SCHOOL OF SOFTWARE, TSINGHUA UNIVERSITY

Optimization Method

homework 5

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CONTENTS

I. Problem 1

Suppose the original problem is

$$\begin{aligned} \min \quad & 4x_1 + 3x_2 + x_3 \\ \text{s.t.} \quad & x_1 - x_2 + x_3 \geq 1 \\ & x_1 + 2x_2 - 3x_3 \geq 2 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

the optimized solution of the dural problem is $(w_1, w_2) = (\frac{5}{3}, \frac{7}{3})$, solve the original problem using dual property. ***Solution:***
Dural problem:

$$\begin{aligned} \max \quad & w_1 + 2w_2 \\ \text{s.t.} \quad & w_1 + w_2 \leq 4 \\ & -w_1 + 2w_2 \leq 3 \\ & w_1 - 3w_2 \leq 1 \\ & w_1, w_2 \geq 0 \end{aligned}$$

Since the optimized solution of the dural problem is $w_1 > 0$, $w_2 > 0$, from the complementary slackness theorem and the constraint on the third inequality in the dural problem is tight, we get

$$\begin{cases} x_1 - x_2 + x_3 = 1 \\ x_1 + 2x_2 - 3x_3 = 2 \\ x_3 = 0 \end{cases}$$

so the optimized solution of the original problem is $x_1 = \frac{4}{3}$, $x_2 = \frac{1}{3}$, $x_3 = 0$, $f_{min} = \frac{19}{3}$

II. Problem 2

Given a linear programming problem:

$$\begin{aligned} \min \quad & 5x_1 + 21x_3 \\ \text{s.t.} \quad & x_1 - x_2 + 6x_3 \geq b_1 \\ & x_1 + x_2 + 2x_3 \geq 1 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

$b_1$is a positive number, and the optimized solution of the problem is $(x_1, x_2, x_3) = (\frac{1}{2}, 0, \frac{1}{4})$.

1. write out the dural problem.

$$
\begin{aligned}
\max \quad & b_1 w_1 + w_2 \\
\text{s.t.} \quad & w_1 + w_2 \leq 5 \\
& -w_1 + w_2 \leq 0 \\
& 6w_1 + 2w_2 \leq 21 \\
& w_1, w_2 \geq 0
\end{aligned}
$$

2. get the optimized solution of the dural problem.
   using the complementary slackness theorem,since $w_1 > 0$, from $x_1 - x_2 + 6x_3 = b_1$ we get $b_1 = 2$.And the optimized solution $x_1 > 0$,$x_3 > 0$, we get

$$
\begin{cases}
w_1 + w_2 = 5 \\
6w_1 + 2w_2 = 21
\end{cases}
$$

   Then, we get the optimized solution $w_1 = \frac{11}{4}$, $w_2 = \frac{9}{4}$,$f_{min} = \frac{31}{4}$

III. PROBLEM 3

Considering the linear problem:

$$
\begin{aligned}
\min \quad & cx \\
\text{s.t.} \quad & Ax = b \\
& x \geq 0
\end{aligned}
$$

A is a $m * m$ symmetrical matrix, $c^T = b$. Prove that if $x^{(0)}$ is a feasible solution, it is optimized.
Proof:
Dural problem:

$$
\begin{aligned}
\max \quad & wb \\
\text{s.t.} \quad & wA \leq c
\end{aligned}
$$

we can know that $w = x^{(0)\,T}$ is a feasible solution and $cx^(0) = w^{(0)}b$, so $x^{(0)}$is optimized.

IV. PROBLEM 4

solve the following LP problem using dual simplex method.

1.

$$
\begin{aligned}
\max \quad & x_1 + x_2 \\
\text{s.t.} \quad & x_1 - x_2 - x_3 = 1 \\
& -x_1 + x_2 + 2x_3 \geq 1 \\
& x_1, x_2, x_3 \geq 0
\end{aligned}
$$

Solution:
the extended problem:

$$
\begin{aligned}
\max \quad & x_1 + x_2 \\
\text{s.t.} \quad & x_1 - x_2 - x_3 = 1 \\
& -x_3 + x_4 = -2 \\
& x_2 + x_3 + x_5 = M \\
& x_j \geq 0, j = 1, \cdots, 5
\end{aligned}
$$

	x_1	x_2	x_3	x_4	x_5	
x_1	1	-1	-1	0	0	1
x_4	0	0	-1	1	0	-2
x_5	0	1	1	0	1	M
f	0	-2	-1	0	0	1
x_1	1	0	0	0	1	M+1
x_4	0	0	-1	1	0	-2
x_2	0	1	1	0	1	M
f	0	0	1	0	2	2M+1
x_1	1	0	0	0	1	M+1
x_3	0	0	1	-1	0	2
x_2	0	1	0	1	1	M-2
f	0	0	0	1	2	2M-1

From the above table, we can see that the optimized solution is $(M+1, M-2, 2, 0, 0)$, $f_{max} = 2M - 1$. Since $M \to \infty$, the problem has no upper bound.

2.

$$
\begin{aligned}
\min \quad & 4x_1 + 3x_2 + 5x_3 + x_4 + 2x_5 \\
\text{s.t.} \quad & -x_1 + 2x_2 - 2x_3 + 3x_4 - 3x_5 + x_6 + x_8 = 1 \\
& x_1 + x_2 - 3x_3 + 2x_4 - 2x_5 + x_8 = 4 \\
& -2x_3 + 3x_4 - 3x_5 + x_7 + x_8 = 2 \\
& x_j \geq 0, j = 1, \cdots, 8
\end{aligned}
$$

Solution:

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	x_8	
x_6	-2	1	1	1	-1	1	0	0	-3
x_8	1	1	-3	2	-2	0	0	1	4
x_7	-1	-1	1	1	-1	0	1	0	-2
f	-4	-3	-5	-1	-2	0	0	0	0
x_5	2	-1	-1	-1	1	-1	0	0	3
x_8	5	-1	-5	0	0	-2	0	1	10
x_7	1	-2	0	0	0	-1	1	0	1
f	0	-5	-7	-3	0	-2	0	0	6

From the above table, we can get the optimized solution **x**=(0,0,0,0,3,0,1,10), $f_{min} = 6$.