Topics: Normal distribution, Functions of Random Variables

1. The time required for servicing transmissions is normally distributed with $\mu = 45$ minutes and $\sigma = 8$ minutes. The service manager plans to have work begin on the transmission of a customer's car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

   A. 0.3875
   B. 0.2676
   C. 0.5
   D. 0.6987

Answer: B

Wok being start after 10 minutes, so the Average time increase from 45 to 55 minutes.

$Z = (X-\mu)/\sigma$

$= (55-45)/8 = 0.625$

Probability = 0.26598552904870054

2. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean $\mu = 38$ and Standard deviation $\sigma=6$. For each statement below, please specify True/False. If false, briefly explain why.
   A. More employees at the processing center are older than 44 than between 38 and 44.
   B. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Answer:

A. False

$Z = (X-\mu)/\sigma$

$= (44-38)/6$

$= 1$

Probability = 0.8413447460685429

Here, ($\mu + \sigma = 38+6=44$) most of the data falls between one std. deviation of the mean, so it is not possible.

B. True

$Z=(X-\mu)/\sigma$

$P(X\leq 30) = p(Z \leq (30-38)/6) = p(Z \leq -1.33) = 0.0918$(using z table)

Expected count = 0.0918*400 = 36.72 (400 is clerical employees)

3. If $X_1 \sim N(\mu, \sigma^2)$ and $X_2 \sim N(\mu, \sigma^2)$ are *iid* normal random variables, then what is the difference between 2 X_1 and $X_1 + X_2$? Discuss both their distributions and parameters.

Answer:

- As we know that if $X \sim N(\mu_1, \sigma_1^2)$, and $Y \sim N(\mu_2, \sigma_2^2)$ are two independent random variables then $X + Y \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$, and $X - Y \sim N(\mu_1 - \mu_2, \sigma_1^2 + \sigma_2^2)$
- Similarly, if $Z = aX + bY$, where X and Y are as defined above, i.e Z is linear combination of X and Y, then $Z \sim N(a\mu_1 + b\mu_2, a^2\sigma_1^2 + b^2\sigma_2^{\wedge 2})$.
- Therefore, in the question, $2X_1 \sim N(2\mu, 4\sigma^2)$ and $X_1+X_2 \sim N(\mu + \mu, \sigma^2 + \sigma^2) \sim N(2\mu, 2\sigma^2)$, $2X1-(X1+X2) = N(4\mu, 6\sigma^2)$.

➢ 2 X_1 will be greater scale version than $X_1 + X_2$. If X_1 and X_2 are normally distributed then the sum of the random sample will be exactly same.

4. Let $X \sim N(100, 20^2)$. Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

A. 90.5, 105.9
B. 80.2, 119.8
C. 22, 78
D. 48.5, 151.5
E. 90.1, 109.9

Answer: D

Alpha= 99% for z-score alpha = 0.995 and 0.005
qnorm (0.995,100,20) = 151.5166
qnorm (0.005,100,20) = 48.48341

5. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions $Profit_1 \sim N(5, 3^2)$ and $Profit_2 \sim N(7, 4^2)$ respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
   A. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.
   B. Specify the 5th percentile of profit (in Rupees) for the company
   C. Which of the two divisions has a larger probability of making a loss in a given year?

Answer:

a.

stats.norm.ppf(0.975, loc=5, scale=3)	10.87	45×10.87	= 489.15
stats.norm.ppf(0.25, loc=5,scale=3)	2.97	45× 2.97	= 133.65
stats.norm.ppf(0.975, loc=7,scale=3)	12.87	45×12.87	= 579.15
stats.norm.ppf (0.25, loc=7, scale=3)	4.97	45× 4.97	= 223.65

Range of $Profit_1$ = (133.65, 489.15)

Range of $Profit_2$ = (223.65, 579.15)

b.

stats.norm.ppf(0.05, loc=5, scale=3)	2.065	45×2.065	= 92.925
stats.norm.ppf(0.05, loc=7,scale=3)	0.065	45×0.065	= 02.925

5th percentile of profit in rupees = 92.925 + 02.925 = 95.25

c. 2nd division has larger probability of making a loss in given year.