Algebra in Lean

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Chapter 1

Basic Definitions

Definition 1.1 (Magma). *A magma consists of a set G equipped with a single binary operation μ. No other properties are imposed.*

Definition 1.2 (Semigroup). *A semigroup G is a magma where the operation μ is associative: For all $a, b, c \in G$, we have $\mu(a, \mu(b * c)) = \mu(\mu(a, b), c)$*

Definition 1.3 (Monoid). *A monoid G is a semigroup that contains an identity element e that satisfies the condition: for all $a \in G, \mu(a, e) = a = \mu(e, a)$.*

Definition 1.4 (Commutative Monoid). *A commutative monoid G is a monoid where the binary operation μ is commutative: for all $a, b \in G, \mu(a, b) = \mu(b, a)$*