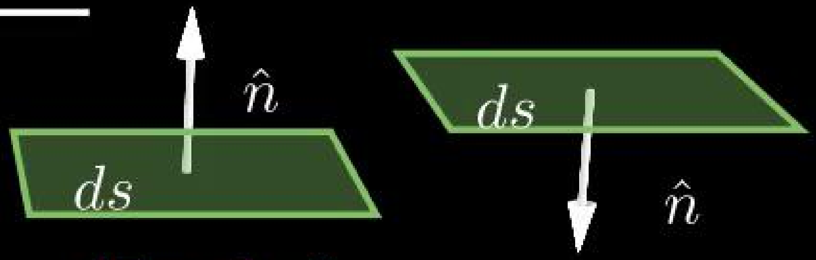
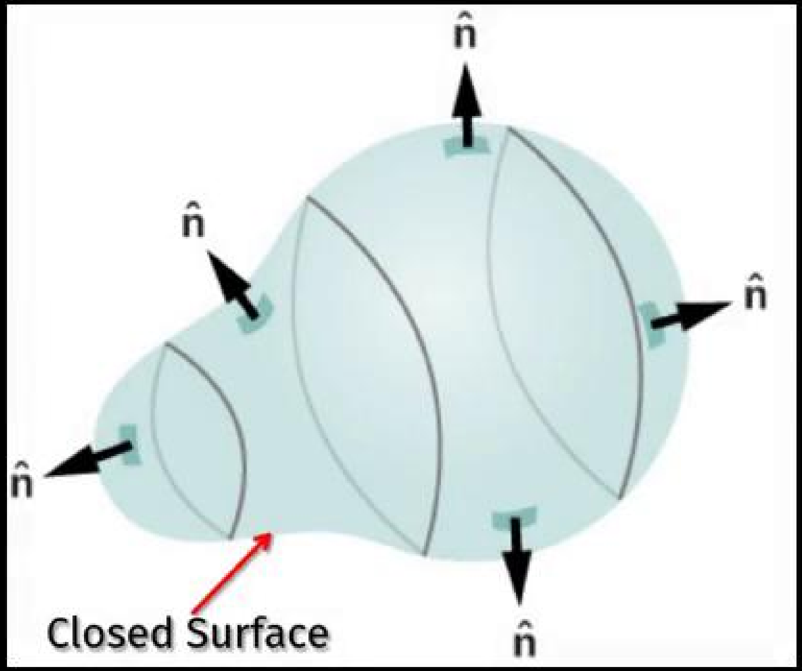
Electric Flux



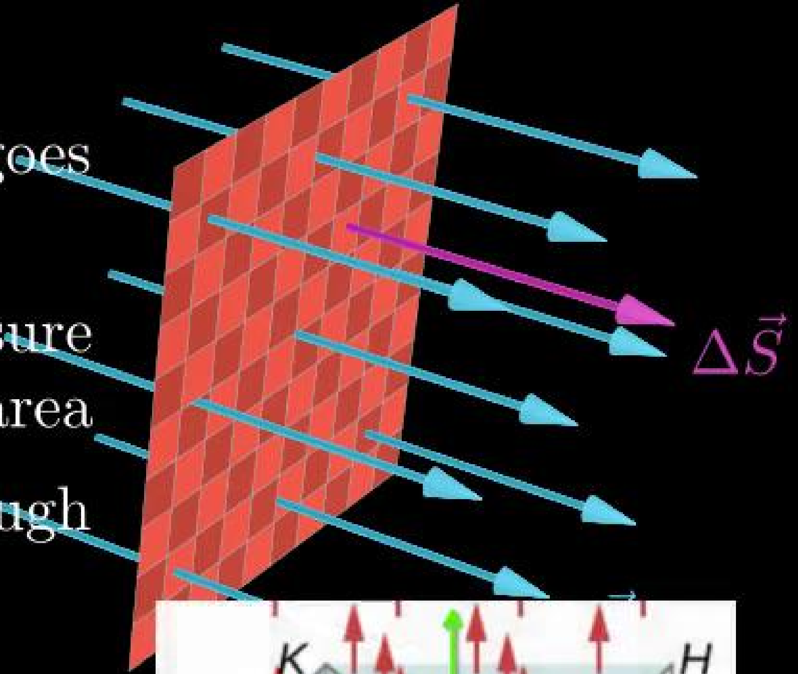
$d\vec{s} = ds\ \hat{n}$

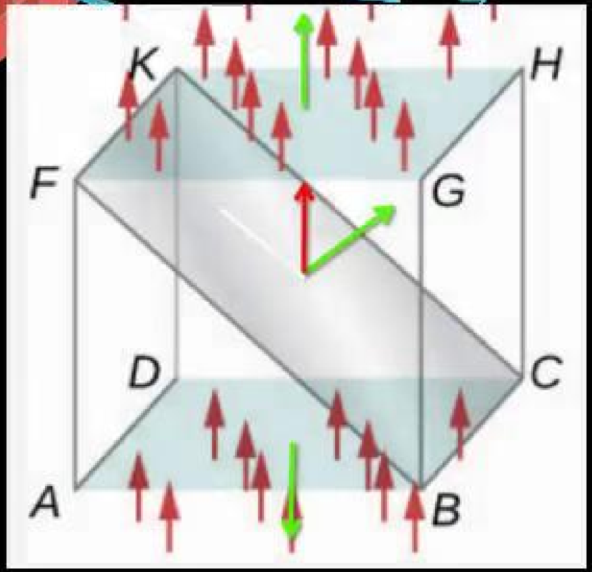
- Sometimes it is useful to treat **area** as a vector. It has magnitude and direction.
- Magnitude is equal to area ds
- Direction is along the normal to the surface ($\hat{n}$); that is, perpendicular to the surface.
- Since $\hat{n}$ is a unit normal to a surface, it has two possible directions at every point on that surface. For an open surface, we can use either direction, as long as we are consistent over the entire surface.
- On a closed surface, $\hat{n}$ is chosen to be the outward normal at every point



Electric Flux

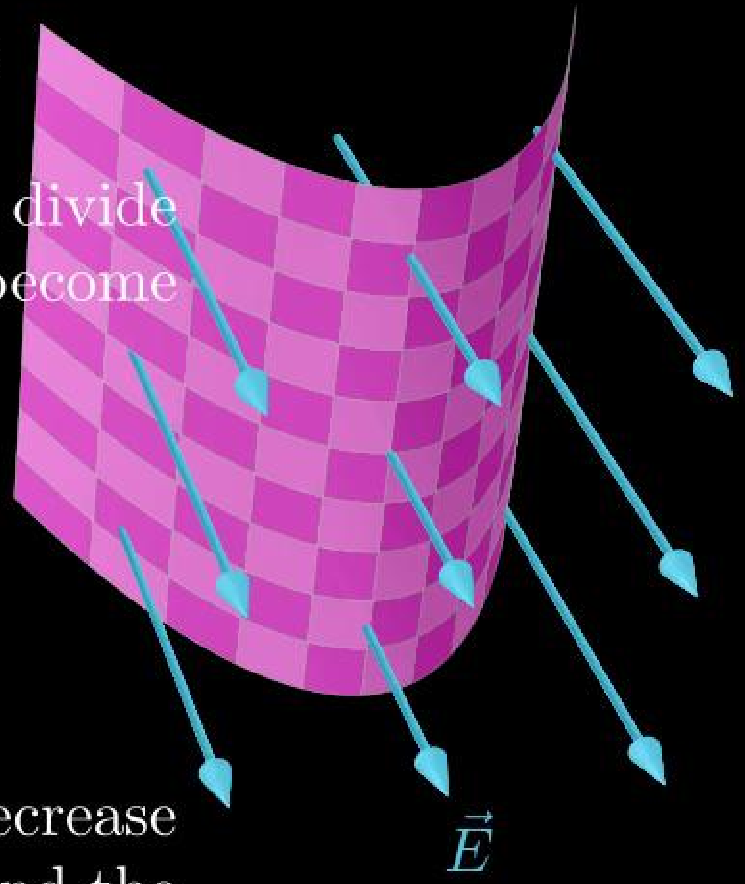
- The concept of **flux** describes how much of something goes through a given area.
- You may conceptualize the Electric Flux ($\Delta\phi$) as a measure of the number of electric field lines passing through an area
- The larger the area (ΔS), the more field lines go through it and, hence, the greater the flux (i.e., $\Delta\phi \propto \Delta S$)
- Similarly, the stronger the electric field is (represented by a greater density of lines), the greater the flux. (i.e., $\Delta\phi \propto E$)
- Similarly, Larger the value of $\cos\theta$ (i.e., at $\theta = 0°$), the more field lines go through it, hence, the greater the flux (i.e., $\Delta\phi \propto \cos\theta$)
- Electric Flux: $\boxed{\Delta\phi = E\Delta S\cos\theta = \vec{E}\cdot\Delta\vec{S}}$





Electric Flux

- Here, θ is the angle between $\vec{E}$ and Area vector $\Delta\vec{S}$
- If $\vec{E}$ is not uniform or if S is a curved surface, we divide S into many small elemetns ΔS, as the elements become smaller, they can be approximated by flat surfaces.
- Then electric flux through the area element ΔS is $\Delta\phi = \vec{E} \cdot \Delta\vec{S}$
- Then the totla flux through entire surface S is $\phi \approx \sum_{i=1}^{n} \vec{E} \cdot \Delta\vec{S}$
- This estimate of the total flux gets better as we decrease the size of the area elements i.e., $(\Delta S \to 0 = dS)$. and the limit of the sum becomes a surface integral.



- $\boxed{\phi = \int_S \vec{E} \cdot d\vec{S} \text{ (For Open surface)}}$

$\boxed{\phi = \oint_S \vec{E} \cdot d\vec{S} \text{ (For Closed surface)}}$

Example 37 : A rectangular surface of sides 10 cm and 15 cm is placed inside a uniform electric field of 25 N/C, such that the surface makes an angle of 30° with the direction of electric field. Find the flux of the electric field through the rectangular surface.

(a) 0.1675 Nm^2C^{-1}

(b) 0.1875 Nm^2C^{-1}

(c) Zero

(d) 0.1075 Nm^2C^{-1}

Solution:

Example 38 : If an electric field is given by $10\hat{i} + 3\hat{j} + 4\hat{k}$, calculate the electric flux through a surface area of 10 units lying in yz plane

(a) 100 units

(b) 10 units

(c) 30 units

(d) 40 units

Solution:

Example 39 : If an electric field in a region is given by $a\hat{i} + b\hat{j}$, where a and b are constants. Find the net flux through a square area of side l parallel to y-z plane.

Solution:

Example 38 : There is a uniform electric field of $8 \times 10^3\ \hat{i}$ N/C. What is the net flux (in S.I. Units) of the uniform electric field throught a cube of side 0.3 m oriented so that its faces are parallel to the coordinate plane?

(a) 16×10^3 (b) 2.4×10^3

(c) Zero (d) 48×10^3

Solution:

Gauss's Law

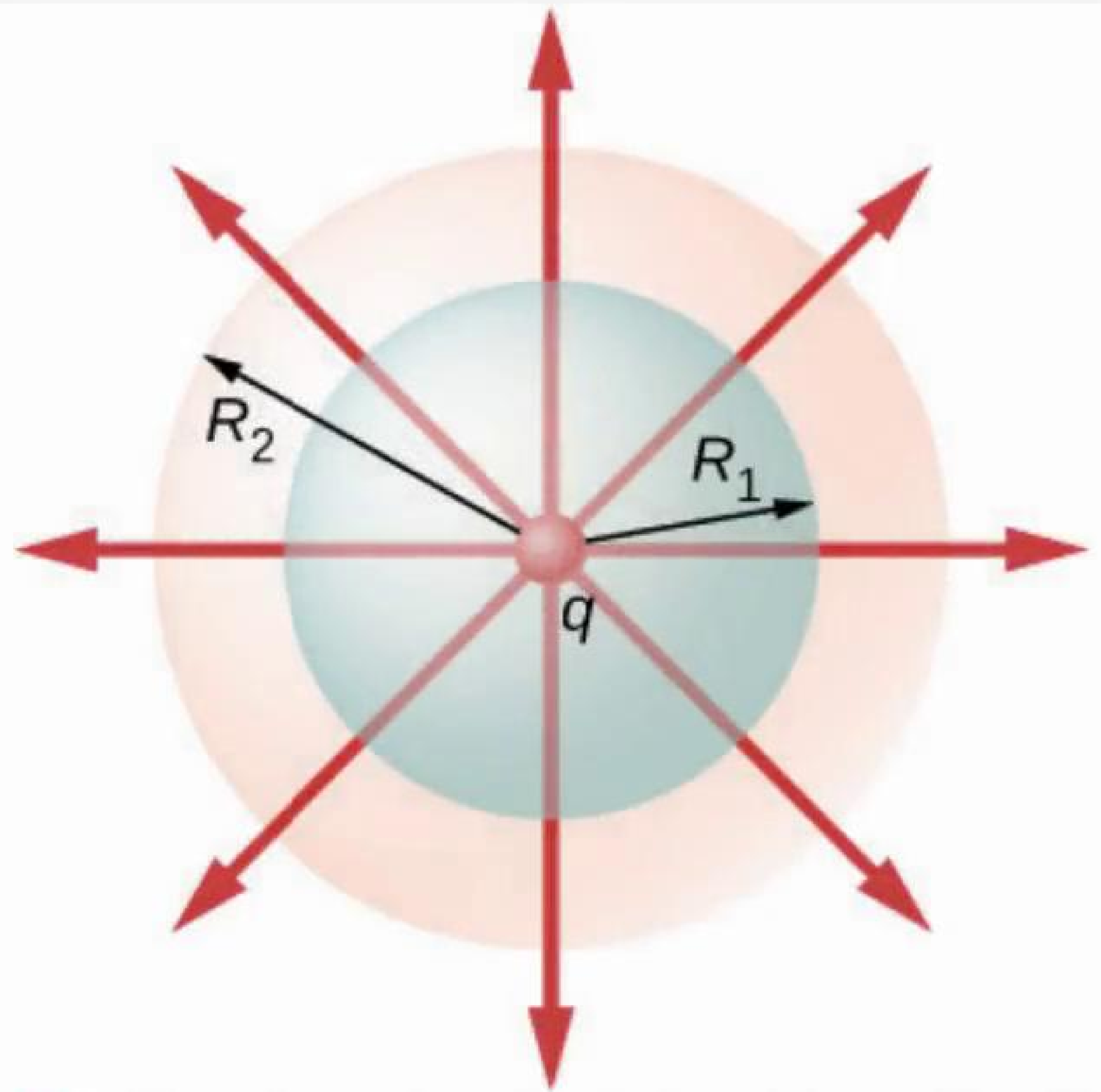
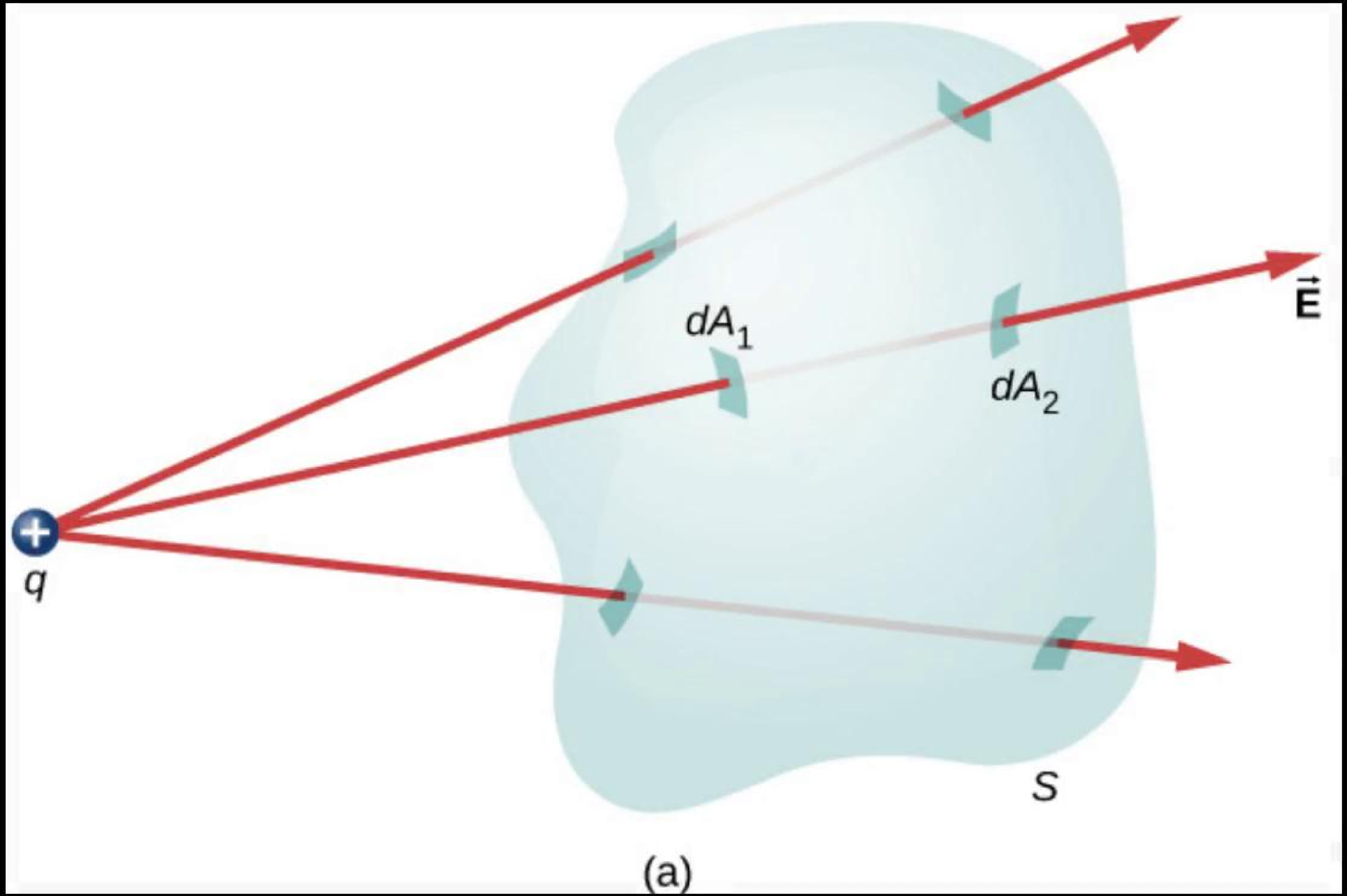


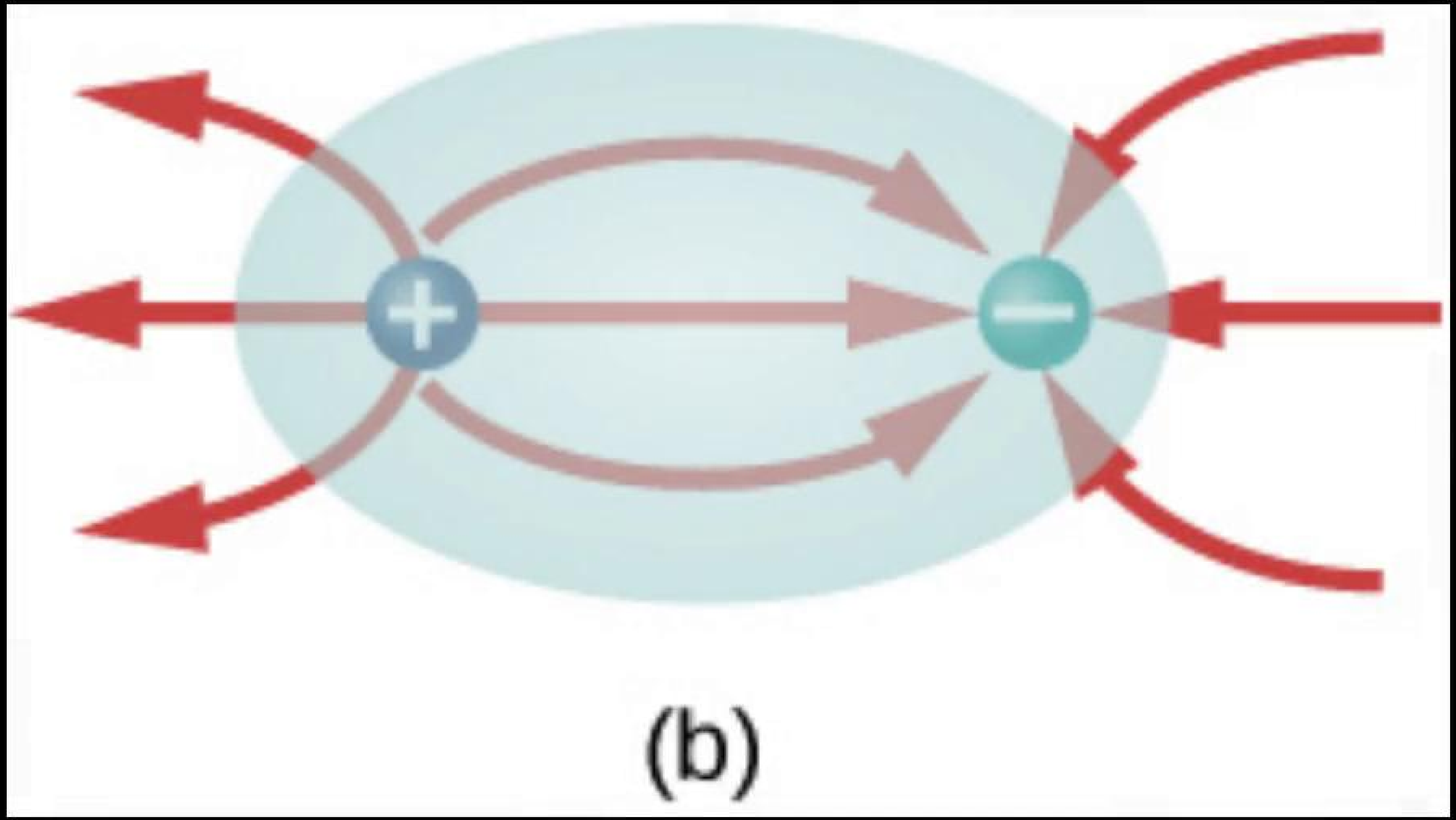
Figure 6.14 Flux through spherical surfaces of radii R_1 and R_2 enclosing a charge q are equal, independent of the size of the surface, since all E-field lines that pierce one surface from the inside to outside direction also pierce the other surface in the same direction.

Gauss's Law



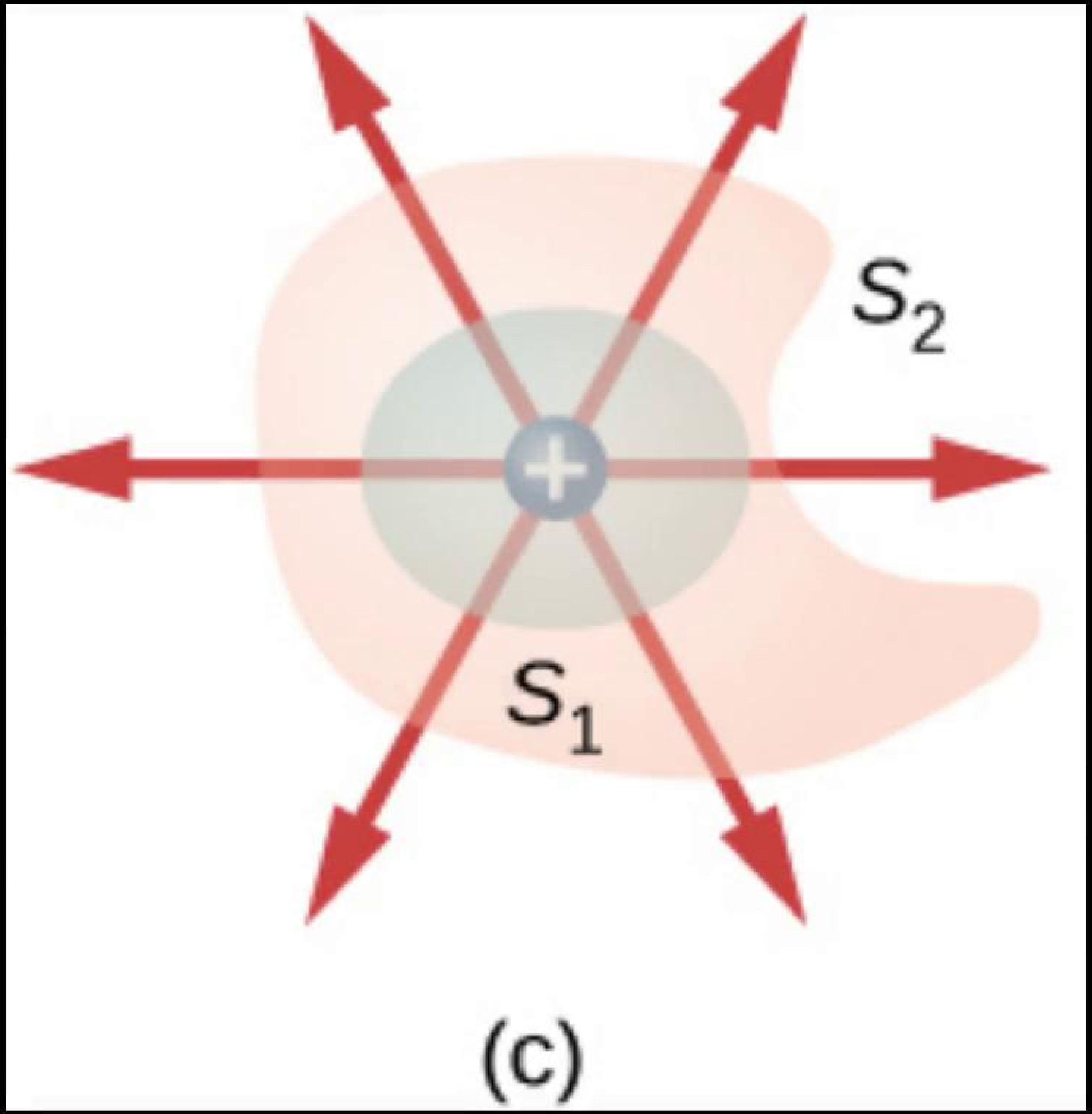
(a) The electric flux through a closed surface due to a charge outside that surface is zero.

Gauss's Law



(b) Charges are enclosed, but because the net charge included is zero, the net flux through the closed surface is also zero.

Gauss's Law



(c) The shape and size of the surfaces that enclose a charge does not matter because all surfacesenclosing the same charge have the same flux.

Gauss's Law

Statement of Gauss's Law :

According to Gauss's law, the flux (Φ) of the electric field (E) through any closed surface(S), also called a Gaussian surface, is equal to $\frac{1}{\epsilon_0}$ times the net charge enclosed (q_{enc}) by the surface.

$$\Phi_{\text{Closed surface}} = \oint_s \vec{E} \cdot d\vec{S} = \frac{q_{enc}}{\epsilon_0}$$

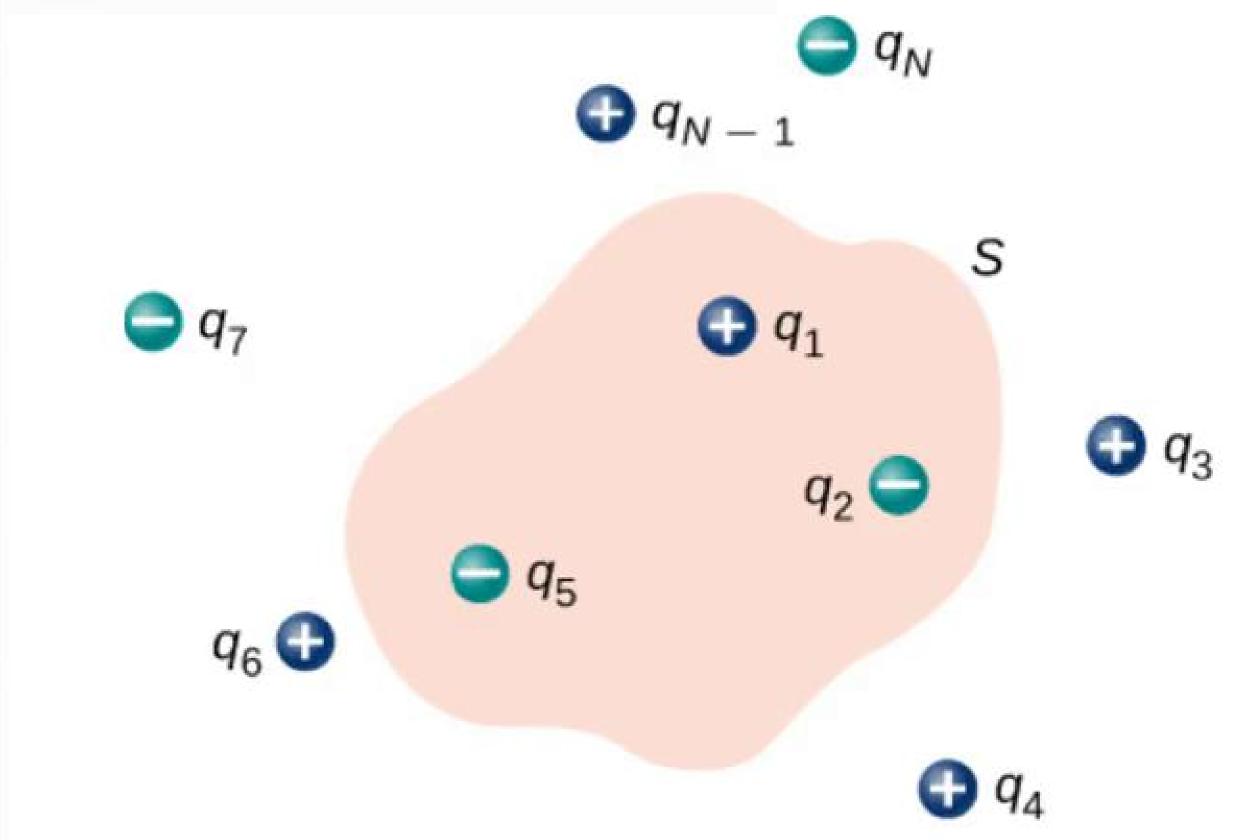


Figure – The flux through the Gaussian surface shown, due to the charge distribution, is $\Phi = (q_1 + q_2 + q_5)/\varepsilon_0$.

Gauss's Law

Important Points Regarding Gauss's Law :

$$\Phi_{\text{Closed surface}} = \oint_s \vec{E} \cdot d\vec{S} = \frac{q_{enc}}{\epsilon_0}$$

- The term q_{enc} in Gauss's law is just the net charge enclosed (or inside) (i.e. q_1, q_2 and q_5) the Gaussian surface(S)
- Charges Outside the surface (i.e., q_3, q_4, q_6, ..., q_n), no matter how large or how nearby it may be, is not included in the term q_{enc} in Gauss law.
- The electric field $\vec{E}$ used in the Gauss's law is the total electric field at every point on the Gaussian surface, due to **all charges inside or outside** the Gaussian surface.

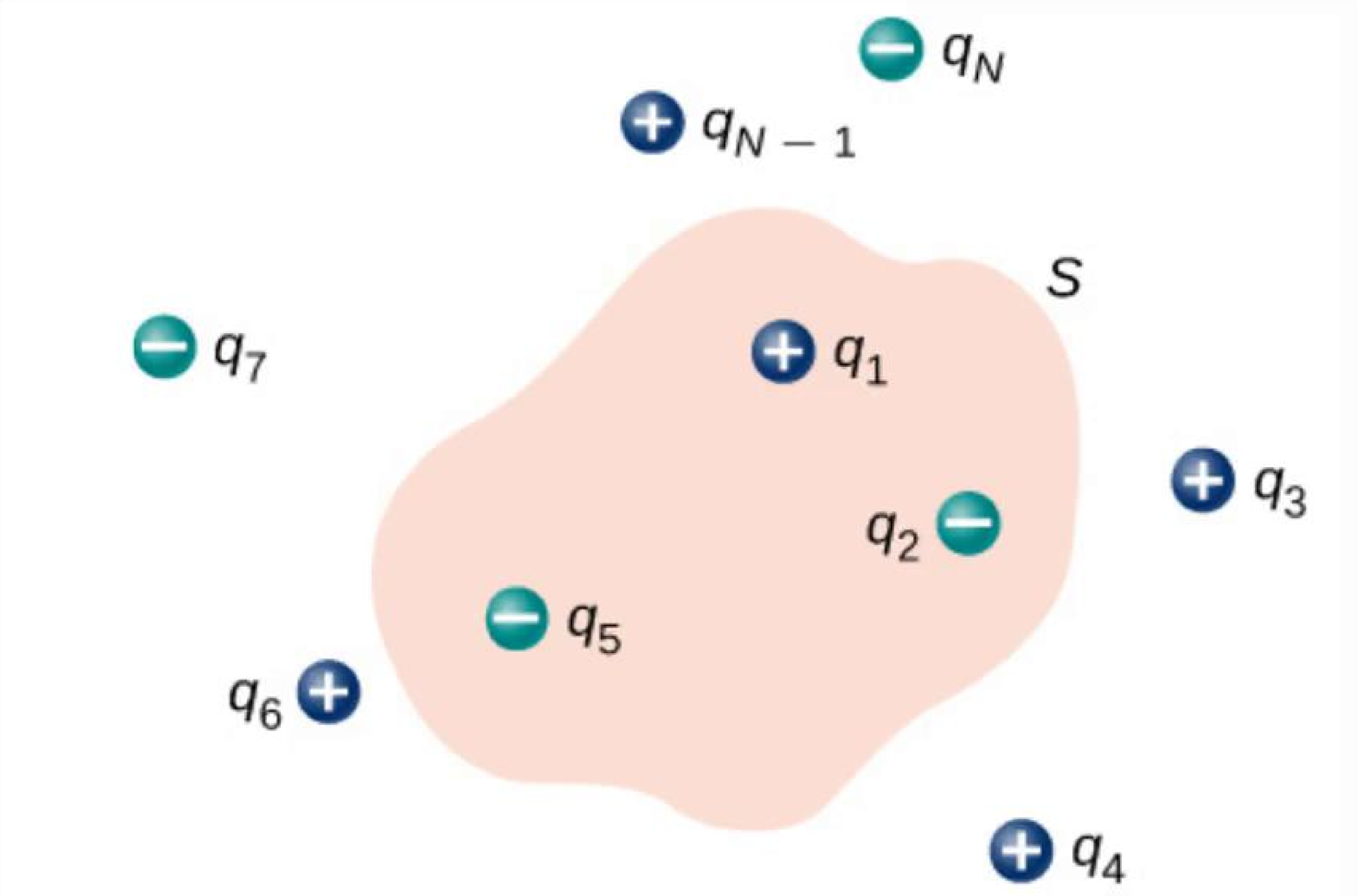


Figure – The flux through the Gaussian surface shown, due to the charge distribution, is $\Phi = (q_1 + q_2 + q_5)/\varepsilon_0$.

Gauss's Law

Important Points Regarding Gauss's Law :

$$\Phi_{\text{Closed surface}} = \oint_s \vec{E} \cdot d\vec{S} = \frac{q_{enc}}{\epsilon_0}$$

- Gaussian surface is any closed surface in space. That surface can coincide with the actual surface of a conductor, or it can be an imaginary geometric surface. The only requirement imposed on a Gaussian surface is that it be closed.
- The Gaussian surface should not pass through any discrete charge because the electric field due to a discrete charge at its location is not defined. However it can pass through a continuous charge distribution.

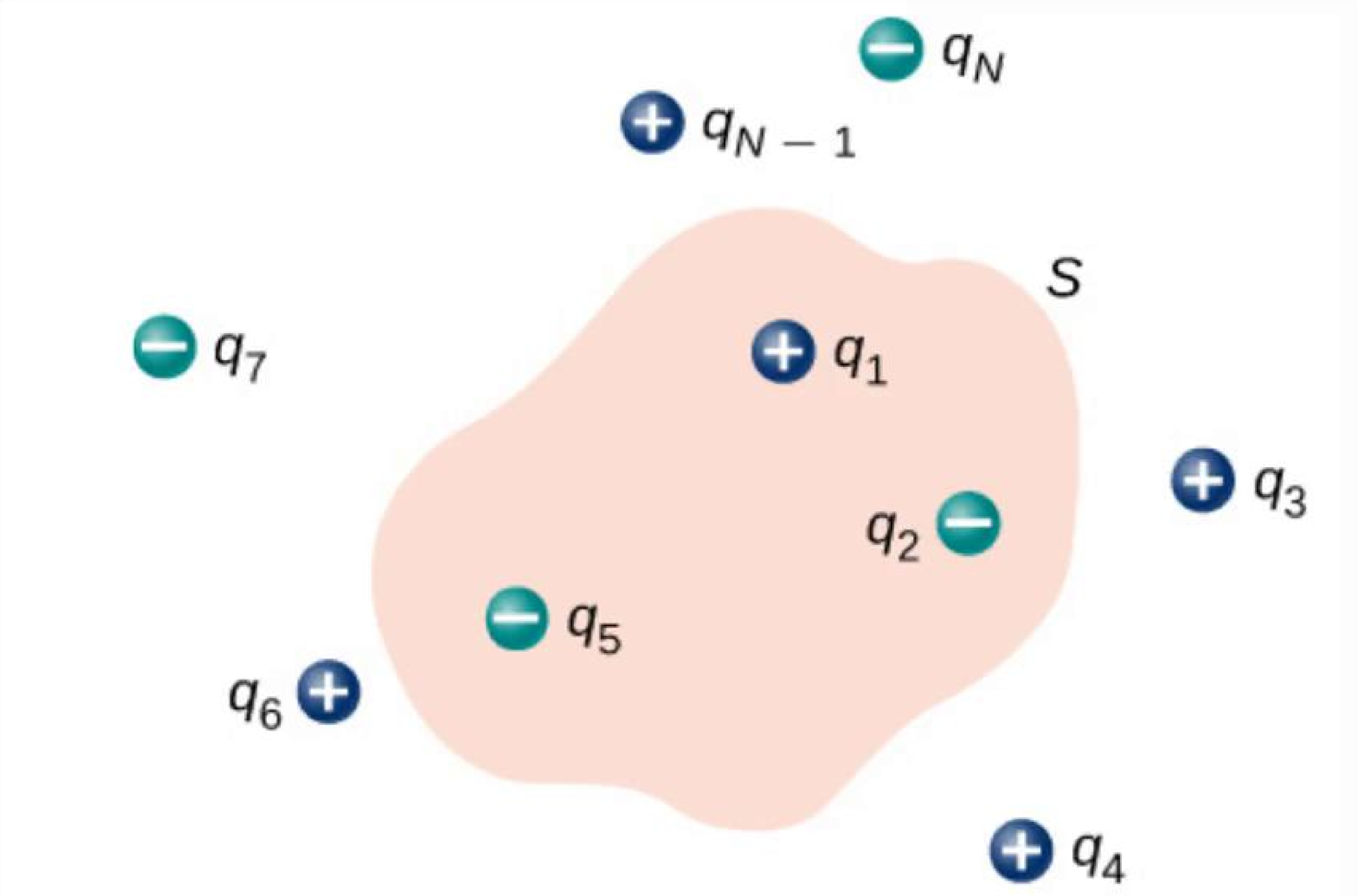


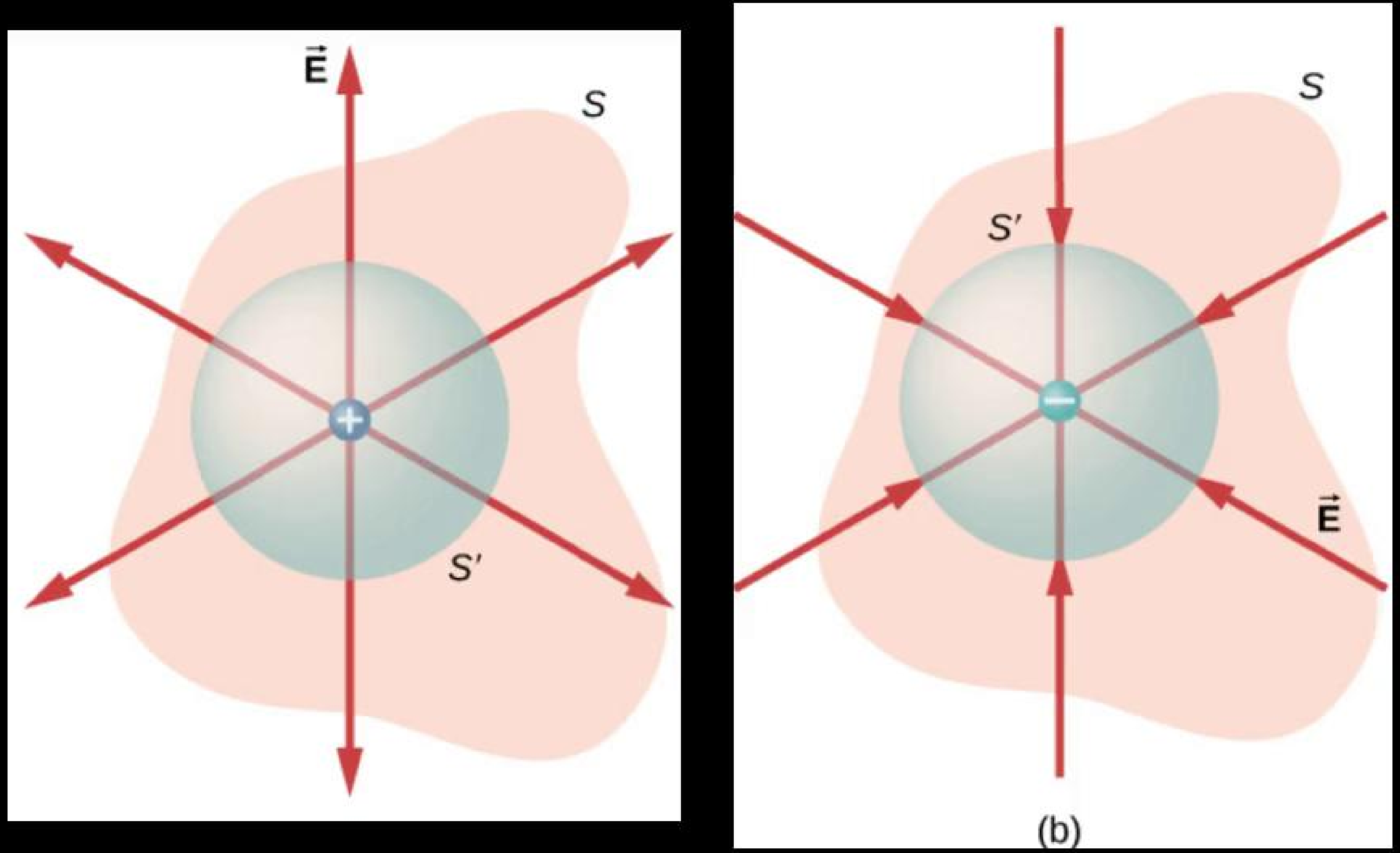
Figure – The flux through the Gaussian surface shown, due to the charge distribution, is $\Phi = (q_1 + q_2 + q_5)/\varepsilon_0$.

Gauss's Law

Important Points Regarding Gauss's Law :

$$\boxed{\Phi_{\text{Closed surface}} = \oint_s \vec{E} \cdot d\vec{S} = \frac{q_{enc}}{\epsilon_0}}$$

- Gauss's law is true for any closed surface, no matter what its shape or size be.
- If q_{enc} is positive, the net flux is outward. If q_{enc} is negative, net flux is inward
- If $q_{enc} = 0$, then $\Phi = \oint_s \vec{E} \cdot d\vec{S} = 0$,But, E may or may not be Zero.
- Gauss's law is commonly used for calculating electric field for symmetric charge configuration.



- Gauss's law and Coulomb's law are equivalent.

Proof of Gauss's Law for Shperically Symmetric Surface :

- Let's calculate the electric flux through a spherical surface around a positive point charge q

  The electric field at a point P on the surface at distance R from the charge at the origin is given by: $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}\hat{r}$

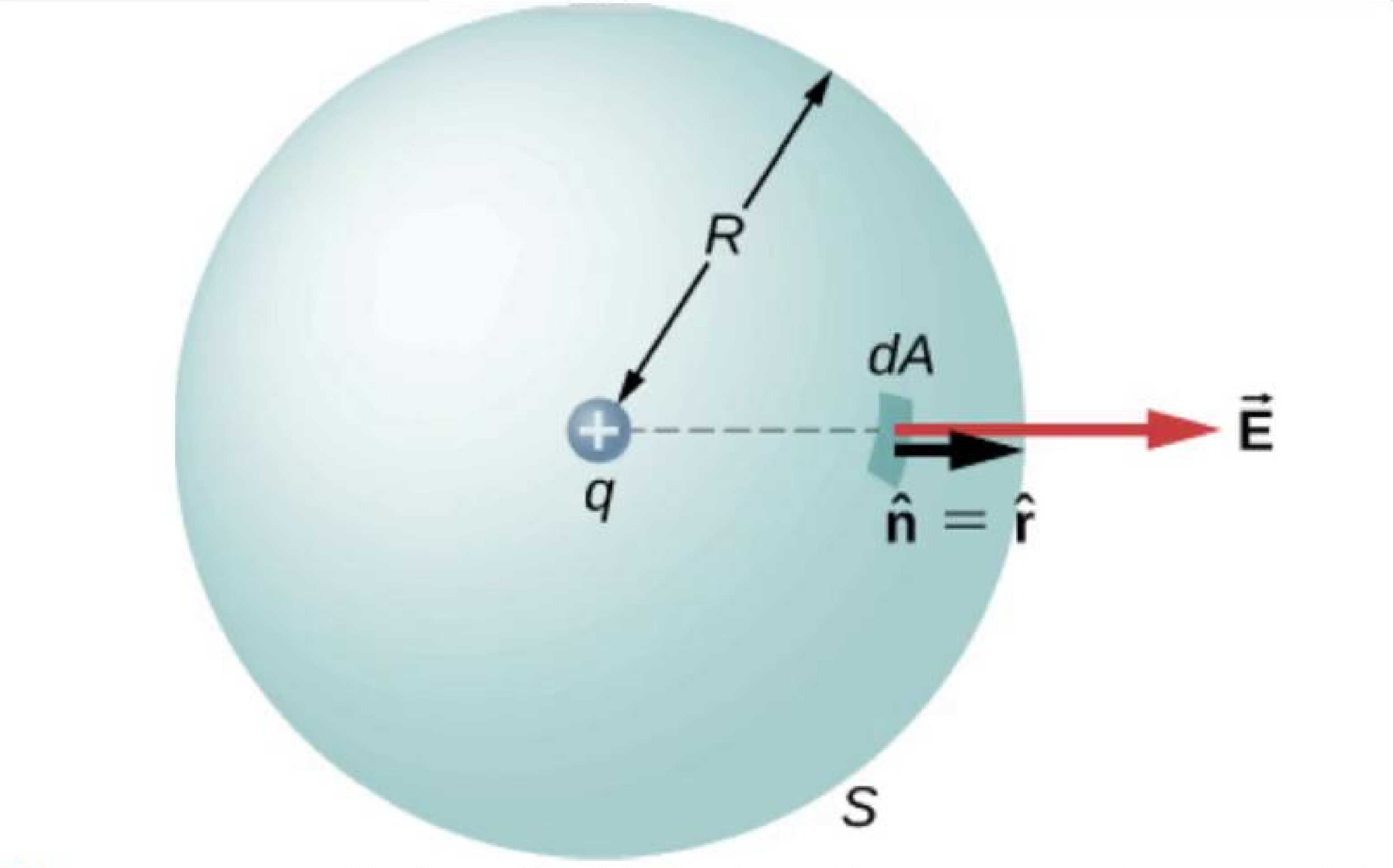


Figure — A closed spherical surface surrounding a point charge q.

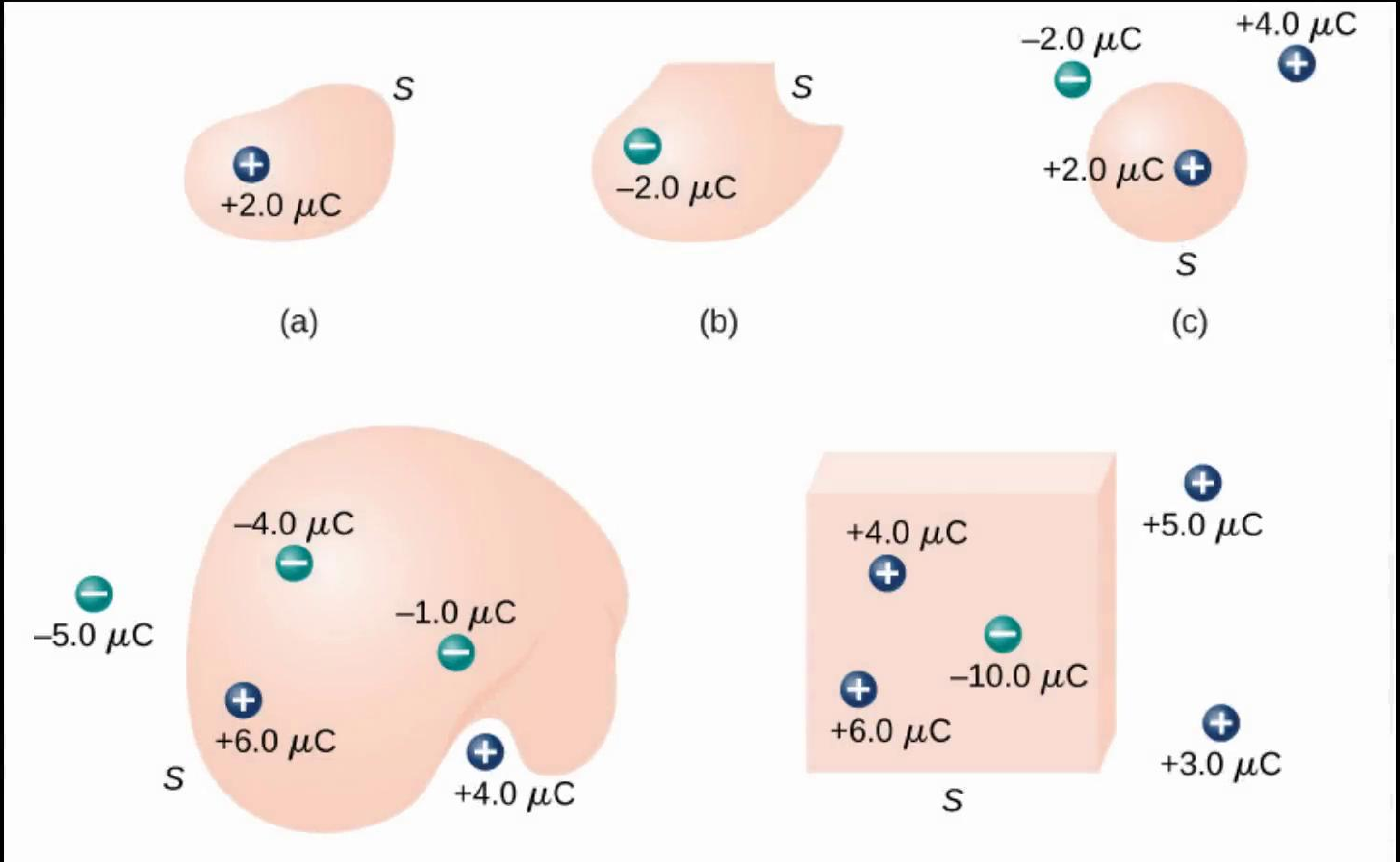
- The total flux (Φ) passing through the spherical surface S is:

$$\Phi = \oint_S \vec{E}\cdot dA\,\hat{n} = \oint_S \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}\hat{r}\cdot\hat{n}dA$$

$$= \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}\oint_S dA = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}\times 4\pi R^2$$

$$\boxed{\Phi = \frac{q}{\epsilon_0}}$$

Example 39 : Calculate the electric flux through each Gaussian surface shown in Figure



Example 40 : A charge q is situated at the centre of a cube. Electric flux through one of the faces of the cube is

(a) $\frac{q}{\epsilon_0}$ (b) $\frac{q}{3\epsilon_0}$

(c) $\frac{q}{6\epsilon_0}$ (d) Zero

Solution:

Example 41 : A chagre q is placed at the centre of the open end of cylindrical vessel. Electric flux through the surface of the vessel is

(a) $\dfrac{q}{2\epsilon_0}$

(b) $\dfrac{q}{\epsilon_0}$

(c) $\dfrac{2q}{\epsilon_0}$

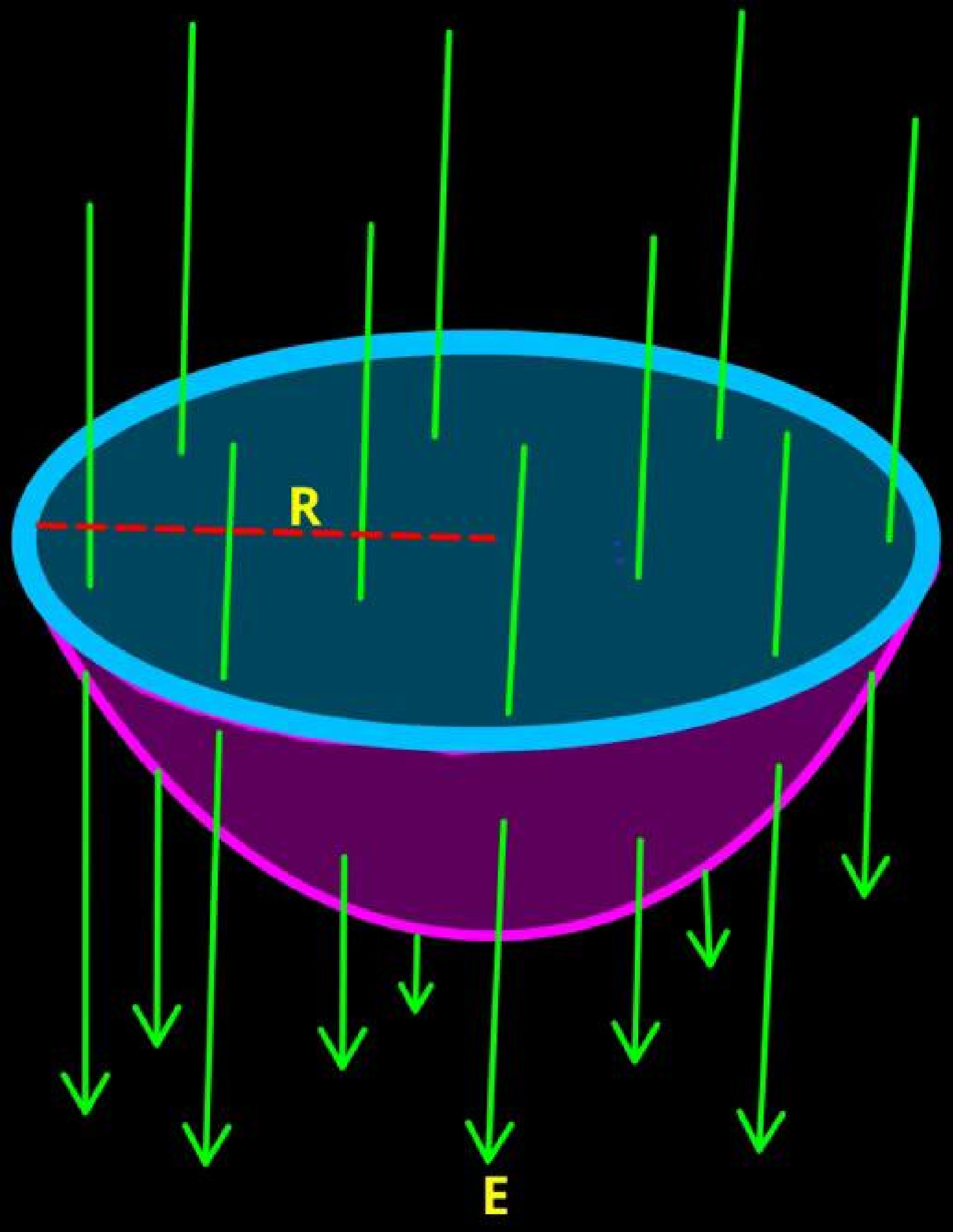
(d) Zero

Solution:

Example 42 : A hemispherical surface of radius R is kept in a uniform electric field E as shown in the figure. The flux through the curved surface is

(a) $E \times 2\pi R^2$ (b) $E \times \pi R^2$

(c) $E \times 4\pi R^2$ (d) Zero



Solution:

Example 43 : A charge of 1 C is located at the centre of a sphere of radius 10 cm and a cube of side 20 cm. The ratio of outgoing flux from the sphre and cube will be

(a) More than one

(b) Less than one

(c) one

(d) Nothing can be said

Solution:

Example 44 : If the nuber of electric lines of force emerging out of a closed surface is 1000, then the charge enclosed by the surface is

(a) 8.854×10^{-9} C

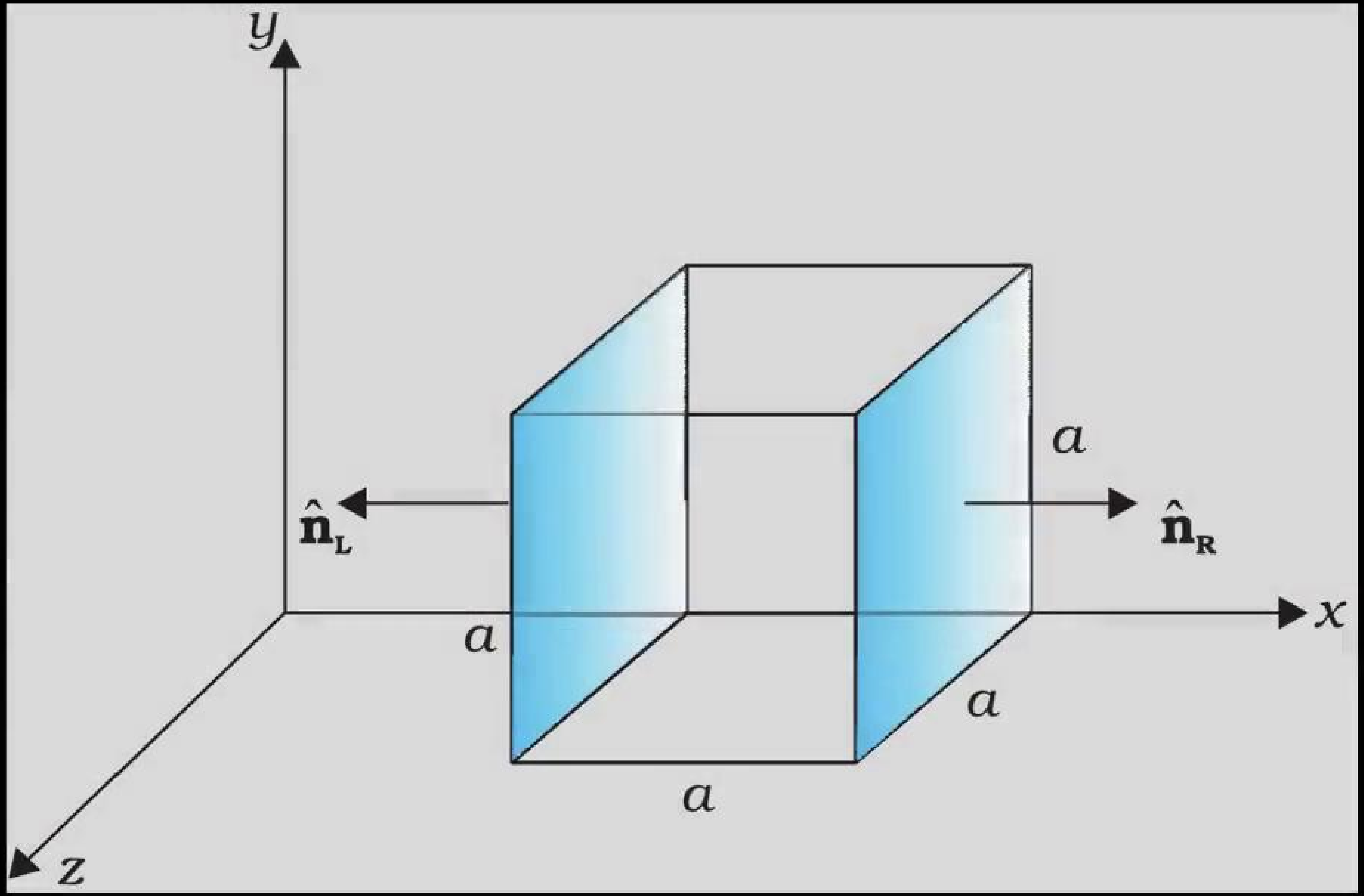
(b) 8.854×10^{-4} C

(c) 8.854×10^{-1} C

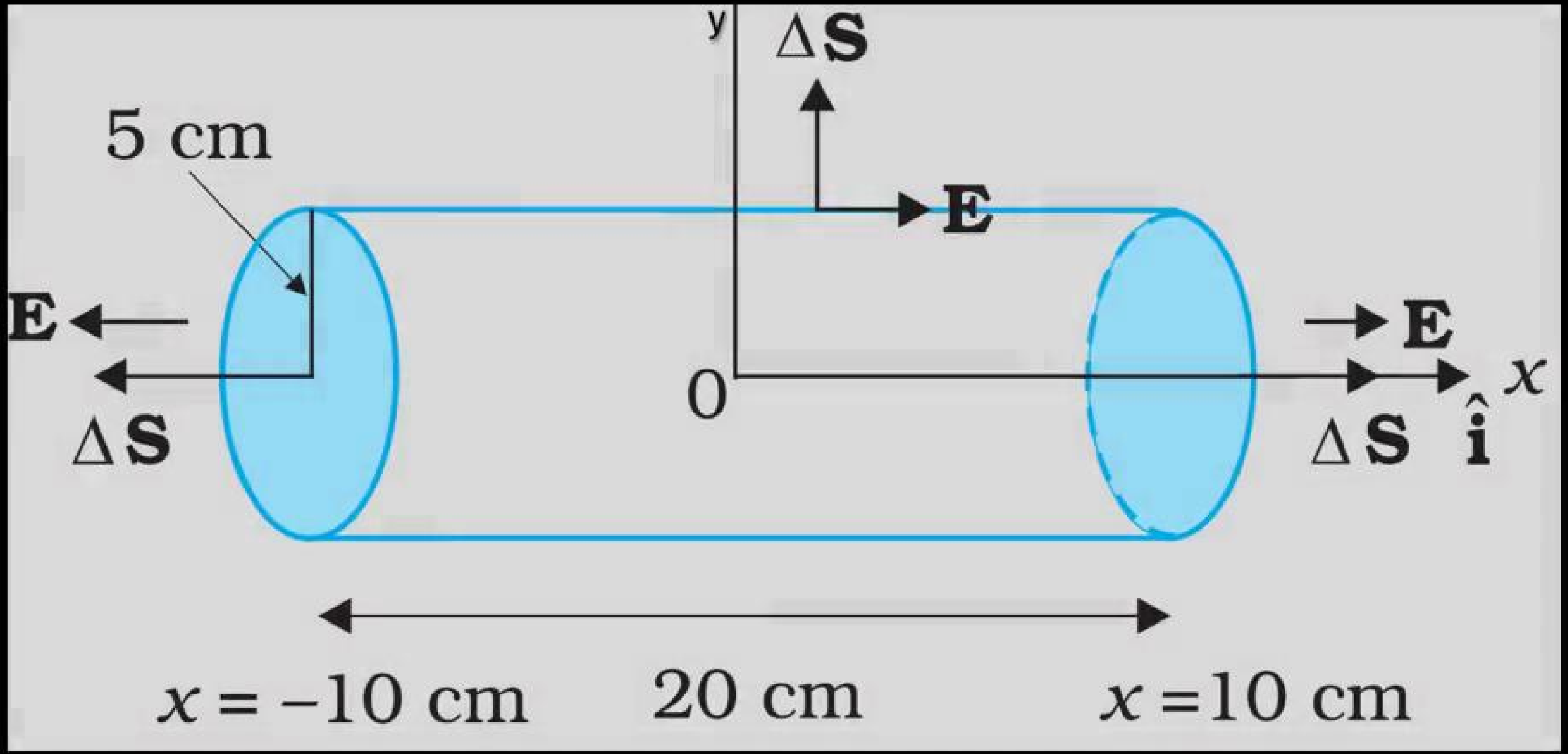
(d) 8.854 C

Solution:

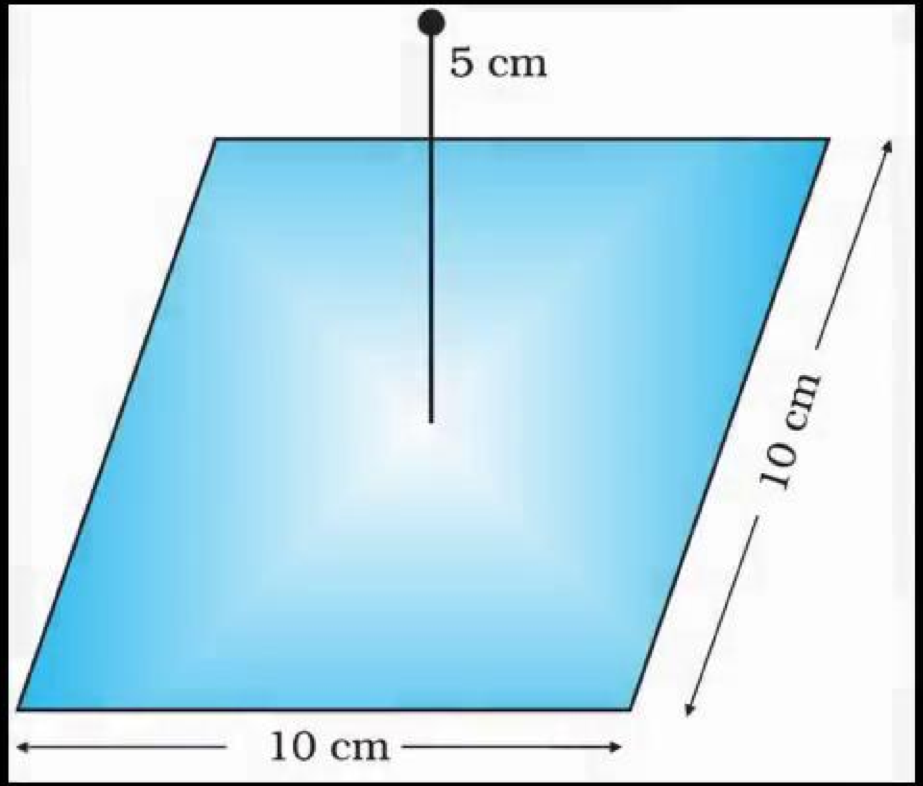
Example 45 : The electric field components in Fig. 1.27 are $E_x = \alpha x^{1/2}$, $E_y = E_z = 0$, in which $\alpha = 800 NC^{-1} m^{-1/2}$. Calculate (a) the flux through the cube, and (b) the charge within the cube. Assume that $a = 0.1$ m



Example 46 : An electric field is uniform, and in the positive x direction for positive x, and uniform with the same magnitude but in the negative x direction for negative x. It is given that $E = 200\ \hat{i}$ N/C for $x > 0$ and $E = -200\ \hat{i}$ N/C for $x < 0$. A right circular cylinder of length 20 cm and radius 5 cm has its centre at the origin and its axis along the x-axis so that one face is at $x = +10$ cm and the other is at $x = 10$ cm (Fig.). (a) What is the net outward flux through each flat face? (b) What is the flux through the side of the cylinder? (c) What is the net outward flux through the cylinder? (d) What is the net charge inside the cylinder?



Example 48 : A point charge $+10\ \mu C$ is a distance 5 cm directly above the centre of a square of side 10 cm, as shown in Fig. 1.34. What is the magnitude of the electric flux through the square? (Hint: Think of the square as one face of a cube with edge 10 cm.)



Example 49 : A point charge of 2.0 μC is at the centre of a cubic Gaussian surface 9.0 cm on edge. What is the net electric flux through the surface?

CONTINUOUS CHARGE DISTRIBUTION

- The charge distributions we have seen so far have been discrete: made up of individual point particles (q_1 q_2, q_3, ... q_n)

- If the region in which charges are closely spaced is said to have continuous distribution of charge.

- For continuous distribution of charges, it is impractical to specify the charge distribution in terms of the locations of the microscopic charged constituents(electrons or protons).

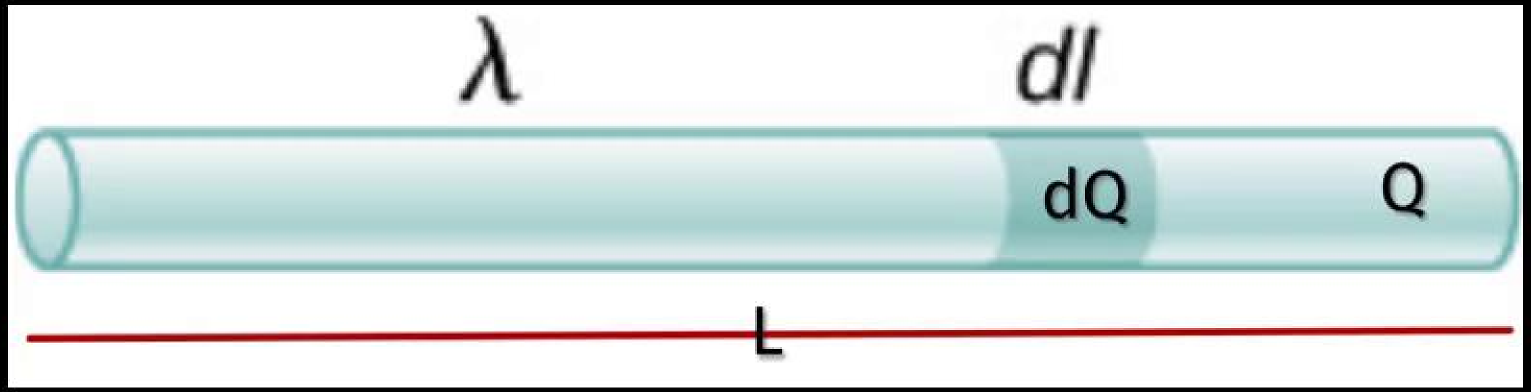
- For continuous charge distribution, we can generalize the definition of the electric field. We simply divide the charge into infinitesimal pieces and treat each piece as a point charge.

- Our first step is to define a charge density for a charge distribution along a line, across a surface, or within a volume

CONTINUOUS CHARGE DISTRIBUTION

Linear/Line Charge distribution

- Let charge Q is uniformly distributed along a line of length L, with **linear charge density (charge per unit length)** λ



- $\lambda = \dfrac{Q}{L}$ S.I unit : C/m

- The charge dQ on a small element dl of the wire will be

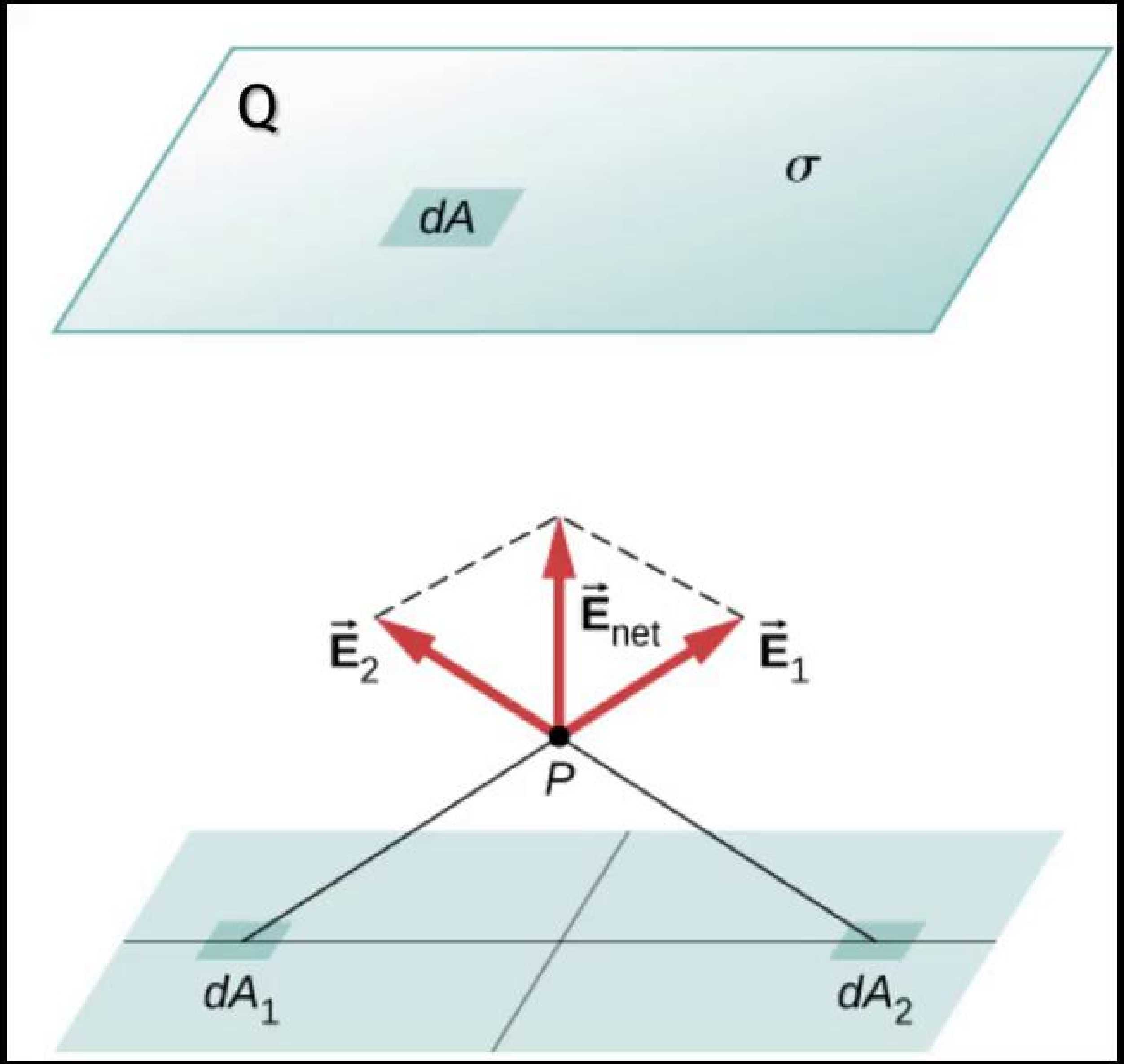
- $dQ = \lambda \times dl$ (We can consider this element as a point charge.)

- Electric field due to small element at any point is $dE = \dfrac{1}{4\pi\epsilon_0}\dfrac{dq}{r^2} = \dfrac{1}{4\pi\epsilon_0}\dfrac{\lambda dl}{r^2}$

CONTINUOUS CHARGE DISTRIBUTION

Surface Charge distribution

- Let charge Q is uniformly distributed uniformly on a surface A, with **surface charge density (charge per unit area)** σ



- $\sigma = \dfrac{Q}{A}$ S.I unit : $\mathrm{C/m^2}$

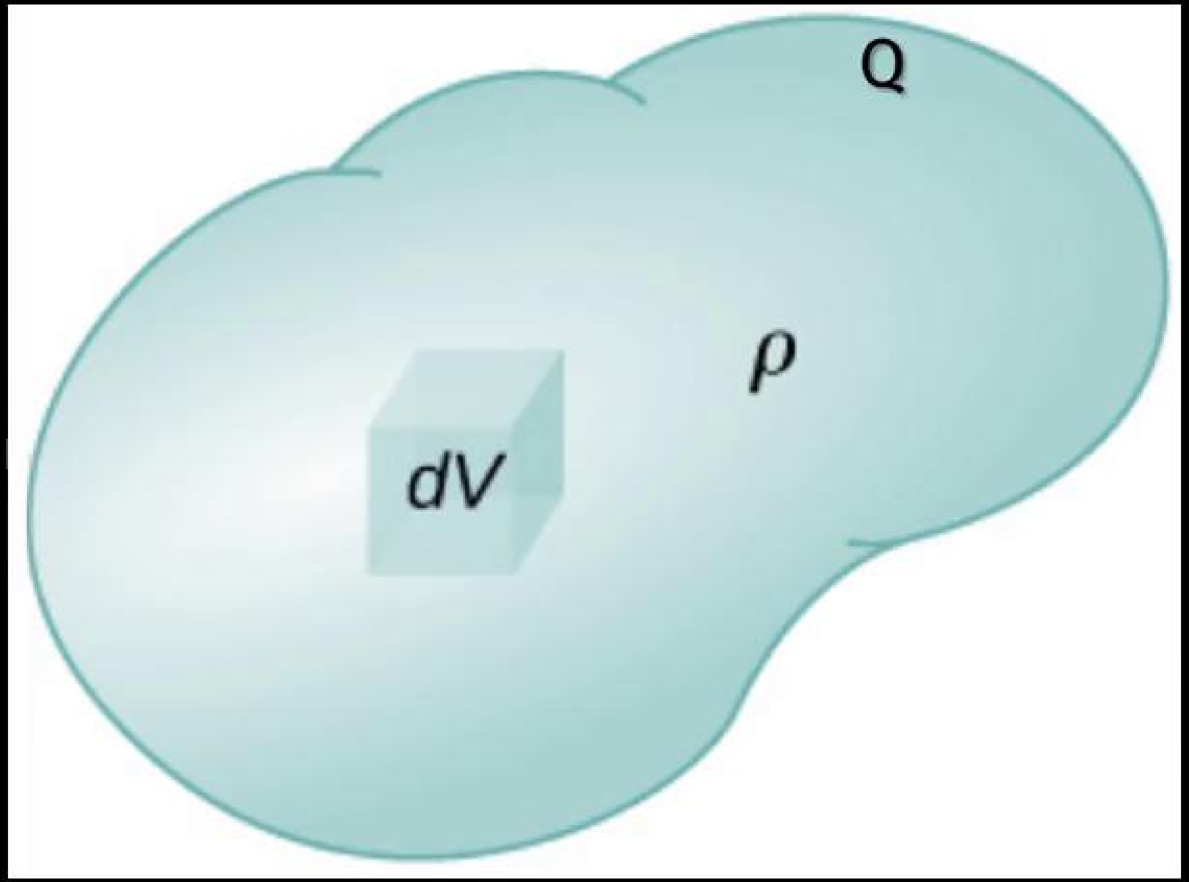
- The charge dQ on a small area element dA will be

- $dQ = \sigma \times dA$ (We can consider this element as a point charge.)

- Electric field due to small element at any point is $dE = \dfrac{1}{4\pi\epsilon_0}\dfrac{dq}{r^2} = \dfrac{1}{4\pi\epsilon_0}\dfrac{\sigma dA}{r^2}$

CONTINUOUS CHARGE DISTRIBUTION

Volume Charge distribution



- If some charge Q is uniformly distributed uniformly om a volume V, with **volume charge density (charge per unit volume)** ρ

- $\rho = \dfrac{Q}{V}$ S.I unit : $\mathrm{C/m^3}$

- The charge dQ on a small volume element dV will be

- $dQ = \sigma \times dV$ (We can consider this element as a point charge.)

- Electric field due to small element at any point is $dE = \dfrac{1}{4\pi\epsilon_0}\dfrac{dq}{r^2} = \dfrac{1}{4\pi\epsilon_0}\dfrac{\rho dV}{r^2}$

Applications of Gauss's Law

(1) Electric Filed Due to Infinite Long Uniformly Charged Straight Wire:

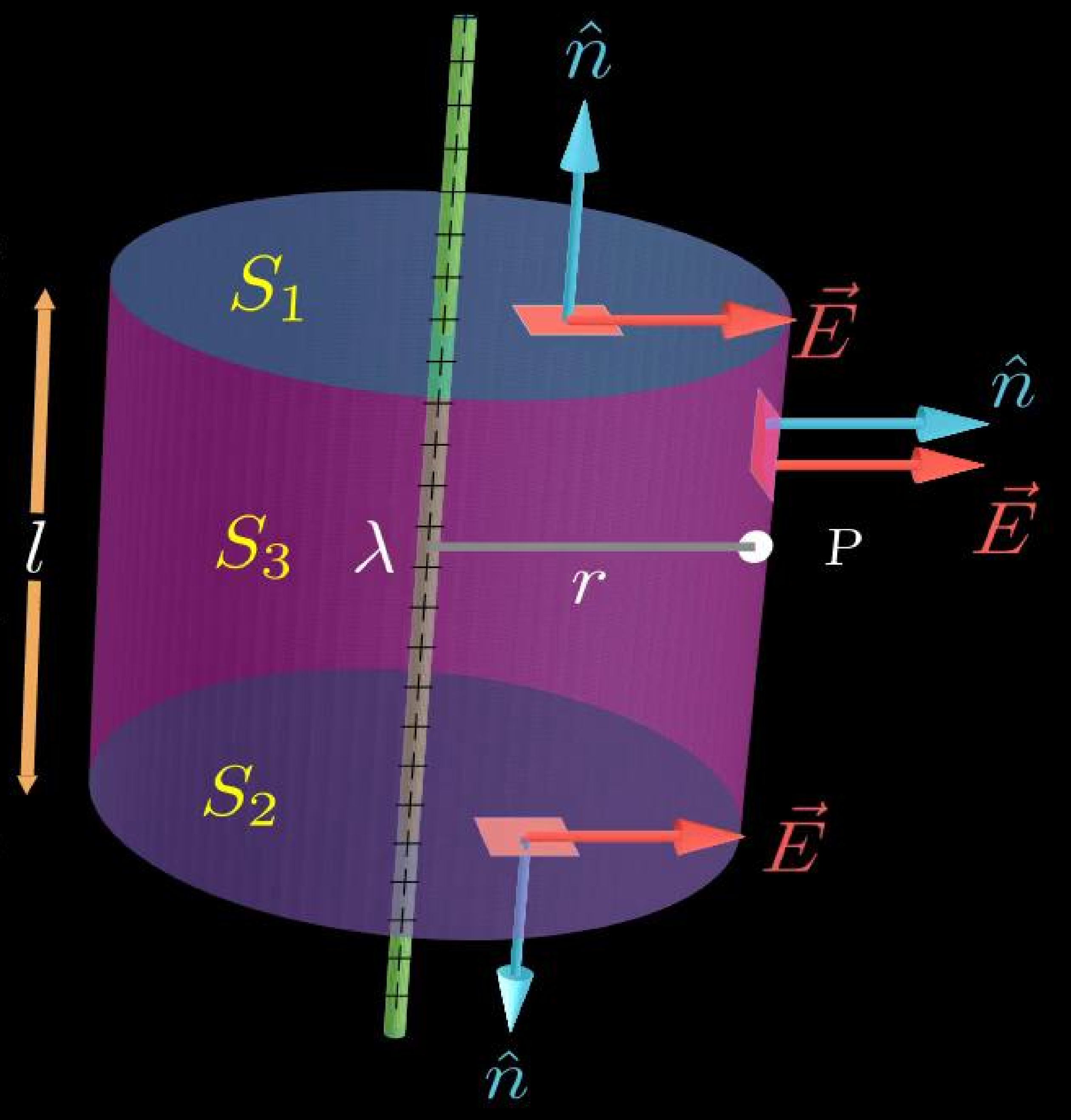
- Consider an infinite long straight wire, with uniform charge density λ

- We have to find electric field (E) at point P using Gauss's Law.

- From symmetry, the electric field is radial everywhere and its magnitude only depends on the radial distance r

- From Gauss's Law, Total electric flux through the cylindrical Gaussian surface is:

- $\Phi = \oint \vec{E} \cdot d\vec{S} = \dfrac{q_{enc}}{\epsilon_0}$



(1) Electric Filed Due to Infinite Long Uniformly Charged Straight Wire:

$$\oint \vec{E}\cdot d\vec{S} = \int_{S_1} \vec{E}\cdot d\vec{S} + \int_{S_2} \vec{E}\cdot d\vec{S} + \int_{S_3} \vec{E}\cdot d\vec{S} = \frac{q_{enc}}{\epsilon_0}$$

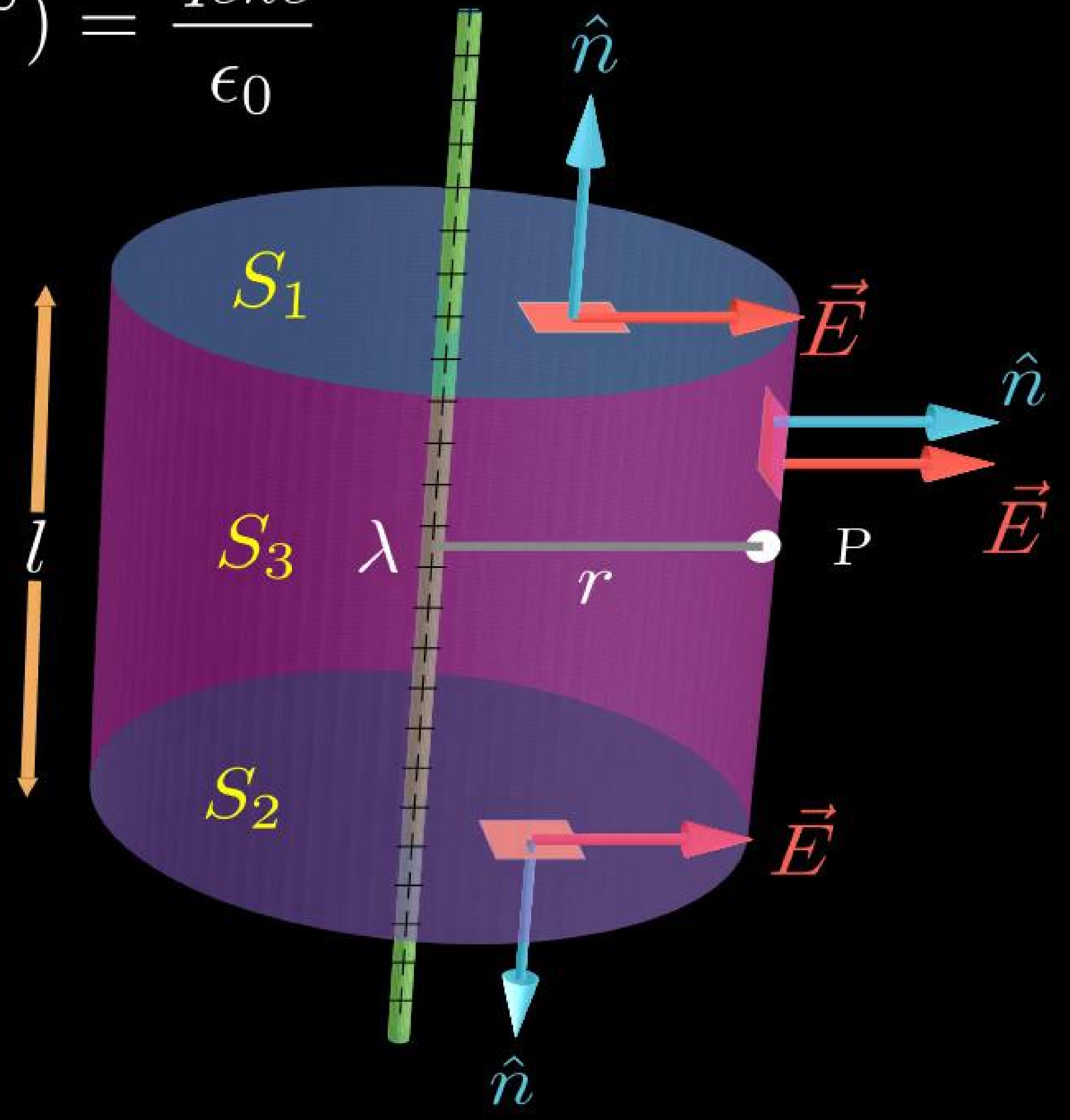
$$\int_{S_1} Eds\cos(90^\circ) + \int_{S_2} Eds\cos(90^\circ) + \int_{S_3} Eds\cos(0^\circ) = \frac{q_{enc}}{\epsilon_0}$$

$$0 + 0 + E\int_{S_3} ds = \frac{q_{enc}}{\epsilon_0}$$

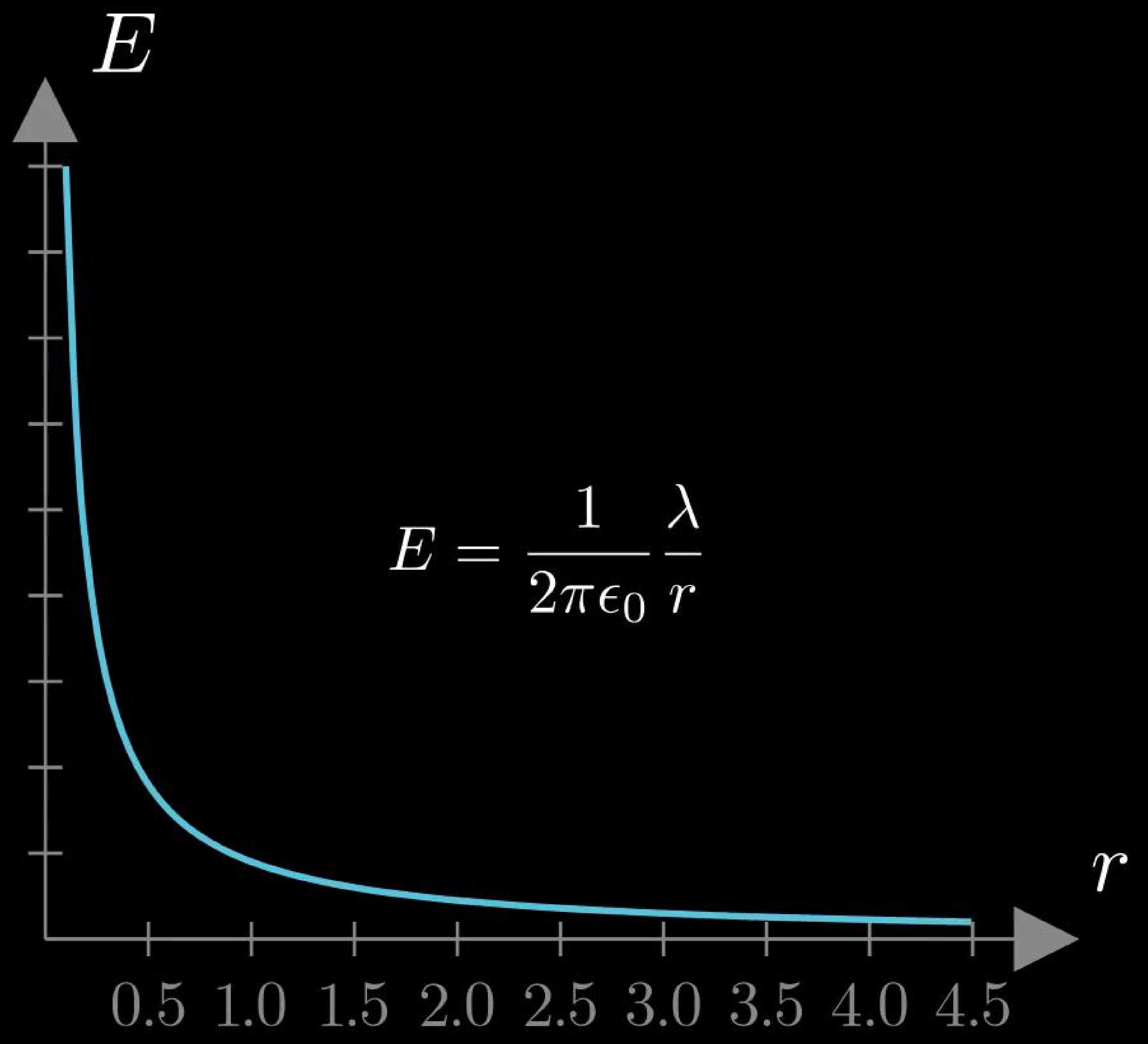
$$E \times 2\pi r l = \frac{\lambda l}{\epsilon_0} \ (\because q_{enc} = \lambda l)$$

$$\boxed{E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}}$$

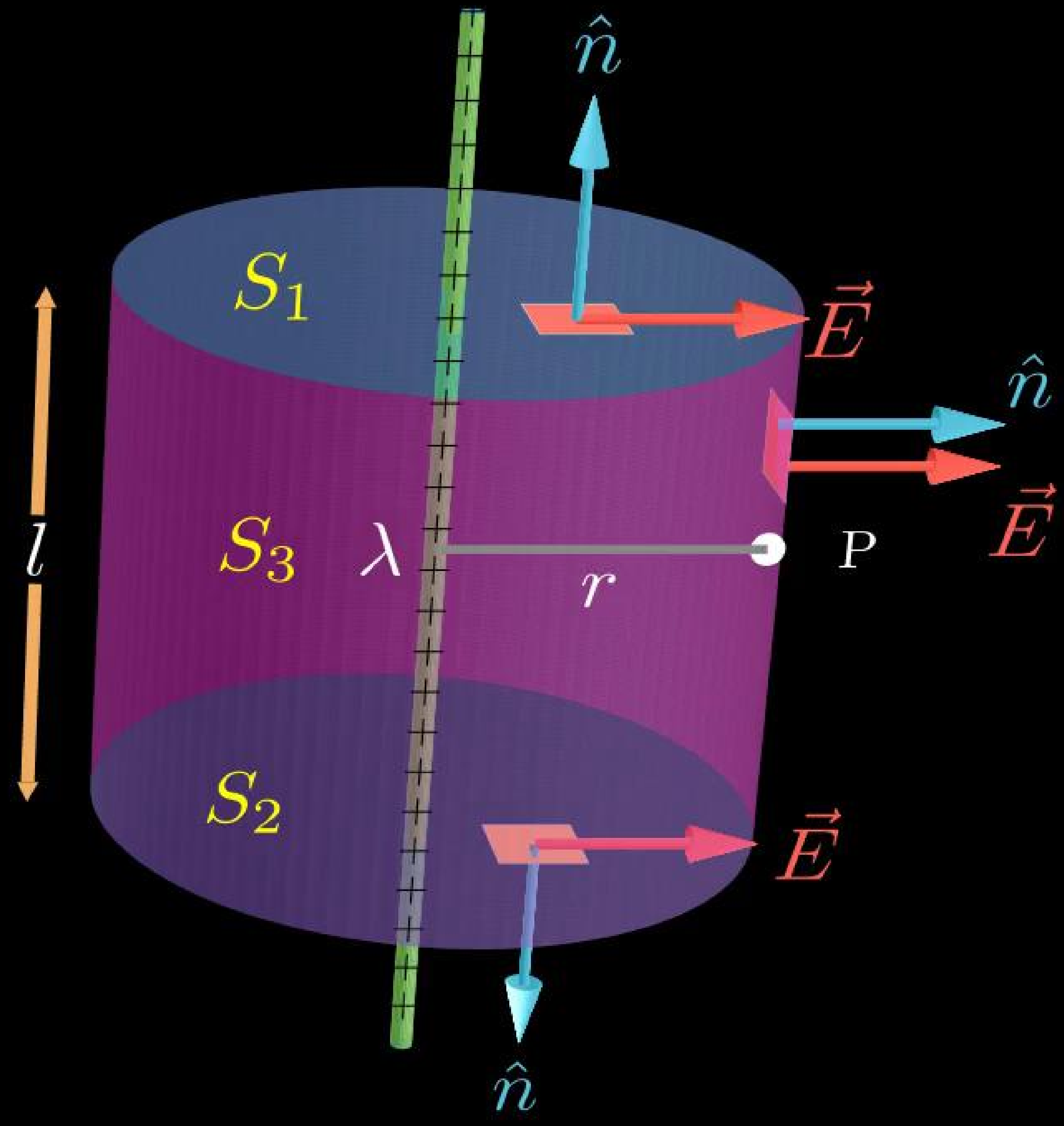
- If charge is $+$Ve $\implies$ Direction : radially outwards
- If charge is $-$Ve $\implies$ Direction : radially inwards



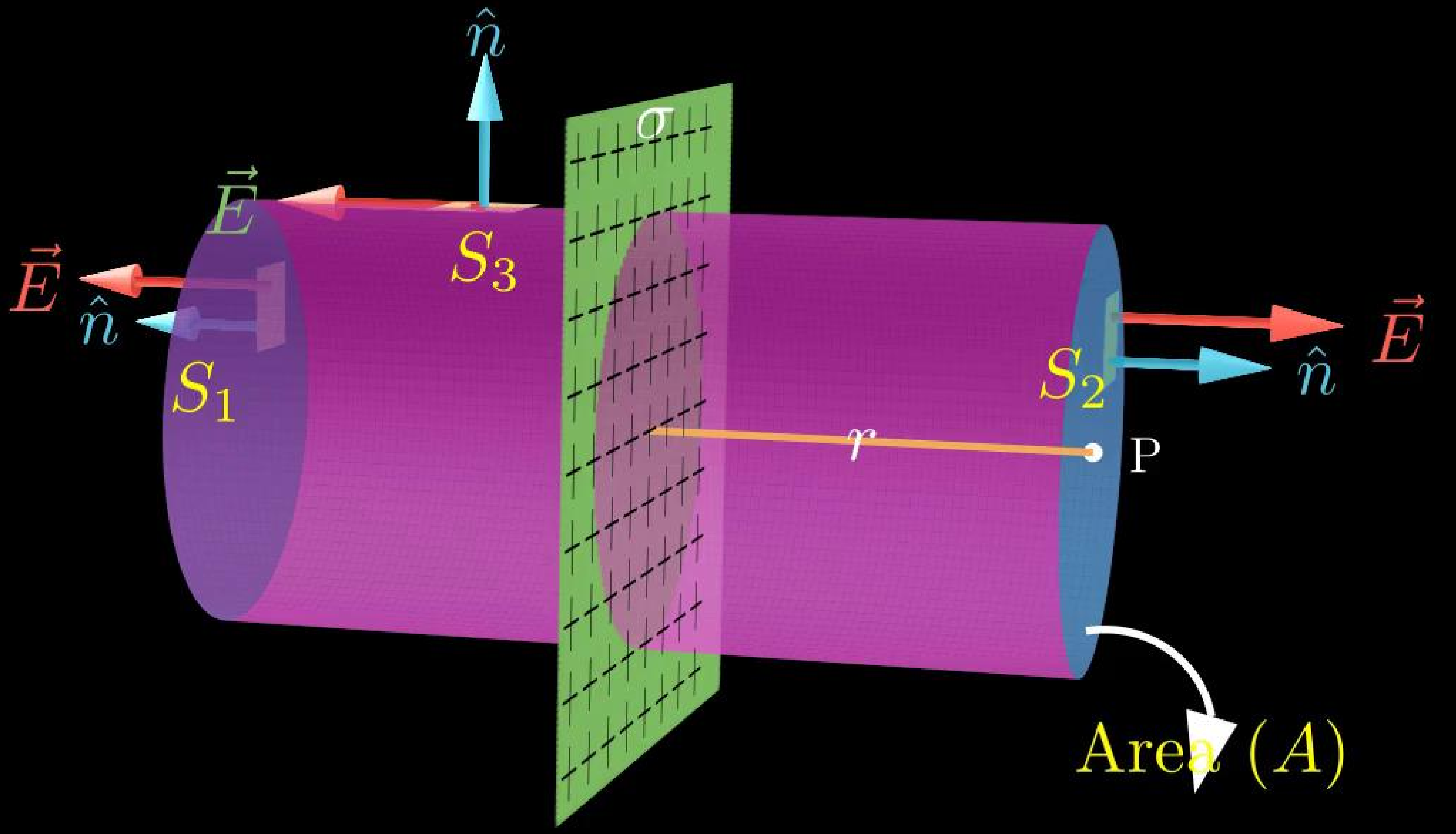
(1) Electric Filed Due to Infinite Long Uniformly Charged Straight Wire:



$$E = \frac{1}{2\pi\epsilon_0}\frac{\lambda}{r}$$



(2) Electric Filed Due to Uniformly Charged Infinite Plane Sheet:



Example 51 : An early model for an atom considered it to have a positively charged point nucleus of charge Ze, surrounded by a uniform density of negative charge up to a radius R. The atom as a whole is neutral. For this model, what is the electric field at a distance r from the nucleus?

Solution :

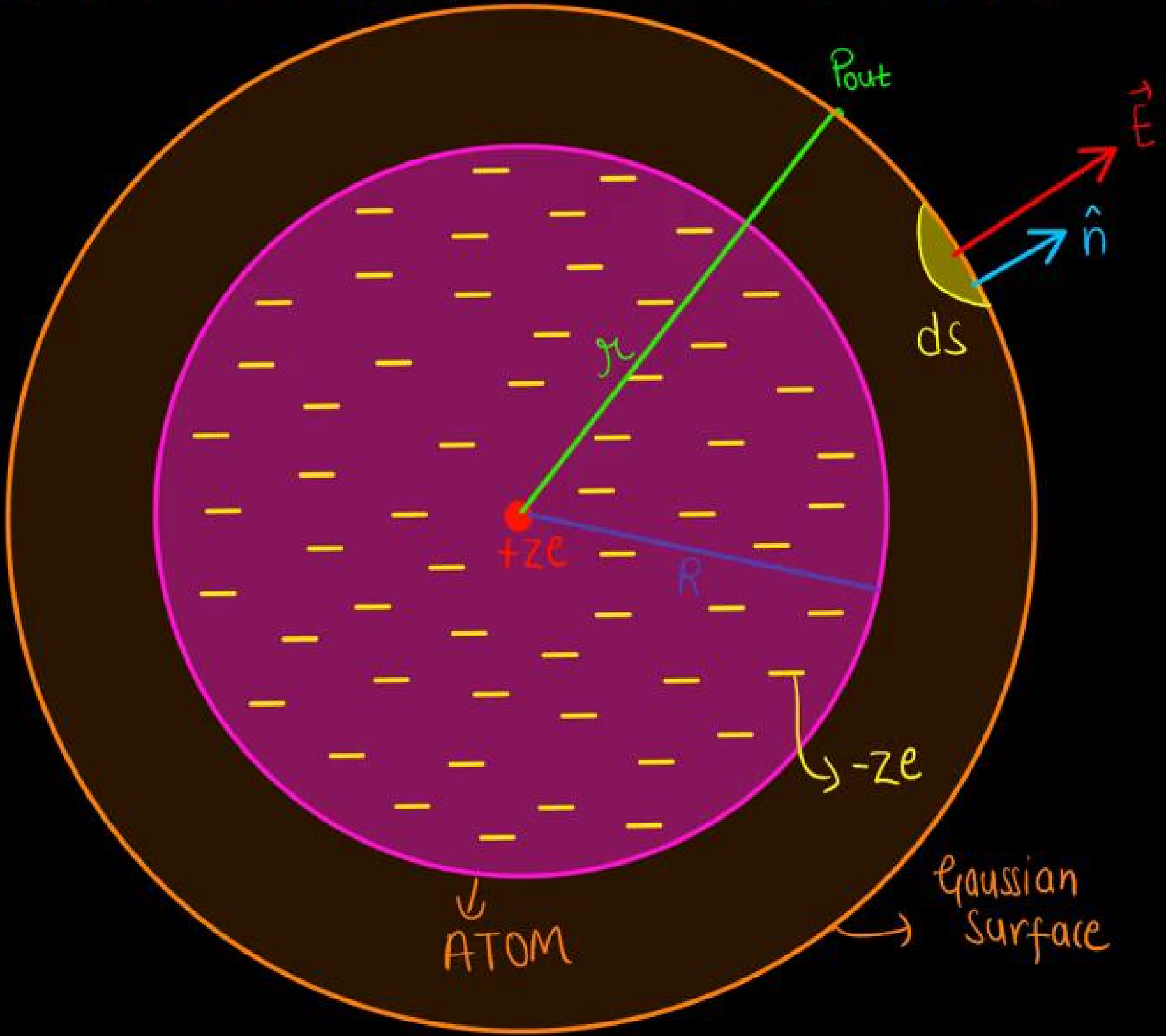
- (1) Filed Outside the Atom ($r > R$)

- Using Gauss's Law : $\oint_S \vec{E} \cdot d\vec{S} = \dfrac{q_{enc}}{\epsilon_0}$

$$E \oint_S ds = \frac{+Ze - Ze}{\epsilon_0}$$

$$E \times 4\pi r^2 = \frac{0}{\epsilon_0}$$

$$\boxed{E_{out} = 0}$$



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Solution :

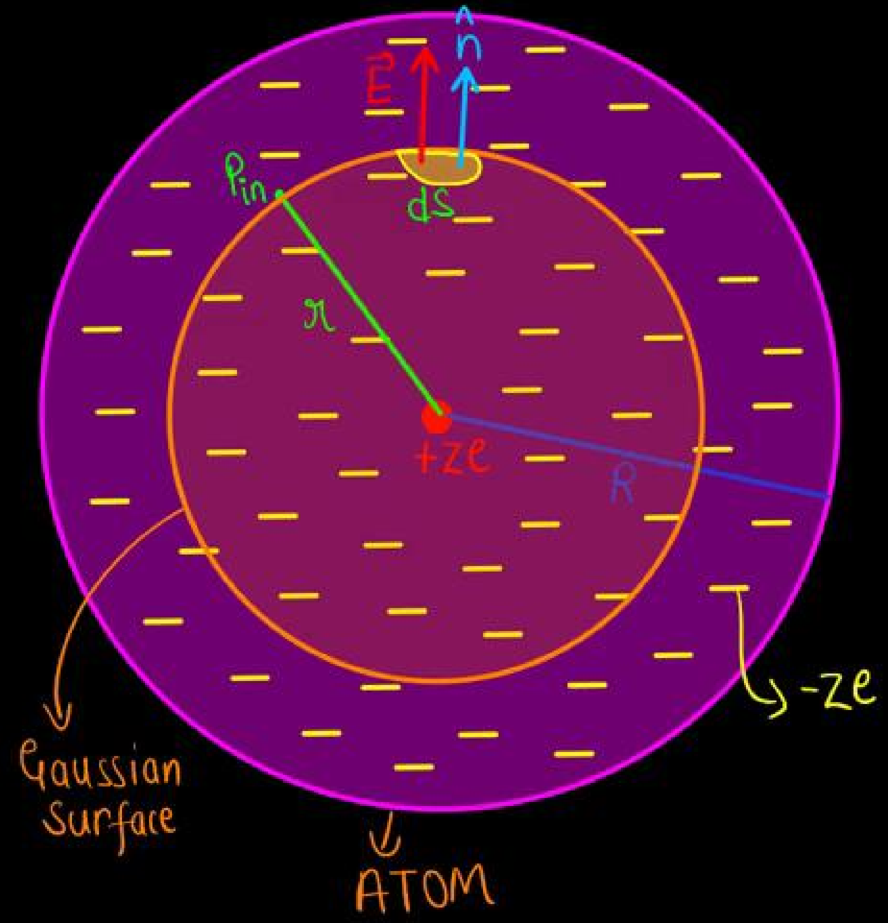
- (2) Filed Inside the Atom ($r < R$)
- Using Gauss's Law : $\oint_S \vec{E} \cdot d\vec{S} = \dfrac{q_{enc}}{\epsilon_0}$

$$E \oint_S ds = \frac{+Ze + q_{-enc}}{\epsilon_0}$$

$$E \times 4\pi r^2 = \frac{+Ze + q_{-enc}}{\epsilon_0}$$

$$q_{-enc} = \rho \times \text{ Volume enclosed}$$

$$q_{-enc} = \frac{-Ze}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3$$



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Solution :

- (2) Filed Inside the Atom ($r < R$)
- Using Gauss's Law : $\oint_S \vec{E} \cdot d\vec{S} = \frac{q_{enc}}{\epsilon_0}$

$$E \oint_S ds = \frac{+Ze + q_{-enc}}{\epsilon_0}$$

$$E \times 4\pi r^2 = \frac{+Ze + q_{-enc}}{\epsilon_0}$$

$$q_{-enc} = \rho \times \text{ Volume enclosed}$$

$$q_{-enc} = \frac{-Ze}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi r^3$$

$$q_{-enc} = -Ze\frac{r^3}{R^3}$$

$$E \times 4\pi r^2 = \frac{+Ze + -Ze\frac{r^3}{R^3}}{\epsilon_0}$$

$$E = \frac{Ze\left[1 - \frac{r^3}{R^3}\right]}{4\pi\epsilon_0 \times r^2}$$

$$\boxed{E = \frac{Ze}{4\pi\epsilon_0} \times \left[\frac{1}{r^2} - \frac{r}{R^3}\right]}$$

Example 52 : A conducting sphere of radius 10 cm has an unknown charge. If the electric field 20 cm from the centre of the sphere is 1.5×10^3 N/C and points radially inward, what is the net charge on the sphere?

Solution :

Example 53 :A uniformly charged conducting sphere of 2.4 m diameter has a surface charge density of 80.0 $\mu C m^{-2}$. (a) Find the charge on the sphere. (b) What is the total electric flux leaving the surface of the sphere?

Solution :

Example 54 :An infinite line charge produces a field of 9×104 N/C at a distance of 2 cm. Calculate the linear charge density.

Solution :

Example 55 :Two large, thin metal plates are parallel and close to each other. On their inner faces, the plates have surface charge densities of opposite signs and of magnitude $17.0 \times 10^{-22}\ C/m^2 2$. What is E: (a) in the outer region of the first plate, (b) in the outer region of the second plate, and (c) between the plates?

Solution :

Example 56 : An electron is rotating around an infinite positive linear charge in a circle of radius 0.1 m, if the linear charge density is 1 μC, then te velocity of electron in m/s will be

(a) 0.562×10^7

(b) 5.62×10^7

(c) 562×10^7 C

(d) 0.0562×10^7

Solution:

Example 57 : Two isolate metallic spheres of radii 2 cm and 4 cm are given equal charge, then the ratio of charge density on the surface of the spheres will be

(a) 4 : 1 (b) 1 : 2

(c) 1 : 4 C (d) 8 : 1

Solution: