SCHOOL OF SOFTWARE, TSINGHUA UNIVERSITY

Optimization Method
homework 15

Qingfu Wen
2015213495

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I. PROBLEM 1

Consider the following problem:

$$\begin{aligned} \min \quad & x_1^3 + x_2^3 \\ \text{s.t.} \quad & x_1 + x_2 = 1 \end{aligned}$$

1. Solve the optimum of problem.
2. Define penalty function

$$F(x, \sigma) = x_1^3 + x_2^3 + \sigma(x_1 + x_2 - 1)^2$$

   Can we get the optimal solution of original problem by solving non-constraint problem $\min F(x, \sigma)$? Why?

Solution:

1. we can use $x_2 = 1 - x_1$ replace into objective function, we change original problem into a non-constraint problem

$$\min \quad 3x_1^2 - 3x_1 + 1$$

   Set $f'(x) = 6x - 3 = 0$, we get $x = \frac{1}{2}$. Then we can easily get the optimal solution $\bar{x} = (\frac{1}{2}, \frac{1}{2})^T$,$f_{min} = \frac{1}{4}$.
2. No, $F(x^{(k)}, \sigma_k) \leq f(\bar{x})$'s solution is not contained in a compact set.

II. PROBLEM 2

Consider the following problem:

$$\begin{aligned} \min \quad & x_1 x_2 \\ \text{s.t.} \quad & g(x) = -2x_1 + x_2 + 3 \geq 0 \end{aligned}$$

1. Prove that

$$\bar{x} = \begin{bmatrix} \frac{3}{4} \\ -\frac{3}{2} \end{bmatrix}$$

   is local optimum, and tell whether it is global optimum or not.

2. Define barrier function

$$G(x,r) = x_1 x_2 - r \ln g(x)$$

Solve this problem using inner point method and explain that sequence generated by inner point method incline to $\bar{x}$.

Solution:

1. at point $\bar{x}$,we have

$$\nabla f(x) = \begin{bmatrix} x_2 \\ x_1 \end{bmatrix} = \begin{bmatrix} -\frac{3}{2} \\ \frac{3}{4} \end{bmatrix}, \nabla g(\bar{x}) = \begin{bmatrix} -2 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -\frac{3}{2} \\ \frac{3}{4} \end{bmatrix} - w \begin{bmatrix} -2 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Solve it and we get $w = \frac{3}{4} > 0$, $\bar{x}$ is a KTT point. we get Lagrange function

$$L(x,w) = x_1 x_2 - w(-2x_1 + x_2 + 3)$$

Then

$$\nabla^2 L(x,w) = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

Set

$$\nabla g(\bar{x})^T d = [-2,1] \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} = 0$$

we get $d_2 = 2d_1$. $\forall d \in G$, we have

$$d^T \nabla^2 L(x,w) d = 4d_1^2 > 0$$

So, $\bar{x}$ is strict local optimum, and it is easy to find out $\bar{x}$ is not global optimum.

2.

$$G(x,r) = x_1 x_2 - r \ln(-2x_1 + x_2 + 3)$$

Set

$$\begin{cases} \dfrac{\partial G(x,r)}{\partial x_1} = x_2 + \dfrac{2r}{-2x_1 + x_2 + 3} = 0 \\ \dfrac{\partial G(x,r)}{\partial x_2} = x_1 - \dfrac{r}{-2x_1 + x_2 + 3} = 0 \end{cases}$$

Solve the above equation, we get

$$\begin{cases} x_1 = \dfrac{3 + \sqrt{9 - 16r}}{8} = 0 \\ x_2 = -\dfrac{3 + \sqrt{9 - 16r}}{4} = 0 \end{cases}$$

Set $r \to 0$, we have

$$\bar{x} = (\frac{3}{4}, -\frac{3}{2})^T$$