times \frac{3}{3.14 \times (1 \times 10^{-3})^2} = 1.52 \times 10^{-2}\Omega$$

$$R_t = R_o(1 + \alpha\,\Delta t) = 1.52 \times 10^{-2}\,(1 + 0.0039 \times 50) = 1.52 \times 10^{-2} \times 1.195 = 1.82 \times 10$$

$$R_t = \boxed{0.0182\Omega}$$

W.P. 13.17 ***The resistance of a tungsten wire used in the filament of a 60 W bulb is 240Ω when the bulb is hot at a temperature of*** $2020\,^{\circ}C^{0-1}$***. What would you estimate its resistance at*** $20\,^{\circ}C$***? (Given*** $\alpha = 0.0046^{\circ}C^{0-1}$***)***

Solution

$R_t' = 240\Omega$ $\quad R_t = ?$ $\quad t' = 2020^{\circ}C,$ $\quad t = 20^{\circ}C.$

$$R_t' = R_t(1 + +\alpha\,\Delta t) \quad \Rightarrow \quad R_t = \frac{R_t'}{1 + \alpha\Delta t}$$

$$R_t' = \frac{240}{1 + 0.0046 \times (2020 - 20)} = \frac{240}{10.2} = \boxed{23.5\Omega}$$

W.P. 13.18 ***A water heater that will deliver 1kg of water per minute is required. The water is supplied at*** $20^{\circ}C$ ***and an output temperature of*** $80^{\circ}C$ ***is desired. What should be the resistance of the heating element in the water if the line voltage is 220V?***
(Given sp. heat capacity of water = 4200 J $kg^{-1}\,K^{-1}$***)***

Solution

$M = 1$ kg, $\quad t = 1$ min $= 1 \times 60$ sec., $\quad T_1 = 20^{\circ}C,$ $\quad T_2 = 80^{\circ}C$

$R = ?,$ $\quad V = 220V.$

Heat developed, $H = \frac{V^2}{R}\,t$ ——————— (1)

Heat $\quad H = mc\Delta T$ ——————— (2)

$$Mc\,(T_2 - T_1) = \frac{V^2}{R} \times t$$

$$1 \times 4200\,(80 - 20) = \frac{(220)^2}{R} \times 1 \times 60$$

$$R = \frac{48400 \times 60}{4200 \times 60} = \boxed{11.5\Omega}$$

W.P. 13.19 ***Prove that the rate of heat production in each of the two resistors connected in parallel are inversely proportional to the resistance.***

Solution

Heat produced in the first resistor, $H_1 = \frac{V^2}{R_1} \times t$ ——————— (1)

Heat produced in the second resistor, $H_2 = \frac{V^2}{R_2} \times t$ ——————— (2)

Dividing eq. (1) and (2), we get: $\frac{H_1}{H_2} = \frac{V^2t/R_1}{(V^2t/R_2)} = \frac{V^2t}{R_1} \times \frac{R_2}{V^2t}$

$\therefore \boxed{\frac{H_1}{H_2} = \frac{R_2}{R_1}}$

W.P. ~~13.20~~ 13.21 ***A 240-V cloth dryer draws a current of 15A. How much energy in kWh and joules does it use in 45 minutes operation and how much will be the cost at the rate of Rs.145 per unit of electric energy?***

Solution

$V = 240$ V; $I = 15$A; $t = 45$ min $= 45 \times 60 = 2700$ s; Energy, $W = ?$

$W = VIt = 240 \times 15 \times 2700 = 9.72 \times 10^6$ J

Also, $W = \frac{9.72 \times 10^6 \text{ J}}{3.6 \times 10^6 \text{ J/kWh}} = \boxed{2.7 \text{ kWh}}$ Cost $= 2.7 \times 1.45 =$ Rs.3.92

W.P. 13.21 ***A resistor is made by winding on a spool a 40m length of constantan wire of diameter 0.8mm. Calculate the resistance of the wire at (a) 0°C (b) 50°C***

[Assume ρ_o at 0°C = 49×10^{-8} Ω.m. and $\alpha = 1 \times 10^{-5}$ C^{-1}].

Solution

$L = 40$m, $d = 0.8$mm, $r = \frac{0.8}{2} = 0.04$mm $= 0.4 \times 10^{-3}$ m, $R_o = ?$, $R_t = ?$

$R_o = \rho_o \frac{L}{A} = \frac{\rho_o L}{\pi r^2} = \frac{49 \times 10^{-8} \times 40}{3.14 \times (0.4 \times 10^{-3})^2} = \frac{1960}{0.502} \times 10^{-2} = 39.01\Omega$

$R_t = R_o(1 + \alpha \Delta t) = (39.01)[1 + (1 \times 10^{-5})(50 - 0)] = 39.01 \times 1.005055$

$\boxed{Rt = 39.03\Omega}$

W.P. 13.22 ***Calculate the resistance of a tungsten wire 20m long that has a radius of 2mm. If the two ends of the wire are connected to a source of 24V, what is the value of current?***

Solution

$L = 20$m, $r = 2$mm, 2×10^{-3} m, $V = 24$ V, $\rho = 5.5 \times 10^{-8}$ Ω.m

$R = \rho \frac{L}{A} = \frac{\rho L}{\pi r^2} = \frac{5.5 \times 10^{-8} \times 20}{3.14 \times (2 \times 10^{-3})^2} = 8.75 \times 10^{-2}$

$$I = \frac{V}{R} = \frac{24}{8.75 \times 10^{-2}} = \boxed{274\ A}$$

W.P. 13.23 *To be safe, we do not want current in the above problem to exceed 100A, while keeping the length same but changing the cross-sectional area. Calculate R and A in this case.*

Solution

$$R = \frac{V}{I} = \frac{24}{100} = 0.24\Omega$$

$$R = \rho\frac{L}{A} \Rightarrow A = \frac{\rho L}{E} = \frac{5.5 \times 10^{-8} \times 20}{0.24} = \boxed{4.58 \times 10^{-6}\ m^2}$$

W.P. 13.24 *The resistance of a tungsten wire at room temperature (20°C) is 1.5Ω. What will be its resistance at 0°C and 100°C? ($\alpha = 0.00450°C^{-1}$)*

Solution

$R = 1.5\Omega$, $T_1 = 20°C$, $T_2 = 0°C$, $R_o = ?$, $R_t = ?$

$$R_o = \frac{R}{1 + \alpha\Delta t} = \frac{1.5}{1 + (0.0045)(20-0)} = \frac{1.5}{1.09} = 1.37\Omega$$

$$R' = R_o(1 + \alpha\,\Delta t) = 1.37\ [1 + 0.0045)(100-0)] = 1.37 \times 1.45$$

$$\boxed{R' = 1.97\Omega}$$

W.P. 13.25 *At what temperature will the value of resistance of the tungsten in the above question be doubled?*

Solution

$R = 1.5\Omega$, $t_1 = 20°C$, $t_2 = ?$, $R^1 = 2 \times 1.5 = 3.0\Omega$

$$R^1 = R\ (1 + \alpha\,\Delta T) \Rightarrow \frac{R^1}{R} = 1 + \alpha\,\Delta T \Rightarrow \alpha\,\Delta T = \frac{R^1}{R} - 1$$

$$\Delta T\ \frac{1}{\alpha}\ \left(\frac{R^1}{R} - 1\right) = \frac{1}{0.0045}\left(\frac{3.0}{1.5} - 1\right) = 222.2\ (2-1) = 222.2°C$$

Now $t_2 - t_1 = \Delta T \Rightarrow t_2 = t_1 + \Delta T$

$$t_2 = 20 + 222.2 = \boxed{242.2°C}$$

W.P. 13.26 *If there are 10^{18} electrons flowing across any cross-section of a wire in 1 min, what is the current in the wire?*

Solution

Answers

(1) both (a) and (c) (2) clockwise (3) 100 (4) magnetic field (5) east (6) inductance (7) NBAω (8) zero (9) resistance (10) a.c alone (11) Long's law (12) 1 werber/meter^2 (13) electric charges at rest (14) south pole (15) perpendicular to the field (16) weber (17) μ_0.1 (18) μ_0n1 (19) different parameter (20) tesla (21) NC^{-1} (22) mutual inductance (23) faraday (24) mechanical energy into electrical energy (25) electrical energy into mechanical energy (26) henry (27) both current and voltage (28) rate of change of flux linkage (29) fleming's right hand rule (30) a.c. alone

EXERCISE – B

WORKED PROBLEMS: APPLICATIONS

W.P. 14.1 *A horizontal strainght wire 5 cm long weighing 1.2* g.m^{-1} *is placed perpendicular to a uniform horizontal field of 0.6 Weber* –m^{-2}. *If the resistance of the wire is 3.8*Ω m^{-1} *, calculate the potential difference to be applied between the end of the wire to make it just self supporting.*

Solution

Data: L = 5 cm = 0.05 m; m = 1.2 g/m; Total; mass, M = mxL = 1.2 x 0.05 = 0.06g
M = 0.06 x 10^{-3} kg = 6.0 x 10^{-5} kg; B = 0.6 T; Resistance, r = 3.8 Ω /m
Total resistance, R = 3.8 x 0.05 = 0.19 Ω; p.d., V = ?

Formula:

$$BIL = Mg \Rightarrow B\left(\frac{V}{R}\right) L = Mg$$

Calculation:

$$V = \frac{MgR}{BL} = \frac{6.0 \times 10^{-5} \times 5.8 \times 0.19}{0.6 \times 0.05} = \frac{11.172}{0.03} \times 10^{-5}$$

$$V = 372.4 \times 10^{-5} = \boxed{3.72 \times 10^{-2} \text{ volts}}$$

W.P 14.2 *A cathode ray tube is set up horizontally with its axis N–S and it is surrounded by magnetic shield. If the voltage across the tube is 900 volts, the distance from electron gun to the screen is 10 cm and vertical component of the earth's field is 0.45 x 10^{-4} weber/m^2, calculate by how*

much the spot on the screen will move when the magnetic field is removed? *(Given* $e/m = 1.8 \times 10^{11}\ C \cdot kg^{-1}$*).*

Solution

Data: V = 900 volts; b = 10 cm = 0.1 m; $B = 0.45 \times 10^{-4}$ T; a = ?

Calculation:

(i) Velocity, $v = \sqrt{\dfrac{2Ve}{m}} = \sqrt{2V(\dfrac{2}{m})} = \sqrt{2 \times 900 \times 1.8 \times 10^{11}} = \sqrt{3.24 \times 10^{14}} = 1.8 \times 10^{7}$ m/s

(ii) $\dfrac{e}{m} = \dfrac{v}{Br} \Rightarrow r = \dfrac{vm}{Be} = \dfrac{v}{B} \times \dfrac{1}{(e/m)} = \dfrac{1.8 \times 10^{7}}{0.45 \times 10^{-4}} \times \dfrac{1}{1.8 \times 10^{11}}$

r = 2.22 m

(ii) $r = \dfrac{b^2}{2a} \Rightarrow a = \dfrac{b^2}{2r} = \dfrac{(0.1)^2}{2 \times 2.22} = \dfrac{0.01}{4.44} = \boxed{2.25 \times 10^{-3} m}$

W.P 14.3 ***What is the flux density at a distance of 0.1m in air from a long straight conductor carrying a current of 6.5 A. Hence calculate the force of repulsion per meter on a similar parallel conductor at a distance of 0.1m from the first and carrying a current of 3A. Explain how the expression of force between two such conductors is used to define ampere.***

Solution

Data: r = 0.1m ; I_1 = 6.5 A; I_2 = 3A; L = 1 m; F = ?

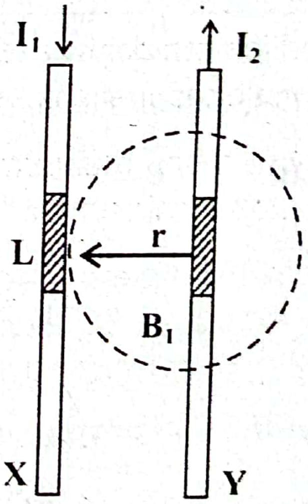


Fig. 14-48

Calculation:

(i) Consider two parallel conductors x and Y.

Field due to x is $B_1 = \dfrac{\mu_o I_1}{2\pi r} = \dfrac{4\pi \times 10^{-7} \times 6.5}{2\pi \times 0.1} = 13 \times 10^{-6}$ N

Force on y is $F_{21} = I_2 LB_1 \sin 90^o = 3 \times 1 \times 13 \times 10^{-6} = 39 \times 10^{-6}$ N

(ii) $\therefore F_{21} = I_2 LB_1\ I_2L\ (\dfrac{\mu_o I_1}{2\pi r})$

If $I_1 = I_2 = 1A$, L = 1m, r = 1m, then $F_{21} = \boxed{\dfrac{\mu_o I_1 I_2 L}{2\pi r}}$

$$F_{21} = \frac{4\pi \times 10^{-7} \times 1 \times 1 \times 1}{2\pi \times 1} = \boxed{2 \times 10^{-7}\ N}$$

Def. of Ampere:

Hence *1 ampere* is defined as the current which, if maintained in two straight parallel conductors of infinite length placed 1 meter apart in vacuum, would produce between them a force equal to 2×10^{-7} N per meter of length.

W.P 14.4 ***A straight metal rod 50 cm long can slide with negligible friction on parallel conducting rails. It moves at right angles to a magnetic field 0.72***

B

webers m^{-2}. The rails are joined to a battery of e.m.f. 3 volts and a fixed series resistance of 1.6Ω. Find the force required to hold the rod at rest.

Solution

Data: L = 50 cm = 0.50 m; θ = 90° ; B = 0.72 T ; V = 3 volts ; R = 1.6 Ω ; F = ?

Calculation:

$$F = BIL \sin\theta = B\left(\frac{V}{R}\right) L \sin\theta = \frac{0.72 \times 3 \times 0.50 \times \sin 90^\circ}{1.6} = \frac{1.08}{1.6}$$

$$\boxed{F = 0.675\ N}$$

W.P 14.5 ***It is required to produce inside a toroid a field of 2 x 10^{-3} webers m^{-2}. The toroid has a radius of 15 cm and 300 turns. Find the current required for this purpose. If the toroid is wound on an iron core of permeability 300 times the permeability of free space, what increase in B will occur for the same current?***

Solution

Data: B = 2 x 10^{-3} T; r = 15cm = 0.15m; N = 300; I = ?

If μ = 300 μo, B' = ?

Calculation: For air core:

(i) $$B = \frac{\mu_o NI}{2\pi r} \quad \text{———— (1)}$$

$$I = \frac{2\pi r B}{\mu_o N} = \frac{2\pi \times 0.15 \times 2 \times 10^{-3}}{4\pi \times 10^{-7} \times 300} = \frac{0.3 \times 10^{-3}}{0.06 \times 10^{-3}} = \boxed{5A}$$

(ii) For iron core: $$B' = \frac{\mu NI}{2\pi r} \quad \text{———— (2)}$$

$$B' = \frac{300\,\mu_o NI}{2\pi r} \quad \text{———— (3)}$$

Dividing eq (3) by (1), we get

$$\frac{B'}{B} = 300$$

$$\therefore \boxed{B' = 300 \times B}$$

W.P 14.6 ***A proton is accelerated by a potential difference of 6 x 10^5 volts. It then enters a uniform field B = 0.3 weber m^{-2} in a direction making an angle of 45° with the magnetic field. What will be the radius of the circular (spiral) path?***

(Karachi Board 1973)

Solution

Data: $V = 6 \times 10^5$ v ; $B = 0.3$ T ; $\theta = 45°$; $m = 1.67 \times 10^{-27}$ kg;
$e = 1.6 \times 10^{-19}$ C ; $r = ?$

Calculation:

(i) $$v = \sqrt{\frac{2Ve}{m}} = \sqrt{\frac{2 \times 6 \times 10^5 \times 1.6 \times 10^{-19}}{1.67 \times 10^{-27}}} = \sqrt{1.149 \times 10^{14}} = 1.072 \times 10^7 \text{ m/s}$$

(ii) Magnetic force = centripetal force

$$Bev \sin\theta = \frac{m (v \sin\theta)^2}{r}$$

$$r = \frac{m v \sin\theta}{Be} = \frac{1.67 \times 10^{-27} \times 1.072 \times 10^7 \times \sin 45°}{0.3 \times 1.6 \times 10^{-19}}$$

$$r = \frac{1.79 \times 0.707}{0.48} \times 10^{-1} = 2.62 \times 10^{-1}$$

$$\boxed{r = 0.262 \text{ m}}$$

W.P 14.7 ***The parallel metal plates separated by 5 cm of air have a potential difference of 220V. A magnetic field B = 5 x 10^{-3} webers m^{-2} is also produced perpendicular to electric field. A beam of electrons travel undeflected through these crossed electric and magnetic fields. Find the speed of electrons.***

Solution

Data: $d = 5$ cm $= 0.05$m ; $V_1 = 220$ volt ; $B = 5 \times 10^{-3}$ T ; $v = ?$

Formula: The electron beam is undeflected, if

Magnetic force = electrostatic force

$$Bev = eE$$

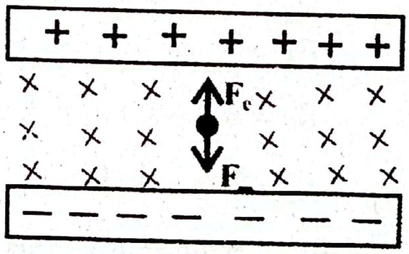


Fig. 14-49

Calculation

$$v = \frac{E}{B} = \frac{V_1}{d} \times \frac{1}{B} = \frac{220}{0.05 \times 5 \times 10^{-3}}$$

$$\boxed{v = 8.8 \times 10^5 \text{ m/s}}$$

W.P 14.8 ***A coil of 50 turns wound on a rectangular frame 2 cm x 4 cm is pivoted to rotate in a magnetic field of 2 webers m^{-2}. the face of the coil is parallel to the field. How much torque acts over the coil when a current of 0.5 A passes through it? What will be the torque when the coil is rotated by 60° from its initial position?***

(Karachi Board 1996, Supp.)

Solution

Data: N = 50 ; A = 2 cm x 4 cm = $\frac{2}{100} \times \frac{4}{100}$ = $8 \times 10^{-4}\ m^{-2}$; B = 2 T;

I = 0.5A ; τ = ?

(i) $\alpha = 0^{\circ}$ (ii) $\alpha = 60^{\circ}$

Calculation:

Part (i): τ = BINA cos α = $2 \times 0.5 \times 50 \times 8 \times 10^{-4} \times \cos 0^{\circ}$.

$\tau = 400 \times 10^{-4} \times 1$ $\boxed{4 \times 10^{-2}\ N.m}$

Part (ii): τ = BINA cos α = $2 \times 0.5 \times 50 \times 8 \times 10^{-4} \times \cos 60^{\circ}$.

$\tau = 400 \times 10^{-4} \times 0.5 = \boxed{2 \times 10^{-2}\ N.m}$

W.P 14.9 ***A cube 100 cm on a side is placed in a uniform magnetic field of flux density 0.2 weber.*** m^{-2} ***as shown in the diagram. Wires A, and C and D move in the direction indicated each at a rate of 50 cm*** s^{-1}***. Determine the induced e.m.f. in each wire.***

Solution

Data: L = 1m; L_A = 100 cm = 1m; B 0.2T; v = 50cm. s^{-1} = 0.5 m/s

Calculations:

Now, in ΔPQR, from the Pythagorean theorem,

$PR^2 = PQ^2 + QR^2$

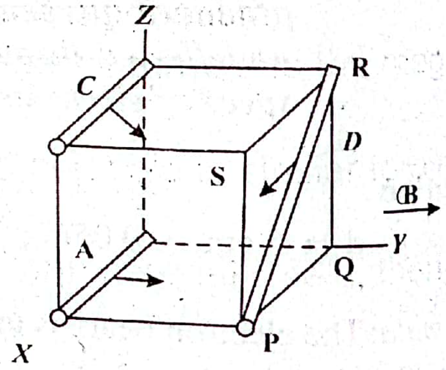


Fig. 14-50

$L_D^2 = l^2 + l^2 \Rightarrow L_D = \sqrt{2}$ m = 1.414 m

(i) $E_A = BvL_A \sin\theta_1 = 0.2 \times 0.5 \times 1 \times \sin 0^{\circ}$

$E_A = 0.10 \times 0 = \boxed{0\ volt}$

(ii) $E_C = BvL_C \sin\theta_2 = 0.2 \times 0.5 \times 1 \times \sin 45^{\circ} = 0.1 \times 0.707 = \boxed{0.707\ V}$

(iii) $E_D = BvL_D \sin\theta_3 = 0.2 \times 0.5 \times 1.414 \times \sin 135^{\circ} = 0.1 \times 1.414 \times 0.707 = \boxed{0.1\ V}$

W.P 14.10 ***What is the mutual inductance of a pair of coils if a current change of 6 ampere in one coil causes the flux in the second coil of 2000 turns to change by 12 x*** 10^{-4} ***webers.***

Solution

Data: M = ? ; ΔI_P = 6A ; N_s = 2000; $\Delta\varnothing = 12 \times 10^{-4}$ H

Calculation:

We know, $M = \frac{N_s \Delta\varnothing}{\Delta I_P} = \frac{2000 \times 12 \times 10^{-4}}{6} = 4000 \times 10^{-4}$ Wb

$M = 4000 \times 10^{-3} = \boxed{400\ mH}$

W.P 14.11 ***An e.m.f. of 45 mV is induced in a coil of 500 turns, when the current in a neighboring coil changes from 10 amperes to 4 amperes in 0.2 second.***

(a) What is the mutual inductance of the coils?

(b) What is the rate of change of flux in the second coil?

Solution

Data: $E = 45 \times 10^{-3}$; $N = 500$; $\Delta I = 10 - 4 = 6A$; $\Delta t = 0.2$ s; $M = ?$; $\Delta\varnothing / \Delta t = ?$

Calculation:

(i) $$M = \frac{\epsilon_s}{\left(\dfrac{\Delta I_P}{\Delta t}\right)} = \frac{45 \times 10^{-3}}{\left(\dfrac{6}{0.2}\right)} = \frac{45 \times 10^{-3}}{30} = 1.5 \times 10^{-3} \text{ H} = \boxed{1.5 \text{ mH}}$$

(ii) $$\epsilon_s = \frac{N_s \Delta\varnothing_s}{\Delta t} = \Rightarrow \frac{\Delta\varnothing}{\Delta t} = \frac{\epsilon_s}{N} = \frac{45 \times 10^{-3}}{500} = 0.09 \times 10^{-3}$$

$$\frac{\Delta\varnothing}{\Delta t} = \boxed{9 \times 10^{-5} \text{ Wb.s}^{-1}}$$

Q. 14.12 ***An iron core solenoid with 400 turns has a cross-section area of 4.0 cm^2. A current of 2 ampere passing through it produces B = 0.5 Wb. m^{-2}. How large an e.m.f. is induced in it, if the current is turned off in 0.1 second. What is the self inductance of the solenoid?***

(Karachi Board, 1980)

Solution

Data: $N = 400$; $\Delta A = 4.0 \text{ cm}^2 = \dfrac{4}{100 \times 100} = 4 \times 10^{-4} \text{ m}^2$; $\Delta I = 2A$; $B = 0.5$ T;

$\Delta t = 0.1$ s; $\epsilon = ?$; $L = ?$

Calculation:

(i) $$\epsilon = N \frac{\Delta\varnothing}{\Delta t} = \frac{NB\Delta A}{\Delta t} = \frac{400 \times 0.5 \times 4 \times 10^{-4}}{0.1}$$

$$\boxed{\epsilon = 0.8 \text{ V}}$$

(ii) Since $$L = \frac{\epsilon}{(\Delta I / \Delta t)} \Rightarrow L = \frac{0.8}{(2/0.1)} = \frac{0.8}{20} = 0.04 \text{ H}$$

$$L = 4 \times 10^{-2} = 40 \times 10^{-3} \text{ H} = \boxed{40 \text{ mH}}$$

W.P 14. 13 ***The current in a coil of 325 turns is changed from zero to 6.32 amperes, thereby producing a flux of 8.46 x 10^{-4} Wb. What is the self inductance of the coil?***

Solution

Data: $N = 325$; $\Delta I = 6.32 - 0 = 6.32$ A; $\Delta\varnothing = 8.46 \times 10^{-4}$ Wb; $L = ?$

Formula: $L = \dfrac{\epsilon}{(\Delta I / \Delta t)} = \dfrac{N(\Delta\varnothing/\Delta t)}{(\Delta I/\Delta t)} = N\dfrac{\Delta\varnothing}{\Delta I}$

Calculation:

$$L = 325 \times \frac{8.46 \times 10^{-4}}{6.32}$$

$$L = 435 \times 10^{-4}\ H = 43.5 \times 10^{-3}\ H = \boxed{43.5\ mH}$$

W.P 14. 14 ***A 100 turn coil in a generator, having an area of 500 cm², rotates in a field with B = 0.6 Wb. m⁻². How fast must the coil be rotated in order to generate a maximum voltage of 150 volts?***

Solution

Data: $N = 100;\ A = 500\,cm^2 = \dfrac{500}{100 \times 100} = 0.05\,m^2;\ B = 0.6\ Wb.\,m^{-2};\ V_o = 150v,\ \omega = ?$

Formula:

$$V_o = BNA\,\omega \Rightarrow \omega = \frac{V}{BNA}$$

Calculation:

$$\omega = \frac{150}{0.6 \times 100 \times 0.05}$$

$$\omega = \frac{150}{3} = \boxed{50\ radian/s}\quad f = \frac{50}{2\pi} = \frac{50}{6.28} = \boxed{7.96\ rev./s\ or\ Hz}$$

W.P 14. 15 ***A step-down transformer at the end of a transmission line reduces the voltage from 2400 V to 1200 V. The power output is 9.0 kW and overall efficiency of the transformer is 95%. The primary winding has 400 turns. How many turns has the secondary coil? What is the power input? What is the current in each of the coils?***

Solution

Data: $V_p = 2400\ V;\ V_s = 1200\ V;\ P_o = 9kW = 9000W;$ =; $\eta = 95\% = 95/100 = 0.95;$

$N_p = 400;\ N_s = ?\ P_i = ?;\ I_p = ?;\ I_s = ?$

Calculation:

(i) $\dfrac{N_s}{N_P} = \dfrac{V_s}{V_P} \Rightarrow N_s = \dfrac{V_s \times N_P}{V_P} = \dfrac{1200 \times 400}{2400} = \boxed{200}$

(ii) $\eta = \dfrac{P_o}{P_i} \Rightarrow P_i = \dfrac{P_o}{\eta} = \dfrac{9000}{0.95} = \boxed{9473.6\ W}$

(iii) $P_i = V_P I_P \Rightarrow I_P = \dfrac{P_i}{V_P} = \dfrac{9473.6}{2400} = \boxed{3.947\ A}$

(iv) $P_o = V_s I_s \Rightarrow I_s = \frac{P_o}{V_s} = \frac{9000}{1200} = \boxed{7.5\text{ A}}$

Q. 14. 16 ***What is the force per meter length on a wire carrying a 0.05A current due to 0.5 tesla magnetic flux density which acts perpendicularly?***

Solution

Data: $I = 0.5$ A; $B = 0.5$ T; $F/L = ?$

Calculation:

$$F = BIL \Rightarrow \frac{F}{L} = BI \Rightarrow \frac{E}{L} = 0.5 \times 0.5 = \boxed{0.25\text{ N/m}}$$

W.P 14. 17 ***A power line parallel to the earth's surface carries a current of 20.0 A straight west. At that point, the earth's magnetic field is 0.80 Gauss parallel to the earth's surface and directed straight north. (a) Find the force due to the field on a 15m length of wire. (b) What is its direction?***

Solution

Data: $I = 20$ A; $B = 0.8$ G; 0.8×10^{-4} T; $L = 15$ m; $F = ?$

Calculation:

$$F = BIL = 0.8 \times 10^{-4} \times 20 \times 15 = \boxed{0.024\text{ N}}$$

W.P 14. 18 ***A wire loop is being pulled out of magnetic field at constant rate. If the magnetic field is uniform (0.30T) in this region and zero elsewhere, what is the induced e.m.f. in the loop? (The speed of the loop is 2.0 m/s and its breadth is 4.0 cm).***

Solution

Date: $B = 0.3$ T; $v = 2$ m/s; $L = 4$ cm $= 0.04$m; $\in = ?$

Calculation:

$$\in = BvL = 0.3 \times 2 \times 0.4 = \boxed{0.024\text{ V}}$$

W.P 14. 19 **Two coils are wound tighly on the same iron core. The cross-sectional area of both is about $4.0\,\text{cm}^2$. When a current of 3.0 A flows in the primary coil, B = 0.20 T. There are 100 turns on the secondary coil. (a) How large an e.m.f. is induced in the secondary coil if the current in the primary coil drops uniformly to zero in 0.0.50s?**
(b) What is the mutual inductance of the coils?

Solution

Date: $N_s = 100$; $B = 0.2$ T; $\Delta I_P = (3 - 0) = 3$A; $\Delta t = 0.05$ s;

SECTION "B"

WORKED PROBLEMS: APPLICATIONS

W.P. 15.1 *A galvanometer has a resistance of 50 ohms and it deflects full scale when a current of 10 milliampers flows in it. How can it be converted into an ammeter of range 10A?*

Solution

$R_g = 50\ \Omega$, $\quad I_g = 10\ mA = 10 \times 10^{-3} A = 0.01A, I = 10A,$ $\quad R_s = ?$

Shunt resistance, $R_s = \dfrac{I_g R_g}{I - I_g} = \dfrac{0.010 \times 50}{10 - 0.01}$ [Vide Fig. 15.16 (a)]

$$R_s = \frac{0.5}{9.99} = \boxed{0.05\Omega}$$

[A current of I_g = 10 mA is to be used to measure a current of I = 10 A using a shunt resistance of R_s = 0.05 Ω. The excess current of 9.99A ($I_s = I - I_g$) can detour around the 50-ohm (= R_g) galvanometer coil].

W.P. 15.2 *A galvanometer whose resistance is 40 ohms deflects full-scale for a potential difference of 100 millivolts across its terminals. How can it be converted into an ammeter of 5 ampere range?*

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Solution

$R_g = 40\ \Omega$, $V_g = 100\text{m V} = 100 \times 10^{-3} = 0.1\text{V}$, $I = 5\text{A}$, $R_s = 10^{-3} = ?$

Now, $I_g = \dfrac{V_g}{R_g} = \dfrac{0.1}{40} = 2.5 \times 10^{-3}\text{A} = 0.0025\ \text{A}$

$$R_s = \left(\frac{I_g R_g}{I - I_g}\right) = \frac{0.0025 \times 40}{5 - 0.0025} = \frac{0.1}{4.9975} = \boxed{0.02\Omega}$$

W.P. 15.3 ***The coil of a galvanometer which has a resistance of 50 ohms and a current of 50 microamperes produces full scale deflection in it. Show by a diagram how can it be converted to (a) an ammeter of 5 ampere range and compute the shunt resistance (b) a voltmeter of 300 volts range and compute the seies resistance. (multiplier resistance)***

Solution

$R_g = 50\ \Omega$, $I_g = 500\ \mu\text{A} = 500 \times 10^{-6}\ \text{A}$, $I = 5\text{A}$, $R_s = ?$

$V = 300\ \text{V}$, $R_x = ?$

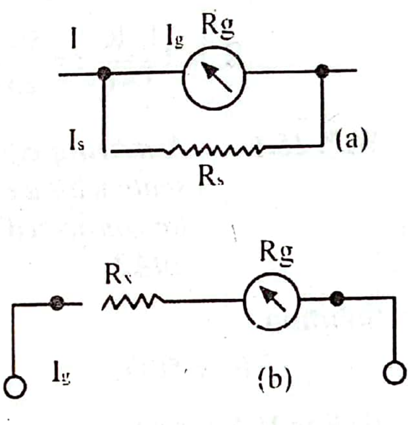

Fig. 15-16

(a) Shunt resistance, $R_s = R_s = \dfrac{I_g R_g}{I - I_g}$

$$R_s = \frac{500 \times 10^{-6} \times 50}{5 - (500 \times 10^{-6})} = \frac{0.025}{4.9995} = \boxed{R_s = 0.005\Omega}$$

(b) The voltage V across the series combination of galvanometer coil and a multiplier resistor is I $(R_g + R_x)$. So the multiplier resistance,

$$Rx = R_x = \left(\frac{V}{I_g}\right) - R_g \qquad R_s = \frac{300}{(500 \times 10^{-6})} - (50) = 600000 - 50$$

$$\boxed{R_x = 599950\Omega}$$

W.P. 15.4 ***A galvanometer of resistance 25 ohms deflects full scale for a current of 0.05 ampere. It is desired to convert this galvanometer into an ammeter reading 25 amperes full-scale. The only shunt available is 0.06 ohm. What resistance R must be included in series with the galvanometer coil, as shown in figure, for using this shun?***

Solution

$R_g = 25\ \Omega$, $I_g = 0.05\ \text{A}$, $I = 25\ \text{A}$, $R_s = 0.06\ \Omega$, $R = ?$

$V_{ab} = I_g R + I_g R_g = I_g (R + R_g)$ __________(1)

$V_{ab} = (I - I_g) R_s$ __________(2)

$\therefore (R + R_g)\, I_g = (I - I_g)\, R_s$ [By comparing eq. (1) & (2)]

$$R + R_g = \left(\frac{I - I_g}{I_g}\right) R_s = \left(\frac{25 - 0.05}{0.05}\right) \times 0.06 = \frac{1.497}{0.05}$$

$R + R_g = 29.94$

$R = 29.94 - R_g = 29.94 - 25 = \boxed{4.94\Omega}$

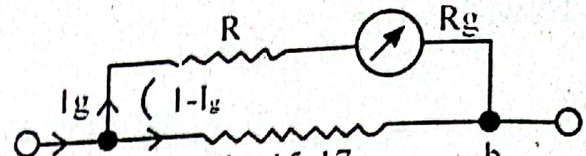

Fig. 15-17

W.P. 15.5 ***An ammeter deflects full scale with a current of 5 amperes and has a total resistance of 0.5 ohms. What shunt resistance must be connected to it to measure 25 amperes full scale?***

Solution

The initial 'ammeter' is treated as a galvanometer.

$I_g = 5A$, $R_g = 0.5\ \Omega$ $I = 25$ A, $R_s = ?$

$$R_s = \left(\frac{I_g R_g}{I - I_g}\right) = \frac{5 \times 0.5}{25 - 5} = \frac{25}{20} = \boxed{0.125\Omega}$$

W.P. 15.6 ***A moving coil galvanometer has a resistance of 50 ohms and deflects full scale with a current of 0.005 amperes. What resistance R_1, R_2, and R_3 must be connected to it as shown in figure to measure currents upto 1A, 5A and 10A?***

Solution

$R_g = 50\Omega$, $I_g = 0.005$ A

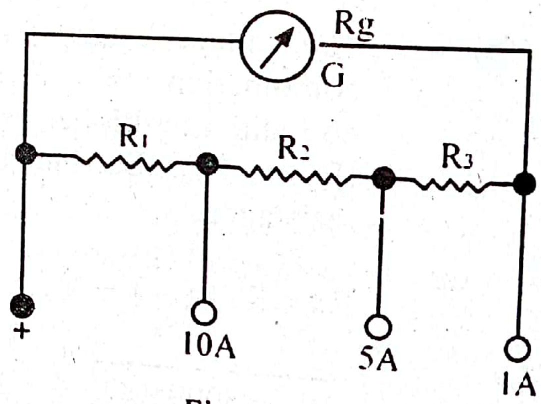

Fig. 15-18

(i) For 10A range

R_2 and R_3 in series with G; and R_1 will be the shunt.

Now, $I = 10A$. [vide Fig. 15.19]

Formula: shunt, $R_1 = \dfrac{I_g\,(R_g + R_2 + R_3)}{(I - I_g)}$

$$R_1 = \frac{0.005(50 + R_2 + R_3)}{(10 - 0.005)}$$

$9.995\ R1 = 0.005\ (50 + R_2 + R_3) \longrightarrow \dfrac{9.995\ R_1}{0.005} = R_2 + R_3 + 50$

$1999\ R_1 - R_2 - R_3 = 50$ ________ (1)

(ii) For 5A range

R_3 in series with G; and '$R_1 + R_2$" will be the shunt. [vide Fig. 15.20]

$$R_1 + R_2 = \frac{I_g\,(R_g + R_3)}{(I - I_g)}$$

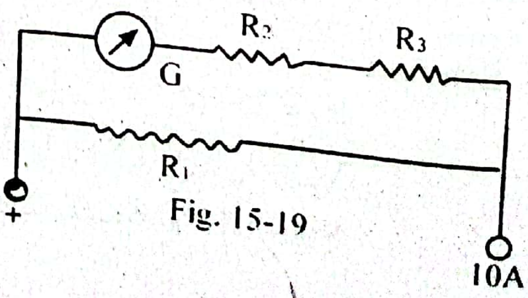

Fig. 15-19

$$R_1+R_2=\frac{0.005\,(50+R_3)}{(5-0.005)}$$

$$4.995\,(R_1+R_2)=0.005\,(50+R_3)$$

$$\frac{4.995}{0.005}\,(R_1+R_2)=50\,R_3 \longrightarrow 999\,R_1+999R_2-R_3=50 \,\_\_\_\_\_\_\_\_ (2)$$

(iii) For 1A range

$R_1+R_2+R_3$ will be the shunt. [vide Fig. 15.18]

$$\therefore\ R_1+R_2+R_3=\frac{I_g\,R_g}{I-I_g}$$

$$R_1+R_2+R_3=\frac{0.005\times 50}{(1-0.005)}$$

$$R_1+R_2+R_3=\frac{0.250}{0.995}$$

$$R_1+R_2+R_3=0.251 \,\_\_\_\_\_\_\_\_ (3)$$

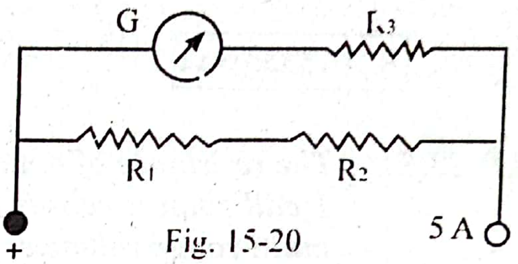

Fig. 15-20

Now, $1999\,R_1-R_2-R_3=50$ ________ (1)

$$R_1+R_2+R_3=0.251 \,\_\_\_\_\_\_\_\_ (3)$$

[By adding eq. (1) and eq. (3)]

$$2000\,R_1=50.251$$

$$R_1=\frac{50.251}{2000}=\boxed{0.0251\,\Omega}$$

Also, $999\,R_1+999\,R_2-R_3=50$ ________ (2)

$$R_1+R_2+R_3=0.251 \,\_\_\_\_\_\_\_\_ (3)$$

[By adding]

$$1000\,R_1+1000\,R_2=50.251$$

$$(1000\times 0.0251+1000\,R_2=50.251$$

$$1000\,R_2=50.251-25.1 \longrightarrow R_2=\frac{25.151}{1000} \longrightarrow \boxed{R_2=0.0251\,\Omega}$$

Putting values of R_1 and R_2 in eq. (3), we get

$$0.0251+0.0251+R_3=0.251 \longrightarrow R_3\ 0.251-0.0502$$

$$\boxed{R_3=0.2008\,\Omega}$$

W.P. 15.7 *A 300-volt voltmeter has a total resistance of 20,000 ohms. What additional series resistance must be connected to it to increase its range to 500 volts?*

Solution

$V_g = 300$ volt, $R_g = 20{,}000\Omega$ $R = ?$ $V = 500v$

$$\text{Now } I_g = \frac{V_g}{R_g} = \frac{300}{20{,}000} = 0.015\text{ A}$$

$$R_x = \left(\frac{V_g}{I_g}\right) - R_g = \left(\frac{500}{0.015}\right) - 20{,}000 = 33333.3 - 20{,}000$$

$$\boxed{R_x = 13333.3\ \Omega}$$

W.P. 15.8 *The resistance of a moving-coil galvanometer is 25 ohms and a current of 1 milliampere causes full scale deflection in it. It is to be converted into a multi-range voltmeter. Find the series resistances R_1, R_2 and R_3 to give the ranges of 5V, 50V and 500V at the range terminals, as shown in the figure.*

Solution

$R_g = 25\ \Omega$, $I_g = 1 \times 10^{-3}$A.

(i) For 5V range

Multiplier, $R_x = R_1$

$$\therefore R_1 = \frac{V}{I_g} - R_g = \left(\frac{5}{1\times10^{-3}}\right) - 25$$

$$\boxed{R_1 = 4975\ \Omega}$$

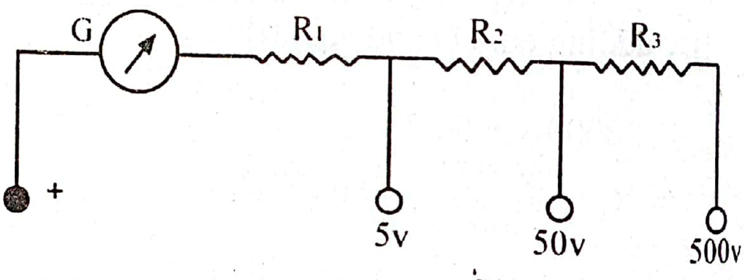

Fig. 15-21

(ii) For 50V range

Multiplier $= R_2$

$$R_2 = \left(\frac{V}{I_g}\right) - (R'_g) = \left(\frac{50}{1\times10^{-3}}\right) - (R_g + R_1)$$

$$R_2 = 50000 - (25 + 4975) = 50{,}000 - 5000 = \boxed{45{,}000\ \Omega}$$

(iii) For 500V range

$R''_g = R_g + R_1 + R_2$

Multiplier $= R_1$

$$R_3 = \left(\frac{V}{I_g}\right) - R''_g = \frac{500}{1\times10^{-3}} - (R + R_1 + R_2)$$

$$R_3 = 500{,}000 - (25 + 4975 + 45000)\ \boxed{450{,}000\ \Omega}$$

W.P. 15.9 ***The galvanometer of the ohmmeter in the figure has a resistance of 25□ and deflects full scale with a current of 2 milli-amperes in it. The e.m.f. of the cell is 1.5 volts.***

(i) What is the value of the series resistor R?

(ii) To what value of X connected to its terminal do the deflection of $^1/_5$, full-scale correspond?

(iii) Is the scale of the ohmmeter linear?

Solution

(i) $R_g = 25\ \Omega$, $E = 1.5V$, $I_g = 2 \times 10^{-3}A$.

$$R = \left(\frac{V}{I_g}\right) - R_g = \left(\frac{E}{I_g}\right) - R_g = \frac{1.5}{2\times10^{-3}} - 25$$

$$R = 0.75 \times 10^3 - 25 = 750 - 25$$

$$\boxed{R = 725\ \Omega}$$

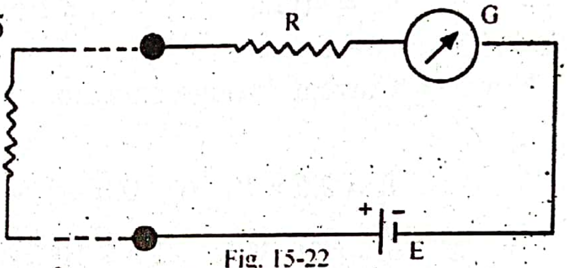

Fig. 15-22

(ii) For $\frac{1}{5}$th deflection, $I_g = \frac{1}{5} \times 2 \times 10^{-3} = 0.4 \times 10^{-3}A$

$$X_1 = \frac{V}{I_g} - (R+R_g) = \left(\frac{1.5}{0.4 \times 10^{-3}}\right) - (725 + 25) = 3750 - 750$$

$$\boxed{X_1 = 3000\ \Omega}$$

(iii) The scale of the ohmmeter is not linear, because its resistance is not proportional to the deflection.

W.P. 15.10 ***A potentiometer is set up to measure the e.m.f. Ex of a cell. The potentiometer wire is 120cm long. E_s is the e.m.f. of a standard cadmium cell, equal to 1.018V. When the key 1 only is closed to include the e.m.f. E_x in the galvanometer circuit, the galvanometer gives no deflection with the sliding contact at C, 56.4cm from A. When the key 2 only is closed to include the e.m.f. Es in the galvanometer circuit, the balance is obtained at C, 43.2 cm from A.***

(a) What is the e.m.f. Ex of the cell?

(b) What is the p.d. across the entire length of the wire AB?

Solution

(a) $$\frac{E_x}{E_s} = \frac{l_x}{l_s} \Rightarrow \frac{E_x}{1.018} = \frac{56.4}{43.2}$$

$E_x = 1.108 \times 1.3055 = \boxed{1.329\ V}$

(b) $\dfrac{V}{E_s} = \dfrac{I}{I_s} \Rightarrow \dfrac{V}{1.018} = \dfrac{120}{43.2}$

$\boxed{V = 2.828 \text{ volts}}$

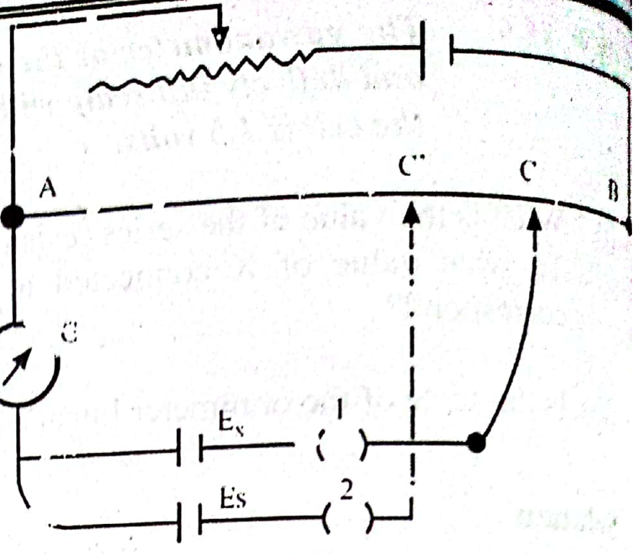

Fig. 15-23

W.P. 15.11 ***A certain galvanometer has a resistance of 40 ohm and deflects full scale for a voltage of 100mV across its terminal. How can it be made into 2-A ammeter?***

Solution

$R_g = 40\ \Omega$, $V_g = 100mV$ (for f.s.d.), $I = 2A$, $R_s = ?$

Now, Current through galvanometer for f.s.d., $I_g = \dfrac{V_g}{R_g} = \dfrac{100 \times 10^{-3}}{40}$

$I_g = 2.5 \times 10^{-3} A = 0.0025\ A$

$$R_s = \frac{I_g R_g}{(2 - I_g)} = \frac{2.5 \times 10^{-3} \times 40}{(2 - 0.0025)} = \frac{0.1}{1.9975} = \boxed{0.05\ \Omega}$$

W.P. 15.12 ***A certain galvanometer has a full scale deflection for 0.003A. How can this galvanometer of 20 ohm resistance be converted into 90-V voltmeter?***

Solution

$I_g = 0.003A$, $R_g = 20\Omega$, $V = 90V$, $R_x = ?$

$$R_x = \left(\frac{V}{I_g}\right) - R_g = \left(\frac{90}{0.003}\right) - (20) = 30000 - 20 = \boxed{29980\ \Omega}$$

W.P. 15.13 ***If the resistance in three successive arms of a balanced bridge be 1, 2 and 36 ohms, respectively, calculate the resistance of the galvanometer in the fourth arm.***

Solution

$R_1 = 1\Omega$, $R_2 = 2\ \Omega$, $R_3 = 36\ \Omega$, $R_4 = ?$

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \Rightarrow \frac{1}{2} = \frac{36}{R_4}$$

$$R_4 = 36 \times \frac{2}{1} = \boxed{72\ \Omega}$$

W.P. 15.14 ***A resistance of 10 ohm is placed in the right gap and one of unknown value is placed in the left gap of metre bridge. The null point is found to be at 40cm. Find the unknown resistance.***

EXERCISE – B

WORKED PROBLEMS: APPLICATION

W.P.16.1 ***Light is said to be a transverse wave phenomenon. What is that varies at right angles to the direction In which a light wave travels?***

Solution

Electric field (E) and magnetic field (B) intensities both are mutually perpendicular and also perpendicular to the direction of propagation of the wave

W.P.16.2. ***A Radar sends out 0.05μs pulses of microwaves whose wavelength is 2.5cm. What is the frequency of these waves? How many waves does each pulse contain?***

Solution:

$$\text{Frequency, } \nu = \frac{v}{\lambda} = \frac{3\times10^{8}\,m/s}{2.5\times10^{-2}\,m} = 1.2 \times 10^{10}\ \text{c/s}$$

Number of waves = v T = $(1.2 \times 10^{10})(0.05 \times 10^{-6}) = 0.060 \times 10^{4} = \boxed{600}$

W.P.16.3. ***In what wavelength range do radar signals lie?***

Solution

Most radars use radio signals of frequency range 100MHz to 10,000 MHz

$$\therefore \lambda = \frac{v}{\nu} = \frac{3\times10^{8}}{100MH_z} = 3 \times \frac{10^{8}}{10^{2} x 10^{6}} = 3\text{ m} = \boxed{3 \times 10^{9}\text{ nm}}$$

$$\lambda = \frac{v}{\nu} = \frac{3x10^8}{10x10^3MH_z} = \frac{3}{10} \times \frac{10^8}{10^3 x10^6} = 0.3 \times 10^{-1} = 0.03m = 3 \times 10^{-2} = \boxed{3 \times 10^7 \text{ nm}}$$

∴ Required wavelength range is of 3×10^7 nm to 3×10^9 nm

W.P.16.4. ***A nano-second is 10^{-9}s (a) what is the frequency of electromagnetic wave whose period is 1ns? (b) What is its wavelength? (c) To what class of electromagnetic waves does it belong?***

Solution

(a) $\nu = \frac{1}{T} = \frac{1}{1x10^{-9s}} = 1 \times 10^9$ c/s $= 10^9$ Hz

(b) $\lambda = vT = 3 \times 10^8 \times 10^{-9} = 0.3$ m

(c) This e.m. wave belongs to *micro-wave.*

W.P.16.5. ***The induced electric field at a distance of 30cm from the circle is 90 N/C. Calculate the rate of change of magnetic flux.***

Solution

$$E = \frac{1}{2\pi r} \cdot \frac{\Delta\phi}{\Delta t} \rightleftharpoons \frac{\Delta\phi}{\Delta t} = 2\pi r . E = 2 \pi x \frac{30}{100} x\ 90$$

$$\frac{\Delta\phi}{\Delta t} = 54 \pi \boxed{\text{Wb/s}}$$

W.P.16.6. ***A wire carries a current of 10 amperes. At what distance from the wire, the magnetic field will be equal to the earth's magnetic field? The earth's magnetic field is 5×10^{-5}T.***

Solution

$$B = \frac{\mu o}{2\pi} \times \frac{I}{d} \Rightarrow d = \frac{\mu o}{2\pi} \times \frac{I}{B}$$

$$d = \frac{4\pi \times 10^{-7}}{2\pi} \times \frac{10}{5 \times 10^{-5}} = 0.04m = \boxed{4 \times 10^{-2} \text{ m}}$$

EXERSCISE – C

SHORT ANSWER QUESTIONS

Q.16.1 ***Under what circumstances does a charge radiate electromagnetic waves?***

Ans: A charge radiates e.m. ...

7. $mc^2 - m_oc^2$ 8. $hv = v_oe + hv_o$ 9. angular momentum 10. infinity 11. $\Delta t \sim \frac{\Delta E}{h}$

12. in which newton's first law of motion is applicable. 13. X-rays
14. independent of the motion of the source and observer 15. photoelectric effect
16. uncertainity principle 17. equal to the frequency of ancident photon
18. greater frequency and greater wave length 19. β-particles

20. $E = \frac{h}{v}$ 21. $p - \frac{h}{\lambda}$ 22. classical physics 23. T^4 24. $h\upsilon < \emptyset_o$

25. 2 MeV 26. positively charged 27. frequency 28. Compton effect
29. 90 % 30. Compton effect

EXERCISE – B

WORDED PROBLEMS: APPLICATION

W.P.17.1. ***In the inertial frame of a pendulum, the time period is measured to be 3 s. What will be the period of the pendulum for an observer moving at a speed of 0.95c with respect to the pendulum?***

Solution

Data: $t_o = 3s$, $V = 0.95c$ $t=?.$

Formula: $t = \dfrac{t_o}{\sqrt{1-\dfrac{v^2}{c^2}}}$

Calculation: $t = \dfrac{3}{\sqrt{(1-\dfrac{0.95c)^2}{c}}} = \dfrac{3}{\sqrt{(1-0.95)^2}} = \dfrac{3}{\sqrt{0.0975}} = \dfrac{3}{0.312}$

$\boxed{T = 9.6\text{ s}}$

W.P. 17.2 *What will be the length of a bar in the stationary frame if its length along the x-direction is 1m and the motion is with a velocity 0.75 c with respect to the observer at rest.*

Solution

$I_o = 1\text{m},\ v = 0.75c,\ 1 = ?$

$I = I_o \cdot \sqrt{1-\dfrac{v^2}{c^2}} = 1\ (\sqrt{(1-\dfrac{(0.75c)^2}{c}}) = \sqrt{0.4375} = \boxed{0.66\text{m}}$

W.P. 17.3 *Given $m_oc^2 = 0.511$ MeV, find the total energy E and the kinetic energy K of an electron moving with a speed v = 0.85 c.*

Solution

Total energy, $E = mc^2 = \dfrac{m_o}{\sqrt{\dfrac{1-v^2}{c^2}}} \times c^2$

$E = \dfrac{m_oc^2}{\sqrt{1-\dfrac{(1-85c)^2}{c^2}}} = \dfrac{0.511\text{Mev}}{\sqrt{0.2775}} = \dfrac{0.511}{0.527}$

$\boxed{E = 0.97\text{ MeV}}$

Total energy = rest mass energy + ke. $\Rightarrow E = E_o + K$

$K = E - E_o = 0.970 - 0.511$

$\boxed{K = 0.459\text{ MeV}}$

W.P. 17.4 *The total energy of a proton of mass 1.67 x 10^{-17} kg is three times it rest energy. Find*
(a) protons rest energy (b) speed of the proton (c) kinetic energy (K) of proton in eV.

Solution

(a) Rest energy, $E_o = m_oc^2 = 1.67 \times 10^{-27} \times (3 \times 10^8)^2 = 1.5 \times 10^{-10}$ J

$= 0.9375 \times 10^9$ eV $= 0.938 \times 10^3 \times 10^6$ eV

$\boxed{= 938 \text{ MeV}}$

(b) For Proton, $E = 3 E_o \Rightarrow mc^2 = 3 m_oc^2$

$$\therefore \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} xc^2 = 3mc^2 \Rightarrow \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} = 3$$

$$3\sqrt{1-\frac{v^2}{c^2}} = 1 \Rightarrow 9\left(1-\frac{v^2}{c^2}\right) = 1 \Rightarrow 1-\frac{v^2}{c^2} = \frac{1}{9}$$

$$\frac{v^2}{c^2} = 1-\frac{1}{9} \Rightarrow \frac{v^2}{c^2} = \frac{8}{9} \Rightarrow v^2 = \frac{8}{9}c^2$$

$$v = \sqrt{\frac{8}{9} c^2} = \sqrt{0.888c} = 0.943 \times 3 \times 10^8 = \boxed{2.83 \times 10^8 \text{ m/s.}}$$

(c) $K = mc^2 - m_oc^2 = 3m_oc^2 - m_oc^2 = 2m_oc^2$

$K = 2 \times 938$ Mev $= \boxed{1876 \text{ Mev}}$

W.P 17.5 *particle of rest mass m_o has a speed $v = 0.8c$. Find its relativistic momentum, its kinetic energy (K) and total energy*

Solution

(a) $$p = m.v = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}}.(0.8c) = \frac{0.8m_0c}{\sqrt{1-\frac{(0.8c)^2}{c^2}}} = \frac{0.8m_0c}{0.6} z$$

$$\boxed{P = \frac{3}{4}m_0c}$$

(b) Total energy $$E = mc^2 = \frac{m_o}{\sqrt{1-\frac{v^2}{c^2}}} xc^2 = \frac{m_0c^2}{\sqrt{1-\frac{(0.8c)^2}{c^2}}}$$

$$E = \frac{m_0c^2}{\sqrt{1-0.64}} = \frac{m_0c^2}{0.6} = \frac{10}{6}m_0c^2 = \boxed{\frac{5}{3}m_0c^2}$$

(c) Kinetic energy, $$K = E - m_oc^2 = \frac{5}{3}m_0c^2 - m_0c^2 = \left(\frac{5-3}{3}\right) m_0c^2$$

$$K = \boxed{\frac{2}{3}m_0c^2}$$

K = E - E_o

W.P. 17.6 ***hat will be the velocity and momentum of a particle whose rest mass is m_0 and whose kinetic energy is equal to its rest mass?***

Solution

Givne: Kinetic energy = m_oc^2

(i) Total energy, $mc^2 = m_oc^2 + k.e. = m_oc^2 + m_oc^2$

$mc^2 = 2\, m_oc^2$

$\therefore m = 2m_o$ ______________ (1)

Also $\dfrac{m_o}{\sqrt{1-\dfrac{v^2}{c^2}}} = 2m_0 \Rightarrow \dfrac{1}{\sqrt{1-\dfrac{v^2}{c^2}}} = 2$

$2\left(\sqrt{1-\dfrac{v^2}{c^2}}\right) = 1 \Rightarrow 1-\dfrac{v^2}{c^2} = \left(\dfrac{1}{2}\right)^2$

$\dfrac{v^2}{c^2} = 1-\dfrac{1}{4} \qquad v^2 = \dfrac{3}{4}c^2$

$V = \dfrac{\sqrt{3}}{2}c$ ______________ (2)

(ii) $p = mv$

$P = 2m_ov$..........................(using 1)

$P = 2\, m.\ \dfrac{\sqrt{3}}{2}c$ ....(using 2)

$\boxed{P = \sqrt{3}m_oc}$

W.P. 17.7 ***he sun radiates energy at a rate 3.8 x 10^{26} W. At what rate does the mass of the sun diminish per year?***

Solution

Rate of energy or power = 3.8 x 10^{26} J/s

Power = 3.8 x 10^{26} J/s $\left(\left(\dfrac{1}{60\times60\times24\times365}\right)year\right.$

P = 3.8 x 10^{26} $(60\times60\times24\times365)$ J/yr.

P = 3.8 x 10^{26} x 3.15 x 10^7 J/yr. = 11.97 x 10^{33} J/yr.

Now, $m = \dfrac{E}{c^2}$ and rate of diminishing of mass, $\dfrac{m}{t} = \dfrac{(E/t)}{c^2}$

$\dfrac{m}{t} = \dfrac{p}{c^2} = \dfrac{11.97\times10^{33}\,J/yr}{(3\times10^8)^2(m/s)^2} = \boxed{1.33 \times 10^{17}\ kg/yr.}$

W.P. 17.8 ***What will be the work function of a substance for a threshold frequency of 43.9 x 10^{13} Hz?***

Solution

$$\varnothing_0 = h\upsilon_0 = 6.625 \times 10^{-34} \times 43.9 \times 10^{13} = 290.8 \times 10^{-21}\ J$$

$$\varnothing_0 = \frac{290.8 \times 10^{-21}}{1.6 \times 10^{-19}} = 181.7 \times 10^{2} = \boxed{1.817\ eV}$$

W.P. 17.9 ***What will be the value of λ_{min} if $V_0 = 10^4$ Volts?***

Solution

$$\lambda_{min} = \frac{hc}{eV_0} = \frac{6.63 \times 10^{-31} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times 10^{4}} = 12.43 \times \frac{10^{26}}{10^{-15}} = 12.43 \times 10^{-11}$$

$$\lambda_{min} = \boxed{1.243 \times 10^{-10} m}$$

W.P. 17.10 ***In a Compton scattering process the fractional change in wavelength of an x-ray photon is 1% at an angle of 120°. Find the wavelength of x-rays used in the experiment.***

Solution

Data : $\frac{\Delta\lambda}{\lambda} = 1\% = \frac{1}{100} = 0.01, \theta = 120°, \lambda = ?$

Formula, $\Delta\lambda = \frac{h}{m_o c}(1 - \cos\theta) = \frac{6.625 \times 10^{34}}{9.10 \times 10^{-31} \times 3 \times 10^{8}}(1 - \cos 120°)$

$\Delta\lambda = 2.426 \times 10^{-12}\ [1-(-0.5)] = 2.426 \times 1.5 \times 10^{-12} = 3.64 \times 10^{-12}$

Since $\frac{\Delta\lambda}{\lambda} = 10.01 \Rightarrow \lambda = \frac{\Delta\lambda}{0.01} = \frac{3.64 \times 10^{-12}}{0.01} = 364.2 \times 10^{-12}$

$\boxed{\lambda = 3.64 \times 10^{-10} m}$

W.P. 17.11 ***Find the wavelength of a 2.0g light ball moving with a velocity (a) 1.0mm per century (b) 1.0 ms^{-1}***

[Given 1 yr = 3.15 x 10^7 s]

Solution

$$\text{(a)}\quad \lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\ J.s.}{\left(\frac{2.0}{1000}kg\right)\left(\frac{1 \times 10^{-3} m}{100 \times 3.15 \times 10^{7} s}\right)} = \frac{6.63 \times 10^{-34}}{0.002 \times (3.17 \times 10^{-13})} = m$$

$$\lambda = 1045.7 \times 10^{-21} = \boxed{1.05 \times 10^{-18} m}$$

(b) $\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\text{ J.s.}}{\left(\frac{2.0}{1000}\text{kg}\right)(1\text{m/s})} = \frac{6.63}{0.002}\times10^{-34} = \text{m}$

$\lambda = 3315\times10^{-34} = \boxed{3.3\times10^{-31}\text{m}}$

W.P. 17.12 ***An electron exists with a region of 10^{-10}m. Find its momentum uncertainty and the approximate kinetic energy***

[Given $h = 1.05\times10^{-34}$ J.s]

Solution

$\Delta p.\ \Delta x = h \Rightarrow \Delta p = \frac{h}{\Delta x} = \frac{1.05\times10^{-34}}{10^{-10}} = 1.05\times10^{-24}$ kg. m/s

$\text{k.e.} = \frac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{\Delta p^2}{2m} = \frac{(1.05\times10^{-24})^2}{2\times9.1\times10^{-31}}$

$\text{k.e.} = \frac{1.1025}{18.2}\times\frac{10^{-18}}{10^{-3}} = 0.0605\times10^{-17} = \boxed{6.05\times10^{-19}\text{ J}}$

W.P. 17.13 ***Sodium surface is shined with light of wavelength 3×10^{-7} m. If the work function of sodium is 2.46 eV, find k.e. of the photoelectrons and also the cut off wavelength (λ_c)***

Solution

Data $\lambda = 3\times10^{-7}$ m; $\varnothing_0 = 2.46\text{ eV} = 2.46\times1.6\times10^{-19}$J; k.e. = ? λ_c = ?

Calculation:

(a) $\text{k.e.} = h\upsilon - \varnothing_0 = \frac{hc}{\lambda} - \varnothing_0 = \left(\frac{6.625\times10^{-34}\times3\times10^{8}}{3\times10^{-7}}\right) - (2.46\times1.6\times10^{-19})$

$\text{k.e.} = 6.625\times10^{-19} - 3.936\times10^{-19} = 10^{-19}(6.625 - 3.936)$

$\text{K.e. } 2.689\times10^{-19}\text{ J} = \frac{2.689\times10^{-19}}{1.6\times10^{-19}} = 1.68\text{eV}$

(b) $\lambda_c = \frac{hc}{\varnothing_0} = \frac{6.625\times10^{-34}\times3\times10^{8}}{2.46\times1.6\times10^{-19}} = \boxed{5.049\times10^{-7}\text{ m}}$

W.P. 17.14 ***X-rays of wavelength $\lambda_0 = 0.2$ nm are scattered from a carbon block at an angle of 45^0 with respect to the incident beam. Find the shift in wavelength.***

$\Delta\lambda = \frac{h}{m_0 c}(1-\cos\theta) = \frac{6.625\times10^{-34}}{9.1\times10^{-31}\times3\times10^{8}}(1-\cos45^0)$

$\Delta\lambda = 2.426\times10^{-12}(1-0.707) = 2.426\times0.293\times10^{-12}$

$$\Delta\lambda = 0.711\times10^{-12} = \boxed{7.11 \times 10^{-13}\ m}$$

W.P. 17.15 ***If the electron beam in a T.V picture tube is accelerated by 10.000 V. what will the de Broglie's wavelength?***

V = 10,0000 volts $\lambda = ?$

$$V = \sqrt{\frac{2Ve}{m}} = \sqrt{\frac{2\times10000\times1.6\times10^{-19}}{9.1\times10^{-31}}}\ \sqrt{3516.48\times10^{12}} = 59.3\times10^{6}\,ms^{-1}$$

$$\lambda = \frac{h}{mv} = \frac{6.625\times10^{-34}}{9.1\times10^{-31}\times59.6\times10^{6}} = 0.01228\times10^{-9} = \boxed{1.228 \times 10^{-11}\ m}$$

W.P. 17.16 ***What minimum energy photon can be used to observe an object of size 2.5 x 10^{-10} m?***

Solution

$E = ?\qquad \Delta x = 2.5\times10^{-10} = m$

$$\Delta P.\ \Delta x \approx h \Rightarrow \Delta P = \frac{h}{\Delta x} = \frac{6.625\times10^{-34}}{2.5\times10^{-10}} = 2.65\times10^{-24}$$

$$E = \Delta P.c = 2.65 \times 10^{-24} \times 3 \times 10^{8} = 7.95 \times 10^{-16}\ J$$

$$E = \frac{7.95\times10^{-10}}{1.6\times10^{-19}} = \boxed{4.95 \times 10^{3}\ eV}$$

W.P. 17.17 ***What will be the de Broglie's wavelength associated with a mass of 0.01 kg moving with a velocity of $10ms^{-1}$***

Solution

$\lambda = ?\qquad m = 0.01\ kg,\qquad v = 10ms^{-1}$

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}}{0.01\times10} = \boxed{6.63 \times 10^{-33}\ m}$$

W.P. 17.18 ***Certain excited state of hydrogen atom has a lifetime 2.5 x 10^{-19}s. What will be the minimum uncertainty in its energy***

Solution

$$\Delta t = 2.5\times10^{-19}, \Delta E = ?$$

$$\Delta E.\ \Delta t \approx h \Rightarrow \Delta E = \frac{h}{\Delta t} = \frac{6.63\times10^{-34}}{2.5\times10^{-19}} = \boxed{2.652 \times 10^{-15}\ J}$$

W.P. 17.19 ***X-rays are scatted from a target material. The scattered radiation is viewed at an angle of 90° with respect to the incident beam. Find the Compton shift in wavelength***

Solution

$$\Delta\lambda = \frac{h}{m.c}(1-\cos\theta) = \frac{6.625\times10^{-19}}{9.1\times10^{-31}\times3\times10^{9}}(1-\cos 90°)$$

$$\Delta\lambda = 2.42\times10^{-12}\times(1.0) = \boxed{2.42\text{ x }10^{-12}\text{ m}}$$

W.P. 17.20 ***Find the frequency of a photon when an electron of 20 keV is brought to rest in a collision with a heavy nucleus***

Solution

$$E = 20\text{ keV} = 20000\text{ x }1.6\text{ x }10^{-19}\text{ J}, \qquad v=?$$

$$hv = E \Rightarrow \; = v = \frac{E}{h} = \frac{20000\times1.6\times10^{-19}}{6.625\times10^{34}} = 4830.18\times10^{15}$$

$$\boxed{v = 4.83\text{ x }10^{18}\text{ Hz}}$$

W.P. 17.21 ***What will be the wavelength and frequency of a 1.0 keV photon?***

Solution

$$E = 1.0\text{ keV} = 1.0\text{ x }10^{3}\text{ x }1.6\text{ x }10^{-19}\text{ J} = 1.6\text{ x }10^{-16}\text{ J}$$

(i) $$E = \frac{hc}{\lambda} \Longrightarrow \lambda = \frac{hc}{\lambda}$$

$$\lambda = \frac{6.625\times10^{-34}\times3\times10^{8}}{1.6\times10^{-16}} = 12.42\times10^{-10}\text{m} = \boxed{12.42A^{o}}$$

(ii) $$\upsilon = \frac{c}{\lambda} = \frac{3\times10^{8}}{12.42\times10^{-10}} = \boxed{0.24\text{ x }10^{18}\text{ Hz}}$$

W.P. 17.22 ***Find the frequency of a photon when an electron of 20 keV is brough to rest in a collision with a heavy nucleus***

Solution

$$E = 20\text{ keV} = 20\text{ x }1000\text{ x }1.6\text{ x }10^{-19}\text{ J} = 3.2\text{ x }10^{-15}\text{ J}$$

$$E = h\upsilon \Longrightarrow \upsilon = \frac{E}{h} = \frac{3.2\times10^{-15}}{6.625\times10^{-34}} = \boxed{4.8\text{ x }10^{18}\text{ Hz}}$$

W.P. 17.23 ***What will be wavelength of a photon in order to separate a molecule whose binding energy is 15eV?***

Solution

$$\varnothing_o = 15\text{eV} = 15\text{ x }1.6\text{ x }10^{-19} = 24\text{ x }10^{-19}\text{ J} \qquad \lambda=?$$

$$\varnothing_o = h\upsilon_0 = \frac{hc}{\lambda_o} \Longrightarrow \lambda_0 = \frac{hc}{\varnothing_o}$$

19. $\frac{nh}{2\pi}$ | 17. applied voltage | 18. X-ray interferener
22. 0.53 Å | 20. Hessenberg | 21. 0.001nm to 1nm
25. electrons | 23. in the field of medical | 24. 10^6 kg m^{-1}

EXERCISE – B

WORKED PROBLEMS: APPLICATION

W.P. 18.1 ***Calculate the following (a) the orbit radius (b) the angular momentum (c) the linear momentum (d) the kinetic energy (e) the potential energy (f) the total energy for the Bohr's hydrogen atomic ground state.***

Solution

(a) $$r = \frac{(4\pi\epsilon_o)\,h^2}{me^2} = \frac{4 \times 3.149 \times 8.85 \times 10^{-12} \times (1.05 \times 10^{-34})^2}{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^2} = \boxed{5.3 \times 10^{-11}\ m}$$

(b) $$L = n\,h\ 1 > A = 1 \times 1.05 \times 10^{-34} = \boxed{1.1 \times 10^{-34}\ J.s.}$$

(c) $$L = mvr = pr \Rightarrow p = \frac{L}{n} = \frac{1.1 \times 10^{-34}}{5.3 \times 10^{-11}} = \boxed{20 \times 10^{-24}\ kg\ \frac{m}{s}}$$

(d) $$k.e. = \frac{ke^2}{2r_1} = \frac{1}{2} \times \frac{1}{4\pi\epsilon_o} \times \frac{e^2}{n} = \frac{1}{2} \times (9 \times 10^9) \times \frac{(1.6 \times 10^{-19})^2}{5.3 \times 10^{-11}}$$

$$k.e. = \frac{23.04}{10.6} \times \frac{10^{-29}}{10^{-11}} = \boxed{218\ \ 10^{-18}\ J}$$

$$k.e. = \frac{21.8 \times 10 \times 10^{-19}}{1.6 \times 10^{-19}} = \boxed{13.6\ eV}$$

(e) $$p.e. = -\frac{ke^2}{r_1} = -\left(\frac{1}{4\pi\epsilon_o}\right)\frac{e^2}{r_1} = -9 \times 10^9 \times \frac{(1.6 \times 10^{-19})^2}{5.3 \times 10^{-11}}$$

$$p.e. = -4.34 \times 10^{-18}\ J = \frac{-4.34 \times 10^{-18}}{1.6 \times 10^{-19}} = \boxed{-27.2\ eV}$$

(f) $$\text{Total energy} = p.e. + k.e. = -\frac{ke^2}{2r} = \boxed{-13.6\ eV}$$

W.P. 18.2 ***What is the wavelength of the radiation that is emitted when a hydrogen atom undergoes a transition from the state n = 3 to n = 1 ?***

Solution

$$\frac{1}{\lambda} = R \propto \left(\frac{1}{p^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{\lambda} = 1.097 \text{ x } 10^7 \left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 1.097 \text{ x } 10^7 \left(\frac{9-1}{9}\right) = \frac{8}{9} \text{ x } 1.097 \text{ x } 10^7$$

$$\frac{1}{\lambda} = 0.975 \text{ x } 10^7$$

$$\lambda = \frac{1}{0.975} \text{ x } 10^{-7}$$

$$\lambda = 1.0256 \text{ x } 10^{-7} \text{ m}$$

$$\lambda = 102.6 \text{ x } 10^{-9} \text{ m} = \boxed{102.6 \text{ nm}}$$

nf:2

W.P. 18.3 ***Light of wavelength 486.3 nm is emitted by a hydrogen atom in Balmar series. What transitions of the hydrogen atom is responsible for this radiation?***

$R\infty = 1.09 \times 10^7$

$\frac{1}{\lambda} = R\infty\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$

Solution

For Balmar series:

$$\frac{1}{\lambda} = R \propto \left(\frac{1}{2^2} - \frac{1}{n}\right) \Rightarrow \frac{1}{\lambda\, R\alpha} = \frac{1}{4} - \frac{1}{n^2}$$

$$\frac{1}{n^2} = \frac{1}{4} - \frac{1}{\lambda\, R\alpha} = \frac{1}{4} - \frac{1}{486.3\times10^{-9}\times1.097\times10^7}$$

$$\frac{1}{n^2} = \frac{1}{4} - \frac{1}{6\ 33} = \frac{5.33-4}{4\times5.33} = \frac{1.33}{21.32}$$

$$n^2 = \frac{21.32}{1.33} \Rightarrow n = \sqrt{16.03} \Rightarrow \boxed{n = 4}$$

W.P. 18.4 ***In the hydrogen atoms an electron experiences a transition from a state whose binding energy is 0.54 eV to another state whose excitation energy is 10.2 eV. (a) What are the quantum numbers for these states? (b) Compute the wavelength of the emitted photon. (c) To what series does this line belong?***

Solution

(a) i, Binding energy for electron, $|E_n| = \left(\frac{1}{n^2}\times13.6\right)$ eV

$$\therefore n^2 = \frac{13.6}{E_n} = \frac{13.6}{0.54} = 25.16$$

$$n = \sqrt{25.16} \Rightarrow n = 5.01 \Rightarrow \boxed{n = 5}$$

ii, $|E_p| = \frac{1}{p^2} \times 13.6 \Rightarrow p^2 = \frac{13.6}{E_p} = \frac{13.6}{3.4} = 4$

$$p = \sqrt{4} \Rightarrow \boxed{p = 2}$$

(b) $\frac{1}{\lambda} = R \propto \left(\frac{1}{2^2} - \frac{1}{5^2}\right)$..............[Here p = 2, n = 5]

$$\frac{1}{\lambda} = 1.097 \times 10^7 \left(\frac{1}{4} - \frac{1}{25}\right) = 1.097 \times 10^7 \left(\frac{25-4}{100}\right)$$

$$\lambda = \frac{1}{1.097 \times 10^7} \times \frac{100}{21}$$

$$\lambda = 0.0434 \times 10^{-5} = 434 \times 10^{-9}\ \text{m}$$

$$\lambda = \boxed{434\ \text{nm}}$$

) The value of p is 2. Hence this line belongs to Balmer series.

P. 18.5 ***Photon of 12.1 eV absorbed by a hydrogen atom, originally in the ground state, raises the atom to an excited state. What is the quantum number of this state?***

olution

$$h\nu = E_n - E_1$$

$$12.1\ \text{eV} = -\frac{1}{n^2} \times K - \left(-\frac{K}{1^2}\right)$$

$$12.1\ \text{eV} = -\frac{13.6\text{eV}}{n^2} + \frac{13.6\text{eV}}{1}$$

$$\frac{13.6}{n^2} = 13.6 - 12.1 \Rightarrow \frac{13.6}{n^2} = 1.5$$

$$n^2 = \frac{13.6}{1.5} \Rightarrow n = \sqrt{9.06} = 3.01$$

$$\boxed{n = 3}$$

$n_f = 1$

P. 18.6 ***Find the wavelength of the first three lines of the Lyman series of hydrogen.*** $R_\infty = 1.09 \times 10^7$

olution

(i) $$\frac{1}{\lambda_1} = R_\infty \left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 1.097 \times 10^7 \left(\frac{1}{1} - \frac{1}{4}\right) = 1.097 \times 10^7 \times \frac{3}{4}$$

$$\frac{1}{\lambda} = 0.82275 \times 10^7 \Rightarrow \lambda_1 = \frac{1}{0.82275 \times 10^7}$$

$$\lambda_1 = 1.215 \times 10^{-7}\ \text{m} = 121.5 \times 10^{-9}\ \text{m} = \boxed{121.5\ \text{nm}}$$

(ii) $$\frac{1}{\lambda_2} = R_\infty \left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 1.097 \times 10^7 \left(\frac{1}{1} - \frac{1}{9}\right)$$

$$\frac{1}{\lambda_2} = 0.97511 \times 10^7 \Rightarrow \lambda_2 = \frac{1}{0.97511 \times 10^7}$$

$$\lambda_2 = 1.0256 \times 10^{-7} = 102.5 \times 10^{-9}\ \text{m}$$

$$\boxed{\lambda_2 = 102.5\ \text{nm}}$$

(iii) $$\frac{1}{\lambda_3} = \frac{1}{R_\infty} \left(\frac{1}{1^2} - \frac{1}{4^2}\right) = 1.097 \times 10^7 \left(\frac{16-1}{16}\right)$$

$$\lambda_3 = \frac{1}{1.097 \times 10^7} \times \frac{16}{15}$$

$\lambda_3 = 0.9723 \times 10^{-7}\ m = 97.23 \times 10^{-9}\ m$

$\boxed{\lambda_3 = 97.23\ nm}$

W.p. 18.7 *In an experiment, the excitation potentials of hydrogen are found at 1.9 V, 10.21 V and 12.10 V and three different spectral lines are emitted. Find their wavelengths.*

Solution

(i) $V = 1.9\ V, \quad E = Ve \Rightarrow h\nu = Ve \Rightarrow \frac{hc}{\lambda} = Ve$

$$\lambda_1 = \frac{hc}{V_1 e} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{1.9 \times 1.6 \times 10^{-19}} = 6.5378 \times 10^{-7}\ m$$

$\lambda_1 = 653.78 \times 10^{-9}\ m = \boxed{653.7\ nm}$

(ii) $$\lambda_2 = \frac{hc}{V_2 e} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{10.21 \times 1.6 \times 10^{-19}} = 1.2166 \times 10^{-7}\ m$$

$\boxed{\lambda_2 = 121.66\ nm}$

(iii) $$\lambda_3 = \frac{hc}{V_3 e} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{12.1 \times 1.6 \times 10^{-19}} = 1.0266 \times 10^{-7}\ m$$

$\boxed{\lambda_3 = 102.66\ nm}$

W.P. 18.8 *What minimum energy is needed in an x-ray tube in order to produce x-rays with a wavelength of 0.1 x 10^{-10} m?*

Solution

$$E = \frac{hc}{\lambda} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{0.1 \times 10^{-10}} = \frac{19.875 \times 10^{-26}}{0.1 \times 10^{-10}}$$

$E = 198.75 \times 10^{-16} = \boxed{1.98 \times 10^{-14}\ J}$

W.P. 18.9 *A certain atom emits spectrum lines at 300, 400 and 1200 nm. Assuming that three energy levels are involved in the corresponding transitions, calculate the quantum of energy emitted at each wavelength.*

Solution

(i) $$E_1 = \frac{hc}{\lambda_1} = \frac{6.625 \times 10^{-34} \times 3 \times 10^8}{300 \times 10^{-9}} = \frac{19.875 \times 10^{-26}}{300 \times 10^{-9}}$$

$E_1 = 0.06625 \times 10^{-17}\ J = \frac{6.625 \times 10^{-19}}{1.6 \times 10^{-19}}$

$E_1 = \boxed{4.1406\ eV}$

(ii) $$E_2 = \frac{hc}{\lambda_2} = \frac{19.875 \times 10^{-26}}{400 \times 10^{-9}} = 0.04968 \times 10^{-17}\ J$$

$$E_2 = \frac{4.968 \times 10^{-19}}{1.6 \times 10^{-19}} = \boxed{3.105\ eV}$$

(iii) $$E_3 = \frac{hc}{\lambda_3} = \frac{19.875 \times 10^{-26}}{1200} = 0.01656 \times 10^{-17}\ J$$

$$E_3 = \frac{1.656 \times 10^{-19}}{1.6 \times 10^{-19}} = \boxed{1.035\ eV}$$

W.P. 18.10 ***Calculate the energy (in eV) of a photon whose frequency is***

(a) (i) 4×10^{14} Hz (ii) 20GHz (iii) 30 MHz.

(b) Describe the corresponding wavelengths for the photons.

Solution

(a) (i) $E_1 = h\nu_1 = 6.625 \times 10^{-34} \times 4 \times 10^{14} = \boxed{26.5 \times 10^{-20}\ J}$

$$E_1 = \frac{26.5 \times 10^{-20}}{1.6 \times 10^{-19}} = \boxed{1.65\ eV}$$

(ii) $E_2 = h\nu_2 = 6.625 \times 10^{-34} \times 20 \times 10^9 = \boxed{132.5 \times 10^{-25}\ J}$

$$E_2 = \frac{132.5 \times 10^{-25}}{1.6 \times 10^{-19}} = \boxed{8.28 \times 10^{-5}\ eV}$$

(iii) $E_3 = h\nu_3 = 6.625 \times 10^{-34} \times 30 \times 10^5 = \boxed{198.75 \times 10^{-28}\ J}$

$$E_3 = \frac{198.75 \times 10^{-28}}{1.6 \times 10^{-19}} = 124.2 \times 10^{-9} = \boxed{1.24 \times 10^{-7}\ eV}$$

(b) (i) $$\lambda_1 = \frac{c}{\nu_1} = \frac{3 \times 10^8}{4 \times 10^{14}} = 7.5 \times 10^{-7}\ m = \boxed{750\ nm}$$

(ii) $$\lambda_2 = \frac{c}{\nu_2} = \frac{3 \times 10^8}{20 \times 10^9} = 0.15 \times 10^{-1} = \boxed{0.015\ m}$$

(iii) $$\lambda_3 = \frac{c}{\nu_3} = \frac{3 \times 10^8}{30 \times 10^6} = \frac{10^8}{10^7} = \boxed{10m}$$

W.P.18.11 ***What is the shortest wavelength radiation in the Balmer series? What value of n must be used?***

Solution

$P = 2$ and $n = \infty$

$$\frac{1}{\lambda_{min}} = R_\infty \left(\frac{1}{2^2} - \frac{1}{(\infty)^2}\right) = 1.097 \times 10^7 \left(\frac{1}{4} - 0\right)$$

$$\lambda_{min} = \frac{1}{1.097 \times 10^7} \times \frac{4}{1} = 3.6449 \times 10^{-7}\ m$$

$$\lambda_{min} = 364.5 \times 10^{-9}\ m = \boxed{364.5\ nm}$$

(18) critical mass (19) producing μ (20) isomers

(21) to take photograph of high velocity ions (22) $_{q2}\mu^{238}$ into $_{q2}P\mu^{239}$

(23) fusion (24) pair production (25) binding energy

EXERCISE – B

WORKED PROBLEMS: APPLICATION

W.P.19.1 ***When the chlorine atom of mass number 35 and charge number 17, is bombarded by protons, the resulting atom disintegrates, emitting an α-particle. Write the equation representing the reaction.***

Solution

$$_{17}Cl^{35} + {}_{1}H^{1} \longrightarrow {}_{16}S^{32} + {}_{2}He^{4}$$

W.P.19.2 ***The half life of radon is 3.80 days. What would be its decay constant?***

Solution

$T_{1/2} = 3.8$ days $= 3.8 \times 24\ 60 \times 60 = 328320$ sec, $\lambda = ?$

$\boxed{\lambda = 2.1\times10^{-6}s^{-1}}$

W.P.19.3 ***The atomic weight of Bromine is 79.938 u and it is composed of two isotopes of mass 78.943 and 80.942 u. Compute the percentage of each isotopes.***

Solution

At. Mass of Bromine = M = 79.938 u

At. Mass of Br-1 = M_1 = 78.943 u

At. Mass of Br-2 = M_2 = 80.942 u

Rel. abundance of Br-1 = X_1 = ? and % X_1 = ?

Rel. abundance of Br-2 = X_2 = 80.942 u

$\therefore X_1 + X_2 = 1$ .... (1)

$M_1X_1 + M_2X_2 = M$ .... (2)

Substituting in eq. (2), we get

$78.943\ X_1 + 80.942\ X_2 = 79.938$ .... (3)

From eq. (1), we have $X_2 = 1 - X_1$ .... (4)

Putting eq. (4) in (3), we get

$78.943\ X_1 + 80.942\ (1 - X_1) = 79.938$

$78.943\ X_1 + 80.942 - 80.942\ X_1 = 79.938$

$-1.999\ X_1 = 79.938 - 80.942$

$-1.999\ X_1 = -1.004$

$$X_1 = \frac{1.004}{1.999}$$

$$\boxed{X_1 = 0.5022}$$

Putting the value of X_1 in eq. (4), we have

$X_2 = 1 - 0.5022 = \boxed{0.4977}$

$\% X_1 = 0.5022 \times 100 = \boxed{50.2\%}$

$\% X_2 = 0.4977 \times 100 = \boxed{49.8\%}$

W.P.19.4 ***The half life of $_{104}P_0^{21}$ is 140 days. By what percent does its activity decrease per week?***

$T_{1/2}$ = 140 days = 140/7 = 20 weeks

Solution

Since $A = \lambda N$ .... (1)

ΔT = 1 week

$\therefore\ \Delta A = \lambda \Delta N$ .... (2)

$\dfrac{\Delta A}{A} = \dfrac{\Delta N}{N}$ .... (3) [By dividing eq. (2) by eq. (1)]

But $\left|\dfrac{\Delta N}{N}\right| = \lambda \Delta T$ .... (4)

$\therefore\ \dfrac{\Delta A}{A} = \lambda \Delta T$ [By dividing eq (4) in (3)]

$T_{1/2} = \dfrac{0.693}{\lambda}$

$\% \dfrac{\Delta A}{A} = \lambda \Delta T \times 100 \Rightarrow \boxed{\% \dfrac{\Delta A}{A} = \dfrac{0.693}{T_{1/2}} \times \Delta T \times 100}$

$\lambda = \dfrac{0.693}{T_{1/2}}$

$T_{1/2} = 140$ days $= \dfrac{140}{7} = 20$ weeks

$\Delta T = 1$ week, $\% \dfrac{\Delta A}{A} = ?$

$\% \dfrac{\Delta A}{A} = \dfrac{0.693}{20} \times 1 \times 100 = \dfrac{69.3}{20} = \boxed{3.465\%}$

W.P.19.5 ***If a neutron be entirely converted into energy, how much energy would be produced?***

Solution

Mass of neutron = m = 1.67493×10^{-27} kg, E = ?

$E = mc^2 = 1.67493 \times 10^{-27} \times (3 \times 10^8)10^2$

$E = 15.074 \times 10^{-27} \times 10^{16}$

$E = 15.074 \times 10^{-11}$

$E = 1.5074 \times 10^{-10}\ J = \boxed{1.5 \times 10^{-10}\ J}$

$E = mc^2$

$E = \dfrac{1.5 \times 10^{-10}}{1.6 \times 10^{-19}} = 0.9375 \times 10^{-10+19}$

$E = 0.9375 \times 1000 \times 106$ ev $= \boxed{937.5\ MeV}$

W.P.19.6 ***Find the binding energy of $_{52}Te^{126}$, given***

$m_p = 1.0078$ u $m_n = 1.0086$ u, $m_{Te} = 125.9033$ u.

Solution

Number of protons = 52, number of neutrons = 126 – 52 = 74

Total mass of protons = $M_p = 52 \times 1.0078 = 52.4\ \mu$

Total mass on neutrons = $m_n = 74 \times 1.0086 = 74.6\ \mu$

Total mass of the nucleus = $M = M_p + M_n = 127.04\ \mu$

Mass defect = $\Delta M = M - M_{Te} = 127.04 - 125.9033 = 1.1387$ u

B.E. = $1.1387 \times 931.5 = 1060.69$ MeV = $\boxed{1.06\times10^3 \text{ MeV}}$

W.P.19.7 ***If the number of atoms per gram of $_{81}Ra^{226}$ is 2.666×10^{21} and it decays with a half life of***

1622 years, find the decay constant and the activity of the sample.

Solution

$N = 2.666 \times 10^{21}$, $T^1/_2 = 1622$ years, $\lambda = ?$, $A = ?$

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1622\times365\times24\times60\times60} = \frac{0.693}{5.1\times10^{10}} = 1.35\times10^{-11}s^{-1}$$

$A = \lambda N = 1.35 \times 10^{-11} \times 2.666 \times 10^{21} = \boxed{3.59\times10^{-10}\text{dis./s.}}$

W.P.19.8 ***What will be the maximum energy of the electron in the beta decay of $_1H^3$ through the reaction?***

$$_1H^3 \longrightarrow {}_2He^3 + \beta^- + v$$

Solution

Max. energy of electron = $(M_H{}^3 - M_{He}{}^3)c^2$

$= (3.016049 - 3.016029)c^2$

$= 0.0002$ u x 93.15 MeV/u

$= 0.0186$ MeV

W.P.19.9 ***Find the Q-value for the nuclear reaction:***

$$_3Li^7 + {}_1H^1 \longrightarrow {}_2He^4 + {}_2He^4$$

Solution

Mass of the reactants = 7.016003 + 1.007825 = 8.023828 u

Mass of the products = 4.002603 + 4.002603 = 8.005206 u

Change in mass = Δm = 0.018622 u

Q-value = Δmc^2 = 0.018622 u x 931.5 MeV/u = $\boxed{17.35 \text{ MeV}}$

W.P.19.10 ***Find the energy released when two deuterium ($_1H^2$) nuclei fuse together to form an alpha particle ($_2He^4$).***

Solution

$$_1H^2 + {}_1H^2 \longrightarrow {}_2He^4 + Q$$

Mass of the reactants = 2(2.014102) = 4.028204 u

Mass of the product = 4.002603 u

Change in mass = Δm = 4.028204 − 4.002603 = 0.025601 u

Q-value (energy released) Δmc^2 = 0.025601 u x 931.5 MeV/u