SCHOOL OF SOFTWARE, TSINGHUA UNIVERSITY

Optimization Method

homework 14

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CONTENTS

I. PROBLEM 1

Consider the following problem:

$$\begin{aligned}
\min \quad & x_1^2 + x_1x_2 + 2x_2^2 - 6x_1 - 2x_2 - 12x_3 \\
\text{s.t.} \quad & x_1 + x_2 + x_3 = 2 \\
& -x_1 + 2x_2 \leq 3 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}$$

Please compute the descent feasible direction at point $\hat{x} = (1, 1, 0)^T$.

Solution:

$\hat{x}$'s active constraint are $x_1 + x_2 + x_3 = 0$ and $x_3 \geq 0$. so feasible direction d holds

$$\begin{cases} d_1 + d_2 + d_3 = 0 \\ d_3 \geq 0 \end{cases}$$

As a descent direction, we have $\nabla f(\hat{x})d < 0$. Since

$$\nabla f(\hat{x}) = \begin{bmatrix} 2x_1 + x_2 - 6 \\ x_1 + 4x_2 - 2 \\ -12 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ -12 \end{bmatrix}$$

So the descent feasible direction holds

$$\begin{cases} d_1 + d_2 + d_3 = 0 \\ d_3 \geq 0 \\ -3d_1 + 3d_2 - 12d_3 < 0 \end{cases}$$

A possible solution is $d = (-1, 0, 1)$.

II. PROBLEM 2

Consider the following problem:

$$\begin{aligned}
\min \quad & f(x) \\
\text{s.t.} \quad & g_i(x) \geq 0, i = 1, 2, \cdots, m \\
& h_j(x) = 0, j = 1, 2, \cdots, l
\end{aligned}$$

Suppose $\hat{x}$ is a feasible point, $I = \{i | g_i(\hat{x}) = 0\}$.

Please prove $\hat{x}$ is a KTT point's Necessary and Sufficient Condition is that the optimum of the following objective function is 0:

$$\begin{aligned} \min \quad & \nabla f(\hat{x})^T d \\ \text{s.t.} \quad & \nabla g_i(\hat{x})^T d \geq 0, i \in I \\ & \nabla h_j(\hat{x})^T d = 0, j = 1, 2, \cdots, l \\ & -1 \leq d_j \leq 1, j = 1, 2, \cdots, n \end{aligned}$$

Proof:

$\hat{x}$ is a KTT point whose Necessary and Sufficient Condition is that $\exists w_i \geq 0 (i \in I)$ and v_j,

$$\nabla f(\hat{x}) - \sum_{i \in I} w_i \nabla g_i(\hat{x}) - \sum_{j=1}^{l} v_j \nabla h_j(\hat{x}) = 0 \tag{ii.1}$$

Mark $A_1 = \left[\nabla g_{i1}(\hat{x}), \cdots, \nabla g_{im}(\hat{x})\right]$, $E = [\nabla h_1(\hat{x}, \cdots, \nabla h_l(\hat{x}]$, $v = p - q (p \geq 0, q \geq 0)$. We can rewrite equation ii.1 into

$$(-A_1, -E, E) \begin{bmatrix} w \\ p \\ q \end{bmatrix} = -\nabla f(\hat{x}), \begin{bmatrix} w \\ p \\ q \end{bmatrix} \geq 0 \tag{ii.2}$$

From Farkas Theorem, we know ii.2 has solution whose Necessary and Sufficient Condition is

$$\begin{bmatrix} -A_1^T \\ -E^T \\ E^T \end{bmatrix} d \leq 0, -\nabla f(\hat{x})^T d > 0 \tag{ii.3}$$

has no solution. That is

$$\begin{cases} \nabla f(\hat{x})^T d < 0 \\ A_1^T d \geq 0 \\ E^T d = 0 \end{cases}$$

has no solution. Thus,

$$\begin{aligned} \min \quad & \nabla f(\hat{x})^T d \\ \text{s.t.} \quad & \nabla g_i(\hat{x})^T d \geq 0, i \in I \\ & \nabla h_j(\hat{x})^T d = 0, j = 1, 2, \cdots, l \\ & -1 \leq d_j \leq 1, j = 1, 2, \cdots, n \end{aligned}$$

has optimal solution 0.