SCHOOL OF SOFTWARE, TSINGHUA UNIVERSITY

Optimization Method
homework 3

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CONTENTS

I. Problem 1

solve the following LP problem using two-step algorithm.

1.

$$
\begin{aligned}
\max \quad & 3x_1 - 5x_2 \\
\text{s.t.} \quad & -x_1 + 2x_2 + 4x_3 \le 4 \\
& x_1 + x_2 + 2x_3 \le 5 \\
& -x_1 + 2x_2 + x_3 \ge 1 \\
& x_1, x_2, x_3 \ge 0
\end{aligned}
$$

Solution:

Lead slack variable x_4, x_5, x_6 into it.

$$
\begin{aligned}
\max \quad & 3x_1 - 5x_2 \\
\text{s.t.} \quad & -x_1 + 2x_2 + 4x_3 + x_4 = 4 \\
& x_1 + x_2 + 2x_3 + x_5 = 5 \\
& -x_1 + 2x_2 + x_3 - x_6 = 1 \\
& x_j \ge 0, j = 1, \cdots, 6
\end{aligned}
$$

solve it using two-step algorithm. Add artificial variable x_7.

$$
\begin{aligned}
\min \quad & x_7 \\
\text{s.t.} \quad & -x_1 + 2x_2 + 4x_3 + x_4 = 4 \\
& x_1 + x_2 + 2x_3 + x_5 = 5 \\
& -x_1 + 2x_2 + x_3 - x_6 + x_7 = 1 \\
& x_j \ge 0, j = 1, \cdots, 7
\end{aligned}
$$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_4	-1	2	4	1	0	0	0	4
x_5	1	1	2	0	1	0	0	5
x_7	-1	2	1	0	0	-1	1	1
f	-1	2	1	0	0	-1	0	1
x_4	0	0	3	1	0	1	-1	3
x_5	$\frac{3}{2}$	0	$\frac{3}{2}$	0	1	$\frac{1}{2}$	-$\frac{1}{2}$	$\frac{9}{2}$
x_2	-$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	-$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
f	0	0	0	0	0	0	-1	0

we get a basic feasible of original LP problem $\mathbf{x} = (0, \frac{1}{2}, 0, 3, \frac{9}{2}, 0)$.

	x_1	x_2	x_3	x_4	x_5	x_6	
x_4	0	0	3	1	0	1	3
x_5	$\frac{3}{2}$	0	$\frac{3}{2}$	0	1	$\frac{1}{2}$	$\frac{9}{2}$
x_2	-$\frac{1}{2}$	1	$\frac{1}{2}$	0	0	-$\frac{1}{2}$	$\frac{1}{2}$
f	-$\frac{1}{2}$	0	$-\frac{5}{2}$	0	0	$\frac{5}{2}$	$-\frac{5}{2}$
x_3	0	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	1
x_5	$\frac{3}{2}$	0	0	-$\frac{1}{2}$	1	0	3
x_2	-$\frac{1}{2}$	1	0	-$\frac{1}{6}$	0	$-\frac{2}{3}$	0
f	$-\frac{1}{2}$	0	0	$\frac{5}{6}$	0	$\frac{10}{3}$	0
x_3	0	0	1	$\frac{1}{3}$	0	$\frac{1}{3}$	1
x_1	1	0	0	-$\frac{1}{3}$	$\frac{2}{3}$	0	2
x_2	0	1	0	-$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	1
f	0	0	0	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{10}{3}$	1

From the above table, we can see that solution **x**=(2,1,1,0,0), $f_{max} = 1$.

2.

$$\begin{aligned} \min \quad & x_1 - 3x_2 + x_3 \\ \text{s.t.} \quad & 2x_1 - x_2 + x_3 = 8 \\ & 2x_1 + x_2 \geq 2 \\ & x_1 + 2x_2 \leq 10 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Solution:

Lead slack variable x_4, x_5 into it.

$$\begin{aligned} \min \quad & x_1 - 3x_2 + x_3 \\ \text{s.t.} \quad & 2x_1 - x_2 + x_3 = 8 \\ & 2x_1 + x_2 - x_4 = 2 \\ & x_1 + 2x_2 + x_5 = 10 \\ & x_j \geq 0, j = 1, \cdots, 5 \end{aligned}$$

solve it using two-step algorithm. Add artificial variable x_6.

$$\begin{aligned} \min \quad & x_6 \\ \text{s.t.} \quad & 2x_1 - x_2 + x_3 = 8 \\ & 2x_1 + x_2 - x_4 + x_6 = 2 \\ & x_1 + 2x_2 + x_5 = 10 \\ & x_j \geq 0, j = 1, \cdots, 6 \end{aligned}$$

	x_1	x_2	x_3	x_4	x_5	x_6	
x_3	2	-1	1	0	0	0	8
x_6	2	1	0	-1	0	1	2
x_5	1	2	0	0	1	0	10
f	2	1	0	0	0	0	2
x_3	0	-2	1	1	0	-1	6
x_1	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	$\frac{1}{2}$	1
x_5	0	$\frac{3}{2}$	0	$\frac{1}{2}$	1	-$\frac{1}{2}$	9
f	0	0	0	0	0	-1	0

we get a basic feasible of original LP problem $\mathbf{x} = (1, 0, 6, 0, 9)$.

	x_1	x_2	x_3	x_4	x_5	
x_3	0	-2	1	1	0	6
x_1	1	$\frac{1}{2}$	0	$-\frac{1}{2}$	0	1
x_5	0	$\frac{3}{2}$	0	$\frac{1}{2}$	1	9
f	0	$\frac{3}{2}$	0	$\frac{1}{2}$	0	7

	x_1	x_2	x_3	x_4	x_5	
x_3	0	0	1	$\frac{5}{3}$	$\frac{4}{3}$	18
x_1	1	0	0	$-\frac{2}{3}$	$-\frac{1}{3}$	-2
x_2	0	1	0	$\frac{1}{3}$	$\frac{2}{3}$	6
f	0	0	0	0	-1	-2

From the above table, we can see that solution $\mathbf{x}$=(-2,6,18,0,0), $f_{min} = -2$.