SCHOOL OF SOFTWARE, TSINGHUA UNIVERSITY

Optimization Method

homework 12

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I. PROBLEM 1

Algorithm mapping is defined as follows:

$$A(x) = \begin{cases} \left[\frac{3}{2} + \frac{1}{4}x, 1 + \frac{1}{2}x\right], when \quad x \geq 2 \\ \frac{1}{2}(x+1), when \quad x < 2 \end{cases}$$

Prove that A is not closed at point $x = 2$.
Proof: we can choose point range $x^{(k)} = 2 - \frac{1}{k}$, when $k \to \infty$, $x^{(k)} \to \hat{x} = 2$.
From the definition of A(x) we have

$$y^{(k)} = \frac{1}{2}(2 - \frac{1}{k} + 1) = \frac{3}{2} - \frac{1}{2k}$$

When $k \to \infty$, $y^{(k)} \to \frac{3}{2} \notin A(\hat{x}) = \{2\}$.
So $A(x)$ is not closed at point $x = 2$.

II. PROBLEM 2

Define a algorithm mapping on set $X = [0, 1]$.

$$A(x) = \begin{cases} [0, x), 0 < x \leq 1 \\ 0, x = 0 \end{cases}$$

Discuss whether A is closed or not at: $x^{(1)} = 0$, $x^{(2)} = \frac{1}{2}$.
Solution:
For $x^{(1)} = 0$, we can choose point range $x^{(k)} = 0 + \frac{1}{k}$, when $k \to \infty$, $x^{(k)} \to \hat{x} = 0$.
From the definition of A(x) we have

$$y^{(k)} = \left[0, \frac{1}{k}\right)$$

When $k \to \infty$, $y^{(k)} \to 0 \in A(\hat{x}) = A(0) = \{0\}$.
Since $A(x)$ is defined on $[0, 1]$,we don't need to consider $0 - \frac{1}{k}$, So $A(x)$ is closed at point $x^{(1)} = 0$.

For $x^{(2)} = 0$, we can choose point range $x^{(k)} = \frac{1}{2} + \frac{1}{k}$, when $k \to \infty$, $x^{(k)} \to \hat{x} = \frac{1}{2}$.
From the definition of A(x) we have

$$y^{(k)} = \left[0, \frac{1}{2} + \frac{1}{k}\right)$$

When $k \to \infty$, $y^{(k)} \to [0, \frac{1}{2}] \notin A(\hat{x}) = A(\frac{1}{2}) = [0, \frac{1}{2})$.
So $A(x)$ is not closed at point $x^{(2)} = \frac{1}{2}$.