## Unit I: Electrostatics

## CHAPTER–1: ELECTRIC CHARGES AND FIELDS

## GIST OF THE CHAPTER:

Electric charges, Conservation of charge, Coulomb's law-force between two-point charges, forces between multiple charges; superposition principle and continuous charge distribution.

Electric field, electric field due to a point charge, electric field lines, electric dipole, electric field due to a dipole, torque on a dipole in uniform electric field.
Electric flux, statement of Gauss's theorem and its applications to find field due to infinitely long straight wire, uniformly charged infinite plane sheet and uniformly charged thin spherical shell (field inside and outside).

## DEFINITIONS & CONCEPTS:-

**1. Charge :** Charge is an intrinsic property of elementary particles of matter which gives rise to electric force between various objects.

**2. Two types of charges:** Positive and negative.

**3. Transference of electrons** is the cause of frictional electricity.

**4. Basic properties of electric charge :**

**i) Additivity of charges :** Total charge is the algebraic sum of individual charges.

**ii) Conservation of charges** : The total charge of an isolated system is always conserved.

**iii) Quantisation of charges** : Charge of an object is always in the form of integral multiple of electronic charge and never its fraction.

**5. Coulomb's Law :** It states that electrostatic force of attraction or repulsion between two stationary point charges kept in free space is given by:

$F = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r^2}\right)$ where '$q_1$' and '$q_2$' are the stationary point charges and 'r' is the separation between them.

$\varepsilon_0$ = permittivity of free space = 8.85 x $10^{-12}$ $C^2/Nm^2$

$\frac{1}{4\pi\varepsilon_0}$= 9 x $10^9$ $Nm^2/C^2$

It states that electrostatic force of attraction or repulsion between two stationary point charges kept in medium is given by:

$F = \frac{1}{4\pi\varepsilon}\left(\frac{q_1 q_2}{r^2}\right)$ where '$q_1$' and '$q_2$' are the stationary point charges and 'r' is the separation between them. $\varepsilon =$ absolute permittivity of medium.

## In vector form

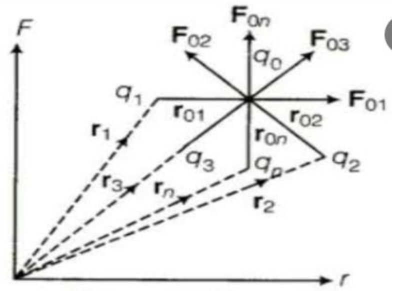
$$\overrightarrow{F} = \frac{1}{4\pi\varepsilon}\left(\frac{q_1 q_2}{r^2}\right)\hat{r}$$

(charge) $q_1$ ———— $\overrightarrow{r}$ ———— $q_2$ (another charge) $\longrightarrow \overrightarrow{F}$

**6. Dielectric constant** = The ratio of force between two charges in vacuum to the force acting between when they are shifted in a medium is called relative permittivity or dielectric constant of the medium.

$K = \dfrac{\varepsilon}{\varepsilon_0}$ Where $K$ is also called the relative permittivity and $\varepsilon$ is the permittivity of medium.

**7. Principle of Superposition of Electrostatic Forces:**
This principle states that the net electric force experienced by a given charge particle $q_o$ due to a system of charged particles is equal to the vector sum of the forces exerted on it due to all the other charged particles of the system.



Superposition of electrostatic forces

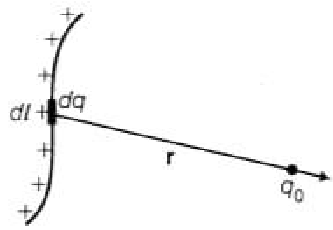
i.e. $\mathbf{F_0 = F_{01} + F_{02} + F_{03} \ldots\ldots + F_{0N}}$

**8. Electrostatic force due to continuous charge distribution:**

**i) linear charge distribution( λ ):**

$\lambda$= Charge/Length =C/m

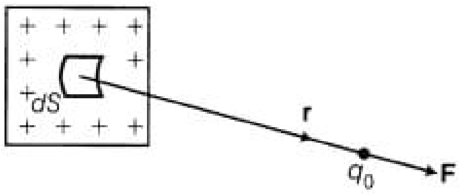
$$\lambda = \frac{dq}{dl}$$




**ii) Surface charge distribution(σ):**

σ= Charge/Area = C$m^{-2}$

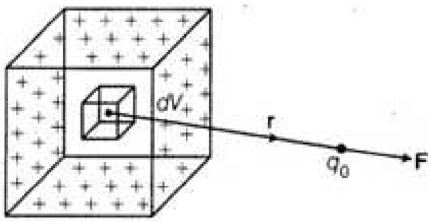
$$\sigma = \frac{dq}{dS}$$




**iii) Volume charge distribution(ρ) :**

ρ = Charge/ Volume = C$m^{-3}$

$$\rho = \frac{dq}{dV}$$



## UNITS OF CHARGE

**(i) SI unit coulomb (C)**
**(ii) CGS system**

(a) Electrostatic unit, esu of charge or stat-coulomb (stat-C)
(b) Electromagnetic unit, emu of charge or ab-C (ab-coulomb) 1 ab-C = 10 C, 1 C = 3 x $10^9$ stat-C

**Relationship of k to $\varepsilon_0$:** $\boldsymbol{k = \dfrac{1}{4\pi\varepsilon_0} = 9X\,10^9\,Nm^2C^2}$

$\boldsymbol{\varepsilon_0}$= permittivity of free space = 8.85 X $10^{-12}\ C^2N^{-1}m^{-2}$

**Force on a point charge in an electric field:** $\vec{F} = q\,\vec{E}$

## Electric Field ($\vec{E}$) :

It may be defined as the space surrounding the electric charge where another charge felt a force of attraction or repulsion.

Q - - - - - - - - - - - - - - - - - - - - - - - - - - - - $q_0$ ⟶

(Source charge) $\vec{r}$ (Test charge) $\vec{E}$

Electric field produced by the charge Q at a distance $\vec{r}$

$\vec{E} = \dfrac{\vec{F}}{q_0}$ Where $\vec{E}$ = electric field; $q_0$ = Test charge; $\vec{F}$ = Electrostatic force

Electric field lines radiate outwards from positive charges. The net electric field is zero inside a conductor.

**Electric field due to point charge**

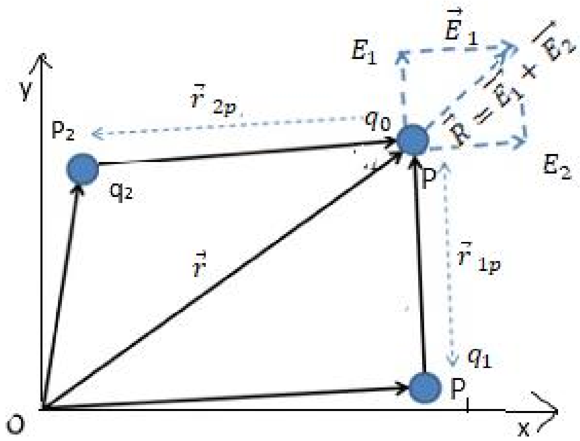
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r^2}\right)\hat{r}$$

(Note that Derivation is available in textbook)

**Field due to system of charge (Multiple Charges):**

$\vec{E} = E_1 + E_2 + \ldots E_n = \frac{1}{4\pi\varepsilon_0}\sum_{i=0}^{n}(q_i\,/r_i^2)\hat{r_i}$

(Note that Derivation is available in textbook)




## NOTES:-

Note1 Electric field is a vector quantity. It takes the direction of force.
Note2 If $q_0$ is negative, then direction of $\vec{E}$ will be in the opposite direction of force.
Note3 SI unit of $\vec{E}$ =newton/coulomb (N/C) or (N/$C^{-1}$)
Note4 If $q_0$ =1 (unity), then electric field is the force that a unit positive charge would experience if placed at that point.

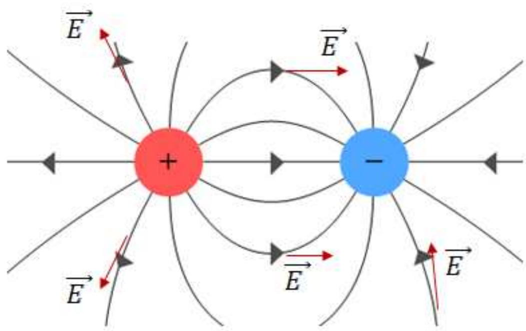
## GRAPH for Electric field due to a point charge


Note : Graph of electric field with distance r

**Electric Field lines**: It is defined as the path pull over by moving test charges. To visualise electric field due to a system of charge, an imaginary field lines are draw.

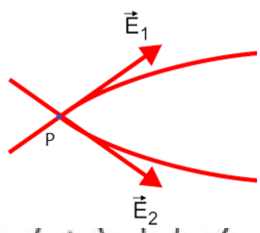
**Properties:**

1. Field lines from a positive charge to negative charge
2. Direction of filed lines shown by the tangent to the field lines.
3. Electric field lines emerge normal from positive charge and terminate at negative charge.

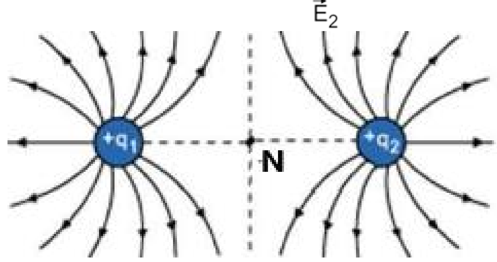



4. They never interest each other.

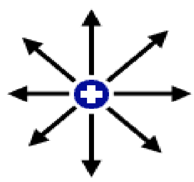
   This can be explained by method of contradiction at two field lines $E_1$ & $E_2$ interest at P as shown in the figure. Then there may be two tangents at $E_1$ and $E_2$. Which is not possible. Since one and only one tangent can be drawn. Therefore, our assumption is wrong.

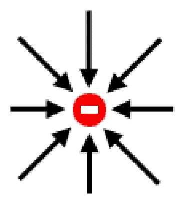



5. Electric field contract in length which shows that opposite charges attract.
6. Electric field lines exert lateral pressure which shows that like charges repel. N is a point which shows field intensity is zero called neutral point.

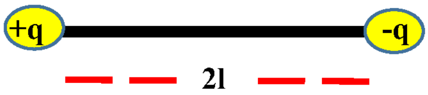



7. Closer field lines indicate the stronger region and rarer field lines shows weaker region.
8. Single positive charge radiates field lines radially outward (q > 0).
9. Single negative charge radiates field lines radially inward (q < 0).





**ELECTRIC DIPOLE:**

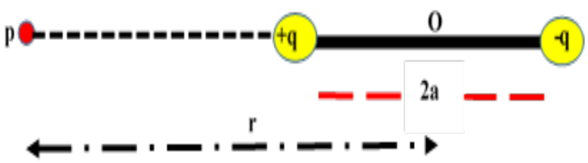



- Two equal and opposite charges separated by small distance forms dipole
- Dipole moment is a vector quantity with direction from negative to positive charge.
  Dipole Moment: $\vec{p} = (q \times 2a)$
- Electric field intensity due to dipole can be calculated at axial as well as equatorial point.

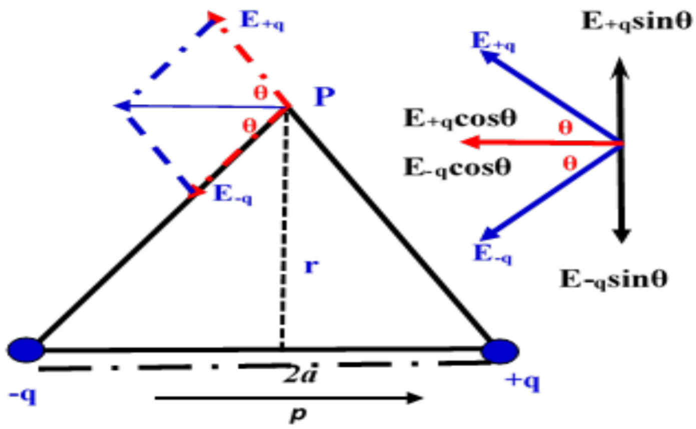
Electric Field Intensity (Axial point): $E_{Axial} = \frac{1}{4\pi\varepsilon_0}\left[\frac{2p}{r^3}\right]$

Electric Field Intensity (equatorial line): $E_{equat} = -\frac{1}{4\pi\varepsilon_0}\left[\frac{p}{r^3}\right]$

i) Axial Point




ii) Equatorial point




**Electric Dipole in a Uniform Electric Field:**

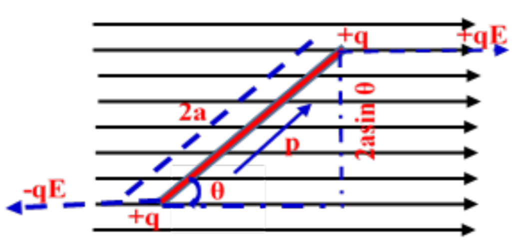
- Electric dipole is placed in a uniform electric field, it experiences torque

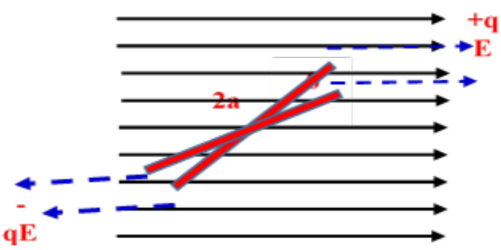
**i)Torque on an Electric Dipole:** $\tau = q\,E\,(2a \sin\theta) = p\,E \sin\theta,\ \ \tau = \vec{p} \times \vec{E}$

**ii) Work done On an Electric Dipole:** $W = p\,E\,(\cos\theta_1 - \cos\theta_2)$

**iii) Potential Energy:** $U = -\,p\,E \cos\theta$

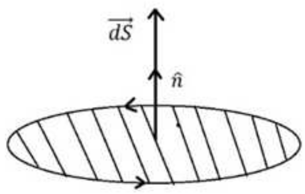
- Potential Energy can be taken zero arbitrarily at any position of the dipole.



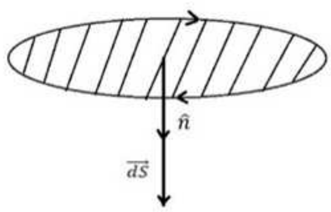



## Electric Flux, Gauss Theorem and Applications:-

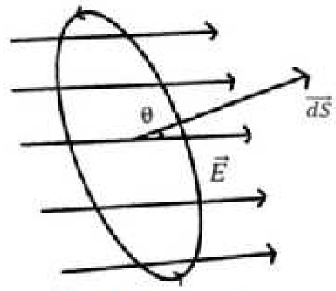
1) **Area Vector** – It is vector associated with small elemental area and expressed as $\overrightarrow{dS}$=dS $\hat{n}$.



**Area traced- Anti-Clockwise**



**Area traced- Clockwise**



**$d\Phi_E$ = E dS cos θ**

2) **Electric Flux**: -Electric flux over an area represents/measures total number of electric field lines crossing the area when it is held normal to the field direction. $\mathbf{\Phi_{E\ Total}} = \oint_s \vec{E}.\overrightarrow{dS} = \oint_s E\, dS \cos\theta$

3) **Gauss Theorem or Gauss's Law of Electrostatics**

It states that total electric flux over closed surfaces enclosing volume V in vacuum is $1/\epsilon_o$ times the total charge enclosed inside closed surface S.

$$\Phi_{E\ Total} = q_{\ Total} / \epsilon_o \quad \text{And} \quad \Phi_{E\ Total} = \oint_s \vec{E}.\overrightarrow{dS}$$

$$\therefore \oint_s \vec{E}.\overrightarrow{dS} = q_{\ Total} / \epsilon_o$$

4) **Gaussian Surface**: -It is an imaginary surface around a point charge or charge distribution such that electric field intensity E at every point of it is same.

1) Electric field due to infinitely long uniformly charged straight wire $E = \lambda/2\pi\epsilon_o r = 2\lambda/4\pi\epsilon_o r$
2) Electric field intensity due to uniformly charged thin infinite plane sheet. $E = \sigma/2\epsilon_o$
3) Electric field at a point due to uniformly charged spherical shell.
   i) Electric field intensity at point 'P' outside shell distant r (r >R).

$$\boxed{\therefore E = q/4\pi\,\epsilon_o\, r^2}$$

   ii) Electric field at a point P on the surface of sphere r= R

$$E = q/4\pi\,\epsilon_o\, R^2 = \sigma\, A/4\pi\,\epsilon_o\, R^2$$

$$= \sigma\,(4\pi R^2)/4\pi\,\epsilon_o\, R^2$$

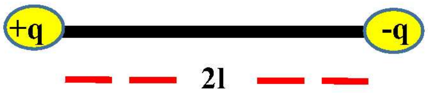
$$\boxed{\therefore E = \sigma/\,\epsilon_o}$$

   iii) Electric field at a point P inside shell (r < R)

$$\boxed{\therefore E=0}$$

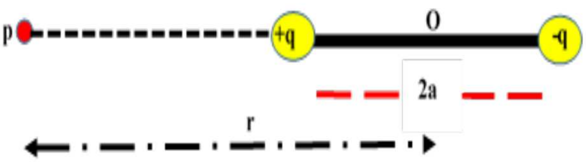
# DIAGRAMS:

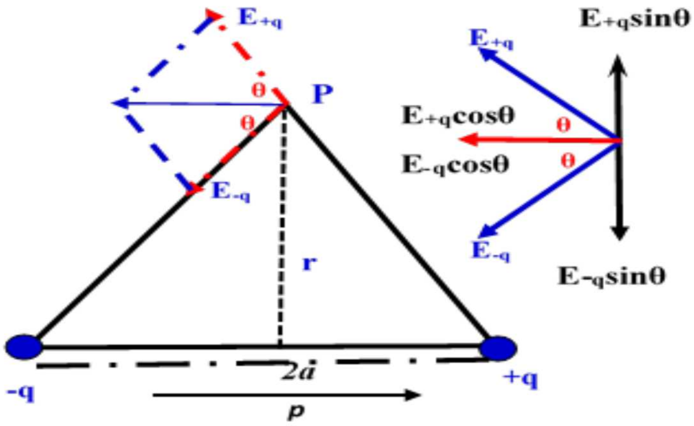
**1. Electric Dipole**



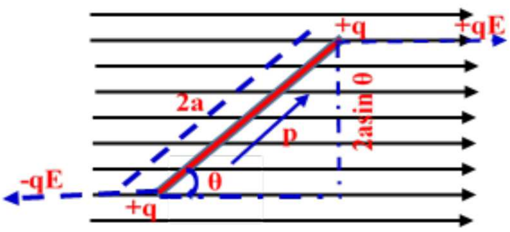

**2. Electric Field Intensity Due to an Electric Dipole:**

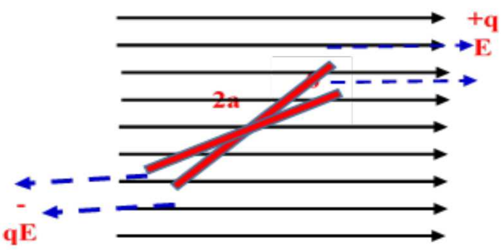
**i) Axial Point** **ii) Equatorial point**



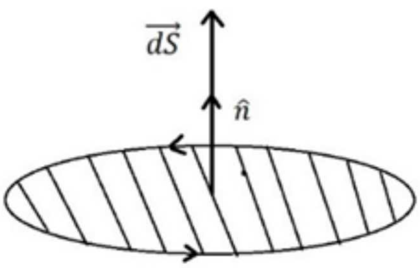



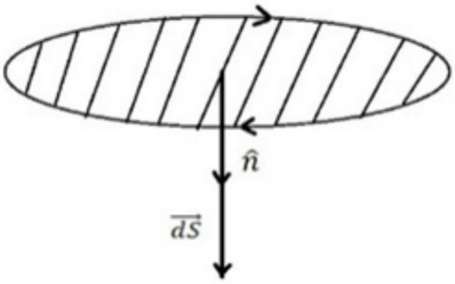

**3. Electric Dipole in a Uniform Electric Field:**




**4. Area Vector** $\overrightarrow{dS}$=dS $\hat{n}$




5.

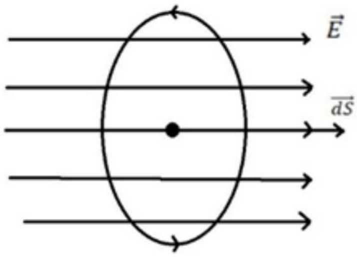
Area traced- Anti-Clockwise

Area traced- Clockwise

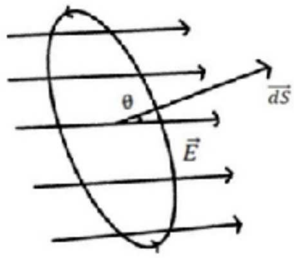
Electric Flux: - Electric

flux over an area represents/measures total number of electric field lines crossing the area when it is held normal to the field direction.

SI Unit of electric flux = SI Unit of E x SI Unit of area = $Nm^2C^{-1}$ or V-m

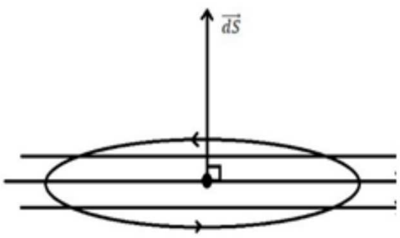



$(d\Phi_E)_{max} = E\ dS$

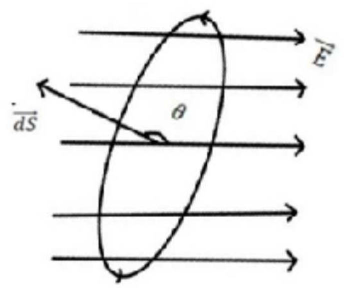



$d\Phi_E = E\ dS \cos\theta$

$d\Phi_E > 0$ for $\theta < 90°$



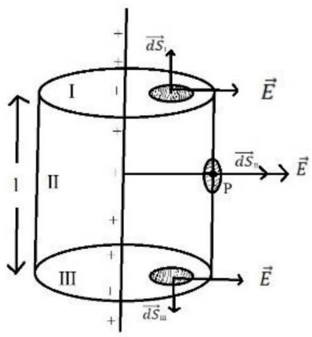

$d\Phi_E = 0$ for $\theta = 90°$




$d\Phi_E < 0$ for $\theta > 90°$

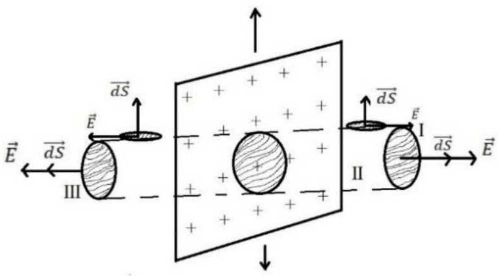
6. Diagrams for Applications of GAUSS THEOREM

a) Electric field due to infinitely long uniformly charged straight wire



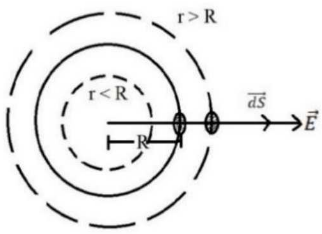

$E = \lambda/2\pi\epsilon_o r = 2\lambda/4\pi\epsilon_o r$

b) Electric field intensity due to uniformly charged thin infinite plane sheet.




$\boxed{E = \sigma/2\epsilon_o}$

c) Electric field intensity due to uniformly charged thin spherical shell

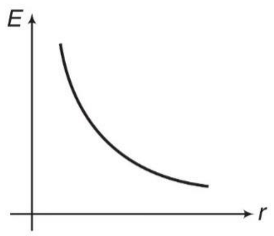


## UNITS

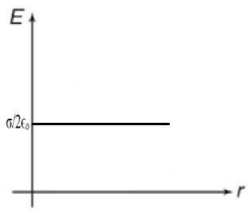
- The SI unit of 'dipole moment' is Coulomb metre (C m)'.
- Electric Field intensity= F/q. Unit of E is $NC^{-1}$ or Vm
- SI unit of torque is newton metre (Nm).
- SI unit of potential energy is joule.
- SI Unit of electric flux = SI Unit of E x SI Unit of area
- = $Nm^2C^{-1}$ or V-m
- Dimensional Formula = $[M^1L^3T^{-3}A^{-1}]$.
- Electric flux is a scalar quantity.

## GRAPHS

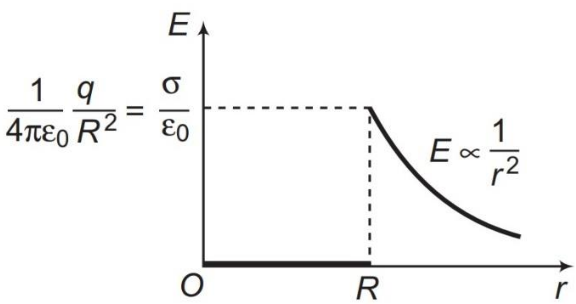
a) Electric field due to infinitely long uniformly charged straight wire



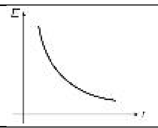
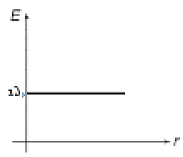
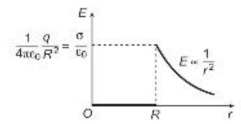

b) Electric field intensity due to uniformly charged thin infinite plane sheet.




c) Electric field intensity due to uniformly charged thin spherical shell



## TABLES:

| **Electric Field Intensity** | $\boldsymbol{\tau} = \vec{p} \times \vec{E}$ | ***U = - p E cos θ*** |
|---|---|---|
| **$E_{Axial}$=2 $E_{Equatorial}$** | **Case i: If θ = 0°, then τ = 0.**<br>**Case ii: If θ = 90°, then τ = pE (maximum value).**<br>**Case iii: If θ = 180°, then τ = 0.** | **Case i: If θ = 0°, then U = - pE (Stable Equilibrium)**<br>**Case ii: If θ = 90°, then U = 0**<br>**Case iii: If θ = 180°, then U = pE (Unstable Equilibrium)** |
| **Electric field at a point due to** | **Formula** | **Graph** |
| infinitely long uniformly charged straight wire | E=λ/2πϵor |  |
| uniformly charged thin infinite plane sheet. | E= σ/2ϵo |  |
| uniformly charged thin spherical shell | E= q/4π ϵo $r^2$ |  |

**Some important points to remember**

1) Total electric flux over closed surface depends only upon total charge enclosed within the surface and is independent of charge distribution within closed surface.

2) Total electric flux through surface is zero if charge enclosed is zero or algebraic sum of charges enclosed is zero.

3) Charges situated outside the closed surface makes no contribution to electric flux over surface boundary.

4) Increasing or decreasing the volume of closed surface S does not affect the flux through closed surface S as long as total charge enclosed remains unchanged/same.

## FORMULAE

**1. Dipole Moment:** $\vec{p} = (q \times 2a)$

**2. Electric Field Intensity (Axial point):** $E_{Axial} = \frac{1}{4\pi\varepsilon_0}\left[\frac{2p}{r^3}\right]$

**3. Electric Field Intensity (equatorial line):** $E_{equat} = -\frac{1}{4\pi\varepsilon_0}\left[\frac{p}{r^3}\right]$

## Electric Dipole in a Uniform Electric Field:

**i)Torque on an Electric Dipole:** $\tau = q\,E\,(2a \sin\theta) = p\,E \sin\theta$, $\tau = \vec{p} \times \vec{E}$

**ii) Work done On an Electric Dipole:** $W = p\,E\,(\cos\theta_1 - \cos\theta_2)$

**iii) Potential Energy:** $U = -\,p\,E \cos\theta$

## GAUSS'S LAW:

- **Total Electric flux through closed surface** $\Phi E_{Total} = q_{Total} / \epsilon_O$
- **Mathematical form of Gauss law** $\oint_s \vec{E}.\,\overrightarrow{dS} = q_{Total} / \epsilon_o$
- **Electric field due to infinitely long uniformly charged straight wire**
  $E = \lambda/2\pi\epsilon_o r = 2\lambda/4\pi\epsilon_o r$
- **Electric field intensity due to uniformly charged thin infinite plane sheet.**
  $E = \sigma/2\epsilon_o$
- **Electric field at a point due to uniformly charged spherical shell.**
- **Electric field intensity at point 'P' outside shell distant r (r >R).**
  $\boxed{\therefore E = q/4\pi\epsilon_o r^2}$
- **Electric field at a point P on the surface of sphere r= R**
  $E = q/4\pi\epsilon_o R^2 = \sigma A/4\pi\epsilon_o R^2$
  $= \sigma (4\pi R^2)/4\pi\epsilon_o R^2$
  $\boxed{\therefore E = \sigma/\epsilon_o}$
- **Electric field at a point P inside shell (r < R)**

  $\boxed{\therefore E = 0}$