SCHOOL OF SOFTWARE, TSINGHUA UNIVERSITY

Optimization Method

homework 7

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CONTENTS

I. Problem 1

Solve the following 0-1 programming problem:

$$\begin{aligned} \min \quad & 2x_1 + 3x_2 + 4x_3 \\ \text{s.t.} \quad & -3x_1 + 5x_2 - 2x_3 \geq -4 \\ & 3x_1 + x_2 + 4x_3 \geq 3 \qquad \text{(i.1)} \\ & x_1 + x_2 \geq 1 \qquad \text{(i.2)} \\ & x_1, x_2, x_3 \in \{0, 1\} \qquad \text{(i.3)} \end{aligned}$$

Solution:

- First, we can try a feasible solution $(1,1,0)$, $f_{min} = 5$.
- we can list all possible solution by enumeration and validate it by constraint condition and present f_{min}.

point	present f_{min}	1	2	3	f
(0,0,0)	5	Y	N		
(1,0,0)	5	Y	Y	Y	2
(0,1,0)	2	Y	N		
(0,0,1)	2	Y	Y	Y	4
(1,1,0)	2	Y	Y	Y	5
(1,0,1)	2	N			
(0,1,1)	2	Y	Y	Y	7
(1,1,1)	2	Y	Y	Y	9

Above all, we can see the optimal solution is $(1,0,0)$, $f_{min} = 2$.

II. Problem 2

Suppose a factory produce two kind of products, product 1 and product 2, and each product's profits are respectively 15 and 25 (hundred yuan). Producing these products need production line A and production line B. For producing one product 1, it takes 1 hour on each line A and line B. For producing one product 2, it takes 3 hours on line A and 1 hour on line B. When making production plan, we need to consider:

1. P_1: profits of every week are no less than 750.
2. P_2: number of product 1 produced every week are no less than 25, product 2 are no less than 10.
3. P_3: work time of line A is no more than 60 hours and line B is no more than 40 hours, or the expense of line A is 3 times than line B for working another hour.

Please model this problem by goal programming.

Solution:

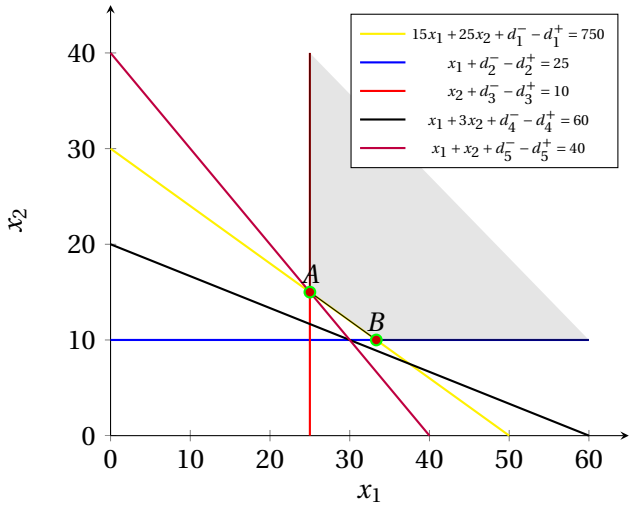
$$
\begin{aligned}
\min \quad & P_1 d_1^- + P_2(d_2^- + d_3^-) + P_3(3d_4^+ + d_5^+) \\
\text{s.t.} \quad & 15x_1 + 25x_2 + d_1^- - d_1^+ = 750 \\
& x_1 + d_2^- - d_2^+ = 25 \\
& x_2 + d_3^- - d_3^+ = 10 \\
& x_1 + 3x_2 + d_4^- - d_4^+ = 60 \\
& x_1 + x_2 + d_5^- - d_5^+ = 40 \\
& x_1, x_2, d_i^-, d_i^+ \geq 0, i = 1, \cdots, 5
\end{aligned}
$$

III. PROBLEM 3

Solve the following goal programming problem using graph method:

$$
\begin{aligned}
\min \quad & P_1 d_1^- + P_2(d_2^- + d_3^-) + P_3(3d_4^+ + d_5^+) \\
\text{s.t.} \quad & 15x_1 + 25x_2 + d_1^- - d_1^+ = 750 \\
& x_1 + d_2^- - d_2^+ = 25 \\
& x_2 + d_3^- - d_3^+ = 10 \\
& x_1 + 3x_2 + d_4^- - d_4^+ = 60 \\
& x_1 + x_2 + d_5^- - d_5^+ = 40 \\
& x_1, x_2, d_i^-, d_i^+ \geq 0, i = 1, \cdots, 5
\end{aligned}
$$

Solution:



Considering constraint condition P_1 and P_2, the solution is in the gray area. By adding P_3, we can see the optimal solution is $B(33.33, 10)$.