

# Algebra in Lean

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June 24, 2024

# Chapter 1

## Basic Definitions

**Definition 1.1** (Magma). *A magma consists of a set  $G$  equipped with a single binary operation  $\mu$ . No other properties are imposed.*

**Definition 1.2** (Semigroup). *A semigroup  $G$  is a magma where the operation  $\mu$  is associative: For all  $a, b, c \in G$ , we have  $\mu(a, \mu(b * c)) = \mu(\mu(a, b), c)$*

**Definition 1.3** (Monoid). *A monoid  $G$  is a semigroup that contains an identity element  $e$  that satisfies the condition: for all  $a \in G$ ,  $\mu(a, e) = a = \mu(e, a)$ .*

**Definition 1.4** (Commutative Monoid). *A commutative monoid  $G$  is a monoid where the binary operation  $\mu$  is commutative: for all  $a, b \in G$ ,  $\mu(a, b) = \mu(b, a)$*

**Definition 1.5** (Group). *A group  $G$  is a monoid along with an inverse map  $\iota : G \rightarrow G$  such that for all  $a \in G$ ,  $\mu((\iota a), a) = e$*

**Definition 1.6** (Abelian Group). *An abelian group  $G$  is a group where the binary operation is commutative: for all  $a, b \in G$ ,  $\mu(a, b) = \mu(b, a)$*