SCHOOL OF SOFTWARE, TSINGHUA UNIVERSITY

Optimization Method

homework 4

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CONTENTS

I. Problem 1

solve the following LP problem using big M algorithm.

1.

$$\begin{aligned} \max \quad & -3x_1 + 2x_2 - x_3 \\ \text{s.t.} \quad & 2x_1 - x_2 + x_3 \le 5 \\ & 4x_1 + 3x_2 + x_3 \ge 3 \\ & -x_1 + x_2 + x_3 = 2 \\ & x_1, x_2, x_3 \ge 0 \end{aligned}$$

Solution:

Lead slack variable x_4, x_5 and artificial variable x_6, x_7 into it.

$$\begin{aligned} \max \quad & -3x_1 + 2x_2 - x_3 + M(x_6 + x_7) \\ \text{s.t.} \quad & 2x_1 - x_2 + x_3 + x_4 = 5 \\ & 4x_1 + 3x_2 + x_3 - x_5 + x_6 = 3 \\ & -x_1 + x_2 + x_3 + x_7 = 2 \\ & x_j \ge 0, j = 1, \cdots, 7 \end{aligned}$$

	x_1	x_2	x_3	x_4	x_5	x_6	x_7	
x_4	2	-1	1	1	0	0	0	5
x_6	4	3	1	-1	1	0	3	
x_7	-1	1	1	0	0	0	1	2
f	-3M+3	-4M-2	-2M+1	0	M	0	0	-5M
x_4	$\frac{10}{3}$	0	$\frac{4}{3}$	1	$-\frac{1}{3}$	$\frac{1}{3}$	0	6
x_2	$\frac{4}{3}$	1	$\frac{1}{3}$	0	$-\frac{1}{3}$	$\frac{1}{3}$	0	1
x_7	-$\frac{7}{3}$	0	$\frac{2}{3}$	0	$\frac{1}{3}$	-$\frac{1}{3}$	1	1
f	$\frac{7}{3}M + \frac{17}{3}$	0	$-\frac{2}{3}M + \frac{5}{3}$	0	$-\frac{M}{3} - \frac{2}{3}$	$-\frac{4}{3}M + \frac{2}{3}$	0	2-M
x_4	8	0	0	1	-1	1	-2	4
x_2	$\frac{5}{2}$	1	0	0	$-\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	$\frac{1}{2}$
x_3	-$\frac{7}{2}$	0	1	0	$\frac{1}{2}$	-$\frac{1}{2}$	$\frac{3}{2}$	$\frac{3}{2}$
f	$\frac{23}{2}$	0	0	0	$-\frac{3}{2}$	$\frac{3}{2} + M$	-$\frac{5}{2} + M$	-$\frac{1}{2}$
x_4	1	0	2	1	0	0	1	7
x_2	-1	1	1	0	0	0	1	2
x_5	-7	0	2	0	1	-1	3	3
f	1	0	3	0	0	M	2+M	4

From the above table, we can see that solution **x**=(0,2,0), $f_{max} = 4$.

2.

$$\begin{aligned} \min \quad & 3x_1 - 2x_2 + x_3 \\ \text{s.t.} \quad & 2x_1 - 3x_2 + x_3 = 1 \\ & 2x_1 + 3x_2 \geq 8 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Solution:
Lead slack variable x_4 and artificial variable x_5 into it.

$$\begin{aligned} \min \quad & 3x_1 - 2x_2 + x_3 \\ \text{s.t.} \quad & 2x_1 - 3x_2 + x_3 = 1 \\ & 2x_1 + 3x_2 - x_4 + x_5 = 8 \\ & x_j \geq 0, j = 1, \cdots, 5 \end{aligned}$$

	x_1	x_2	x_3	x_4	x_5	
x_3	2	-3	1	0	0	1
x_5	2	3	0	-1	1	8
f	2M-1	3M-1	0	-M	0	8M+1
x_3	4	0	1	-1	1	9
x_2	$\frac{2}{3}$	1	0	$-\frac{1}{3}$	$\frac{1}{3}$	$\frac{8}{3}$
f	-$\frac{1}{3}$	0	0	-$\frac{1}{3}$	$\frac{1}{3}$-M	$\frac{11}{3}$

From the above table, we can see that solution **x**=(0,$\frac{8}{3}$,9), $f_{min} = \frac{11}{3}$.