# NCERT ANALOG 12.7.17

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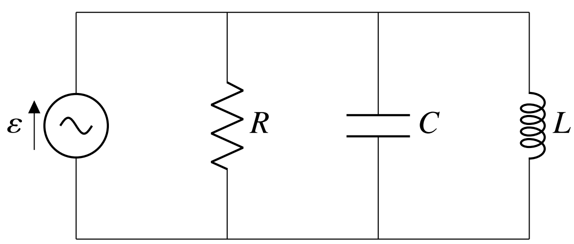
**Question 17:** Keeping the source frequency equal to the resonating frequency of the series $LCR$ circuit, if the three elements, $L, C$, and $R$ are arranged in parallel, show that the total current in the parallel $LCR$ circuit is minimum at this frequency. Obtain the current rms value in each branch of the circuit for the elements and source specified in Exercise 7.11 for this frequency
$\varepsilon = 230V$, $L = 5.0H$, $C = 80\mu\text{F}$, $R = 40\Omega$

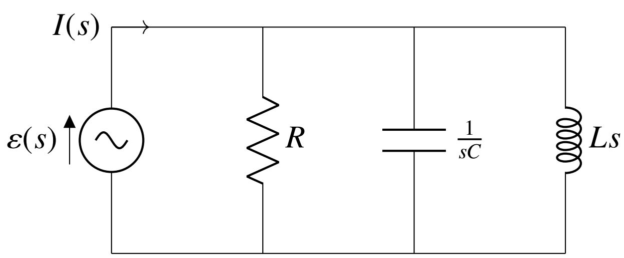



**Solution:**

| Symbol | Value | Description |
|---|---|---|
| $L$ | $50H$ | Inductance |
| $C$ | $80\mu F$ | Capacitance |
| $R$ | $40\Omega$ | Resistance |
| $\varepsilon$ | $230\,V$ | Voltage source |
| $\omega_0$ | $\frac{1}{\sqrt{LC}}$ | Resonant Angular Frequency |
| $I_R$ | $5.75A$ | Rms current value in Resistance |
| $I_L$ | $0.92A$ | Rms current value in Inductor |
| $I_C$ | $0.92A$ | Rms current value in Capacitor |

TABLE 1: Input Parameter

(a)




The impedance of the circuit is.

$$\frac{1}{z} = \frac{1}{R} + sC + \frac{1}{Ls} \tag{1}$$

$$I(s) = V(s)\left(\frac{1}{R} + sC + \frac{1}{Ls}\right) \tag{2}$$

At resonance, the circuit becomes purely resistive The admittance of capacitor and inductor cancel out as follows:

$$sC + \frac{1}{Ls} = 0 \tag{3}$$

$$\implies s = j\frac{1}{\sqrt{LC}} \tag{4}$$

$s$ can be expressed in terms of angular resonance frequency as

$$s = j\omega_0 \tag{5}$$

Comparing (4) and (5), we get

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{6}$$

Plot of Impedance vs Angular Frequency
Impedance is defined as

$$H(s) = \frac{V(s)}{I(s)} \tag{7}$$

Using (5),

$$H(s) = \frac{1}{\frac{1}{R} + sC + \frac{1}{Ls}} \tag{8}$$

$$\implies H(j\omega) = \frac{1}{\frac{1}{R} + j\omega C + \frac{1}{j\omega L}} \tag{9}$$

$$\implies |H(j\omega)| = \frac{1}{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}} \tag{10}$$

Hence it is proved that in parallel RLC circuit the total current is minimum at resonance frequency

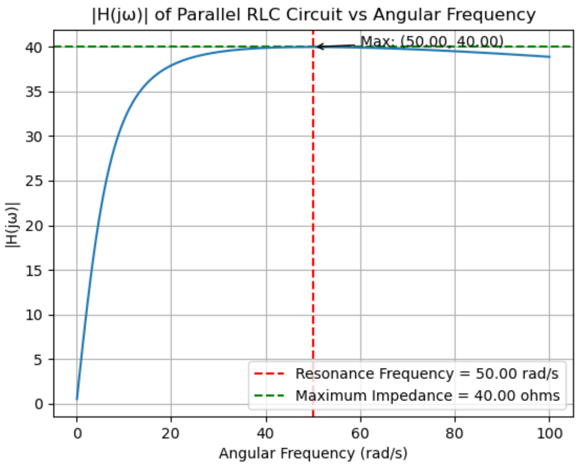



Fig. 0: Maximum impedance at resonating frequency in parallel RLC circuit

(b)

At resonance frequency, Rms currents are

$$\omega_0 = \frac{1}{\sqrt{LC}} = 50 rad/s \tag{11}$$

$$I_L = \frac{V}{\omega_0 L} = \frac{230}{250} = 0.92A \tag{12}$$

$$I_C = \omega_0 CV = 0.92A \tag{13}$$

$$I_R = \frac{V}{R} = \frac{230}{40} = 5.75A \tag{14}$$