```
import numpy as np
import pandas as pd
from scipy.stats import multivariate_normal as mvn
import matplotlib.pyplot as plt
```

Load MNIST data

```
train_data = pd.read_csv('/content/drive/MyDrive/MNIST_train.csv')
test_data = pd.read_csv('/content/drive/MyDrive/MNIST_test.csv')
```

```
test_data
```

	Unnamed: 0	index	labels	0	1	2	3	4	5	6	...	774	775	776	777	778	779	780	781	782	783
0	0	0	7	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	0	0
1	1	1	2	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	0	0
2	2	2	1	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	0	0
3	3	3	0	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	0	0
4	4	4	4	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	0	0
...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...
9995	9995	9995	2	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	0	0
9996	9996	9996	3	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	0	0
9997	9997	9997	4	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	0	0
9998	9998	9998	5	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	0	0
9999	9999	9999	6	0	0	0	0	0	0	0	...	0	0	0	0	0	0	0	0	0	0

10000 rows × 787 columns

Data Transformations

```
X_train = train_data.drop('labels', axis=1).to_numpy()
y_train = train_data['labels'].to_numpy()
X_test = test_data.drop('labels', axis=1).to_numpy()
y_test = test_data['labels'].to_numpy()
```

```
X_train = train_data.iloc[:, 3:].to_numpy() / 255.0
X_test = test_data.iloc[:, 3:].to_numpy() / 255.0
```

```
X_train.shape
```

```
(60000, 784)
```

```
def check_duplicates(X):

    unique_rows = np.unique(X, axis=0)
    if unique_rows.shape[0] == X.shape[0]:
        print("No duplicate rows found")
    else:
        print(f"Found {X.shape[0] - unique_rows.shape[0]} duplicate rows")

# Check  NaN/Null values
def check_nan(X, y=None):
    if np.isnan(X).any():
        print("Null values found in the feature matrix")
    else:
        print("No Null values found in the feature matrix")

    if y is not None and np.isnan(y).any():
        print("Null values found in the labels")
    else:
        print("No Null values found in the labels")


check_duplicates(X_train)
check_nan(X_train, y_train)
check_duplicates(X_test)
```

```
check_nan(X_test, y_test)
```

```
No duplicate rows found
No Null values found in the feature matrix
No Null values found in the labels
No duplicate rows found
No Null values found in the feature matrix
No Null values found in the labels
```

```
# X_train_min = X_train.min(axis=0)
# X_train_max = X_train.max(axis=0)
# X_train_scaled = (X_train - X_train_min) / (X_train_max - X_train_min + 1e-8)
# X_test_scaled = (X_test - X_train_min) / (X_train_max - X_train_min + 1e-8)
```

Methods

Accuracy Calculation

```
def accuracy(y, y_hat):
    return np.mean(y == y_hat)
```

Display a single image

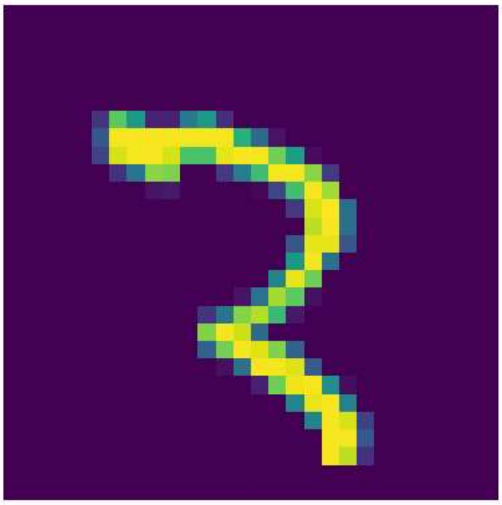
```
def show_me(X):
    plt.imshow(X.reshape(28, 28))
    plt.axis('off')
    plt.show()
```

Calculate and display the mean image for a given class `k`

```
def show_me_all_mean(X, y, k):
    class_images = X[y == k, :]
    mean_image = np.mean(class_images, axis=0)  # Compute mean of images in class `k`
    show_me(mean_image)
```

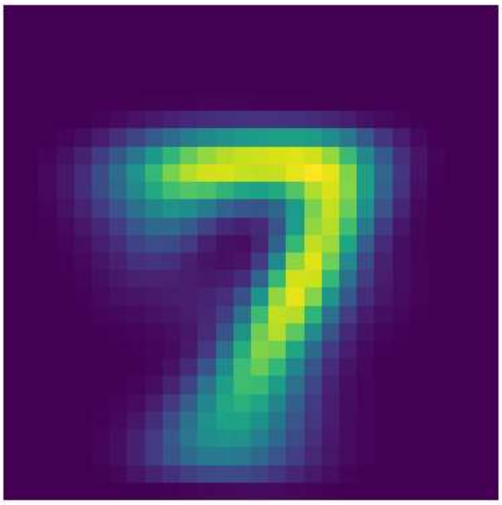
```
print("Displaying a sample image from the dataset:")
show_me(X_train[500])
```

```
Displaying a sample image from the dataset:
```



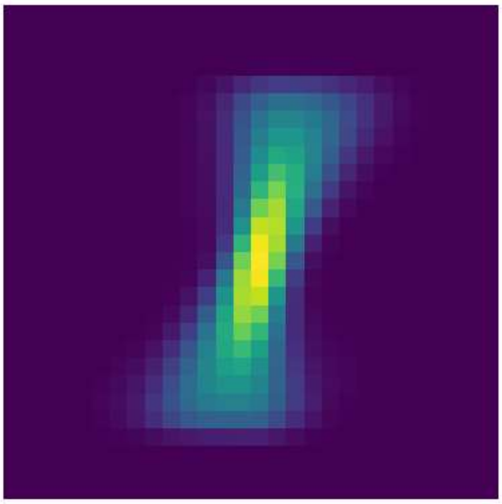
```
print("Displaying mean image for Class 7:")
show_me_all_mean(X_train, y_train, 7)
```

Displaying mean image for Class 7:



```
print("Displaying mean image for Class 1:")
show_me_all_mean(X_train, y_train, 1)
```

Displaying mean image for Class 1:

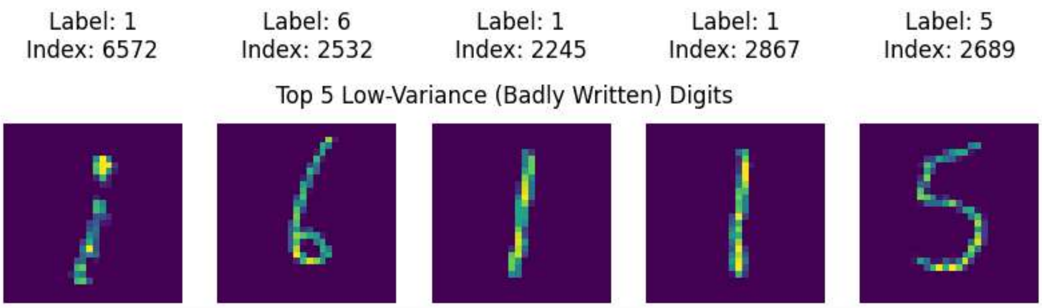


Identify the top 5 values

```
# Calculate pixel intensity variance for each image
variances = X_test.var(axis=1)

low_variance_indices = np.argsort(variances)[:5]

worst_index = np.argmin(variances)
```

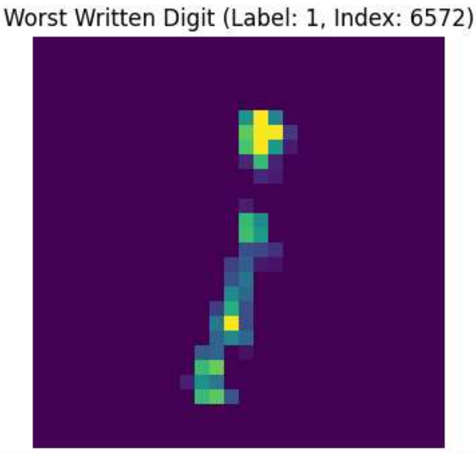
Display "badly written" digits(5)

```
plt.figure(figsize=(10, 2))
for i, idx in enumerate(low_variance_indices):
    img = X_test[idx].reshape(28, 28)
    plt.subplot(1, 5, i + 1)
    plt.imshow(img)
    plt.axis('off')
    plt.title(f"Label: {y_test[idx]}\nIndex: {idx}\n\n ")
plt.suptitle("Top 5 Low-Variance (Badly Written) Digits")
plt.show()
```



Display the "worst" written digit

```
plt.figure(figsize=(4, 4))
plt.imshow(X_test[worst_index].reshape(28, 28))
plt.axis('off')
plt.title(f"Worst Written Digit (Label: {y_test[worst_index]}, Index: {worst_index})")
plt.show()
```



Naive Bayes Classifier

```
class GaussNB:
    def fit(self, X, y, epsilon=1e-3):
        self.likelihoods = {}
        self.priors = {}
        self.K = set(y.astype(int))

        for k in self.K:
            X_k = X[y == k]
            self.likelihoods[k] = {"mean": X_k.mean(axis=0), "cov": X_k.var(axis=0) + epsilon}
            self.priors[k] = len(X_k) / len(X)

    def predict(self, X):
        N, D = X.shape
        P_hat = np.zeros((N, len(self.K)))

        for k, l in self.likelihoods.items():
            P_hat[:, k] = mvn.logpdf(X, l["mean"], l["cov"]) + np.log(self.priors[k])

        return P_hat.argmax(axis=1)
```

```
gnb = GaussNB()
gnb.fit(X_train, y_train)
y_hat_tr = gnb.predict(X_train)
print("Naive Bayes Accuracy:", accuracy(y_train, y_hat_tr))
```

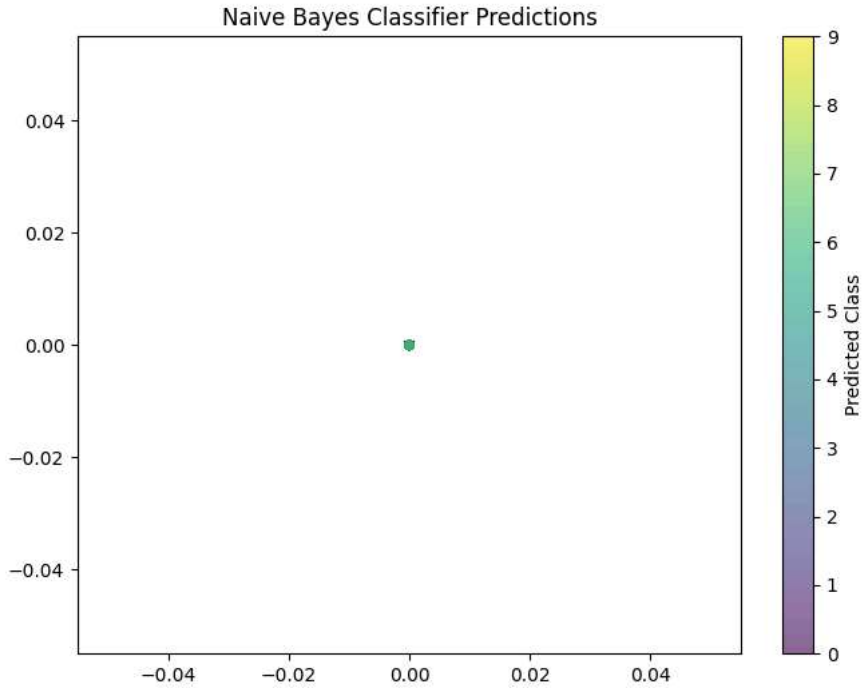
```
Naive Bayes Accuracy: 0.7682333333333333
```

Instantiate and test each classifier on MNIST

```
gnb = GaussNB()
gnb.fit(X_train, y_train)
y_hat_nb = gnb.predict(X_test)
print("Naive Bayes Accuracy:", accuracy(y_test, y_hat_nb))
```

```
Naive Bayes Accuracy: 0.7746
```

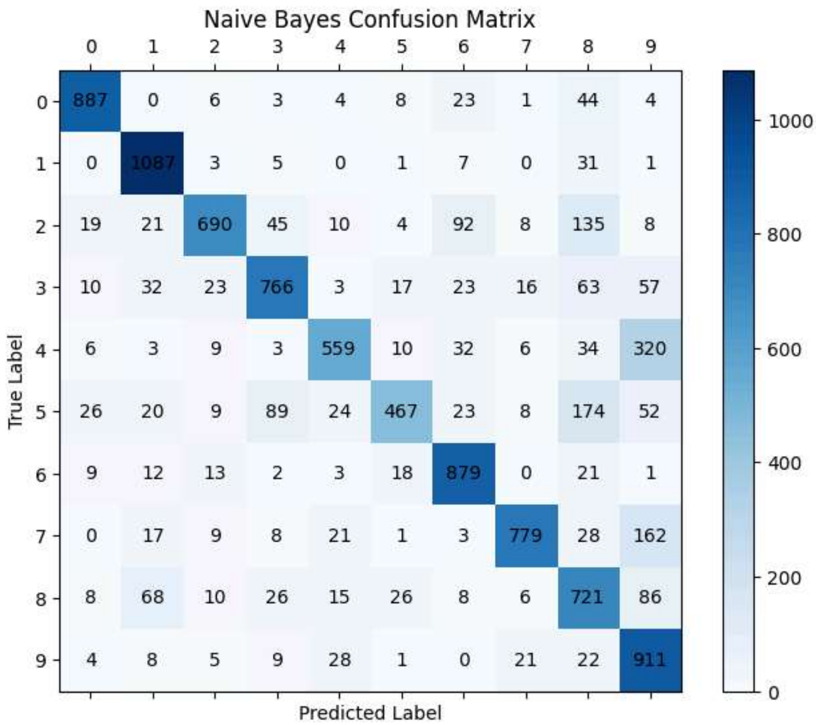
```
plt.figure(figsize=(8, 6))
scatter = plt.scatter(X_test[:, 0], X_test[:, 1], c=y_hat_nb, alpha=0.6, s=20)
plt.colorbar(scatter, label='Predicted Class')
plt.title("Naive Bayes Classifier Predictions")
plt.show()
```



Confusion Matrix

```
def compute_confusion_matrix(y_true, y_pred):
    num_classes = len(np.unique(y_true))
    confusion_matrix = np.zeros((num_classes, num_classes), dtype=int)

    for true, pred in zip(y_true, y_pred):
        confusion_matrix[int(true), int(pred)] += 1

    return confusion_matrix

def plot_confusion_matrix(y_true, y_pred, model_name):
    conf_matrix = compute_confusion_matrix(y_true, y_pred)


    plt.figure(figsize=(8, 6))
    cax = plt.matshow(conf_matrix, cmap='Blues', fignum=1)
    plt.colorbar(cax)
    plt.title(f"{model_name} Confusion Matrix")
    plt.xlabel("Predicted Label")
    plt.ylabel("True Label")

    num_classes = conf_matrix.shape[0]
    plt.xticks(np.arange(num_classes))
    plt.yticks(np.arange(num_classes))


    for (i, j), value in np.ndenumerate(conf_matrix):
        plt.text(j, i, value, ha="center", va="center", color="black")

    plt.show()
```

```
plot_confusion_matrix(y_test, y_hat_nb, "Naive Bayes")
```



Non-Naive Bayesian Classifier

```
class GaussBayes:
    def fit(self, X, y, epsilon=1e-3):
        self.likelihoods = {}
        self.priors = {}
        self.K = set(y.astype(int))

        for k in self.K:
            X_k = X[y == k]
            N_k, D = X_k.shape
            mu_k = X_k.mean(axis=0)
            self.likelihoods[k] = {"mean": mu_k, "cov": (1 / (N_k - 1)) * np.matmul((X_k - mu_k).T, X_k - mu_k) + epsilon * np.identity(
            self.priors[k] = len(X_k) / len(X)

    def predict(self, X):
        N, D = X.shape
        P_hat = np.zeros((N, len(self.K)))

        for k, l in self.likelihoods.items():
            P_hat[:, k] = mvn.logpdf(X, l["mean"], l["cov"]) + np.log(self.priors[k])

        return P_hat.argmax(axis=1)
```

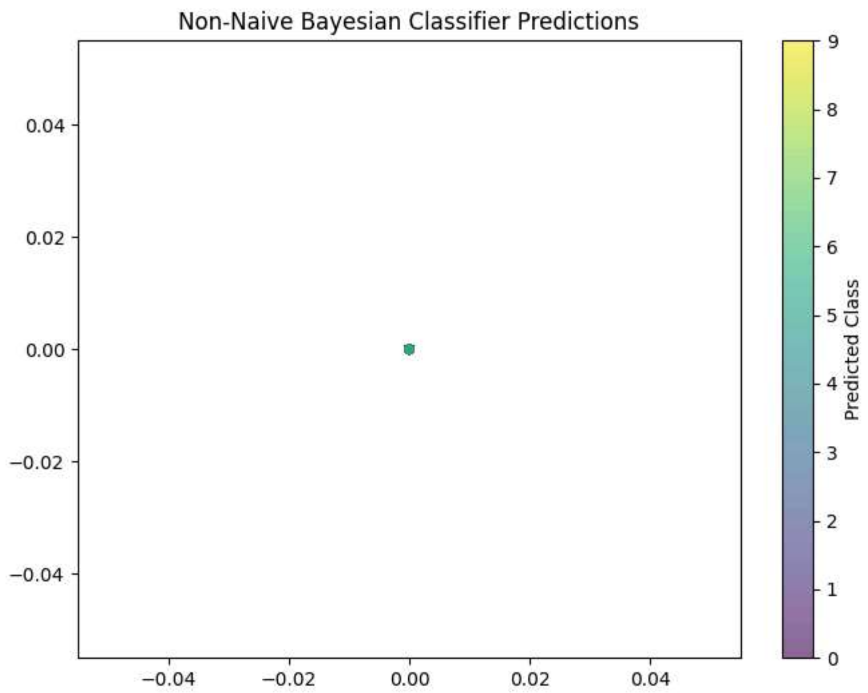
```
gauss_b = GaussBayes()
gauss_b.fit(X_train, y_train)
y_hat_train = gauss_b.predict(X_train)
print("Non-Naive Bayesian Accuracy:", accuracy(y_train, y_hat_train))
```

```
Non-Naive Bayesian Accuracy: 0.9306333333333333
```

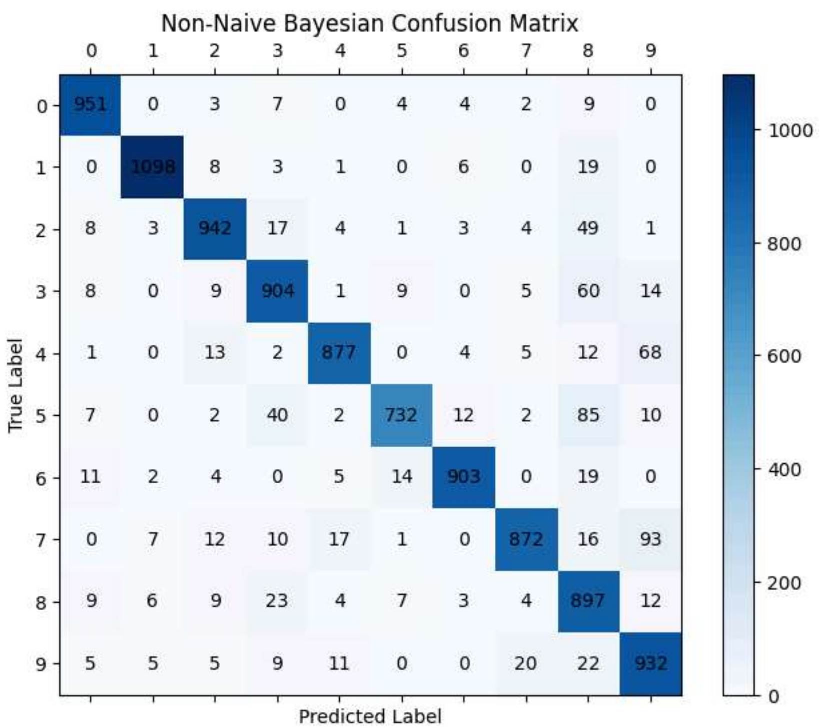
```
gauss_b = GaussBayes()
gauss_b.fit(X_train, y_train)
y_hat_bayes = gauss_b.predict(X_test)
print("Non-Naive Bayesian Accuracy:", accuracy(y_test, y_hat_bayes))
```

```
Non-Naive Bayesian Accuracy: 0.9108
```

```
plt.figure(figsize=(8, 6))
scatter = plt.scatter(X_test[:, 0], X_test[:, 1], c=y_hat_bayes, alpha=0.6, s=20)
plt.colorbar(scatter, label='Predicted Class')
plt.title("Non-Naive Bayesian Classifier Predictions")
plt.show()
```




```
plot_confusion_matrix(y_test, y_hat_bayes, "Non-Naive Bayesian")
```



K-Nearest Neighbors Classifier

```
class KNNClassifier:
    def fit(self, X, y):
        self.X = X
        self.y = y

    def predict(self, X, K=3, epsilon=1e-3):
        N = len(X)
        y_hat = np.zeros(N)

        for i in range(N):
            dist2 = np.sum((self.X - X[i]) ** 2, axis=1)
            idxt = np.argsort(dist2)[:K]
            gamma_k = 1 / (np.sqrt(dist2[idxt] + epsilon))
            y_hat[i] = np.bincount(self.y[idxt].astype(int), weights=gamma_k).argmax(
```

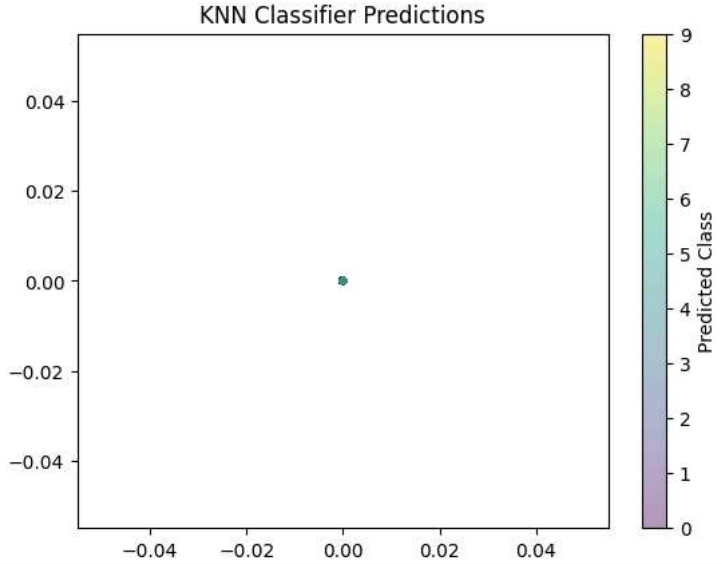
```
knn = KNNClassifier()
knn.fit(X_train, y_train)
y_knn_train = knn.predict(X_train, K=5)
print("KNN Accuracy:", accuracy(y_train, y_knn_train))
```

```
KNN Accuracy: 1.0
```

```
knn = KNNClassifier()
knn.fit(X_train, y_train)
y_hat_knn = knn.predict(X_test, K=5)
print("KNN Accuracy:", accuracy(y_test, y_hat_knn))
```

```
KNN Accuracy: 0.9691
```

```
plt.figure()
scatter = plt.scatter(X_test[:, 0], X_test[:, 1], c=y_hat_knn, alpha=0.4, s=10)
plt.colorbar(scatter, label='Predicted Class')
plt.title("KNN Classifier Predictions")
plt.show()
```




```
plot_confusion_matrix(y_test, y_hat_knn, "KNN")
```