11.9.2 Q 4

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Question 4: How many terms of the A.P. -6,$-\frac{11}{2}$, -5, ... are needed to give the sum -25?

Solution:

Symbol	Value	Description
$x(0)$	-6	first term of AP
d	$\frac{1}{2}$	common difference of AP
$n+1$	?	number of terms
$x(n)$	$x(0)+nd$	nth term of the AP

TABLE 1
INPUT DATA

$$y(n) = x(n) * u(n) \tag{1}$$

$$Y(z) = X(z)\,U(z) \tag{2}$$

$$Y(z) = \frac{x(0)}{(1-z^{-1})^2} + \frac{dz^{-1}}{(1-z^{-1})^3} \quad |z| > 1 \tag{3}$$

$$Y(z) = \frac{-6}{(1-z^{-1})^2} + \frac{0.5z^{-1}}{(1-z^{-1})^3} \quad |z| > 1 \tag{4}$$

Some Results:

$$(n+1) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{1}{(1-z^{-1})^2} \quad |z| > 1 \tag{5}$$

$$(n^2+n) \overset{\mathcal{Z}}{\longleftrightarrow} \frac{2z^{-1}}{(1-z^{-1})^3} \quad |z| > 1 \tag{6}$$

Using (5) and (6) and taking inverse Z-transform

$$y(n) = (-6(n+1) + \frac{1}{4}(n^2+n))u(n) \tag{7}$$

$$\implies -25 = \frac{1}{4}n^2 - \frac{23}{4}n - 6 \tag{8}$$

$$\implies 0 = n^2 - 23n + 76 \tag{9}$$

$$n = 19 or 4 \tag{10}$$

Hence number of terms required is 5 or 20.

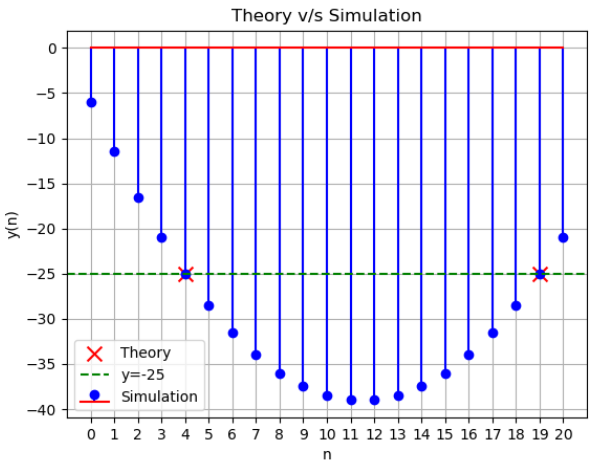

Fig. 0. Theory matches with simulated values