https://MAO3J1m0Op.github.io/algebra-in-lean https://github.com/MAO3J1m0Op/algebra-in-lean https://MAO3J1m0Op.github.io/algebra-in-lean/docs

Algebra in Lean

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Chapter 1

Basic Definitions

Definition 1.1 (Magma)**.** *Defs.Magma A magma consists of a set G equipped with a single binary operation μ. No other properties are imposed.*

Definition 1.2 (Semigroup)**.** *Defs.Semigroup definition : magma A semigroup G is a magma where the operation μ is associative: For all $a, b, c \in G$, we have $\mu(a, \mu(b * c)) = \mu(\mu(a, b), c)$*

Definition 1.3 (Monoid)**.** *Defs.Monoid definition : semigroup A monoid G is a semigroup that contains an identity element e that satisfies the condition: for all $a \in G$, $\mu(a, e) = a = \mu(e, a)$.*

Definition 1.4 (Commutative Monoid)**.** *Defs.CommMonoid definition : monoid A commutative monoid G is a monoid where the binary operation μ is commutative: for all $a, b \in G$, $\mu(a, b) = \mu(b, a)$*

Definition 1.5 (Group)**.** *Defs.Group definition : monoid A group G is a monoid along with an inverse map $\iota : G \to G$ such that for all $a \in G$, $\mu((\iota\ a), a) = e$*

Definition 1.6 (Abelian Group)**.** *Defs.AbelianGroup definition: group An abelian group G is a group where the binary operation is commutative: for all $a, b \in G$, $\mu(a, b) = \mu(b, a)$*