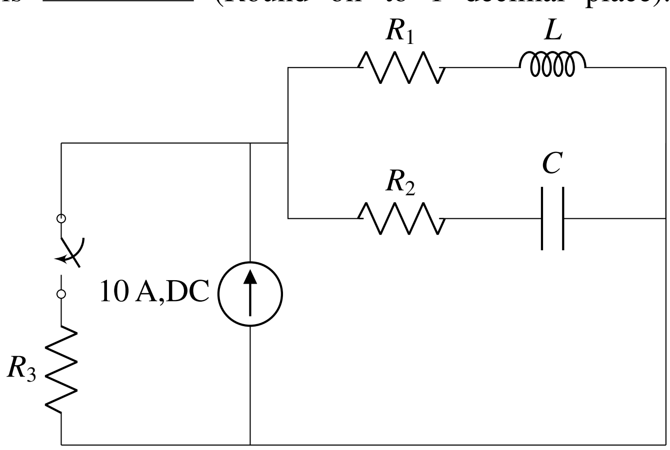
GATE-2023 (EE)
Q 29

MANOJ KUMAR (EE23BTECH11211)

Q29: The value of parameters of the circuit shown in the figure are

$$R_1 = 2\Omega, R_2 = 2\Omega, R_3 = 3\Omega, L = 10mH, C = 100\mu\text{F}$$

For time $t < 0$, the circuit is at steady state with the switch $'K'$ in closed condition. If the switch is opened at $t = 0$, the value of the voltage across the inductor (V_L) at $t = 0^+$ in Volts is __________ (Round off to 1 decimal place).

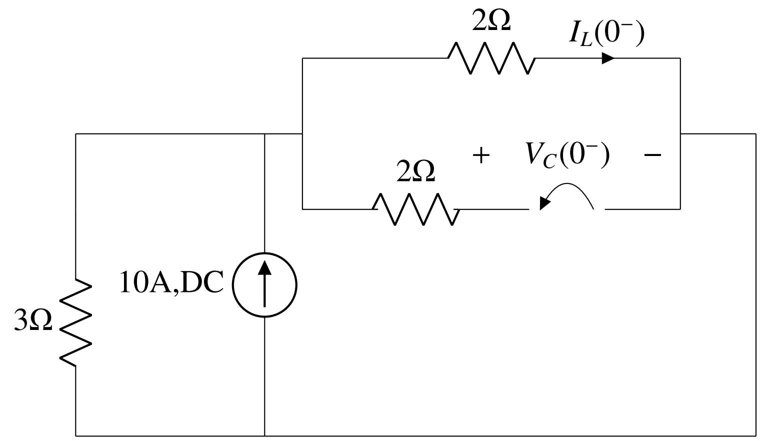


Solution:

Symbol	Value	Description
L	$10mH$	Inductance
C	$100muF$	Capacitance
R_1	2Ω	Resistance
R_2	2Ω	Resistance
R_3	3Ω	Resistance
V_L	??	Voltage across the inductor
V_C	??	Voltage across the capacitor
I_0	$10A$	DC current source
I_L	??	Current in inductor

TABLE 1: Input Parameter

At t=0$^-$, inductor behaves as wire and capacitor as open switch,

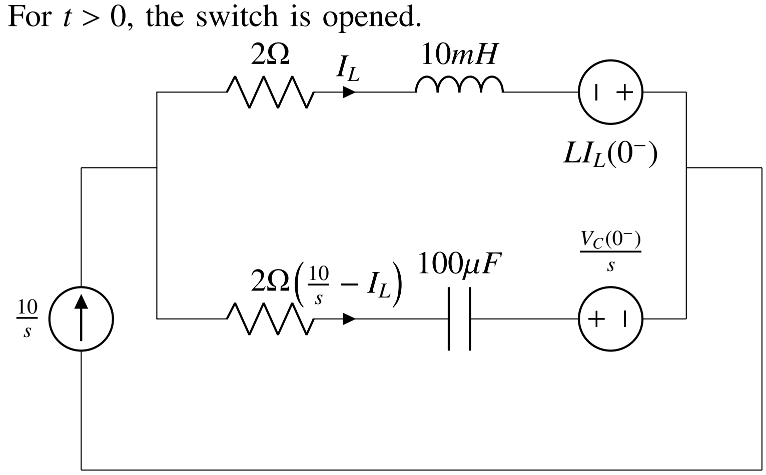


after current distribution

$$I_L(0^-) = 10A\left(\frac{3}{3+2}\right) = 6A \tag{1}$$

$$V_C(0^-) = 6 \times 2 = 12V \tag{2}$$

For $t > 0$, the switch is opened.



Using KVL,

$$2I_L + LsI_L - LI_L(0^-) - \frac{V_C(0^-)}{s} - \frac{1}{Cs}\left(\frac{10}{s} - I_L\right) - 2\left(\frac{10}{s} - I_L\right) = 0 \tag{3}$$

From (1), (2), (3)

$$I_L = \frac{6s^2 + 3200s + 10^7}{s\left(s^2 + 400s + 10^6\right)} \tag{4}$$

$$V_L(s) = I_L(sL) \tag{5}$$

Using (4)

$$V_L(s) = \frac{0.06s^2 + 32s + 10^5}{(s^2 + 400s + 10^6)} \tag{6}$$

Some Result:

$$\frac{1}{s^2 + 400s + 10^6} \overset{\mathcal{L}}{\longleftrightarrow} \left(e^{-200t}\right) \frac{\sin(400\sqrt{6}t)}{400\sqrt{6}} \tag{7}$$

$$\frac{s}{s^2 + 400s + 10^6} \overset{\mathcal{L}}{\longleftrightarrow} \left(e^{-200t}\right) \frac{\left(2\sqrt{6}\cos(400\sqrt{6}t) - \sin(400\sqrt{6}t)\right)}{2\sqrt{6}} \tag{8}$$

$$\frac{s^2}{s^2 + 400s + 10^6} \overset{\mathcal{L}}{\longleftrightarrow} \left(-e^{-200t}\right) \frac{\left(2300\sin(400\sqrt{6}t) + 400\sqrt{6}\cos(400\sqrt{6}t)\right)}{\sqrt{6}} \tag{9}$$

Inverse Laplace transform of (6) Using (7),(8), (9)

$$V_L(t) = e^{-200t}\left(-0.06\left(\frac{\left(2300\sin(400\sqrt{6}t) + 400\sqrt{6}\cos(400\sqrt{6}t)\right)}{\sqrt{6}}\right) + 32\left(\frac{\left(2\sqrt{6}\cos(400\sqrt{6}t) - \sin(400\sqrt{6}t)\right)}{2\sqrt{6}}\right)\right)$$
$$+ e^{-200t}\left(10^5\frac{\sin(400\sqrt{6}t)}{400\sqrt{6}}\right) \tag{10}$$

at t=0^+

$$V_L(0^+) = -24 + 32 = 8V \tag{11}$$

Hence at t=0^+ voltage across inductor is 8V