# Package 'VEMIRT'

March 15, 2025

Title Variational Expectation Maximization for High-Dimensional IRT Models

```
Description
      VEMIRT is created to assist researchers in conducting high-dimensional exploratory and confir-
      matory multidimensional item response theory (MIRT) analysis and corresponding differen-
      tial item functioning (DIF) analysis. The core computation engine of VEMIRT is a family of Gaus-
      sian Variational EM algorithms that are considerably more efficient than currently available algo-
      rithms in other statistical packages, especially when the number of latent factors exceeds four.
License GPL-3
Imports abind,
      GPArotation,
      MASS,
      Matrix,
      mirt,
      mvQuad,
      mvnfast,
      polycor,
      psych,
      Rcpp,
      RcppArmadillo,
      testit,
      tibble,
      torch,
      shiny,
      shinyjs,
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      DT,
      callr,
      openxlsx
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```

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Type Package

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## RoxygenNote 7.3.2

 $\textbf{URL} \ \text{https://MAP-LAB-UW.github.io/VEMIRT}, \text{https://github.com/MAP-LAB-UW/VEMIRT}$ 

Suggests knitr, rmarkdown

VignetteBuilder knitr

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VEMIRT-package

VEMIRT: A Package for High-Dimensional IRT Models

#### **Description**

VEMIRT is created to assist researchers to conduct exploratory and confirmatory multidimensional item response theory (MIRT) analysis and cooresponding item differential functioning (DIF) analysis. The core computation engine of VEMIRT is a family of Gaussian Variational EM algorithms that are considerably more efficient than currently available algorithms in other software packages, especially when the number of latent factors exceeds four.

#### Identifying the number of factors

pa\_poly identifies the number of factors via parallel analysis.

#### **Exploratory factor analysis**

- E2PL\_gvem\_rot conducts M2PL Analysis with post-hoc rotation (Promax & CF-Quartimax)
- E2PL\_gvem\_lasso conducts M2PL Analysis with Lasso penalty
- E2PL\_gvem\_adaptlasso conducts M2PL Analysis with adaptive Lasso penalty
- E2PL\_iw conducts importance sampling to correct bias for M2PL analysis
- E3PL\_sgvem\_rot conducts stochastic GVEM to further improve the computational effficiency for exploratory M3PL analysis
- E3PL\_sgvem\_lasso conducts M3PL Analysis with Lasso penalty
- E3PL\_sgvem\_adaptlasso conducts M3PL Analysis with adaptive Lasso penalty
- MGRM\_gvem conducts GVEM for the multidimensional graded response model with post-hoc rotation
- MGPCM\_gvem conducts GVEM for the multidimensional partial credit model with post-hoc rotation

#### Confirmatory factor analysis

- C2PL\_gvem conducts GVEM for confirmatory M2PL analysis
- C2PL\_bs conducts bootstrap sampling to correct bias and produce standard errors for confirmatory M2PL analysis
- C2PL\_iw conducts importance sampling to correct bias for M2PL analysis
- C2PL\_iw2 conducts IW-GVEM for confirmatory M2PL analysis (alternative implementation to C2PL\_iw)
- C3PL\_sgvem conducts stochastic GVEM for confirmatory M3PL analysis
- MGRM\_gvem conducts GVEM for the multidimensional graded response model
- MGPCM\_gvem conducts GVEM for the multidimensional partial credit model

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### Differential item functioning analysis

- D1PL\_em conducts DIF analysis for M1PL models using EM algorithms
- D1PL\_gvem conducts DIF analysis for M1PL models using GVEM algorithms
- D2PL\_em conducts DIF analysis for M2PL models using EM algorithms
- D2PL\_pair\_em conducts DIF analysis for 2PL models using EM algorithms with group pairwise truncated  $L_1$  penalty
- D2PL\_gvem conducts DIF analysis for M2PL models using GVEM algorithms
- D2PL\_1rt conducts DIF analysis for M2PL models using the likelihood ratio test

### Shiny app for VEMIRT

• shinyVEMIRT Run the shiny app for VEMIRT

#### Author(s)

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### See Also

Useful links:

- https://MAP-LAB-UW.github.io/VEMIRT
- https://github.com/MAP-LAB-UW/VEMIRT

C1PL\_data

Simulated Data Set for Confirmatory M1PL Analysis

#### **Description**

Responses are simulated based on an M1PL model with 2 factors. The true factor correlations are set as 0.8.

## Usage

C1PL\_data

C2PL\_bs 5

#### **Format**

A list of components of the data set:

data Item responses

model Loading indicators

params True parameters used for generating the item responses

### Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

C2PL\_bs

Bootstrap Version of GVEM Confirmatory Analysis for M2PL

## **Description**

A bootstrap version of GVEM (i.e., GVEM-BS) can be implemented to correct the bias on item parameters and compute standard errors under confirmatory M2PL models

### Usage

```
C2PL_bs(gvem_result, boots = 5)
```

### **Arguments**

gvem\_result a list that includes exploratory or confirmatory GVEM results for M2PL models. boots the number of bootstrap samples; default is 5

#### Value

a list containing the following objects:

boots\_a item discrimination parameters corrected by bootstrap sampling, a  $J \times K$  matrix boots\_b item difficulty parameters corrected by bootstrap sampling, a vector of length J stardard errors of item discrimination parameters, a  $J \times K$  matrix sd\_b stardard errors of item difficulty parameters, a vector of length J

### Author(s)

Jiaying Xiao <jxiao6@uw.edu>

#### See Also

```
C2PL_gvem, C2PL_iw
```

### **Examples**

```
## Not run:
gvem_result <- with(C2PL_data, C2PL_gvem(data, model))
C2PL_bs(gvem_result, boots=10)
## End(Not run)</pre>
```

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C2PL_data	Simulated Data Set for Confirmatory M2PL Analysis	
C2PL_data	Simulated Data Set for Confirmatory M2PL Analysis	

## Description

Responses are simulated based on an M2PL model with 2 factors. The true factor correlations are set as 0.8.

## Usage

C2PL\_data

### **Format**

A list of components of the data set:

data Item responses

model Loading indicators

params True parameters used for generating the item responses

## Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

C2PL_gvem	Confirmatory M2PL Analysis

## Description

Confirmatory M2PL Analysis

## Usage

```
C2PL_gvem(u, indic, max.iter = 5000, SE.est = FALSE)
```

## Arguments

u	an $N \times J$ matrix or a data. frame that consists of binary responses of $N$ individuals to $J$ items. The missing values are coded as NA
indic	a $J \times K$ matrix or a data. frame that describes the factor loading structure of $J$ items to $K$ factors. It consists of binary values where 0 refers to the item is irrelevant with this factor, 1 otherwise
max.iter	the maximum number of iterations for the EM cycle; default is 5000
SE.est	whether to estimate SE for item parameters using the updated supplemented expectation maximization (USEM); default is FALSE

#### Value

a list containing the following objects:

ra	item discrimination parameters, a $J \times K$ matrix
rb	item difficulty parameters, vector of length $J$
reta	variational parameters $\eta(\xi)$ , a $N \times J$ matrix
reps	variational parameters $\xi$ , a $N \times J$ matrix
rsigma	population variance-covariance matrix, a $K \times K$ matrix
mu_i	mean parameter for each person, a $K \times N$ matrix
sig_i	covariance matrix for each person, a $K \times K \times N$ array
n	the number of iterations for the EM cycle
Q_mat	factor loading structure, a $J \times K$ matrix
GIC	model fit index
AIC	model fit index
BIC	model fit index
SE	Standard errors of item parameters, a $J \times (K+1)$ matrix where the last column

includes SE estimates for item difficulty parameters

### Author(s)

Jiaying Xiao <jxiao6@uw.edu>

### See Also

```
C3PL_sgvem, C2PL_bs, C2PL_iw
```

## **Examples**

```
## Not run:
with(C2PL_data, C2PL_gvem(data, model))
## End(Not run)
```

C2PL\_iw

Importance Weighted Version of GVEM Analysis for M2PL Models

### **Description**

An importance weighted version of GVEM (i.e., IW-GVEM) can be implemented to correct the bias on item parameters under M2PL models

## Usage

```
C2PL_iw(u, gvem_result, S = 10, M = 10, max.iter = 10)
E2PL_iw(u, gvem_result, S = 10, M = 10, max.iter = 10)
```

#### **Arguments**

u a  $N \times J$  matrix or a data. frame that consists of binary responses of N indi-

viduals to J items. The missing values are coded as NA

gvem\_result a list that includes exploratory or confirmatory GVEM results for M2PL models.

S the number of times to draw samples; default is 10

M the number of samples drawn from the variational distributions; default is 10

max.iter the maximum number of iterations for the EM cycle; default is 10

#### Value

a list containing the following objects:

ra item discrimination parameters estimated by GVEM, a  $J \times K$  matrix

rb item difficulty parameters estimated by GVEM, vector of length J

reta variational parameters  $\eta(\xi)$ , a  $N \times J$  matrix reps variational parameters  $\xi$ , a  $N \times J$  matrix

rsigma population variance-covariance matrix estimated by GVEM, a  $K \times K$  matrix

mu\_i mean parameter for each person, a  $K \times N$  matrix sig\_i covariance matrix for each person, a  $K \times K \times N$  array

n the number of iterations for the EM cycle

rk factor loadings, a  $J \times K$  matrix, for exploratory analysis only

Q\_mat factor loading structure, a  $J \times K$  matrix

GIC model fit index

AIC model fit index

BIC model fit index

SE Standard errors of item parameters, a  $J \times (K+1)$  matrix where the last column

includes SE estimates for item difficulty parameters, for confirmatory analysis

only

ur\_a item discrimination parameters before conducting the rotation, a  $J \times K$  matrix,

for exploratory analysis only

new\_a item discrimination parameters estimated by IW-GVEM, a  $J \times K$  matrix new\_b item difficulty parameters estimated by IW-GVEM, vector of length J

new\_Sigma\_theta

population variance-covariance matrix estimated by IW-GVEM, a  $K \times K$  matrix

best\_lr The learning rate used for importance sampling
best\_lb The lower bound value for importance sampling

## Author(s)

Jiaying Xiao <jxiao6@uw.edu>

#### See Also

```
C2PL_gvem, E2PL_gvem_rot, C2PL_bs
```

### **Examples**

```
## Not run:
CFA_result <- with(C2PL_data, C2PL_gvem(data, model))
C2PL_iw(C2PL_data$data, CFA_result)
## End(Not run)
## Not run:
EFA_result <- with(E2PL_data_C1, E2PL_gvem_lasso(data, model, constrain = constrain, non_pen = non_pen))
E2PL_iw(E2PL_data_C1$data, EFA_result)
## End(Not run)</pre>
```

C2PL\_iw2

IW-GVEM Algorithm for Confirmatory M2PL Analysis

### **Description**

IW-GVEM Algorithm for Confirmatory M2PL Analysis

## Usage

```
C2PL_iw2(
   data,
   model = matrix(1, ncol(data)),
   criterion = "BIC",
   iter = 200,
   eps = 0.001,
   c = 1,
   S = 10,
   M = 10,
   lr = 0.1,
   SE.level = NULL
)
```

## Arguments

data	An $N \times J$ binary matrix of item responses (missing responses should be coded as NA)
model	A $J \times K$ binary matrix of loading indicators (all items load on the only dimension by default)
criterion	Information criterion for model selection, one of 'GIC' (recommended), 'BIC', or 'AIC' $$
iter	Maximum number of iterations
eps	Termination criterion on numerical accuracy
С	Constant for computing GIC
S	Sample size for approximating the expected lower bound
М	Sample size for approximating a tighter lower bound
lr	Learning rate for the Adam optimizer
SE.level	Accuracy level of Gaussian quadrature for mvQuad to compute standard errors (SEs are not computed if SE.level is NULL)

### Value

An object of class vemirt\_DIF, which is a list containing the following elements:

N	Number of respondents
niter0	Number(s) of iterations for initialization
fit	The only element of all
best	Equal to 1
all	A list of model which has one element:
\$niter	Number(s) of iterations
\$SIGMA	Person-level posterior covariance matrices
\$MU	Person-level posterior mean vectors
\$Sigma	Population covariance matrix
\$Mu	Population mean vector
\$a	Slopes
\$b	Intercepts
\$SE.a	Standard errors of a
\$SE.b	Standard errors of b
\$11	Estimated lower bound of log-likelihood
\$10	Number of nonzero elements in model
\$AIC	Akaike Information Criterion: -2*11+10*2
\$BIC	Bayesian Information Criterion: -2*11+10*log(N)
\$GIC	Generalized Information Criterion: -2*11+c*10*log(N)*log(log(N))

### Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

### See Also

```
C2PL_gvem, C2PL_iw, D2PL_gvem, coef.vemirt_DIF, print.vemirt_DIF, summary.vemirt_DIF
```

## **Examples**

```
## Not run:
with(C2PL_data, C2PL_iw2(data, model, SE = TRUE))
## End(Not run)
```

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C3PL\_data

Simulated Data Set for Confirmatory M3PL Analysis

### **Description**

Responses are simulated based on an M3PL model with 2 factors. The true factor correlations are set as 0.8.

### Usage

```
C3PL_data
```

### **Format**

A list of components of the data set:

data Item responses

model Loading indicators

params True parameters used for generating the item responses

### Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

 ${\tt C3PL\_sgvem}$ 

Stochastic GVEM for Confirmatory M3PL Analysis

## Description

Stochastic GVEM for Confirmatory M3PL Analysis

## Usage

```
C3PL_sgvem(
    u,
    indic,
    samp = 50,
    forgetrate = 0.51,
    mu_b,
    sigma2_b,
    Alpha,
    Beta,
    max.iter = 5000
)
```

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### **Arguments**

u an  $N \times J$  matrix or a data. frame that consists of binary responses of N indi-

viduals to J items. The missing values are coded as NA

indic a  $J \times K$  matrix or a data. frame that describes the factor loading structure of

J items to K factors. It consists of binary values where 0 refers to the item is

irrelevant with this factor, 1 otherwise

samp a subsample for each iteration; default is 50

forgetrate the forget rate for the stochastic algorithm. The value should be within the range

from 0.5 to 1. Default is 0.51

mu\_b the mean parameter for the prior distribution of item difficulty parameters sigma2\_b the variance parameter for the prior distribution of item difficulty parameters

Alpha the  $\alpha$  parameter for the prior distribution of guessing parameters

Beta the  $\beta$  parameter for the prior distribution of guessing parameters

max.iter the maximum number of iterations for the EM cycle; default is 5000

#### Value

a list containing the following objects:

ra item discrimination parameters, a  $J \times K$  matrix rb item difficulty parameters, vector of length J rc item guessing parameters, vector of length J rs variational parameters s, a  $N \times J$  matrix reta variational parameters  $\eta(\xi)$ , a  $N \times J$  matrix variational parameters  $\xi$ , a  $N \times J$  matrix

rsigma population variance-covariance matrix, a  $K \times K$  matrix mu\_i mean parameter for each person, a  $K \times N$  matrix sig\_i covariance matrix for each person, a  $K \times K \times N$  array

n the number of iterations for the EM cycle

Q\_mat factor loading structure, a  $J \times K$  matrix

GIC model fit index
AIC model fit index
BIC model fit index

#### Author(s)

Jiaying Xiao <jxiao6@uw.edu>

#### References

Cho, A. E., Wang, C., Zhang, X., & Xu, G. (2021). Gaussian variational estimation for multidimensional item response theory. *British Journal of Mathematical and Statistical Psychology*, 74, 52-85.

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. *Psychometrika*. https://doi.org/10.1007/s11336-022-09874-6

coef.vemirt\_DIF

#### See Also

```
C2PL_gvem
```

### **Examples**

```
## Not run:
with(C3PL_data, C3PL_sgvem(data, model, samp=50, forgetrate=0.51, mu_b=0, sigma2_b=4, Alpha=10, Beta=40))
## End(Not run)
```

coef.vemirt\_DIF

Extract Parameter Estimates from DIF 2PL Analysis

## Description

Extract Parameter Estimates from DIF 2PL Analysis

### Usage

```
coef(object, criterion = NULL)
```

## Arguments

object An object of class vemirt\_DIF

criterion Information criterion for model selection, one of 'AIC', 'BIC', 'GIC', or the

constant for computing GIC, otherwise use the criterion specified when fitting

the model(s)

### Author(s)

Weicong Lyu <wlyu4@uw.edu>

### See Also

```
D2PL_em, D2PL_pair_em, D2PL_gvem, print.vemirt_DIF, summary.vemirt_DIF
```

```
coef.vemirt_DIF_summary
```

Extract DIF 2PL Items

## Description

Extract DIF 2PL Items

#### Usage

```
coef(object)
```

### **Arguments**

object An object of class vemirt\_DIF\_summary

D1PL\_data

### Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

#### See Also

```
summary.vemirt_DIF, print.vemirt_DIF_summary
```

coef.vemirt\_FA

Extract Parameter Estimates from Explanatory or Confirmatory Analysis

## Description

Extract Parameter Estimates from Explanatory or Confirmatory Analysis

### Usage

```
coef(object)
```

### **Arguments**

object

An object of class vemirt\_FA

#### Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

#### See Also

```
C2PL_gvem, C2PL_bs, C2PL_iw, C3PL_sgvem, E2PL_gvem_adaptlasso, E2PL_gvem_lasso, E2PL_gvem_rot, E2PL_IS, E3PL_sgvem_adaptlasso, E3PL_sgvem_lasso, E3PL_sgvem_rot, print.vemirt_FA
```

D1PL\_data

Simulated Data Set for DIF M1PL Analysis

## Description

Simulated Data Set for DIF M1PL Analysis

## Usage

D1PL\_data

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#### **Format**

A list of components of the data set:

data	Item responses
model	Loading indicators
group	Group indicators
j	Number of DIF items (the first j items have DIF)
params	A list of true parameters used for generating the item responses:
\$a	Slopes
\$b	Negated intercepts
\$theta	Latent traits

## Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

D1PL\_em

EM Algorithms for DIF Detection in M1PL Models

## Description

EM Algorithms for DIF Detection in M1PL Models

## Usage

```
D1PL_em(
    data,
    model = matrix(1, ncol(data)),
    group = rep(1, nrow(data)),
    a = 1,
    method = "EMM",
    Lambda0 = if (length(unique(group)) == 1) 0 else seq(0.1, 1, by = 0.1),
    level = 10,
    criterion = "BIC",
    iter = 200,
    eps = 0.001,
    c = 1,
    verbose = TRUE
)
```

D1PL\_em

### **Arguments**

An  $N \times J$  binary matrix of item responses (missing responses should be coded data A  $J \times K$  binary matrix of loading indicators (all items load on the only dimenmodel sion by default) An N dimensional vector of group indicators from 1 to G (all respondents are in group the same group by default) A scalar indicating the common discrimination parameter for all the dimensions а of all the items (takes 1 by default) method Estimation algorithm, one of 'EM' or 'EMM' Lambda0 A vector of lambda0 values for  $L_1$  penalty (lambda equals sqrt(N) \* lambda0) level Accuracy level, either a number for mvQuad or a vector indicating the grid for each latent dimension Information criterion for model selection, one of 'BIC' (recommended), 'AIC', criterion or 'GIC' iter Maximum number of iterations Termination criterion on numerical accuracy eps Constant for computing GIC С

### Value

verbose

An object of class vemirt\_DIF, which is a list containing the following elements:

Whether to show the progress

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### Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

### See Also

```
D1PL_gvem, coef.vemirt_DIF, print.vemirt_DIF, summary.vemirt_DIF
```

## **Examples**

```
## Not run:
with(D1PL_data, D1PL_em(data, model, group))
## End(Not run)
```

D1PL\_gvem

GVEM Algorithms for DIF Detection in M1PL Models

### **Description**

GVEM Algorithms for DIF Detection in M1PL Models

## Usage

```
D1PL_gvem(
   data,
   model = matrix(1, ncol(data)),
   group = rep(1, nrow(data)),
   a = 1,
   method = "IWGVEMM",
   Lambda0 = if (length(unique(group)) == 1) 0 else seq(0.1, 1, by = 0.1),
   criterion = "GIC",
   iter = 200,
   eps = 0.001,
   c = 1,
   S = 10,
   M = 10,
   lr = 0.1,
   verbose = TRUE
)
```

## Arguments

data	An $N \times J$ binary matrix of item responses (missing responses should be coded as NA)
model	A $J \times K$ binary matrix of loading indicators (all items load on the only dimension by default)
group	An $N$ dimensional vector of group indicators from 1 to ${\bf G}$ (all respondents are in the same group by default)
а	A scalar indicating the common discrimination parameter for all the dimensions of all the items (takes 1 by default)
method	Estimation algorithm, one of 'GVEM' or 'IWGVEMM'

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Lambda0 A vector of lambda0 values for  $L_1$  penalty (lambda equals sqrt(N) \* lambda0) criterion Information criterion for model selection, one of 'GIC' (recommended), 'BIC',

or 'AIC'

iter Maximum number of iterations

eps Termination criterion on numerical accuracy

c Constant for computing GIC

S Sample size for approximating the expected lower bound ('IWGVEMM' only)

M Sample size for approximating a tighter lower bound ('IWGVEMM' only)

1r Learning rate for the Adam optimizer ('IWGVEMM' only)

verbose Whether to show the progress

#### Value

An object of class vemirt\_DIF, which is a list containing the following elements:

N Number of respondents

niter0 Number(s) of iterations for initialization

fit The best (with lowest information criterion) model, which is an element of all

best The index of fit in all

all A list of models which has the same length as Lambda0:

...\$lambda0 Corresponding element in Lambda0

 $\dots$ \$lambda sqrt(N) \* lambda0  $\dots$ \$niter Number(s) of iterations

...\$SIGMA Person-level posterior covariance matrices

...\$MU Person-level posterior mean vectors
...\$Sigma Group-level covariance matrices

...\$Mu Group-level mean vectors

...\$a Slopes for group 1
...\$b Intercepts for group 1

...\$gamma D1PL parameters for the slopes (all elements are zero)

...\$beta D1PL parameters for the intercepts

...\$RMSE Root mean square error of fitted probability of each item for each group

...\$11 Estimated lower bound of log-likelihood
...\$10 Number of nonzero D1PL parameters in beta
...\$AIC Akaike Information Criterion: -2\*11+10\*2

...\$BIC Bayesian Information Criterion: -2\*11+10\*log(N)

...\$GIC Generalized Information Criterion: -2\*11+c\*10\*log(N)\*log(log(N))

#### Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

#### See Also

```
D1PL_em, coef.vemirt_DIF, print.vemirt_DIF, summary.vemirt_DIF
```

D2PL\_data

## **Examples**

```
## Not run:
with(D1PL_data, D1PL_gvem(data, model, group))
## End(Not run)
```

D2PL\_data

Simulated Data Set for DIF M2PL Analysis

## Description

Simulated Data Set for DIF M2PL Analysis

## Usage

D2PL\_data

### **Format**

A list of components of the data set:

data	Item responses
model	Loading indicators
group	Group indicators
j	Number of DIF items (the first j items have DIF)
params	A list of true parameters used for generating the item responses:
\$a	Slopes
\$b	Negated intercepts
\$theta	Latent traits

## Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

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D2PL\_em

EM Algorithms for DIF Detection in M2PL Models

### **Description**

EM Algorithms for DIF Detection in M2PL Models

### Usage

```
D2PL_em(
  data,
  model = matrix(1, ncol(data)),
  group = rep(1, nrow(data)),
  method = "EMM",
  Lambda0 = if (length(unique(group)) == 1) 0 else seq(0.1, 1, by = 0.1),
  level = 10,
  criterion = "BIC",
  iter = 200,
  eps = 0.001,
  c = 1,
  verbose = TRUE
)
```

### **Arguments**

data	An $N \times J$ binary matrix of item responses (missing responses should be coded as NA)
model	A $J \times K$ binary matrix of loading indicators (all items load on the only dimension by default)
group	An $N$ dimensional vector of group indicators from 1 to ${\rm G}$ (all respondents are in the same group by default)
method	Estimation algorithm, one of 'EM' or 'EMM'
Lambda0	A vector of lambda0 values for $L_1$ penalty (lambda equals $sqrt(N) * lambda0$ )
level	Accuracy level, either a number for mvQuad or a vector indicating the grid for each latent dimension
criterion	Information criterion for model selection, one of 'BIC' (recommended), 'AIC', or 'GIC'
iter	Maximum number of iterations
eps	Termination criterion on numerical accuracy
С	Constant for computing GIC
verbose	Whether to show the progress

#### Value

An object of class vemirt\_DIF, which is a list containing the following elements:

N Number of respondents niter0 Number(s) of iterations for initialization D2PL\_gvem 21

fit	The best (with lowest information criterion) model, which is an element of all
best	The index of fit in all
all	A list of models which has the same length as Lambda0:
\$lambda0	Corresponding element in Lambda0
\$lambda	sqrt(N) * lambda0
\$niter	Number(s) of iterations
\$Sigma	Group-level covariance matrices
\$Mu	Group-level mean vectors
\$a	Slopes for group 1
\$b	Intercepts for group 1
\$gamma	D2PL parameters for the slopes
\$beta	D2PL parameters for the intercepts
\$11	Log-likelihood
\$10	Number of nonzero D2PL parameters in gamma and beta
\$AIC	Akaike Information Criterion: -2*11+10*2
\$BIC	Bayesian Information Criterion: -2*11+10*log(N)
\$GIC	Generalized Information Criterion: -2*11+c*10*log(N)*log(log(N))

## Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

#### See Also

```
D2PL_pair_em, D2PL_gvem, D2PL_lrt, coef.vemirt_DIF, print.vemirt_DIF, summary.vemirt_DIF
```

## **Examples**

```
## Not run:
with(D2PL_data, D2PL_em(data, model, group))
## End(Not run)
```

D2PL\_gvem

GVEM Algorithms for DIF Detection in M2PL Models

## Description

GVEM Algorithms for DIF Detection in M2PL Models

D2PL\_gvem

### Usage

```
D2PL_gvem(
   data,
   model = matrix(1, ncol(data)),
   group = rep(1, nrow(data)),
   method = "IWGVEMM",
   Lambda0 = if (length(unique(group)) == 1) 0 else seq(0.1, 1, by = 0.1),
   criterion = "GIC",
   iter = 200,
   eps = 0.001,
   c = 1,
   S = 10,
   M = 10,
   lr = 0.1,
   verbose = TRUE
)
```

### **Arguments**

data	An $N\times J$ binary matrix of item responses (missing responses should be coded as NA)
model	A $J\times K$ binary matrix of loading indicators (all items load on the only dimension by default)
group	An $N$ dimensional vector of group indicators from 1 to ${\rm G}$ (all respondents are in the same group by default)
method	Estimation algorithm, one of 'GVEM' or 'IWGVEMM'
Lambda0	A vector of lambda0 values for $L_1$ penalty (lambda equals sqrt(N) * lambda0)
criterion	Information criterion for model selection, one of 'GIC' (recommended), 'BIC', or 'AIC' $$
iter	Maximum number of iterations
eps	Termination criterion on numerical accuracy
С	Constant for computing GIC
S	Sample size for approximating the expected lower bound ('IWGVEMM' only)
М	Sample size for approximating a tighter lower bound ('IWGVEMM' only)
lr	Learning rate for the Adam optimizer ('IWGVEMM' only)
verbose	Whether to show the progress

### Value

An object of class vemirt\_DIF, which is a list containing the following elements:

N	Number of respondents
niter0	Number(s) of iterations for initialization
fit	The best (with lowest information criterion) model, which is an element of all
best	The index of fit in all
all	A list of models which has the same length as Lambda0:
\$lambda0	Corresponding element in Lambda0

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```
sqrt(N) * lambda0
...$lambda
...$niter
                 Number(s) of iterations
...$SIGMA
                 Person-level posterior covariance matrices
...$MU
                 Person-level posterior mean vectors
                 Group-level covariance matrices
...$Sigma
                 Group-level mean vectors
...$Mu
...$a
                 Slopes for group 1
...$b
                 Intercepts for group 1
...$gamma
                 D2PL parameters for the slopes
...$beta
                 D2PL parameters for the intercepts
...$RMSE
                 Root mean square error of fitted probability of each item for each group
...$11
                 Estimated lower bound of log-likelihood
...$10
                 Number of nonzero D2PL parameters in gamma and beta
...$AIC
                 Akaike Information Criterion: -2*11+10*2
...$BIC
                 Bayesian Information Criterion: -2*11+10*log(N)
...$GIC
                 Generalized Information Criterion: -2*11+c*10*log(N)*log(log(N))
```

### Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

#### See Also

```
D2PL_pair_em, D2PL_em, D2PL_lrt, coef.vemirt_DIF, print.vemirt_DIF, summary.vemirt_DIF
```

### **Examples**

```
## Not run:
with(D2PL_data, D2PL_gvem(data, model, group))
## End(Not run)
```

D2PL\_lrt

Likelihood Ratio Test for DIF Detection in M2PL Models

### **Description**

Likelihood Ratio Test for DIF Detection in M2PL Models

## Usage

```
D2PL_lrt(data, model, group, unif = F)
```

## Arguments

data	An $N \times J$ binary matrix of item responses
model	A $J \times K$ binary matrix of loading indicators
group	An $N$ dimensional vector of group indicators (integers from 1 to $G$ )
unif	Whether to detect uniform D2PL only

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#### Value

A list:

Sigma Group-level posterior covariance matrices

Mu Group-level posterior mean vectors

a Slopes for group 1b Intercepts for group 1

gamma D2PL parameters for the slopes beta D2PL parameters for the intercepts

### Author(s)

Ruoyi Zhu <zhux0445@uw.edu>

#### See Also

```
D2PL_em, D2PL_pair_em, D2PL_gvem
```

#### **Examples**

```
## Not run:
with(D2PL_data, D2PL_lrt(data, model, group))
## End(Not run)
```

D2PL\_pair\_em

EM Algorithm with ADMM for DIF Detection Using Group Pairwise Truncated  $L_1$  Penalty in 2PL Models

### **Description**

EM Algorithm with ADMM for DIF Detection Using Group Pairwise Truncated  $L_1$  Penalty in 2PL Models

## Usage

```
D2PL_pair_em(
  data,
  group = rep(1, nrow(data)),
  Lambda0 = if (length(unique(group)) == 1) 0 else seq(0.5, 1.5, by = 0.1),
  Tau = if (length(unique(group)) == 1) 0 else c(Inf, seq(0.05, 0.3, by = 0.05)),
  rho0 = 0.5,
  level = 10,
    criterion = "BIC",
  iter = 200,
  eps = 0.001,
    c = 1,
  verbose = TRUE
)
```

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### Arguments

data An  $N \times J$  binary matrix of item responses (missing responses should be coded

as NA)

group An N dimensional vector of group indicators from 1 to G (all respondents are in

the same group by default)

Lambda $\emptyset$  A vector of lambda $\emptyset$  values for truncated  $L_1$  penalty (lambda equals sqrt(N) /

G \* lambda0)

Tau A vector of tau values for truncated  $L_1$  penalty (becomes  $L_1$  penalty when tau

equals Inf)

rho0 A value of rho for augmented Lagrangian in ADMM (tau equals sqrt(N) / G

\* tau0)

level Accuracy level of Gaussian quadrature for mvQuad

criterion Information criterion for model selection, one of 'BIC' (recommended), 'AIC',

or 'GIC'

iter Maximum number of iterations

eps Termination criterion on numerical accuracy

c Constant for computing GIC verbose Whether to show the progress

#### Value

An object of class vemirt\_DIF, which is a list containing the following elements:

N Number of respondents

niter0 Number(s) of iterations for initialization

fit The best (with lowest information criterion) model, which is an element of all

best The index of fit in all

all A list of models which has the same length as Lambda0:

...\$lambda0 Corresponding element in Lambda0

...\$lambda sqrt(N) / G \* lambda0 ...\$tau Corresponding element in

...\$tau Corresponding element in Tau ...\$rho0 Same as rho0 in input

 $\dots$ \$rho Same as rhoe in input  $\dots$ \$rho sqrt(N) / G \* rho0  $\dots$ \$niter Number(s) of iterations

...\$Sigma Group-level covariance matrices

...\$Mu Group-level mean vectors

...\$a Slopes...\$b Intercepts

...\$d.a Group pairwise differences of slopes
...\$d.b Group pairwise differences of intercepts

...\$u.a Lagrangian multipliers of corresponding elements in d.a ...\$u.b Lagrangian multipliers of corresponding elements in d.b

...\$11 Log-likelihood

...\$10 Number of nonzero D2PL parameters in gamma and beta

...\$AIC Akaike Information Criterion: -2\*11+10\*2

...\$BIC Bayesian Information Criterion: -2\*11+10\*log(N)

...\$GIC Generalized Information Criterion: -2\*11+c\*10\*log(N)\*log(log(N))

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#### Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

### See Also

```
D2PL_em, D2PL_gvem, D2PL_lrt, coef.vemirt_DIF, print.vemirt_DIF, summary.vemirt_DIF
```

## **Examples**

```
## Not run:
with(D2PL_data, D2PL_pair_em(data, group, Tau = c(Inf, seq(0.01, 0.05, by = 0.01))))
## End(Not run)
```

E2PL\_data\_C1

Simulated Data Set for Exploratory M2PL Analysis Under C1 Constraint

### **Description**

Responses are simulated based on an M2PL model with 3 factors. The true factor correlations are set as 0.5.

## Usage

```
E2PL_data_C1
```

### **Format**

A list of components of the data set:

data	Item responses
model	Loading indicators for (adaptive) lasso penalty
constrain	Constraint for model identification ('C1')
non_pen	Index of an item that is associated with all the factors (NULL under C1)
params	True parameters used for generating the item responses

### Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

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	Simulated Data Set for Exploratory M2PL Analysis Under C2 Constraint
--	--

## Description

Responses are simulated based on an M2PL model with 3 factors. The true factor correlations are set as 0.5.

## Usage

```
E2PL_data_C2
```

#### **Format**

A list of components of the data set:

data	Item responses
model	Loading indicators for (adaptive) lasso penalty
constrain	Constraint for model identification ('C2')
non_pen	Index of an item that is associated with all the factors
params	True parameters used for generating the item responses

## Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

## Description

Exploratory M2PL Analysis with Adaptive Lasso Penalty

## Usage

```
E2PL_gvem_adaptlasso(
    u,
    indic,
    max.iter = 5000,
    constrain = "C1",
    non_pen = NULL,
    gamma = 2
)
```

#### **Arguments**

u

an  $N \times J$  matrix or a data. frame that consists of binary responses of N individuals to J items. The missing values are coded as NA

indic

a  $J \times K$  matrix or a data. frame that describes the factor loading structure of J items to K factors. It consists of binary values where 0 refers to the item is irrelevant to this factor, and 1 otherwise. For exploratory factor analysis with adaptive lasso penalty, indic should include constraints on the a  $K \times K$  submatrix to ensure identifiability. The remaining parts do not assume any prespecified zero structure but instead, the appropriate lasso penalty would recover the true zero structure. Also see constrain

max.iter

the maximum number of iterations for the EM cycle; default is 5000

constrain

the constraint setting: "C1" or "C2". To ensure identifiability, "C1" sets a  $K \times K$  sub-matrix of indic to be an identity matrix. This constraint anchor K factors by designating K items that load solely on each factor respectively. Note that the  $K \times K$  matrix does not have to appear at the top of the indic matrix. "C2" sets the  $K \times K$  sub-matrix to be a lower triangular matrix with the diagonal being ones. That is, there are test items associated with each factor for sure and they may be associated with other factors as well. Nonzero entries (in the lower triangular part) except for the diagonal entries of the sub-matrix are penalized during the estimation procedure. For instance, assume K = 3, then the "C2"

constraint will imply the following submatrix:  $C2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ . As shown,

item 1 is allowed to only load on the first factor, item 2 will for sure load on the second factor but it may also load on the first factor (hence a penalty is added on the (2,1) element of "C2", i.e.,  $C2_{2,1}$ ). Item 3 will for sure load on the third factor but it may also load on the first two factors. However, note that for all remaining items their loading vector will all be (1,1,1) hence indistinguishable from the third anchor item. Therefore, we need to alert the algorithm that this third anchor item will for sure load on the third factor, and whether or not it loads on the first two factors depends on the regularization results. Therefore, we need to specify "non\_pen=" to identify the Kth anchor item. Although, "C2" is much weaker than "C1", it still ensures empirical identifiability. Default is "C1". During estimation, under both the "C1" and "C2" constraints, the population means and variances are constrained to be 0 and 1, respectively.

non\_pen

the index of an item that is associated with every factor under constraint "C2". For C1, the input can be NULL

gamma

a numerical value of adaptive lasso parameter. Zou (2006) recommended three values, 0.5, 1, and 2. The default value is 2.

#### Value

a list containing the following objects:

ra item discrimination parameters, a  $J \times K$  matrix rb item difficulty parameters, vector of length J reta variational parameters  $\eta(\xi)$ , a  $N \times J$  matrix reps variational parameters  $\xi$ , a  $N \times J$  matrix

rsigma population variance-covariance matrix, a  $K \times K$  matrix mu\_i mean parameter for each person, a  $K \times N$  matrix

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sig_i	covariance matrix for each person, a $K \times K \times N$ array
n	the number of iterations for the EM cycle
Q_mat	factor loading structure, a $J \times K$ matrix
GIC	model fit index
AIC	model fit index
BIC	model fit index
1bd	numerical value of lasso penalty parameter $\lambda$

### Author(s)

Jiaying Xiao <jxiao6@uw.edu>

#### References

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. *Psychometrika*. https://doi.org/10.1007/s11336-022-09874-6

Zou, H. (2006). The adaptive LASSO and its oracle properties. *Journal of the American Statistical Association*, 7, 1011418–1429.

#### See Also

```
E2PL_gvem_rot, E2PL_gvem_lasso, exampleIndic_efa2pl_c1, exampleIndic_efa2pl_c2
```

#### **Examples**

```
## Not run:
with(E2PL_data_C1, E2PL_gvem_adaptlasso(data, model, constrain = constrain, non_pen = non_pen, gamma=2))
with(E2PL_data_C2, E2PL_gvem_adaptlasso(data, model, constrain = constrain, non_pen = non_pen, gamma=2))
## End(Not run)
```

E2PL\_gvem\_lasso

Exploratory M2PL Analysis with Lasso Penalty

## Description

Exploratory M2PL Analysis with Lasso Penalty

## Usage

```
E2PL_gvem_lasso(u, indic, max.iter = 5000, constrain = "C1", non_pen = NULL)
```

#### **Arguments**

u

an  $N \times J$  matrix or a data. frame that consists of binary responses of N individuals to J items. The missing values are coded as NA

indic

a  $J \times K$  matrix or a data. frame that describes the factor loading structure of J items to K factors. It consists of binary values where 0 refers to the item is irrelevant with this factor, 1 otherwise. For exploratory factor analysis with lasso penalty, indic should be imposed certain constraints on the a  $K \times K$  sub-matrix to ensure identifiability. The remaining parts do not assume any pre-specified zero structure but instead, the appropriate lasso penalty would recover the true zero structure. Also see constrain

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max.iter

the maximum number of iterations for the EM cycle; default is 5000

constrain

the constraint setting: "C1" or "C2". To ensure identifiablity, "C1" sets a  $K \times K$  sub-matrix of indic to be an identity matrix. This constraint anchor K factors by designating K items that load solely on each factor respectively. Note that the  $K \times K$  matrix does not have to appear at the top of the indic matrix. "C2" sets the  $K \times K$  sub-matrix to be a lower triangular matrix with the diagonal being ones. That is, there are test items associated with each factor for sure and they may be associated with other factors as well. Nonzero entries (in the lower triangular part) except for the diagonal entries of the sub-matrix are penalized during the estimation procedure. For instance, assume K = 3, then the "C2"

constraint will imply the following submatrix:  $C2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ . As shown,

item 1 is allowed to only load on the first factor, item 2 will for sure load on the second factor but it may also load on the first factor (hence a penalty is added on the (2,1) element of "C2", i.e.,  $C2_{2,1}$ ). Item 3 will for sure load on the third factor but it may also load on the first two factors. However, note that for all remaining items their loading vector will all be (1,1,1) hence indistinguishable from the third anchor item. Therefore, we need to alert the algorithm that this third anchor item will for sure load on the third factor, and and whether or not it loads on the first two factors depends on the regularization results. Therefore, we need to specify "non\_pen=" to identify the Kth anchor item. Although, "C2" is much weaker than "C1", it still ensures empirical identifiability. Default is "C1". During estimation, under both the "C1" and "C2" constraints, the population means and variances are constrained to be 0 and 1, respectively.

non\_pen

the index of an item that is associated with every factor under constraint "C2". For C1, the input can be NULL

### Value

a list containing the following objects:

ra item discrimination parameters, a  $J \times K$  matrix rb item difficulty parameters, vector of length J reta variational parameters  $\eta(\xi)$ , a  $N \times J$  matrix reps variational parameters  $\xi$ , a  $N \times J$  matrix

rsigma population variance-covariance matrix, a  $K \times K$  matrix mu\_i mean parameter for each person, a  $K \times N$  matrix sig\_i covariance matrix for each person, a  $K \times K \times N$  array

n the number of iterations for the EM cycle Q\_mat factor loading structure, a  $J \times K$  matrix

GIC model fit index
AIC model fit index
BIC model fit index

1bd numerical value of lasso penalty parameter  $\lambda$ 

#### Author(s)

Jiaying Xiao <jxiao6@uw.edu>

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#### References

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. *Psychometrika*. https://doi.org/10.1007/s11336-022-09874-6

#### See Also

```
E2PL_gvem_rot, E2PL_gvem_adaptlasso, exampleIndic_efa2pl_c1, exampleIndic_efa2pl_c2
```

#### **Examples**

```
## Not run:
with(E2PL_data_C1, E2PL_gvem_lasso(data, model, constrain = constrain, non_pen = non_pen))
with(E2PL_data_C2, E2PL_gvem_lasso(data, model, constrain = constrain, non_pen = non_pen))
## End(Not run)
```

E2PL\_gvem\_rot

Exploratory M2PL Analysis with Post-hoc Rotation

#### **Description**

Exploratory M2PL Analysis with Post-hoc Rotation

#### Usage

```
E2PL_gvem_rot(u, domain, max.iter = 5000, rot = "Promax")
```

#### **Arguments**

u an  $N \times J$  matrix or a data. frame that consists of binary responses of N indi-

viduals to J items. The missing values are coded as NA

domain the number of factors

max.iter the maximum number of iterations for the EM cycle; default is 5000

rot the post-hoc rotation method: Promax or CF-Quartimax; default is "Promax",

but may also be "cfQ" for conducting the CF-Quartimax rotation

## Value

a list containing the following objects:

ra	item discrimination parameters, a $J\times K$ matrix
rb	item difficulty parameters, vector of length ${\cal J}$
reta	variational parameters $\eta(\xi)$ , a $N \times J$ matrix
reps	variational parameters $\xi$ , a $N \times J$ matrix
rsigma	population variance-covariance matrix, a $K \times K$ r

rsigma population variance-covariance matrix, a  $K \times K$  matrix mu\_i mean parameter for each person, a  $K \times N$  matrix sig\_i covariance matrix for each person, a  $K \times K \times N$  array

n the number of iterations for the EM cycle

rk factor loadings, a  $J \times K$  matrix

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Q_mat	factor loading structure, a $J \times K$ matrix
GIC	model fit index
AIC	model fit index
BIC	model fit index
ur_a	item discrimination parameters before conducting the rotation, a $J \times K$ matrix

### Author(s)

Jiaying Xiao <jxiao6@uw.edu>

### See Also

```
E2PL_gvem_lasso, E2PL_gvem_adaptlasso
```

### **Examples**

```
## Not run:
E2PL_gvem_rot(E2PL_data_C1$data, domain=5,max.iter=3000)
E2PL_gvem_rot(E2PL_data_C1$data, domain=5,rot="cfQ")
## End(Not run)
```

E3PL\_data\_C1 Simulated Data Set for Exploratory M3PL Analysis Under C1 Constraint

## Description

Responses are simulated based on an M3PL model with 3 factors. The true factor correlations are set as 0.5.

## Usage

```
E3PL_data_C1
```

## **Format**

A list of components of the data set:

data Item responses

model Loading indicators for (adaptive) lasso penalty

constrain Constraint for model identification ('C1')

non\_pen Index of an item that is associated with all the factors (NULL under C1)

params True parameters used for generating the item responses

## Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

E3PL\_data\_C2

E3PL_data_C2 Simulated Data Set for Exploratory M3PL Analysis Under C2 Constraint
---

### **Description**

Responses are simulated based on an M3PL model with 3 factors. The true factor correlations are set as 0.5.

### Usage

```
E3PL_data_C2
```

#### **Format**

A list of components of the data set:

data	Item responses
model	Loading indicators for (adaptive) lasso penalty
constrain	Constraint for model identification ('C2')
non_pen	Index of an item that is associated with all the factors
params	True parameters used for generating the item responses

## Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

## Description

Stochastic GVEM with Adaptive Lasso Penalty for Exploratory M3PL Analysis

### Usage

```
E3PL_sgvem_adaptlasso(
    u,
    indic,
    samp = 50,
    forgetrate = 0.51,
    mu_b,
    sigma2_b,
    Alpha,
    Beta,
```

```
max.iter = 5000,
constrain = "C1".
non_pen = NULL,
gamma = 2
```

#### **Arguments**

an  $N \times J$  matrix or a data. frame that consists of binary responses of N individuals to J items. The missing values are coded as NA

indic a  $J \times K$  matrix or a data. frame that describes the factor loading structure of J items to K factors. It consists of binary values where 0 refers to the item is irrelevant with this factor, 1 otherwise. For exploratory factor analysis with lasso penalty, indic should be imposed certain constraints on the a  $K \times K$  sub-matrix to ensure identifiability. The remaining parts do not assume any pre-specified zero structure but instead, the appropriate lasso penalty would recover the true

zero structure. Also see constrain

a subsample for each iteration; default is 50 samp

the forget rate for the stochastic algorithm. The value should be within the range forgetrate

from 0.5 to 1. Default is 0.51

the mean parameter for the normal prior distribution of item difficulty parame-

ters

the variance parameter for the normal prior distribution of item difficulty paramsigma2\_b

the  $\alpha$  parameter for the beta prior distribution of guessing parameters

the  $\beta$  parameter for the beta prior distribution of guessing parameters

the maximum number of iterations for the EM cycle; default is 5000 max.iter

sub-matrix of indic to be an identity matrix. This constraint anchor K factors by designating K items that load solely on each factor respectively. Note that the  $K \times K$  matrix does not have to appear at the top of the indic matrix. "C2"

sets the  $K \times K$  sub-matrix to be a lower triangular matrix with the diagonal being ones. That is, there are test items associated with each factor for sure and they may be associated with other factors as well. Nonzero entries (in the lower triangular part) except for the diagonal entries of the sub-matrix are penalized during the estimation procedure. For instance, assume K=3, then the "C2"

the constraint setting: "C1" or "C2". To ensure identifiablity, "C1" sets a  $K \times K$ 

 $[1 \quad 0 \quad 0]$ constraint will imply the following submatrix: C2 = $|1 \quad 1 \quad 0|$ . As shown, 1 1 1

item 1 is allowed to only load on the first factor, item 2 will for sure load on the second factor but it may also load on the first factor (hence a penalty is added on the (2,1) element of "C2", i.e.,  $C2_{2,1}$ ). Item 3 will for sure load on the third factor but it may also load on the first two factors. However, note that for all remaining items their loading vector will all be (1, 1, 1) hence indistinguishable from the third anchor item. Therefore, we need to alert the algorithm that this third anchor item will for sure load on the third factor, and and whether or not it loads on the first two factors depends on the regularization results. Therefore, we need to specify "non\_pen=" to identify the Kth anchor item. Although, "C2" is much weaker than "C1", it still ensures empirical identifiability. Default is "C1". During estimation, under both the "C1" and "C2" constraints, the population means and variances are constrained to be 0 and 1, respectively.

mu\_b

Alpha

Beta

constrain

non\_pen the index of an item which is associated with each factor to satisfy "C2". For

C1, the input can be NULL

gamma a numerical value of adaptive lasso parameter. Zou (2006) recommended three

values, 0.5, 1, and 2. The default value is 2.

#### Value

a list containing the following objects:

ra	item discrimination parameters, a $J \times K$ matrix
rb	item difficulty parameters, vector of length ${\cal J}$
rc	item guessing parameters, vector of length ${\cal J}$
rs	variational parameters $s$ , a $N \times J$ matrix
reta	variational parameters $\eta(\xi)$ , a $N \times J$ matrix
reps	variational parameters $\xi$ , a $N \times J$ matrix
rsigma	population variance-covariance matrix, a $K \times K$ matrix
mu_i	mean parameter for each person, a $K \times N$ matrix
sig_i	covariance matrix for each person, a $K\times K\times N$ array
n	the number of iterations for the EM cycle
Q_mat	factor loading structure, a $J \times K$ matrix
GIC	model fit index

GIC model fit index
AIC model fit index
BIC model fit index

lbd numerical value of lasso penalty parameter  $\lambda$ 

#### Author(s)

Jiaying Xiao <jxiao6@uw.edu>

#### References

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. *Psychometrika*. https://doi.org/10.1007/s11336-022-09874-6

Zou, H. (2006). The adaptive LASSO and its oracle properties. *Journal of the American Statistical Association*, 7, 1011418–1429.

### See Also

```
E3PL_sgvem_rot, E3PL_sgvem_lasso, exampleIndic_efa3pl_c1, exampleIndic_efa3pl_c2
```

### **Examples**

```
## Not run:
with(E3PL_data_C1, E3PL_sgvem_adaptlasso(data, model,samp=50,forgetrate=0.51,mu_b=0,sigma2_b=4,Alpha=10,Bet
with(E3PL_data_C2, E3PL_sgvem_adaptlasso(data, model,samp=50,forgetrate=0.51,mu_b=0,sigma2_b=4,Alpha=10,Bet
## End(Not run)
```

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E3PL\_sgvem\_lasso

Stochastic GVEM with Lasso Penalty for Exploratory M3PL Analysis

### **Description**

Stochastic GVEM with Lasso Penalty for Exploratory M3PL Analysis

#### Usage

```
E3PL_sgvem_lasso(
    u,
    indic,
    samp = 50,
    forgetrate = 0.51,
    mu_b,
    sigma2_b,
    Alpha,
    Beta,
    max.iter = 5000,
    constrain = "C1",
    non_pen = NULL
)
```

#### **Arguments**

u an  $N \times J$  matrix or a data. Frame that consists of binary responses of N indi-

viduals to J items. The missing values are coded as NA

indic a  $J \times K$  matrix or a data. frame that describes the factor loading structure of

J items to K factors. It consists of binary values where 0 refers to the item is irrelevant with this factor, 1 otherwise. For exploratory factor analysis with lasso penalty, indic should be imposed certain constraints on the a  $K \times K$  sub-matrix to ensure identifiability. The remaining parts do not assume any pre-specified zero structure but instead, the appropriate lasso penalty would recover the true

zero structure. Also see constrain

samp a subsample for each iteration; default is 50

forgetrate the forget rate for the stochastic algorithm. The value should be within the range

from 0.5 to 1. Default is 0.51

mu\_b the mean parameter for the normal prior distribution of item difficulty parame-

ters

sigma2\_b the variance parameter for the normal prior distribution of item difficulty param-

eters

Alpha the  $\alpha$  parameter for the beta prior distribution of guessing parameters the  $\beta$  parameter for the beta prior distribution of guessing parameters

max.iter the maximum number of iterations for the EM cycle; default is 5000

constrain the constraint setting: "C1" or "C2". To ensure identifiablity, "C1" sets a  $K \times K$ 

sub-matrix of indic to be an identity matrix. This constraint anchor K factors by designating K items that load solely on each factor respectively. Note that the  $K \times K$  matrix does not have to appear at the top of the indic matrix. "C2"

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sets the  $K \times K$  sub-matrix to be a lower triangular matrix with the diagonal being ones. That is, there are test items associated with each factor for sure and they may be associated with other factors as well. Nonzero entries (in the lower triangular part) except for the diagonal entries of the sub-matrix are penalized during the estimation procedure. For instance, assume K=3, then the "C2"

constraint will imply the following submatrix:  $C2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$ . As shown,

item 1 is allowed to only load on the first factor, item 2 will for sure load on the second factor but it may also load on the first factor (hence a penalty is added on the (2,1) element of "C2", i.e.,  $C2_{2,1}$ ). Item 3 will for sure load on the third factor but it may also load on the first two factors. However, note that for all remaining items their loading vector will all be (1,1,1) hence indistinguishable from the third anchor item. Therefore, we need to alert the algorithm that this third anchor item will for sure load on the third factor, and and whether or not it loads on the first two factors depends on the regularization results. Therefore, we need to specify "non\_pen=" to identify the Kth anchor item. Although, "C2" is much weaker than "C1", it still ensures empirical identifiability. Default is "C1". During estimation, under both the "C1" and "C2" constraints, the population means and variances are constrained to be 0 and 1, respectively.

non\_pen

the index of an item which is associated with each factor to satisfy "C2". For C1, the input can be NULL

#### Value

a list containing the following objects:

ra	item discrimination parameters, a $J\times K$ matrix
rb	item difficulty parameters, vector of length ${\cal J}$
rc	item guessing parameters, vector of length $\boldsymbol{J}$
rs	variational parameters $s$ , a $N \times J$ matrix
reta	variational parameters $\eta(\xi)$ , a $N \times J$ matrix
reps	variational parameters $\xi$ , a $N \times J$ matrix
rsigma	population variance-covariance matrix, a $K\times K$ matrix
mu_i	mean parameter for each person, a $K \times N$ matrix
sig_i	covariance matrix for each person, a $K\times K\times N$ array
n	the number of iterations for the EM cycle
Q_mat	factor loading structure, a $J \times K$ matrix
GIC	model fit index
AIC	model fit index
BIC	model fit index
lbd	numerical value of lasso penalty parameter $\boldsymbol{\lambda}$

### Author(s)

Jiaying Xiao <jxiao6@uw.edu>

38 E3PL\_sgvem\_rot

#### References

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. Psychometrika. https://doi.org/10.1007/s11336-022-09874-6

#### See Also

```
E3PL_sgvem_rot, E3PL_sgvem_adaptlasso, exampleIndic_efa3pl_c1, exampleIndic_efa3pl_c2
```

### **Examples**

```
## Not run:
with(E3PL_data_C1, E3PL_sgvem_lasso(data, model, samp=50, forgetrate=0.51, mu_b=0, sigma2_b=4, Alpha=10, Beta=40, mu_b=0, sigma2_b=4, Alpha=10, sigma2_b=4, 
with(E3PL_data_C2, E3PL_sgvem_lasso(data, model, samp=50, forgetrate=0.51, mu_b=0, sigma2_b=4, Alpha=10, Beta=40, mu_b=0, sigma2_b=4, Alpha=10, sigma2_b=4, 
## End(Not run)
```

E3PL\_sgvem\_rot

Stochastic GVEM for Exploratory M3PL Analysis

### **Description**

Stochastic GVEM for Exploratory M3PL Analysis

### Usage

```
E3PL_sgvem_rot(
  u,
  domain,
  samp = 50,
  forgetrate = 0.51,
  mu_b,
  sigma2_b,
  Alpha,
  Beta,
  max.iter = 5000,
  rot = "Promax"
```

#### Arguments

u	an $N\times J$ matrix or a data.frame that consists of binary responses of $N$ individuals to $J$ items. The missing values are coded as NA
domain	the number of factors

the number of factors

a subsample for each iteration; default is 50 samp

the forget rate for the stochastic algorithm. The value should be within the range forgetrate

from 0.5 to 1. Default is 0.51

mu\_b the mean parameter for the prior distribution of item difficulty parameters the variance parameter for the prior distribution of item difficulty parameters sigma2\_b

Alpha the  $\alpha$  parameter for the prior distribution of guessing parameters the  $\beta$  parameter for the prior distribution of guessing parameters Beta

E3PL\_sgvem\_rot 39

max.iter	the maximum number of iterations for the EM cycle; default is 5000
rot	the post-hoc rotation method: Promax or CF-Quartimax; default is "Promax",
	but may also be "cfQ" for conducting the CF-Quartimax rotation

## Value

a list containing the following objects:

ra	item discrimination parameters, a $J \times K$ matrix
rb	item difficulty parameters, vector of length ${\cal J}$
rc	item guessing parameters, vector of length ${\cal J}$
rs	variational parameters $s$ , a $N \times J$ matrix
reta	variational parameters $\eta(\xi)$ , a $N \times J$ matrix
reps	variational parameters $\xi$ , a $N \times J$ matrix
rsigma	population variance-covariance matrix, a $K \times K$ matrix
mu_i	mean parameter for each person, a $K \times N$ matrix
sig_i	covariance matrix for each person, a $K \times K \times N$ array
n	the number of iterations for the EM cycle
Q_mat	factor loading structure, a $J \times K$ matrix
rk	factor loadings, a $J \times K$ matrix
GIC	model fit index
AIC	model fit index
BIC	model fit index
ur_a	item discrimination parameters before conducting the rotation, a $J\times K$ matrix

## Author(s)

Jiaying Xiao <jxiao6@uw.edu>

## See Also

```
E3PL_sgvem_lasso, E3PL_sgvem_adaptlasso
```

## **Examples**

```
## Not run:
E3PL_sgvem_rot(E3PL_data_C1$data, 3,samp=50,forgetrate=0.51,
mu_b=0,sigma2_b=4,Alpha=10,Beta=40,max.iter=5000,rot="Promax")
## End(Not run)
```

40 MGPCM\_gvem

MGPCM\_data

Simulated Data Set for the Generalized Partial Credit Model

## Description

Simulated Data Set for the Generalized Partial Credit Model

## Usage

MGPCM\_data

#### **Format**

An object of class list of length 3.

 ${\tt MGPCM\_gvem}$ 

GVEM Algorithm for the Generalized Partial Credit Model

## Description

GVEM Algorithm for the Generalized Partial Credit Model

## Usage

```
MGPCM_gvem(
  data,
  model = matrix(1, nrow = J, ncol = 4),
  group = rep(1, nrow(data)),
  iter = 2000,
  eps = 1e-05,
  SE = FALSE,
  verbose = TRUE,
  EFA = FALSE
)
```

## Arguments

data	An $N\times J$ matrix of item responses where 0 is the minimal partial credit score (missing responses should be coded as NA)
model	A $J \times K$ matrix of loading indicators (K is the Number of latent dimension)(all items load on the only dimension by default)
iter	Maximum number of iterations
eps	Termination criterion on numerical accuracy
SE	Whether to calculate the standard errors
verbose	Whether to show the progress
EFA	Whether to rotate the output

MGRM\_data 41

#### Value

An object of class vemirt\_DIF, which is a list containing the following elements:

...\$Sigma Group-level covariance matrices

#'

...\$MU Person-level posterior mean vectors

...\$aSlopes for group 1...\$bIntercepts for group 1

...\$11 Estimated lower bound of log-likelihood

#### Author(s)

Yijun Cheng <chengxb@uw.edu>

## **Examples**

```
with(MGPCM_gvem, MGPCM_gvem(data, model))
```

MGRM\_data

Simulated Data Set for the Graded Response Model

## Description

Simulated Data Set for the Graded Response Model

## Usage

MGRM\_data

## **Format**

An object of class list of length 3.

MGRM\_gvem

GVEM Algorithm for the Graded Response Model

## Description

GVEM Algorithm for the Graded Response Model

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## Usage

```
MGRM_gvem(
  data,
  model = matrix(1, ncol(data)),
  method = "GVEM",
  iter = 200,
  tol = 1e-04,
  S = 10,
  M = 10,
  MinDim = 0,
  MaxDim = 0,
  verbose = FALSE,
  EFA = FALSE
)
```

## Arguments

data	An $N \times J$ matrix of item responses where 0 is the minimal partial credit score (missing responses should be coded as NA)
model	A $J \times K$ matrix of loading indicators (K is the Number of latent dimension)(all items load on the only dimension by default)
iter	Maximum number of iterations
tol	Termination criterion on numerical accuracy
S	Sample size for approximating the expected lower bound ('IWGVEM' only)
М	Sample size for approximating a tighter lower bound ('IWGVEM' only)
MinDim	Minimum num of possible dimensions ('EFA' only)
MaxDim	Maximum num of possible dimensions ('EFA' only)
verbose	Whether to show the progress
EFA	Whether to run EFA or CFA
criterion	Information criterion for model selection, one of 'GIC' (recommended), 'BIC', or 'AIC' $$
С	Constant for computing GIC

## Value

An object of class vemirt\_DIF, which is a list containing the following elements:

\$SIGMA	Person-level posterior covariance matrices
\$MU	Person-level posterior mean vectors
\$Sigma	Group-level covariance matrices
\$Mu	Group-level mean vectors
\$ksi1	Variational parameter 1
\$ksi2	Variational parameter 2
\$dim	Num of dimension between latent variables
\$a	Slopes
\$b	Intercepts
\$n2vlb	Bayesian Information Criterion: -2*11+10*log(N)
iter	Number(s) of iterations for initialization

pa\_poly 43

### Author(s)

Yijun Cheng <chengxb@uw.edu>

### **Examples**

```
## Not run:
with(MGRM_data, MGRM_gvem(data, method = "IWGVEM", model, EFA = FALSE))
## End(Not run)
```

pa\_poly

Parallel analysis using polychoric correlation

## Description

Identify the number of factors

#### Usage

```
pa_poly(data, n.iter = 10, figure = TRUE)
```

## **Arguments**

data a  $N \times J$  matrix or a data. frame that consists of the responses of N individuals

to J items without any missing values. The responses are binary or polytomous.

n.iter Number of simulated analyses to perform

figure By default, pa\_poly draws an eigenvalue plot. If FALSE, it suppresses the

graphic output

#### Value

pa\_poly returns a data.frame with the eigenvalues for the real data and the simulated data.

### Author(s)

Jiaying Xiao <jxiao6@uw.edu>

## **Examples**

```
## Not run:
pa_poly(C2PL_data$data, n.iter=20)
## End(Not run)
```

print.vemirt\_DIF

Print DIF 2PL Items by Group

### **Description**

Print DIF 2PL Items by Group

## Usage

```
print(x, criterion = NULL, max = 99999L, digits = 3, ...)
```

### **Arguments**

x An object of class vemirt\_DIF

criterion Information criterion for model selection, one of 'AIC', 'BIC', 'GIC', or the

constant for computing GIC, otherwise use the criterion specified when fitting

the model(s)

### Author(s)

Weicong Lyu <wlyu4@uw.edu>

#### See Also

```
D2PL_em, D2PL_pair_em, D2PL_gvem, coef.vemirt_DIF, summary.vemirt_DIF
```

```
print.vemirt_DIF_summary
```

Print Summary of DIF 2PL Items

## Description

Print Summary of DIF 2PL Items

### Usage

```
print(x, max = 99999L, ...)
```

### **Arguments**

Х

An object of class vemirt\_DIF\_summary

## Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

### See Also

```
summary.vemirt_DIF, coef.vemirt_DIF_summary
```

print.vemirt\_FA 45

print.vemirt\_FA

Print Parameter Estimates from Explanatory or Confirmatory Analysis

### **Description**

Print Parameter Estimates from Explanatory or Confirmatory Analysis

## Usage

print(x)

## Arguments

Χ

An object of class vemirt\_FA

### Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

#### See Also

shinyVEMIRT

Shiny App for VEMIRT

## Description

Shiny App for VEMIRT

## Usage

shinyVEMIRT()

### Author(s)

Weicong Lyu <weiconglyu@um.edu.mo>

46 summary.vemirt\_DIF

summary.vemirt\_DIF

Summarize DIF 2PL Items

## Description

Summarize DIF 2PL Items

### Usage

```
summary(x, criterion = NULL)
```

### **Arguments**

criterion Information criterion for model selection, one of 'AIC', 'BIC', 'GIC', or the

constant for computing GIC, otherwise use the criterion specified when fitting

the model(s)

x An object of class vemirt\_DIF

### Author(s)

Weicong Lyu <wlyu4@uw.edu>

#### See Also

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