Package ‘VEMIRT’

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Type Package

Title Variational Expectation Maximization for High-dimensional IRT Models

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Description VEMIRT is created to assist researchers in conducting exploratory and confirmatory multidimensional item response theory (MIRT) analysis and corresponding differential item functioning (DIF) analysis. The core computation engine of VEMIRT is a family of Gaussian Variational EM algorithms that are considerably more efficient than currently available algorithms in other software packages, especially when the number of latent factors exceeds four.

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Imports abind,
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tibble,
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R topics documented:

`VEMIRT-package` *VEMIRT: A package for high-dimensional IRT models*

Description

VEMIRT is created to assist researchers to conduct exploratory and confirmatory multidimensional item response theory (MIRT) analysis and cooresponding item differential functioning (DIF) analysis. The core computation engine of VEMIRT is a family of Gaussian Variational EM algorithms that are considerably more efficient than currently available algorithms in other software packages, especially when the number of latent factors exceeds four.

Identifying the number of factors

`pa_poly` identifies the number of factors via parallel analysis.

Exploratory factor analysis

- `gvem_2PLEFA_rot` conducts M2PL Analysis with post-hoc rotation (Promax & CF-Quartimax)
- `gvem_2PLEFA_lasso` conducts M2PL Analysis with Lasso penalty
- `gvem_2PLEFA_adaptlasso` conducts M2PL Analysis with adaptive Lasso penalty
- `sgvem_3PLEFA_rot` conducts stochastic GVEM to futher imporve the computational effficiency for exploratory M3PL analysis
- `sgvem_3PLEFA_lasso` conducts M3PL Analysis with Lasso penalty
- `sgvem_3PLEFA_adaptlasso` conducts M3PL Analysis with adaptive Lasso penalty

Confirmatory factor analysis

- gvem_2PLCFA conducts GVEM for confirmatory M2PL analysis
- sgvem_3PLCFA conducts stochastic GVEM for confirmatory M3PL analysis
- bs_2PLCFA conducts bootstrap sampling to correct bias and produce standard errors for confirmatory M2PL analysis
- importanceSampling conducts importance sampling to correct bias for M2PL analysis

Differential item functioning analysis

- em_DIF conducts DIF analysis for M2PL models using EM algorithms
- gvemm_DIF conducts DIF analysis for M2PL models using GVEMM algorithms
- lrt_DIF conducts DIF analysis for M2PL models using the likelihood ratio test

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bs_2PLCFA *Bootstrap Version of GVEM Confirmatory Analysis for M2PL*

Description

A bootstrap version of GVEM (i.e., GVEM-BS) can be implemented to correct the bias on item parameters and compute standard errors under M2PL models

Usage

```
bs_2PLCFA(gvem_result, boots = 5)
```

Arguments

gvem_result	a list that includes exploratory or confirmatory GVEM results for M2PL models.
boots	the number of bootstrap samples; default is 5

Value

a list containing the following objects:

boots_a	item discrimination parameters corrected by bootstrap sampling, a $J \times K$ matrix
boots_b	item difficulty parameters corrected by bootstrap sampling, a vector of length J
sd_a	stardard errors of item discrimination parameters, a $J \times K$ matrix
sd_b	stardard errors of item difficulty parameters, a vector of length J

See Also

gvem_2PLCFA,importanceSampling

Examples

```
## Not run:
gvem_result <- gvem_2PLCFA(exampleData_2pl, exampleIndic_cfa2pl)
bs_2PLCFA(gvem_result, boots=10)
## End(Not run)
```

---

coef.vemirt_DIF *Extract Parameter Estimates from DIF Analysis*

Description

Extract Parameter Estimates from DIF Analysis

Usage

```
coef(object, criterion = NULL)
```

Arguments

`object`	An object of class `vemirt_DIF`
`criterion`	Information criterion for model selection, one of `'AIC'`, `'BIC'` and `'GIC'`, otherwise use the criterion specified when fitting the model(s)

See Also

em_DIF, gvemm_DIF, lrt_DIF, print.vemirt_DIF

---

em_DIF *EM Algorithms for DIF Detection in 2PL Models*

Description

EM Algorithms for DIF Detection in 2PL Models

Usage

```
em_DIF(
  Y,
  D,
  X,
  method = "EMM",
  unif = F,
  Lambda0 = seq(0.2, 0.7, by = 0.1),
  criterion = "BIC",
  iter = 1000,
  eps = 0.001,
```

```
  c = 0.7,
  eta = 1
)
```

Arguments

`Y`	An $N \times J$ binary matrix of item responses
`D`	A $J \times K$ binary matrix of loading indicators
`X`	An N dimensional vector of group indicators (integers from `1` to `G`)
`method`	Estimation algorithm, one of `'EM'`, `'EMM'` and `'Adapt'`
`unif`	Whether to detect uniform DIF only
`Lambda0`	A vector of `lambda0` values for L_1 penalty (`lambda` is `sqrt(N) * lambda0`)
`criterion`	Information criterion for model selection, one of `'BIC'` (recommended), `'AIC'` and `'GIC'`
`iter`	Maximum number of iterations
`eps`	Termination criterion on numerical accuracy
`c`	Constant for computing GIC
`eta`	Tuning constant for adaptive lasso (`'Adapt'` only)

Value

An object of class `vemirt_DIF`, which is a list containing three elements:

`fit`	The best (with lowest information criterion) model, which is an element of `all`
`best`	The location of `fit` in `all`
`all`	A list of models whose length is equal to `Lambda0`:
`...$lambda0`	Corresponding element in `Lambda0`
`...$lambda`	`sqrt(N) * lambda0`
`...$iter`	Number(s) of iterations
`...$Sigma`	Group-level posterior covariance matrices
`...$Mu`	Group-level posterior mean vectors
`...$a`	Slopes for group 1
`...$b`	Intercepts for group 1
`...$gamma`	DIF parameters for the slopes
`...$beta`	DIF parameters for the intercepts
`...$ll`	Log-likelihood
`...$l0`	Number of nonzero parameters in `gamma` and `beta`
`...$AIC`	Akaike Information Criterion
`...$BIC`	Bayesian Information Criterion
`...$GIC`	Generalized Information Criterion

See Also

`gvemm_DIF`, `lrt_DIF`, `coef.vemirt_DIF`, `print.vemirt_DIF`

Examples

```
## Not run:
with(exampleDIF, em_DIF(Y, D, X))
## End(Not run)
```

`exampleData_2pl` *Response data set for M2PL*

Description

The response data set is simulated based on a between-item M2PL model with 5 factors. The true factor correlations are set as 0.1.

Usage

```
exampleData_2pl
```

Format

A data frame with 2000 respondents and 75 items

`exampleData_3pl` *Response data set for M3PL*

Description

The response data set is simulated based on a within-item M3PL model with 3 factors. The true factor correlations are set as 0.1.

Usage

```
exampleData_3pl
```

Format

A data frame with 2000 respondents and 45 items

exampleDIF — *Simulated Data Set for DIF Analysis*

Description

Simulated Data Set for DIF Analysis

Usage

```
exampleDIF
```

Format

A list of components of the data set:

`Y`	Item responses
`D`	Loading indicators
`X`	Group indicators
`j`	Number of DIF items (the first `j` items have DIF)
`params`	A list of true parameters used for generating the item responses:
`...$a`	Slopes
`...$b`	Negated intercepts
`...$theta`	Latent traits

exampleIndic_cfa2pl — *Factor-loading indicator matrix for M2PL-CFA*

Description

The factor-loading indicator matrix can be used as an input for confirmatory factor analysis.

Usage

```
exampleIndic_cfa2pl
```

Format

A data frame with 75 items and 5 factors

exampleIndic_cfa3pl *Factor-loading indicator matrix for M3PL-CFA*

Description

The factor-loading indicator matrix can be used as an input for confirmatory factor analysis.

Usage

```
exampleIndic_cfa3pl
```

Format

A data frame with 45 items and 3 factors

exampleIndic_efa2pl_c1 *Factor-loading indicator matrix for M2PL-EFA with lasso/ adaptive penalty under constraint 1*

Description

The factor-loading indicator matrix can be used as an input for exploratory factor analysis with lasso/ adaptive lasso penalty under constraint 1.

Usage

```
exampleIndic_efa2pl_c1
```

Format

A data frame with 75 items and 5 factors. Items 1, 16, 31, 46 and 61 can be combined as an identity matrix to satisfy constraint 1

exampleIndic_efa2pl_c2 *Factor-loading indicator matrix for M2PL-EFA with lasso/ adaptive penalty under constraint 2*

Description

The factor-loading indicator matrix can be used as an input for exploratory factor analysis with lasso/ adaptive lasso penalty for constraint 1.

Usage

```
exampleIndic_efa2pl_c2
```

Format

A data frame with 75 items and 5 factors. Items 1, 16, 31, 46 and 61 can be combined as a triangular matrix to satisfy constraint 2

`exampleIndic_efa3pl_c1`

Factor-loading indicator matrix for M3PL-EFA with lasso/ adaptive penalty under constraint 1

Description

The factor-loading indicator matrix can be used as an input for exploratory factor analysis with lasso/ adaptive lasso penalty under constraint 1.

Usage

```
exampleIndic_efa3pl_c1
```

Format

A data frame with 45 items and 3 factors. Items 1, 16, and 19 can be combined as an identity matrix to satisfy constraint 1

`exampleIndic_efa3pl_c2`

Factor-loading indicator matrix for M3PL-EFA with lasso/ adaptive penalty under constraint 2

Description

The factor-loading indicator matrix can be used as an input for exploratory factor analysis with lasso/ adaptive lasso penalty for constraint 1.

Usage

```
exampleIndic_efa3pl_c2
```

Format

A data frame with 45 items and 3 factors. Items 1, 16, and 19 can be combined as a triangular matrix to satisfy constraint 2

`exampleItem_2pl` *True item parameters for M2PL*

Description

True item parameters for M2PL

Usage

```
exampleItem_2pl
```

Format

An object of class `data.frame` with 75 rows and 6 columns.

`exampleItem_3pl` *True item parameters for M3PL*

Description

True item parameters for M3PL

Usage

```
exampleItem_3pl
```

Format

An object of class `data.frame` with 45 rows and 5 columns.

`gvemm_DIF` *GVEMM Algorithms for DIF Detection in 2PL Models*

Description

GVEMM Algorithms for DIF Detection in 2PL Models

Usage

```
gvemm_DIF(
  Y,
  D,
  X,
  method = "IWGVEMM",
  Lambda0 = seq(0.2, 0.7, by = 0.1),
  criterion = "GIC",
  iter = 1000,
  eps = 0.001,
  c = 0.7,
  S = 10,
  M = 10,
  lr = 0.1
)
```

Arguments

Y	An $N \times J$ binary matrix of item responses
D	A $J \times K$ binary matrix of loading indicators
X	An N dimensional vector of group indicators (integers from 1 to G)
method	Estimation algorithm, one of 'GVEMM' and 'IWGVEMM'
Lambda0	A vector of lambda0 values for L_1 penalty (lambda is sqrt(N) * lambda0)
criterion	Information criterion for model selection, one of 'GIC' (recommended), 'BIC' and 'AIC'
iter	Maximum number of iterations
eps	Termination criterion on numerical accuracy
c	Constant for computing GIC
S	Sample size for approximating the expected lower bound ('IWGVEMM' only)
M	Sample size for approximating a tighter lower bound ('IWGVEMM' only)
lr	Learning rate for the Adam optimizer ('IWGVEMM' only)

Value

An object of class vemirt_DIF, which is a list containing three elements:

fit	The best (with lowest information criterion) model, which is an element of all
best	The location of fit in all
all	A list of models whose length is equal to Lambda0:
...$lambda0	Corresponding element in Lambda0
...$lambda	sqrt(N) * lambda0
...$iter	Number(s) of iterations
...$SIGMA	Person-level posterior covariance matrices
...$MU	Person-level posterior mean vectors
...$Sigma	Group-level posterior covariance matrices
...$Mu	Group-level posterior mean vectors

`...$a`	Slopes for group 1
`...$b`	Intercepts for group 1
`...$gamma`	DIF parameters for the slopes
`...$beta`	DIF parameters for the intercepts
`...$ll`	Log-likelihood
`...$l0`	Number of nonzero parameters in `gamma` and `beta`
`...$AIC`	Akaike Information Criterion
`...$BIC`	Bayesian Information Criterion
`...$GIC`	Generalized Information Criterion

See Also

`em_DIF`, `lrt_DIF`, `coef.vemirt_DIF`, `print.vemirt_DIF`

Examples

```
## Not run:
with(exampleDIF, gvemm_DIF(Y, D, X))
## End(Not run)
```

`gvem_2PLCFA` *Confirmatory M2PL Analysis*

Description

Confirmatory M2PL Analysis

Usage

```
gvem_2PLCFA(u, indic, max.iter = 5000, SE.est = FALSE)
```

Arguments

`u`	a $N \times J$ `matrix` or a `data.frame` that consists of binary responses of N individuals to J items. The missing values are coded as `NA`
`indic`	a $J \times K$ `matrix` or a `data.frame` that describes the factor loading structure of J items to K factors. It consists of binary values where 0 refers to the item is irrelevant with this factor, 1 otherwise
`max.iter`	the maximum number of iterations for the EM cycle; default is 5000
`SE.est`	whether to estimate SE for item parameters using the updated supplemented expectation maximization (USEM); default is FALSE

Value

a list containing the following objects:

ra	item discrimination parameters, a $J \times K$ matrix
rb	item difficulty parameters, vector of length J
reta	variational parameters $\eta(\xi)$, a $N \times J$ matrix
reps	variational parameters ξ, a $N \times J$ matrix
rsigma	population variance-covariance matrix, a $K \times K$ matrix
mu_i	mean parameter for each person, a $K \times N$ matrix
sig_i	covariance matrix for each person, a $K \times K \times N$ array
n	the number of iterations for the EM cycle
Q_mat	factor loading structure, a $J \times K$ matrix
GIC	model fit index
AIC	model fit index
BIC	model fit index
SE	Standard errors of item parameters, a $J \times (K+1)$ matrix where the last column includes SE estimates for item difficulty parameters

See Also

sgvem_3PLCFA,importanceSampling,bs_2PLCFA

Examples

```
## Not run:
gvem_2PLCFA(exampleData_2pl, exampleIndic_cfa2pl)
## End(Not run)
```

---

gvem_2PLEFA_adaptlasso *Exploratory M2PL Analysis with Adaptive Lasso Penalty*

Description

Exploratory M2PL Analysis with Adaptive Lasso Penalty

Usage

```
gvem_2PLEFA_adaptlasso(
  u,
  indic,
  max.iter = 5000,
  constrain = "C1",
  non_pen = NULL,
  gamma = 2
)
```

Arguments

u a $N \times J$ `matrix` or a `data.frame` that consists of binary responses of N individuals to J items. The missing values are coded as `NA`

indic a $J \times K$ `matrix` or a `data.frame` that describes the factor loading structure of J items to K factors. It consists of binary values where 0 refers to the item is irrelevant with this factor, 1 otherwise. For exploratory factor analysis with adaptive lasso penalty, `indic` should be imposed certain constraints on the a $K \times K$ sub-matrix to ensure identifiability. The remaining parts do not assume any pre-specified zero structure but instead, the appropriate lasso penalty would recover the true zero structure. Also see `constrain`

max.iter the maximum number of iterations for the EM cycle; default is 5000

constrain the constraint setting: `"C1"` or `"C2"`. To ensure identifiablity, `"C1"` sets a $K \times K$ sub-matrix of `indic` to be an identity matrix.This constraint anchor K factors by designating K items that load solely on each factor respectively. Note that the $K \times K$ matrix does not have to appear at the top of the `indic` matrix. `"C2"` sets the $K \times K$ sub-matrix to be a lower triangular matrix with the diagonal being ones. That is, there are test items associated with each factor for sure and they may be associated with other factors as well. Nonzero entries (in the lower triangular part) except for the diagonal entries of the sub-matrix are penalized during the estimation procedure. For instance, assume $K = 3$, then the `"C2"` constraint will imply the following submatrix: $C2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. As shown, item 1 is allowed to only load on the first factor, item 2 will for sure load on the second factor but it may also load on the first factor (hence a penalty is added on the $(2,1)$ element of `"C1"`, i.e., $C2_{2,1}$). Item 3 will for sure load on the third factor but it may also load on the first two factors. However, note that for all remaining items their loading vector will all be $(1,1,1)$ hence indistinguishable from the third anchor item. Therefore, we need to alert the algorithm that this third anchor item will for sure load on the third factor, and and whether or not it loads on the first two factors depends on the regularization results. Therefore, we need to specify `"non_pen="` to identify the Kth anchor item. Although, `"C2"` is much weaker than `"C1"`, it still ensures empirical identifiability. Default is `"C1"`. During estimation, under both the `"C1"` and `"C1"` constraints, the population means and variances are constrained to be 0 and 1, respectively.

non_pen the index of an item which is associated with each factor to satisfy `"C2"`. For `C1`, the input can be `NULL`

gamma a numerical value of adaptive lasso parameter. Zou (2006) recommended three values, 0.5, 1, and 2. The default value is 2.

Value

a list containing the following objects:

ra item discrimination parameters, a $J \times K$ `matrix`

rb item difficulty parameters, vector of length J

reta variational parameters $\eta(\xi)$, a $N \times J$ matrix

reps variational parameters ξ, a $N \times J$ matrix

rsigma population variance-covariance matrix, a $K \times K$ matrix

mu_i mean parameter for each person, a $K \times N$ matrix

`sig_i`	covariance matrix for each person, a $K \times K \times N$ array
`n`	the number of iterations for the EM cycle
`Q_mat`	factor loading structure, a $J \times K$ matrix
`GIC`	model fit index
`AIC`	model fit index
`BIC`	model fit index
`lbd`	numerical value of lasso penalty parameter λ

References

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. *Psychometrika*. https://doi.org/10.1007/s11336-022-09874-6

Zou, H. (2006). The adaptive LASSO and its oracle properties. *Journal of the American Statistical Association, 7*, 1011418–1429.

See Also

`gvem_2PLEFA_rot`, `gvem_2PLEFA_lasso`, `exampleIndic_efa2pl_c1`, `exampleIndic_efa2pl_c2`

Examples

```
## Not run:
gvem_2PLEFA_adaptlasso(exampleData_2pl, exampleIndic_efa2pl_c1,constrain="C1",non_pen=NULL,gamma=2)
gvem_2PLEFA_adaptlasso(exampleData_2pl, exampleIndic_efa2pl_c2,constrain="C2",non_pen=61,gamma=2)
## End(Not run)
```

`gvem_2PLEFA_lasso` *Exploratory M2PL Analysis with Lasso Penalty*

Description

Exploratory M2PL Analysis with Lasso Penalty

Usage

```
gvem_2PLEFA_lasso(u, indic, max.iter = 5000, constrain = "C1", non_pen = NULL)
```

Arguments

`u`	a $N \times J$ `matrix` or a `data.frame` that consists of binary responses of N individuals to J items. The missing values are coded as `NA`
`indic`	a $J \times K$ `matrix` or a `data.frame` that describes the factor loading structure of J items to K factors. It consists of binary values where 0 refers to the item is irrelevant with this factor, 1 otherwise. For exploratory factor analysis with lasso penalty, `indic` should be imposed certain constraints on the a $K \times K$ sub-matrix to ensure identifiability. The remaining parts do not assume any pre-specified zero structure but instead, the appropriate lasso penalty would recover the true zero structure. Also see `constrain`
`max.iter`	the maximum number of iterations for the EM cycle; default is 5000

constrain — the constraint setting: "C1" or "C2". To ensure identifiablity, "C1" sets a $K \times K$ sub-matrix of indic to be an identity matrix.This constraint anchor K factors by designating K items that load solely on each factor respectively. Note that the $K \times K$ matrix does not have to appear at the top of the indic matrix. "C2" sets the $K \times K$ sub-matrix to be a lower triangular matrix with the diagonal being ones. That is, there are test items associated with each factor for sure and they may be associated with other factors as well. Nonzero entries (in the lower triangular part) except for the diagonal entries of the sub-matrix are penalized during the estimation procedure. For instance, assume $K = 3$, then the "C2" constraint will imply the following submatrix: $C2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. As shown, item 1 is allowed to only load on the first factor, item 2 will for sure load on the second factor but it may also load on the first factor (hence a penalty is added on the $(2, 1)$ element of "C1", i.e., $C2_{2,1}$). Item 3 will for sure load on the third factor but it may also load on the first two factors. However, note that for all remaining items their loading vector will all be $(1, 1, 1)$ hence indistinguishable from the third anchor item. Therefore, we need to alert the algorithm that this third anchor item will for sure load on the third factor, and and whether or not it loads on the first two factors depends on the regularization results. Therefore, we need to specify "non_pen=" to identify the Kth anchor item. Although, "C2" is much weaker than "C1", it still ensures empirical identifiability. Default is "C1". During estimation, under both the "C1" and "C1" constraints, the population means and variances are constrained to be 0 and 1, respectively.

non_pen — the index of an item which is associated with each factor to satisfy "C2". For C1, the input can be NULL

Value

a list containing the following objects:

ra	item discrimination parameters, a $J \times K$ matrix
rb	item difficulty parameters, vector of length J
reta	variational parameters $\eta(\xi)$, a $N \times J$ matrix
reps	variational parameters ξ, a $N \times J$ matrix
rsigma	population variance-covariance matrix, a $K \times K$ matrix
mu_i	mean parameter for each person, a $K \times N$ matrix
sig_i	covariance matrix for each person, a $K \times K \times N$ array
n	the number of iterations for the EM cycle
Q_mat	factor loading structure, a $J \times K$ matrix
GIC	model fit index
AIC	model fit index
BIC	model fit index
lbd	numerical value of lasso penalty parameter λ

References

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. *Psychometrika*. https://doi.org/10.1007/s11336-022-09874-6

See Also

gvem_2PLEFA_rot, gvem_2PLEFA_adaptlasso, exampleIndic_efa2pl_c1, exampleIndic_efa2pl_c2

Examples

```
## Not run:
gvem_2PLEFA_lasso(exampleData_2pl, exampleIndic_efa2pl_c1,constrain="C1")
gvem_2PLEFA_lasso(exampleData_2pl, exampleIndic_efa2pl_c2,constrain="C2",non_pen=61)
## End(Not run)
```

---

gvem_2PLEFA_rot *Exploratory M2PL Analysis with Post-hoc Rotation*

Description

Exploratory M2PL Analysis with Post-hoc Rotation

Usage

```
gvem_2PLEFA_rot(u, domain, max.iter = 5000, rot = "Promax")
```

Arguments

u	a $N \times J$ matrix or a data.frame that consists of binary responses of N individuals to J items. The missing values are coded as NA
domain	the number of factors
max.iter	the maximum number of iterations for the EM cycle; default is 5000
rot	the post-hoc rotation method: Promax or CF-Quartimax; default is "Promax", but may also be "cfQ" for conducting the CF-Quartimax rotation

Value

a list containing the following objects:

ra	item discrimination parameters, a $J \times K$ matrix
rb	item difficulty parameters, vector of length J
reta	variational parameters $\eta(\xi)$, a $N \times J$ matrix
reps	variational parameters ξ, a $N \times J$ matrix
rsigma	population variance-covariance matrix, a $K \times K$ matrix
mu_i	mean parameter for each person, a $K \times N$ matrix
sig_i	covariance matrix for each person, a $K \times K \times N$ array
n	the number of iterations for the EM cycle
rk	factor loadings, a $J \times K$ matrix
Q_mat	factor loading structure, a $J \times K$ matrix
GIC	model fit index
AIC	model fit index
BIC	model fit index
ur_a	item discrimination parameters before conducting the rotation, a $J \times K$ matrix

See Also

gvem_2PLEFA_lasso, gvem_2PLEFA_adaptlasso

Examples

```
## Not run:
gvem_2PLEFA_rot(exampleData_2pl, domain=5,max.iter=3000)
gvem_2PLEFA_rot(exampleData_2pl, domain=5,rot="cfQ")
## End(Not run)
```

---

importanceSampling *Importance Weighted Version of GVEM Analysis for M2PL Models*

Description

An importance weighted version of GVEM (i.e., IW-GVEM) can be implemented to correct the bias on item parameters under M2PL models

Usage

```
importanceSampling(u, gvem_result, S = 10, M = 10, max.iter = 10)
```

Arguments

u	a $N \times J$ matrix or a data.frame that consists of binary responses of N individuals to J items. The missing values are coded as NA
gvem_result	a list that includes exploratory or confirmatory GVEM results for M2PL models.
S	the number of times to draw samples;default is 10
M	the number of samples drawn from the variational distributions;default is 10
max.iter	the maximum number of iterations for the EM cycle; default is 10

Value

a list containing the following objects:

ra	item discrimination parameters estimated by GVEM, a $J \times K$ matrix
rb	item difficulty parameters estimated by GVEM, vector of length J
reta	variational parameters $\eta(\xi)$, a $N \times J$ matrix
reps	variational parameters ξ, a $N \times J$ matrix
rsigma	population variance-covariance matrix estimated by GVEM, a $K \times K$ matrix
mu_i	mean parameter for each person, a $K \times N$ matrix
sig_i	covariance matrix for each person, a $K \times K \times N$ array
n	the number of iterations for the EM cycle
rk	factor loadings, a $J \times K$ matrix, for exploratory analysis only
Q_mat	factor loading structure, a $J \times K$ matrix
GIC	model fit index
AIC	model fit index

BIC	model fit index
SE	Standard errors of item parameters, a $J \times (K+1)$ matrix where the last column includes SE estimates for item difficulty parameters, for confirmatory analysis only
ur_a	item discrimination parameters before conducting the rotation, a $J \times K$ `matrix`, for exploratory analysis only
new_a	item discrimination parameters estimated by IW-GVEM, a $J \times K$ `matrix`
new_b	item difficulty parameters estimated by IW-GVEM, vector of length J
new_Sigma_theta	population variance-covariance matrix estimated by IV-GVEM, a $K \times K$ matrix
best_lr	The learning rate used for importance sampling
best_lb	The lower bound value for importance sampling

See Also

gvem_2PLCFA, gvem_2PLEFA_rot,bs_2PLCFA

Examples

```
## Not run:
CFA_result <- gvem_2PLCFA(exampleData_2pl, exampleIndic_cfa2pl)
importanceSampling(exampleData_2pl,CFA_result)

## End(Not run)
```

`lrt_DIF` *Likelihood Ratio Test for DIF Detection in 2PL Models*

Description

Likelihood Ratio Test for DIF Detection in 2PL Models

Usage

```
lrt_DIF(Y, D, X, unif = F)
```

Arguments

Y	An $N \times J$ binary matrix of item responses
D	A $J \times K$ binary matrix of loading indicators
X	An N dimensional vector of group indicators (integers from 1 to G)
unif	Whether to detect uniform DIF only

Value

A list:

`Sigma`	Group-level posterior covariance matrices
`Mu`	Group-level posterior mean vectors
`a`	Slopes for group 1
`b`	Intercepts for group 1
`gamma`	DIF parameters for the slopes
`beta`	DIF parameters for the intercepts

See Also

`em_DIF`, `gvemm_DIF`, `coef.vemirt_DIF`, `print.vemirt_DIF`

Examples

```
## Not run:
with(exampleDIF, lrt_DIF(Y, D, X))
## End(Not run)
```

`pa_poly` *Parallel analysis using polychoric correlation*

Description

Identify the number of factors

Usage

```
pa_poly(data, n.iter = 10, figure = TRUE)
```

Arguments

`data`	a $N \times J$ `matrix` or a `data.frame` that consists of the responses of N individuals to J items without any missing values. The responses are binary or polytomous.
`n.iter`	Number of simulated analyses to perform
`figure`	By default, `pa_poly` draws an eigenvalue plot. If FALSE, it suppresses the graphic output

Value

`pa_poly` returns a `data.frame` with the eigenvalues for the real data and the simulated data.

Examples

```
## Not run:
pa_poly(exampleData_2pl, n.iter=20)
## End(Not run)
```

`print.vemirt_DIF` *Print DIF Items*

Description

Print DIF Items

Usage

```
print(x, criterion = NULL)
```

Arguments

`x`	An object of class `vemirt_DIF`
`criterion`	Information criterion for model selection, one of `'AIC'`, `'BIC'` and `'GIC'`, otherwise use the criterion specified when fitting the model(s)

See Also

`em_DIF`, `gvemm_DIF`, `lrt_DIF`, `coef.vemirt_DIF`

`sgvem_3PLCFA` *Stochastic GVEM for Confirmatory M3PL Analysis*

Description

Stochastic GVEM for Confirmatory M3PL Analysis

Usage

```
sgvem_3PLCFA(
  u,
  indic,
  samp = 50,
  forgetrate = 0.51,
  mu_b,
  sigma2_b,
  Alpha,
  Beta,
  max.iter = 5000
)
```

Arguments

`u`	a $N \times J$ `matrix` or a `data.frame` that consists of binary responses of N individuals to J items. The missing values are coded as `NA`
`indic`	a $J \times K$ `matrix` or a `data.frame` that describes the factor loading structure of J items to K factors. It consists of binary values where 0 refers to the item is irrelevant with this factor, 1 otherwise

samp	a subsample for each iteration; default is 50
forgetrate	the forget rate for the stochastic algorithm. The value should be within the range from 0.5 to 1. Default is 0.51
mu_b	the mean parameter for the prior distribution of item difficulty parameters
sigma2_b	the variance parameter for the prior distribution of item difficulty parameters
Alpha	the α parameter for the prior distribution of guessing parameters
Beta	the β parameter for the prior distribution of guessing parameters
max.iter	the maximum number of iterations for the EM cycle; default is 5000

Value

a list containing the following objects:

ra	item discrimination parameters, a $J \times K$ matrix
rb	item difficulty parameters, vector of length J
rc	item guessing parameters, vector of length J
rs	variational parameters s, a $N \times J$ matrix
reta	variational parameters $\eta(\xi)$, a $N \times J$ matrix
reps	variational parameters ξ, a $N \times J$ matrix
rsigma	population variance-covariance matrix, a $K \times K$ matrix
mu_i	mean parameter for each person, a $K \times N$ matrix
sig_i	covariance matrix for each person, a $K \times K \times N$ array
n	the number of iterations for the EM cycle
Q_mat	factor loading structure, a $J \times K$ matrix
GIC	model fit index
AIC	model fit index
BIC	model fit index

References

Cho, A. E., Wang, C., Zhang, X., & Xu, G. (2021). Gaussian variational estimation for multidimensional item response theory. *British Journal of Mathematical and Statistical Psychology, 74*, 52-85.

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. *Psychometrika*. https://doi.org/10.1007/s11336-022-09874-6

See Also

gvem_2PLCFA

Examples

```
## Not run:
sgvem_3PLCFA(exampleData_3pl, exampleIndic_cfa3pl,samp=50,forgetrate=0.51,
mu_b=0,sigma2_b=4,Alpha=10,Beta=40)
## End(Not run)
```

sgvem_3PLEFA_adaptlasso

Stochastic GVEM with Adaptive Lasso Penalty for Exploratory M3PL Analysis

Description

Stochastic GVEM with Adaptive Lasso Penalty for Exploratory M3PL Analysis

Usage

```
sgvem_3PLEFA_adaptlasso(
  u,
  indic,
  samp = 50,
  forgetrate = 0.51,
  mu_b,
  sigma2_b,
  Alpha,
  Beta,
  max.iter = 5000,
  constrain = "C1",
  non_pen = NULL,
  gamma = 2
)
```

Arguments

`u`	a $N \times J$ `matrix` or a `data.frame` that consists of binary responses of N individuals to J items. The missing values are coded as `NA`
`indic`	a $J \times K$ `matrix` or a `data.frame` that describes the factor loading structure of J items to K factors. It consists of binary values where 0 refers to the item is irrelevant with this factor, 1 otherwise. For exploratory factor analysis with lasso penalty, `indic` should be imposed certain constraints on the a $K \times K$ sub-matrix to ensure identifiability. The remaining parts do not assume any pre-specified zero structure but instead, the appropriate lasso penalty would recover the true zero structure. Also see `constrain`
`samp`	a subsample for each iteration; default is 50
`forgetrate`	the forget rate for the stochastic algorithm. The value should be within the range from 0.5 to 1. Default is 0.51
`mu_b`	the mean parameter for the prior distribution of item difficulty parameters
`sigma2_b`	the variance parameter for the prior distribution of item difficulty parameters
`Alpha`	the α parameter for the prior distribution of guessing parameters
`Beta`	the β parameter for the prior distribution of guessing parameters
`max.iter`	the maximum number of iterations for the EM cycle; default is 5000
`constrain`	the constraint setting: `"C1"` or `"C2"`. To ensure identifiablity, `"C1"` sets a $K \times K$ sub-matrix of `indic` to be an identity matrix.This constraint anchor K factors by designating K items that load solely on each factor respectively. Note that

the $K \times K$ matrix does not have to appear at the top of the indic matrix. "C2" sets the $K \times K$ sub-matrix to be a lower triangular matrix with the diagonal being ones. That is, there are test items associated with each factor for sure and they may be associated with other factors as well. Nonzero entries (in the lower triangular part) except for the diagonal entries of the sub-matrix are penalized during the estimation procedure. For instance, assume $K = 3$, then the "C2" constraint will imply the following submatrix: $C2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. As shown, item 1 is allowed to only load on the first factor, item 2 will for sure load on the second factor but it may also load on the first factor (hence a penalty is added on the $(2, 1)$ element of "C1", i.e., $C2_{2,1}$). Item 3 will for sure load on the third factor but it may also load on the first two factors. However, note that for all remaining items their loading vector will all be $(1, 1, 1)$ hence indistinguishable from the third anchor item. Therefore, we need to alert the algorithm that this third anchor item will for sure load on the third factor, and and whether or not it loads on the first two factors depends on the regularization results. Therefore, we need to specify "non_pen=" to identify the Kth anchor item. Although, "C2" is much weaker than "C1", it still ensures empirical identifiability. Default is "C1". During estimation, under both the "C1" and "C1" constraints, the population means and variances are constrained to be 0 and 1, respectively.

non_pen — the index of an item which is associated with each factor to satisfy "C2". For C1, the input can be NULL

gamma — a numerical value of adaptive lasso parameter. Zou (2006) recommended three values, 0.5, 1, and 2. The default value is 2.

Value

a list containing the following objects:

ra	item discrimination parameters, a $J \times K$ matrix
rb	item difficulty parameters, vector of length J
rc	item guessing parameters, vector of length J
rs	variational parameters s, a $N \times J$ matrix
reta	variational parameters $\eta(\xi)$, a $N \times J$ matrix
reps	variational parameters ξ, a $N \times J$ matrix
rsigma	population variance-covariance matrix, a $K \times K$ matrix
mu_i	mean parameter for each person, a $K \times N$ matrix
sig_i	covariance matrix for each person, a $K \times K \times N$ array
n	the number of iterations for the EM cycle
Q_mat	factor loading structure, a $J \times K$ matrix
GIC	model fit index
AIC	model fit index
BIC	model fit index
lbd	numerical value of lasso penalty parameter λ

References

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. *Psychometrika*. https://doi.org/10.1007/s11336-022-09874-6

Zou, H. (2006). The adaptive LASSO and its oracle properties. *Journal of the American Statistical Association, 7*, 1011418–1429.

See Also

sgvem_3PLEFA_rot, sgvem_3PLEFA_lasso, exampleIndic_efa3pl_c1, exampleIndic_efa3pl_c2

Examples

```
## Not run:
sgvem_3PLEFA_adaptlasso(exampleData_3pl, exampleIndic_efa3pl_c1,samp=50,
forgetrate=0.51,mu_b=0,sigma2_b=4,Alpha=10,Beta=40,max.iter=5000,
constrain="C1",non_pen=NULL,gamma=2)
sgvem_3PLEFA_adaptlasso(exampleData_3pl, exampleIndic_efa3pl_c2,samp=50,
forgetrate=0.51,mu_b=0,sigma2_b=4,Alpha=10,Beta=40,max.iter=5000,
constrain="C2",non_pen=19,gamma=2)
## End(Not run)
```

---

sgvem_3PLEFA_lasso   *Stochastic GVEM with Lasso Penalty for Exploratory M3PL Analysis*

Description

Stochastic GVEM with Lasso Penalty for Exploratory M3PL Analysis

Usage

```
sgvem_3PLEFA_lasso(
  u,
  indic,
  samp = 50,
  forgetrate = 0.51,
  mu_b,
  sigma2_b,
  Alpha,
  Beta,
  max.iter = 5000,
  constrain = "C1",
  non_pen = NULL
)
```

Arguments

u	a $N \times J$ `matrix` or a `data.frame` that consists of binary responses of N individuals to J items. The missing values are coded as `NA`

indic
: a $J \times K$ `matrix` or a `data.frame` that describes the factor loading structure of J items to K factors. It consists of binary values where 0 refers to the item is irrelevant with this factor, 1 otherwise. For exploratory factor analysis with lasso penalty, `indic` should be imposed certain constraints on the a $K \times K$ sub-matrix to ensure identifiability. The remaining parts do not assume any pre-specified zero structure but instead, the appropriate lasso penalty would recover the true zero structure. Also see `constrain`

samp
: a subsample for each iteration; default is 50

forgetrate
: the forget rate for the stochastic algorithm. The value should be within the range from 0.5 to 1. Default is 0.51

mu_b
: the mean parameter for the prior distribution of item difficulty parameters

sigma2_b
: the variance parameter for the prior distribution of item difficulty parameters

Alpha
: the α parameter for the prior distribution of guessing parameters

Beta
: the β parameter for the prior distribution of guessing parameters

max.iter
: the maximum number of iterations for the EM cycle; default is 5000

constrain
: the constraint setting: `"C1"` or `"C2"`. To ensure identifiablity, `"C1"` sets a $K \times K$ sub-matrix of `indic` to be an identity matrix.This constraint anchor K factors by designating K items that load solely on each factor respectively. Note that the $K \times K$ matrix does not have to appear at the top of the `indic` matrix. `"C2"` sets the $K \times K$ sub-matrix to be a lower triangular matrix with the diagonal being ones. That is, there are test items associated with each factor for sure and they may be associated with other factors as well. Nonzero entries (in the lower triangular part) except for the diagonal entries of the sub-matrix are penalized during the estimation procedure. For instance, assume $K = 3$, then the `"C2"` constraint will imply the following submatrix: $C2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. As shown, item 1 is allowed to only load on the first factor, item 2 will for sure load on the second factor but it may also load on the first factor (hence a penalty is added on the $(2, 1)$ element of `"C1"`, i.e., $C2_{2,1}$). Item 3 will for sure load on the third factor but it may also load on the first two factors. However, note that for all remaining items their loading vector will all be $(1, 1, 1)$ hence indistinguishable from the third anchor item. Therefore, we need to alert the algorithm that this third anchor item will for sure load on the third factor, and and whether or not it loads on the first two factors depends on the regularization results. Therefore, we need to specify `"non_pen="` to identify the Kth anchor item. Although, `"C2"` is much weaker than `"C1"`, it still ensures empirical identifiability. Default is `"C1"`. During estimation, under both the `"C1"` and `"C1"` constraints, the population means and variances are constrained to be 0 and 1, respectively.

non_pen
: the index of an item which is associated with each factor to satisfy `"C2"`. For `C1`, the input can be `NULL`

Value

a list containing the following objects:

ra
: item discrimination parameters, a $J \times K$ `matrix`

rb
: item difficulty parameters, vector of length J

rc
: item guessing parameters, vector of length J

rs
: variational parameters s, a $N \times J$ matrix

reta	variational parameters $\eta(\xi)$, a $N \times J$ matrix
reps	variational parameters ξ, a $N \times J$ matrix
rsigma	population variance-covariance matrix, a $K \times K$ matrix
mu_i	mean parameter for each person, a $K \times N$ matrix
sig_i	covariance matrix for each person, a $K \times K \times N$ array
n	the number of iterations for the EM cycle
Q_mat	factor loading structure, a $J \times K$ matrix
GIC	model fit index
AIC	model fit index
BIC	model fit index
lbd	numerical value of lasso penalty parameter λ

References

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. *Psychometrika*. https://doi.org/10.1007/s11336-022-09874-6

See Also

sgvem_3PLEFA_rot, sgvem_3PLEFA_adaptlasso, exampleIndic_efa3pl_c1, exampleIndic_efa3pl_c2

Examples

```
## Not run:
sgvem_3PLEFA_lasso(exampleData_3pl, exampleIndic_efa3pl_c1,samp=50,
forgetrate=0.51,mu_b=0,sigma2_b=4,Alpha=10,Beta=40,max.iter=5000,
constrain="C1",non_pen=NULL)
sgvem_3PLEFA_lasso(exampleData_3pl, exampleIndic_efa3pl_c2,samp=50,
forgetrate=0.51,mu_b=0,sigma2_b=4,Alpha=10,Beta=40,max.iter=5000,
constrain="C2",non_pen=19)
## End(Not run)
```

---

sgvem_3PLEFA_rot *Stochastic GVEM for Exploratory M3PL Analysis*

Description

Stochastic GVEM for Exploratory M3PL Analysis

Usage

```
sgvem_3PLEFA_rot(
  u,
  domain,
  samp = 50,
  forgetrate = 0.51,
  mu_b,
  sigma2_b,
  Alpha,
```

```
  Beta,
  max.iter = 5000,
  rot = "Promax"
)
```

Arguments

u	a $N \times J$ matrix or a data.frame that consists of binary responses of N individuals to J items. The missing values are coded as NA
domain	the number of factors
samp	a subsample for each iteration; default is 50
forgetrate	the forget rate for the stochastic algorithm. The value should be within the range from 0.5 to 1. Default is 0.51
mu_b	the mean parameter for the prior distribution of item difficulty parameters
sigma2_b	the variance parameter for the prior distribution of item difficulty parameters
Alpha	the α parameter for the prior distribution of guessing parameters
Beta	the β parameter for the prior distribution of guessing parameters
max.iter	the maximum number of iterations for the EM cycle; default is 5000
rot	the post-hoc rotation method: Promax or CF-Quartimax; default is "Promax", but may also be "cfQ" for conducting the CF-Quartimax rotation

Value

a list containing the following objects:

ra	item discrimination parameters, a $J \times K$ matrix
rb	item difficulty parameters, vector of length J
rc	item guessing parameters, vector of length J
rs	variational parameters s, a $N \times J$ matrix
reta	variational parameters $\eta(\xi)$, a $N \times J$ matrix
reps	variational parameters ξ, a $N \times J$ matrix
rsigma	population variance-covariance matrix, a $K \times K$ matrix
mu_i	mean parameter for each person, a $K \times N$ matrix
sig_i	covariance matrix for each person, a $K \times K \times N$ array
n	the number of iterations for the EM cycle
Q_mat	factor loading structure, a $J \times K$ matrix
rk	factor loadings, a $J \times K$ matrix
GIC	model fit index
AIC	model fit index
BIC	model fit index
ur_a	item discrimination parameters before conducting the rotation, a $J \times K$ matrix

See Also

sgvem_3PLEFA_lasso, sgvem_3PLEFA_adaptlasso

Examples

```
## Not run:
sgvem_3PLEFA_rot(exampleData_3pl, 3,samp=50,forgetrate=0.51,
mu_b=0,sigma2_b=4,Alpha=10,Beta=40,max.iter=5000,rot="Promax")
## End(Not run)
```

Index