Package 'VEMIRT'

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Title Variational Expectation Maximization for High-Dimensional IRT Models

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      VEMIRT is created to assist researchers in conducting high-dimensional exploratory and confir-
      matory multidimensional item response theory (MIRT) analysis and corresponding differen-
      tial item functioning (DIF) analysis. The core computation engine of VEMIRT is a family of Gaus-
      sian Variational EM algorithms that are considerably more efficient than currently available algo-
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```

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VEMIRT-package

VEMIRT: A package for high-dimensional IRT models

Description

VEMIRT is created to assist researchers to conduct exploratory and confirmatory multidimensional item response theory (MIRT) analysis and cooresponding item differential functioning (DIF) analysis. The core computation engine of VEMIRT is a family of Gaussian Variational EM algorithms that are considerably more efficient than currently available algorithms in other software packages, especially when the number of latent factors exceeds four.

Identifying the number of factors

pa_poly identifies the number of factors via parallel analysis.

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Exploratory factor analysis

- E2PL_gvem_rot conducts M2PL Analysis with post-hoc rotation (Promax & CF-Quartimax)
- E2PL_gvem_lasso conducts M2PL Analysis with Lasso penalty
- E2PL_gvem_adaptlasso conducts M2PL Analysis with adaptive Lasso penalty
- E2PL_iw conducts importance sampling to correct bias for M2PL analysis
- E3PL_sgvem_rot conducts stochastic GVEM to futher imporve the computational effficiency for exploratory M3PL analysis
- E3PL_sgvem_lasso conducts M3PL Analysis with Lasso penalty
- E3PL_sgvem_adaptlasso conducts M3PL Analysis with adaptive Lasso penalty

Confirmatory factor analysis

- C2PL_gvem conducts GVEM for confirmatory M2PL analysis
- C2PL_bs conducts bootstrap sampling to correct bias and produce standard errors for confirmatory M2PL analysis
- C2PL_iw conducts importance sampling to correct bias for M2PL analysis
- C3PL_sgvem conducts stochastic GVEM for confirmatory M3PL analysis

Differential item functioning analysis

- D2PL_em conducts DIF analysis for M2PL models using EM algorithms
- D2PL_gvem conducts DIF analysis for M2PL models using GVEM algorithms
- D2PL_1rt conducts DIF analysis for M2PL models using the likelihood ratio test

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See Also

Useful links:

• https://map-lab-uw.github.io/VEMIRT/

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C2PL_bs

Bootstrap Version of GVEM Confirmatory Analysis for M2PL

Description

A bootstrap version of GVEM (i.e., GVEM-BS) can be implemented to correct the bias on item parameters and compute standard errors under confirmatory M2PL models

Usage

```
C2PL_bs(gvem_result, boots = 5)
```

Arguments

gvem_result a list that includes exploratory or confirmatory GVEM results for M2PL models.

boots the number of bootstrap samples; default is 5

Value

a list containing the following objects:

boots_a	item discrimination parameters corrected by bootstrap sampling, a $J \times K$ matrix
boots_b	item difficulty parameters corrected by bootstrap sampling, a vector of length \boldsymbol{J}
sd_a	stardard errors of item discrimination parameters, a $J \times K$ matrix
sd_b	stardard errors of item difficulty parameters, a vector of length J

Author(s)

```
Jiaying Xiao <jxiao6@uw.edu>
```

See Also

```
C2PL_gvem, C2PL_iw
```

Examples

```
## Not run:
gvem_result <- with(C2PL_data, C2PL_gvem(data, model))
C2PL_bs(gvem_result, boots=10)
## End(Not run)</pre>
```

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C2PL_data	Simulated Data Set for Confirmatory M2PL Analysis	

Description

Responses are simulated based on a between-item M2PL model with 5 factors. The true factor correlations are set as 0.1.

Usage

C2PL_data

Format

A list of components of the data set:

data Item responses

model Loading indicators

params True parameters used for generating the item responses

C2PL_gvem	Confirmatory M2PL Analysis	

Description

Confirmatory M2PL Analysis

Usage

```
C2PL_gvem(u, indic, max.iter = 5000, SE.est = FALSE)
```

Arguments

u	an $N \times J$ matrix or a data. frame that consists of binary responses of N individuals to J items. The missing values are coded as NA
indic	a $J \times K$ matrix or a data.frame that describes the factor loading structure of J items to K factors. It consists of binary values where 0 refers to the item is irrelevant with this factor, 1 otherwise
max.iter	the maximum number of iterations for the EM cycle; default is 5000
SE.est	whether to estimate SE for item parameters using the updated supplemented expectation maximization (USEM); default is FALSE

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Value

a list containing the following objects:

ra	item discrimination parameters, a $J \times K$ matrix
rb	item difficulty parameters, vector of length ${\cal J}$
reta	variational parameters $\eta(\xi)$, a $N \times J$ matrix
reps	variational parameters ξ , a $N \times J$ matrix
rsigma	population variance-covariance matrix, a $K \times K$ matrix
mu_i	mean parameter for each person, a $K \times N$ matrix
sig_i	covariance matrix for each person, a $K \times K \times N$ array
n	the number of iterations for the EM cycle
Q_mat	factor loading structure, a $J \times K$ matrix
GIC	model fit index
AIC	model fit index
BIC	model fit index

includes SE estimates for item difficulty parameters

Author(s)

SE

Jiaying Xiao <jxiao6@uw.edu>

See Also

```
C3PL_sgvem, C2PL_bs, C2PL_iw
```

Examples

```
## Not run:
with(C2PL_data, C2PL_gvem(data, model))
## End(Not run)
```

C2PL_iw

Importance Weighted Version of GVEM Analysis for M2PL Models

Standard errors of item parameters, a $J \times (K+1)$ matrix where the last column

Description

An importance weighted version of GVEM (i.e., IW-GVEM) can be implemented to correct the bias on item parameters under M2PL models

Usage

```
C2PL_iw(u, gvem_result, S = 10, M = 10, max.iter = 10)
E2PL_iw(u, gvem_result, S = 10, M = 10, max.iter = 10)
```

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Arguments

u a $N \times J$ matrix or a data. frame that consists of binary responses of N indi-

viduals to J items. The missing values are coded as NA

gvem_result a list that includes exploratory or confirmatory GVEM results for M2PL models.

S the number of times to draw samples; default is 10

M the number of samples drawn from the variational distributions; default is 10

max.iter the maximum number of iterations for the EM cycle; default is 10

Value

a list containing the following objects:

ra item discrimination parameters estimated by GVEM, a $J \times K$ matrix

rb item difficulty parameters estimated by GVEM, vector of length J

reta variational parameters $\eta(\xi)$, a $N \times J$ matrix reps variational parameters ξ , a $N \times J$ matrix

rsigma population variance-covariance matrix estimated by GVEM, a $K \times K$ matrix

mu_i mean parameter for each person, a $K \times N$ matrix sig_i covariance matrix for each person, a $K \times K \times N$ array

n the number of iterations for the EM cycle

rk factor loadings, a $J \times K$ matrix, for exploratory analysis only

Q_mat factor loading structure, a $J \times K$ matrix

GIC model fit index

AIC model fit index

BIC model fit index

SE Standard errors of item parameters, a $J \times (K+1)$ matrix where the last column

includes SE estimates for item difficulty parameters, for confirmatory analysis

only

ur_a item discrimination parameters before conducting the rotation, a $J \times K$ matrix,

for exploratory analysis only

new_a item discrimination parameters estimated by IW-GVEM, a $J \times K$ matrix new_b item difficulty parameters estimated by IW-GVEM, vector of length J

new_Sigma_theta

population variance-covariance matrix estimated by IV-GVEM, a $K \times K$ matrix

best_lr The learning rate used for importance sampling
best_lb The lower bound value for importance sampling

Author(s)

Jiaying Xiao <jxiao6@uw.edu>

See Also

```
C2PL_gvem, E2PL_gvem_rot,C2PL_bs
```

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Examples

```
## Not run:
CFA_result <- with(C2PL_data, C2PL_gvem(data, model))
C2PL_iw(C2PL_data$data, CFA_result)
## End(Not run)
## Not run:
EFA_result <- with(E2PL_data_C1, E2PL_gvem_lasso(data, model, constrain = constrain, non_pen = non_pen))
E2PL_iw(E2PL_data_C1$data, EFA_result)
## End(Not run)</pre>
```

C3PL_data

Simulated Data Set for Confirmatory M3PL Analysis

Description

Responses are simulated based on a within-item M3PL model with 3 factors. The true factor correlations are set as 0.1.

Usage

C3PL_data

Format

A list of components of the data set:

data Item responses

model Loading indicators

params True parameters used for generating the item responses

C3PL_sgvem

Stochastic GVEM for Confirmatory M3PL Analysis

Description

Stochastic GVEM for Confirmatory M3PL Analysis

Usage

```
C3PL_sgvem(
    u,
    indic,
    samp = 50,
    forgetrate = 0.51,
    mu_b,
    sigma2_b,
    Alpha,
    Beta,
    max.iter = 5000
)
```

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Arguments

u an $N \times J$ matrix or a data. frame that consists of binary responses of N indi-

viduals to J items. The missing values are coded as NA

indic a $J \times K$ matrix or a data. frame that describes the factor loading structure of

J items to K factors. It consists of binary values where 0 refers to the item is

irrelevant with this factor, 1 otherwise

samp a subsample for each iteration; default is 50

forgetrate the forget rate for the stochastic algorithm. The value should be within the range

from 0.5 to 1. Default is 0.51

mu_b the mean parameter for the prior distribution of item difficulty parameters sigma2_b the variance parameter for the prior distribution of item difficulty parameters

Alpha the α parameter for the prior distribution of guessing parameters

Beta the β parameter for the prior distribution of guessing parameters

max.iter the maximum number of iterations for the EM cycle; default is 5000

Value

a list containing the following objects:

ra item discrimination parameters, a $J \times K$ matrix rb item difficulty parameters, vector of length J rc item guessing parameters, vector of length J rs variational parameters s, a $N \times J$ matrix reta variational parameters $\eta(\xi)$, a $N \times J$ matrix variational parameters ξ , a $N \times J$ matrix

rsigma population variance-covariance matrix, a $K \times K$ matrix mu_i mean parameter for each person, a $K \times N$ matrix sig_i covariance matrix for each person, a $K \times K \times N$ array

n the number of iterations for the EM cycle Q_mat factor loading structure, a $J \times K$ matrix

GIC model fit index
AIC model fit index
BIC model fit index

Author(s)

Jiaying Xiao <jxiao6@uw.edu>

References

Cho, A. E., Wang, C., Zhang, X., & Xu, G. (2021). Gaussian variational estimation for multidimensional item response theory. *British Journal of Mathematical and Statistical Psychology*, 74, 52-85.

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. *Psychometrika*. https://doi.org/10.1007/s11336-022-09874-6

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See Also

```
C2PL_gvem
```

Examples

```
## Not run:
with(C3PL_data, C3PL_sgvem(data, model, samp=50, forgetrate=0.51, mu_b=0, sigma2_b=4, Alpha=10, Beta=40))
## End(Not run)
```

coef.vemirt_DIF

Extract Parameter Estimates from D2PL Analysis

Description

Extract Parameter Estimates from D2PL Analysis

Usage

```
coef(object, criterion = NULL)
```

Arguments

object An object of class vemirt_DIF

criterion Information criterion for model selection, one of 'AIC', 'BIC', 'GIC', or the

constant for computing GIC, otherwise use the criterion specified when fitting

the model(s)

Author(s)

Weicong Lyu <wlyu4@uw.edu>

See Also

```
em_D2PL, gvemm_D2PL, lrt_D2PL, print.vemirt_DIF, summary.vemirt_DIF
```

coef.vemirt_FA

Extract Parameter Estimates from Explanatory or Confirmatory Analysis

Description

Extract Parameter Estimates from Explanatory or Confirmatory Analysis

Usage

```
coef(object)
```

Arguments

object An object of class vemirt_FA

D2PL_data

Author(s)

Weicong Lyu <wlyu4@uw.edu>

See Also

D2PL_data

Simulated Data Set for DIF M2PL Analysis

Description

Simulated Data Set for DIF M2PL Analysis

Usage

D2PL_data

Format

A list of components of the data set:

data	Item responses
model	Loading indicators
group	Group indicators
j	Number of DIF items (the first j items have DIF)
params	A list of true parameters used for generating the item responses:
\$a	Slopes
\$b	Negated intercepts
\$theta	Latent traits

D2PL_em

D2PL_em

EM Algorithms for DIF Detection in M2PL Models

Description

EM Algorithms for DIF Detection in M2PL Models

Usage

```
D2PL_em(
   data,
   model = matrix(1, ncol(data)),
   group = rep(1, nrow(data)),
   method = "EMM",
   Lambda0 = if (length(unique(group)) == 1) 0 else seq(0.2, 0.8, by = 0.1),
   level = 10,
   criterion = "BIC",
   iter = 200,
   eps = 0.001,
   c = 1
)
```

Arguments

data	An $N \times J$ binary matrix of item responses (missing responses should be coded as NA)
model	A $J \times K$ binary matrix of loading indicators (all items load on the only dimension by default)
group	An N dimensional vector of group indicators from 1 to ${\rm G}$ (all respondents are in the same group by default)
method	Estimation algorithm, one of 'EM' or 'EMM'
Lambda0	A vector of lambda0 values for L_1 penalty (lambda equals $sqrt(N) * lambda0$)
level	Accuracy level, either a number for mvQuad or a vector indicating the grid for each latent dimension
criterion	Information criterion for model selection, one of 'BIC' (recommended), 'AIC', or 'GIC' $$
iter	Maximum number of iterations
eps	Termination criterion on numerical accuracy
С	Constant for computing GIC

Value

An object of class vemirt_DIF, which is a list containing the following elements:

N	Number of respondents
niter0	Number(s) of iterations for initialization
fit	The best (with lowest information criterion) model, which is an element of all
best	The index of fit in all

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```
all
                 A list of models which has the same length as Lambda0:
...$lambda0
                 Corresponding element in Lambda0
...$lambda
                  sqrt(N) * lambda0
...$niter
                 Number(s) of iterations
...$Sigma
                 Group-level posterior covariance matrices
...$Mu
                 Group-level posterior mean vectors
...$a
                 Slopes for group 1
                 Intercepts for group 1
...$b
...$gamma
                 D2PL parameters for the slopes
...$beta
                 D2PL parameters for the intercepts
                 Log-likelihood
...$11
...$10
                 Number of nonzero D2PL parameters in gamma and beta
...$AIC
                 Akaike Information Criterion: -2*11+10*2
...$BIC
                 Bayesian Information Criterion: -2*11+10*log(N)
...$GIC
                 Generalized Information Criterion: -2*11+c*10*log(N)*log(log(N))
```

Author(s)

Weicong Lyu <wlyu4@uw.edu>

See Also

```
D2PL_gvem, D2PL_lrt, coef.vemirt_DIF, print.vemirt_DIF, summary.vemirt_DIF
```

Examples

```
## Not run:
with(D2PL_data, D2PL_em(data, model, group))
## End(Not run)
```

D2PL_gvem

GVEM Algorithms for DIF Detection in M2PL Models

Description

GVEM Algorithms for DIF Detection in M2PL Models

Usage

```
D2PL_gvem(
  data,
  model = matrix(1, ncol(data)),
  group = rep(1, nrow(data)),
  method = "IWGVEMM",
  Lambda0 = if (length(unique(group)) == 1) 0 else seq(0.2, 0.8, by = 0.1),
  criterion = "GIC",
  iter = 200,
  eps = 0.001,
```

D2PL_gvem

```
c = 1,
S = 10,
M = 10,
lr = 0.1
```

Arguments

data	An $N \times J$ binary matrix of item responses (missing responses should be coded as NA)
model	A $J \times K$ binary matrix of loading indicators (all items load on the only dimension by default)
group	An N dimensional vector of group indicators from 1 to ${\rm G}$ (all respondents are in the same group by default)
method	Estimation algorithm, one of 'GVEM' or 'IWGVEMM'
Lambda0	A vector of lambda0 values for L_1 penalty (lambda equals $sqrt(N) * lambda0$)
criterion	Information criterion for model selection, one of 'GIC' (recommended), 'BIC', or 'AIC' $$
iter	Maximum number of iterations
eps	Termination criterion on numerical accuracy
С	Constant for computing GIC
S	Sample size for approximating the expected lower bound ('IWGVEMM' only)
М	Sample size for approximating a tighter lower bound ('IWGVEMM' only)
lr	Learning rate for the Adam optimizer ('IWGVEMM' only)

Value

An object of class vemirt_DIF, which is a list containing the following elements:

· ·	-
N	Number of respondents
niter0	Number(s) of iterations for initialization
fit	The best (with lowest information criterion) model, which is an element of all
best	The index of fit in all
all	A list of models which has the same length as Lambda0:
\$lambda0	Corresponding element in Lambda0
\$lambda	sqrt(N) * lambda0
\$niter	Number(s) of iterations
\$SIGMA	Person-level posterior covariance matrices
\$MU	Person-level posterior mean vectors
\$Sigma	Group-level posterior covariance matrices
\$Mu	Group-level posterior mean vectors
\$a	Slopes for group 1
\$b	Intercepts for group 1
\$gamma	D2PL parameters for the slopes
\$beta	D2PL parameters for the intercepts

D2PL_lrt

\$RMSE	Root mean square error of fitted probability of each item for each group
\$11	Estimated lower bound of log-likelihood
\$10	Number of nonzero D2PL parameters in gamma and beta
\$AIC	Akaike Information Criterion: -2*11+10*2
\$BIC	Bayesian Information Criterion: -2*11+10*log(N)
\$GIC	Generalized Information Criterion: -2*11+c*10*log(N)*log(log(N))

Author(s)

Weicong Lyu <wlyu4@uw.edu>

See Also

```
D2PL_em, D2PL_lrt, coef.vemirt_DIF, print.vemirt_DIF, summary.vemirt_DIF
```

Examples

```
## Not run:
with(D2PL_data, D2PL_gvem(data, model, group))
## End(Not run)
```

D2PL_lrt

Likelihood Ratio Test for DIF Detection in M2PL Models

Description

Likelihood Ratio Test for DIF Detection in M2PL Models

Usage

```
D2PL_lrt(data, model, group, unif = F)
```

Arguments

data	An $N \times J$ binary matrix of item responses
model	A $J \times K$ binary matrix of loading indicators
	A 37 1

group An N dimensional vector of group indicators (integers from 1 to G)

unif Whether to detect uniform D2PL only

Value

A list:

Sigma Group-level posterior covariance matrices

Mu Group-level posterior mean vectors

a Slopes for group 1b Intercepts for group 1

gamma D2PL parameters for the slopes beta D2PL parameters for the intercepts 16 E2PL_data_C2

Author(s)

Ruoyi Zhu <zhux0445@uw.edu>

See Also

```
D2PL_em, D2PL_gvem
```

Examples

```
## Not run:
with(D2PL_data, D2PL_lrt(data, model, group))
## End(Not run)
```

E2PL_data_C1

Simulated Data Set for Exploratory M2PL Analysis Under C1 Constraints

Description

Responses are simulated based on a between-item M2PL model with 5 factors. The true factor correlations are set as 0.1.

Usage

```
E2PL_data_C1
```

Format

A list of components of the data set:

data Item responses

model Loading indicators for (adaptive) lasso penalty

constrain Constraint for model identification ('C1')

non_pen Index of an item that is associated with all the factors (NULL under C1)

E2PL_data_C2 Simulated Data Set for Exploratory M2PL Analysis Under C2 Constraints

Description

Responses are simulated based on a between-item M2PL model with 5 factors. The true factor correlations are set as 0.1.

Usage

```
E2PL_data_C2
```

Format

A list of components of the data set:

data Item responses

model Loading indicators for (adaptive) lasso penalty

constrain Constraint for model identification ('C2')

non_pen Index of an item that is associated with all the factors

E2PL_gvem_adaptlasso Exploratory M2PL Analysis with Adaptive Lasso Penalty

Description

Exploratory M2PL Analysis with Adaptive Lasso Penalty

Usage

```
E2PL_gvem_adaptlasso(
    u,
    indic,
    max.iter = 5000,
    constrain = "C1",
    non_pen = NULL,
    gamma = 2
)
```

Arguments

u

an $N \times J$ matrix or a data. frame that consists of binary responses of N individuals to J items. The missing values are coded as NA

indic

a $J \times K$ matrix or a data. frame that describes the factor loading structure of J items to K factors. It consists of binary values where 0 refers to the item is irrelevant to this factor, and 1 otherwise. For exploratory factor analysis with adaptive lasso penalty, indic should include constraints on the a $K \times K$ submatrix to ensure identifiability. The remaining parts do not assume any prespecified zero structure but instead, the appropriate lasso penalty would recover the true zero structure. Also see constrain

max.iter

the maximum number of iterations for the EM cycle; default is 5000

constrain

the constraint setting: "C1" or "C2". To ensure identifiability, "C1" sets a $K \times K$ sub-matrix of indic to be an identity matrix. This constraint anchor K factors by designating K items that load solely on each factor respectively. Note that the $K \times K$ matrix does not have to appear at the top of the indic matrix. "C2" sets the $K \times K$ sub-matrix to be a lower triangular matrix with the diagonal being ones. That is, there are test items associated with each factor for sure and they may be associated with other factors as well. Nonzero entries (in the lower triangular part) except for the diagonal entries of the sub-matrix are penalized

during the estimation procedure. For instance, assume K=3, then the "C2" constraint will imply the following submatrix: $C2=\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. As shown,

item 1 is allowed to only load on the first factor, item 2 will for sure load on the second factor but it may also load on the first factor (hence a penalty is added on the (2,1) element of "C2", i.e., $C2_{2,1}$). Item 3 will for sure load on the third factor but it may also load on the first two factors. However, note that for all remaining items their loading vector will all be (1,1,1) hence indistinguishable from the third anchor item. Therefore, we need to alert the algorithm that this third anchor item will for sure load on the third factor, and whether or not it loads on the first two factors depends on the regularization results. Therefore, we need to specify "non_pen=" to identify the Kth anchor item. Although, "C2" is much weaker than "C1", it still ensures empirical identifiability. Default is "C1". During estimation, under both the "C1" and "C2" constraints, the population means and variances are constrained to be 0 and 1, respectively.

non_pen the index of an item that is associated with every factor under constraint "C2".

For C1, the input can be NULL

gamma a numerical value of adaptive lasso parameter. Zou (2006) recommended three

values, 0.5, 1, and 2. The default value is 2.

Value

a list containing the following objects:

ra item discrimination parameters, a $J \times K$ matrix rb item difficulty parameters, vector of length J reta variational parameters $\eta(\xi)$, a $N \times J$ matrix reps variational parameters ξ , a $N \times J$ matrix

rsigma population variance-covariance matrix, a $K \times K$ matrix mu_i mean parameter for each person, a $K \times N$ matrix sig_i covariance matrix for each person, a $K \times K \times N$ array

n the number of iterations for the EM cycle Q_mat factor loading structure, a $J \times K$ matrix

GIC model fit index
AIC model fit index
BIC model fit index

lbd numerical value of lasso penalty parameter λ

Author(s)

Jiaying Xiao <jxiao6@uw.edu>

References

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. *Psychometrika*. https://doi.org/10.1007/s11336-022-09874-6

Zou, H. (2006). The adaptive LASSO and its oracle properties. *Journal of the American Statistical Association*, 7, 1011418–1429.

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See Also

```
E2PL_gvem_rot, E2PL_gvem_lasso, exampleIndic_efa2pl_c1, exampleIndic_efa2pl_c2
```

Examples

```
## Not run:
with(E2PL_data_C1, E2PL_gvem_adaptlasso(data, model, constrain = constrain, non_pen = non_pen, gamma=2))
with(E2PL_data_C2, E2PL_gvem_adaptlasso(data, model, constrain = constrain, non_pen = non_pen, gamma=2))
## End(Not run)
```

E2PL_gvem_lasso

Exploratory M2PL Analysis with Lasso Penalty

Description

Exploratory M2PL Analysis with Lasso Penalty

Usage

```
E2PL_gvem_lasso(u, indic, max.iter = 5000, constrain = "C1", non_pen = NULL)
```

Arguments

u

an $N \times J$ matrix or a data. frame that consists of binary responses of N individuals to J items. The missing values are coded as NA

indic

a $J \times K$ matrix or a data. frame that describes the factor loading structure of J items to K factors. It consists of binary values where 0 refers to the item is irrelevant with this factor, 1 otherwise. For exploratory factor analysis with lasso penalty, indic should be imposed certain constraints on the a $K \times K$ sub-matrix to ensure identifiability. The remaining parts do not assume any pre-specified zero structure but instead, the appropriate lasso penalty would recover the true zero structure. Also see constrain

max.iter

the maximum number of iterations for the EM cycle; default is 5000

constrain

the constraint setting: "C1" or "C2". To ensure identifiablity, "C1" sets a $K \times K$ sub-matrix of indic to be an identity matrix. This constraint anchor K factors by designating K items that load solely on each factor respectively. Note that the $K \times K$ matrix does not have to appear at the top of the indic matrix. "C2" sets the $K \times K$ sub-matrix to be a lower triangular matrix with the diagonal being ones. That is, there are test items associated with each factor for sure and they may be associated with other factors as well. Nonzero entries (in the lower triangular part) except for the diagonal entries of the sub-matrix are penalized during the estimation procedure. For instance, assume K = 3, then the "C2"

constraint will imply the following submatrix: $C2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. As shown,

item 1 is allowed to only load on the first factor, item 2 will for sure load on the second factor but it may also load on the first factor (hence a penalty is added on the (2,1) element of "C2", i.e., $C2_{2,1}$). Item 3 will for sure load on the third factor but it may also load on the first two factors. However, note that for all remaining items their loading vector will all be (1,1,1) hence indistinguishable

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from the third anchor item. Therefore, we need to alert the algorithm that this third anchor item will for sure load on the third factor, and and whether or not it loads on the first two factors depends on the regularization results. Therefore, we need to specify "non_pen=" to identify the Kth anchor item. Although, "C2" is much weaker than "C1", it still ensures empirical identifiability. Default is "C1". During estimation, under both the "C1" and "C2" constraints, the population means and variances are constrained to be 0 and 1, respectively.

non_pen the index of an item that

the index of an item that is associated with every factor under constraint "C2".

For C1, the input can be NULL

Value

a list containing the following objects:

ra item discrimination parameters, a $J \times K$ matrix rb item difficulty parameters, vector of length J reta variational parameters $\eta(\xi)$, a $N \times J$ matrix reps variational parameters ξ , a $N \times J$ matrix

rsigma population variance-covariance matrix, a $K \times K$ matrix mu_i mean parameter for each person, a $K \times N$ matrix sig_i covariance matrix for each person, a $K \times K \times N$ array

n the number of iterations for the EM cycle Q_mat factor loading structure, a $J \times K$ matrix

GIC model fit index
AIC model fit index
BIC model fit index

1bd numerical value of lasso penalty parameter λ

Author(s)

Jiaying Xiao <jxiao6@uw.edu>

References

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. *Psychometrika*. https://doi.org/10.1007/s11336-022-09874-6

See Also

```
E2PL_gvem_rot, E2PL_gvem_adaptlasso, exampleIndic_efa2pl_c1, exampleIndic_efa2pl_c2
```

Examples

```
## Not run:
with(E2PL_data_C1, E2PL_gvem_lasso(data, model, constrain = constrain, non_pen = non_pen))
with(E2PL_data_C2, E2PL_gvem_lasso(data, model, constrain = constrain, non_pen = non_pen))
## End(Not run)
```

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E2PL_gvem_rot	Exploratory M2PL Analysis with Post-hoc Rotation	

Description

Exploratory M2PL Analysis with Post-hoc Rotation

Usage

```
E2PL_gvem_rot(u, domain, max.iter = 5000, rot = "Promax")
```

Arguments

u an $N \times J$ matrix or a data. frame that consists of binary responses of N individuals to J items. The missing values are coded as NA domain the number of factors

max.iter the maximum number of iterations for the EM cycle; default is 5000

rot the post-hoc rotation method: Promax or CF-Quartimax; default is "Promax",

but may also be "cfQ" for conducting the CF-Quartimax rotation

Value

a list containing the following objects:

ra item discrimination parameters, a $J \times K$ matrix rb item difficulty parameters, vector of length J reta variational parameters $\eta(\xi)$, a $N \times J$ matrix reps variational parameters ξ , a $N \times J$ matrix population variance-covariance matrix, a $K \times K$ matrix

mu_i mean parameter for each person, a $K \times N$ matrix

 sig_i covariance matrix for each person, a $K \times K \times N$ array

n the number of iterations for the EM cycle

 ${\it rk} \hspace{1cm} {\it factor loadings, a} \ J \times K \ {\it matrix}$

Q_mat factor loading structure, a $J \times K$ matrix

GIC model fit index
AIC model fit index
BIC model fit index

ur_a item discrimination parameters before conducting the rotation, a $J \times K$ matrix

Author(s)

Jiaying Xiao <jxiao6@uw.edu>

See Also

```
E2PL_gvem_lasso, E2PL_gvem_adaptlasso
```

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Examples

```
## Not run:
E2PL_gvem_rot(E2PL_data_C1$data, domain=5,max.iter=3000)
E2PL_gvem_rot(E2PL_data_C1$data, domain=5,rot="cfQ")
## End(Not run)
```

E3PL_data_C1

Simulated Data Set for Exploratory M3PL Analysis Under C1 Constraints

Description

Responses are simulated based on a within-item M3PL model with 3 factors. The true factor correlations are set as 0.1.

Usage

```
E3PL_data_C1
```

Format

A list of components of the data set:

data Item responses

model Loading indicators for (adaptive) lasso penalty

constrain Constraint for model identification ('C1')

non_pen Index of an item that is associated with all the factors (NULL under C1)

E3PL_data_C2	Simulated Data Set for Exploratory M3PL Analysis Under C2 Con-
	straints

Description

Responses are simulated based on a within-item M3PL model with 3 factors. The true factor correlations are set as 0.1.

Usage

```
{\tt E3PL\_data\_C2}
```

Format

A list of components of the data set:

data	Item responses
model	Loading indicators for (adaptive) lasso penalty
constrain	Constraint for model identification ('C2')
non_pen	Index of an item that is associated with all the factors

Description

Stochastic GVEM with Adaptive Lasso Penalty for Exploratory M3PL Analysis

Usage

```
E3PL_sgvem_adaptlasso(
    u,
    indic,
    samp = 50,
    forgetrate = 0.51,
    mu_b,
    sigma2_b,
    Alpha,
    Beta,
    max.iter = 5000,
    constrain = "C1",
    non_pen = NULL,
    gamma = 2
)
```

Arguments

u an $N \times J$ matrix or a data. frame that consists of binary responses of N indi-

viduals to J items. The missing values are coded as NA

indic a $J \times K$ matrix or a data. frame that describes the factor loading structure of

J items to K factors. It consists of binary values where 0 refers to the item is irrelevant with this factor, 1 otherwise. For exploratory factor analysis with lasso penalty, indic should be imposed certain constraints on the a $K \times K$ sub-matrix to ensure identifiability. The remaining parts do not assume any pre-specified zero structure but instead, the appropriate lasso penalty would recover the true

zero structure. Also see constrain

samp a subsample for each iteration; default is 50

forgetrate the forget rate for the stochastic algorithm. The value should be within the range

from 0.5 to 1. Default is 0.51

mu_b the mean parameter for the normal prior distribution of item difficulty parame-

ters

sigma2_b the variance parameter for the normal prior distribution of item difficulty param-

eters

Alpha the α parameter for the beta prior distribution of guessing parameters

Beta the β parameter for the beta prior distribution of guessing parameters

max.iter the maximum number of iterations for the EM cycle; default is 5000

constrain the constraint setting: "C1" or "C2". To ensure identifiablity, "C1" sets a $K \times K$

sub-matrix of indic to be an identity matrix. This constraint anchor K factors by designating K items that load solely on each factor respectively. Note that the $K \times K$ matrix does not have to appear at the top of the indic matrix. "C2" sets the $K \times K$ sub-matrix to be a lower triangular matrix with the diagonal being ones. That is, there are test items associated with each factor for sure and they may be associated with other factors as well. Nonzero entries (in the lower triangular part) except for the diagonal entries of the sub-matrix are penalized during the estimation procedure. For instance, assume K=3, then the "C2"

constraint will imply the following submatrix: $C2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. As shown,

item 1 is allowed to only load on the first factor, item 2 will for sure load on the second factor but it may also load on the first factor (hence a penalty is added on the (2,1) element of "C2", i.e., $C2_{2,1}$). Item 3 will for sure load on the third factor but it may also load on the first two factors. However, note that for all remaining items their loading vector will all be (1,1,1) hence indistinguishable from the third anchor item. Therefore, we need to alert the algorithm that this third anchor item will for sure load on the third factor, and and whether or not it loads on the first two factors depends on the regularization results. Therefore, we need to specify "non_pen=" to identify the Kth anchor item. Although, "C2" is much weaker than "C1", it still ensures empirical identifiability. Default is "C1". During estimation, under both the "C1" and "C2" constraints, the population means and variances are constrained to be 0 and 1, respectively.

non_pen the index of an item which is associated with each factor to satisfy "C2". For

C1, the input can be NULL

gamma a numerical value of adaptive lasso parameter. Zou (2006) recommended three

values, 0.5, 1, and 2. The default value is 2.

Value

a list containing the following objects:

ra item discrimination parameters, a $J \times K$ matrix rb item difficulty parameters, vector of length J rc item guessing parameters, vector of length J rs variational parameters s, a $N \times J$ matrix reta variational parameters $\eta(\xi)$, a $N \times J$ matrix variational parameters ξ , a $N \times J$ matrix

rsigma population variance-covariance matrix, a $K \times K$ matrix mu_i mean parameter for each person, a $K \times N$ matrix sig_i covariance matrix for each person, a $K \times K \times N$ array

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n	the number of iterations for the EM cycle
Q_mat	factor loading structure, a $J \times K$ matrix
GIC	model fit index
AIC	model fit index
BIC	model fit index
lbd	numerical value of lasso penalty parameter λ

Author(s)

Jiaying Xiao <jxiao6@uw.edu>

References

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. *Psychometrika*. https://doi.org/10.1007/s11336-022-09874-6

Zou, H. (2006). The adaptive LASSO and its oracle properties. *Journal of the American Statistical Association*, 7, 1011418–1429.

See Also

```
E3PL_sgvem_rot, E3PL_sgvem_lasso, exampleIndic_efa3pl_c1, exampleIndic_efa3pl_c2
```

Examples

```
## Not run:
with(E3PL_data_C1, E3PL_sgvem_adaptlasso(data, model,samp=50,forgetrate=0.51,mu_b=0,sigma2_b=4,Alpha=10,Bet
with(E3PL_data_C2, E3PL_sgvem_adaptlasso(data, model,samp=50,forgetrate=0.51,mu_b=0,sigma2_b=4,Alpha=10,Bet
## End(Not run)
```

E3PL_sgvem_lasso

Stochastic GVEM with Lasso Penalty for Exploratory M3PL Analysis

Description

Stochastic GVEM with Lasso Penalty for Exploratory M3PL Analysis

Usage

```
E3PL_sgvem_lasso(
    u,
    indic,
    samp = 50,
    forgetrate = 0.51,
    mu_b,
    sigma2_b,
    Alpha,
    Beta,
    max.iter = 5000,
    constrain = "C1",
    non_pen = NULL
)
```

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Arguments

constrain

u an $N \times J$ matrix or a data. frame that consists of binary responses of N individuals to J items. The missing values are coded as NA

indic a $J \times K$ matrix or a data. frame that describes the factor loading structure of

J items to K factors. It consists of binary values where 0 refers to the item is irrelevant with this factor, 1 otherwise. For exploratory factor analysis with lasso penalty, indic should be imposed certain constraints on the a $K \times K$ sub-matrix to ensure identifiability. The remaining parts do not assume any pre-specified zero structure but instead, the appropriate lasso penalty would recover the true

zero structure. Also see constrain

samp a subsample for each iteration; default is 50

forgetrate the forget rate for the stochastic algorithm. The value should be within the range

from 0.5 to 1. Default is 0.51

mu_b the mean parameter for the normal prior distribution of item difficulty parame-

ters

sigma2_b the variance parameter for the normal prior distribution of item difficulty param-

eters

Alpha the α parameter for the beta prior distribution of guessing parameters

Beta the β parameter for the beta prior distribution of guessing parameters

max.iter the maximum number of iterations for the EM cycle; default is 5000

the constraint setting: "C1" or "C2". To ensure identifiablity, "C1" sets a $K \times K$ sub-matrix of indic to be an identity matrix. This constraint anchor K factors by designating K items that load solely on each factor respectively. Note that the $K \times K$ matrix does not have to appear at the top of the indic matrix. "C2" sets the $K \times K$ sub-matrix to be a lower triangular matrix with the diagonal being ones. That is, there are test items associated with each factor for sure and they may be associated with other factors as well. Nonzero entries (in the lower triangular part) except for the diagonal entries of the sub-matrix are penalized during the estimation procedure. For instance, assume K=3, then the "C2"

constraint will imply the following submatrix: $C2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. As shown,

item 1 is allowed to only load on the first factor, item 2 will for sure load on the second factor but it may also load on the first factor (hence a penalty is added on the (2,1) element of "C2", i.e., $C2_{2,1}$). Item 3 will for sure load on the third factor but it may also load on the first two factors. However, note that for all remaining items their loading vector will all be (1,1,1) hence indistinguishable from the third anchor item. Therefore, we need to alert the algorithm that this third anchor item will for sure load on the third factor, and and whether or not it loads on the first two factors depends on the regularization results. Therefore, we need to specify "non_pen=" to identify the Kth anchor item. Although, "C2" is much weaker than "C1", it still ensures empirical identifiability. Default is "C1". During estimation, under both the "C1" and "C2" constraints, the population means and variances are constrained to be 0 and 1, respectively.

non_pen the index of an item which is associated with each factor to satisfy "C2". For C1, the input can be NULL

Value

a list containing the following objects:

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ra	item discrimination parameters, a $J\times K$ matrix
rb	item difficulty parameters, vector of length ${\cal J}$
rc	item guessing parameters, vector of length \boldsymbol{J}
rs	variational parameters s , a $N \times J$ matrix
reta	variational parameters $\eta(\xi)$, a $N \times J$ matrix
reps	variational parameters ξ , a $N \times J$ matrix
rsigma	population variance-covariance matrix, a $K \times K$ matrix
mu_i	mean parameter for each person, a $K \times N$ matrix
sig_i	covariance matrix for each person, a $K\times K\times N$ array
n	the number of iterations for the EM cycle
Q_mat	factor loading structure, a $J \times K$ matrix
GIC	model fit index
AIC	model fit index
BIC	model fit index
lbd	numerical value of lasso penalty parameter $\boldsymbol{\lambda}$

Author(s)

Jiaying Xiao <jxiao6@uw.edu>

References

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. *Psychometrika*. https://doi.org/10.1007/s11336-022-09874-6

See Also

```
E3PL_sgvem_rot, E3PL_sgvem_adaptlasso, exampleIndic_efa3pl_c1, exampleIndic_efa3pl_c2
```

Examples

```
## Not run:
with(E3PL_data_C1, E3PL_sgvem_lasso(data,model,samp=50,forgetrate=0.51,mu_b=0,sigma2_b=4,Alpha=10,Beta=40,n
with(E3PL_data_C2, E3PL_sgvem_lasso(data,model,samp=50,forgetrate=0.51,mu_b=0,sigma2_b=4,Alpha=10,Beta=40,n
## End(Not run)
```

E3PL_sgvem_rot Stochastic GVEM for Exploratory M3PL Analysis

Description

Stochastic GVEM for Exploratory M3PL Analysis

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Usage

```
E3PL_sgvem_rot(
    u,
    domain,
    samp = 50,
    forgetrate = 0.51,
    mu_b,
    sigma2_b,
    Alpha,
    Beta,
    max.iter = 5000,
    rot = "Promax"
)
```

Arguments

an $N \times J$ matrix or a data. frame that consists of binary responses of N indi-

viduals to J items. The missing values are coded as NA

domain the number of factors

samp a subsample for each iteration; default is 50

forgetrate the forget rate for the stochastic algorithm. The value should be within the range

from 0.5 to 1. Default is 0.51

mu_b the mean parameter for the prior distribution of item difficulty parameters sigma2_b the variance parameter for the prior distribution of item difficulty parameters

Alpha the α parameter for the prior distribution of guessing parameters

Beta the β parameter for the prior distribution of guessing parameters

max.iter the maximum number of iterations for the EM cycle; default is 5000

rot the post-hoc rotation method: Promax or CF-Quartimax; default is "Promax",

but may also be "cfQ" for conducting the CF-Quartimax rotation

Value

a list containing the following objects:

ra item discrimination parameters, a $J \times K$ matrix rb item difficulty parameters, vector of length J rc item guessing parameters, vector of length J rs variational parameters s, a $N \times J$ matrix reta variational parameters $\eta(\xi)$, a $N \times J$ matrix variational parameters ξ , a $N \times J$ matrix

rsigma population variance-covariance matrix, a $K \times K$ matrix mu_i mean parameter for each person, a $K \times N$ matrix sig_i covariance matrix for each person, a $K \times K \times N$ array

n the number of iterations for the EM cycle Q_mat factor loading structure, a $J \times K$ matrix

rk factor loadings, a $J \times K$ matrix

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GIC	model fit index
AIC	model fit index
BIC	model fit index

ur_a item discrimination parameters before conducting the rotation, a $J \times K$ matrix

Author(s)

Jiaying Xiao <jxiao6@uw.edu>

See Also

```
E3PL_sgvem_lasso, E3PL_sgvem_adaptlasso
```

Examples

```
## Not run:
E3PL_sgvem_rot(E3PL_data_C1$data, 3,samp=50,forgetrate=0.51,
mu_b=0,sigma2_b=4,Alpha=10,Beta=40,max.iter=5000,rot="Promax")
## End(Not run)
```

pa_poly

Parallel analysis using polychoric correlation

Description

Identify the number of factors

Usage

```
pa_poly(data, n.iter = 10, figure = TRUE)
```

Arguments

data a $N \times J$ matrix or a data. frame that consists of the responses of N individuals

to J items without any missing values. The responses are binary or polytomous.

n.iter Number of simulated analyses to perform

figure By default, pa_poly draws an eigenvalue plot. If FALSE, it suppresses the

graphic output

Value

 $\verb"pa_poly" returns" a \ \mathsf{data.frame} \ with the eigenvalues for the real \ \mathsf{data} \ \mathsf{and} \ \mathsf{the} \ \mathsf{simulated} \ \mathsf{data}.$

Author(s)

Jiaying Xiao <jxiao6@uw.edu>

Examples

```
## Not run:
pa_poly(C2PL_data$data, n.iter=20)
## End(Not run)
```

print.vemirt_FA

print.vemirt_DIF

Print D2PL Items by Group

Description

Print D2PL Items by Group

Usage

```
print(x, criterion = NULL, max = 99999L, digits = 3, ...)
```

Arguments

x An object of class vemirt_DIF

criterion Information criterion for model selection, one of 'AIC', 'BIC', 'GIC', or the

constant for computing GIC, otherwise use the criterion specified when fitting

the model(s)

Author(s)

Weicong Lyu <wlyu4@uw.edu>

See Also

```
D2PL_em, D2PL_gvem, D2PL_lrt, coef.vemirt_DIF, summary.vemirt_DIF
```

print.vemirt_FA

Print D2PL Items by Group

Description

Print D2PL Items by Group

Usage

print(x)

Arguments

Χ

An object of class vemirt_FA

Author(s)

Weicong Lyu <wlyu4@uw.edu>

See Also

```
\label{lem:c2PL_gvem_c2PL_gvem_lasso} C2PL\_iw, C3PL\_sgvem, E2PL\_gvem\_adaptlasso, E2PL\_gvem\_lasso, E2PL\_gvem\_rot, E2PL\_IS, E3PL\_sgvem\_adaptlasso, E3PL\_sgvem\_lasso, E3PL\_sgvem\_rot, coef.vemirt\_FA
```

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summary.vemirt_DIF Summarize D2PL Items

Description

Summarize D2PL Items

Usage

```
summary(x, criterion = NULL, max = 99999L, digits = 3, ...)
```

Arguments

x An object of class vemirt_DIF

criterion Information criterion for model selection, one of 'AIC', 'BIC', 'GIC', or the

constant for computing GIC, otherwise use the criterion specified when fitting

the model(s)

Author(s)

Weicong Lyu <wlyu4@uw.edu>

See Also

```
D2PL_em, D2PL_gvem, D2PL_lrt, coef.vemirt_DIF, print.vemirt_DIF
```

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