

<https://MAO3J1m0Op.github.io/algebra-in-lean> <https://github.com/MAO3J1m0Op/algebra-in-lean> <https://MAO3J1m0Op.github.io/algebra-in-lean/docs>

Algebra in Lean

Will Harris, Justin Morrill, Dominic King, Adam Kern, Clara Henne, Arim Lim, William Ho

June 25, 2024

Chapter 1

Basic Definitions

Definition 1.1 (Magma). *Defs.Magma* A magma consists of a set G equipped with a single binary operation μ . No other properties are imposed.

Definition 1.2 (Semigroup). *Defs.Semigroup definition* : magma A semigroup G is a magma where the operation μ is associative: For all $a, b, c \in G$, we have $\mu(a, \mu(b * c)) = \mu(\mu(a, b), c)$

Definition 1.3 (Monoid). *Defs.Monoid definition* : semigroup A monoid G is a semigroup that contains an identity element e that satisfies the condition: for all $a \in G$, $\mu(a, e) = a = \mu(e, a)$.

Definition 1.4 (Commutative Monoid). *Defs.CommMonoid definition* : monoid A commutative monoid G is a monoid where the binary operation μ is commutative: for all $a, b \in G$, $\mu(a, b) = \mu(b, a)$

Definition 1.5 (Group). *Defs.Group definition* : monoid A group G is a monoid along with an inverse map $\iota : G \rightarrow G$ such that for all $a \in G$, $\mu((\iota a), a) = e$

Definition 1.6 (Abelian Group). *Defs.AbelianGroup definition*: group, definition: commutative monoid An abelian group G is a group where the binary operation is commutative: for all $a, b \in G$, $\mu(a, b) = \mu(b, a)$