NCERT ANALOG 12.7.17

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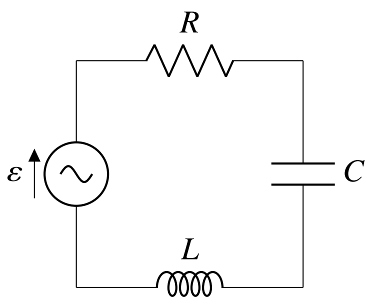
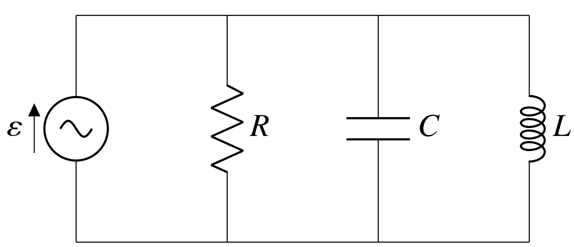
Question 17: Keeping the source frequency equal to the resonating frequency of the series LCR circuit, if the three elements, L, C, and R are arranged in parallel, show that the total current in the parallel LCR circuit is minimum at this frequency. Obtain the current rms value in each branch of the circuit for the elements and source specified in Exercise 7.11 for this frequency
$\varepsilon = 230V$, $L = 5.0H$, $C = 80\mu\text{F}$, $R = 40\Omega$



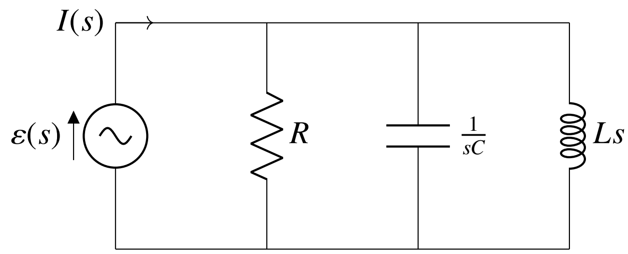


Solution:

Symbol	Value	Description
L	$50H$	Inductance
C	$80\mu F$	Capacitance
R	40Ω	Resistance
ε	$230\,V$	Voltage source
ω_0	$\frac{1}{\sqrt{LC}}$	Resonant Angular Frequency
I_R	$5.75A$	Rms current value in Resistance
I_L	$0.92A$	Rms current value in Inductor
I_C	$0.92A$	Rms current value in Capacitor

TABLE 1: Input Parameter

(a)



The impedance of the circuit is.

$$\frac{1}{z} = \frac{1}{R} + sC + \frac{1}{Ls} \tag{1}$$

$$I(s) = V(s)\left(\frac{1}{R} + sC + \frac{1}{Ls}\right) \tag{2}$$

At resonance, the circuit becomes purely resistive The admittance of capacitor and inductor cancel out as follows:

$$sC + \frac{1}{Ls} = 0 \tag{3}$$

$$\implies s = j\frac{1}{\sqrt{LC}} \tag{4}$$

s can be expressed in terms of angular resonance frequency as

$$s = j\omega_0 \tag{5}$$

Comparing (**??**) and (**??**), we get

$$\omega_0 = \frac{1}{\sqrt{LC}} \tag{6}$$

Plot of Impedance vs Angular Frequency Impedance is defined as

$$H(s) = \frac{V(s)}{I(s)} \tag{7}$$

Using (**??**),

$$H(s) = \frac{1}{\frac{1}{R} + sC + \frac{1}{Ls}} \tag{8}$$

$$\implies H(j\omega) = \frac{1}{\frac{1}{R} + j\omega C + \frac{1}{j\omega L}} \tag{9}$$

$$\implies |H(j\omega)| = \frac{1}{\sqrt{\frac{1}{R^2} + \left(\omega C - \frac{1}{\omega L}\right)^2}} \tag{10}$$

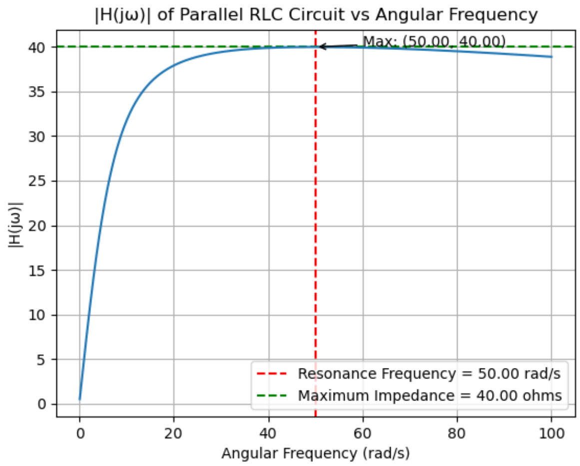


Fig. 0: Maximum impedance at resonating frequency in parallel RLC circuit

Hence it is proved that in parallel RLC circuit the total current is minimum at resonance frequency
(b)
At resonance frequency, Rms currents are

$$\omega_0 = \frac{1}{\sqrt{LC}} = 50 rad/s \tag{11}$$

$$I_L = \frac{V}{\omega_0 L} = \frac{230}{250} = 0.92A \tag{12}$$

$$I_C = \omega_0 C V = 0.92A \tag{13}$$

$$I_R = \frac{V}{R} = \frac{230}{40} = 5.75A \tag{14}$$

$$\tag{15}$$