CHAPTER 2 : ELECTROSTATIC POTENTIAL AND CAPACITANCE

Learning Objectives :

- INTRODUCTION
- ELECTROSTATIC POTENTIAL
- POTENTIAL DUE TO A POINT CHARGE
- POTENTIAL DUE TO AN ELECTRIC DIPOLE
- POTENTIAL DUE TO A SYSTEM OF CHARGES
- EQUIPOTENTIAL SURFACES
- RELATION BETWEEN FIELD AND POTENTIAL
- POTENTIAL ENERGY IN AN EXTERNAL FIELD
- POTENTIAL ENERGY OF A DIPOLE IN AN EXTERNAL FIELD
- ELECTROSTATICS OF CONDUCTORS
- DIELECTRICS AND POLARISATION
- CAPACITORS AND CAPACITANCE
- COMBINATION OF CAPACITORS
- ENERGY STORED IN A CAPACITOR

Introduction

Potential energy and Conservative forces :

- When an external force does work in taking a body from a point to another against a force like spring force or gravitational force, that work gets stored as potential energy of the body.

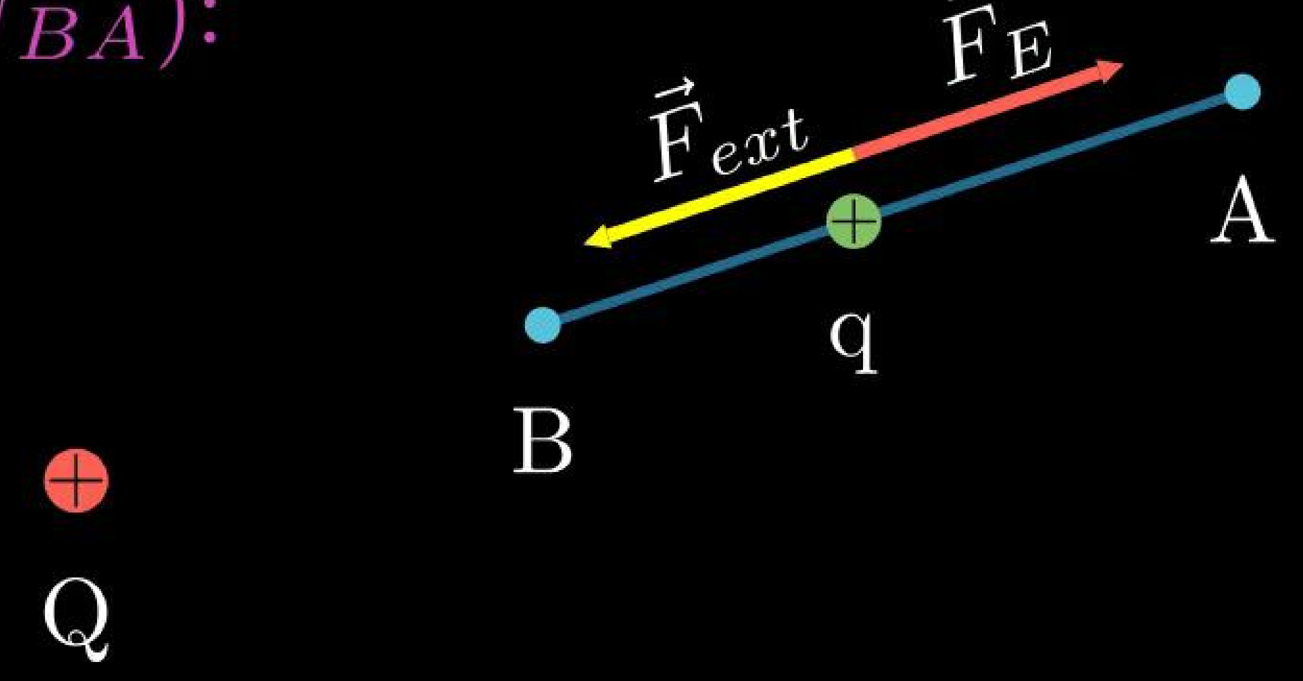
- When the external force is removed, the body moves, gaining kinetic energy and losing an equal amount of potential energy.

- The sum of kinetic and potential energies is thus conserved. Forces of this kind are called conservative forces.

- Spring force and gravitational force are examples of conservative forces.

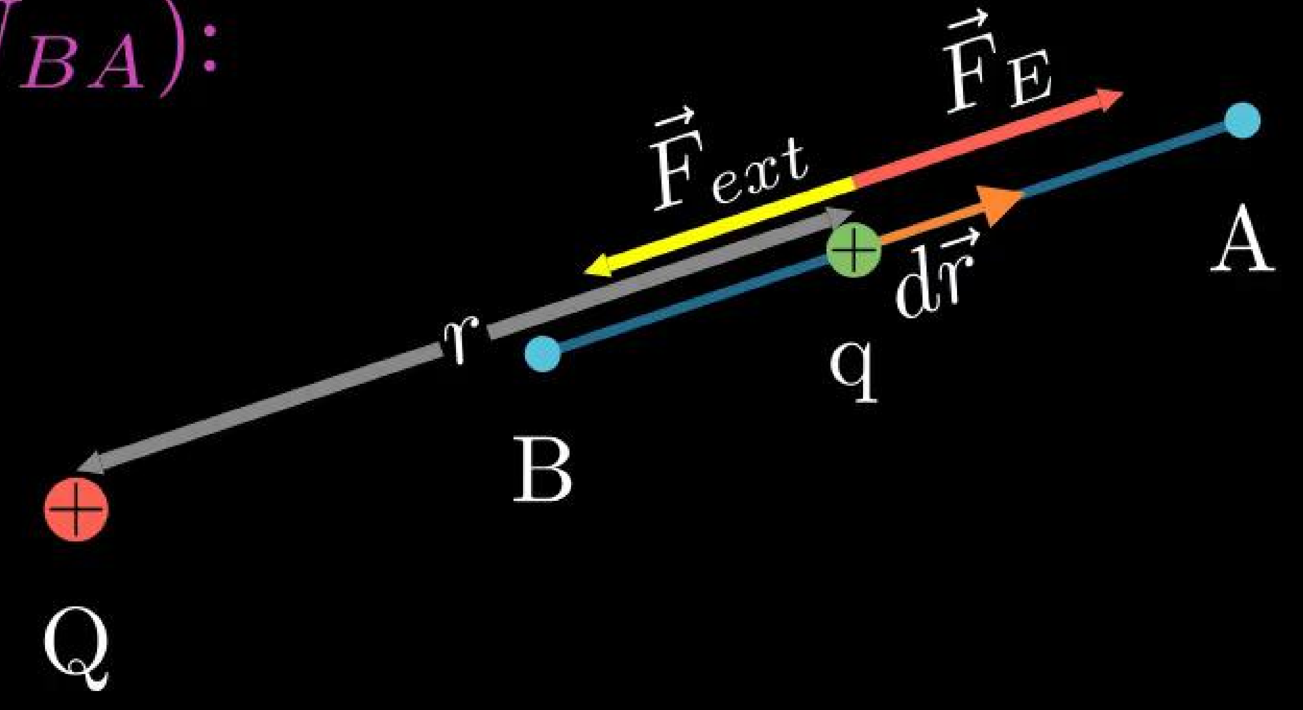
- Since, Coulomb force and Gravitational force both have inverse square dependence on distance. So, we can say that Coulomb force is also a conservative force.

Electrostatic Potential Energy Difference (ΔU_{BA}):



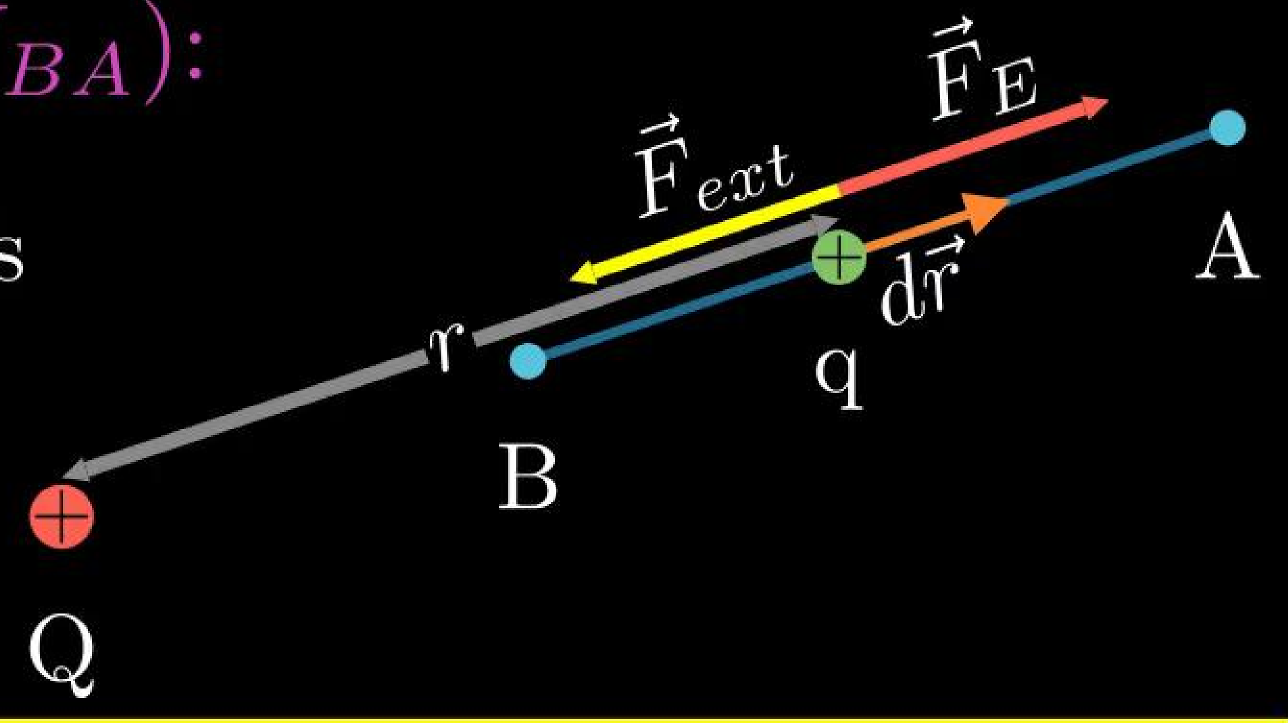
- Consider a charge Q fixed at the origin.
- We are bringing a charge q by applying an external force $\vec{F}_{ext}$ just enough to counter the repulsive electric force $\vec{F}_E$ (i.e, $\vec{F}_{ext} = -\vec{F}_E$).
- This means there is no net force on or acceleration of the charge q when it is brought from A to B, i.e., it is brought with infinitesimally slow constant speed.
- In this situation, work done by the external force is the negative of the work done by the electric force, and gets fully stored in the form of potential energy of the charge q.
- If the external force is removed on reaching B, the stored energy (potential energy) at B is used to provide kinetic energy to the charge q in such away that the sum of the kinetic and potential energies is conserved.

Electrostatic Potential Energy Difference (ΔU_{BA}):



- Thus, work done by external forces in moving a charge q from A to B is.
- $W_{AB} = \int_A^B \vec{F}_{ext} \cdot d\vec{r} = -\int_{r_A}^{r_B} \vec{F}_E \cdot d\vec{r}$
- This work done increases its potential energy by an amount equal to potential energy difference between points B and A.
- $\Delta U_{BA} = U_B - U_A = W_{AB} = -\int_{r_A}^{r_B} \vec{F}_E \cdot d\vec{r}$
- We can define electric potential energy difference between two points as the work required to be done by an external force in moving (without accelerating) charge q from one point to another against the electrostatic field.

Electrostatic Potential Energy Difference (ΔU_{BA}):



- Thus, Potential energy difference (Change in P.E.) is

$$\Delta U_{BA} = W_{AB} = -\int_{r_A}^{r_B} \vec{F}_E \cdot d\vec{r}$$

- $F_E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qq}{r^2}$

$$\Delta U_{BA} = -\int_{r_A}^{r_B} \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2} dr$$

$$\Delta U_{BA} = \frac{-Qq}{4\pi\epsilon_0}\int_{r_A}^{r_B} r^{-2} dr$$

$$\Delta U_{BA} = \frac{-Qq}{4\pi\epsilon_0}\left[\frac{r^{(-2+1)}}{-2+1}\right]_{r_A}^{r_B} = \frac{-Qq}{4\pi\epsilon_0}\left[\frac{-1}{r}\right]_{r_A}^{r_B}$$

$$\boxed{\Delta U_{BA} = U_B - U_A = \frac{Qq}{4\pi\epsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]}$$

- Here, $U_A \rightarrow$ P.E. at A
- And, $U_B \rightarrow$ P.E. at B

Electrostatic Potential Energy Difference (ΔU_{BA}):

$$\boxed{\Delta U_{BA} = U_B - U_A = \frac{Qq}{4\pi\epsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]}$$

- The work done by an electrostatic field in moving a charge from one point to another depends only on the initial and the final points and is independent of the path taken to go from one point to the other. This is the fundamental characteristic of a conservative force.

- The concept of the potential energy would not be meaningful if the work depended on the path(or if force is not conservative).

- The actual value of potential energy is not physically significant; it is only the difference of potential energy that is significant.
  $\Delta U_{BA} = U_B - U_A = (U_B + \alpha) - (U_A + \alpha) = W_A$

- There is a freedom in choosing the point where potential energy is zero. A convenient choice is to have electrostatic potential energy zero at infinity. (i.e., $U_\infty = 0$)

Electrostatic Potential Energy Difference (ΔU_{BA}):

$$\boxed{\Delta U_{BA} = U_B - U_A = \frac{Qq}{4\pi\epsilon_0}\left[\frac{1}{r_B} - \frac{1}{r_A}\right]} \longrightarrow \text{Potential Energy Difference}$$

- With this choice, if we take the point A at infinity
- $U_B - U_\infty = W_{\infty B} = \frac{Qq}{4\pi\epsilon_0}\left[\frac{1}{r_B} - \frac{1}{\infty}\right]$
- $U_B = W_{\infty B} = \frac{Qq}{4\pi\epsilon_0}\frac{1}{r_B} \quad (\because U_{\infty=0})$ $\longrightarrow$ Potential Energy
- Potential Energy (U) : Potential energy of charge q at a point is the work done by the external force (equal and opposite to the electric force) in bringing the charge q from infinity to that point.

ELECTROSTATIC POTENTIAL DIFFERENCE (ΔV) And POTENTIAL ()

- The potential energy (or work) we just defined is proportional to the test charge q .

- It is, therefore, convenient to divide the work by the amount of charge q, so that the resulting quantity is independent of q

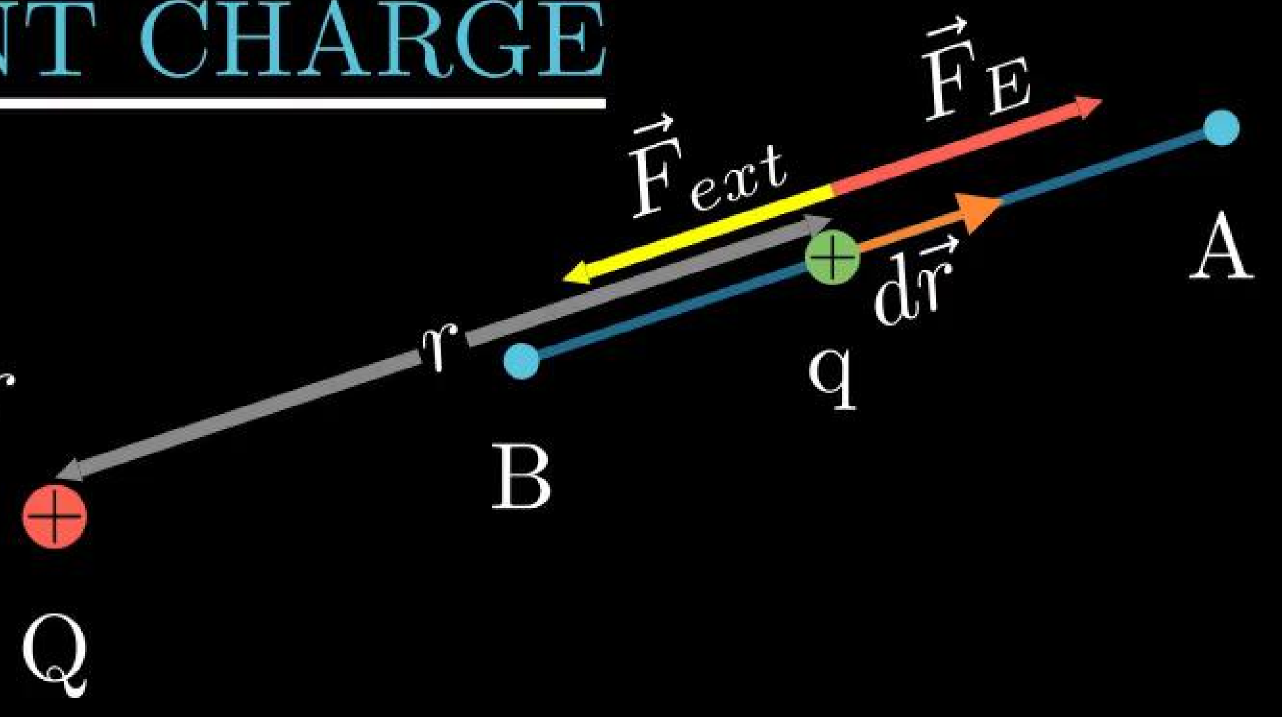
- Electrostatic Potential Difference (ΔV_{BA}) : It is defined as the work done by external force in bringing a unit positive charge from point one point (A) to another (B)

$$\boxed{\Delta V_{BA} = V_B - V_A = \frac{W_{AB}}{q} = \frac{U_{BA}}{q}} \longrightarrow \quad \text{Potential Difference}$$

- Electrostatic Potential (V_B) : It is defined as the work done by external force in bringing a unit positive charge (without acceleration) from infinity to that point (B).

$$\boxed{V_B = \frac{W_{\infty B}}{q} = \frac{U_B}{q}} \longrightarrow \quad \text{Potential}$$

POTENTIAL DUE TO A POINT CHARGE



- Consider a charge $+Q$ at the origin.
- Thus, Potential (V) at any point P at a distance r from the origin is

Potential $V = \dfrac{W_{\infty P}}{q}$

- At some point electrostatic force on the unit positive test charge is $F_E = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qq}{r^2}$

$$V = -\int_{\infty}^{r} F_E \cdot dr = -\int_{\infty}^{r} \frac{1}{4\pi\epsilon_0}\frac{Qq}{r^2}dr$$

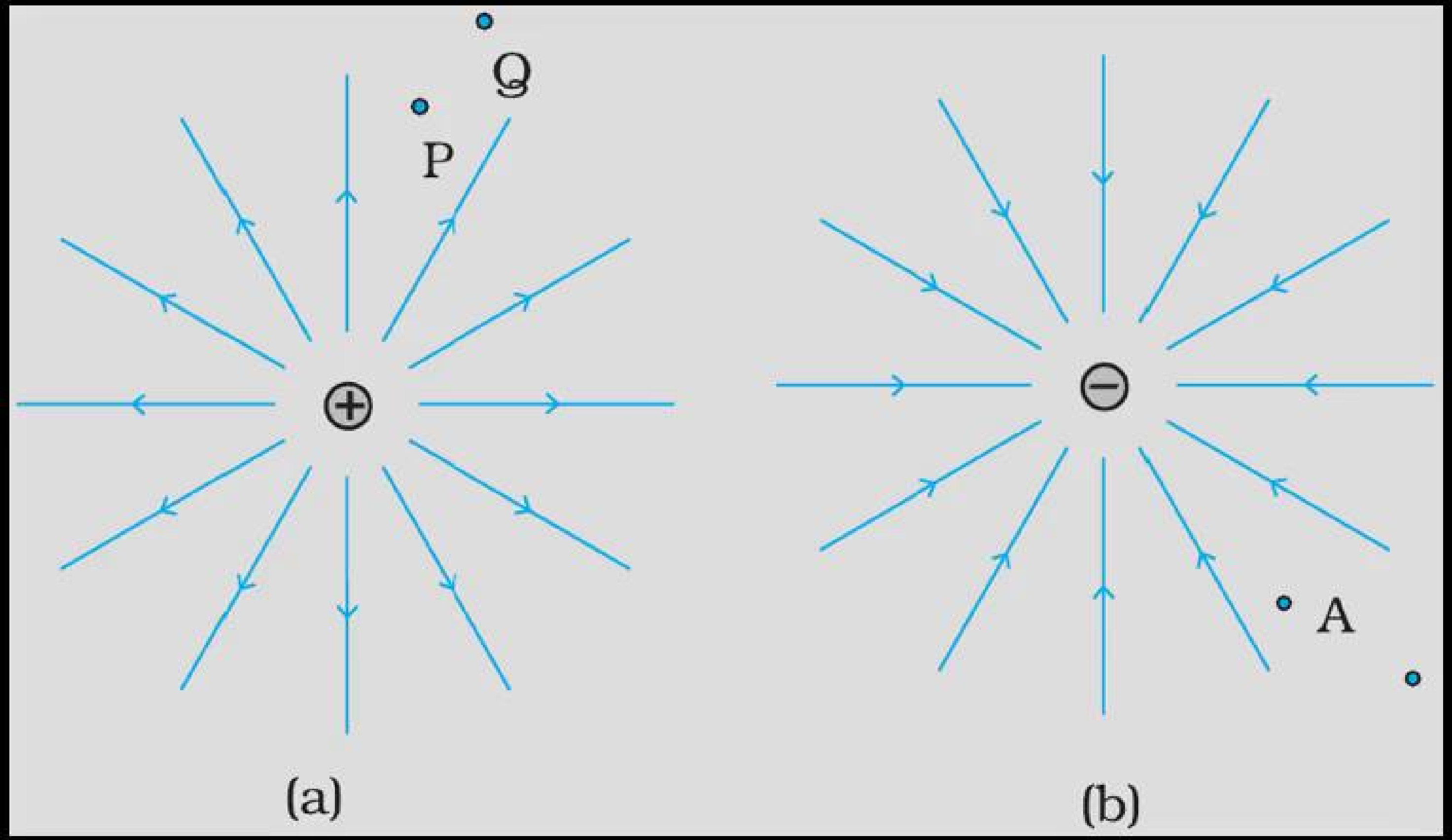
$$V = \frac{-Qq}{4\pi\epsilon_0}\int_{\infty}^{r} r^{-2}dr = \frac{-Qq}{4\pi\epsilon_0}\left[\frac{r^{(-2+1)}}{-2+1}\right]_{\infty}^{r} = \frac{-Qq}{4\pi\epsilon_0}\left[\frac{-1}{r}\right]_{\infty}^{r}$$

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{Q}{r}}$$

Example 1 :(a) Calculate the potential at a point P due to a charge of 4×10^{-7} C located 9 cm away.

(b) Hence obtain the work done in bringing a charge of 2×10^{-9} C from infinity to the point P. Does the answer depend on the path along which the charge is brought?

Example 2 : Figures 2.8 (a) and (b) show the field lines of a positive and negative point charge respectively.



(a) Give the signs of the potential difference $V_P - V_Q$; $V_B - V_A$.

(b) Give the sign of the potential energy difference of a small negative charge between the points Q and P; A and B.

(c) Give the sign of the work done by the field in moving a small positive charge from Q to P.

(d) Give the sign of the work done by the external agency in moving a small negative charge from B to A.

(e) Does the kinetic energy of a small negative charge increase or decrease in going from B to A?

POTENTIAL DUE TO A SYSTEM OF CHARGES

Example 3 :Two charges 3×10^{-8} C and -2×10^{-8} C are located 15 cm apart. At what point on the line joining the two charges is the electric potential zero? Take the potential at infinity to be zero.

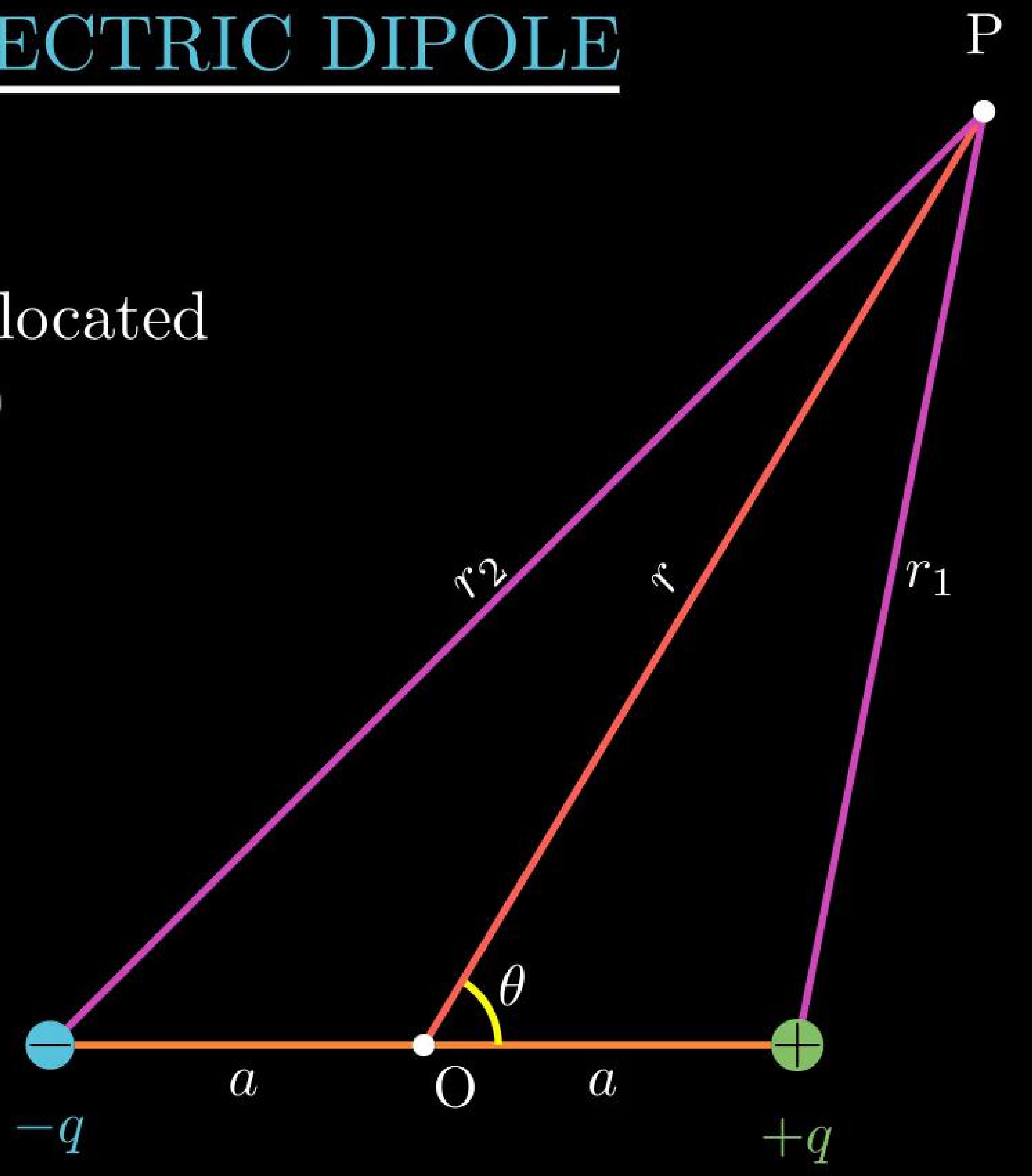
POTENTIAL DUE TO AN ELECTRIC DIPOLE

- Consider a dipole of dipole moment $p = q \times 2a$
- We have to find the electric potential at point P located at a distance r from the centre of the dipole (O)
- $\theta \rightarrow$ Angle between r and dipole axis.
- Potential at point P due to $+q$ charge :

$$V_{+q} = \frac{1}{4\pi\epsilon_0}\frac{q}{r_1}$$

- Potential at point P due to $-q$ charge :

$$V_{-q} = \frac{1}{4\pi\epsilon_0}\frac{-q}{r_2}$$



POTENTIAL DUE TO AN ELECTRIC DIPOLE

- Net potential at (P) due to the dipole:

$$V = V_{+q} + V_{-q} = \frac{1}{4\pi\epsilon_0}\frac{q}{r_1} - \frac{1}{4\pi\epsilon_0}\frac{q}{r_2}$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right] ......(1)$$

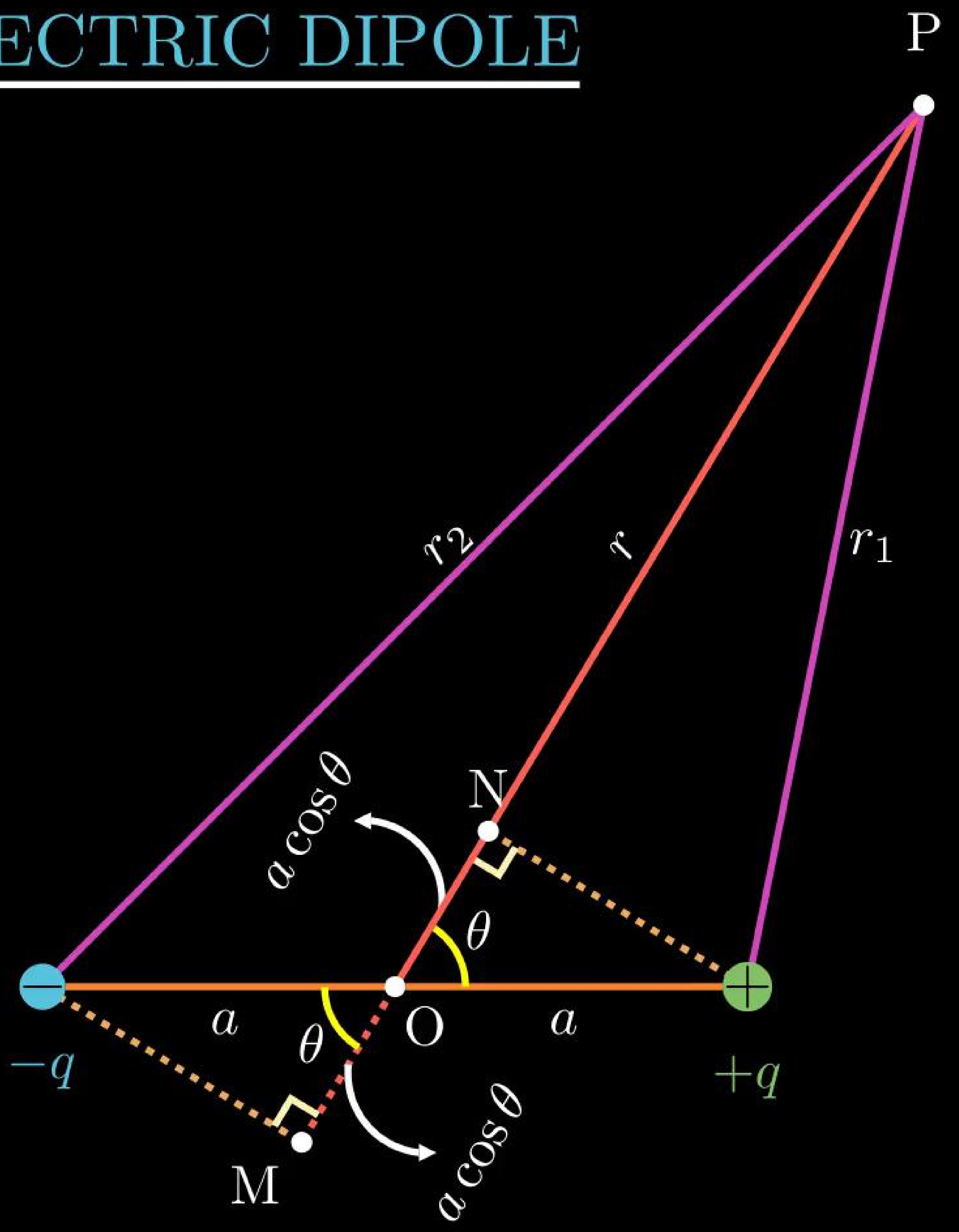
- By Geometry :

$ON = a\cos\theta$ and $OM = a\cos\theta$

- For small/point dipole $(a <<< r)$:

$r_1 = r - a\cos\theta$ and $r_2 = r + a\cos\theta$

- Substituting values of r_1 and r_2 in eq(1)



POTENTIAL DUE TO AN ELECTRIC DIPOLE

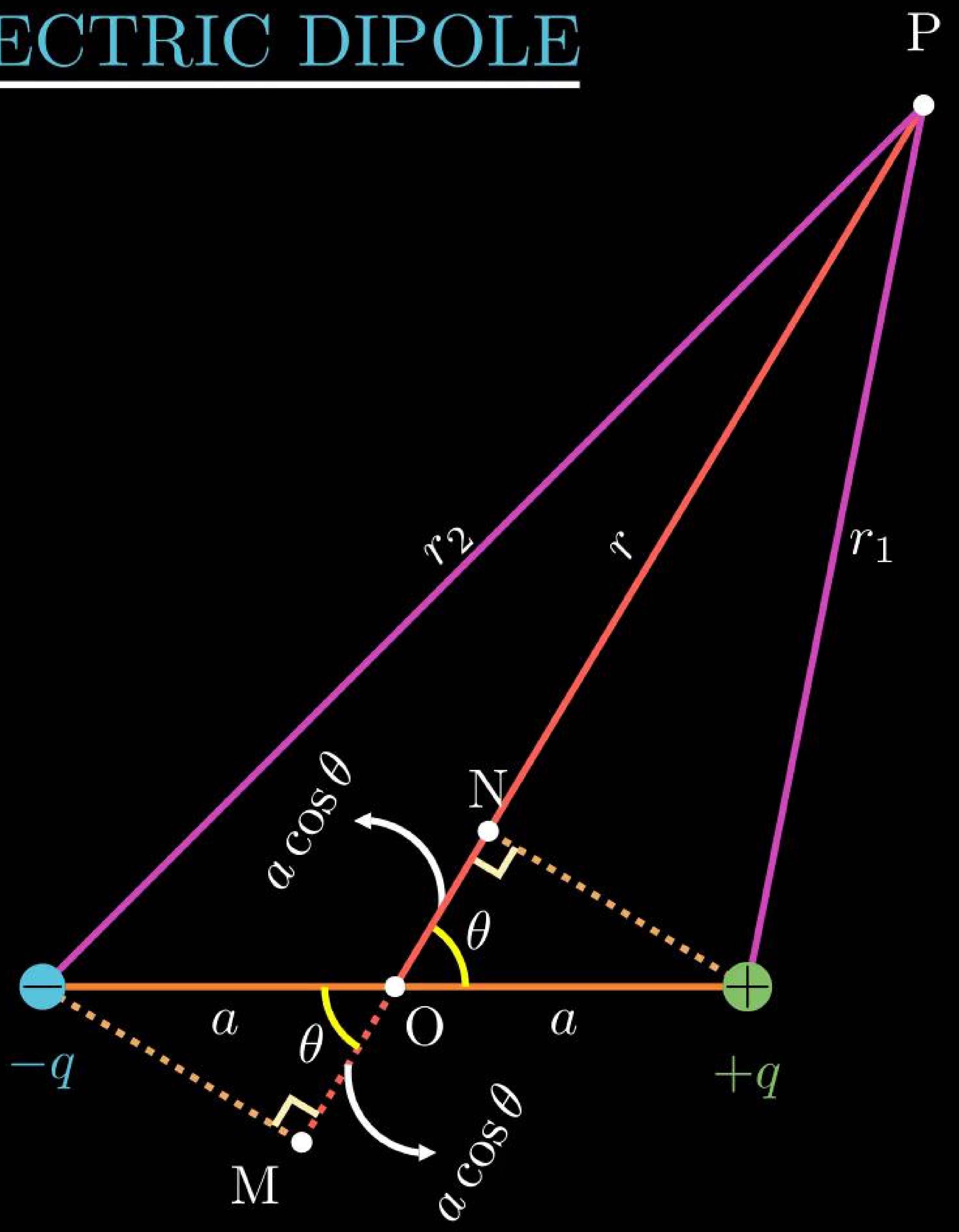
$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{(r - a\cos\theta)} - \frac{1}{(r + a\cos\theta)}\right]$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{r + a\cos\theta - (r - a\cos\theta)}{r^2 - a^2\cos^2\theta}\right]$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{r + a\cos\theta - r + a\cos\theta}{r^2 - a^2\cos^2\theta}\right]$$

$$V = \frac{q}{4\pi\epsilon_0}\left[\frac{2a\cos\theta}{r^2 - a^2\cos^2\theta}\right]$$

$$V = \frac{1}{4\pi\epsilon_0}\left[\frac{p\cos\theta}{r^2 - a^2\cos^2\theta}\right] \quad (\because p = q \times 2a)$$



POTENTIAL DUE TO AN ELECTRIC DIPOLE

- Since, $r >>> a$

$\therefore r^2 - a^2 \cos^2\theta \approx r^2$

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\left[\frac{p\cos\theta}{r^2}\right]}$$

- Special Cases -

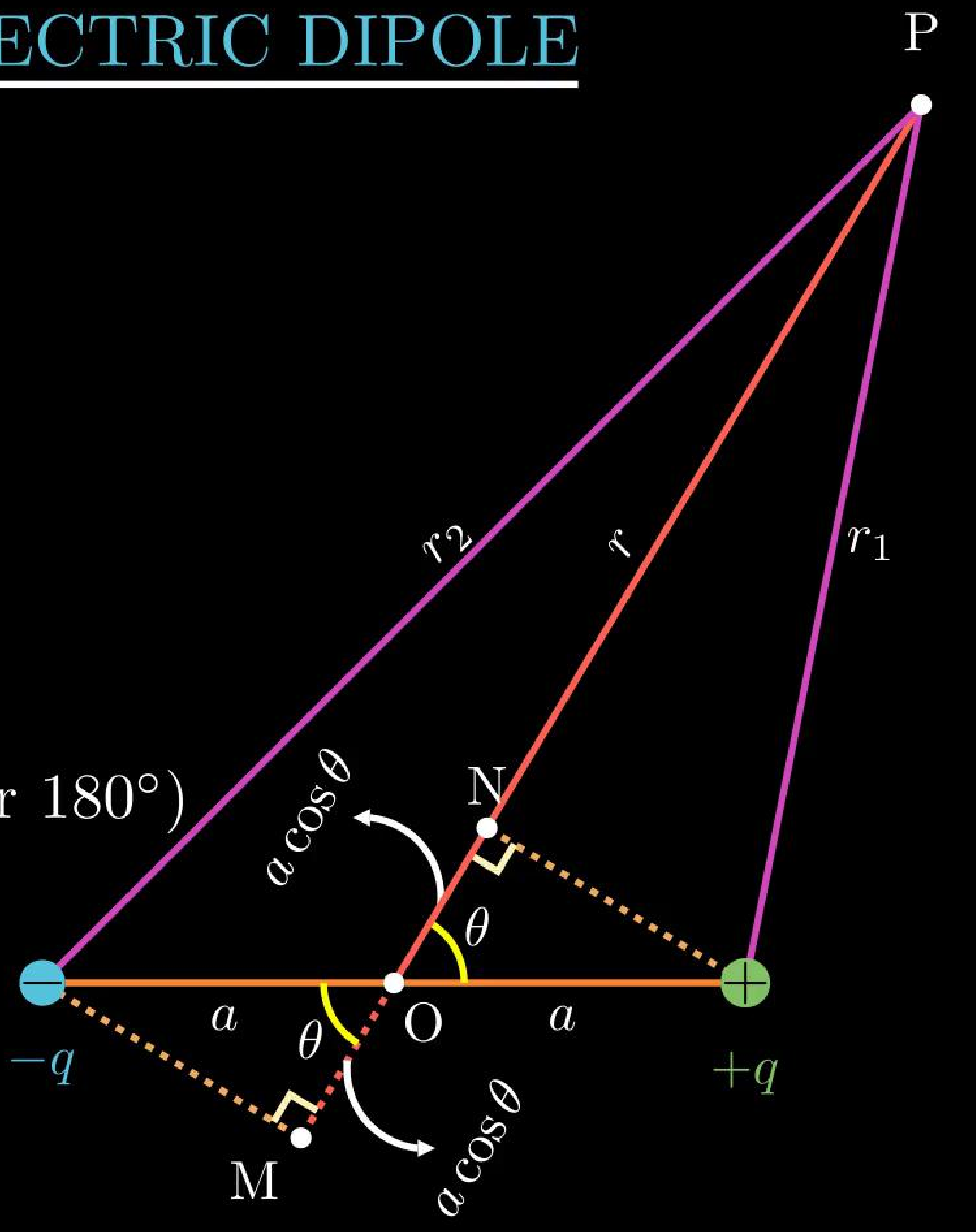
(i) When P lies on axial line of dipole ($\theta = 0°$ Or $180°$)

$$V_{ax} = \frac{1}{4\pi\epsilon_0}\left[\frac{p\cos(0° \ or \ 180°)}{r^2}\right]$$

$$\boxed{V_{ax} = \pm\frac{1}{4\pi\epsilon_0}\left[\frac{p}{r^2}\right]}$$

$+$ if $\theta = 0°$

$-$ if $\theta = 180°$



POTENTIAL DUE TO AN ELECTRIC DIPOLE

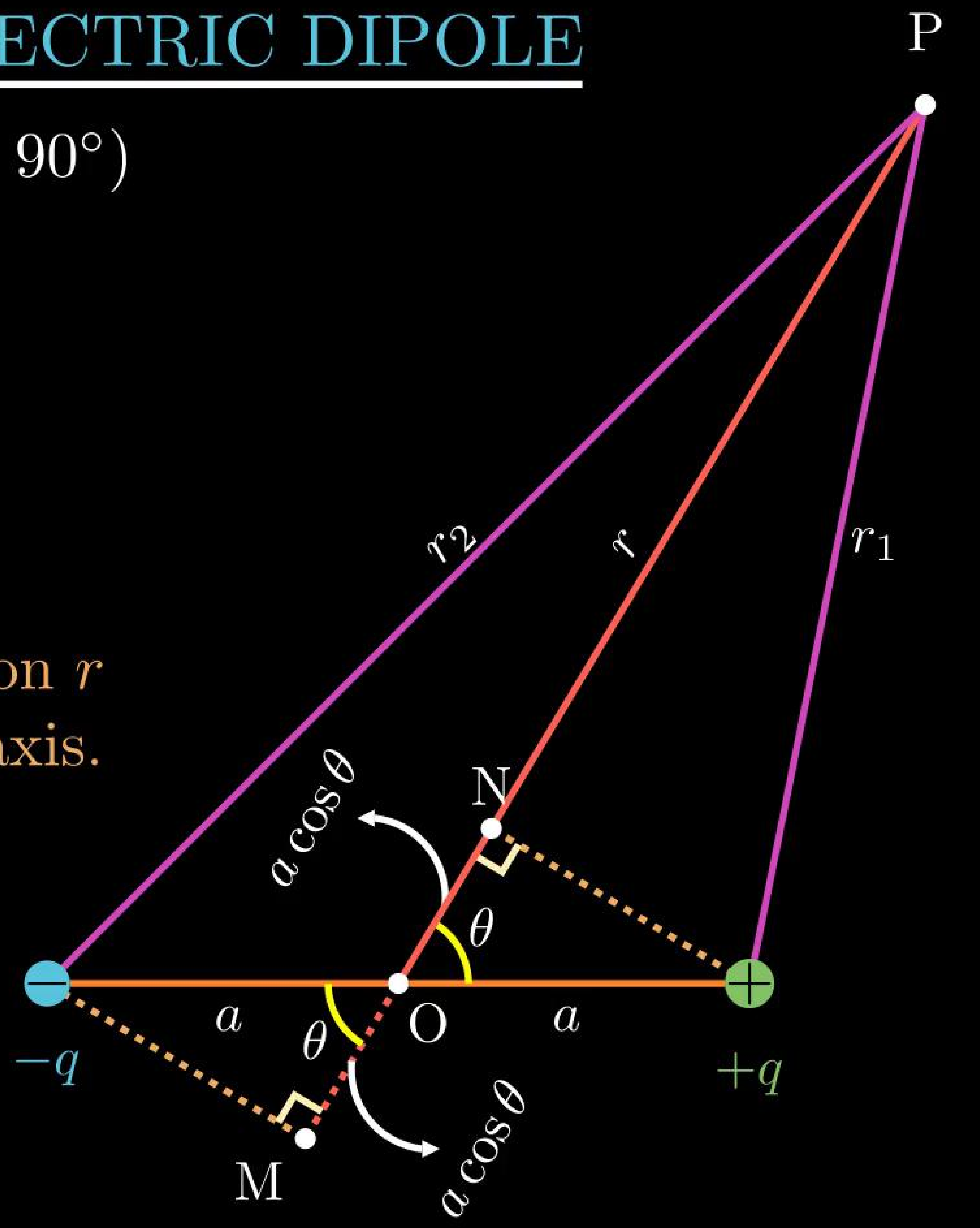
(ii) When P lies on equitorial line of dipole ($\theta = 90°$)

$$V_{eq} = \frac{1}{4\pi\epsilon_0}\left[\frac{p\cos(90°)}{r^2}\right]$$

$$\boxed{V_{eq} = 0}$$

- Potential due to electric dipole not just depend on r but also on the angle (θ) between r and dipole axis.

- $V_{dipole} \propto \frac{1}{r^2}$ But, $V_{\text{point charge}} \propto \frac{1}{r}$



Example 4 :The electric potential at a distance 3 m on the axis of a short dipole of dipole moment 4×10^{-2} coulomb-metre is

(a) 1.33 mV

(b) 4 mV

(c) 12 mV

(d) 27 mV

Solution:

Example 5 :The electric potential in volts due to an electric dipole of dipole moment 2×10^{-8} coulomb-metre at a distance of 3 m on a line making an angle of 60° with the axis of the dipole is

(a) Zero

(b) 10

(c) 20

(d) 40

Solution:

POTENTIAL DUE TO A UNIFORMLY CHARGED SPHERICAL SHELL

- Consider a uniformly charged spherical shell of Radius (R) and having charge (Q).

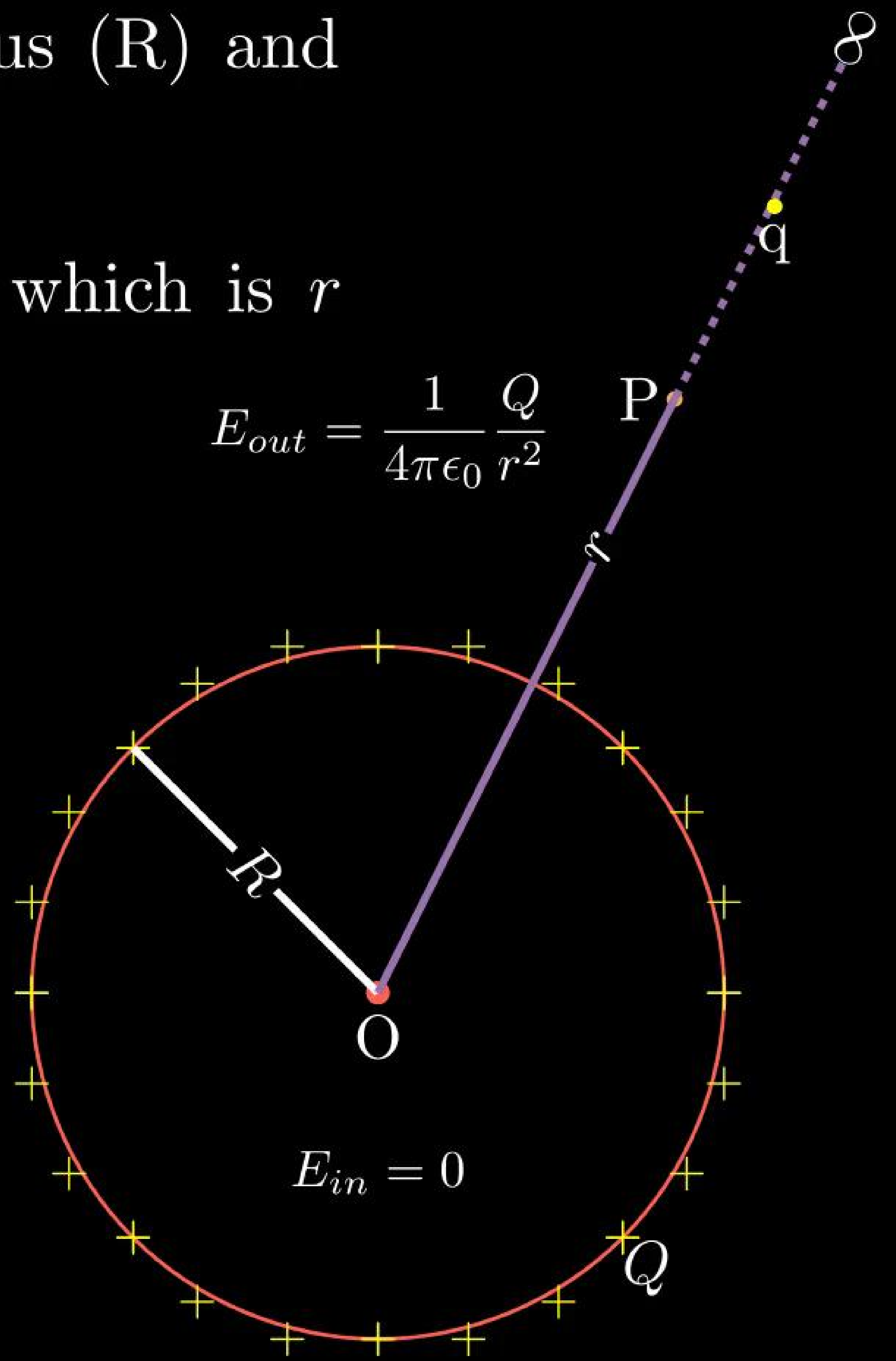
- We have to find the electric potential at a point P which is r distance from the centre of the shell.

- Case (1) : For point(P) outside the shell ($r > R$)

$$V_{out} = \frac{W_{\infty P}}{q} = -\int_{\infty}^{r} \frac{F_E}{q}\ dr$$

$$V_{out} = -\int_{\infty}^{P} E\ dr = -\int_{\infty}^{r} \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2} dr$$

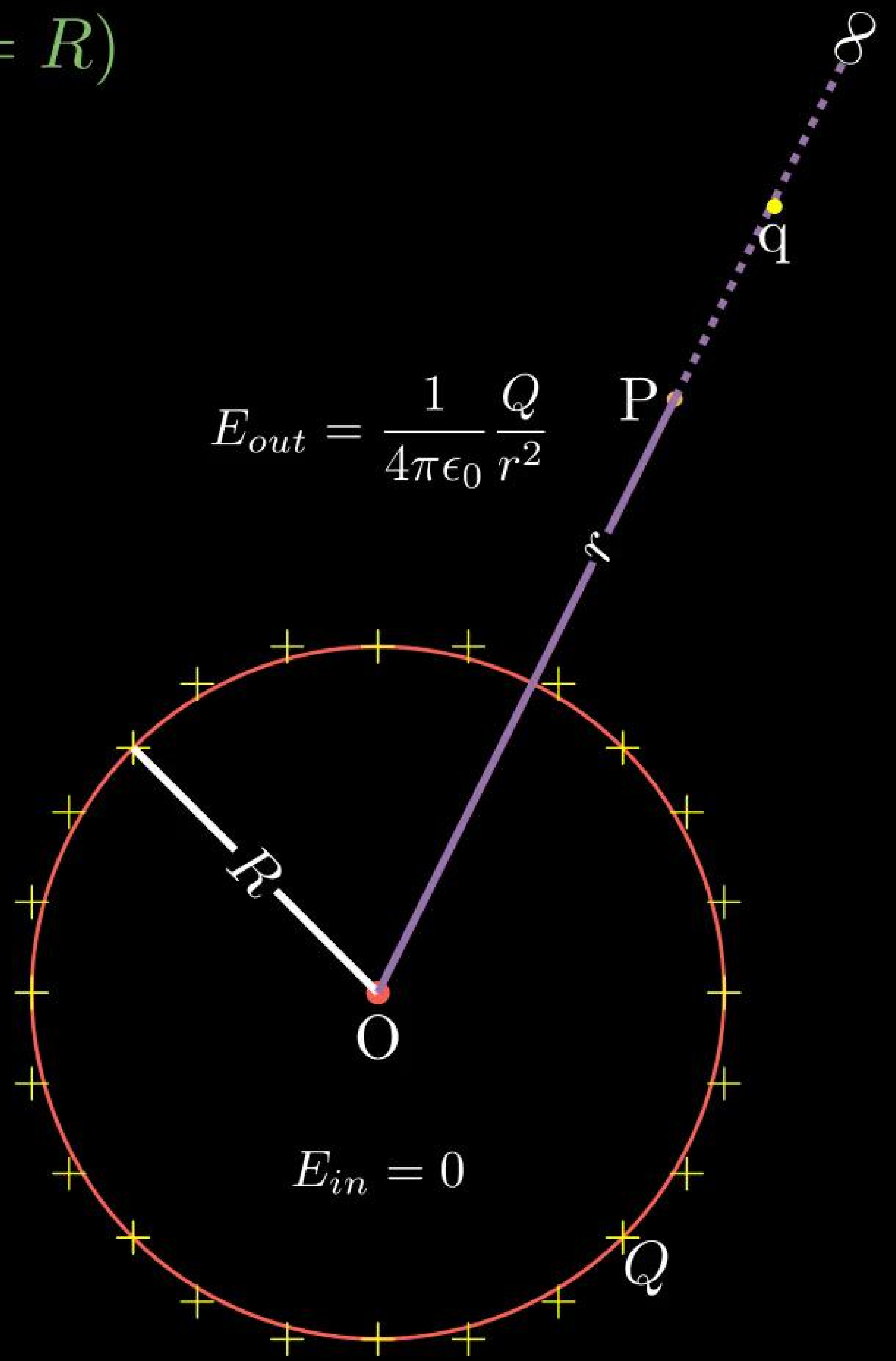
$$\boxed{V_{out} = \frac{1}{4\pi\epsilon_0}\frac{Q}{r} \quad (r > R)}$$ (Same as point charge.)



POTENTIAL DUE TO A UNIFORMLY CHARGED SPHERICAL SHELL

- Case (2) : For point(P) at the surface of the shell $(r = R)$

$$\boxed{V_{\text{surf}} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} \quad (r = R)}$$



POTENTIAL DUE TO A UNIFORMLY CHARGED SPHERICAL SHELL

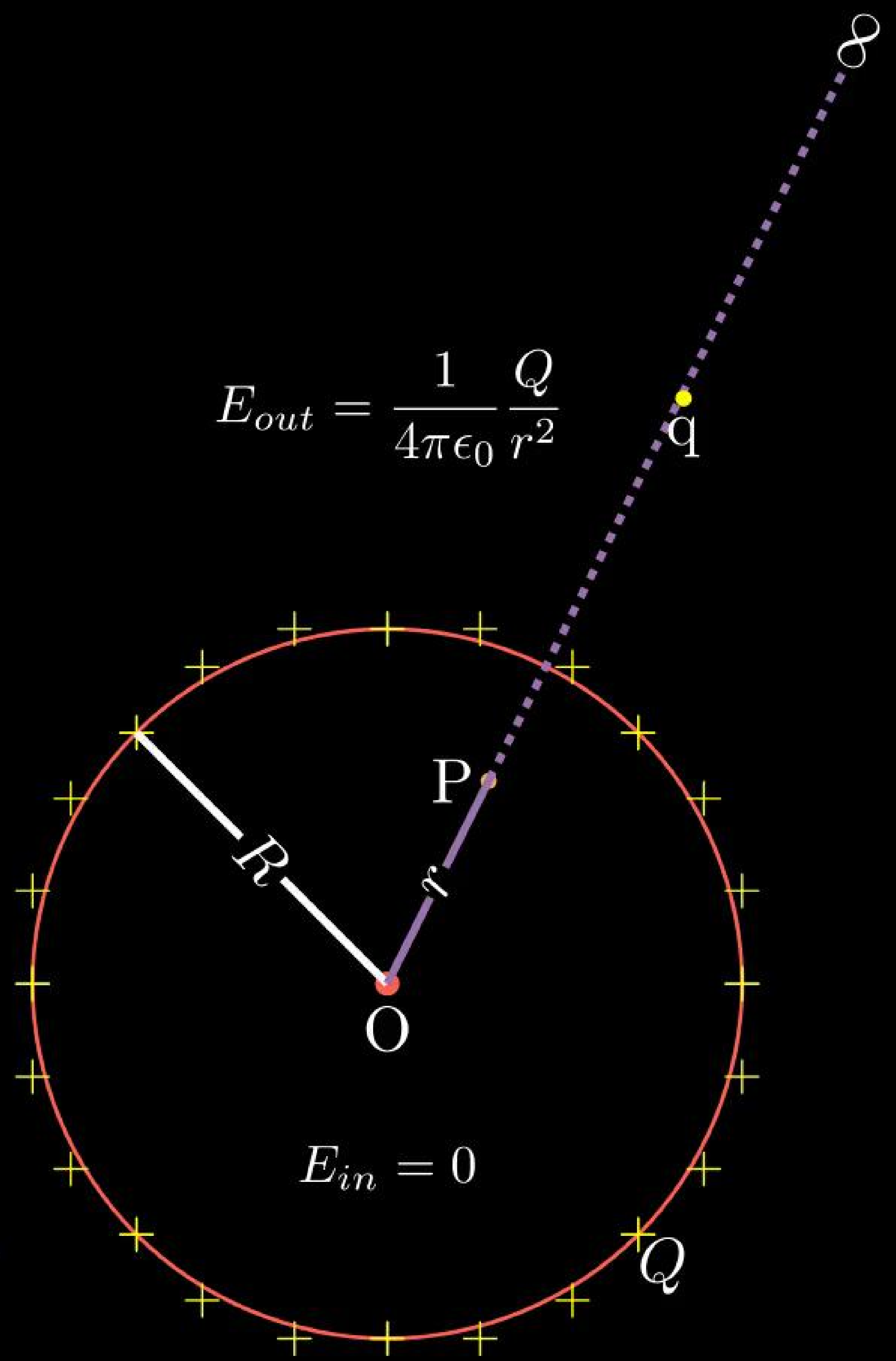
- Case (3) : For point(P) inside the shell ($r < R$)

$$V_{in} = \frac{W_{\infty P}}{q} = -\int_{\infty}^{r} \frac{F_E}{q}\, dr$$

$$V_{in} = -\int_{\infty}^{P} E\, dr = -\int_{\infty}^{R} E_{out}\, dr - \int_{R}^{r} E_{in}\, dr$$

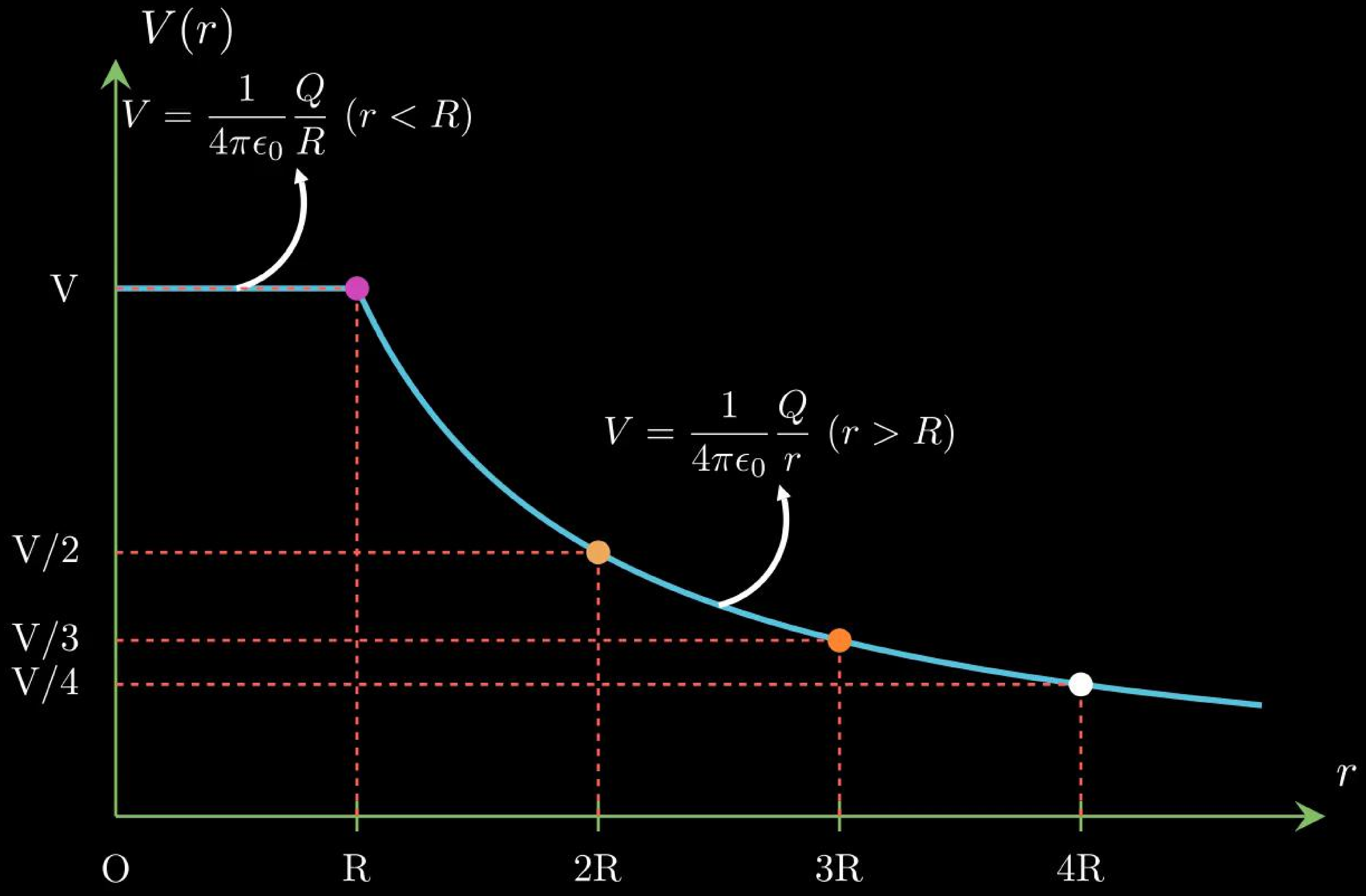
$$V_{in} = -\int_{\infty}^{R} \frac{1}{4\pi\epsilon_0}\frac{Q}{r^2}\, dr - \int_{R}^{r} 0\, dr \quad (\because E_{in} = 0)$$

$$\boxed{V_{in} = \frac{1}{4\pi\epsilon_0}\frac{Q}{R} = V_{\text{surf}} \text{ (Constant)}}$$



- Hence, potential remains constant inside the spherical shell, and is equal to the value at the surface.

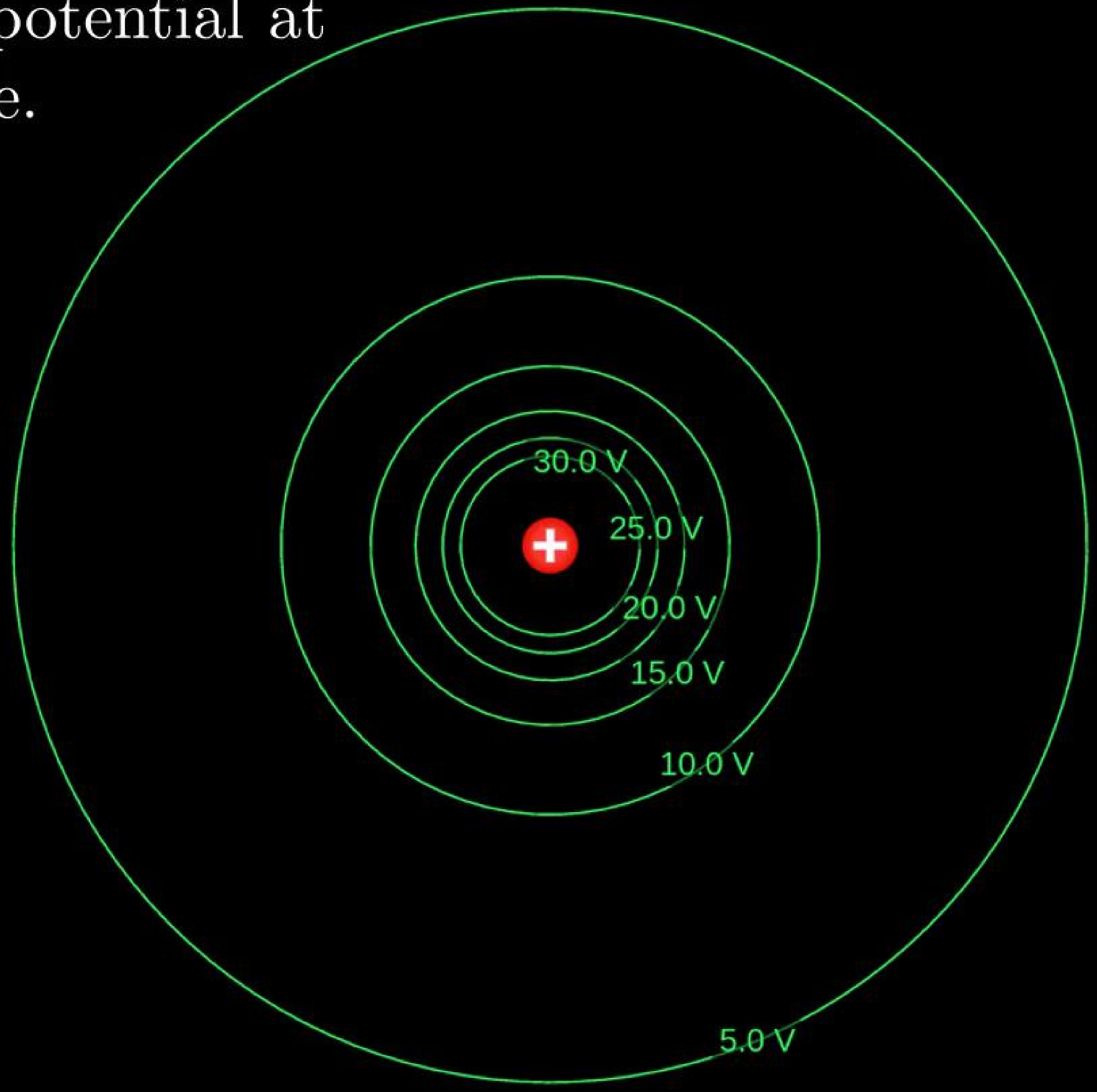
POTENTIAL DUE TO A UNIFORMLY CHARGED SPHERICAL SHELL



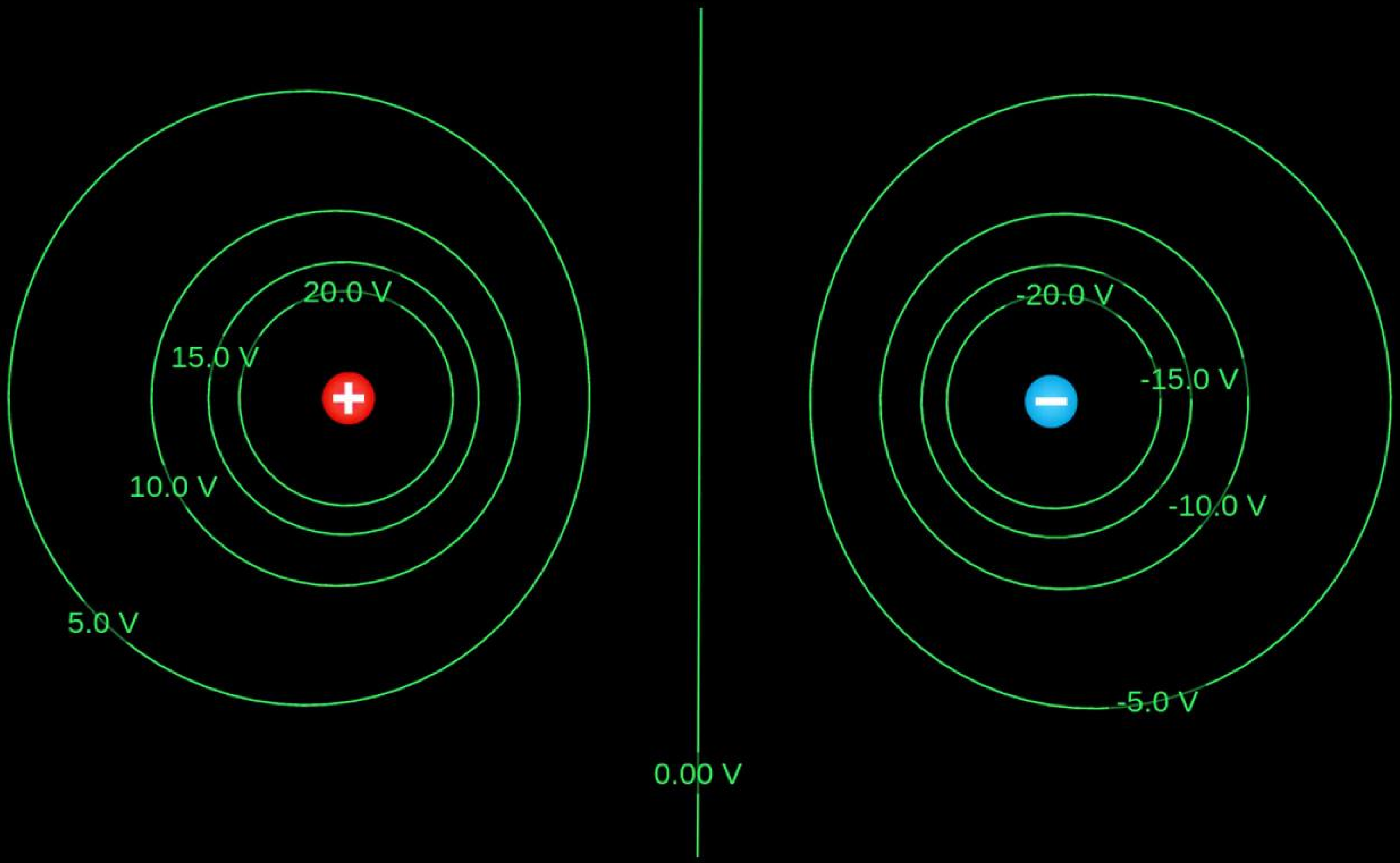
Graph of potential (V) versus r for spherical shell.

EQUIPOTENTIAL SURFACES

- A surface, which has same electrostatic potential at every point is called equipotential surface.
- For a single point charge (Q)
- $V = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q}{r}$
- if r is same (constant) the V is also same (constant)
- Thus equipotential surfaces of a single point charge are concentric spherical surfaces centred at the charge

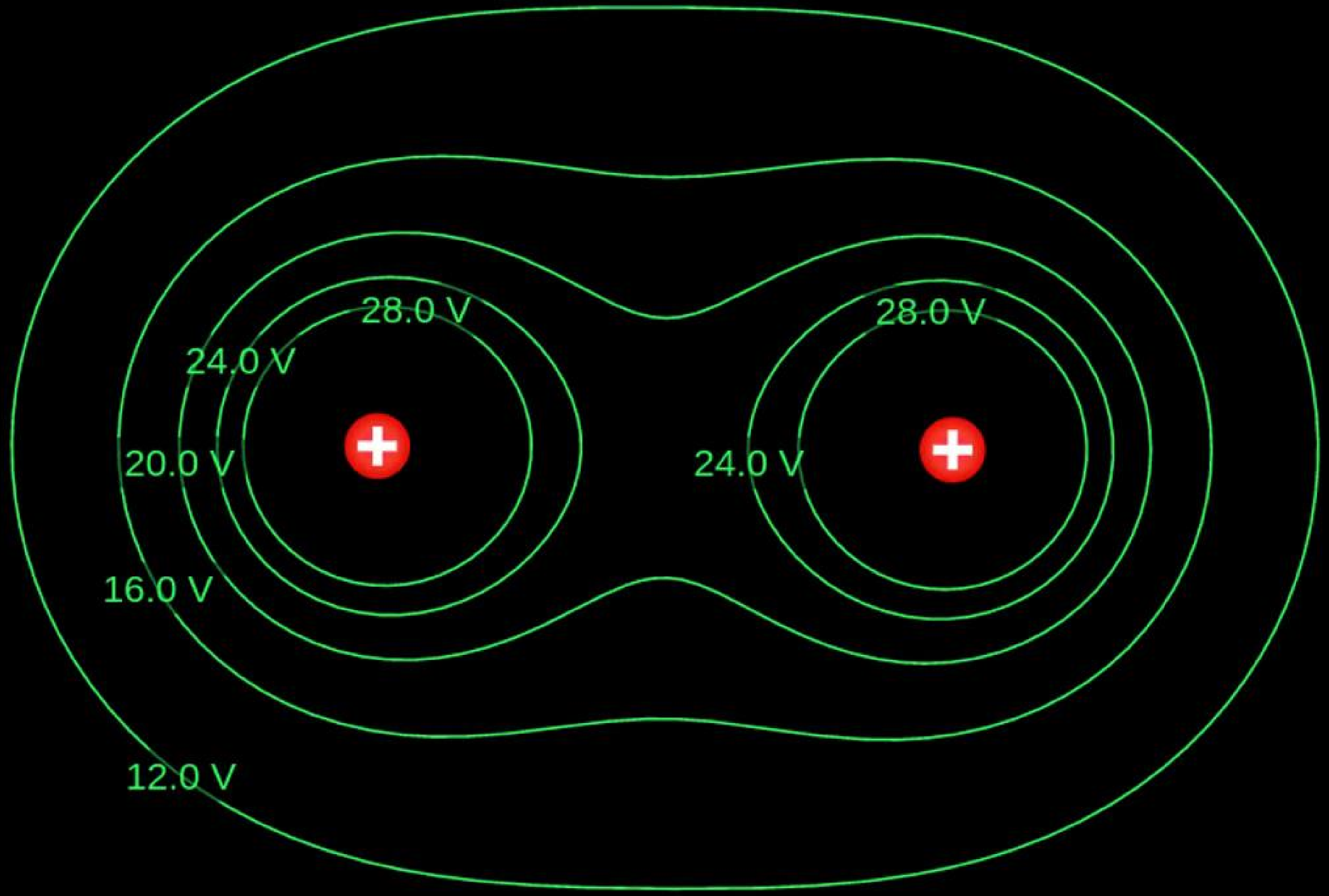


EQUIPOTENTIAL SURFACES



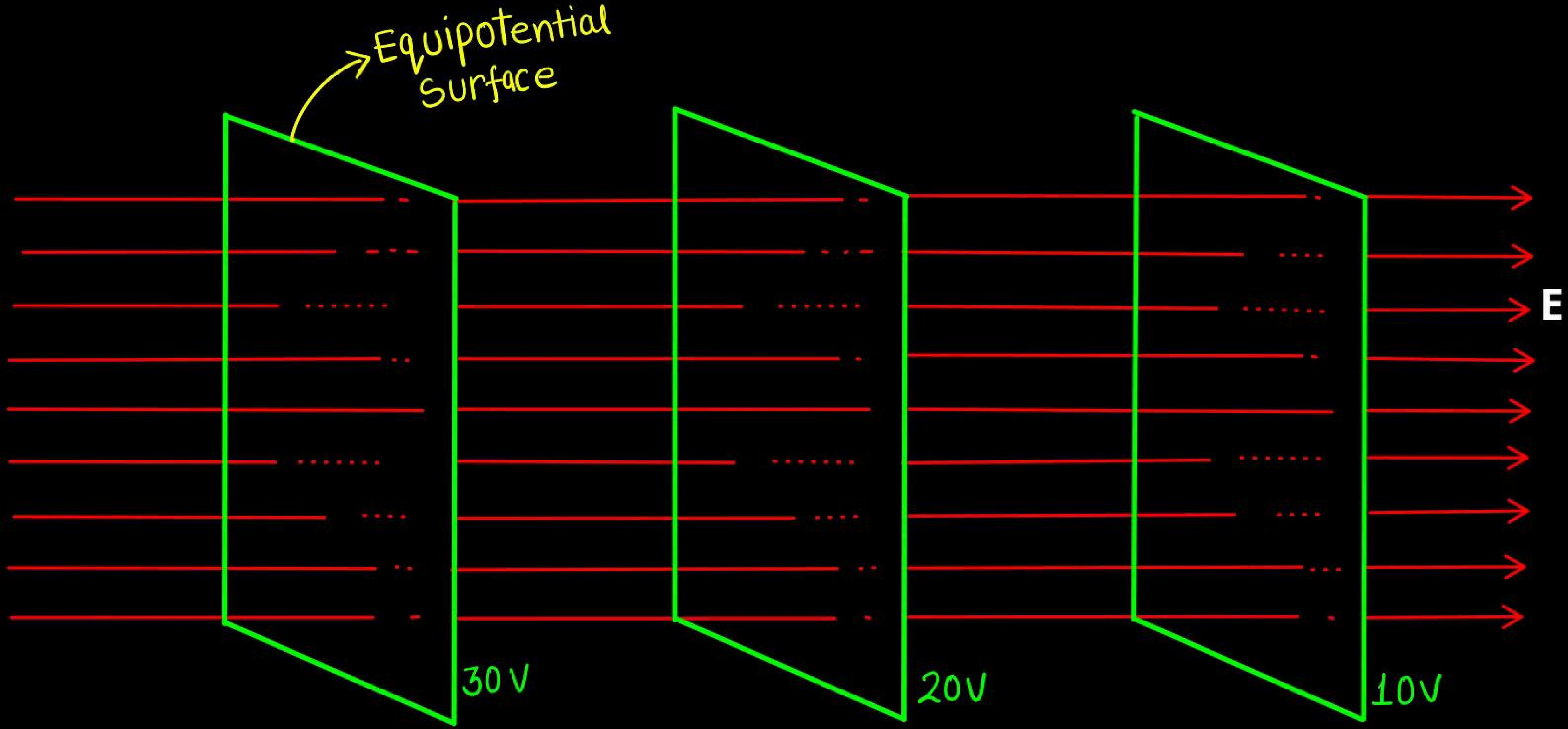
Equipotential surfaces for a dipole

EQUIPOTENTIAL SURFACES



Equipotential surfaces for two identical positive charges.

EQUIPOTENTIAL SURFACES



Equipotential surfaces for a uniform electric field.

EQUIPOTENTIAL SURFACES

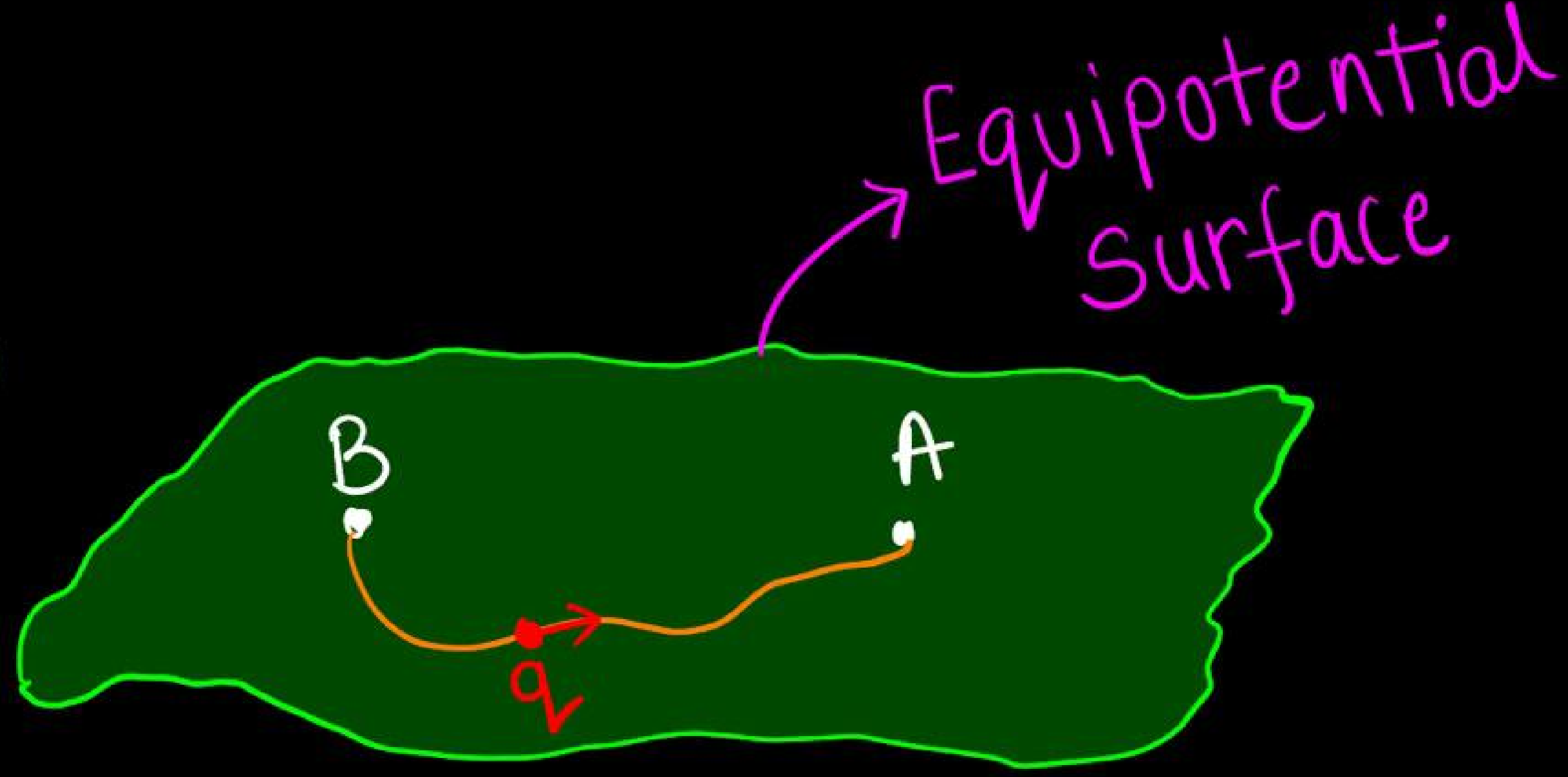
Properties of equipotential surfaces

- The work done in moving any charge(q) over an equipotential surface is always zero.

We know that $\Delta V_{AB} = (V_A - V_B) = \dfrac{W_{BA}}{q}$

$\because V_A = V_B$ (Points on equipotential surface)

$\therefore W_{BA} = q(V_A - V_B) = 0$



EQUIPOTENTIAL SURFACES

Properties of equipotential surfaces

- The electric field at every point is normal to the equipotential surface passing through that point.

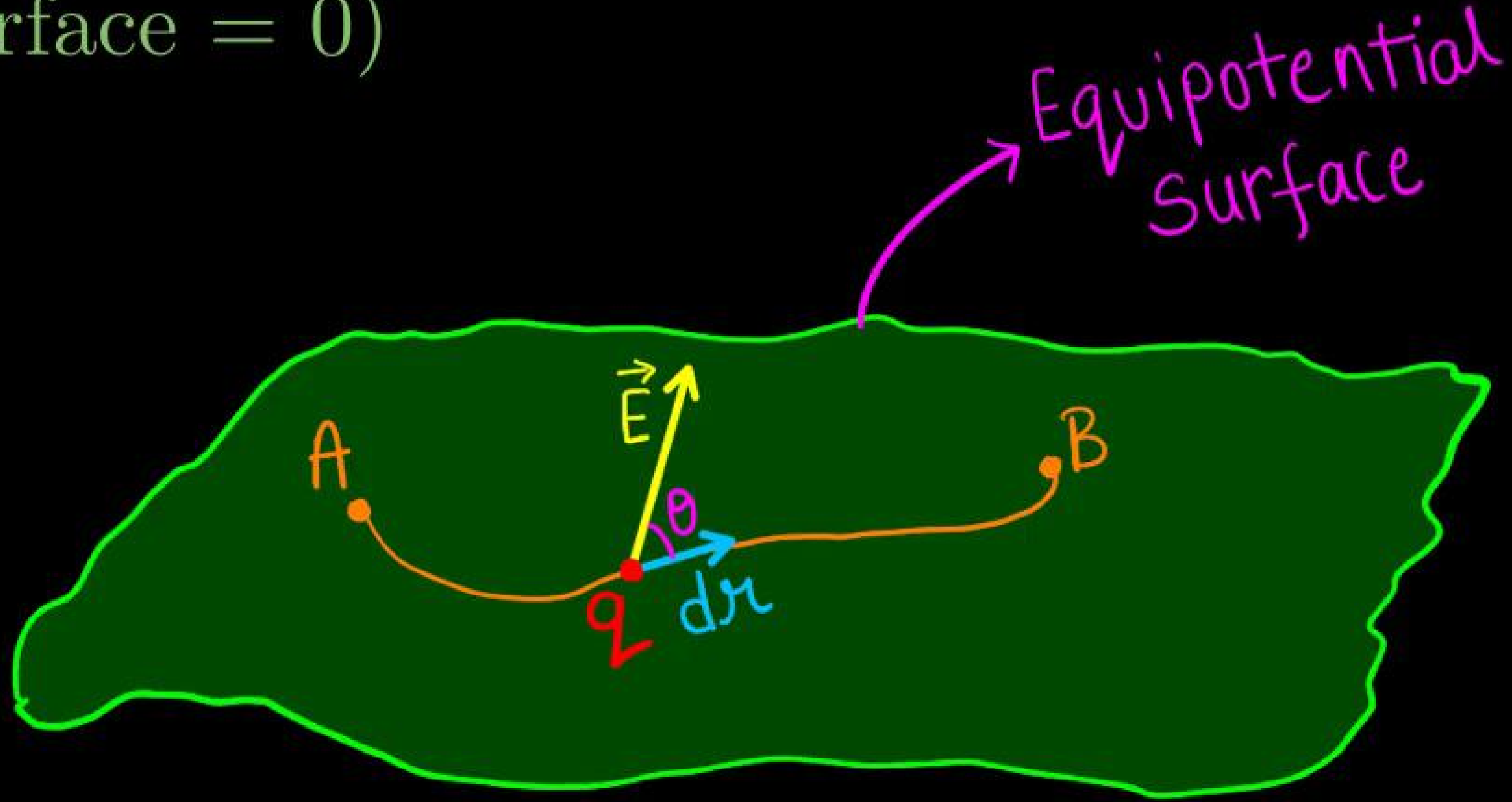
We know that $dW = Fdr\cos\theta = qEdr\cos\theta$

$\because dW = 0$ (Work done over equipotential surface $= 0$)

$\therefore dW = qEdr\cos\theta = 0$

Neither q nor E is zero; dr is also not zero.

So, $\cos\theta = 0$ Or $\theta = 90°$



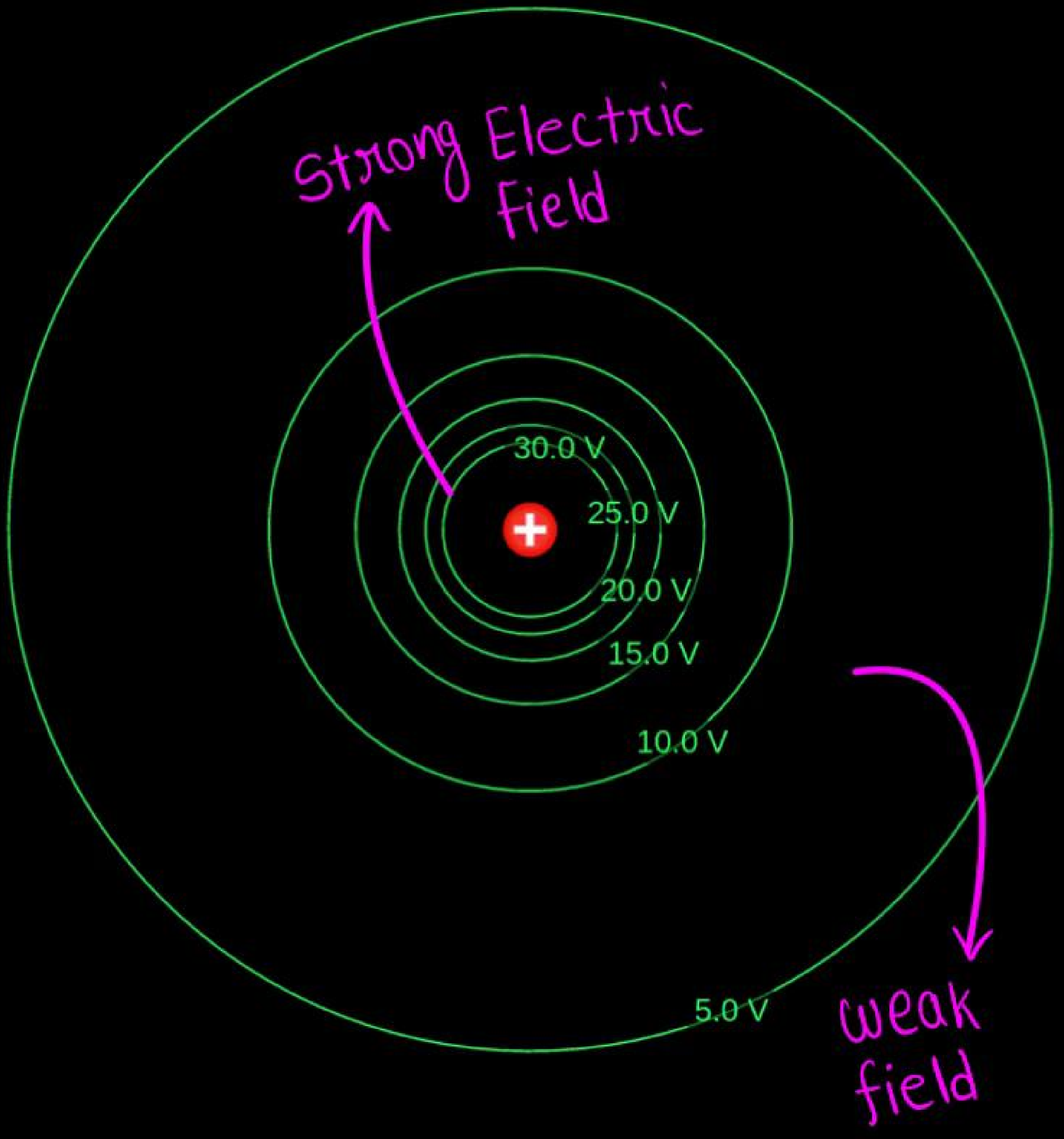
EQUIPOTENTIAL SURFACES

Properties of equipotential surfaces

- In a region, where electric field ($\vec{E}$) is strong equipotential surfaces are close together, and where ($\vec{E}$) is weaker, the equipotential surfaces are farther apart.

- Two equipotential surfaces never intersect each other.

  As electric field is perpendicular to the equipotential surface their intersection means that there are two directions of electric field at the intersection point which is not possible.

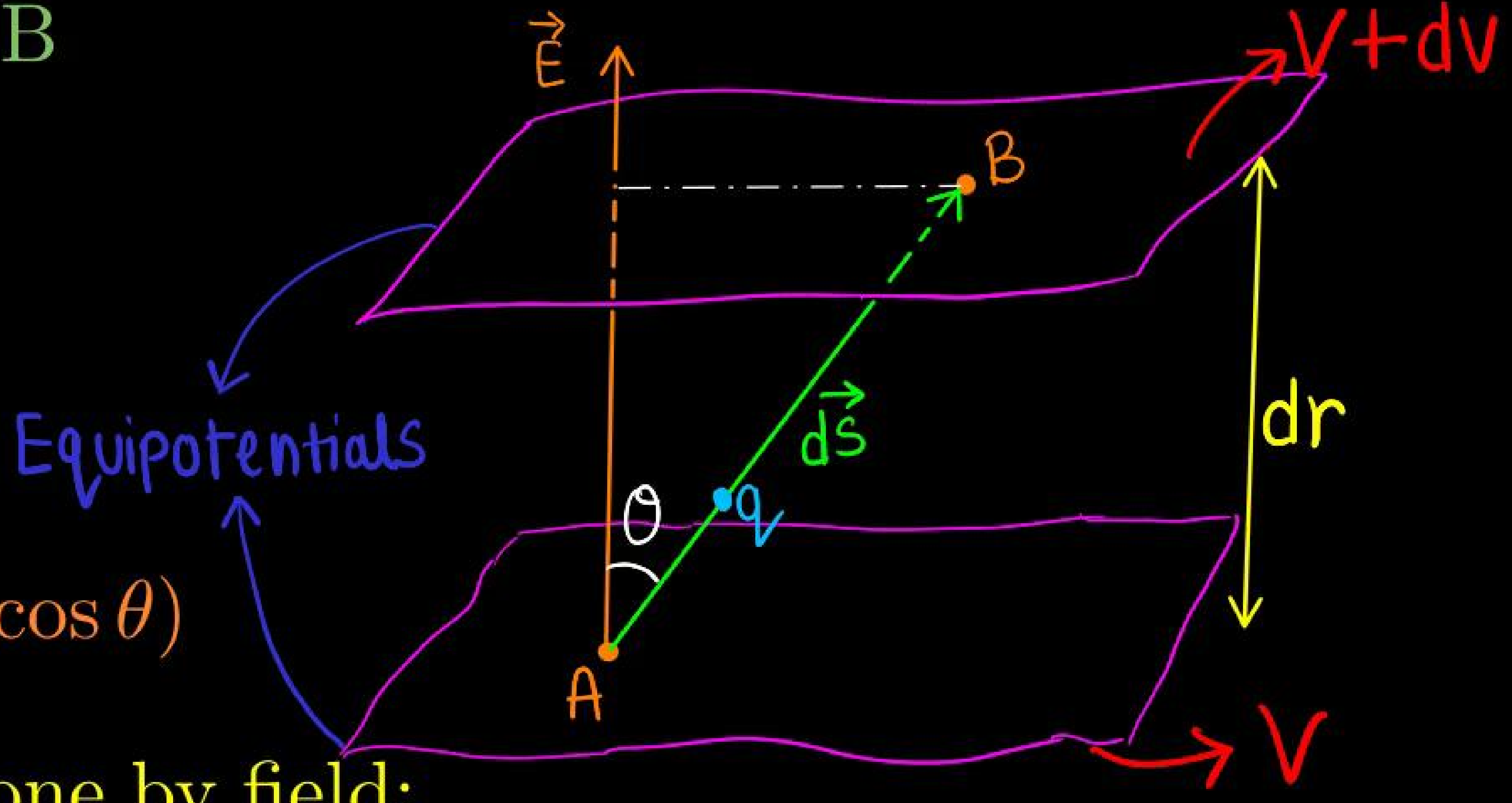


RELATION BETWEEN FIELD AND POTENTIAL

- Consider two closely spaced equipotential surfaces with potential values V and $V+dV$.
- dr is the perpendicular distance between the two equipotential surfaces.
- Imagine that a unit positive charge (q) is moved from point A to point B
- $d\vec{s}$ is the displacement vector from A to B
- Work done by the electric field $\vec{E}$ is

$$dW_{\text{field}} = \vec{F_E} \cdot d\vec{s} = q\vec{E} \cdot d\vec{s}$$

$$dW_{\text{field}} = qEds\cos\theta = qEdr \ (\because dr = ds\cos\theta)$$



- Change in Potential energy $= -$ Work done by field:

$$dU = -dW_{\text{field}} = -qEdr$$

RELATION BETWEEN FIELD AND POTENTIAL

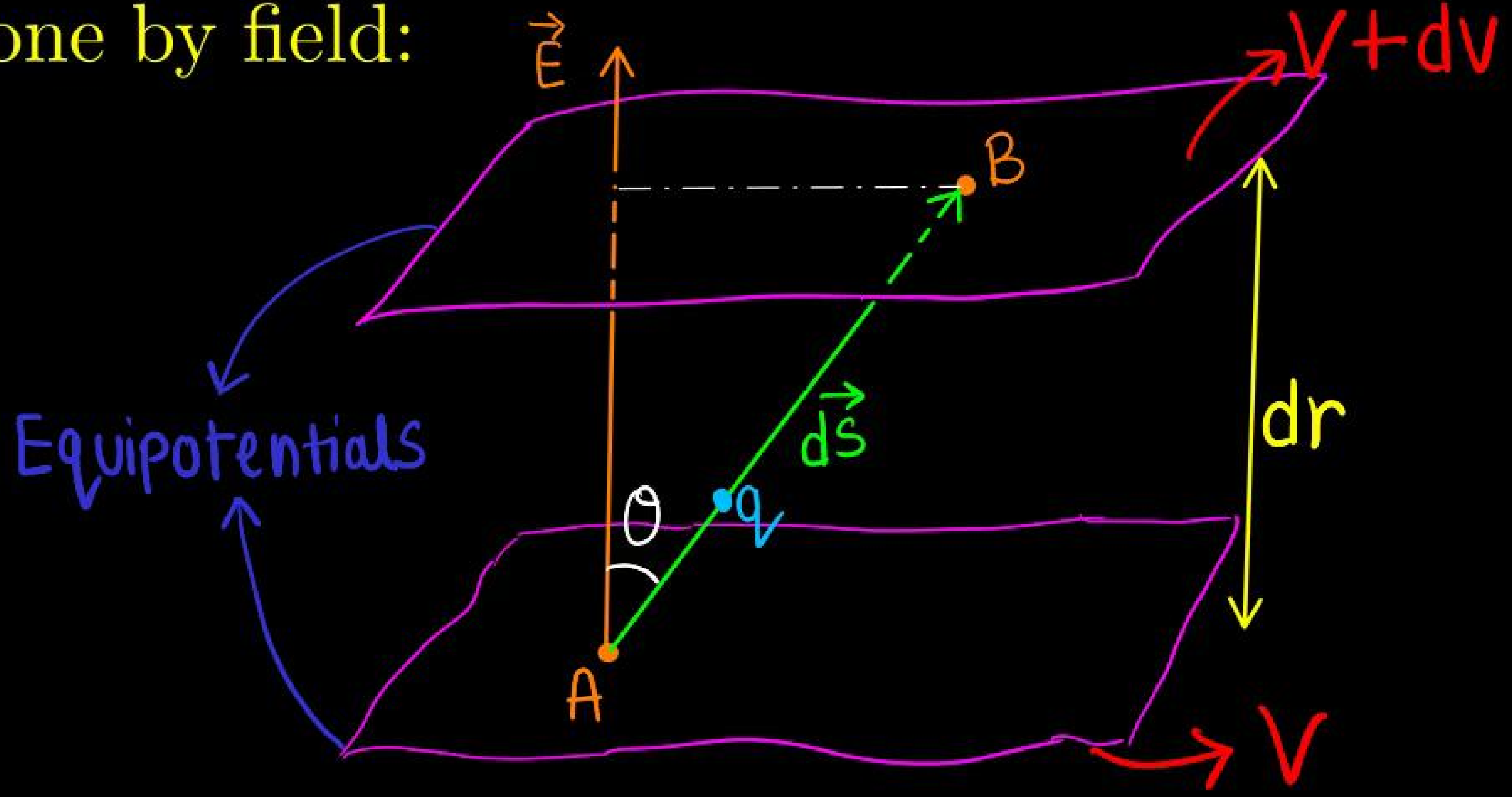
- Change in Potential energy = − Work done by field:

$$dU = -dW_{\text{field}} = -qEdr$$

- Change in Potential $dV = \frac{dU}{q}$

$$\boxed{dV = -Edr}$$

$$\boxed{E = -\frac{dV}{dr}}$$



- Electric field is in the direction in which the potential decreases steepest.
- Magnitude of Electric field is given by the change in the magnitude of potential (dV) per unit displacement (dr) normal to the equipotential surface at the point.

Example 6 :The electric potential V at a point $P(x, y, z)$ in space is given by $V = 4x^2$ volt. Electric field at point (1m, 0, 2m) in V/m is

(a) 8 along -ve x-axis

(b) 8 along +ve x-axis

(c) 16 along -ve x-axis

(d) 16 along +ve x-axis

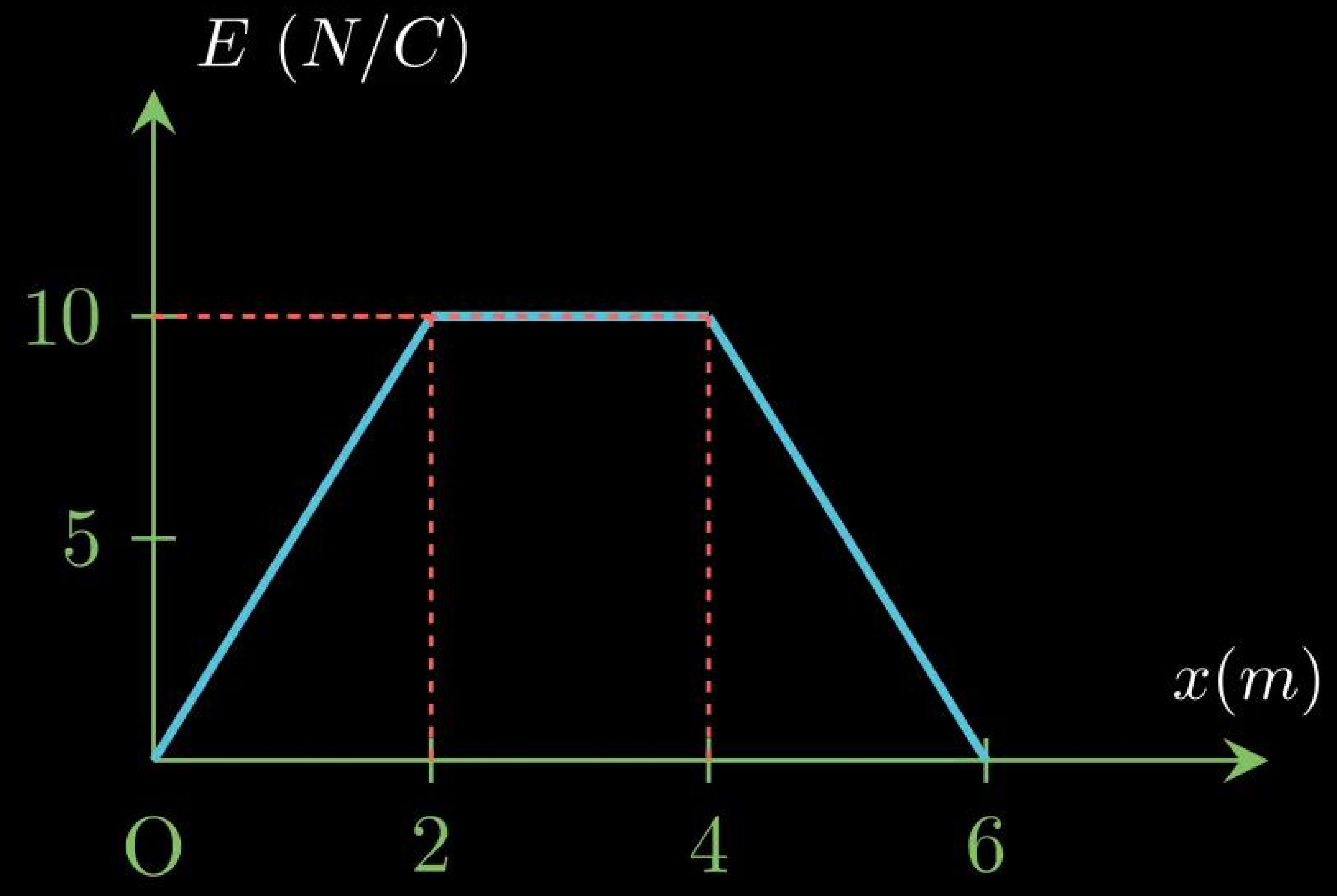
Solution:

Example 7 :Figure shows the variation of electric field intensity E versus distance x. What is the potential difference between the points at $x = 2$ m and $x = 6$ m from O?

(a) 30 V (b) 60 V

(c) 40 V (d) 80 V

Solution:



Example 8 :An infinite plane sheet of charge density $10^{-8}\ Cm^{-2}$ is held in air. In this situation how far apart are two equipotential surfaces, whose potential difference is 5 V?

(a) 2.25 mm

(b) 3.52 mm

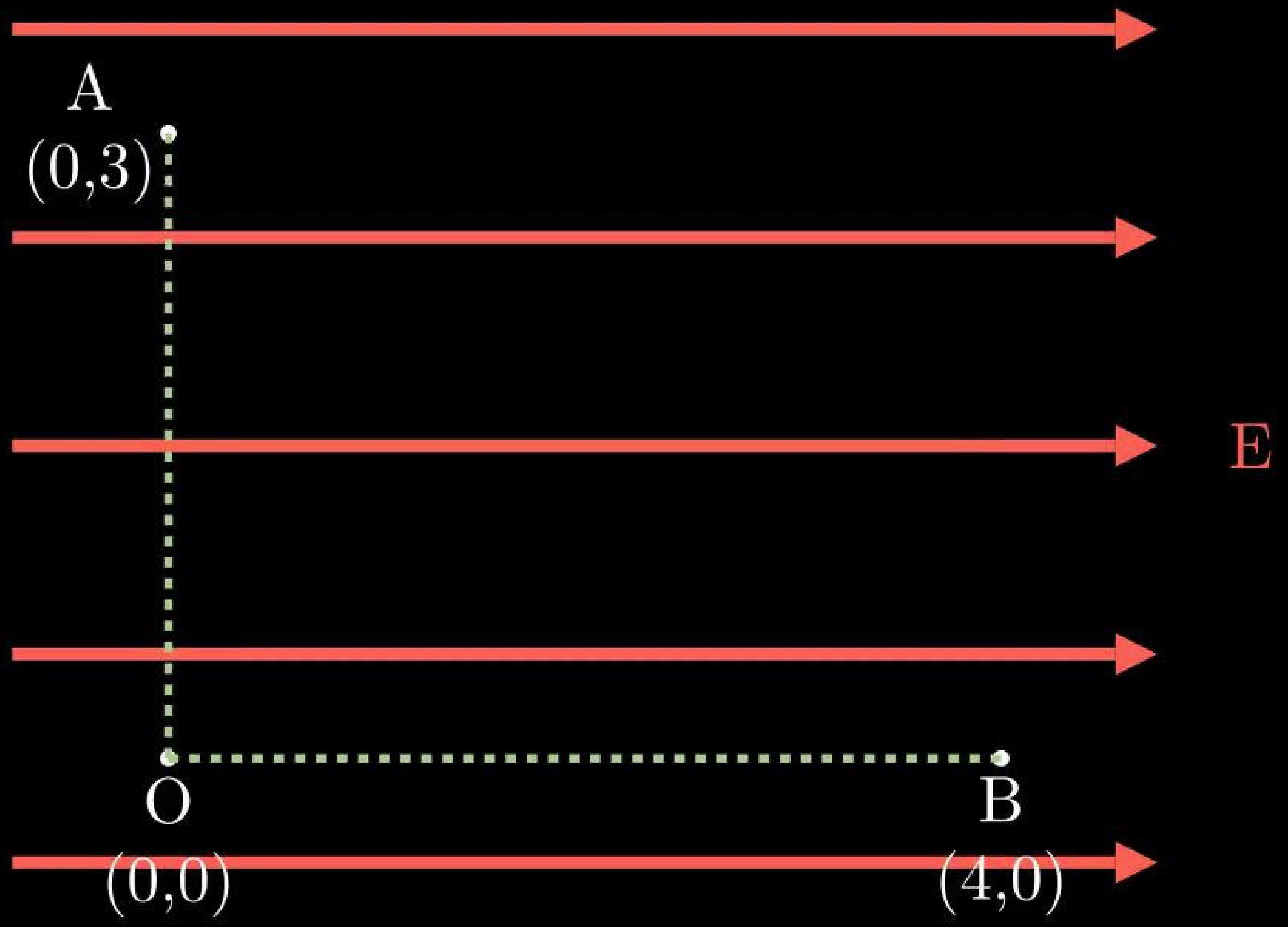
(c) 6 mm

(d) 8.85 mm

Solution:

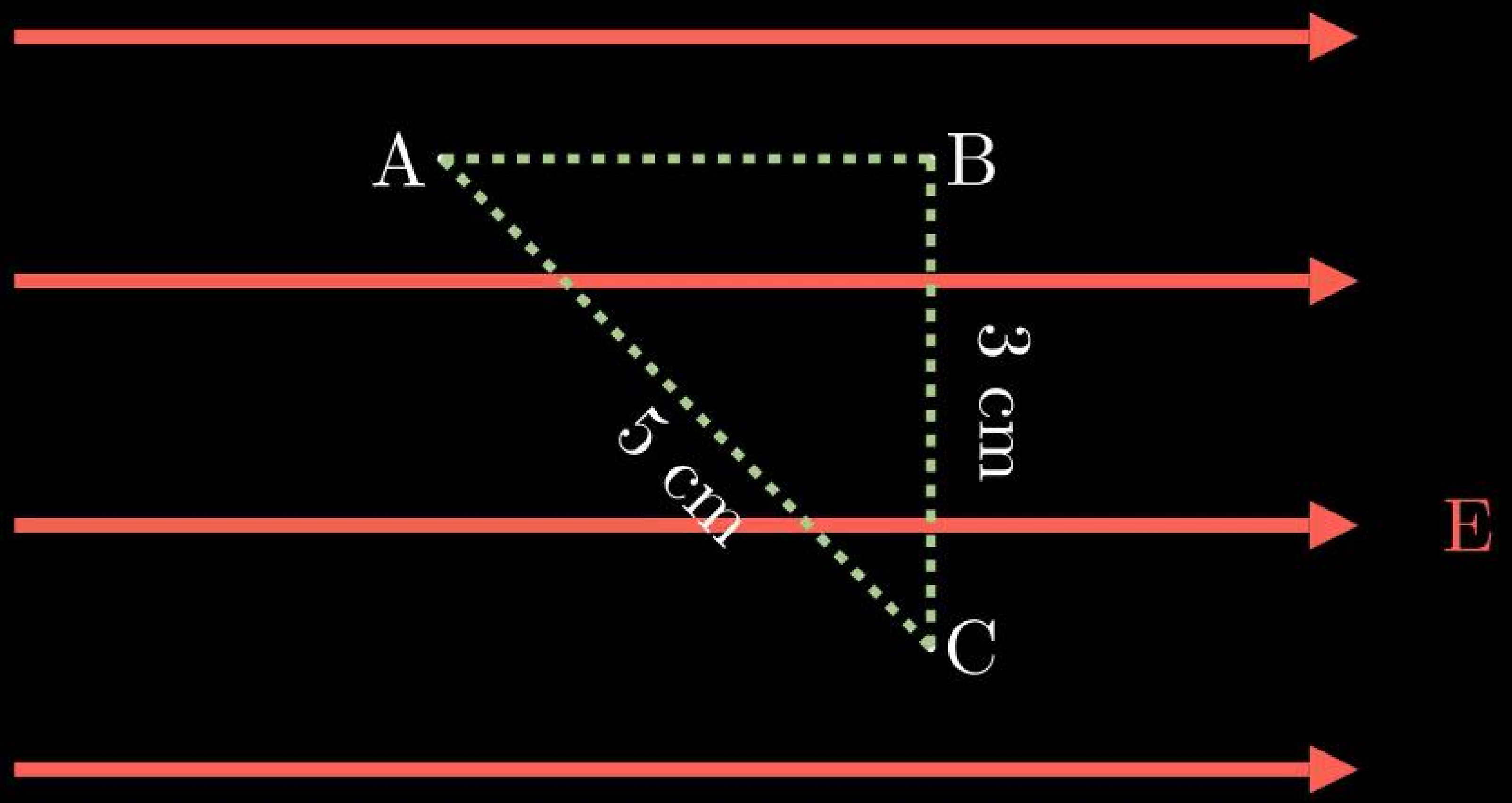
Example 9 :A uniform electric field E of 500 N/C is directed along +x-axis. O, B and A are three points in the field having x- and y-coordinates (in cm) (0, 0), (4, 0) and (0, 3) respectively. Calculate the potential difference between the points (i) O and A, and (ii) O and B. [CBSE 23C]

Solution:



Example 10 :Three points A, B and C lie in a uniform electric field (E) of $5 \times 10^3\ NC^{-1}$ as shown in the figure. Find the potential difference between A and C. [CBSE F 09]

Solution:



Example 11 :If the potential in the region of space around the point (-1 m, 2 m, 3 m) is given by $V = (10x^2 + 5y^2 - 3z^2)$ volt, calculate the three components of electric field at this point.

Solution: