CHAPTER 1 : ELECTRIC CHARGES AND FILEDS

Learning Objectives :

- Introduction
- Electric Charge
- Basic properties of electric charges
- Conductors and Insulators
- Charging by induction
- Coulombs Law
- Forces between multiple charges
- Superposition Principle
- Electric filed
- Electric Field Lines
- Electric Dipole and Dipole moment
- Electric Field due to an electric dipole
- Dipole in a Uniform External Field
- Electric Flux
- Gauss's Law
- Applications of Gauss's Law

Introduction

Daily Observations :

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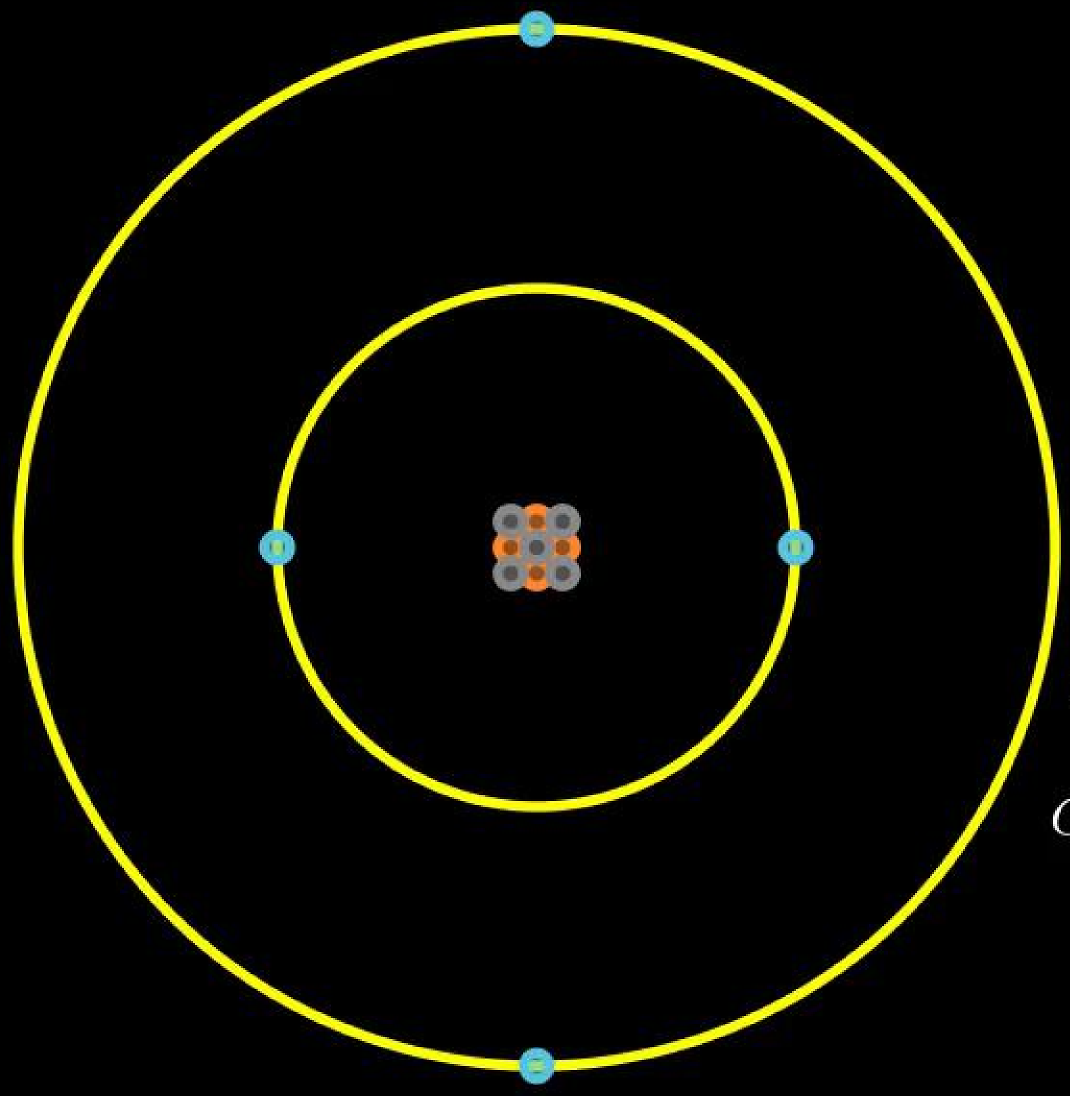
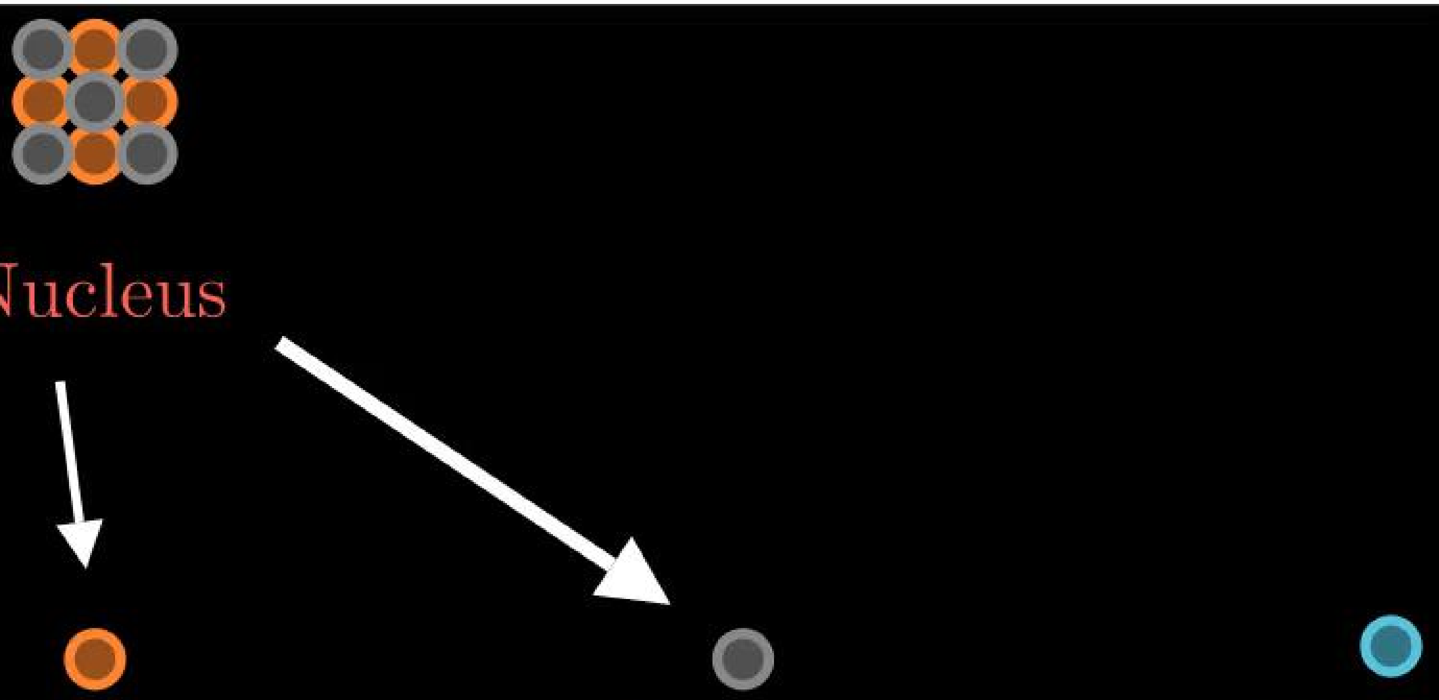
- Another common example of electric discharge is the lightning that we see in the sky during thunderstorms.
- We also experience a sensation of an electric shock either while opening the door of a car or holding the iron bar of a bus after sliding from our seat.
- This is due to generation of static electricity
- Static means anything that does not move or change with time
- Electrostatics deals with the study of forces, fields and potentials arising from static charges.

Electric Charges and Their Properties

- All matter is made of Atoms.



Atom



	Proton	Neutron	Electron
Mass :	$M_p = 1.67 \times 10^{-27}\ kg$	$M_n = 1.68 \times 10^{-27}\ kg$	$M_e = 9.11 \times 10^{-31}\ kg$
Charge :	$q_p = 1.602 \times 10^{-19}\ C$	$q_n = 0\ C\ (Neutral)$	$q_e = -1.602 \times 10^{-19}\ C$

Electric Charges and Their Properties

- Charge is a fundamental property of matter by virtue of which it produces and experience electromagnetic force.
- Electromagnetic forces can be attractive or repulsive. In contrast with the gravitational froce between masses which is always attractive.
- There are two kinds of electric charges which are distinguished form each other by calling one kind as 'posiitve' and the other as 'negative', these names are arbitraily chosen by Benjamin Franklin.
- Like chagres repel each other and Unlike charges attract each other

Electric Charges and Their Properties

- Charge is a scalar quantitiy.
- S.I unit of charge is coulomb (C)
- C.G.S. unit of charge is esu (electrostatic unit) or static coloumb (stat C or franklin)
- $1 \text{ C} = 3 \times 10^9$ stat C
- Dimension formula of charge [Q]=[AT] ($\because Q = It$)
- A charge cannot exist without masss (however a mass can exist without charge e.g. neutron)

Electric Charges and Their Properties

Charge is transferable :

- Charge can be transfered from one body to another.
- Neutral Body + electron $\rightarrow$ Negatively charge body
- Neutral Body $-$ electron $\rightarrow$ Positively charge body
- When we charge an object, the mass of the body changes because wherever there is charge, there is mass.

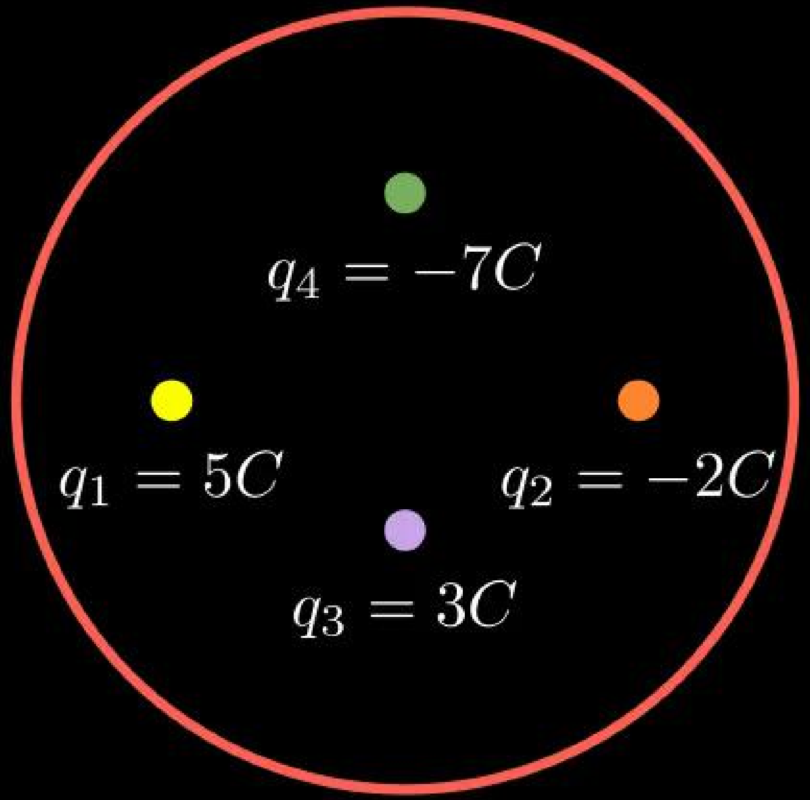
Frictional Electricity :

- When two bodies are rubbed together under friction electrons are transferred from one body to the other. As a result one body becomes positively charged while the other gets negatively charged.

Electric Charges and Their Properties

Additivity of Charges :

- If a system contains n charges $q_1,\ q_2,\ q_3,\ ...,q_n$, then the total charge of the system is $q_1 + q_2 + q_3 + ... + q_n$. i.e., charges add up like real numbers or they are scalars
- Proper signs have to be used while adding the charges in a system.
- Example: Total charge $(Q) = q_1 + q_2 + q_3 + q_4$
- $(Q) = 5\ C + (-2\ C) + 3\ C + (-7\ C) = -1\ C$



Electric Charges and Their Properties

Conservation of Charges :

- Charge can neither be created nor destroyed; it can only be transferred from place to place, from one object to another.
- Within an isolated system consisting of many charged bodies, due to interactions among the bodies, charges may get redistributed but it is found that the total charge of the isolated system is always conserved.
- Sometimes nature creates charged particles from an uncharged particle: a neutron turns into a proton and an electron.
- Neutron $(q_n = 0) \Rightarrow$ Proton $(q_p = +e)$ + Electron $(q_e = -e)$
- Pair Production:
- Gamma Ray photon $(q_\gamma = 0) \Rightarrow$ Positron $(q_{e^+} = +e)$ + Electron $(q_e = -e)$

Electric Charges and Their Properties

Quantisation of charge:

- Quantisation of charge means that electric charge comes in discrete amounts, and there is a smallest possible amount of charge ($e = 1.602 \times 10^{-19}$ C) that an object can have. No free particle can have less charge than this, and, therefore, the charge (q) on any object must be an integer multiple of this amount (e).

$$q = ne \text{ (Where } n = \pm 1,\ \pm 2,\ \pm 3,\ .....)$$

- For macroscopic charges for which n is a very large number, quantisation of charge can be ignored and charge appears to be continuous.

Example 1: If a body has positive charge on it, then it means it has

(a) Gained some proton

(b) Lost some protons

(c) Gained some electrons

(d) Lost some electrons

Example 2: Which of the following is not true about electric charge

(a) Charge on a body is always integral muliple of certain charge known as charge of electron

(b) Charge is a scalar quantity

(c) Net charge of an isolated system is always conservesd

(d) Charge can be converted into energy and energy can be converted into charge

Example 3: Consider three point objects P, Q and R. P and Q repel each other, while P and R attract. What is the nature of force between Q and R?

(a) Repulsive force

(b) Attractive force

(c) No force

(d) None of these

Example 4: When 10^{14} electrons are removed from a neutral metal sphere, the charge on the sphere becomes:

(a) $16\ \mu$ C (b) $-16\ \mu$ C

(c) $32\ \mu$ C (d) $-32\ \mu$ C

Solution :

Given: No. of electron removed $n = 10^{14}$

Find: Charge on the sphere $q = ?$

Using $q = ne$

$$q = 10^{14} \times 1.6 \times 10^{-19}\ \mathrm{C} = 1.6 \times 10^{-5}\ \mathrm{C}$$

$$q = 16 \times 10^{-6}\ \mathrm{C} = 16\ \mu\ \mathrm{C}$$

Example 5: A conductor has 14.4×10^{-19} C positive charge. The conductor has

(a) 9 electron in excess

(b) 27 electrons in short

(c) 27 electrons in excess

(d) 9 electrons in short

Solution :

Given: Charge on conductor $q = 14.4 \times 10^{-19}$ C

Find: No. of electrons short or excess $n = ?$

Using $q = ne$ Or $n = \dfrac{q}{e}$

$$n = \frac{14.4 \times 10^{-19}\ C}{1.6 \times 10^{-19}\ C} = \frac{14.4}{1.6}$$

$$n = 9$$

Example 6: If 10^9 electrons move out of a body to another body every second, how much time is required to get a total charge of 1 C on the other body?

Solution :

Given: Number of elctrons moved out in 1 s $= 10^9$

$\therefore$ Charge moved out in 1 s $= ne = 10^9 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-10}$ C

Time required to get a charge of 1.6×10^{-10} C $= 1$ s

So, time required to get a charge of 1 C $= \dfrac{1}{1.6 \times 10^{-10}}$ s $= 6.25 \times 10^9$ s

Converting this time in s to years we get $t = \dfrac{6.25 \times 10^9}{365 \times 24 \times 3600} = 198.186$ years

Example 7: How much positive and negative charge is there in a cup of water (250 g)?. Given molecular mass of water is 18 g.

Solution :

Given: Mass of a cup of water $M = 250$ g

Molar mass of water $m = 18$ g

Number of water molecules in 1 cup of water
$$= \frac{M}{m} \times N_A = \frac{250}{18} \times 6.02 \times 10^{23} = 83.64 \times 10^{23}$$

Number of electron or protons in 1 molecule of water $= 10$

$\therefore$ Number of electrons or protons in 1 cup of water
$n = 83.64 \times 10^{23} \times 10 = 83.64 \times 10^{24}$

Now, Amount of positive or negative charge in 1 cup of water
$q = ne = 83.64 \times 10^{24} \times 1.6 \times 10^{-19}\ \mathrm{C} = 133.8 \times 10^{5}\ \mathrm{C}$

Example 8: A polythene piece rubbed with wool is found to have a negative charge of 3×10^{-7} C.

(a) Estimate the number of electrons transferred (from which to which?)

(b) Is there a transfer of mass from wool to polythene?

Solution :

Do it yourself !

Conductors and Insulators

Conductors :

- Those substances which allow electricity to pass through them easily are called conductors.
- In conductors outermost electron are loosely bound to the atoms nucleus that are comparatively free to move inside the material.
- Examples: Metals, humans and earth
- When some charge is transferred to a conductor, it readily gets distributed over the entire surface of the conductor.

Conductors and Insulators

Insulators :

- Insulators, in contrast, are made from materials that have bounded electrons(they are not free and it is hard to dislodge these elctrons from the atoms)
- In insulators charge flows only with great difficulty, if at all.
- If some charge is put on an insulator, it stays at the same place.
- Examples: Most of the non-metals like glass, porcelain, plastic, nylon, wood offer high resistance to the passage of electricity through them.

Conductors and Insulators

Earthing Or Grounding :

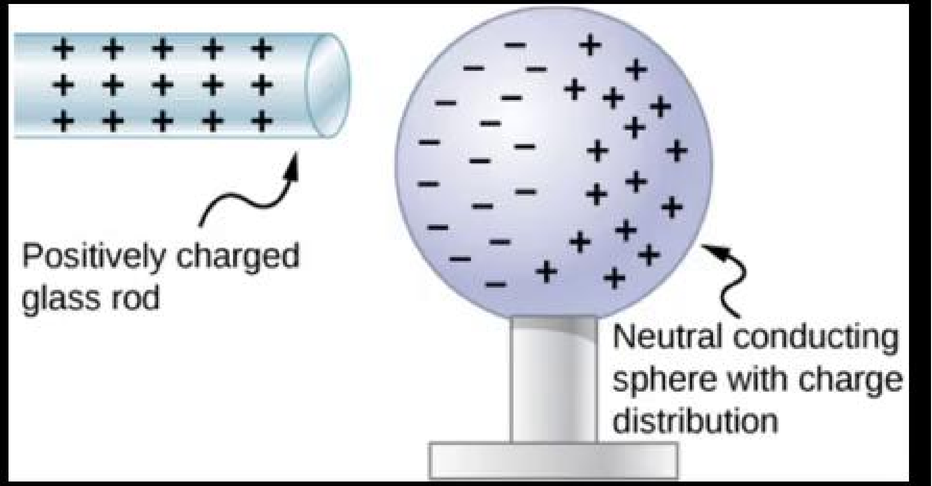
- When we bring a charged body in contact with the earth, all the excess charge on the body disappears by causing a momentary current to pass to the ground through the connecting conductor (such as our body)
- This process of sharing the charges with the earth is called grounding or earthing.

Charging by Induction

Electrostatic Induction :

- When an electrically charged object is brought close to the conductor, the charge on the insulator exerts an electric force on the free electrons of the conductor.

- Since the rod is positively charged, the free electrons are attracted, flowing toward the rod to the near side of the conductor



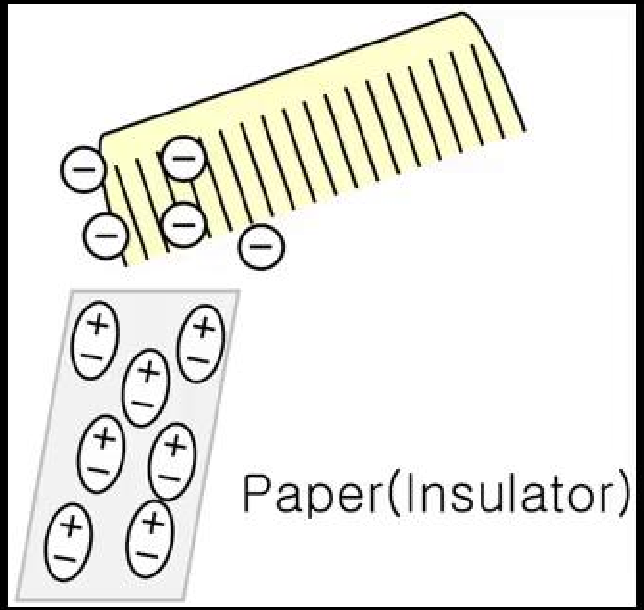
- Now, the conductor is still overall electrically neutral. However, the conductor now has a charge distribution; the near end now has more negative charge than positive charge,

- The result is the formation of what is called an electric dipole.

Charging by Induction

Electrostatic Induction :

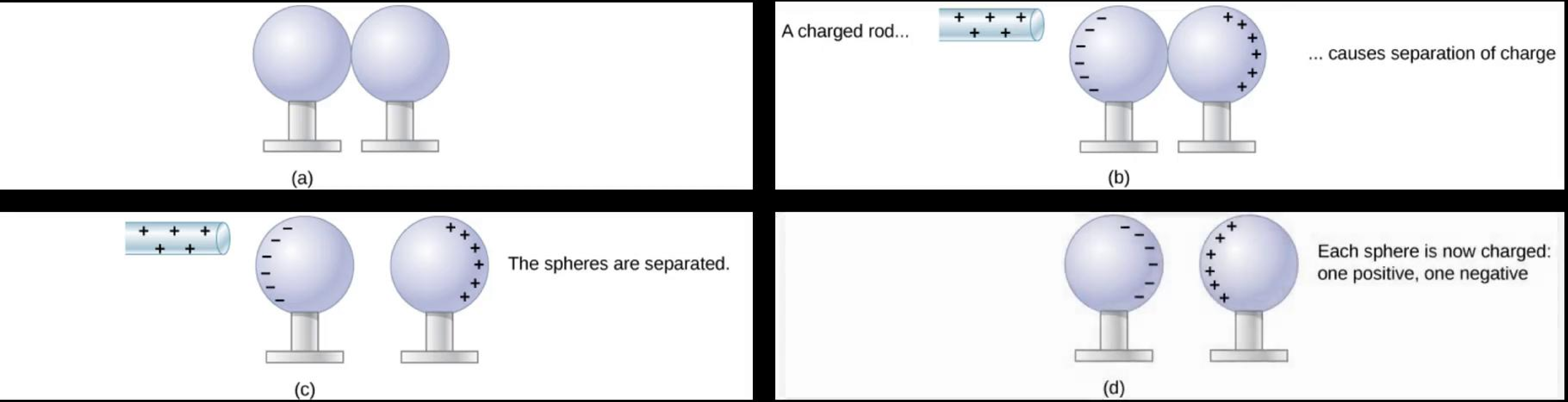
- Neutral objects can be attracted to any charged object.
- If you run a plastic comb through your hair, the charged comb can pick up neutral pieces of paper.
- When a charged comb is brought near a neutral insulator(paper), the distribution of charge in atoms and molecules is shifted slightly.
- Opposite charge is attracted nearer the external charged rod, while like charge is repelled. Since the electrostatic force decreases with distance, the repulsion of like charges is weaker than the attraction of unlike charges, and so there is a net attraction.



Charging by Induction

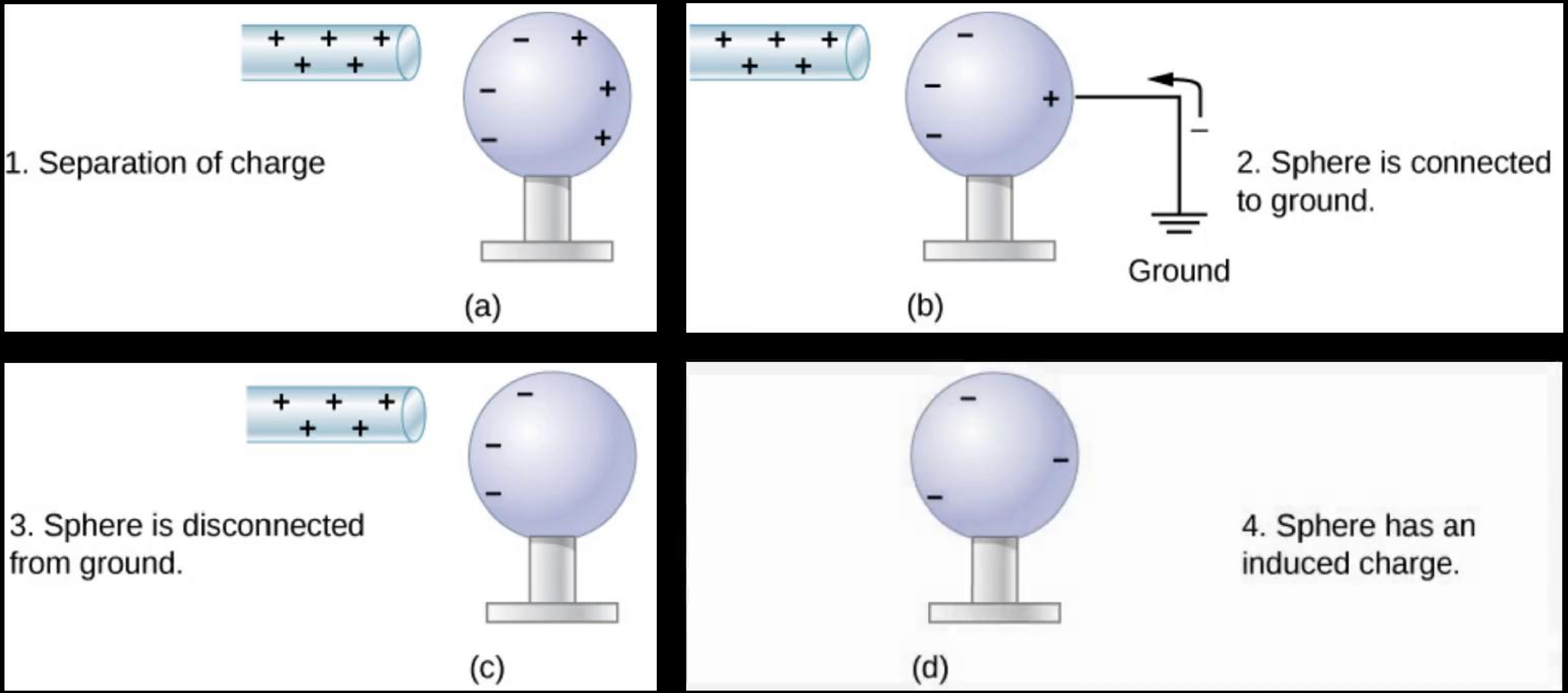
Charging by induction (1st Method)

- The process of charging neutral body by bringing a charged object nearby it without making contact between the two bodies is known as charging by induction



Charging by Induction

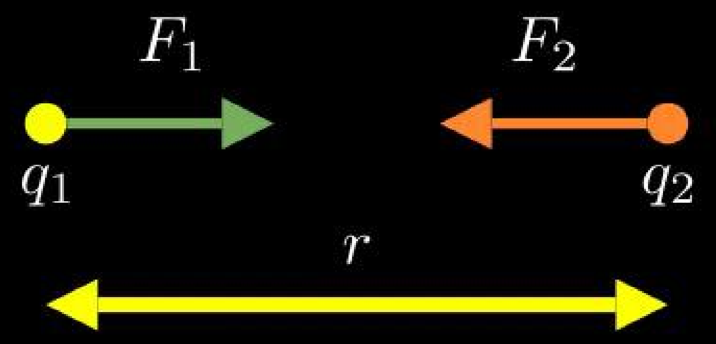
Charging by induction (2nd Method)



Coulomb's Law

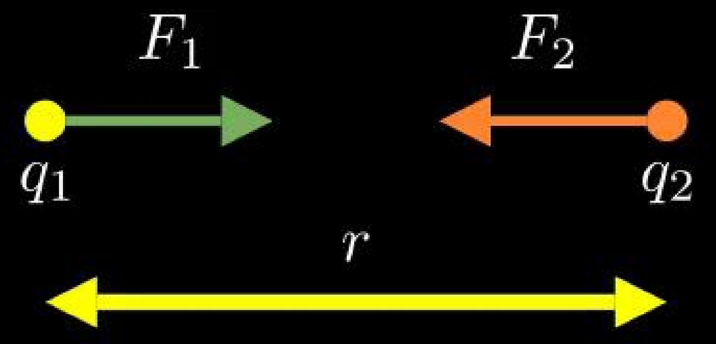
- Coulomb's law is a quantitative statement about the force between two point charges.
- Coulomb measured the force between two point charges and found that it varied inversely as the square of the distance between the charges and was directly proportional to the product of the magnitude of the two charges and acted along the line joining the two charges.
- Mathematically, magnitude of electrostatic force (F) between two stationary charges (q_1, q_2) seperated by a distance r in vacuum is

$$F \propto \frac{|q_1 q_2|}{r^2} \qquad \text{Or} \qquad F = k\frac{|q_1 q_2|}{r^2}$$



Coulomb's Law

- $$F \propto \frac{|q_1 q_2|}{r^2} \qquad \text{Or} \qquad F = k\frac{|q_1 q_2|}{r^2}$$
- Where, k is a proportionality constant.
- In S.I. unit $k = \dfrac{1}{4\pi\epsilon_0} = 9 \times 10^9 \ Nm^2C^{-2}$
- Where $\epsilon_0 = 8.854 \times 10^{-12} \ C^2N^{-1}m^{-2}$ and is called the permittivity of free space (vacuum)
- If $q_1 = q_2 = 1 \ C$ and $r = 1$ m. Then, $F = 9 \times 10^9 \ N$
- That is, 1 C is the charge that when placed at a distance of 1 m from another charge of the same magnitude in vacuum experiences an electrical force of repulsion of magnitude 9×10^9 N.



Absoloute and Relative Permittivity (Dielectric constant) of Medium:

- If the point charges are kept in some other medium (say water) then coulomb's law gives

$$F = \frac{1}{4\pi\epsilon}\frac{q_1q_2}{r^2}$$

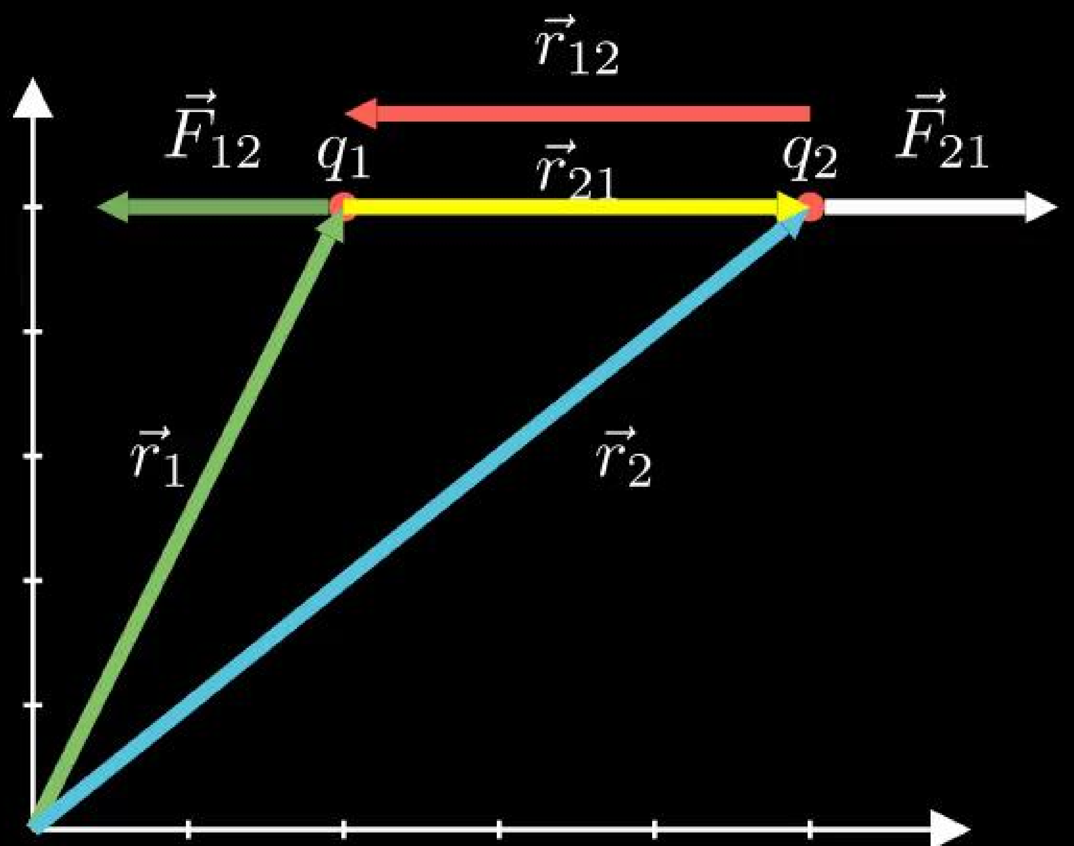
- Where, ϵ is absolute permitivity of the medium

$$\frac{F_{vacuum}}{F_{medium}} = \frac{\frac{1}{4\pi\epsilon_0}\frac{q_1q_2}{r^2}}{\frac{1}{4\pi\epsilon}\frac{q_1q_2}{r^2}} = \frac{\epsilon}{\epsilon_0} = \epsilon_r (or K)$$

- Where, ϵ_r is called relative permittivity of the medium and also known as Dielectric constant(K) of the medium.

Coulomb's Law in Vector Form

- $q_1,\ q_2 \rightarrow$ Two point charges
- $\vec{r}_1,\ \vec{r}_2 \rightarrow$ Position vectors of q_1 and q_2
- $\vec{F}_{12} \rightarrow$ Force on q_1 due to q_2
- $\vec{F}_{21} \rightarrow$ Force on q_2 due to q_1
- $\vec{r}_{21} = \vec{r}_2 - \vec{r}_1$ (Vector leading from 1 to 2)
- $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2$ (Vector leading from 2 to 1)
- $\vec{r}_{21} = -\vec{r}_{12}$ and $|\vec{r}_{21}| = |\vec{r}_{12}| = r$
- To denote the direction from 1 to 2 (or from 2 to 1), we define the unit vectors: $\hat{r}_{21} = \dfrac{\vec{r}_{21}}{|\vec{r}_{21}|}$ and $\hat{r}_{12} = \dfrac{\vec{r}_{12}}{|\vec{r}_{12}|}$



- $\vec{F}_{21} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{|r_{21}|^2}\ \hat{r}_{21}$
- $\vec{F}_{12} = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{|r_{12}|^2}\ \hat{r}_{12}$
- $\vec{F}_{21} = -\vec{F}_{12}$

Exercise 1.1: What is the force between two small charged spheres having charges of 2×10^{-7} C and 3×10^{-7} C placed 30 cm apart in air?

Solution :

Given: $q_1 = 2 \times 10^{-7}$ C and $q_2 = 3 \times 10^{-7}$ C

$r = 30 \text{ cm} = 30 \times 10^{-2} \text{ m} = 3 \times 10^{-1} \text{ m}$

Find : $F = ?$

Using Coulomb's Law $F = \dfrac{1}{4\pi\epsilon_0} \dfrac{q_1 q_2}{r^2}$

$$F = 9 \times 10^9 \times \frac{2 \times 10^{-7} \times 3 \times 10^{-7}}{(3 \times 10^{-1})^2} = 9 \times 10^9 \times \frac{2 \times 10^{-7} \times 3 \times 10^{-7}}{9 \times 10^{-2}}$$

$$F = 10^9 \times 2 \times 10^{-7} \times 3 \times 10^{-7} \times 10^2 = 6 \times 10^{-3} \text{ N}$$

This force is repulsive, since the spheres have same charges.

Example 9: The sum of two point charges is 7 μ C. They repel each other with a force with a force of 1 N when kept at 30 cm apart in free space. Calculate the value of each charge.

Solution :

Given F = 1 N and $q_1 + q_2 = 7\ \mu$ C

$r = 30\ cm = 3 \times 10^{-1}$ m

Find: q_1 and q_2

Let one of the two charges be $x\ \mu$ C.

$\therefore$ Other charge will be $(7 - x)\ \mu$ C

Using Coulomb's Law $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{r^2}$

$$1N = 9 \times 10^9\ Nm^2C^{-2} \times \frac{x \times 10^{-6} \times (7 - x) \times 10^{-6}\ C^2}{(3 \times 10^{-1}\ m)^2}$$

$$1N = 9 \times 10^9\ Nm^2C^{-2} \times \frac{x \times 10^{-6} \times (7 - x) \times 10^{-6}\ C^2}{9 \times 10^{-2}\ m^2}$$

$$1 = 10^9 \times x(7 - x) \times 10^{-12} \times 10^2 = x(7 - x) \times 10^{-1}$$

$$10 = -x^2 + 7x \text{ Or } x^2 - 7x + 10 = 0$$

$$(x - 2)(x - 5) = 0$$

$$x = 2 \text{ Or } x = 5$$

$\therefore$ Two point charges are 2 μ C and 5 μC.

Exercise 1.2: The electrostatic force on a small sphere of charge 0.4 μC due to another small sphere of charge -0.8 μC in air is 0.2 N. (a) What is the distance between the two spheres? (b) What is the force on the second sphere due to the first?

Solution :

Given: : $q_1 = 0.4\ \mu\text{C}$, $q_2 = -0.8\ \mu\text{C}$ and $F = 0.2$ N

Find : (a) Distance between two charged sphere $r =?$

Using Coulomb's Law $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{r^2}$

$$r^2 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{F}$$

$$r^2 = 9\times 10^9\ Nm^2C^{-2} \times \frac{0.4\times 10^{-6}\times 0.8\times 10^{-6}\ C^2}{0.2\ N}$$

$$r^2 = 9\times 10^9 \times 2\times 0.8\times 10^{-12}\ m^2$$

$$r^2 = 14.4\times 10^{-3}\ m^2$$

$$r^2 = 144\times 10^{-4}$$

$$r = \sqrt{144\times 10^{-2}\ m^2}$$

$$r = 12\times 10^{-2}\ m$$

Example 10: There are two charges $+2\ \mu$ C and $-3\ \mu$C. The ratio of forces acting on them will be

(a) $2:3$

(b) $1:1$

(c) $3:2$

(d) $4:9$

Example 11: What is the minimum electric force between two charged particles 1 m apart in free space?

Solution :

Given: : $r = 1$ m

Find : (a) Minimum force between two charged particles

Force will be minimum when the charge on both particle is minimum,

$i.e.,\ q_1 = q_2 = e = 1.6 \times 10^{-19}$ C

Using Coulomb's Law $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{r^2}$

$$F = 9 \times 10^9 \times \frac{1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{1^2}$$

$$F = 23.04 \times 10^{-29} \text{ N}$$

Example 1.4 :Coulomb's law for electrostatic force between two point charges and Newton's law for gravitational force between two stationary point masses, both have inverse-square dependence on the distance between the charges and masses respectively. **(a)** Compare the strength of these forces by determining the ratio of their magnitudes (i) for an electron and a proton and (ii) for two protons. (b) Estimate the accelerations of electron and proton due to the electrical force of their mutual attraction when they are $1\mathring{A}(= 10^{-10}\mathrm{m})$ apart? ($m_p = 1.67 \times 10^{-27}$ kg, $m_e = 9.11 \times 10^{-31}$ kg)

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Solution :

(a) (i) for an electron and a proton:

$$q_1 = q_2 = e \text{ and } m_1 = m_e,\ m_2 = m_p$$

$$\frac{F_e}{F_g} = \frac{\frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}}{G\frac{m_e m_p}{r^2}} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{G m_e m_p}$$

$$\frac{F_e}{F_g} = \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{6.67 \times 10^{-11} \times 9.11 \times 10^{-31} \times 1.67 \times 10^{-27}}$$

$$\frac{F_e}{F_g} = 2.27 \times 10^{39} \approx 10^{39}$$

(a) (ii) for two protons:

$$q_1 = q_2 = e \text{ and } m_1 = m_2 = m_p$$

$$\frac{F_e}{F_g} = \frac{\frac{1}{4\pi\epsilon_0}\frac{e^2}{r^2}}{G\frac{m_p m_p}{r^2}} = \frac{1}{4\pi\epsilon_0}\frac{e^2}{G m_p m_p}$$

$$\frac{F_e}{F_g} = \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{6.67 \times 10^{-11} \times 1.67 \times 10^{-27} \times 1.67 \times 10^{-27}}$$

$$\frac{F_e}{F_g} = 1.24 \times 10^{36} \approx 10^{36}$$

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Solution :

(b) Given : $q_1 = q_2 = e$ and $r = 10^{-10}$ m

Find: Acceleration of electron (a_e) and proton (a_p) due to electric force (F)

$$F_e = F_p = \frac{1}{4\pi\epsilon_0}\frac{e \times e}{r^2}$$

$$F_e = F_p = \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{(10^{-10})^2}$$

$$F_e = F_p = 23.04 \times 10^{-9} \text{ N}$$

Now, using $F_e = m_e a_e$ acceleration of electron

$$a_e = \frac{F_e}{m_e} = \frac{23.04 \times 10^{-9}}{9.11 \times 10^{-31}} = 2.53 \times 10^{22}\ ms^{-2}$$

Similarly, acceleration of proton:

$$a_p = \frac{F_p}{m_p} = \frac{23.04 \times 10^{-9}}{1.67 \times 10^{-27}} = 13.79 \times 10^{18}\ ms^{-2}$$

Example 1.5 : A charged metallic sphere A is suspended by a nylon thread. Another charged metallic sphere B held by an insulating handle is brought close to A such that the distance between their centres is 10 cm, as shown in Fig. 1.7(a). The resulting repulsion of A is noted (for example, by shining a beam of light and measuring the deflection of its shadow on a screen). Spheres A and B are touched by uncharged spheres C and D respectively, as shown in Fig. 1.7(b). C and D are then removed and B is brought closer to A to a distance of 5.0 cm between their centres, as shown in Fig. 1.7(c). What is the expected repulsion of A on the basis of Coulomb's law? Spheres A and C and spheres B and D have identical sizes. Ignore the sizes of A and B in comparison to the separation between their centres.

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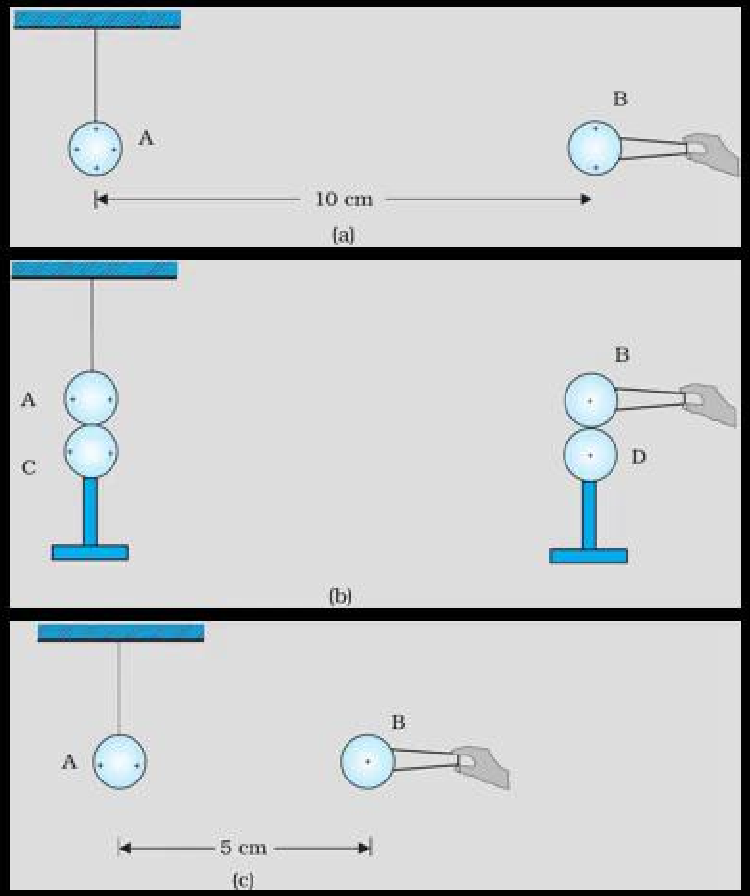


Solution :

Let, $q_1 \rightarrow$ initial charge on A

$q_2 \rightarrow$ initial charge on B

$r = 10\ cm \rightarrow$ initial seperation b/w A and B

$\therefore$ Initial Force $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{r^2}$

Now, $q_1' = \dfrac{q_1}{2} \rightarrow$ charge on A after touching

$q_2' = \dfrac{q_2}{2} \rightarrow$ charge on B after touching

$r' = \dfrac{r}{2} = 5\ cm \rightarrow$ final seperation b/w A and B

$$\therefore \text{Final Force } F' = \frac{1}{4\pi\epsilon_0}\frac{q_1' q_2'}{r'^2} = \frac{1}{4\pi\epsilon_0}\frac{\frac{q_1}{2}\frac{q_2}{2}}{\left(\frac{r}{2}\right)^2} = \frac{1}{4\pi\epsilon_0}\frac{\frac{q_1 q_2}{4}}{\frac{r^2}{4}}$$

$$F' = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2} = F$$

Example 12 : Two point charges q_1 and q_2 exert a force F on each other when kept certain distance apart. If the charge on each particle is halved and the distance between the two particles is doubled, then the new force between the two particles would be

(a) $\frac{F}{2}$ (b) $\frac{F}{4}$

(c) $\frac{F}{8}$ (d) $\frac{F}{16}$

Solution :

Let, $q_1 \rightarrow$ initial charge

$q_2 \rightarrow$ initial charge

$r \rightarrow$ initial seperation

$\therefore$ Initial Force $F = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}$

Now, $q_1' = \frac{q_1}{2} \rightarrow$ charge is halved

$q_2' = \frac{q_2}{2} \rightarrow$ charge is halved

$r' = 2r \rightarrow$ distance is doubled

$$\therefore \text{Final Force } F' = \frac{1}{4\pi\epsilon_0}\frac{q_1' q_2'}{r'^2} = \frac{1}{4\pi\epsilon_0}\frac{\frac{q_1}{2}\frac{q_2}{2}}{(2r)^2} = \frac{1}{4\pi\epsilon_0}\frac{\frac{q_1 q_2}{4}}{4r^2}$$

$$F' = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{16r^2} = \frac{F}{16}$$

Example 13 : Two point charges having equal charges seperated by 1 m distance experience a force of 8 N. What will be the force experienced by them, if they are held in water, at the same distance? (Given, $K_{water} = 80$)

Solution :

Given $F_{air} = 8$ N and Dielectric constant of water $K_{water} = 80$

Find : $F_{water} = ?$

We know that $\dfrac{F_{air}}{F_{medium}} = K_{medium}$

Here, K is the dielectric constant of the medium

$$\frac{8}{F_{water}} = 80$$

$$\implies F_{water} = \frac{8\ N}{80} = 0.1 \text{ N}$$

Example 14: Two same balls having equal positive charge q C are suspended by two insulating strings of equal lengths. What would be the effect on the force when a plastic sheet is inserted between the two?

Solution :

From Coulomb's law, electrostatic force between the two charged boides in a medium,

$$F_{medium} = \frac{1}{4\pi\epsilon}\frac{q_1 q_2}{r^2} = \frac{1}{4\pi\epsilon_0 K}\frac{q_1 q_2}{r^2} \quad (\because \frac{\epsilon}{\epsilon_0} = K)$$

Where, K is the dielectric constant of the medium

For vacuum, $K = 1$

for plastic, $K > 1$

$\therefore$ after insertion of plastic sheet, the force between the two charge balls will reduce.

Exercise 1.12: (a) Two insulated charged copper spheres A and B have their centres separated by a distance of 50 cm. What is the mutual force of electrostatic repulsion if the charge on each is 6.5×10^{-7} C? The radii of A and B are negligible compared to the distance of separation.
(b) What is the force of repulsion if each sphere is charged double the above amount, and the distance between them is halved?

Solution :

Given: : $q_1 = q_2 = 6.5 \times 10^{-7}\mathrm{C}$

$r = 50\ cm = 5 \times 10^{-1}\ m$

Using Coulomb's Law $F = \dfrac{1}{4\pi\epsilon_0}\dfrac{q_1 q_2}{r^2}$

$$F = 9 \times 10^9\ Nm^2C^{-2} \times \frac{6.5 \times 10^{-7} \times 6.5 \times 10^{-7}\ C^2}{(5 \times 10^{-1}\ m)^2}$$

$$F = 9 \times 10^9 \times \frac{6.5 \times 6.5 \times 10^{-14}}{25 \times 10^{-2}}\ N$$

$$F = 15.21 \times 10^{-3}\ N$$

(b) $$F' = \frac{1}{4\pi\epsilon_0}\frac{2q_1 \times 2q_2}{\left(\frac{r}{2}\right)^2} = \frac{1}{4\pi\epsilon_0}\frac{4q_1q_2}{\frac{r^2}{4}}$$

$$F' = 16F$$

Exercise 1.13: Suppose the spheres A and B in Exercise 1.12 have identical sizes. A third sphere of the same size but uncharged is brought in contact with the first, then brought in contact with the second, and finally removed from both. What is the new force of repulsion between A and B?

Solution :

Given : $q_1 = q_2 = q = 6.5 \times 10^{-7}\mathrm{C}$

$r = 50\ cm = 5 \times 10^{-1}\ m$

From previous question $F = 15.21 \times 10^{-3}\ N$

New charge on A and C after touching both sphere

$$q_1' = q_3' = \frac{q}{2}$$

New charge on B after touching sphere C

$$q_2' = \frac{q_2 + q_3'}{2} = \frac{q + \frac{q}{2}}{2} = \frac{3q}{4}$$

New force between A and B

$$F' = \frac{1}{4\pi\epsilon_0}\frac{q_1' \times q_2'}{r^2} = \frac{1}{4\pi\epsilon_0}\frac{\frac{q}{2} \times \frac{3q}{4}}{r^2} = \frac{3}{8}F$$

$$F' = \frac{3}{8} \times 15.21 \times 10^{-3} N = 5.7 \times 10^{-3}\ N$$

Example 2: Five balls marked a to e are suspended using separated threads. Pair $(b,\ c)$ and $(d,\ e)$ show electrostatic repulsion while pairs $(a,\ b)$, $(c,\ e)$ and $(a,\ e)$ show electrostatic attraction. The ball marked a must be

(a) Negatively charged

(b) positively charged

(c) Uncharged

(d) Any of the above is possible

Solution :

$\because (b,\ c)$ repell each other they have same charge

$\because (d,\ e)$ repell each other they have same charge

$(\ b,\ c,\ d$ and $e)$ all are charged particles.

$\because (c,\ e)$ attract each other they have opposite charge

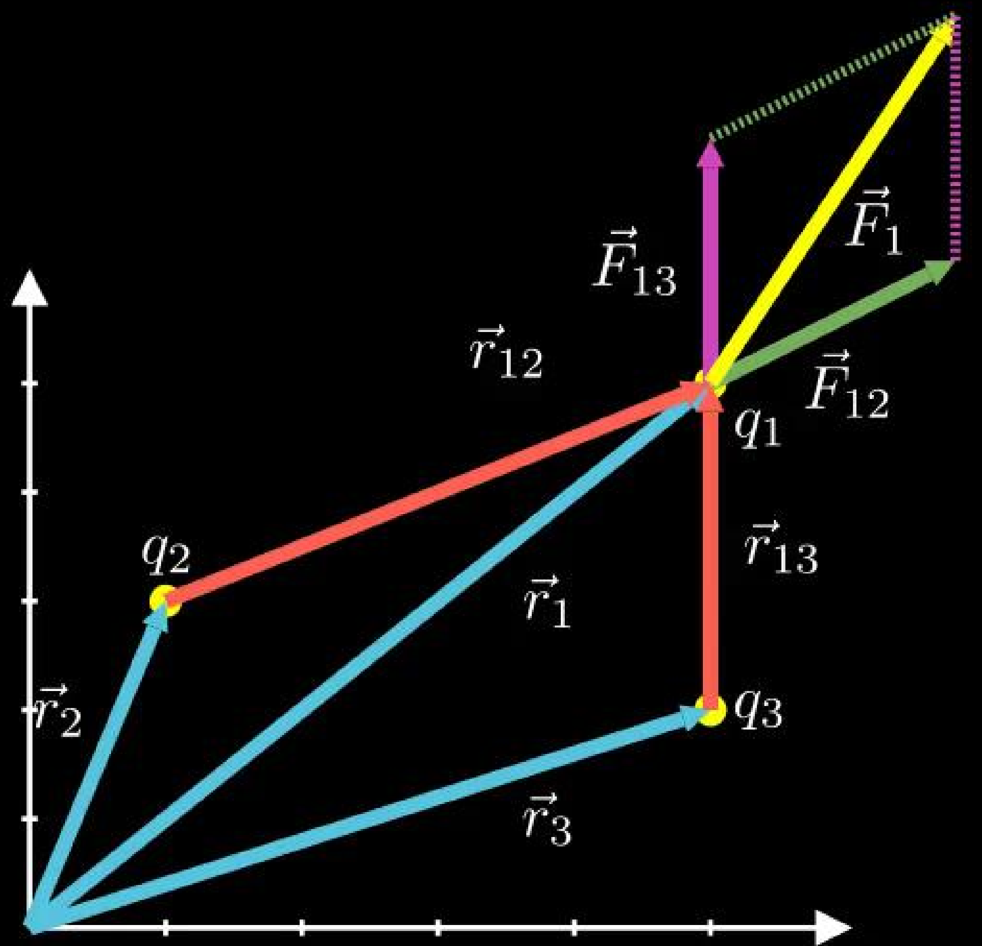
But, e and b both are oppositely charged and attracts a

This is only possible when a is neutral

They are attracting due to indction.

Example 16: Plot a graph showing the variation of magnitude of Coulomb's force (F) versus $\frac{1}{r^2}$, where r is the distance between the two charges of each pair of charges $(1\ \mu C,\ 2\ \mu C)$ and $(1\ \mu C,\ -3\ \mu C)$. Interpret the graphs obtained

Solution :



The superposition principle :

- This pricnciple tells us that if charge q_1 is acted upon by several charges $q_2,\ q_3, ...,\ q_n$, then the force on q_1 can be found out by calculating separately the force $\vec{F}_{12},\ \vec{F}_{13},\ ...,\ \vec{F}_{1n}$ exerted by $q_2,\ q_3,\ ...,\ q_n$, respenctively on q_1, then adding these forces vectorially.

- Their resultant $\vec{F}_1$ is that total force on q_1 due to the collection of charges.

$$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + ...\vec{F}_{1n}$$

- $$\vec{F}_1 = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\vec{r}_{12}|^2}\hat{r}_{12} + \frac{1}{4\pi\epsilon_0}\frac{q_1 q_3}{|\vec{r}_{13}|^2}\hat{r}_{13} + ... + \frac{1}{4\pi\epsilon_0}\frac{q_1 q_n}{|\vec{r}_{1n}|^2}\hat{r}_{1n}$$

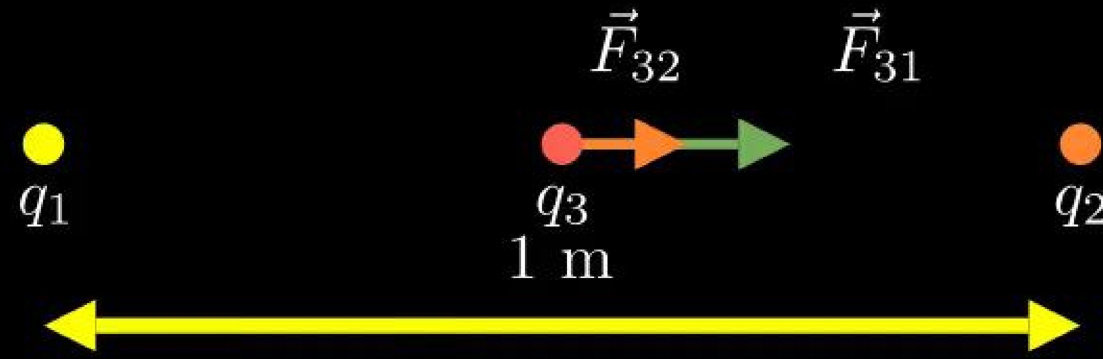
- $$\vec{F}_1 = \frac{q_1}{4\pi\epsilon_0}\left[\frac{q_2}{|\vec{r}_{12}|^2}\hat{r}_{12} + \frac{q_3}{|\vec{r}_{13}|^2}\hat{r}_{13} + ... + \frac{q_n}{|\vec{r}_{1n}|^2}\hat{r}_{1n}\right]$$

- $$\vec{F}_1 = \frac{q_1}{4\pi\epsilon_0}\left[\sum_{i=2}^{n}\frac{q_i}{|\vec{r}_{1i}|^2}\hat{r}_{1i}\right]$$

Example 17: Point charges $q_1 = 50\ \mu$ C and $q_2 = -25\ \mu$C are placed 1.0 m apart. What is the force on a third charge $q_3 = 20\ \mu$C placed midway between q_1 and q_2 ?

Solution :

- Given: $q_1 = 50\ \mu\ \text{C} = 50 \times 10^{-6}$ C
- $q_2 = -25\ \mu\ \text{C} = -25 \times 10^{-6}$ C
- $q_3 = 20\ \mu\ \text{C} = 20 \times 10^{-6}$ C
- $|\vec{r}_{12}| = 1$ m
- $\therefore |\vec{r}_{31}| = |\vec{r}_{32}| = 0.5$ m
- Find: Force on charge q_3, $F_3 =?$
- In the fig. forces $\vec{F}_{31}$ and $\vec{F}_{32}$ are acting in the same direction.



- $\therefore$ The magnitude of $\vec{F}_3$

$$|\vec{F}_3| = |\vec{F}_{31}| + |\vec{F}_{32}| = \frac{q_3}{4\pi\epsilon_0}\left[\frac{q_1}{|\vec{r}_{31}|^2} + \frac{q_2}{|\vec{r}_{32}|^2}\right]$$

$$|\vec{F}_3| = 9{\times}10^9{\times}20{\times}10^{-6}\left[\frac{50 \times 10^{-6}}{(5 \times 10^{-1})^2} + \frac{25 \times 10^{-6}}{(5 \times 10^{-1})^2}\right]$$

$$|\vec{F}_3| = 180{\times}10^3\left[\frac{75 \times 10^{-6}}{25 \times 10^{-2}}\right] = 18{\times}10^4{\times}3{\times}10^{-4} = 54\ N$$

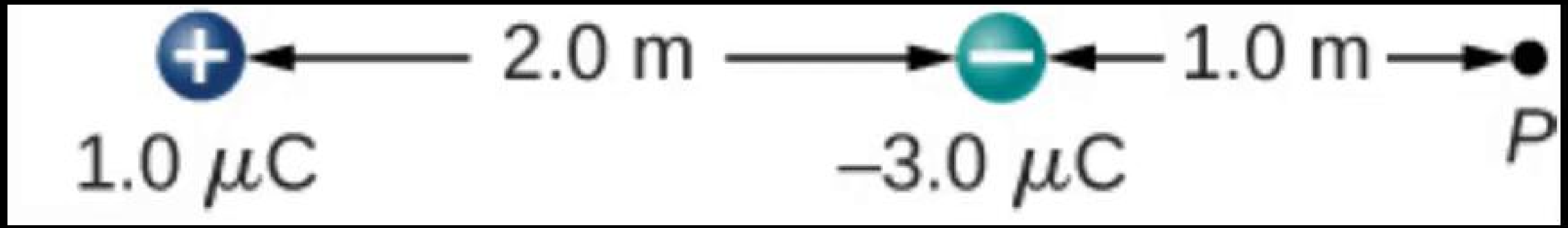
Example 18: Point charges $Q_1 = 2.0\ \mu\mathrm{C}$ and $Q_2 = 4.0\ \mu\mathrm{C}$ are located at $\vec{r}_1 = \left(4\hat{i} - 2\hat{j} + 5\hat{k}\right)\mathrm{m}$ and $\vec{r}_2 = \left(8\hat{i} + 5\hat{j} - 9\hat{k}\right)\mathrm{m}$. What is the force of Q_2 on Q_1 ?

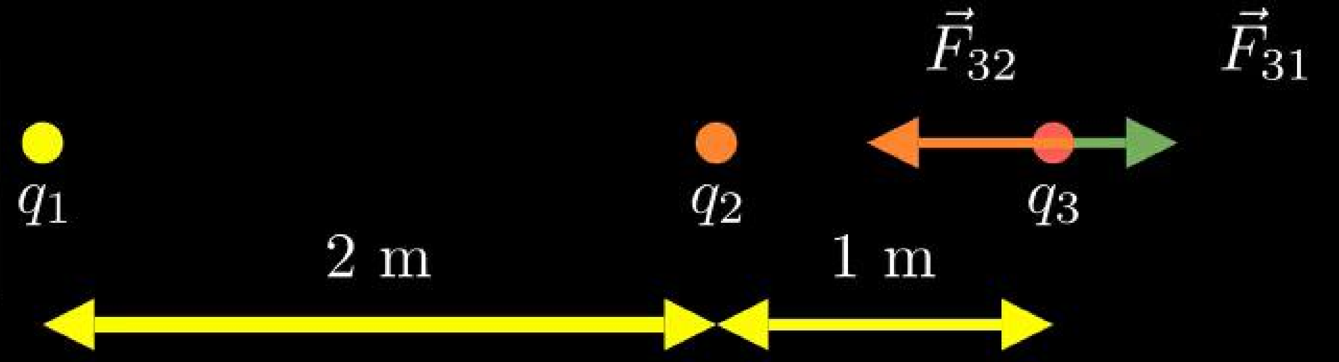
Solution :

- Given: $Q_1 = 2\ \mu\ \mathrm{C} = 2 \times 10^{-6}\ \mathrm{C}$
- $Q_2 = 45\ \mu\ \mathrm{C} = 4 \times 10^{-6}\ \mathrm{C}$
- $\vec{r}_1 = \left(4\hat{i} - 2\hat{j} + 5\hat{k}\right)\ \mathrm{m}$
- $\vec{r}_2 = \left(8\hat{i} + 5\hat{j} - 9\hat{k}\right)\mathrm{m}$
- Find: Force on charge Q_1 due to Q_2, $\vec{F}_{12} = ?$
- $\vec{F}_{12} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Q_1 Q_2}{|\vec{r}_{12}|^2}\ \hat{r}_{12}$

- $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2 = \left(4\hat{i} - 2\hat{j} + 5\hat{k}\right)\ \mathrm{m} - \left(8\hat{i} + 5\hat{j} - 9\hat{k}\right)\ \mathrm{m}$
- $\vec{r}_{12} = (-4\ \hat{i} - 7\ \hat{j} + 14\ \hat{k})\ \mathrm{m}$
- $|\vec{r}_{12}| = \sqrt{(-4)^2 + (-7)^2 + (14)^2} = \sqrt{261}\ \mathrm{m}$
- $\hat{r}_{12} = \dfrac{\vec{r}_{12}}{|\vec{r}_{12}|} = \dfrac{(-4\ \hat{i} - 7\ \hat{j} + 14\ \hat{k})}{\sqrt{261}}$
- $\vec{F}_{12} = 9 \times 10^9 \times \dfrac{2 \times 10^{-6} \times 4 \times 10^{-6}}{261}\dfrac{(-4\ \hat{i} - 7\ \hat{j} + 14\ \hat{k})}{\sqrt{261}}$
- $\vec{F}_{12} = 72 \times 10^{-3} \times \dfrac{(-4\ \hat{i} - 7\ \hat{j} + 14\ \hat{k})}{261 \times \sqrt{261}}$

Example 19: A charge $q_3 = 2.0\ \mu$C is placed at the point P shown below. What is the force on q_3?





Solution :

- Given: $q_1 = 1\ \mu$ C $= 10^{-6}$ C
- $q_2 = -3\ \mu$ C $= -3 \times 10^{-6}$ C
- $q_3 = 2\ \mu$ C $= 2 \times 10^{-6}$ C
- $|\vec{r}_{31}| = 3$ m, $|\vec{r}_{32}| = 1$ m
- Find: Force on charge q_3, $F_3 =?$
- In the fig. forces $\vec{F}_{31}$ and $\vec{F}_{32}$ are acting in the opposite direction.

- $\therefore$ The magnitude of $\vec{F}_3$

$$|\vec{F}_3| = |\vec{F}_{32}| - |\vec{F}_{31}| = \frac{q_3}{4\pi\epsilon_0}\left[\frac{q_2}{|\vec{r}_{32}|^2} + \frac{q_1}{|\vec{r}_{31}|^2}\right]$$

$$|\vec{F}_3| = 9 \times 10^9 \times 2 \times 10^{-6}\left[\frac{3 \times 10^{-6}}{(1)^2} + \frac{10^{-6}}{(2)^2}\right]$$

$$|\vec{F}_3| = 18 \times 10^3\left[3 - \frac{1}{4}\right] \times 10^{-6} = 18 \times 10^{-3} \times \frac{11}{4}$$

- $|\vec{F}_3| = 49.5 \times 10^{-3}$ N (Along -ve x-axis)

Example 20: Two charges $+3\ \mu$C and $+12\ \mu$C are fixed 1 m apart, with the second one to the right. Find the magnitude and direction of the net force on a -2 nC charge when placed at the following locations: (a) halfway between the two (b) half a meter to the left of the $+3\ \mu$C charge (c) half a meter above the $+12\ \mu$C charge in a direction perpendicular to the line joining the two fixed charges?

Solution :

- Given: $q_1 = +3\ \mu\ \mathrm{C} = 3 \times 10^{-6}$ C
- $q_2 = +12\ \mu\ \mathrm{C} = 12 \times 10^{-6}$ C
- $q_3 = -2\ \ \mathrm{nC} = -2 \times 10^{-9}$ C
- $|\vec{r}_{12}| = 1$ m
- (a) $|\vec{r}_{32}| = |\vec{r}_{31}| = 0.5$ m
- $|\vec{F}_{31}| = 2.16 \times 10^{-4}$ N (to the left)

- $|\vec{F}_{net}| = 6.47 \times 10^{-4}$ N (to the right)
- (b) $|\vec{r}_{32}| = 1.5$ m , $|\vec{r}_{31}| = 0.5$ m
- $|\vec{F}_{31}| = 2.16 \times 10^{-4}$ N (to the right)
- $|\vec{F}_{32}| = 0.96 \times 10^{-4}$ N (to the right)
- $|\vec{F}_{net}| = 3.12 \times 10^{-4}$ N (to the right)

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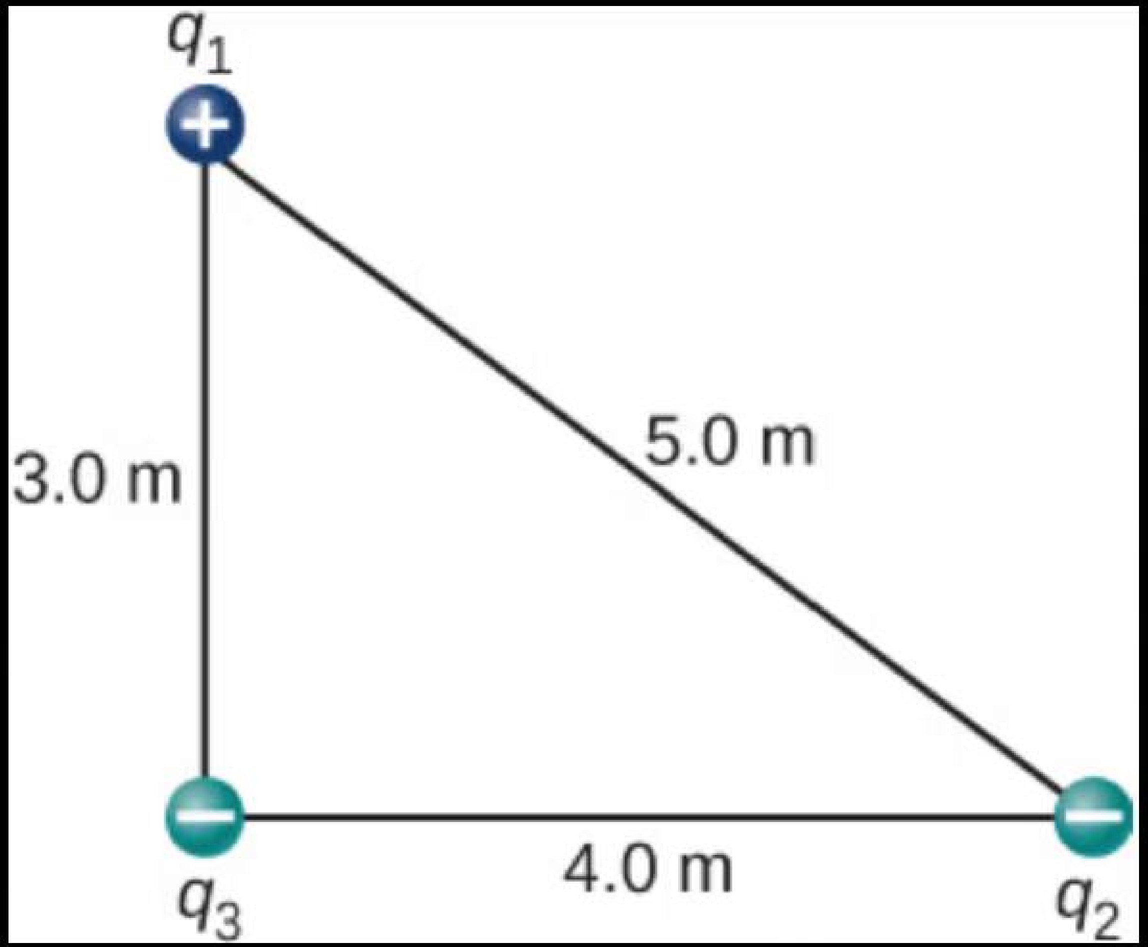
Solution :

- $|\vec{F}_{net}| = 6.47 \times 10^{-4}$ N (to the right)
- (b) $|\vec{r}_{32}| = 1.5$ m , $|\vec{r}_{31}| = 0.5$ m
- $|\vec{F}_{31}| = 2.16 \times 10^{-4}$ N (to the right)
- $|\vec{F}_{32}| = 0.96 \times 10^{-4}$ N (to the right)
- $|\vec{F}_{net}| = 3.12 \times 10^{-4}$ N (to the right)

- (c) $\vec{F}_{31} = -3.86 \times 10^{-5}\ \hat{i} - 0.193 \times 10^{-5}\ \hat{j}$ N
- $\vec{F}_{31} = -8.63 \times 10^{-5}\ \hat{j}$ N
- $\vec{F}_{net} = -3.86 \times 10^{-5}\ \hat{i} - 8.82 \times 10^{-5}\ \hat{j}$ N

Example 21: The charges $q_1 = 2.0 \times 10^{-7}$ C, $q_2 = -4.0 \times 10^{-7}$ C, and $q_3 = -1.0 \times 10^{-7}$ C are placed at the corners of the triangle shown below. What is the force on q_1 ?



Solution :

- Let us cosinder the origin at q_3
- Position Vector of q_1, q_2, and q_3
  $\vec{r}_1 = 0\ \hat{i} + 3\ \hat{j}$
  $\vec{r}_2 = 4\ \hat{i} + 0\ \hat{j}$
  $\vec{r}_3 = 0\ \hat{i} + 0\ \hat{j}$
- $\vec{r}_{12} = \vec{r}_1 - \vec{r}_2 = -4\ \hat{i} + 3\ \hat{j}$
- $\vec{r}_{13} = \vec{r}_1 - \vec{r}_3 = 0\ \hat{i} + 3\ \hat{j}$
- $|\vec{r}_{12}| = \sqrt{(-4)^2 + 3^2} = \sqrt{25} = 5$
- $|\vec{r}_{13}| = \sqrt{(0)^2 + 3^2} = \sqrt{9} = 3$

- $\hat{r}_{12} = \dfrac{\vec{r}_{12}}{|\vec{r}_{12}|} = \dfrac{-4\ \hat{i} + 3\ \hat{j}}{5}$
- $\hat{r}_{13} = \dfrac{\vec{r}_{13}}{|\vec{r}_{13}|} = \dfrac{0\ \hat{i} + 3\ \hat{j}}{3}$
  $\hat{r}_{13} = \hat{i}$
- $\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} = \dfrac{q_1}{4\pi\epsilon_0}\left[\dfrac{q_2}{|\vec{r}_{12}|^2}\ \hat{r}_{12} + \dfrac{q_3}{|\vec{r}_{13}|^2}\ \hat{r}_{13}\right]$

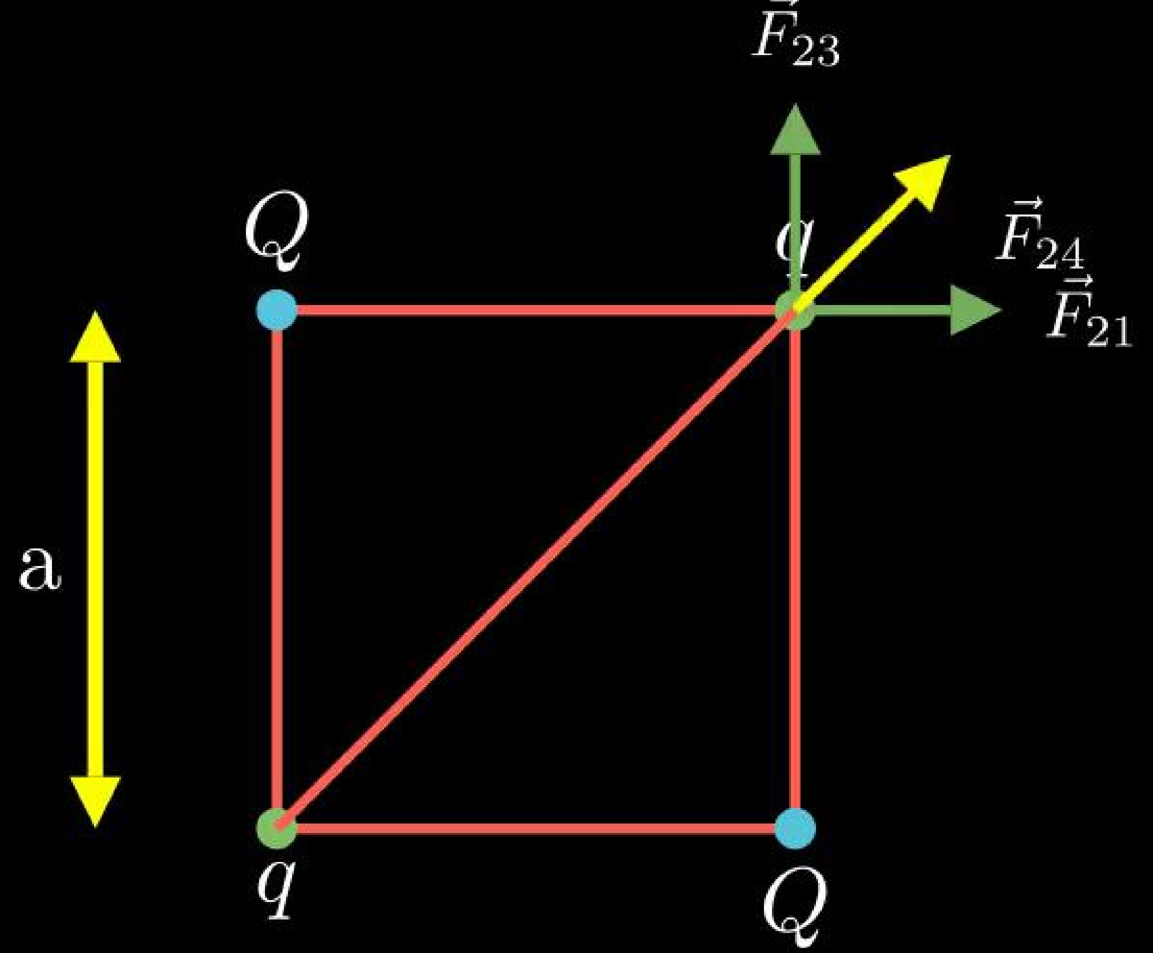
Example 22: Two fixed charges $+4q$ and $+1q$ are at a distance 3 m apart. At what point between the charges, a third charge $+q$ must be placed to keep it in equilibrium?

Solution :

Example 23: Four charges Q, q, Q, and q are kept at the four corners of a square as shown below. What is the relation between Q and q so that the net force on a charge q is zero.

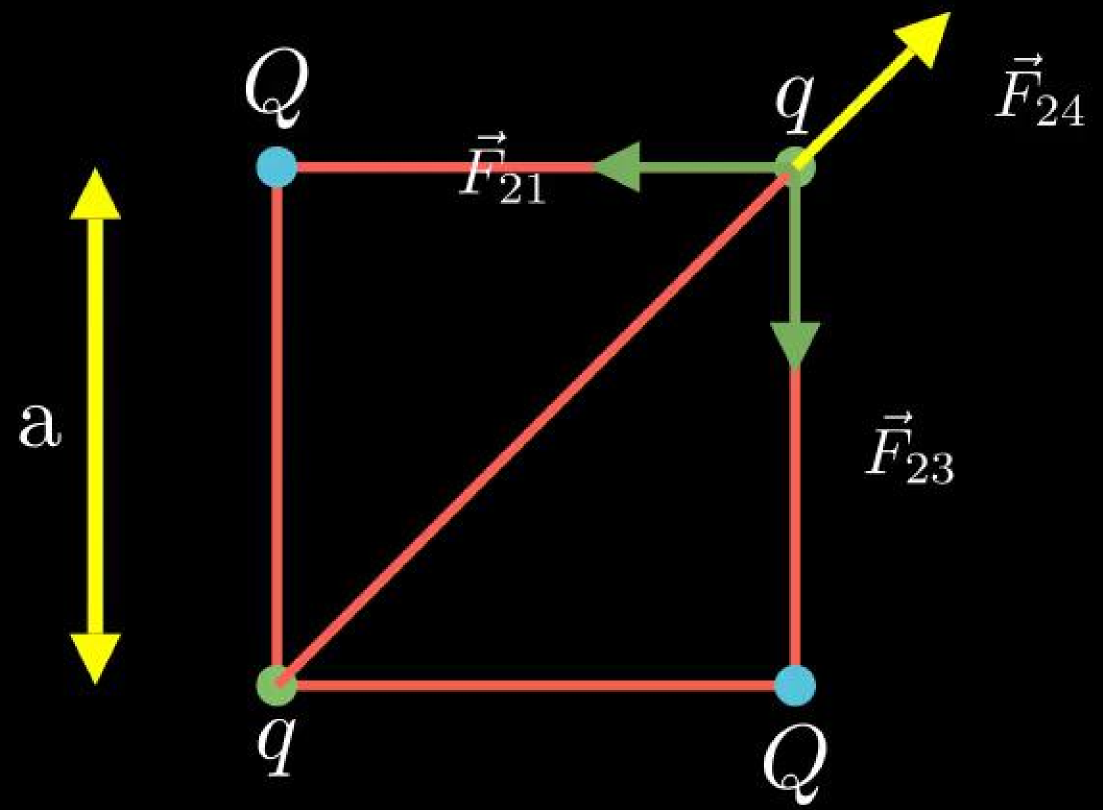
Solution :

- Case 1: Q and q are of same sign



Solution :

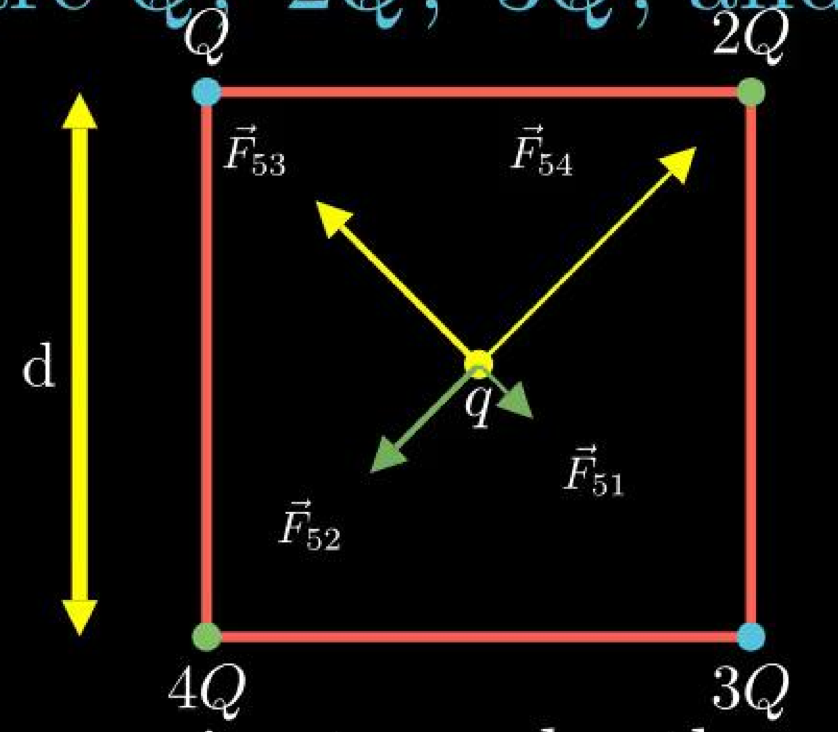
- Case 1: Q and q are of same sign
- This case is not possible since the forces never cancel out each other.
- Case 2: Q and q are of opposite sign
- $\vec{F}_{21} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qq}{a^2}\,(-\hat{i})$
- $\vec{F}_{23} = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qq}{a^2}\,(-\hat{j})$
- $\vec{F}_{24} = \dfrac{1}{4\pi\epsilon_0}\dfrac{qq}{2a^2}\,\dfrac{(\hat{i}+\hat{j})}{\sqrt{2}}$



Solution :

- Case 1: Q and q are of same sign
- This case is not possible since the forces never cancel out each other.
- Case 2: Q and q are of opposite sign
- $\vec{F}_{21} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{a^2}\;(-\hat{i})$
- $\vec{F}_{23} = \frac{1}{4\pi\epsilon_0}\frac{Qq}{a^2}\;(-\hat{j})$
- $\vec{F}_{24} = \frac{1}{4\pi\epsilon_0}\frac{qq}{2a^2}\;\frac{(\hat{i}+\hat{j})}{\sqrt{2}}$
- $\vec{F}_{net} = \vec{F}_{21} + \vec{F}_{23} + \vec{F}_{24}$
- $\vec{F}_{net} = \frac{q}{4\pi\epsilon_0}\left[-\frac{Q}{a^2}\;\hat{i} - \frac{Q}{a^2}\;\hat{j} + \frac{q}{2a^2}\;\frac{(\hat{i}+\hat{j})}{\sqrt{2}}\right]$
- $\vec{F}_{net} = \frac{q}{4\pi\epsilon_0 \times a^2}\left[-2\sqrt{2}Q\;\hat{i} - 2\sqrt{2}Q\;\hat{j} + q\hat{i} + q\hat{j}\right]$
- $0 \quad = \quad \frac{q}{4\pi\epsilon_0 \times a^2}\left[(q-2\sqrt{2}Q)\;\hat{i} + (q-2\sqrt{2}Q)\;\hat{j}\right]$
  $\vec{F}_{net} = 0)$
- $(q-2\sqrt{2}Q)\;\hat{i} + (q-\sqrt{2}Q)\;\hat{j}$
- $q - 2\sqrt{2}Q = 0$ OR $q = 2\sqrt{2}Q$
- $\therefore$ Q and q must have opposite sign and $q = 2\sqrt{2}Q$

Example 24: Find the force on the charge q kept at the centre of a square of side 'd'. The charges on the four corners of the square aare Q, $2Q$, $3Q$, and $4Q$ respectively as shown in the figure below:



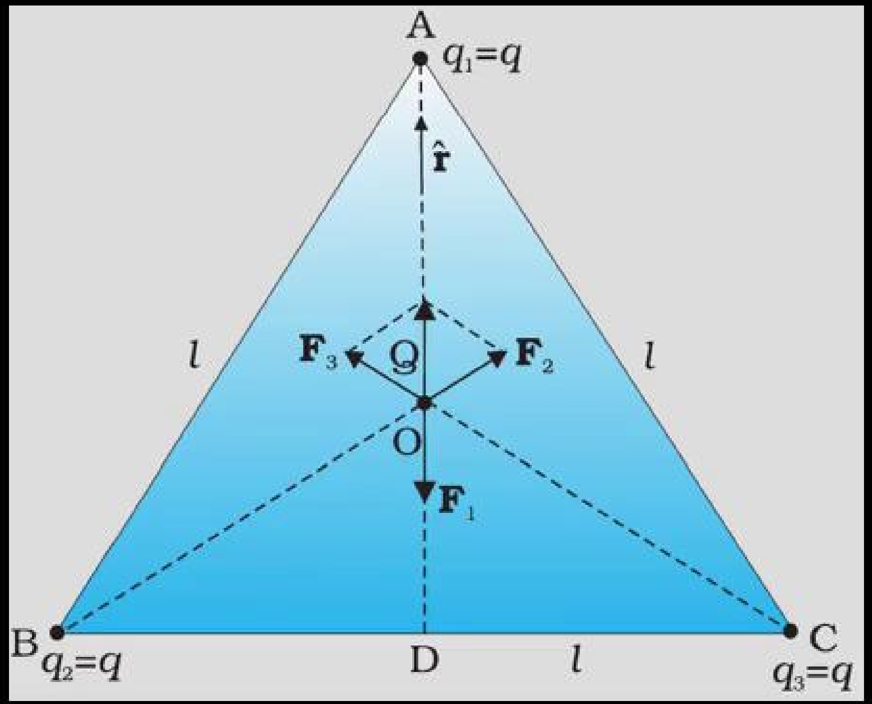
Solution :

- $|\vec{F}_{51}| = \dfrac{1}{4\pi\epsilon_0}\dfrac{Qq}{r^2} = F$
- $|\vec{F}_{52}| = \dfrac{1}{4\pi\epsilon_0}\dfrac{2Qq}{r^2} = 2F$
- $|\vec{F}_{53}| = \dfrac{1}{4\pi\epsilon_0}\dfrac{3Qq}{r^2} = 3F$
- $|\vec{F}_{54}| = \dfrac{1}{4\pi\epsilon_0}\dfrac{4Qq}{r^2} = 4F$
- We can see, $\vec{F}_{51}$ and $\vec{F}_{53}$ are exactly opposite to each other so its net effect will be $2F$ towards

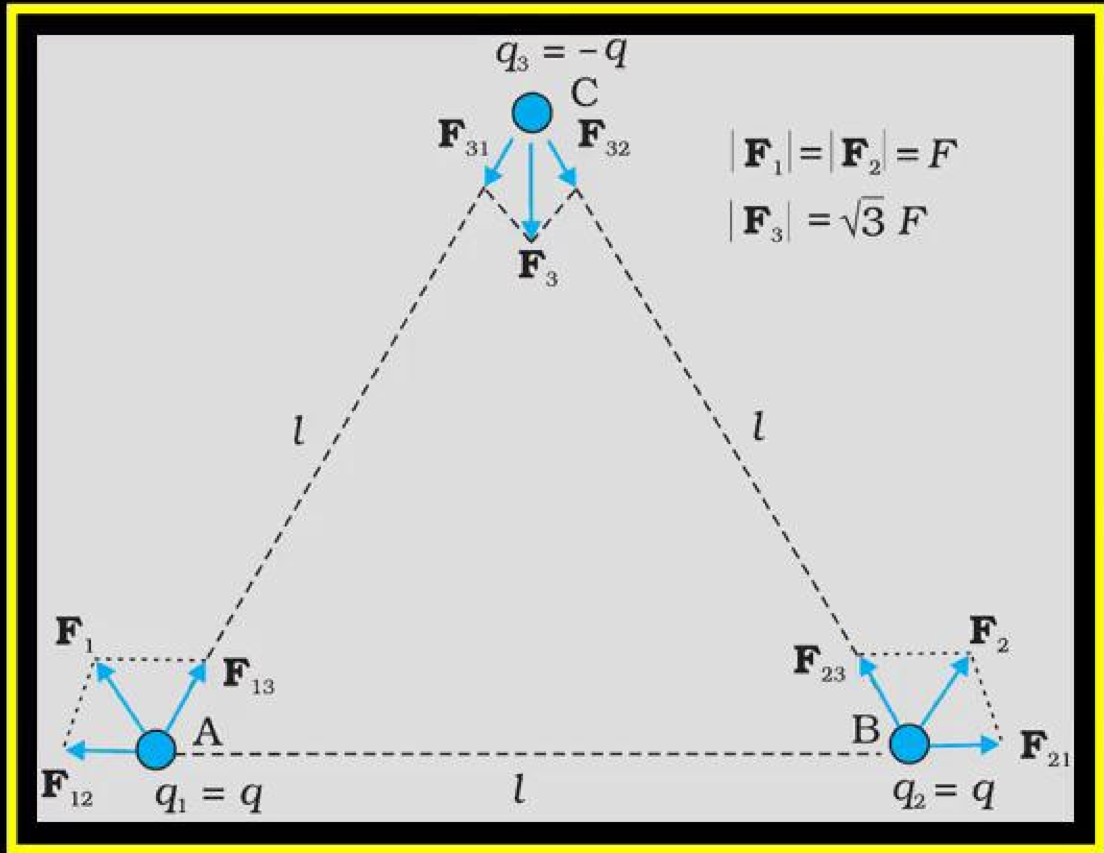
- Also, $\vec{F}_{52}$ and $\vec{F}_{54}$ are exactly opposite to each other so its r effect will be $2F$ towards $2Q$
- So, the resultant force of $2F$ and $2F$ will be (using Pythagor theorem) $2\sqrt{2}F$
- Magnitude of resultant force $= \dfrac{2\sqrt{2}}{4\pi\epsilon_0}\dfrac{Qq}{(d/\sqrt{2})^2} = \dfrac{4\sqrt{2}}{4\pi\epsilon_0}\dfrac{Qq}{d^2}$

Example 1.6: Consider three charges q_1, q_2, q_3 each equal to q at the vertices of an equilateral triangle of side l. What is the force on a charge Q (with the same sign as q) placed at the centroid of the triangle, as shown in Fig.?

Solution :



Example 1.7: Consider the charges q, q, and $-q$ placed at the vertices of an equilateral triangle, as shown in Fig. . What is the force on each charge?



Solution :

(a) Calculation for Force on q_1 (F_1)

$$|\vec{F}_{12}| = |\vec{F}_{13}| = \frac{1}{4\pi\epsilon_0}\frac{q^2}{l^2} = F$$

Magnitude of net force on q_1 $(|\vec{F}_1|)$ is

$$|\vec{F}_1| = \sqrt{|\vec{F}_{12}|^2 + |\vec{F}_{13}|^2 + 2|\vec{F}_{12}||\vec{F}_{13}|\cos\theta}$$

$$|\vec{F}_1| = \sqrt{F^2 + F^2 + 2F^2\cos(120)}$$

$$|\vec{F}_1| = \sqrt{2F^2 - 2F^2 \times \frac{1}{2}} = \sqrt{2F^2 - F^2}$$

$$|\vec{F}_1| = F$$

(b) Similarly, Magnitude of net force on q_2 $|\vec{F}_2| = F$

(c) Calculation for Force on q_3 (F_3)

$$|\vec{F}_{31}| = |\vec{F}_{32}| = F$$

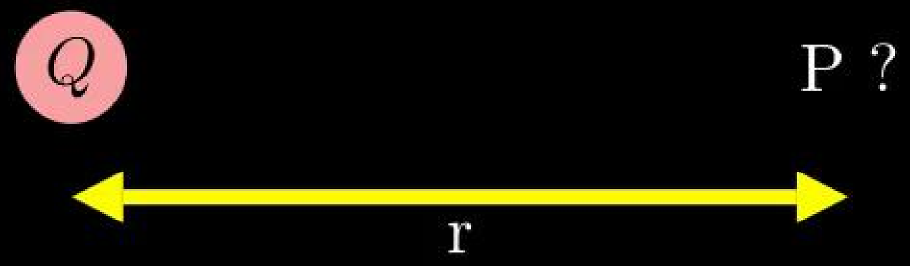
Magnitude of net force on q_3 is

$$|\vec{F}_3| = \sqrt{F^2 + F^2 + 2F^2\cos(60)}$$
$$|\vec{F}_3| = \sqrt{3}F$$

Electric Field

Electric Field

If charge q is removed, then what is left in the surrounding? Is there nothing?



If there is nothing at the point P, then how does a force act when we place the charge q at P.

In order to answer such questions, the early scientists introduced the concept of field.

According to this, we say that the charge Q produces an electric field everywhere in the surrounding. When another charge q is brought at some point P, the field there acts on it and produces a force.

The term field in physics generally refers to a quantity that is defined at every point in space and may vary from point to point.

Temperature, for example, is a scalar field, which we write as $T(x, y, z)$.

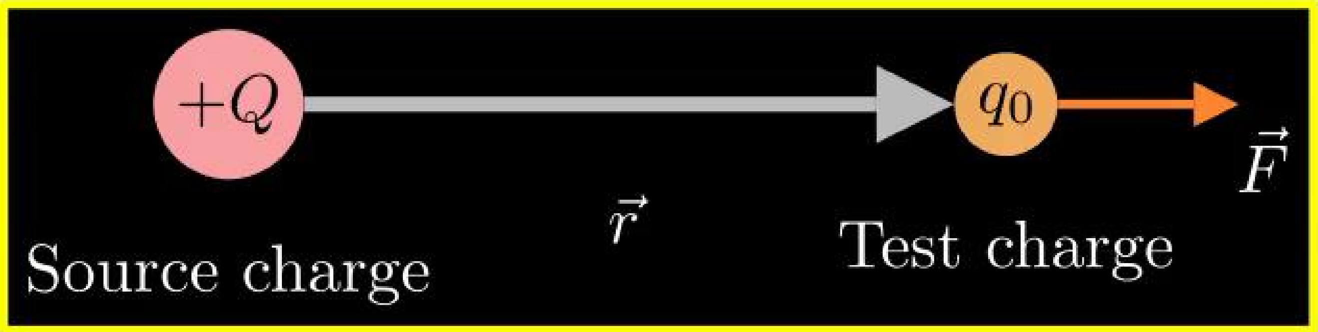
Electric Field

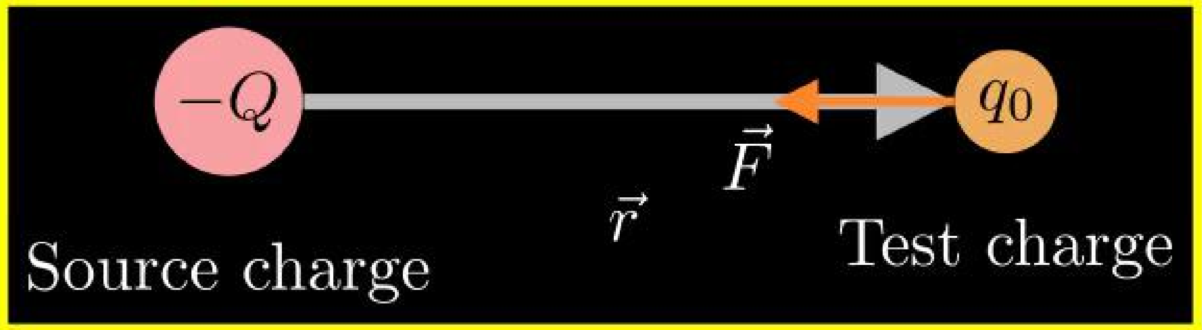
Electric Field:

Electric field is the region of space around a charge in which its influence (force) can be experienced by other charges.

Electric Field Intensity :

The intensity of electric field at any point P is defined as the elctric force on a unit positive test charge placed at the point P.



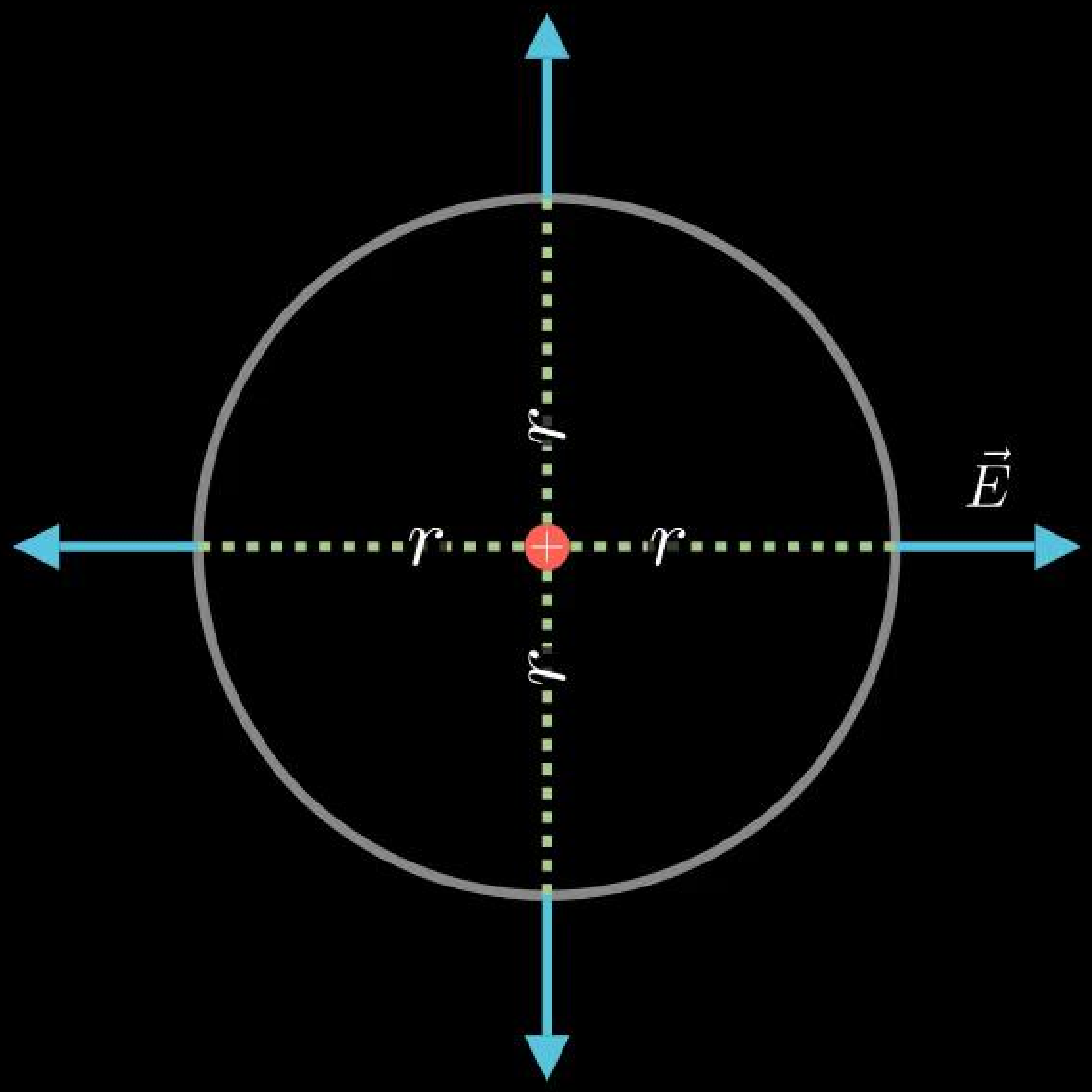


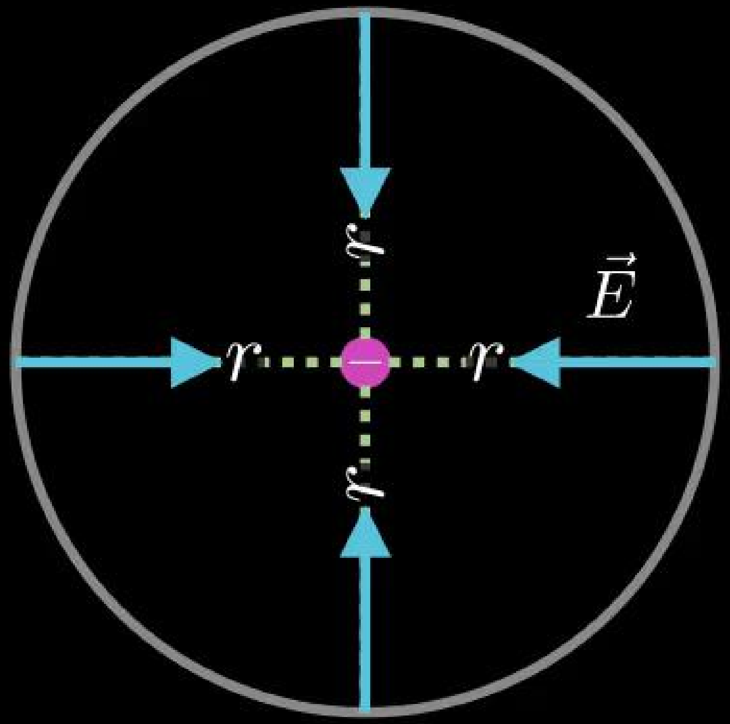
$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \lim_{q_0 \to 0} \frac{1}{4\pi\epsilon_0} \frac{Q q_0}{r^2 \times q_0} \hat{r} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} \hat{r}$$

- A test charge (q_0) is a charge of small magnitude such that it does not disturb the Source charge (Q) which produces the electric filed .

- Though, $\vec{E} = (\vec{F}/q_0)$, but $\vec{E}$ does not depend on test charge q_0.

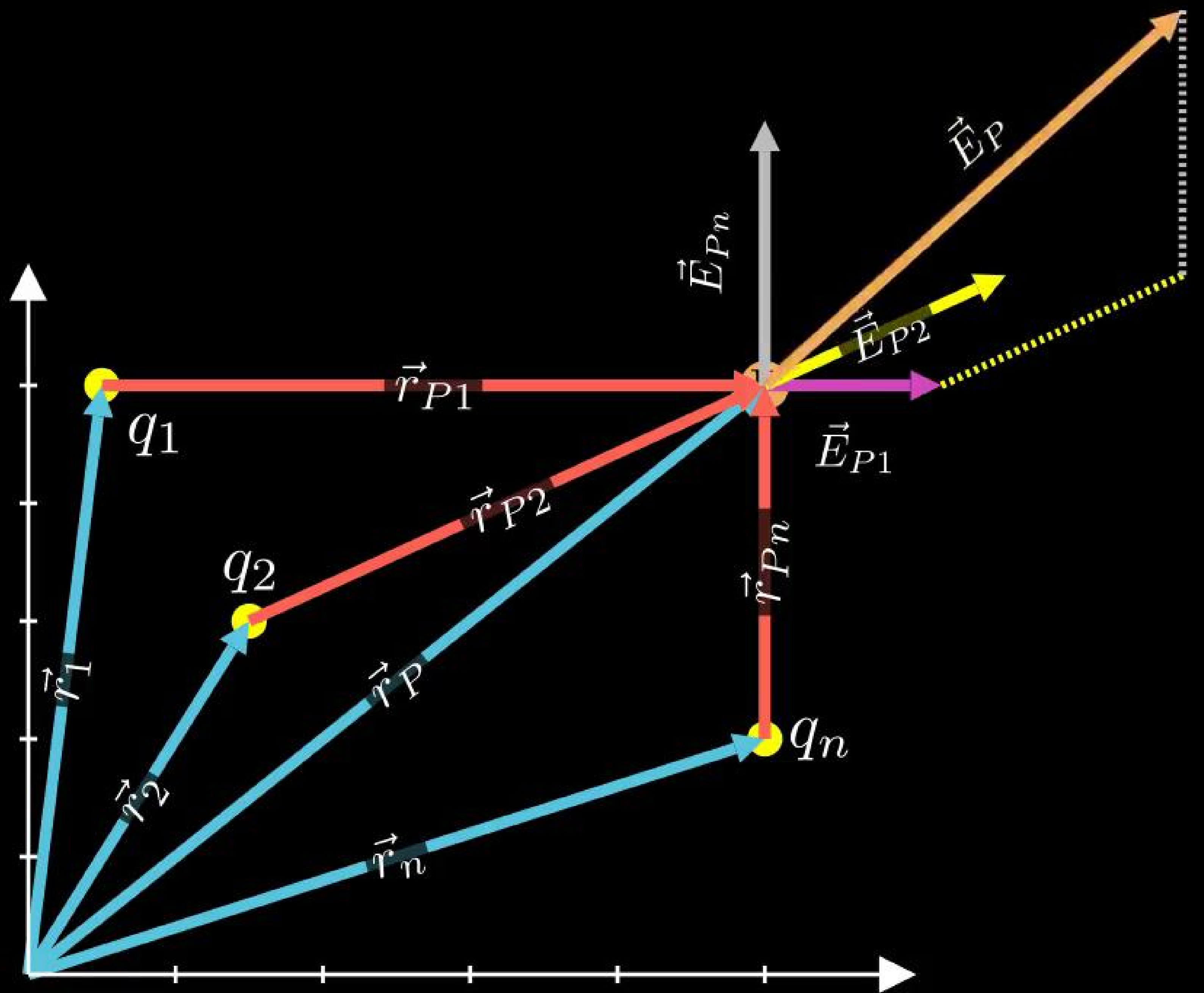
- Electric Field is a vector quantity and its S.I unit : NC^{-1}
- For a positive charge, the electric field will be directed radially outwards from the charge.
- if the source charge is negative, the electric field vector, at each point, points radially inwards.
- At equal distances from the charge Q, the magnitude of its electric field E is same. The magnitude of electric field E due to a point charge is thus same on a sphere with the point charge at its centre; in other words, it has a spherical symmetry.





Electric Field due to a system of charge:

Suppose we have to find the electric field $\vec{E}$ at point P due to point charges $q_1,\ q_2, ...,\ q_n$,with position vectors $r_1, r_2, ..., r_n$



Electric filed $\vec{E}_{P1}$ at P due to q_1:

$$\vec{E}_{P1} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{|\vec{r}_{P1}|^2}\ \hat{r}_{P1}$$

Electric filed $\vec{E}_{P2}$ at P due to q_2:

$$\vec{E}_{P2} = \frac{1}{4\pi\epsilon_0}\frac{q_2}{|\vec{r}_{P2}|^2}\ \hat{r}_{P2}$$

Electric filed $\vec{E}_{Pn}$ at P due to q_n:

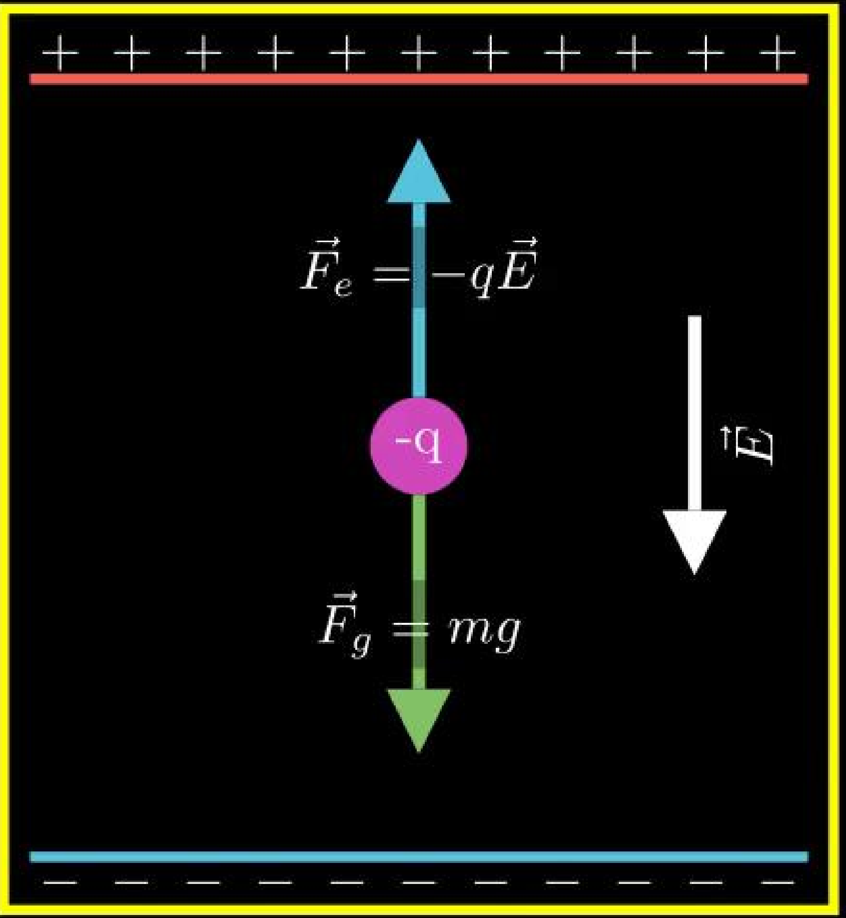
$$\vec{E}_{Pn} = \frac{1}{4\pi\epsilon_0}\frac{q_n}{|\vec{r}_{Pn}|^2}\ \hat{r}_{Pn}$$

By the superposition principle, the electric field $\vec{E}_P$ at P:

$$\vec{E}_P = \vec{E}_{P1} + \vec{E}_{P2} + \vec{E}_{Pn}$$

$$\vec{E}_P = \frac{1}{4\pi\epsilon_0}\left[\frac{q_1}{|\vec{r}_{P1}|^2}\hat{r}_{P1} + \frac{q_2}{|\vec{r}_{P2}|^2}\hat{r}_{P2} + ... + \frac{q_n}{|\vec{r}_{Pn}|^2}\hat{r}_{Pn}\right]$$

Example 25 : A negatively charged oil drop is suspended in uniform field of 3×10^4 N/C, so that it neither falls not rises. The charge on the drop will be: (given the mass of the oil drop $m = 9.9 \times 10^{-15}$ kg and $g = 10$ ms^{-2})



Solution :

Given: $E = 3 \times 10^4$ N/C,
$m = 9.9 \times 10^{-15}$ kg, and $g = 10$ ms^{-2}

Find: Charge on the oil drop $q = ?$

There are two force acting on the oil drop:

Force due to elctric field in upward direction:

$$|\vec{F}_e| = qE$$

Gravitational force in downward direction:

$$|\vec{F}_g| = mg$$

Since, the oil drop is neither falling or rising :

$$\therefore \ |\vec{F}_e| = |\vec{F}_g|$$

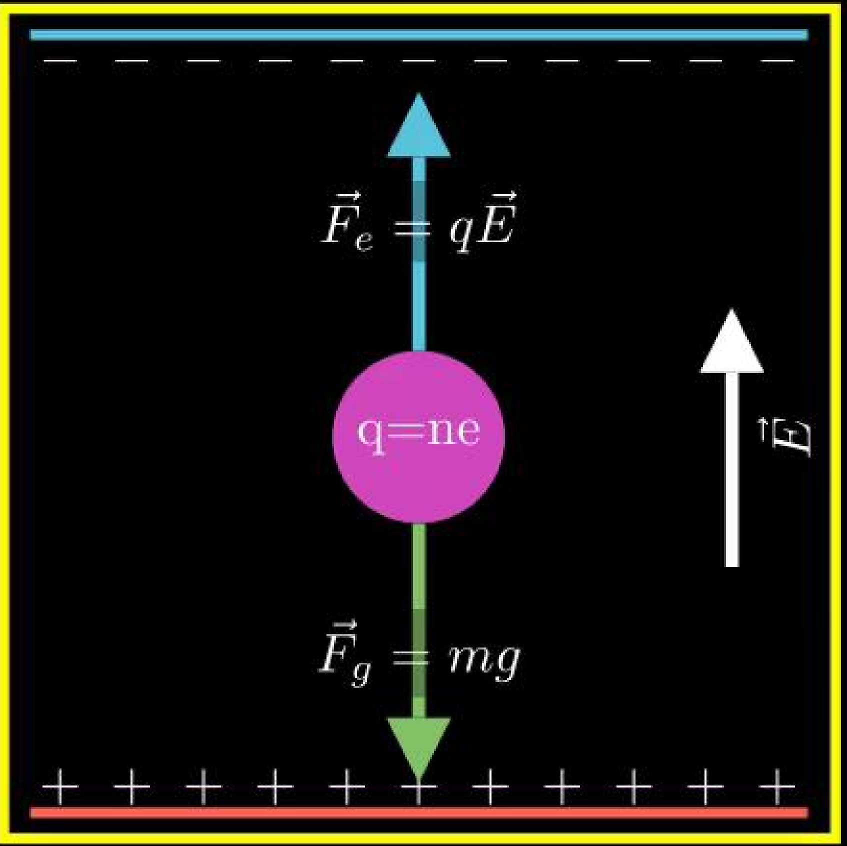
$$qE = mg$$

$$q = \frac{mg}{E} = \frac{\cancel{9.9} \times 10^{-15} \times 10}{\cancel{3} \times 10^4}$$

$$= 3.3 \times 10^{-14} \times 10^{-4}$$

$$\boxed{q = 3.3 \times 10^{-18} \text{ C}}$$

Example 26 : How many electrons should be removed from a coin of mass 1.6 g, so that it may float in an electric field of intensity 10^9 N/C directed upward? (take g = 9.8 ms^2)



(a) 9.8×10^7 (b) 9.8×10^5

(c) 9.8×10^3 (d) 9.8×10^1

Solution :

Given: $E = 10^9$ N/C,
$m = 1.6\ \text{g} = 1.6 \times 10^{-3}$ kg, and $g = 9.8\ \text{ms}^{-2}$

Number of electron removed $n = ?$

There are two force acting on the oil drop:

Since, the oil drop is neither falling or rising :

$\therefore\ |\vec{F}_e| = |\vec{F}_g|$

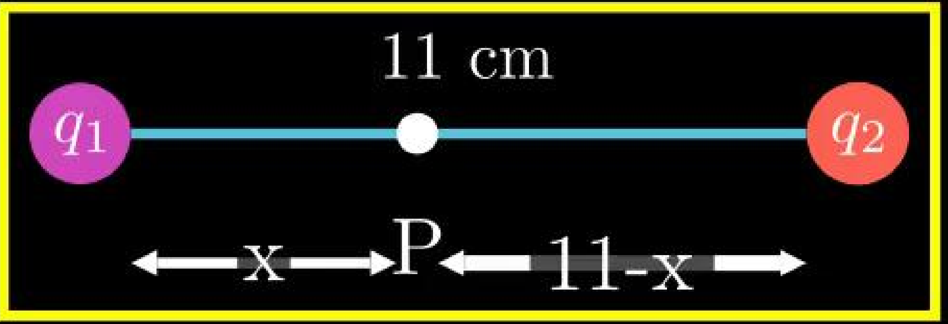
$$neE = mg$$

$$n = \frac{mg}{eE} = \frac{\cancel{1.6} \times 10^{-3} \times 9.8}{\cancel{1.6} \times 10^{-19} \times 10^9}$$

$$= 9.8 \times 10^{-3} \times 10^{10}$$

$$\boxed{n = 9.8 \times 10^7}$$

Example 27 : The distance between the two charges 25 μ C and 36 μ C is 11 cm. At what point on the line joining the two charges the intensity will be zero at a distance of



(a) 4 cm from 36 μ C

(b) 4 cm from 25 μ C

(c) 5 cm from 36 μ C

(d) 5 cm from 25 μ C

Solution :

Given: $q_1 = 25\ \mu$ C, $q_2 = 36\ \mu$ C, and $r = 11$ cm

Let the distance of point P (from q1=25 μC) where the intensity will zero be $r_{P1} = x$ cm

So, the distance of point P from q_2, $r_{P2} = 11 - x$ cm

Since elctric filed intensity is zero at P

$$\therefore |\vec{E}_{P1}| = |\vec{E}_{P2}|$$

$$\frac{1}{\cancel{4\pi\epsilon_0}} \frac{q_1}{r_{P1}^2} = \frac{1}{\cancel{4\pi\epsilon_0}} \frac{q_2}{r_{P2}^2}$$

$$\frac{25\ \cancel{\mu C}}{x^2\ \cancel{cm^2}} = \frac{36\ \cancel{\mu C}}{(11-x)^2\ \cancel{cm^2}}$$

$$\frac{5}{x} = \frac{6}{11-x}$$

$$55 - 5x = 6x$$

$$55 = 6x + 5x$$

$$\boxed{x = 5}$$

Example 1.8 :An electron falls through a distance of 1.5 cm in a uniform electric field of magnitude 2.0×10^4 NC^{-1} [Fig. 1.13(a)]. The direction of the field is reversed keeping its magnitude unchanged and a proton falls through the same distance [Fig. 1.13(b)]. Compute the time of fall in each case. Contrast the situation with that of 'free fall under gravity'.

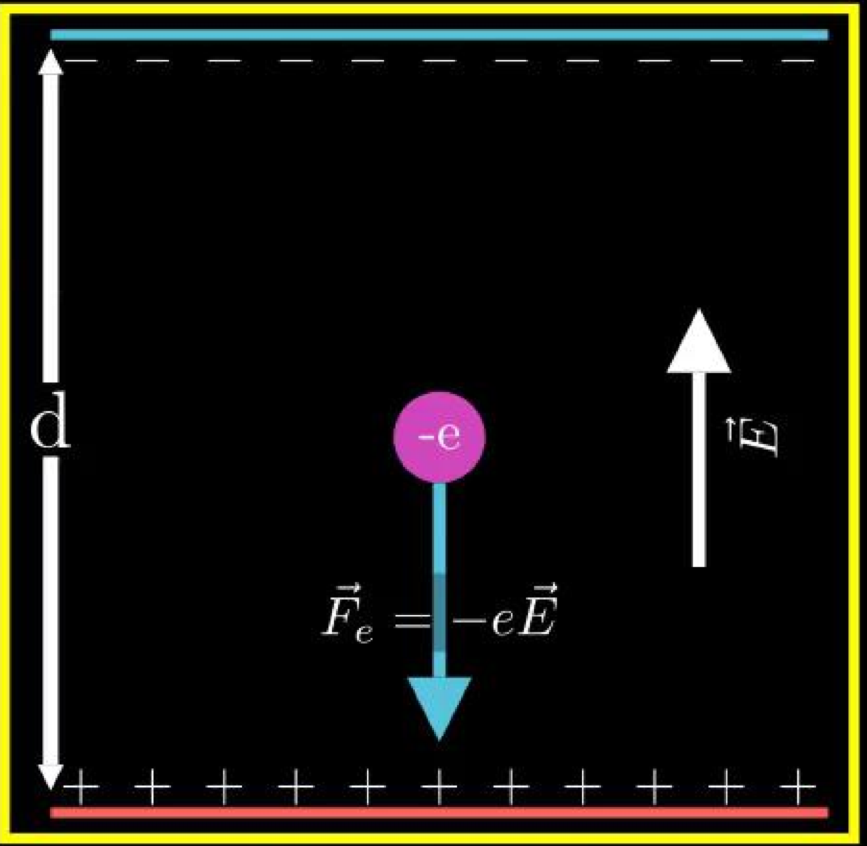


Fig 1.13(a)

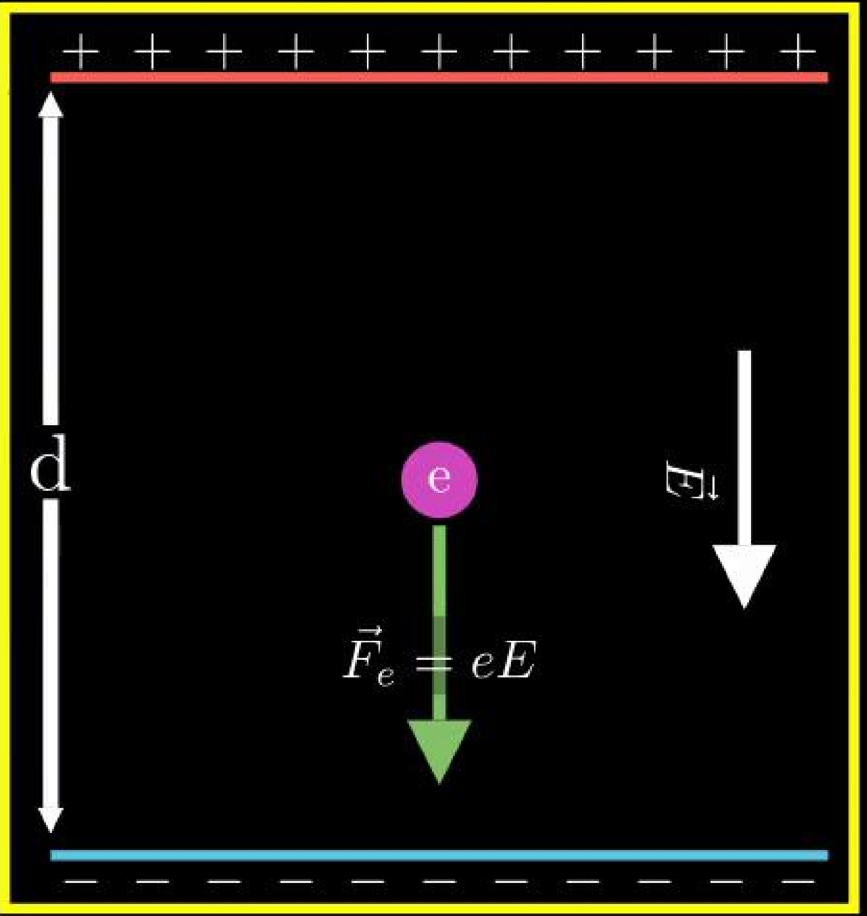


Fig 1.13(b)

Solution :

Given: $E = 2 \times 10^4$ N/C,
$d = 1.5$ cm $= 1.5 \times 10^{-2}$ m

Find: Time of fall of elctron (t_e) and proton (t_p)

Force on electron $F_e = e \times E$

$m_e a_e = eE \quad (\because F = ma)$

$$a_e = \frac{eE}{m_e} = \frac{1.6 \times 10^{-19} \times 2 \times 10^4}{9.11 \times 10^{-31}}$$

$$a_e = 3.51 \times 10^{15}\ \text{ms}^{-2}$$

$$S = ut + \frac{1}{2}at^2 (\text{2nd eq. of motion})$$

$$d = 0 \times t + \frac{1}{2}a_e t_e^2$$

$$t_e = \sqrt{\frac{2d}{a_e}} = \sqrt{\frac{2 \times 1.5 \times 10^{-2}}{3.51 \times 10^{15}}}$$

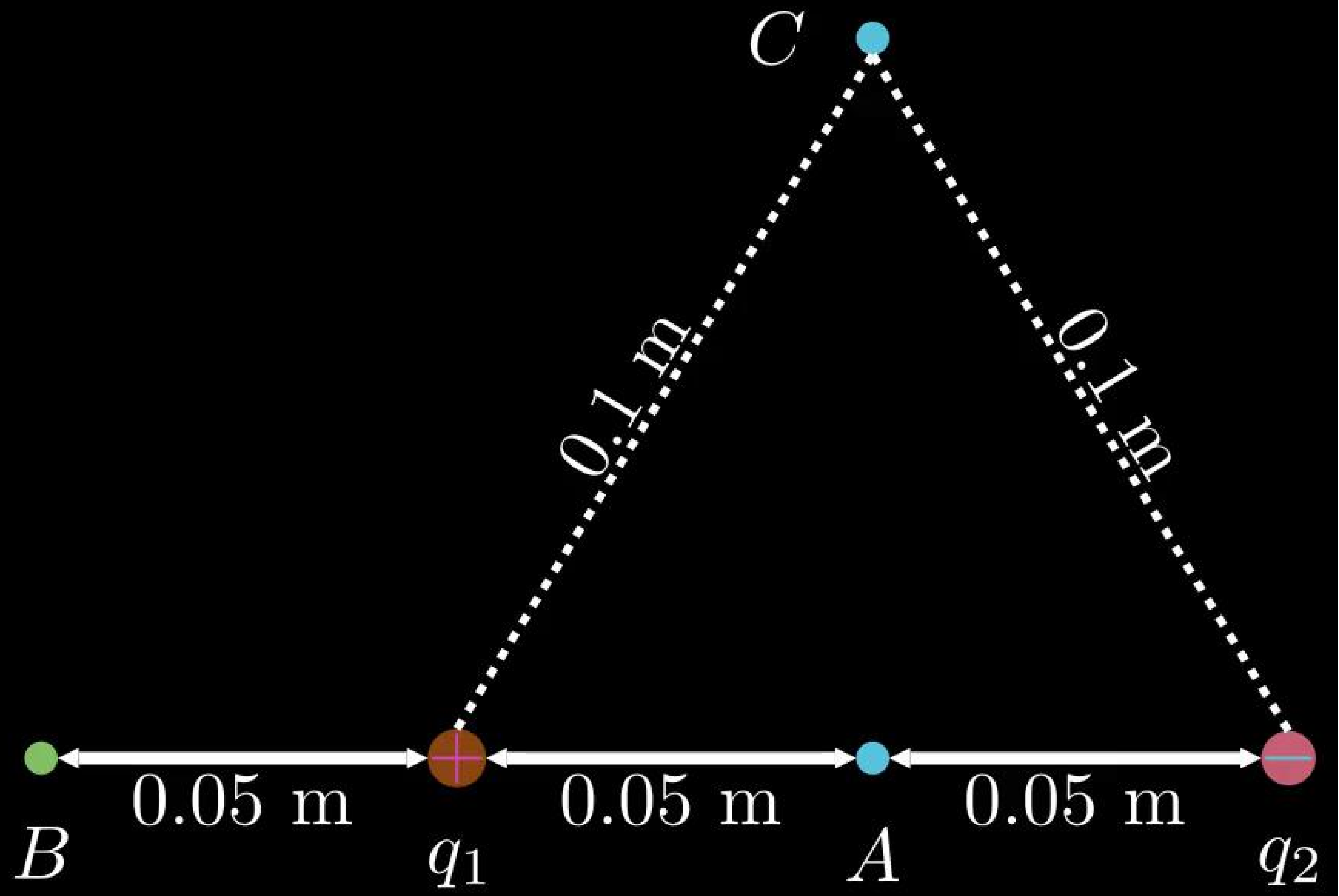
$$\boxed{t_e = 2.96 \times 10^{-9}\ s}$$

Example 1.8 : Two point charges q_1 and q_2, of magnitude $+10^{-8}$ C and -10^{-8} C, respectively, are placed 0.1 m apart. Calculate the electric fields at points A, B and C shown in Fig. 1.14.

Solution :

Given: $q_1 = +10^{-8}$ C and $q_2 = -10^{-8}$ C

Find: Electric field at Point A, B and C

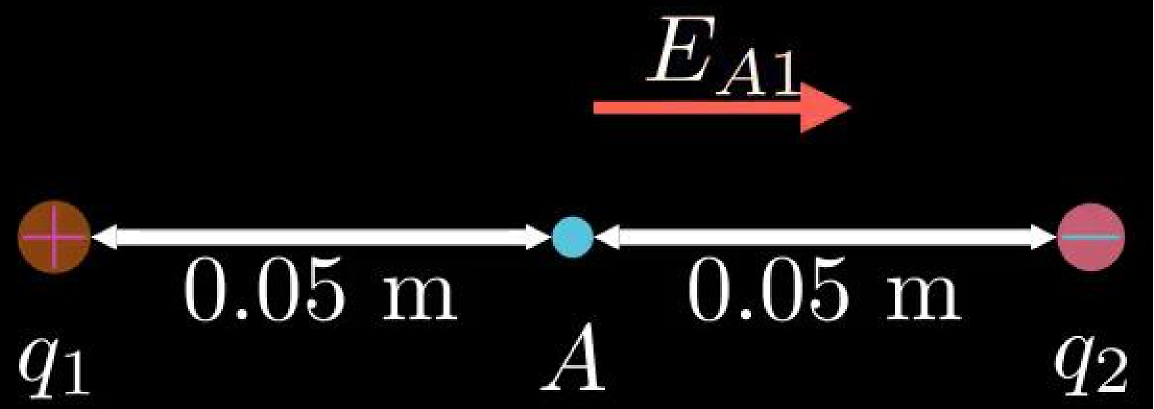


Solution :

Given: $q_1 = +10^{-8}$ C and $q_2 = -10^{-8}$ C

Find: Electric field at Point A, B and C

(i) Magnitude of Electric field at A due to q_1



Solution :

Given: $q_1 = +10^{-8}$ C and $q_2 = -10^{-8}$ C

Find: Electric field at Point A, B and C

(i) Magnitude of Electric field at A due to q_1

$$E_{A1} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_{A1}^2} = \frac{9\times10^9\times10^{-8}}{(5\times10^{-2})^2}$$

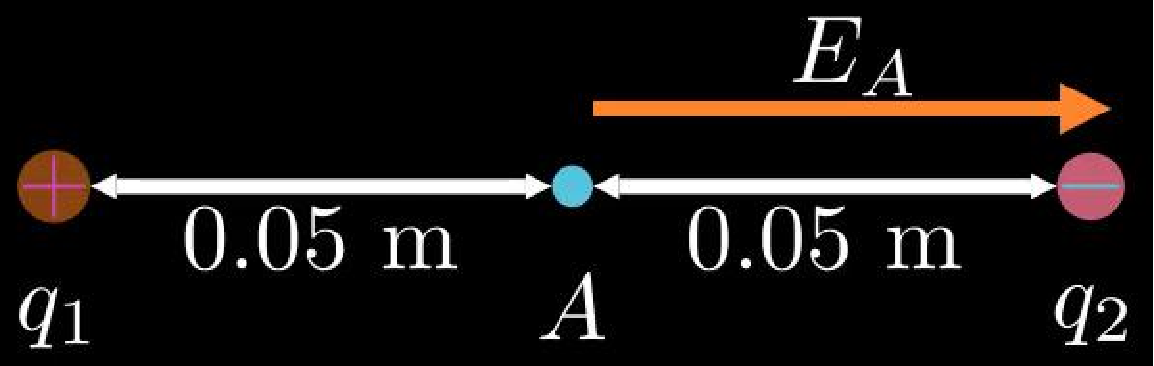
$$E_{A1} = \frac{9\times10^1}{25\times10^{-4}} = 3.6\times10^4 \text{ N/C}$$

(Directed toward Right)

Similarly, magnitude of Electric field at A due to q_2

$$E_{A2} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_{A2}^2} = E_{A1} = 3.6\times10^4 \text{ N/C}$$

(Directed toward Right)



Magnitude of total electric field at A

$$E_A = E_{A1} + E_{A2} = 7.2\times10^4 \text{ N/C}$$

(Directed toward Right)

Solution :

Given: $q_1 = +10^{-8}$ C and $q_2 = -10^{-8}$ C

Find: Electric field at Point A, B and C

(ii) Magnitude of Electric field at B due to q_1

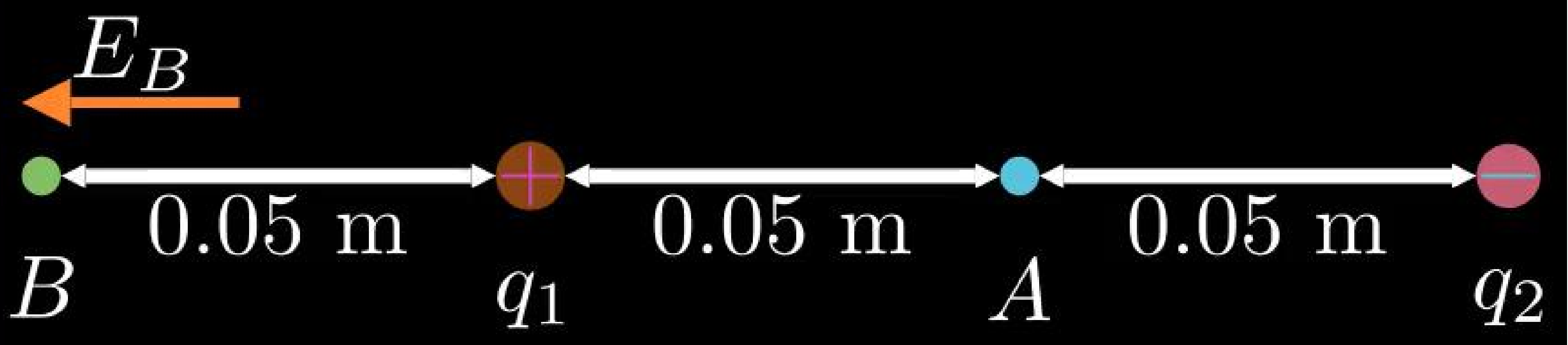
$$E_{B1} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_{B1}^2} = \frac{9 \times 10^9 \times 10^{-8}}{(5 \times 10^{-2})^2}$$

$$E_{B1} = \frac{9 \times 10^1}{25 \times 10^{-4}} = 3.6 \times 10^4 \text{ N/C}$$

(Directed toward Left)

Similarly, magnitude of Electric field at B due to q_2

$$E_{B2} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_{B2}^2} = \frac{9 \times 10^9 \times 10^{-8}}{(15 \times 10^{-2})^2}$$



$$E_{B2} = \frac{9 \times 10^1}{225 \times 10^{-4}} = 0.4 \times 10^4 \text{ N/C}$$

(Directed toward Right)

Magnitude of total electric field at B

$$E_B = E_{B1} - E_{B2} = 3.2 \times 10^4 \text{ N/C}$$

(Directed toward Left)

Solution :

(iii) Magnitude of Electric field at C due to q_1

$$E_{C1} = \frac{1}{4\pi\epsilon_0}\frac{q_1}{r_{C1}^2} = \frac{9\times10^9\times10^{-8}}{(10^{-1})^2} = \frac{9\times10^1}{10^{-2}}$$

$$E_{C1} = E = 9\times10^3 \text{ N/C (Direction indicated in fig.)}$$

Magnitude of Electric field at C due to q_2

$$E_{C2} = E = 9\times10^3 \text{ N/C (Direction indicated in fig.)}$$

Magnitude of total electric field at C

$$E_C = \sqrt{E_{C1}^2 + E_{C2}^2 + 2E_{C1}E_{C2}\cos(120°)}$$

$$E_C = \sqrt{E^2 + E^2 - 2E^2\times\frac{1}{2}} = \sqrt{2E^2 - E^2} = \sqrt{E^2} = E$$

$$E_C = 9\times10^3 \text{ N/C (Directed towards Right)}$$