CHAPTER 2 : ELECTROSTATIC POTENTIAL AND CAPACITANCE

Learning Objectives :

- INTRODUCTION
- ELECTROSTATIC POTENTIAL
- POTENTIAL DUE TO A POINT CHARGE
- POTENTIAL DUE TO A SYSTEM OF CHARGES
- POTENTIAL DUE TO AN ELECTRIC DIPOLE
- EQUIPOTENTIAL SURFACES
- RELATION BETWEEN FIELD AND POTENTIAL
- POTENTIAL ENERGY OF A SYSTEM OF CHARGES
- POTENTIAL ENERGY IN AN EXTERNAL FIELD
- POTENTIAL ENERGY OF A DIPOLE IN AN EXTERNAL FIELD
- ELECTROSTATICS OF CONDUCTORS
- DIELECTRICS AND POLARISATION
- CAPACITORS AND CAPACITANCE
- COMBINATION OF CAPACITORS
- ENERGY STORED IN A CAPACITOR

POTENTIAL ENERGY OF A SYSTEM OF CHARGES

- The electric potential energy of a system of fixed point charges is equal to the work that must be done by an external agent to assemble the system, bring the charges in from an infinite distance.

- Work done in moving charge q_1 first from ∞ to A:

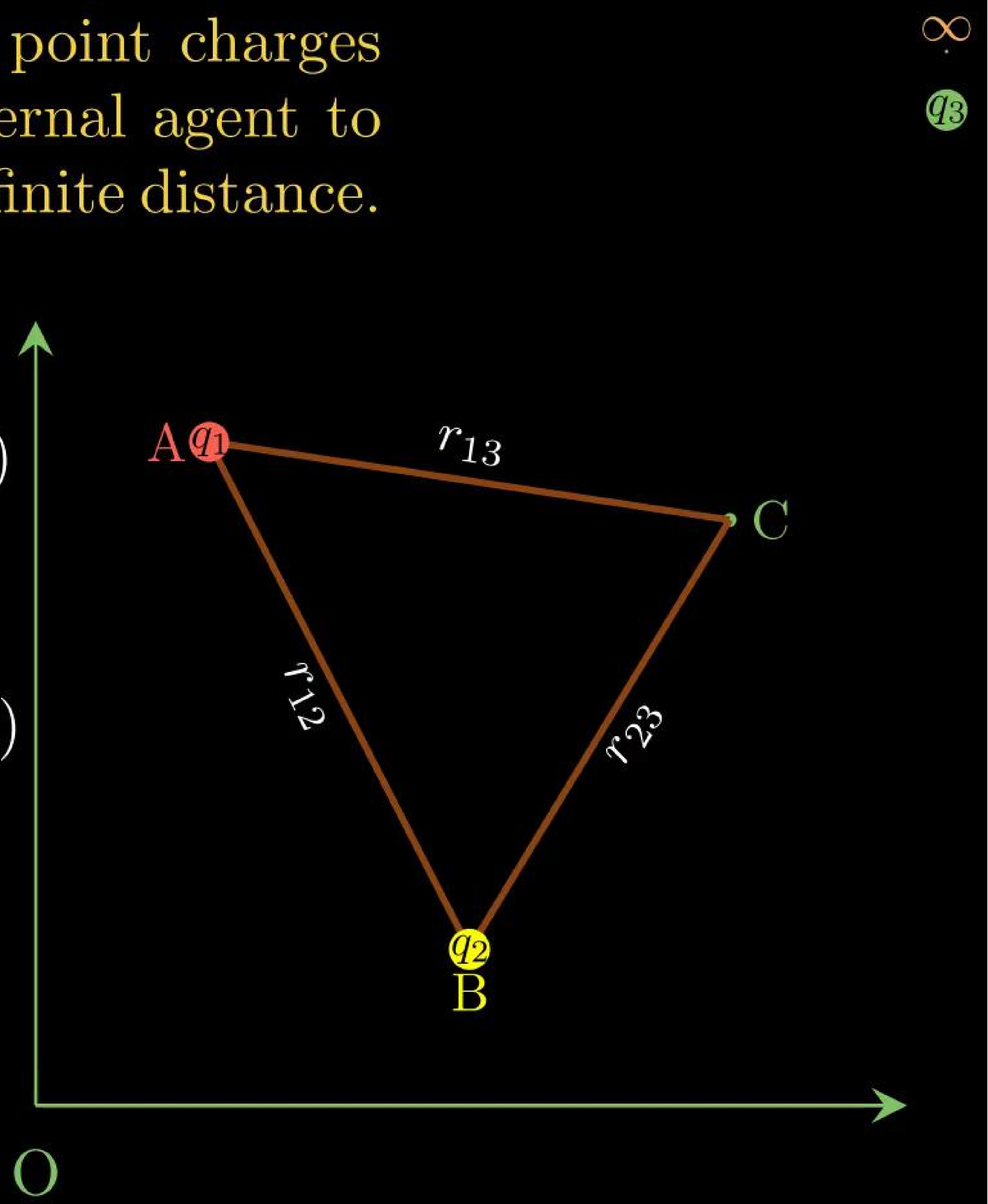
$$W_{q_1} = 0 \ ...(1) \ (\because \text{ no electrostatic force acting on it})$$

- Work done in moving charge q_2 from ∞ to B:

$$W_{q_2} = q_2 \times V_{q_1 B} \ (V_{q_1 B} \rightarrow \text{Potential due to } q_2 \text{ at B.})$$

$$W_{q_2} = q_2 \times \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_{12}}$$

$$W_{q_2} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}} \ ...(2)$$



POTENTIAL ENERGY OF A SYSTEM OF CHARGES

- Work done in moving charge q_3 from ∞ to C:

$W_{q_3} = q_3 \times V_{q_1 q_2 C}$ ($V_{q_1 q_2 C} \rightarrow$ Potential due to q_1 and q_2 at C.)

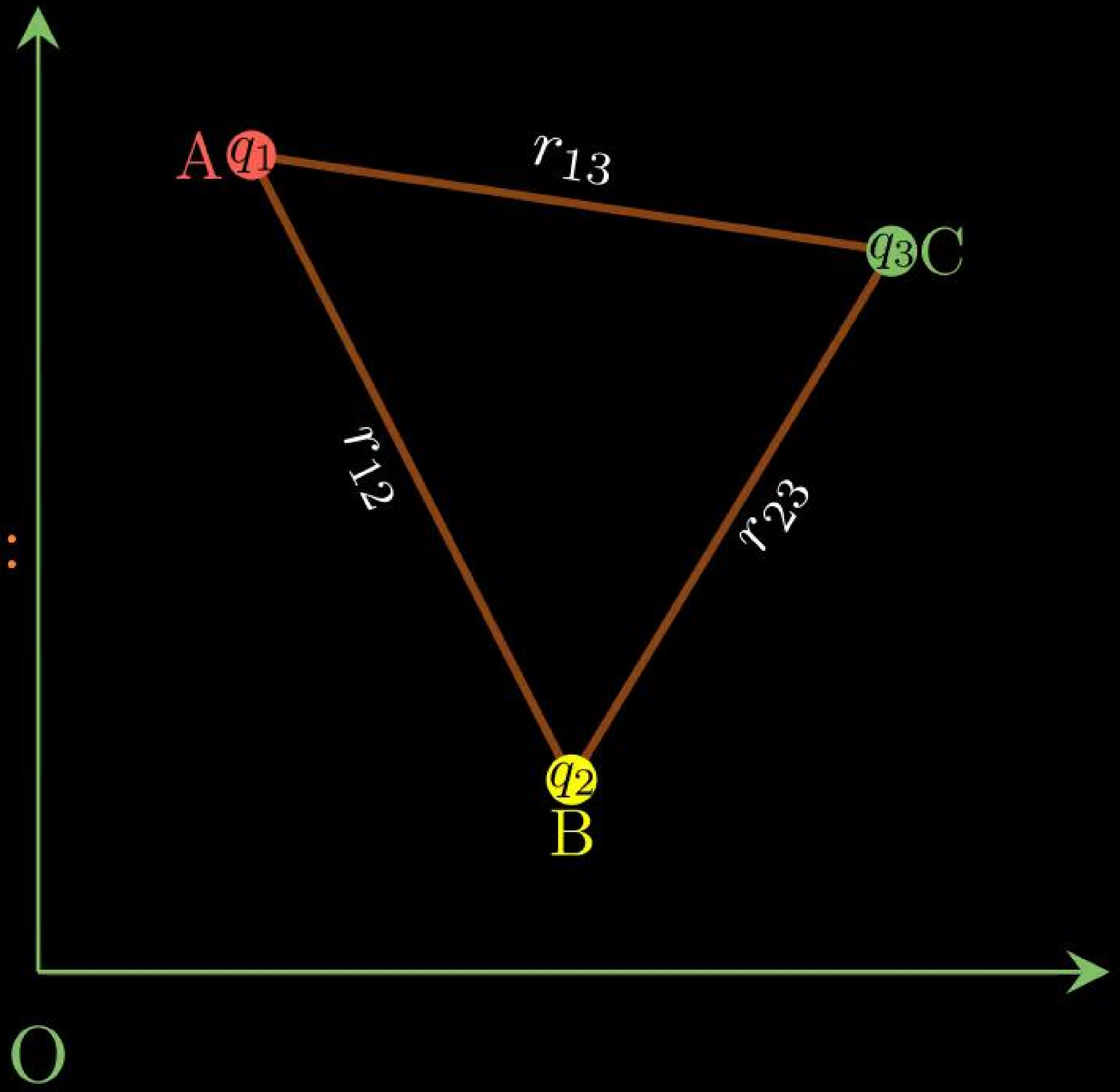
$$W_{q_3} = q_3 \times \left[\frac{1}{4\pi\epsilon_0}\frac{q_1}{r_{13}} + \frac{1}{4\pi\epsilon_0}\frac{q_3}{r_{23}}\right]$$

$$W_{q_3} = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right] \quad ...(3)$$

- Total work done in assembling the system of charges:

$$W = W_{q_1} + W_{q_2} + W_{q_3}$$

$$W = 0 + \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$

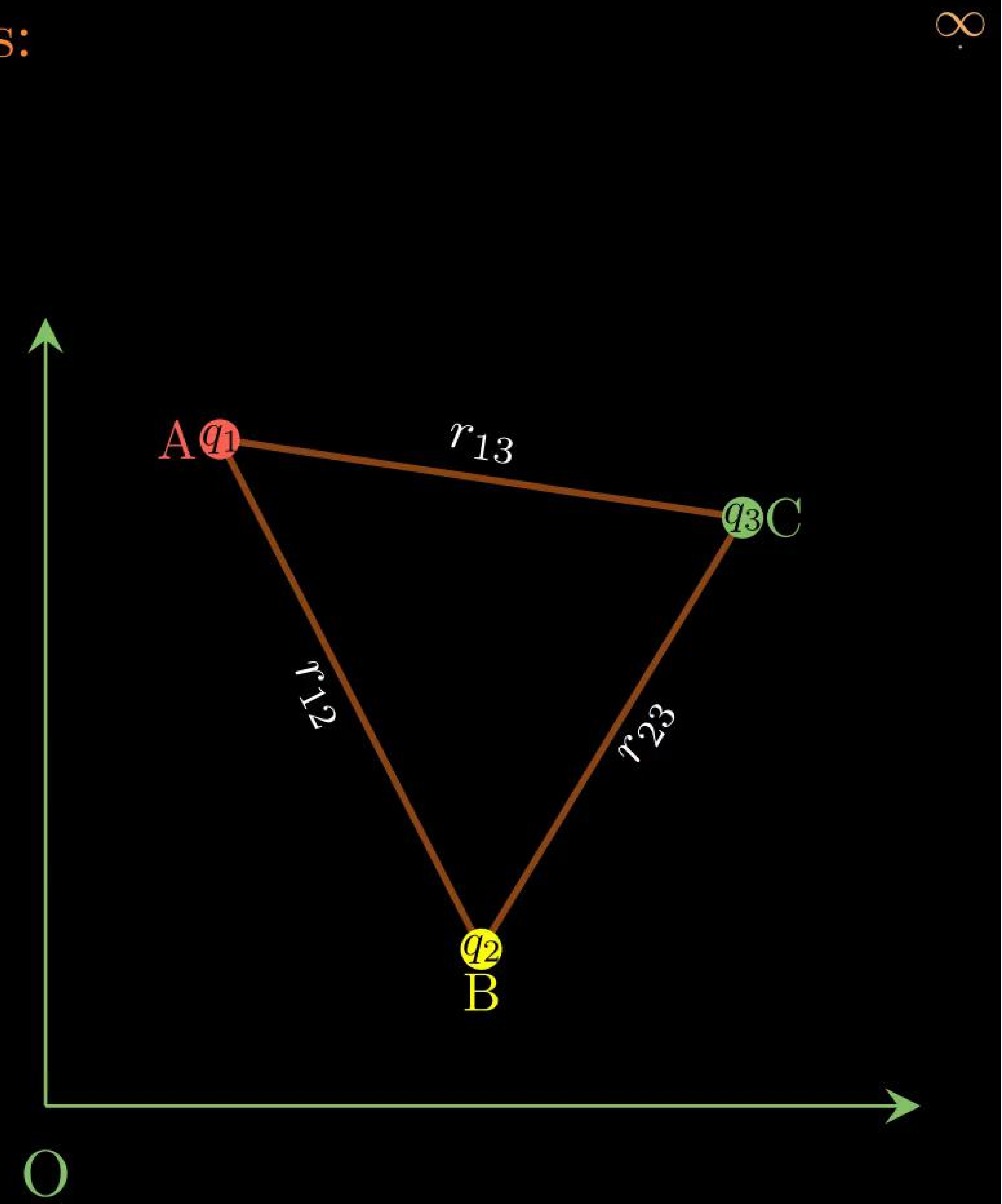


POTENTIAL ENERGY OF A SYSTEM OF CHARGES

- Total work done in assembling the system of charges:

$$W = W_{q_1} + W_{q_2} + W_{q_3}$$

$$W = 0 + \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]$$



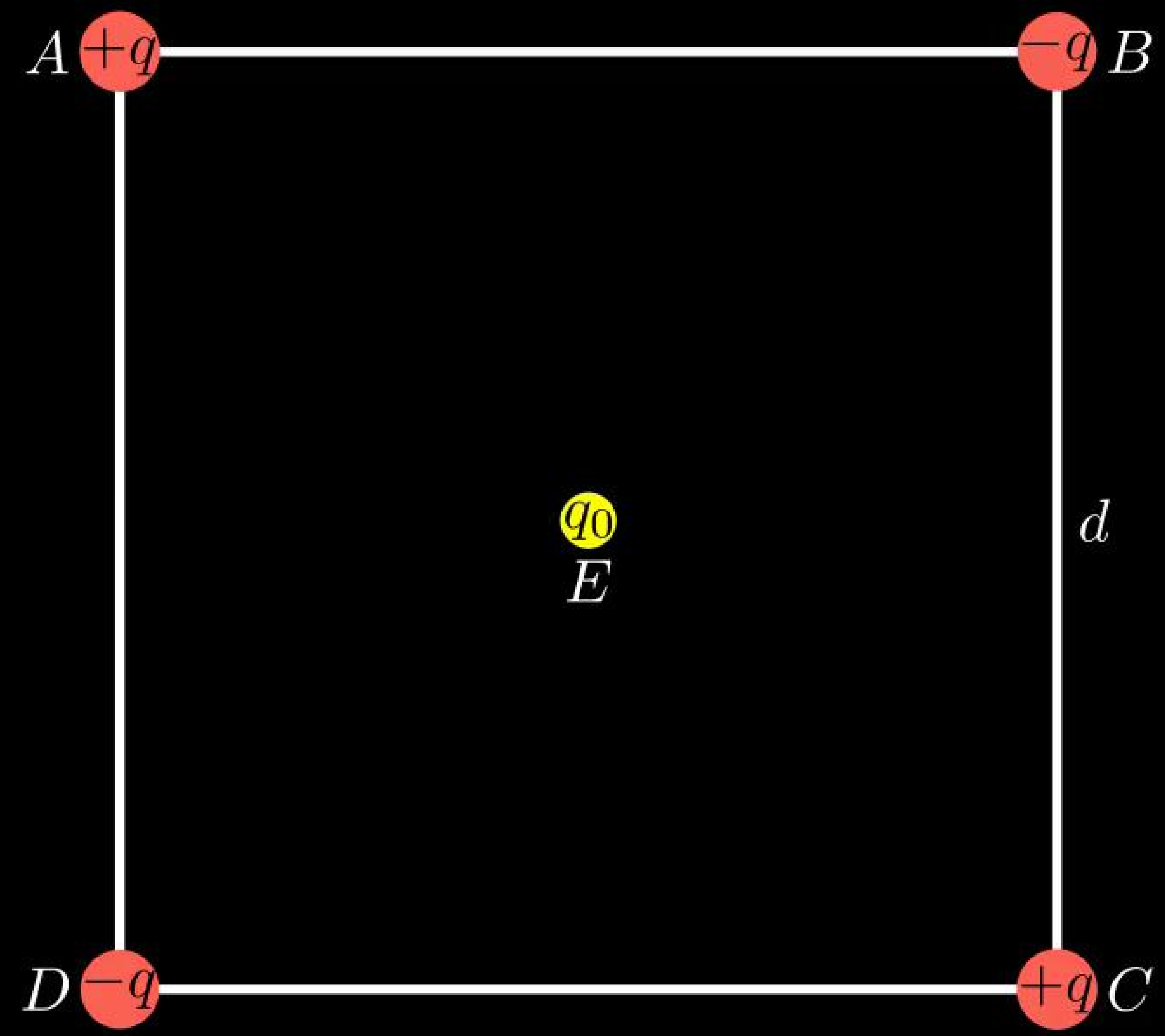
  - This work don by external agent gets stored in the form of potential energy (U) of the system of charges

$$\boxed{U = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right]}$$

  - The potential energy (U) of a system of charges is independent of the manner in which the configuration of charges is assembled.

Example 12 :Four charges are arranged at the corners of a square ABCD of side d, as shown in Fig.
(a) Find the work required to put together this arrangement.
(b) A charge q_0 is brought to the centre E of the square, the four charges being held fixed at its corners. How much extra work is needed to do this? [NCERT]

Solution:



Example 13 :(a) Determine the electrostatic potential energy of a system consisting of two charges 7 μC and $-2\ \mu C$ (and with no external field) placed at (-9 cm, 0, 0) and (9 cm, 0, 0) respectively.

(b) How much work is required to separate the two charges infinitely away from each other?

(c) Suppose that the same system of charges is now placed in an external electric field $E = A\left(\frac{1}{r^2}\right)$; $A = 9 \times 10^5\ NC^{-1}m^2$. What would the electrostatic energy of the configuration be? [NCERT]

Solution:

Example 14 :Two protons are separated by a distance R. What will be the speed of each proton when they reach infinity under their mutual repulsion?

Solution:

Example 14 :Two particles have equal masses of 5 g each and opposite charges of 4×10^{-5} C and -4×10^{-5} C. They are released from rest with a separation of 1 m between them. Find the speed of particles when the separation is reduced to 50 cm.

Solution:

POTENTIAL ENERGY OF A DIPOLE IN AN EXTERNAL FIELD

- Consider a dipole of dipole moment $p = q \times 2a$, placed in a uniform external field $\vec{E}$.

- Torque experienced by dipole $\tau = pE\sin\theta$

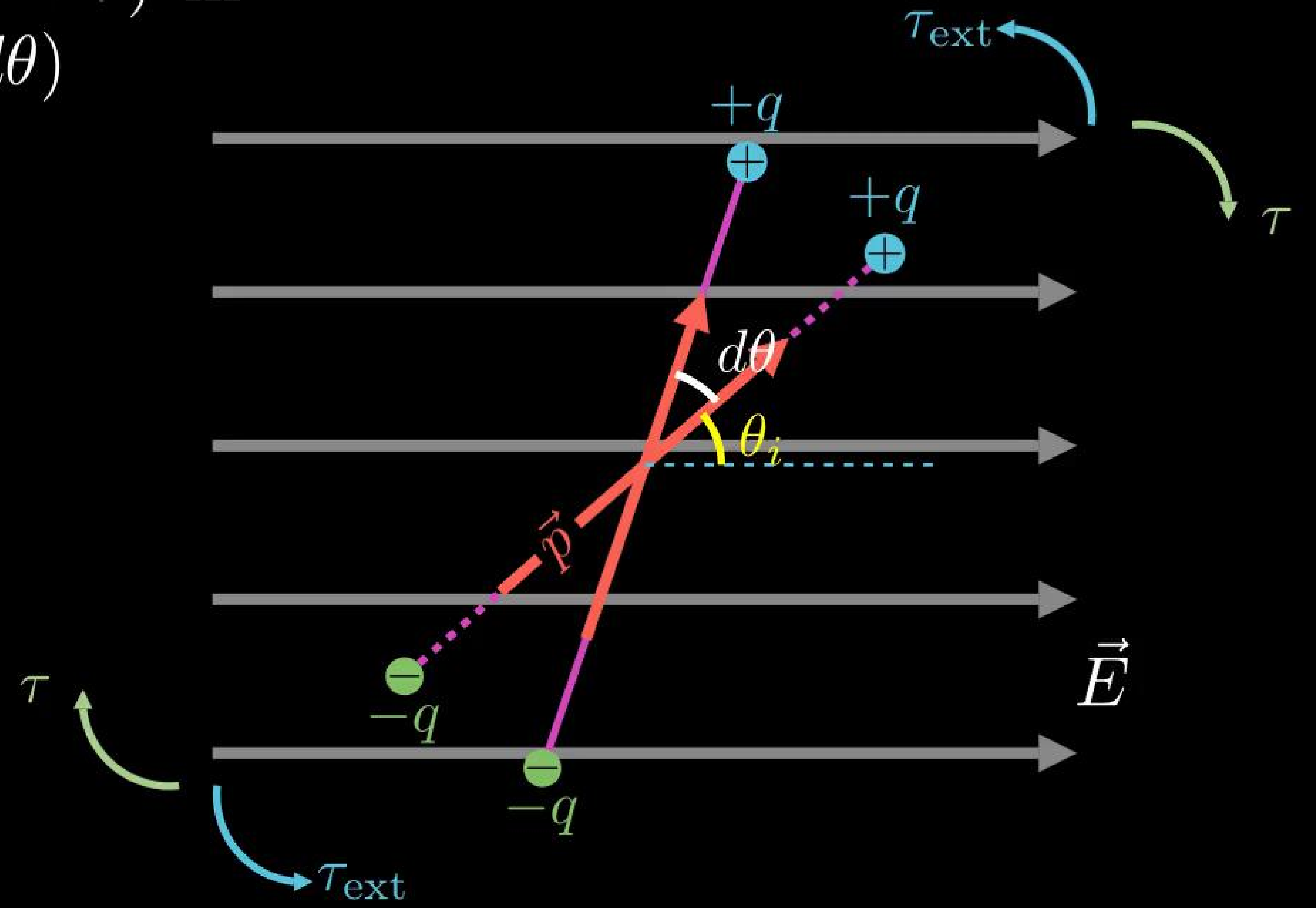
- Work done(dW) by external torque ($\vec{\tau}_{ext} = \vec{\tau}$) in rotating the dipole from angle θ_i to $(\theta_i + d\theta)$

$$dW = \tau_{ext}\ d\theta = pE\sin\theta\ d\theta$$

Now, total work done by external torque in rotating the dipole from θ_i to θ_f

$$W = \int_{\theta_i}^{\theta_f} pE\sin\theta\ d\theta = -pE\left[\cos\theta\right]_{\theta_i}^{\theta_f}$$

$$W = -pE\left[\cos\theta_f - \cos\theta_i\right]$$



POTENTIAL ENERGY OF A DIPOLE IN AN EXTERNAL FIELD

$$W = -pE\left[\cos\theta_f - \cos\theta_i\right]$$

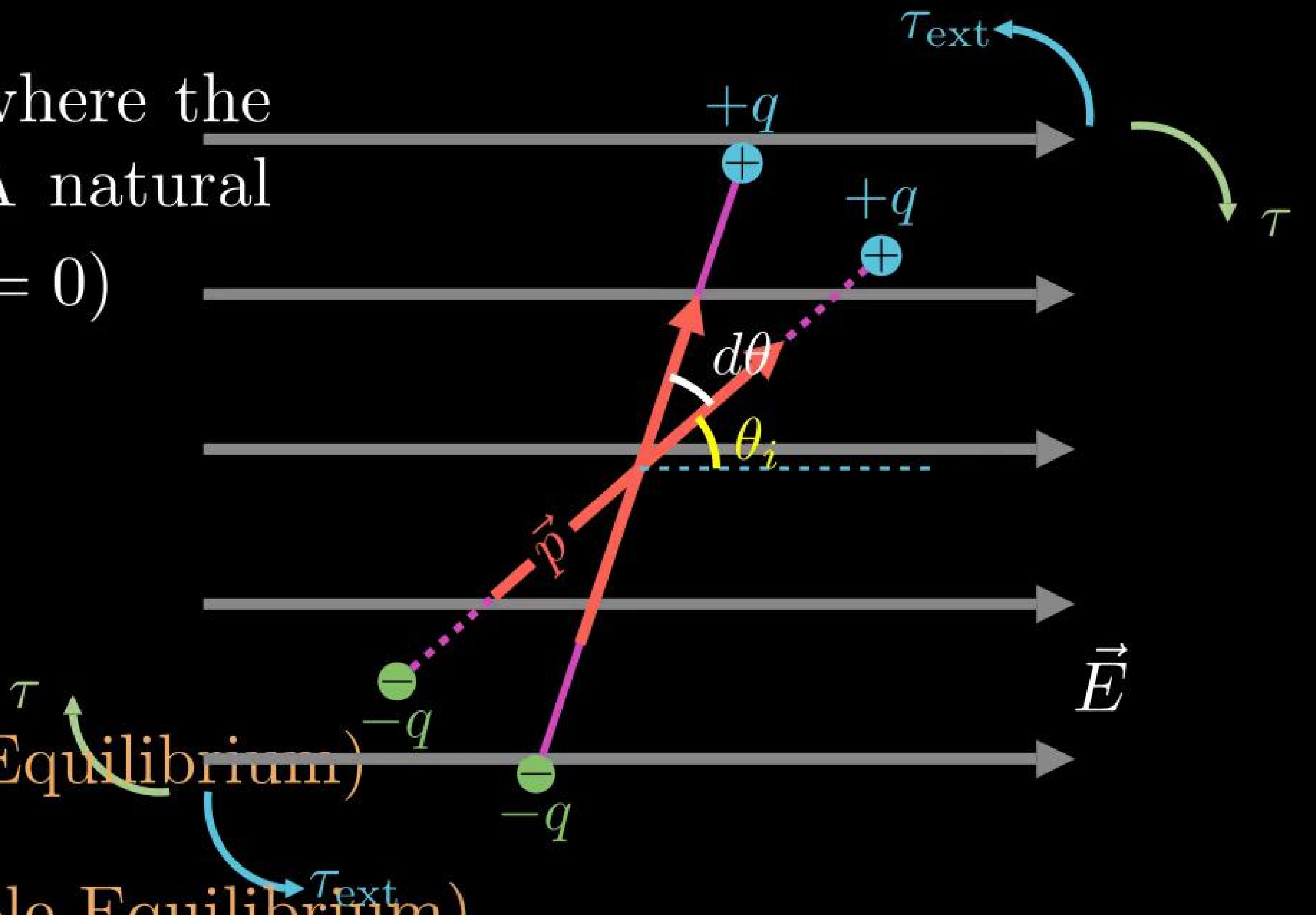
- This work done (W) is equal to the change in potential energy (ΔU) of the dipole

$\boxed{\Delta U = U_{\theta_f} - U_{\theta_i} = -pE\left[\cos\theta_f - \cos\theta_i\right]}$ ⟶ Change in Potential Energy

- There is a freedom in choosing the angle where the potential energy U is taken to be zero. A natural choice is to take $\theta_i = \dfrac{\pi}{2}$ Or 90°. Or ($U_{90^\circ} = 0$)

$$U_\theta - U_{90^\circ} = -pE\left[\cos\theta - \cos(90^\circ)\right]$$

$\boxed{U_\theta = -pEcos\theta}$ ⟶ Potential Energy



- Case 1: if $\theta = 0^\circ \rightarrow\ U_{min} = -PE$(Stable Equilibrium)
- Case 2: if $\theta = 180^\circ \rightarrow\ U_{max} = PE$(Unstable Equilibrium)

Example 16 :An electric dipole of length 2 cm is placed with its axis making an angle of 30° to a uniform electric field 10^5 N/C. If it experiences a torque of $10\sqrt{3}$ Nm, then potential energy of the dipole

(a) -10 J

(b) -20 J

(c) -30 J

(d) -40 J

Solution:

Example 17 : An electric dipole in a uniform electric field E is turned from $\theta = 0^\circ$ position to $\theta = 60^\circ$ position. Find work done by the field.

Solution:

Example 17 : A molecule of a substance has a permanent electric dipole moment of magnitude 10^{-29} C m. A mole of this substance is polarized (at low temperature) by applying a strong electrostatic field of magnitude $10^6\ Vm^{-1}$. The direction of the field is suddenly changed by an angle of 60°. Estimate the heat released by the substance in aligning its dipoles along the new direction of the field. For simplicity, assume 100% polarisation of the sample.

Solution:

ELECTROSTATICS OF CONDUCTORS

Conductors :

- Conductors contains mobile/free charge carriers.

- In metals charge carriers are outer (valance) electrons, They are free to move within the metal but not free to leave the metal.

- In an external field the free electrons drifts in opposite direction of field. But the nuclei and bound electron remain held in their fixed position.

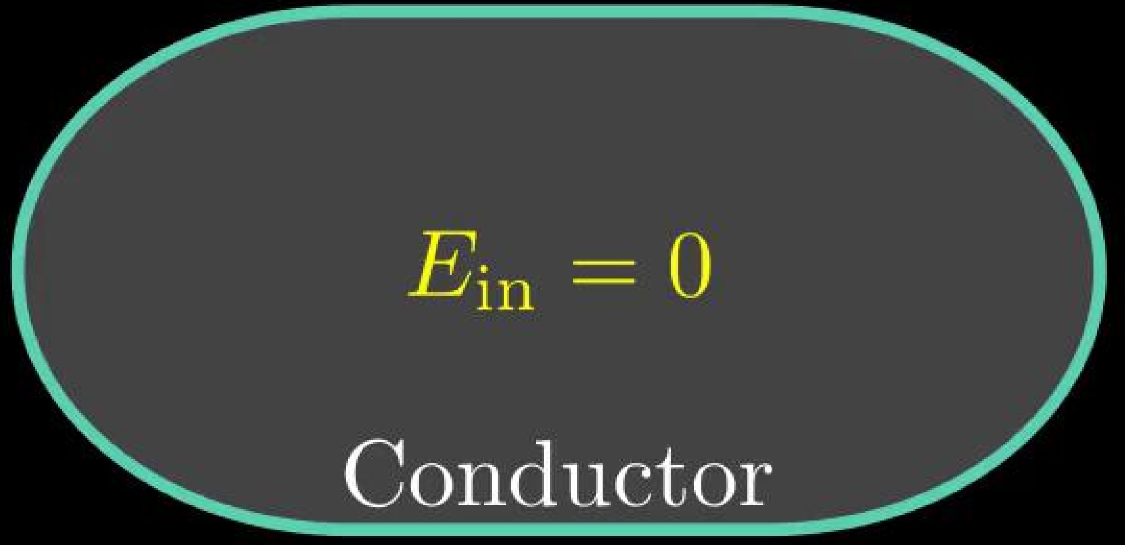
- In electrolytic conductors the charge carriers are both positive and negative ions.

- Electrostatic Condition $\rightarrow$ Assuming charges at rest.

ELECTROSTATICS OF CONDUCTORS

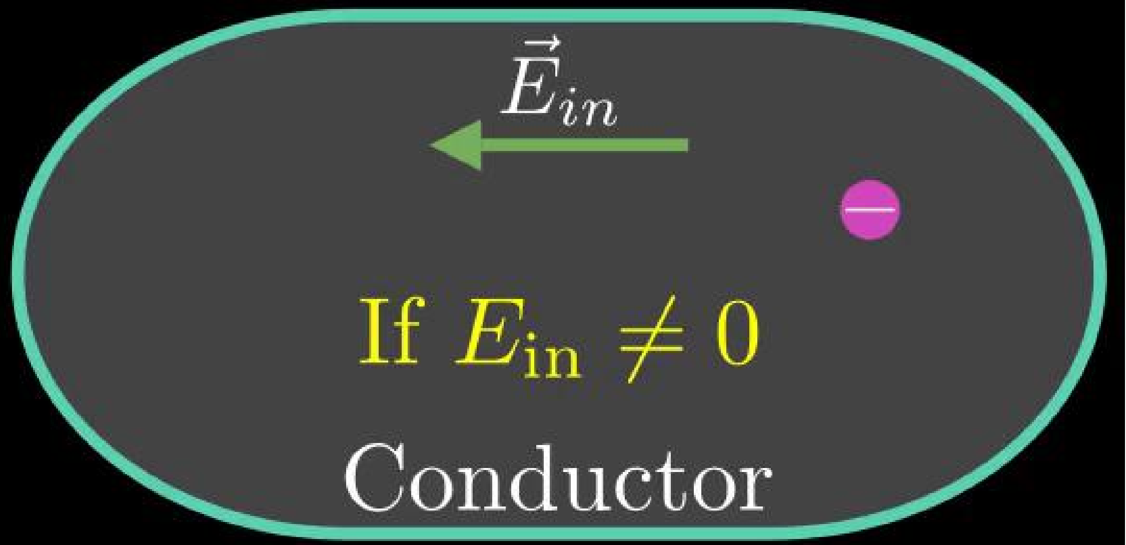
- Under electrostatic condition, the conductors have following properties-

- (1) Inside a conductor, electrostatic field is zero.



Consider a charged/neutral conductor. There may also be an external field.

If there is a net electric field inside a conductor the free electrons will experience force and starts moving which results in the flow of current and violates the static condition.



Therefore, In static situation there is no current inside or at the surface, the free charges have so distribute themselves that electric field is zero everywhere inside the conductor.

ELECTROSTATICS OF CONDUCTORS

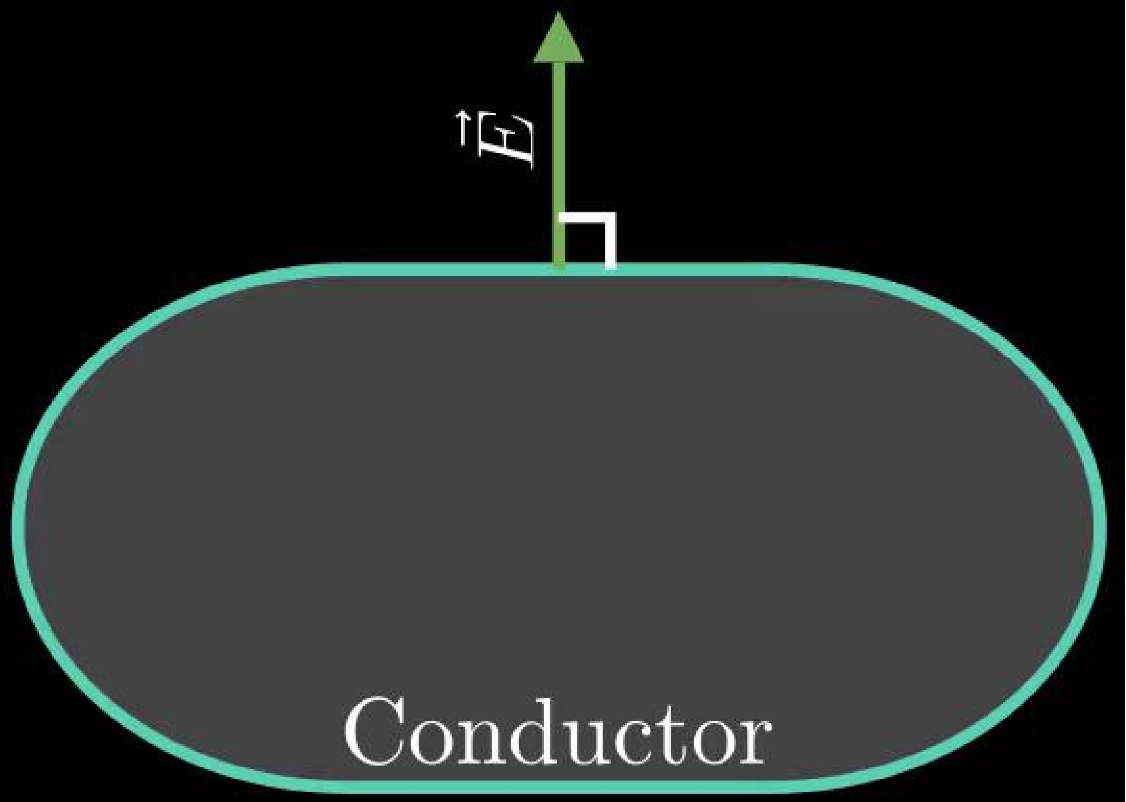
- (2) At the surface of a charged conductor, electrostatic field must be normal to the surface at every point

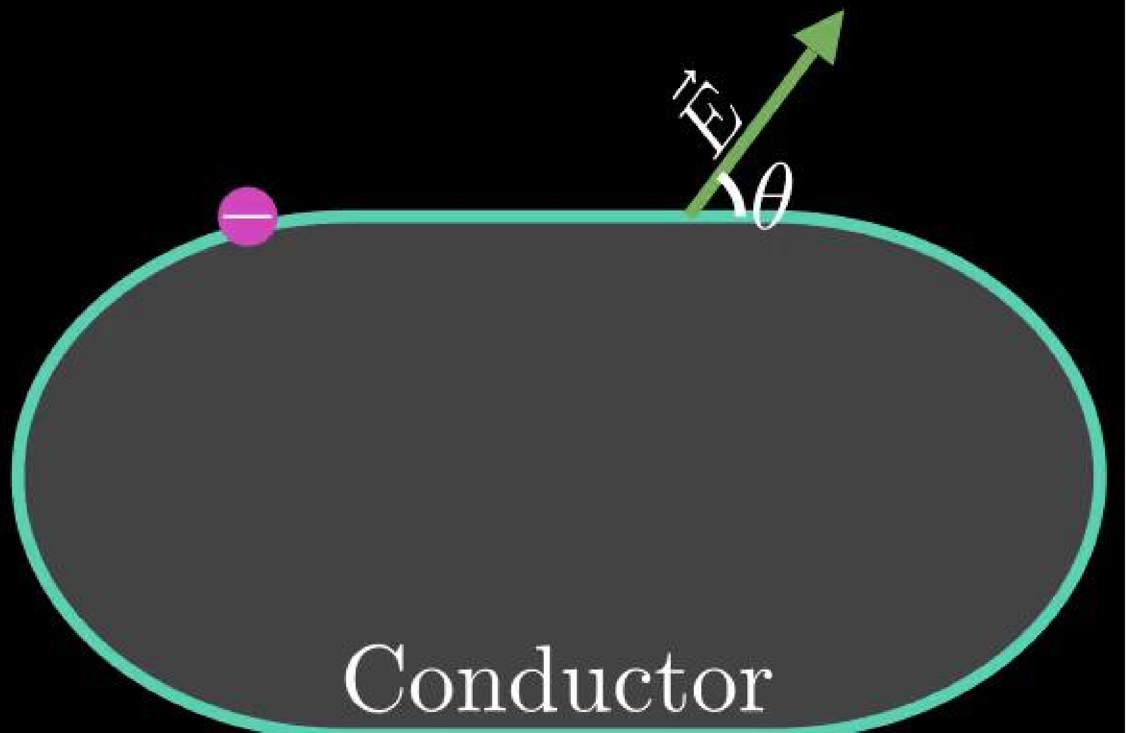
  If E were not normal to the surface, it would have some non-zero component along the surface. Free charges on the surface of the conductor would then experience force and move.

  In the static situation, therefore, E should have no tangential component.

  $E\cos\theta = 0 \quad \text{Or } \cos\theta = 0$

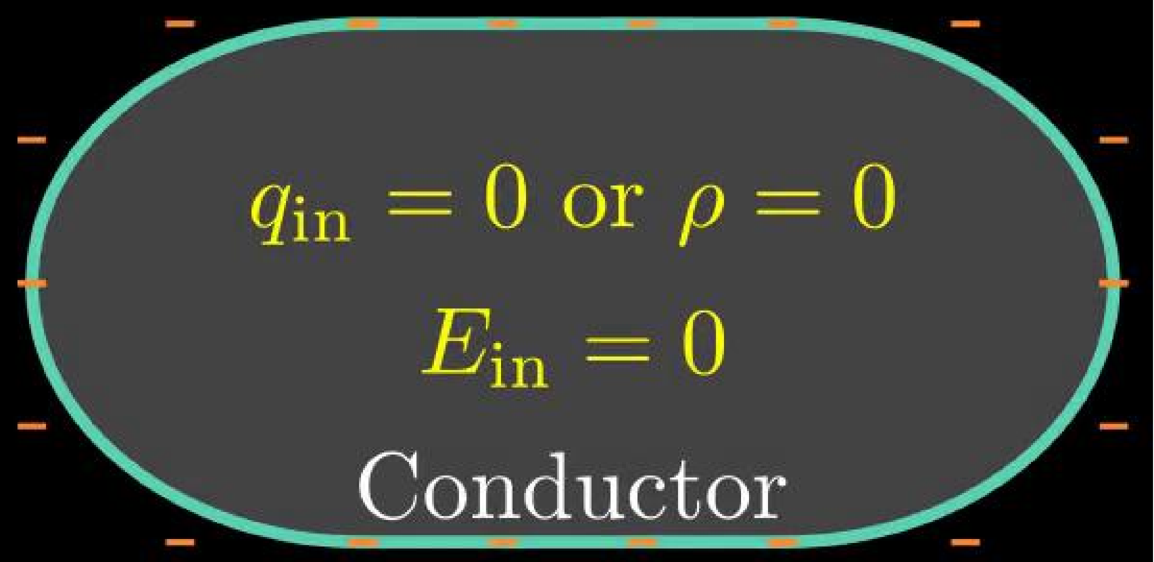
  $\theta = 90^\circ$





ELECTROSTATICS OF CONDUCTORS

- (3) The interior of a conductor can have no excess charge in the static situation, any excess charge must reside at the surface.



ELECTROSTATICS OF CONDUCTORS

- (4) Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface

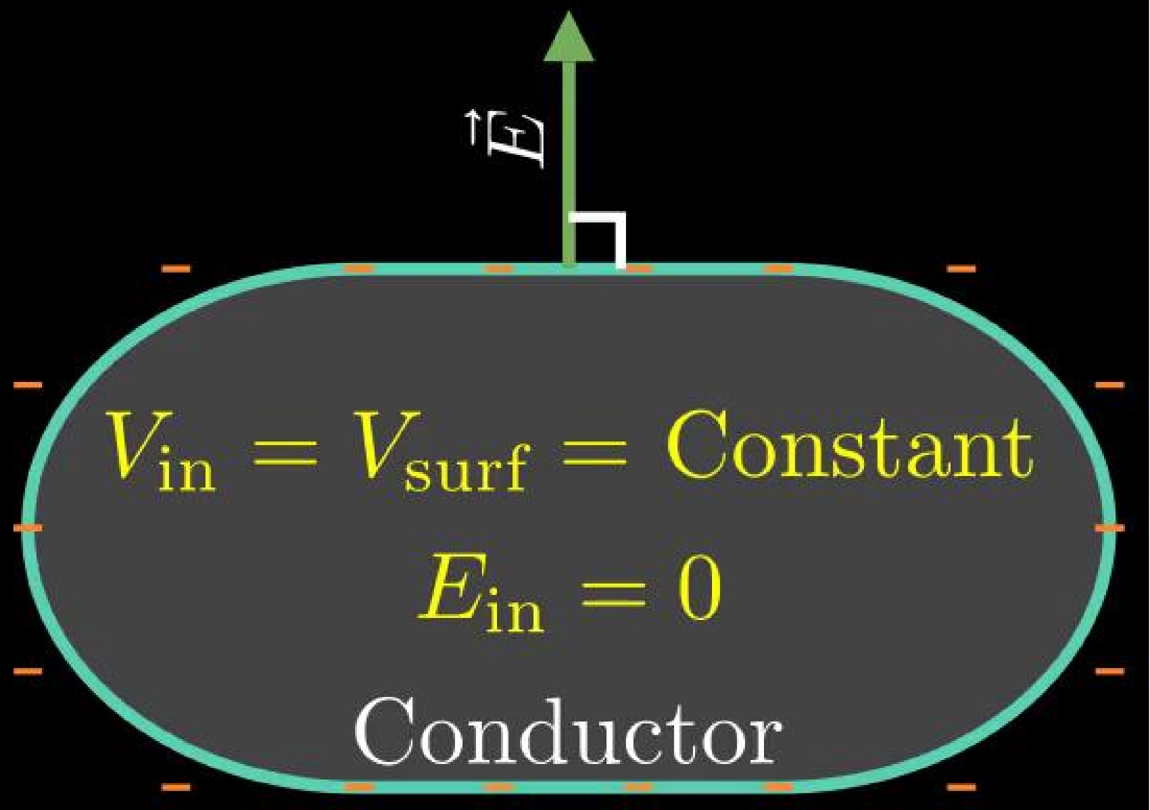
- At each point on the surface of a conductor electric potential is same Or the surface of conductor is an equipotential surface.

Using $dV = -E\ dr$

$$\int_{V_{\text{surf}}}^{V_{in}} dV = \int_{R}^{r} -E\ dr$$

$$V_{\text{in}} - V_{\text{surf}} = 0 \quad (\because E_{\text{in}} = 0)$$

$$V_{\text{in}} = V_{\text{surf}} = \text{Constant}$$



ELECTROSTATICS OF CONDUCTORS

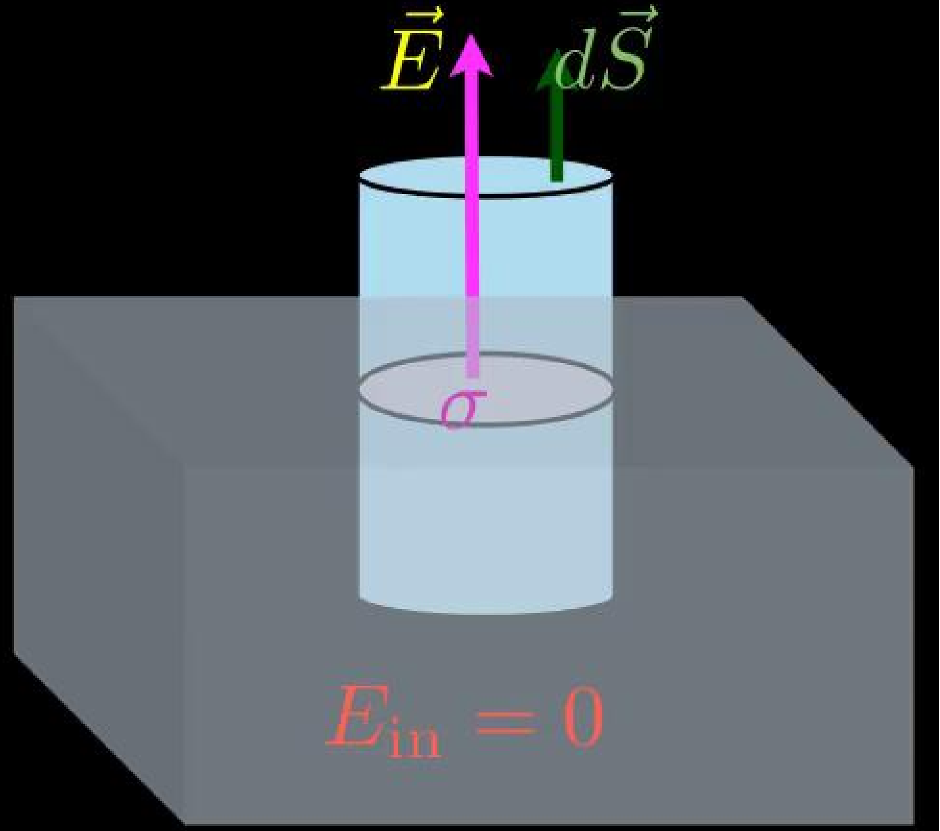
- (5) Electric field at the surface of a charged conductor: $\vec{E} = \dfrac{\sigma}{\epsilon_0}\hat{n}$

Using Gauss's law : $\oint E \, dS = \dfrac{q}{\epsilon_0}$

$$E \oint dS = \frac{\sigma A}{\epsilon_0}$$

$$EA = \frac{\sigma A}{\epsilon_0}$$

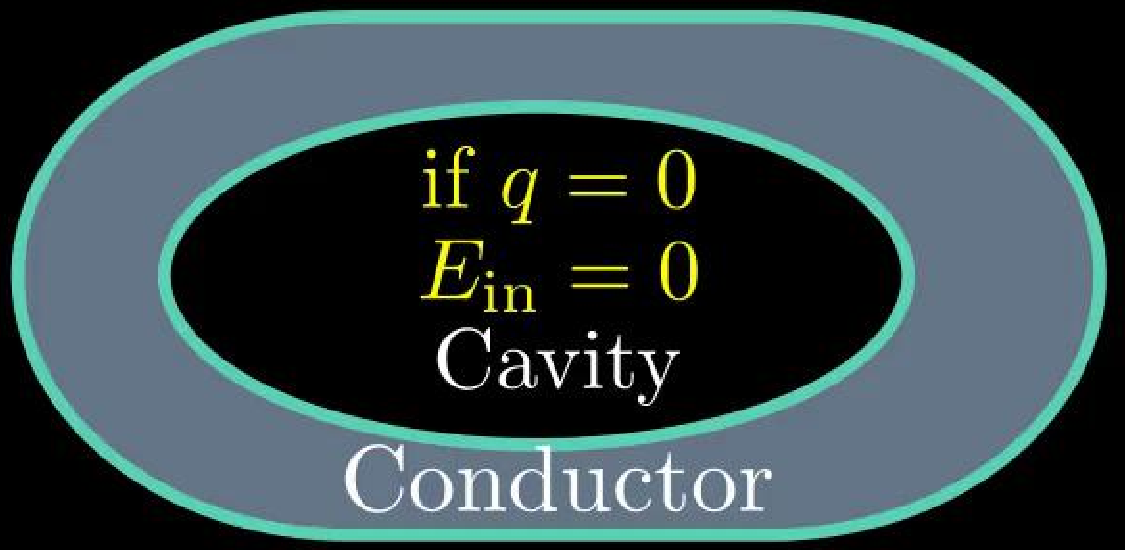
$$E = \frac{\sigma}{\epsilon_0}$$



ELECTROSTATICS OF CONDUCTORS

- (6) Electrostatic shielding

- Whatever be the charge and field configuration outside, any cavity (having no charges) in a conductor remains shielded from outside electric influence: the field inside the cavity is always zero. This is known as electrostatic shielding.



- Shielding effect can be made use of in protecting sensitive instruments from outside electrical influence.

DIELECTRICS AND POLARISATION

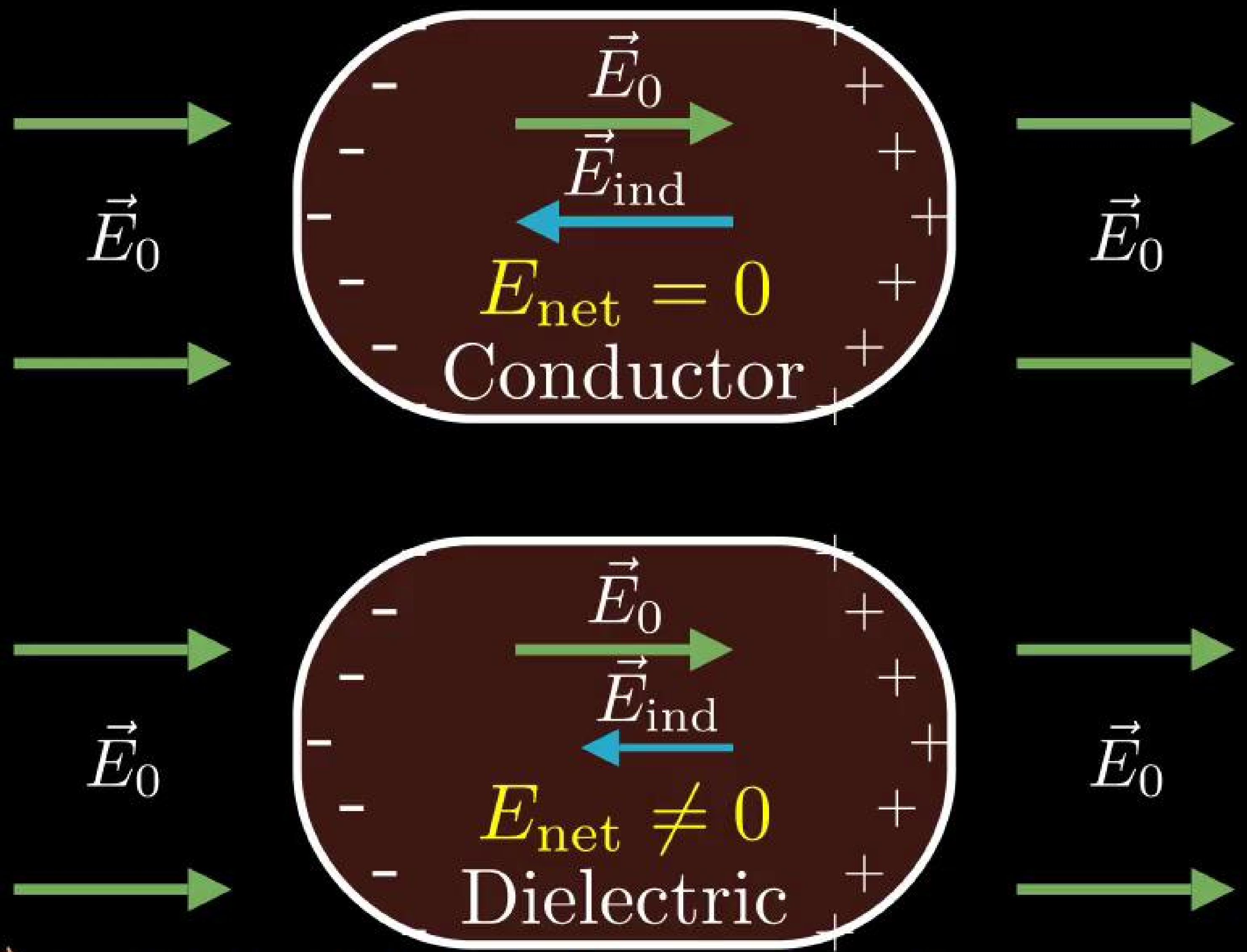
- Dielectrics are non-conducting substances. They have no (or negligible number of) charge carriers.

- Examples: Glass, wax, water, air, wood, rubber, plastic, etc.

DIELECTRICS AND POLARISATION

Behaviour of a conductor and dielectric in the presence of an external field :

- When a conductor is placed in an external field $(\vec{E}_0)$. The free charge carriers moves and charge distribution adjust itself itself in such a way that the electric field due to induced charge $\vec{E}_{\text{ind}}$ cancels the external field $(\vec{E}_0)$ which results in net zero electrostatic field inside.



- In dielectric charges are not free to move because they are tightly bounded to the atom.

- When dielectric is placed in an external field $(\vec{E}_0)$. This field induces dipole moment by stretching or reorienting molecules of the dielectric. Due to the induced dipoles charges builds on the surface which produces an electric field $(\vec{E}_{\text{ind}})$ that opposes the external field but does not exactly cancel the external field. It only reduces it.

DIELECTRICS AND POLARISATION

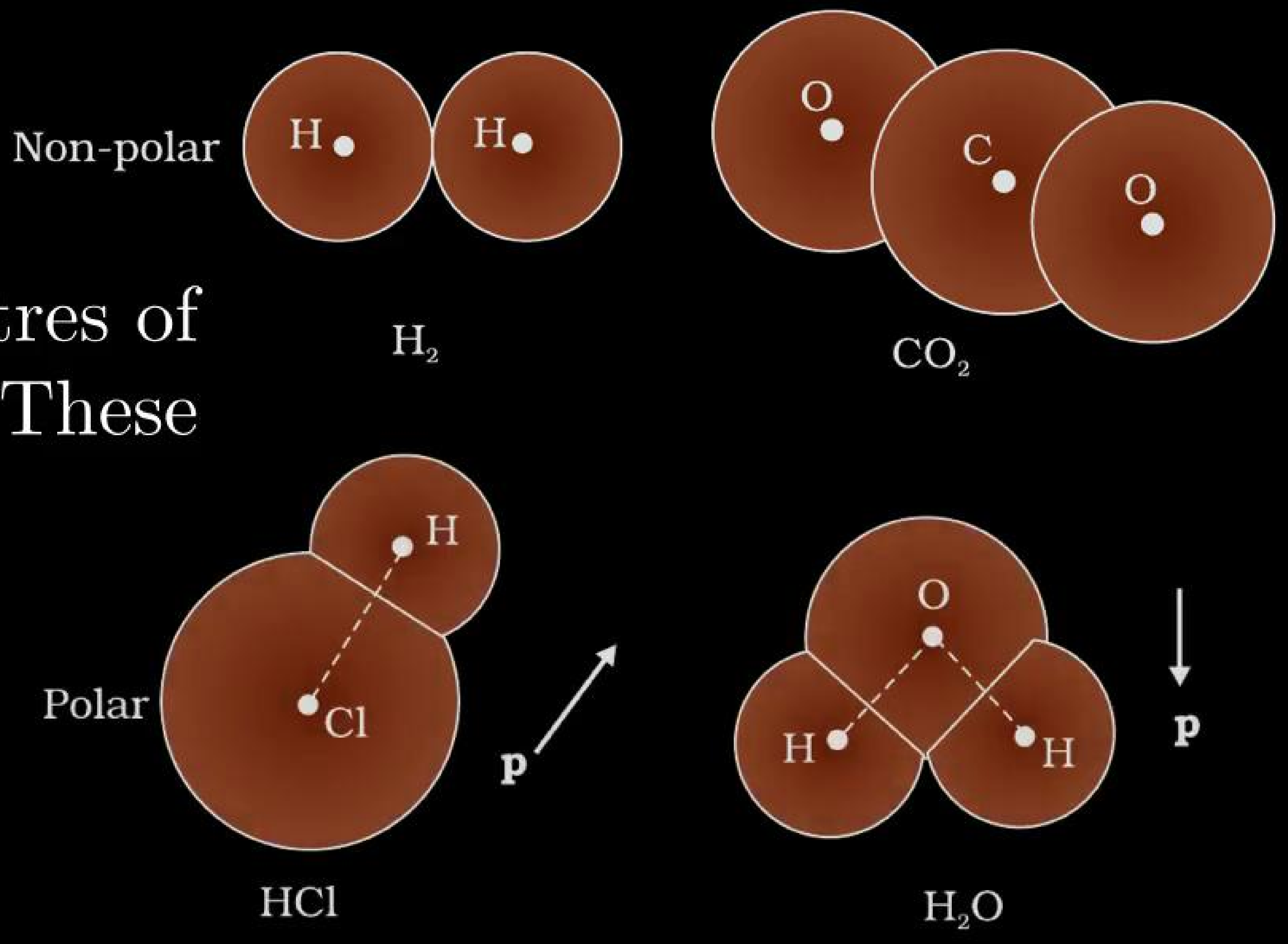
Polar and Non-Polar Dielectrics :

- Non-Polar Dielectric : In a non-polar molecule, the centres of positive and negative charges coincide. These molecules do not have any permanent dipole moment.

- Examples: H_2, O_2, N_2, CO_2, CH_4 etc.

- Polar Dielectric : In a polar molecule, the centres of positive and negative charges are separated. These molecules have permanent dipole moment.

- Examples: H_2O, NH_3, HCl etc.



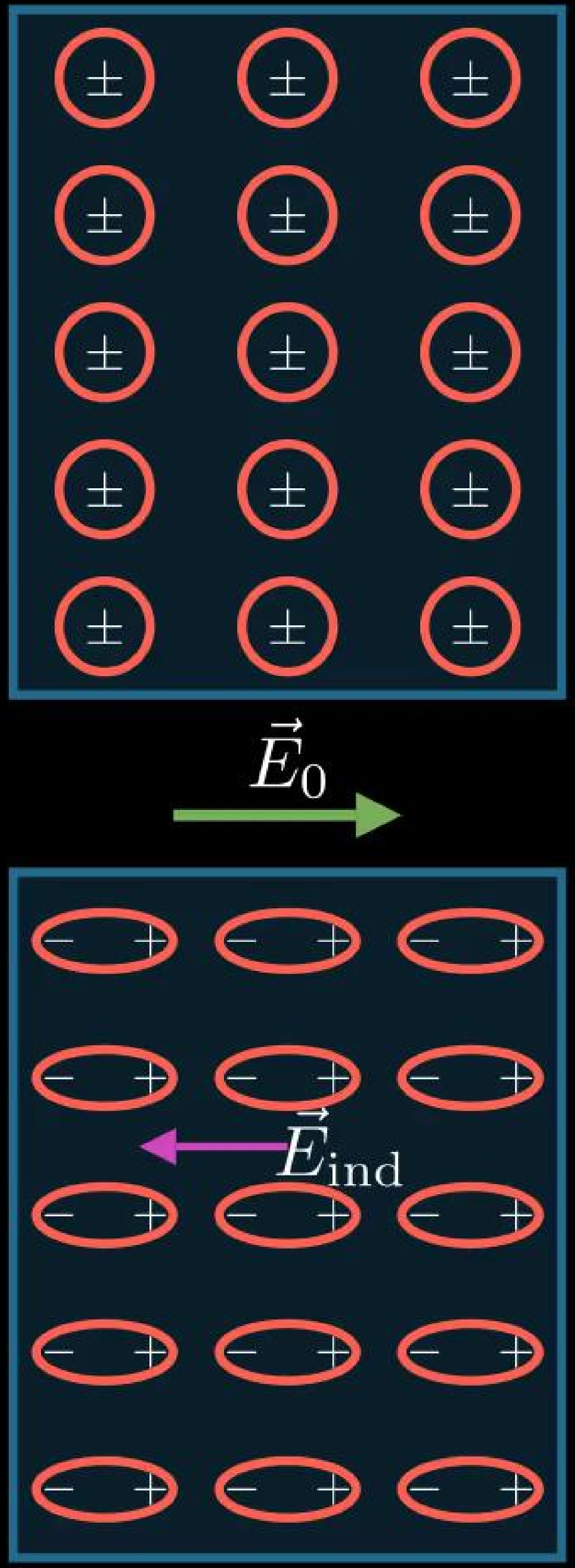
DIELECTRICS AND POLARISATION

Non-Polar Dielectrics in external field :

- In an external electric field, the positive and negative charges of a non- polar molecule are displaced in opposite directions

- The non-polar molecule thus develops an induced dipole moment. The dielectric is said to be polarised by the external field.

- Linear isotropic dielectrics : Dielectrics for which the induced dipole moment is in the direction of the field and is proportional to the field strength



$\vec{E}_{\text{net}} = 0$
$\vec{p}_{\text{net}} = 0$

$\vec{E}_{\text{net}} \neq 0$
$\vec{p}_{\text{net}} \neq 0$

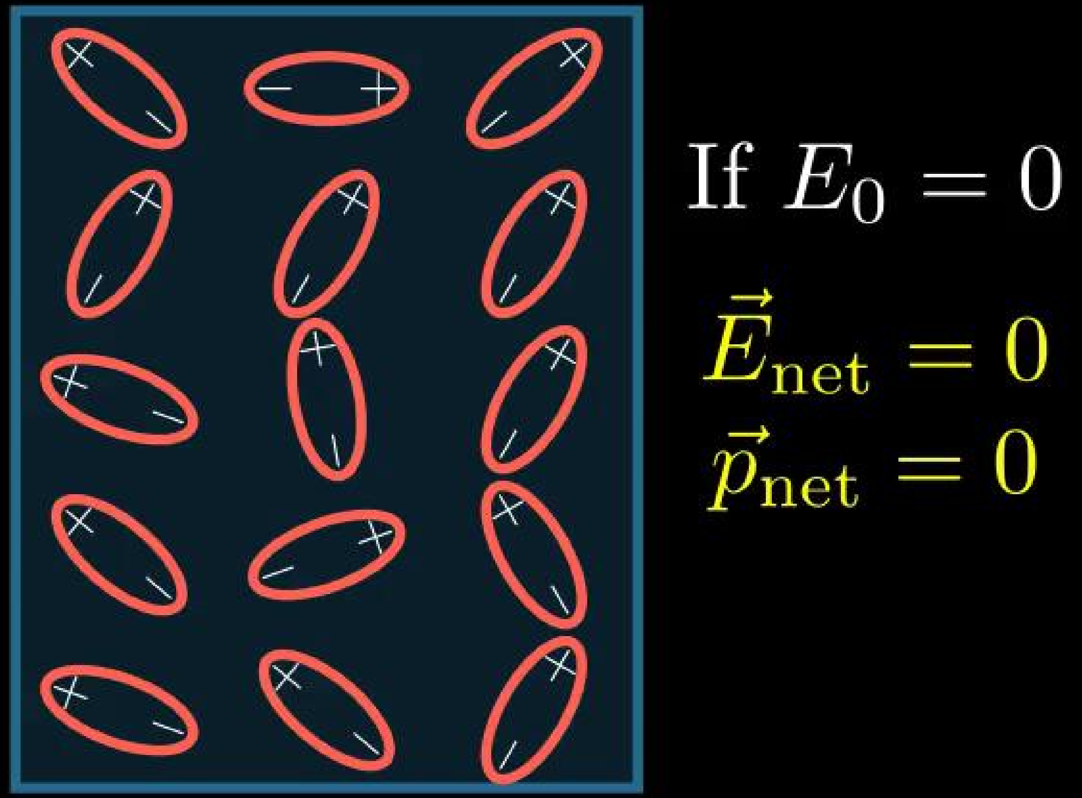
DIELECTRICS AND POLARISATION

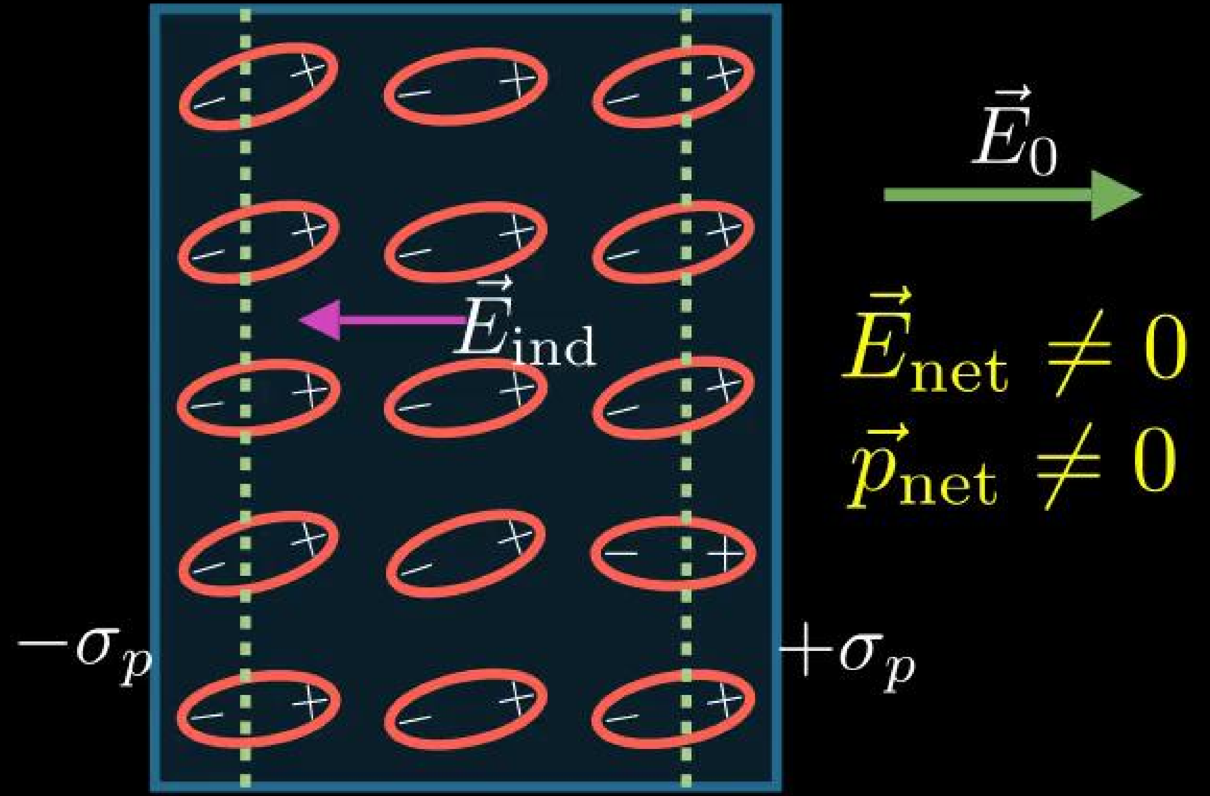
Polar Dielectrics in external field :

- In a dielectric with polar molecules, in the absence of external field, the different permanent dipoles are oriented randomly due to thermal agitation (energy). So, the total dipole moment is zero.

- But, When external field $\vec{E}_0$ is applied, each dipole molecule tend to align with the field which result in a net dipole moment in the direction of electric field.

- The thermal energy tends to disturb the alignment and the external field tends to align the dipole.





DIELECTRICS AND POLARISATION

Polarisation ($\vec{P}$) :

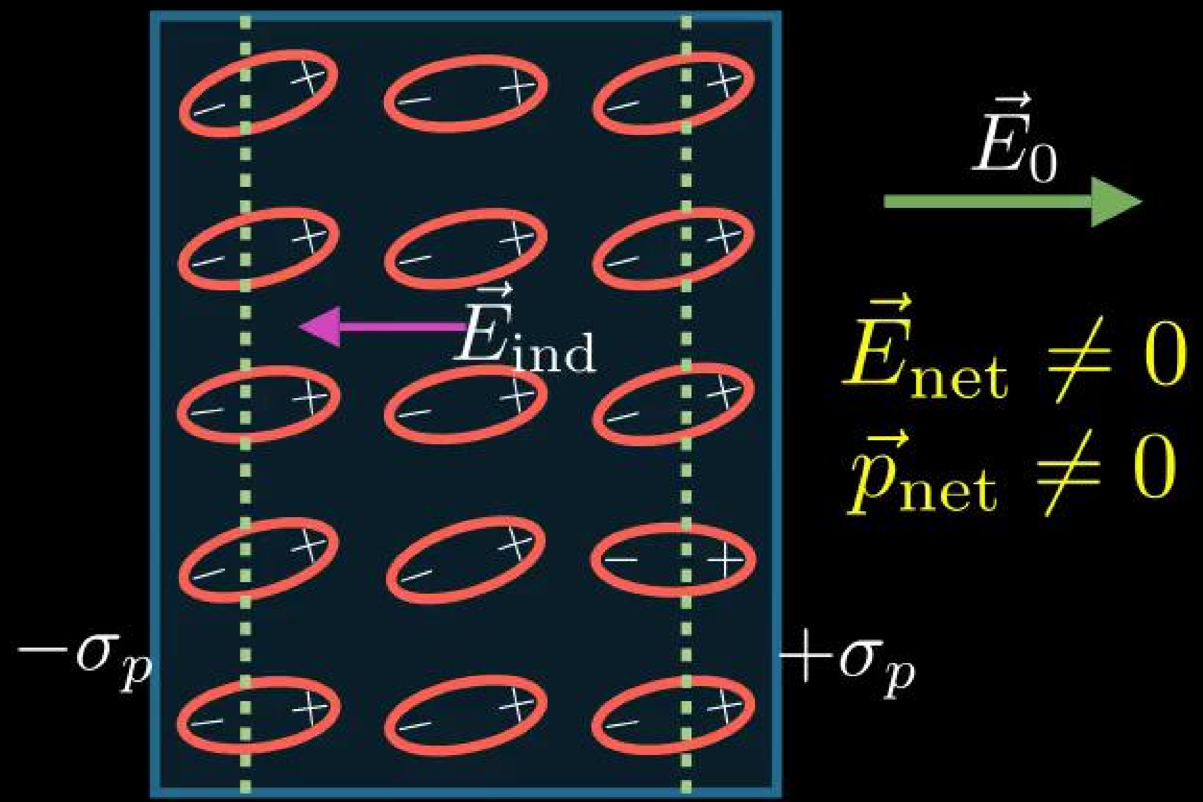
- A dielectric developes a net dipole moment ($\vec{p}_{\text{net}}$) in the presence of external field ($\vec{E}_0$).

- The dipole moment per unit volume is called Polarisation ($\vec{P}$)

$$\vec{P} \propto \vec{E}$$

$$\vec{P} = \epsilon_0 \chi_e \vec{E}$$

- where χ_e is a constant characteristic of the dielectric and is known as the **electric susceptibility** of the dielectric medium



CAPACITORS AND CAPACITANCE

CAPACITORS AND CAPACITANCE

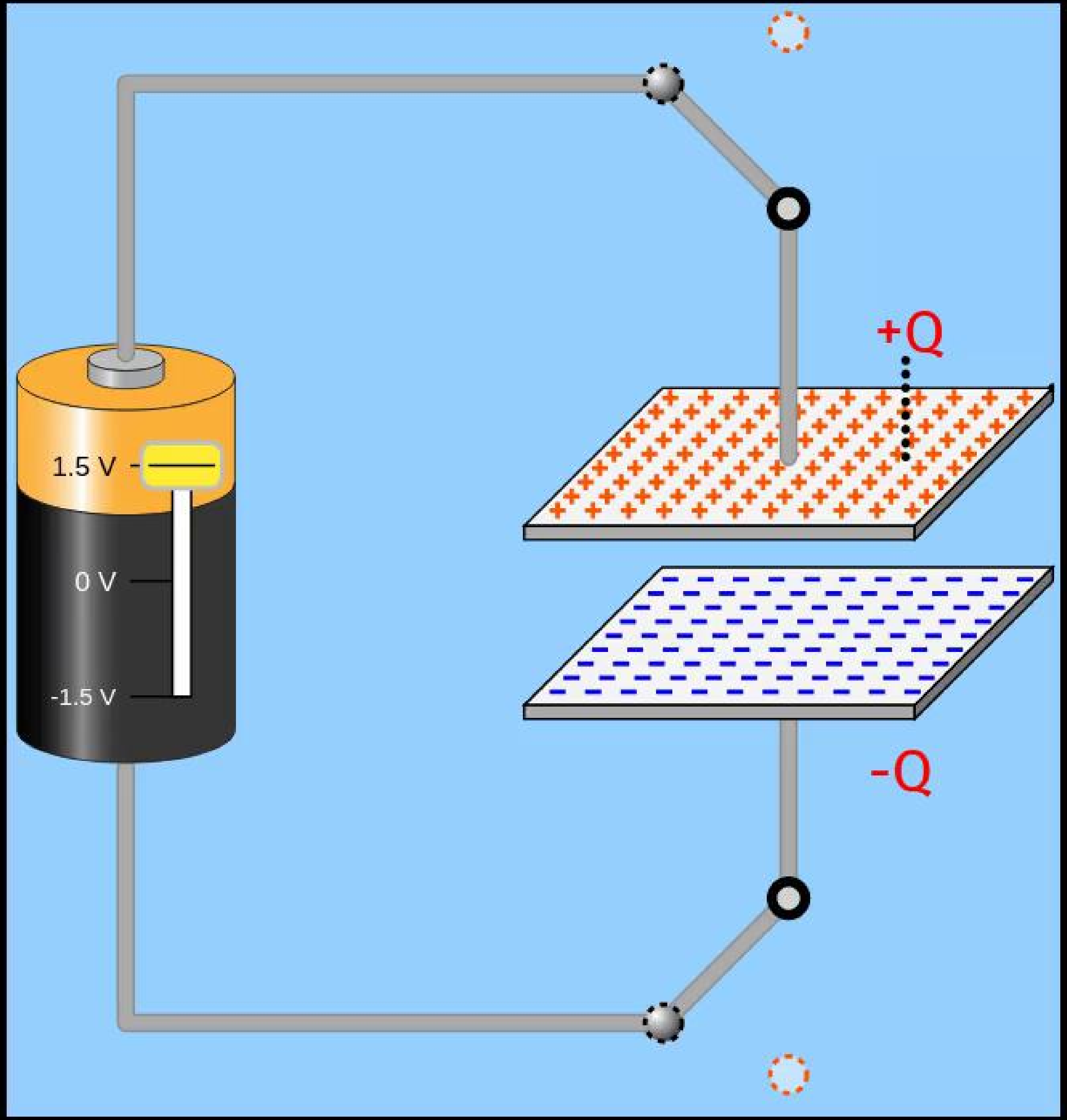
Capacitor :

- A capacitor is a device used to store electrical charge and electrical energy.

- A capacitor is consists of at least two electrical conductors separated by a distance. The space between capacitors may simply be a vacuum or an insulating material (dielectric)

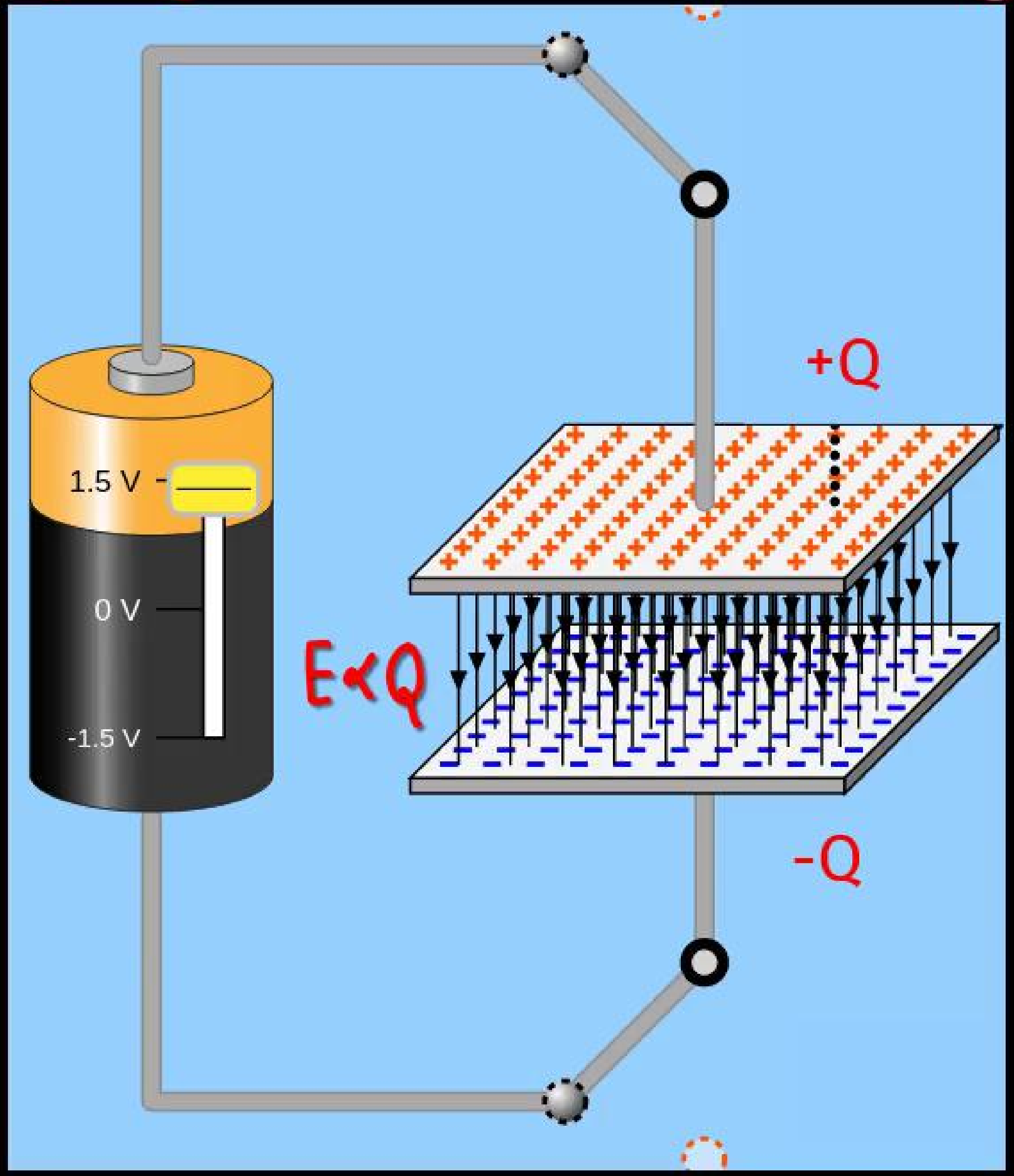
- When battery terminals are connected to an un-charged capacitor, the battery potential moves a small amount of charge Q from positive plate to the negative plate.

- The net charge on the capacitor is zero, but one plate gets $+Q$ charge and other gets $-Q$ charge.



CAPACITORS AND CAPACITANCE

- The electric field in the region between the conductors is proportional to the charge Q.



- Also, the potential difference (V) between the plates is directly proportional to the charge (Q) of the capacitor.

$$Q \propto V$$

$$Q = CV \text{ OR } C = \frac{Q}{V}$$

- The constant C is called capacitance of the capacitor.

- The capacitance C is independent of Q or V and, depends only on the geometrical configuration (shape, size, separation) of the conductors and nature of the insulator (dielectric) between the two conductors.

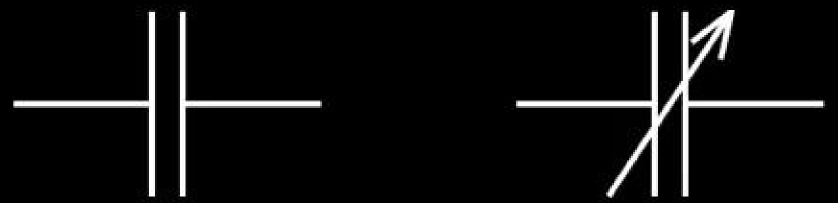
CAPACITORS AND CAPACITANCE

- S.I. unit of capacitance : farad (F) $\quad 1F = \dfrac{1\ C}{1\ V}$

- In practice, a farad is a very big unit; the most common units are

$1\ \mu F = 10^{-6}\ F, \quad 1\ nF = 10^{-9}\ F, \quad 1\ pF = 10^{-12}\ F\ etc.$

- Symbols of Capacitor

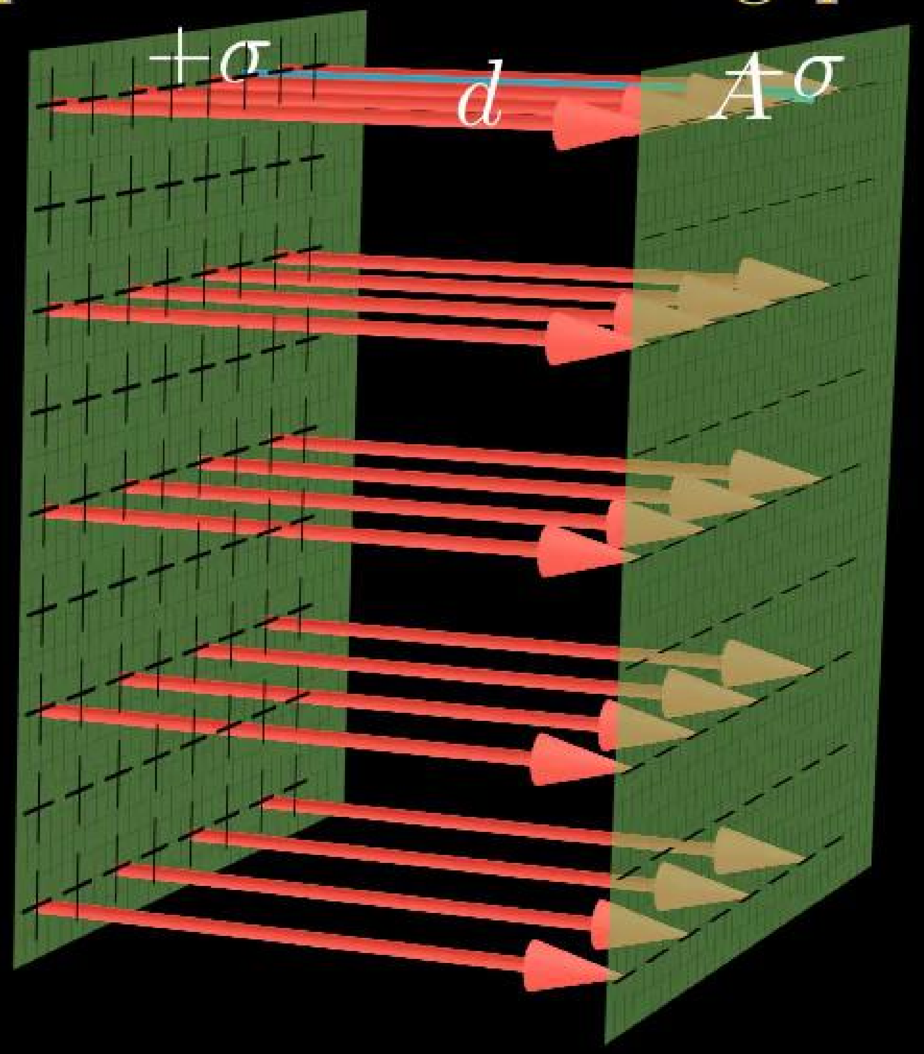


Fixed Capacitor   Variable Capacitor

- Dielectric strength : The maximum electric field that a dielectric medium can withstand without breakdown (of its insulating property) is called dielectric strength. There is a limit to the amount of charge that can be stored on a given capacitor without significant leaking.

PARALLEL PLATE CAPACITOR

- A parallel plate capacitor consist of two large plane parallel conducting plates each of Area (A) and separated by a small distance (d)



- The two plates have charges Q and $-Q$. We first take the medium between the plates to be vacuum.
- if $d^2 <<< A$ $\quad E = \dfrac{\sigma}{\epsilon_0} = \dfrac{Q}{A\epsilon_0}$
- So, Potential difference between the plates: $V = Ed$
- $V = \dfrac{Qd}{A\epsilon_0}$
- The capacitance C of the parallel plate capacitor is:
- $C = \dfrac{Q}{V} = \dfrac{Q\epsilon_0 A}{Qd} = \dfrac{\epsilon_0 A}{d}$

Example 19 : A parallel plate capacitor with air between the plates has a capacitance of 8 pF (1 pF $= 10^{-12}$ F). What will be the capacitance if the distance between the plates is reduced by half, and the space between them is filled with a substance of dielectric constant 6. [NCERT]

Example 20 : In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3}\ m^2$ and the distance between the plates is 3 mm. Calculate the capacitance of the capacitor. If this capacitor is connected to a 100 V supply, what is the charge on each plate of the capacitor? [NCERT]

Example 21 : Explain what would happen if in the capacitor given in Exercise 2.8, a 3 mm thick mica sheet (of dielectric constant = 6) were inserted between the plates,

(a) while the voltage supply remained connected.

(b) after the supply was disconnected. [NCERT]

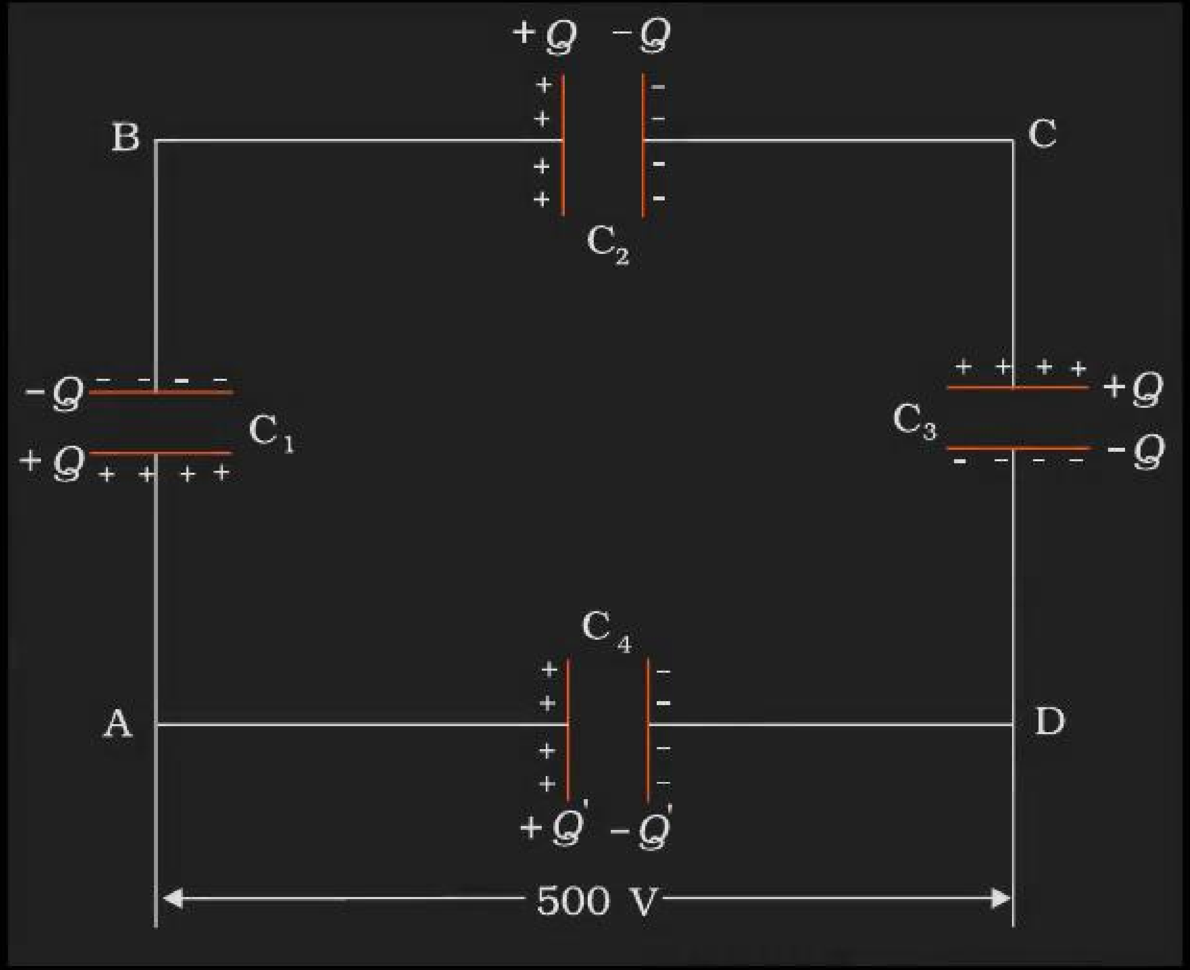
Example 22 : Three capacitors each of capacitance 9 pF are connected in series.
(a) What is the total capacitance of the combination?
(b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply? [NCERT]

Example 23 : Three capacitors of capacitances 2 pF, 3 pF and 4 pF are connected in parallel.

(a) What is the total capacitance of the combination?

(b) Determine the charge on each capacitor if the combination is connected to a 100 V supply. [NCERT]

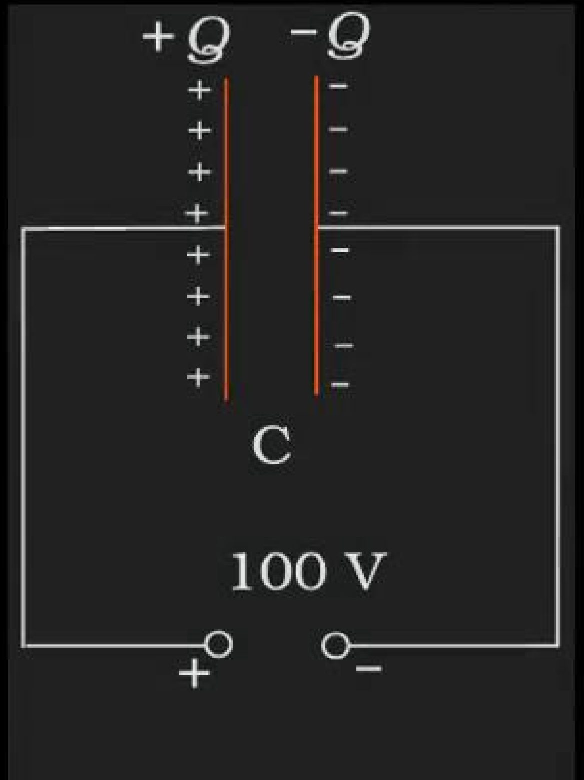
Example 24 : A network of four 10 μF capacitors is connected to a 500 V supply, as shown in Fig. 2.29. Determine
(a) the equivalent capacitance of the network and
(b) the charge on each capacitor. (Note, the charge on a capacitor is the charge on the plate with higher potential, equal and opposite to the charge on the plate with lower potential.) [NCERT]



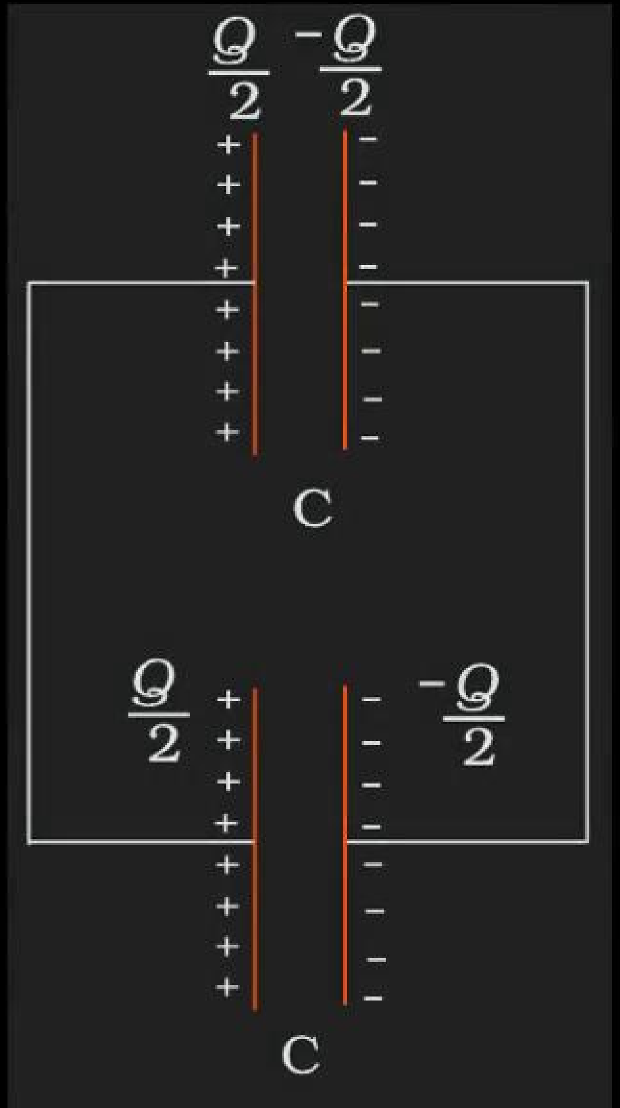
Example 25 : A 12pF capacitor is connected to a 50V battery. How much electrostatic energy is stored in the capacitor?. [NCERT]

Example 26 : (a) A 900 pF capacitor is charged by 100 V battery [Fig. 2.31(a)]. How much electrostatic energy is stored by the capacitor?

(b) The capacitor is disconnected from the battery and connected to another 900 pF capacitor [Fig. 2.31(b)]. What is the electrostatic energy stored by the system? [NCERT]



(a)



Example 27 : A 600pF capacitor is charged by a 200V supply. It is then disconnected from the supply and is connected to another uncharged 600 pF capacitor. How much electrostatic energy is lost in the process? [NCERT]