Package 'VEMIRT'

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Description VEMIRT is created to assist researchers in conducting exploratory and confirmatory multidimensional item response theory (MIRT) analysis and corresponding differential item functioning (DIF) analysis. The core computation engine of VEMIRT is a family of Gaussian Variational EM algorithms that are considerably more efficient than currently available algorithms in other software packages, especially when the number of latent factors exceeds four.
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VEMIRT-package

VEMIRT: A package for high-dimensional IRT models

Description

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VEMIRT is created to assist researchers to conduct exploratory and confirmatory multidimensional item response theory (MIRT) analysis and cooresponding item differential functioning (DIF) analysis. The core computation engine of VEMIRT is a family of Gaussian Variational EM algorithms that are considerably more efficient than currently available algorithms in other software packages, especially when the number of latent factors exceeds four.

Identifying the number of factors

pa_poly identifies the number of factors via parallel analysis.

Exploratory factor analysis

- gvem_2PLEFA_rot conducts M2PL Analysis with post-hoc rotation (Promax & CF-Quartimax)
- gvem_2PLEFA_lasso conducts M2PL Analysis with Lasso penalty
- gvem_2PLEFA_adaptlasso conducts M2PL Analysis with adaptive Lasso penalty
- sgvem_3PLEFA_rot conducts stochastic GVEM to futher imporve the computational effficiency for exploratory M3PL analysis
- sgvem_3PLEFA_lasso conducts M3PL Analysis with Lasso penalty
- sgvem_3PLEFA_adaptlasso conducts M3PL Analysis with adaptive Lasso penalty

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Confirmatory factor analysis

- gvem_2PLCFA conducts GVEM for confirmatory M2PL analysis
- sgvem_3PLCFA conducts stochastic GVEM for confirmatory M3PL analysis
- bs_2PLCFA conducts bootstrap sampling to correct bias and produce standard errors for confirmatory M2PL analysis
- importanceSampling conducts importance sampling to correct bias for M2PL analysis

Differential item functioning analysis

- em_DIF conducts DIF analysis for M2PL models using EM algorithms
- gvemm_DIF conducts DIF analysis for M2PL models using GVEMM algorithms
- lrt_DIF conducts DIF analysis for M2PL models using the likelihood ratio test

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bs 2PLCFA

Bootstrap Version of GVEM Confirmatory Analysis for M2PL

Description

A bootstrap version of GVEM (i.e., GVEM-BS) can be implemented to correct the bias on item parameters and compute standard errors under M2PL models

Usage

```
bs_2PLCFA(gvem_result, boots = 5)
```

Arguments

gvem_result a list that includes exploratory or confirmatory GVEM results for M2PL models. boots the number of bootstrap samples; default is 5

Value

a list containing the following objects:

boots_a	item discrimination parameters corrected by bootstrap sampling, a $J \times K$ matrix
boots_b	item difficulty parameters corrected by bootstrap sampling, a vector of length \boldsymbol{J}
sd_a	stardard errors of item discrimination parameters, a $J \times K$ matrix
sd_b	stardard errors of item difficulty parameters, a vector of length J

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See Also

```
gvem_2PLCFA,importanceSampling
```

Examples

```
## Not run:
gvem_result <- gvem_2PLCFA(exampleData_2pl, exampleIndic_cfa2pl)
bs_2PLCFA(gvem_result, boots=10)
## End(Not run)</pre>
```

coef.vemirt_DIF

Extract Parameter Estimates from DIF Analysis

Description

Extract Parameter Estimates from DIF Analysis

Usage

```
coef(object, criterion = NULL)
```

Arguments

object An object of class vemirt_DIF

criterion Information criterion for model selection, one of 'AIC', 'BIC' and 'GIC', oth-

erwise use the criterion specified when fitting the model(s)

See Also

```
em_DIF, gvemm_DIF, lrt_DIF, print.vemirt_DIF
```

em_DIF

EM Algorithms for DIF Detection in 2PL Models

Description

EM Algorithms for DIF Detection in 2PL Models

Usage

```
em_DIF(
    Y,
    D,
    X,
    method = "EMM",
    unif = F,
    Lambda0 = seq(0.2, 0.7, by = 0.1),
    criterion = "BIC",
    iter = 1000,
    eps = 0.001,
```

em_DIF 5

```
c = 0.7,
eta = 1
```

Arguments

Y An $N \times J$ binary matrix of item responses D A $J \times K$ binary matrix of loading indicators

X An N dimensional vector of group indicators (integers from 1 to G)

method Estimation algorithm, one of 'EM', 'EMM' and 'Adapt'

unif Whether to detect uniform DIF only

Lambda0 A vector of lambda0 values for L_1 penalty (lambda is sqrt(N) * lambda0) criterion Information criterion for model selection, one of 'BIC' (recommended), 'AIC'

and 'GIC'

iter Maximum number of iterations

eps Termination criterion on numerical accuracy

c Constant for computing GIC

eta Tuning constant for adaptive lasso ('Adapt' only)

Value

An object of class vemirt_DIF, which is a list containing three elements:

fit The best (with lowest information criterion) model, which is an element of all

best The location of fit in all

all A list of models whose length is equal to Lambda0:

...\$lambda0 Corresponding element in Lambda0

 \dots \$lambda sqrt(N) * lambda0 \dots \$iter Number(s) of iterations

...\$Sigma Group-level posterior covariance matrices

...\$Mu Group-level posterior mean vectors

...\$aSlopes for group 1...\$bIntercepts for group 1

...\$gamma DIF parameters for the slopes
...\$beta DIF parameters for the intercepts

...\$11 Log-likelihood

...\$10 Number of nonzero parameters in gamma and beta

...\$AIC Akaike Information Criterion...\$BIC Bayesian Information Criterion...\$GIC Generalized Information Criterion

See Also

```
gvemm_DIF, lrt_DIF, coef.vemirt_DIF, print.vemirt_DIF
```

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Examples

```
## Not run:
with(exampleDIF, em_DIF(Y, D, X))
## End(Not run)
```

exampleData_2pl

Response data set for M2PL

Description

The response data set is simulated based on a between-item M2PL model with 5 factors. The true factor correlations are set as 0.1.

Usage

```
exampleData_2pl
```

Format

A data frame with 2000 respondents and 75 items

exampleData_3pl

Response data set for M3PL

Description

The response data set is simulated based on a within-item M3PL model with 3 factors. The true factor correlations are set as 0.1.

Usage

```
exampleData_3pl
```

Format

A data frame with 2000 respondents and 45 items

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exampleDIF

Simulated Data Set for DIF Analysis

Description

Simulated Data Set for DIF Analysis

Usage

exampleDIF

Format

A list of components of the data set:

D Loading indicators

X Group indicators

j Number of DIF items (the first j items have DIF)

params A list of true parameters used for generating the item responses:

...\$a Slopes

...\$b Negated intercepts

...\$theta Latent traits

exampleIndic_cfa2pl

Factor-loading indicator matrix for M2PL-CFA

Description

The factor-loading indicator matrix can be used as an input for confirmatory factor analysis.

Usage

exampleIndic_cfa2pl

Format

A data frame with 75 items and 5 factors

exampleIndic_cfa3pl

Factor-loading indicator matrix for M3PL-CFA

Description

The factor-loading indicator matrix can be used as an input for confirmatory factor analysis.

Usage

```
exampleIndic_cfa3pl
```

Format

A data frame with 45 items and 3 factors

```
exampleIndic_efa2pl_c1
```

Factor-loading indicator matrix for M2PL-EFA with lasso/ adaptive penalty under constraint 1

Description

The factor-loading indicator matrix can be used as an input for exploratory factor analysis with lasso/ adaptive lasso penalty under constraint 1.

Usage

```
exampleIndic_efa2pl_c1
```

Format

A data frame with 75 items and 5 factors. Items 1, 16, 31, 46 and 61 can be combined as an identity matrix to satisfy constraint 1

```
exampleIndic_efa2pl_c2
```

Factor-loading indicator matrix for M2PL-EFA with lasso/ adaptive penalty under constraint 2

Description

The factor-loading indicator matrix can be used as an input for exploratory factor analysis with lasso/ adaptive lasso penalty for constraint 1.

Usage

```
exampleIndic_efa2pl_c2
```

Format

A data frame with 75 items and 5 factors. Items 1, 16, 31, 46 and 61 can be combined as a triangular matrix to satisfy constraint 2

exampleIndic_efa3pl_c1

Factor-loading indicator matrix for M3PL-EFA with lasso/ adaptive penalty under constraint 1

Description

The factor-loading indicator matrix can be used as an input for exploratory factor analysis with lasso/ adaptive lasso penalty under constraint 1.

Usage

```
exampleIndic_efa3pl_c1
```

Format

A data frame with 45 items and 3 factors. Items 1, 16, and 19 can be combined as an identity matrix to satisfy constraint 1

exampleIndic_efa3pl_c2

Factor-loading indicator matrix for M3PL-EFA with lasso/ adaptive penalty under constraint 2

Description

The factor-loading indicator matrix can be used as an input for exploratory factor analysis with lasso/ adaptive lasso penalty for constraint 1.

Usage

```
exampleIndic_efa3pl_c2
```

Format

A data frame with 45 items and 3 factors. Items 1, 16, and 19 can be combined as a triangular matrix to satisfy constraint 2

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exampleItem_2pl

True item parameters for M2PL

Description

True item parameters for M2PL

Usage

exampleItem_2pl

Format

An object of class data. frame with 75 rows and 6 columns.

exampleItem_3pl

True item parameters for M3PL

Description

True item parameters for M3PL

Usage

exampleItem_3pl

Format

An object of class data. frame with 45 rows and 5 columns.

gvemm_DIF

GVEMM Algorithms for DIF Detection in 2PL Models

Description

GVEMM Algorithms for DIF Detection in 2PL Models

gvemm_DIF

Usage

```
gvemm_DIF(
    Y,
    D,
    X,
    method = "IWGVEMM",
    Lambda0 = seq(0.2, 0.7, by = 0.1),
    criterion = "GIC",
    iter = 1000,
    eps = 0.001,
    c = 0.75,
    S = 10,
    M = 10,
    lr = 0.1
)
```

Arguments

Υ	An $N\times J$ binary matrix of item responses (missing responses should be coded as NA)
D	A $J \times K$ binary matrix of loading indicators
Χ	An N dimensional vector of group indicators (integers from 1 to G)
method	Estimation algorithm, one of 'GVEMM' or 'IWGVEMM'
Lambda0	A vector of lambda0 values for L_1 penalty (lambda is sqrt(N) * lambda0)
criterion	Information criterion for model selection, one of 'GIC' (recommended), 'BIC', or 'AIC' $$
iter	Maximum number of iterations
eps	Termination criterion on numerical accuracy
С	Constant for computing GIC
S	Sample size for approximating the expected lower bound ('IWGVEMM' only)
М	Sample size for approximating a tighter lower bound ('IWGVEMM' only)
lr	Learning rate for the Adam optimizer ('IWGVEMM' only)

Value

An object of class vemirt_DIF, which is a list containing three elements:

fit	The best (with lowest information criterion) model, which is an element of all
best	The location of fit in all
all	A list of models whose length is equal to Lambda0:
\$lambda0	Corresponding element in Lambda0
\$lambda	sqrt(N) * lambda0
\$niter	Number(s) of iterations
\$SIGMA	Person-level posterior covariance matrices
\$MU	Person-level posterior mean vectors
\$Sigma	Group-level posterior covariance matrices
\$Mu	Group-level posterior mean vectors

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```
Slopes for group 1
...$a
...$b
                 Intercepts for group 1
                 DIF parameters for the slopes
...$gamma
                 DIF parameters for the intercepts
...$beta
...$11
                 Log-likelihood
...$10
                 Number of nonzero parameters in gamma and beta
                 Number of respondents
...$N
...$AIC
                 Akaike Information Criterion
...$BIC
                 Bayesian Information Criterion
                 Generalized Information Criterion
...$GIC
```

See Also

```
em_DIF, lrt_DIF, coef.vemirt_DIF, print.vemirt_DIF
```

Examples

```
## Not run:
with(exampleDIF, gvemm_DIF(Y, D, X))
## End(Not run)
```

gvem_2PLCFA

Confirmatory M2PL Analysis

Description

Confirmatory M2PL Analysis

Usage

```
gvem_2PLCFA(u, indic, max.iter = 5000, SE.est = FALSE)
```

Arguments

u	a $N \times J$ matrix or a data. frame that consists of binary responses of N individuals to J items. The missing values are coded as NA
indic	a $J \times K$ matrix or a data.frame that describes the factor loading structure of J items to K factors. It consists of binary values where 0 refers to the item is irrelevant with this factor, 1 otherwise
max.iter	the maximum number of iterations for the EM cycle; default is 5000
SE.est	whether to estimate SE for item parameters using the updated supplemented expectation maximization (USEM); default is FALSE

Value

a list containing the following objects:

```
ra
                    item discrimination parameters, a J \times K matrix
                    item difficulty parameters, vector of length J
rb
reta
                    variational parameters \eta(\xi), a N \times J matrix
                    variational parameters \xi, a N \times J matrix
reps
                    population variance-covariance matrix, a K \times K matrix
rsigma
                    mean parameter for each person, a K \times N matrix
mu_i
sig_i
                    covariance matrix for each person, a K \times K \times N array
                    the number of iterations for the EM cycle
Q_mat
                    factor loading structure, a J \times K matrix
                    model fit index
GIC
                    model fit index
AIC
BIC
                    model fit index
SE
                    Standard errors of item parameters, a J \times (K+1) matrix where the last column
                    includes SE estimates for item difficulty parameters
```

See Also

```
sgvem_3PLCFA,importanceSampling,bs_2PLCFA
```

Examples

```
## Not run:
gvem_2PLCFA(exampleData_2pl, exampleIndic_cfa2pl)
## End(Not run)
```

```
gvem_2PLEFA_adaptlasso
```

Exploratory M2PL Analysis with Adaptive Lasso Penalty

Description

Exploratory M2PL Analysis with Adaptive Lasso Penalty

Usage

```
gvem_2PLEFA_adaptlasso(
    u,
    indic,
    max.iter = 5000,
    constrain = "C1",
    non_pen = NULL,
    gamma = 2
```

Arguments

u

a $N \times J$ matrix or a data.frame that consists of binary responses of N individuals to J items. The missing values are coded as NA

indic

a $J \times K$ matrix or a data.frame that describes the factor loading structure of J items to K factors. It consists of binary values where 0 refers to the item is irrelevant with this factor, 1 otherwise. For exploratory factor analysis with adaptive lasso penalty, indic should be imposed certain constraints on the a $K \times K$ sub-matrix to ensure identifiability. The remaining parts do not assume any pre-specified zero structure but instead, the appropriate lasso penalty would recover the true zero structure. Also see constrain

max.iter

the maximum number of iterations for the EM cycle; default is 5000

constrain

the constraint setting: "C1" or "C2". To ensure identifiablity, "C1" sets a $K \times K$ sub-matrix of indic to be an identity matrix. This constraint anchor K factors by designating K items that load solely on each factor respectively. Note that the $K \times K$ matrix does not have to appear at the top of the indic matrix. "C2" sets the $K \times K$ sub-matrix to be a lower triangular matrix with the diagonal being ones. That is, there are test items associated with each factor for sure and they may be associated with other factors as well. Nonzero entries (in the lower triangular part) except for the diagonal entries of the sub-matrix are penalized during the estimation procedure. For instance, assume K=3, then the "C2"

constraint will imply the following submatrix: $C2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. As shown,

item 1 is allowed to only load on the first factor, item 2 will for sure load on the second factor but it may also load on the first factor (hence a penalty is added on the (2,1) element of "C1", i.e., $C2_{2,1}$). Item 3 will for sure load on the third factor but it may also load on the first two factors. However, note that for all remaining items their loading vector will all be (1,1,1) hence indistinguishable from the third anchor item. Therefore, we need to alert the algorithm that this third anchor item will for sure load on the third factor, and and whether or not it loads on the first two factors depends on the regularization results. Therefore, we need to specify "non_pen=" to identify the Kth anchor item. Although, "C2" is much weaker than "C1", it still ensures empirical identifiability. Default is "C1". During estimation, under both the "C1" and "C1" constraints, the population means and variances are constrained to be 0 and 1, respectively.

non_pen

the index of an item which is associated with each factor to satisfy "C2". For C1, the input can be NULL

gamma

a numerical value of adaptive lasso parameter. Zou (2006) recommended three values, 0.5, 1, and 2. The default value is 2.

Value

a list containing the following objects:

ra item discrimination parameters, a $J \times K$ matrix rb item difficulty parameters, vector of length J reta variational parameters $\eta(\xi)$, a $N \times J$ matrix reps variational parameters ξ , a $N \times J$ matrix

rsigma population variance-covariance matrix, a $K \times K$ matrix mu_i mean parameter for each person, a $K \times N$ matrix

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sig_i	covariance matrix for each person, a $K \times K \times N$ array
n	the number of iterations for the EM cycle
Q_mat	factor loading structure, a $J \times K$ matrix
GIC	model fit index
AIC	model fit index
BIC	model fit index
1bd	numerical value of lasso penalty parameter λ

References

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. Psychometrika. https://doi.org/10.1007/s11336-022-09874-6

Zou, H. (2006). The adaptive LASSO and its oracle properties. Journal of the American Statistical Association, 7, 1011418-1429.

See Also

```
gvem_2PLEFA_rot, gvem_2PLEFA_lasso, exampleIndic_efa2pl_c1, exampleIndic_efa2pl_c2
```

Examples

```
## Not run:
gvem_2PLEFA_adaptlasso(exampleData_2pl, exampleIndic_efa2pl_c1,constrain="C1",non_pen=NULL,gamma=2)
gvem_2PLEFA_adaptlasso(exampleData_2pl, exampleIndic_efa2pl_c2,constrain="C2",non_pen=61,gamma=2)
## End(Not run)
```

gvem_2PLEFA_lasso

Exploratory M2PL Analysis with Lasso Penalty

Description

Exploratory M2PL Analysis with Lasso Penalty

Usage

```
gvem_2PLEFA_lasso(u, indic, max.iter = 5000, constrain = "C1", non_pen = NULL)
```

Arguments

indic

u	a $N \times J$ matrix or a data. frame that consists of binary responses of N indi-
	viduals to I itams. The missing values are goded as NA

viduals to J items. The missing values are coded as NA

a $J \times K$ matrix or a data. frame that describes the factor loading structure of J items to K factors. It consists of binary values where 0 refers to the item is irrelevant with this factor, 1 otherwise. For exploratory factor analysis with lasso penalty, indic should be imposed certain constraints on the a $K \times K$ sub-matrix to ensure identifiability. The remaining parts do not assume any pre-specified zero structure but instead, the appropriate lasso penalty would recover the true

zero structure. Also see constrain

the maximum number of iterations for the EM cycle; default is 5000 max.iter

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constrain

the constraint setting: "C1" or "C2". To ensure identifiablity, "C1" sets a $K \times K$ sub-matrix of indic to be an identity matrix. This constraint anchor K factors by designating K items that load solely on each factor respectively. Note that the $K \times K$ matrix does not have to appear at the top of the indic matrix. "C2" sets the $K \times K$ sub-matrix to be a lower triangular matrix with the diagonal being ones. That is, there are test items associated with each factor for sure and they may be associated with other factors as well. Nonzero entries (in the lower triangular part) except for the diagonal entries of the sub-matrix are penalized during the estimation procedure. For instance, assume K = 3, then the "C2"

constraint will imply the following submatrix: $C2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. As shown,

item 1 is allowed to only load on the first factor, item 2 will for sure load on the second factor but it may also load on the first factor (hence a penalty is added on the (2,1) element of "C1", i.e., $C2_{2,1}$). Item 3 will for sure load on the third factor but it may also load on the first two factors. However, note that for all remaining items their loading vector will all be (1,1,1) hence indistinguishable from the third anchor item. Therefore, we need to alert the algorithm that this third anchor item will for sure load on the third factor, and and whether or not it loads on the first two factors depends on the regularization results. Therefore, we need to specify "non_pen=" to identify the Kth anchor item. Although, "C2" is much weaker than "C1", it still ensures empirical identifiability. Default is "C1". During estimation, under both the "C1" and "C1" constraints, the population means and variances are constrained to be 0 and 1, respectively.

non_pen

the index of an item which is associated with each factor to satisfy "C2". For C1, the input can be NULL

Value

a list containing the following objects:

ra item discrimination parameters, a $J \times K$ matrix rb item difficulty parameters, vector of length J reta variational parameters $\eta(\xi)$, a $N \times J$ matrix reps variational parameters ξ , a $N \times J$ matrix

rsigma population variance-covariance matrix, a $K \times K$ matrix mu_i mean parameter for each person, a $K \times N$ matrix

sig_i covariance matrix for each person, a $K \times K \times N$ array

n the number of iterations for the EM cycle Q_mat factor loading structure, a $J \times K$ matrix

GIC model fit index
AIC model fit index
BIC model fit index

lbd numerical value of lasso penalty parameter λ

References

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. *Psychometrika*. https://doi.org/10.1007/s11336-022-09874-6

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See Also

```
gvem_2PLEFA_rot, gvem_2PLEFA_adaptlasso, exampleIndic_efa2pl_c1, exampleIndic_efa2pl_c2
```

Examples

```
## Not run:
gvem_2PLEFA_lasso(exampleData_2pl, exampleIndic_efa2pl_c1,constrain="C1")
gvem_2PLEFA_lasso(exampleData_2pl, exampleIndic_efa2pl_c2,constrain="C2",non_pen=61)
## End(Not run)
```

gvem_2PLEFA_rot

Exploratory M2PL Analysis with Post-hoc Rotation

Description

Exploratory M2PL Analysis with Post-hoc Rotation

Usage

```
gvem_2PLEFA_rot(u, domain, max.iter = 5000, rot = "Promax")
```

Arguments

u a $N \times J$ matrix or a data. frame that consists of binary responses of N indi-

viduals to J items. The missing values are coded as NA

domain the number of factors

max.iter the maximum number of iterations for the EM cycle; default is 5000

rot the post-hoc rotation method: Promax or CF-Quartimax; default is "Promax",

but may also be "cfQ" for conducting the CF-Quartimax rotation

Value

a list containing the following objects:

ra item discrimination parameters, a $J \times K$ matrix rb item difficulty parameters, vector of length J reta variational parameters $\eta(\xi)$, a $N \times J$ matrix reps variational parameters ξ , a $N \times J$ matrix

rsigma population variance-covariance matrix, a $K \times K$ matrix mu_i mean parameter for each person, a $K \times N$ matrix sig_i covariance matrix for each person, a $K \times K \times N$ array

n the number of iterations for the EM cycle

 ${\it rk} \hspace{1cm} {\it factor loadings, a} \hspace{1cm} J \times K \hspace{1cm} {\it matrix}$

Q_mat factor loading structure, a $J \times K$ matrix

GIC model fit index
AIC model fit index
BIC model fit index

ur_a item discrimination parameters before conducting the rotation, a $J \times K$ matrix

See Also

```
{\tt gvem\_2PLEFA\_1asso, gvem\_2PLEFA\_adaptlasso}
```

Examples

```
## Not run:
gvem_2PLEFA_rot(exampleData_2pl, domain=5,max.iter=3000)
gvem_2PLEFA_rot(exampleData_2pl, domain=5,rot="cfQ")
## End(Not run)
```

importanceSampling

Importance Weighted Version of GVEM Analysis for M2PL Models

Description

An importance weighted version of GVEM (i.e., IW-GVEM) can be implemented to correct the bias on item parameters under M2PL models

Usage

```
importanceSampling(u, gvem_result, S = 10, M = 10, max.iter = 10)
```

Arguments

u	a $N \times J$ matrix or a data.frame that consists of binary responses of N individuals to J items. The missing values are coded as NA
gvem_result	a list that includes exploratory or confirmatory GVEM results for M2PL models.
S	the number of times to draw samples; default is 10
М	the number of samples drawn from the variational distributions; default is 10
max.iter	the maximum number of iterations for the EM cycle; default is 10

Value

a list containing the following objects:

ra	item discrimination parameters estimated by GVEM, a $J \times K$ matrix
rb	item difficulty parameters estimated by GVEM, vector of length ${\cal J}$
reta	variational parameters $\eta(\xi)$, a $N \times J$ matrix
reps	variational parameters ξ , a $N \times J$ matrix
rsigma	population variance-covariance matrix estimated by GVEM, a $K \times K$ matrix
mu_i	mean parameter for each person, a $K \times N$ matrix
sig_i	covariance matrix for each person, a $K \times K \times N$ array
n	the number of iterations for the EM cycle
rk	factor loadings, a $J \times K$ matrix, for exploratory analysis only
Q_mat	factor loading structure, a $J \times K$ matrix
GIC	model fit index
AIC	model fit index

lrt_DIF

BIC	model fit index
SE	Standard errors of item parameters, a $J \times (K+1)$ matrix where the last column includes SE estimates for item difficulty parameters, for confirmatory analysis only
ur_a	item discrimination parameters before conducting the rotation, a $J\times K$ matrix, for exploratory analysis only
new_a	item discrimination parameters estimated by IW-GVEM, a $J \times K$ matrix
new_b	item difficulty parameters estimated by IW-GVEM, vector of length \boldsymbol{J}
new_Sigma_theta	
	population variance-covariance matrix estimated by IV-GVEM, a $K \times K$ matrix
best_lr	The learning rate used for importance sampling
best_lb	The lower bound value for importance sampling

See Also

```
gvem_2PLCFA, gvem_2PLEFA_rot,bs_2PLCFA
```

Examples

```
## Not run:
CFA_result <- gvem_2PLCFA(exampleData_2pl, exampleIndic_cfa2pl)
importanceSampling(exampleData_2pl,CFA_result)
## End(Not run)</pre>
```

lrt_DIF

Likelihood Ratio Test for DIF Detection in 2PL Models

Description

Likelihood Ratio Test for DIF Detection in 2PL Models

Usage

```
lrt_DIF(Y, D, X, unif = F)
```

Arguments

Υ	An $N \times J$ binary matrix of item responses
D	A $J \times K$ binary matrix of loading indicators
Χ	An N dimensional vector of group indicators (integers from 1 to $\ensuremath{\mathrm{G}})$
unif	Whether to detect uniform DIF only

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Value

A list:

Sigma Group-level posterior covariance matrices

Mu Group-level posterior mean vectors

a Slopes for group 1b Intercepts for group 1

gamma DIF parameters for the slopes beta DIF parameters for the intercepts

See Also

```
em_DIF, gvemm_DIF, coef.vemirt_DIF, print.vemirt_DIF
```

Examples

```
## Not run:
with(exampleDIF, lrt_DIF(Y, D, X))
## End(Not run)
```

pa_poly

Parallel analysis using polychoric correlation

Description

Identify the number of factors

Usage

```
pa_poly(data, n.iter = 10, figure = TRUE)
```

Arguments

data a $N \times J$ matrix or a data. frame that consists of the responses of N individuals

to J items without any missing values. The responses are binary or polytomous.

n.iter Number of simulated analyses to perform

figure By default, pa_poly draws an eigenvalue plot. If FALSE, it suppresses the

graphic output

Value

pa_poly returns a data.frame with the eigenvalues for the real data and the simulated data.

Examples

```
## Not run:
pa_poly(exampleData_2pl, n.iter=20)
## End(Not run)
```

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print.vemirt_DIF

Print DIF Items

Description

Print DIF Items

Usage

```
print(x, criterion = NULL)
```

Arguments

x An object of class vemirt_DIF

criterion

Information criterion for model selection, one of 'AIC', 'BIC' and 'GIC', oth-

erwise use the criterion specified when fitting the model(s)

See Also

```
em_DIF, gvemm_DIF, lrt_DIF, coef.vemirt_DIF
```

sgvem_3PLCFA

Stochastic GVEM for Confirmatory M3PL Analysis

Description

Stochastic GVEM for Confirmatory M3PL Analysis

Usage

```
sgvem_3PLCFA(
    u,
    indic,
    samp = 50,
    forgetrate = 0.51,
    mu_b,
    sigma2_b,
    Alpha,
    Beta,
    max.iter = 5000
)
```

Arguments

u a $N \times J$ matrix or a data. frame that consists of binary responses of N indi-

viduals to J items. The missing values are coded as NA

indic a $J \times K$ matrix or a data. frame that describes the factor loading structure of J items to K factors. It consists of binary values where 0 refers to the item is

irrelevant with this factor, 1 otherwise

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samp a subsample for each iteration; default is 50

forgetrate the forget rate for the stochastic algorithm. The value should be within the range

from 0.5 to 1. Default is 0.51

mu_b the mean parameter for the prior distribution of item difficulty parameters sigma2_b the variance parameter for the prior distribution of item difficulty parameters

Alpha the α parameter for the prior distribution of guessing parameters

Beta the β parameter for the prior distribution of guessing parameters

max.iter the maximum number of iterations for the EM cycle; default is 5000

Value

a list containing the following objects:

ra item discrimination parameters, a $J \times K$ matrix rb item difficulty parameters, vector of length J rc item guessing parameters, vector of length J rs variational parameters s, a $N \times J$ matrix reta variational parameters $\eta(\xi)$, a $N \times J$ matrix variational parameters ξ , a $N \times J$ matrix

rsigma population variance-covariance matrix, a $K \times K$ matrix mu_i mean parameter for each person, a $K \times N$ matrix sig_i covariance matrix for each person, a $K \times K \times N$ array

n the number of iterations for the EM cycle ${\it Q_mat} \qquad \qquad {\it factor loading structure, a } J \times K \; {\it matrix}$

GIC model fit index
AIC model fit index
BIC model fit index

References

Cho, A. E., Wang, C., Zhang, X., & Xu, G. (2021). Gaussian variational estimation for multidimensional item response theory. *British Journal of Mathematical and Statistical Psychology*, 74, 52-85.

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. *Psychometrika*. https://doi.org/10.1007/s11336-022-09874-6

See Also

```
gvem_2PLCFA
```

Examples

```
## Not run:
sgvem_3PLCFA(exampleData_3pl, exampleIndic_cfa3pl,samp=50,forgetrate=0.51,
mu_b=0,sigma2_b=4,Alpha=10,Beta=40)
## End(Not run)
```

```
sgvem_3PLEFA_adaptlasso
```

Stochastic GVEM with Adaptive Lasso Penalty for Exploratory M3PL Analysis

Description

Stochastic GVEM with Adaptive Lasso Penalty for Exploratory M3PL Analysis

Usage

```
sgvem_3PLEFA_adaptlasso(
    u,
    indic,
    samp = 50,
    forgetrate = 0.51,
    mu_b,
    sigma2_b,
    Alpha,
    Beta,
    max.iter = 5000,
    constrain = "C1",
    non_pen = NULL,
    gamma = 2
)
```

Arguments

viduals to J items. The missing values are coded as NA

indic a $J \times K$ matrix or a data. frame that describes the factor loading structure of

J items to K factors. It consists of binary values where 0 refers to the item is irrelevant with this factor, 1 otherwise. For exploratory factor analysis with lasso penalty, indic should be imposed certain constraints on the a $K \times K$ sub-matrix to ensure identifiability. The remaining parts do not assume any pre-specified zero structure but instead, the appropriate lasso penalty would recover the true

zero structure. Also see constrain

samp a subsample for each iteration; default is 50

forgetrate the forget rate for the stochastic algorithm. The value should be within the range

from 0.5 to 1. Default is 0.51

mu_b the mean parameter for the prior distribution of item difficulty parameters

sigma2_b the variance parameter for the prior distribution of item difficulty parameters

Alpha the α parameter for the prior distribution of guessing parameters

Beta the β parameter for the prior distribution of guessing parameters

max.iter the maximum number of iterations for the EM cycle; default is 5000

constrain the constraint setting: "C1" or "C2". To ensure identifiablity, "C1" sets a $K \times K$

sub-matrix of indic to be an identity matrix. This constraint anchor K factors by designating K items that load solely on each factor respectively. Note that

the $K \times K$ matrix does not have to appear at the top of the indic matrix. "C2" sets the $K \times K$ sub-matrix to be a lower triangular matrix with the diagonal being ones. That is, there are test items associated with each factor for sure and they may be associated with other factors as well. Nonzero entries (in the lower triangular part) except for the diagonal entries of the sub-matrix are penalized during the estimation procedure. For instance, assume K = 3, then the "C2"

constraint will imply the following submatrix: $C2 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$. As shown,

item 1 is allowed to only load on the first factor, item 2 will for sure load on the second factor but it may also load on the first factor (hence a penalty is added on the (2,1) element of "C1", i.e., $C2_{2,1}$). Item 3 will for sure load on the third factor but it may also load on the first two factors. However, note that for all remaining items their loading vector will all be (1,1,1) hence indistinguishable from the third anchor item. Therefore, we need to alert the algorithm that this third anchor item will for sure load on the third factor, and and whether or not it loads on the first two factors depends on the regularization results. Therefore, we need to specify "non_pen=" to identify the Kth anchor item. Although, "C2" is much weaker than "C1", it still ensures empirical identifiability. Default is "C1". During estimation, under both the "C1" and "C1" constraints, the population means and variances are constrained to be 0 and 1, respectively.

non_pen the index of an item which is associated with each factor to satisfy "C2". For

C1, the input can be NULL

gamma a numerical value of adaptive lasso parameter. Zou (2006) recommended three

values, 0.5, 1, and 2. The default value is 2.

Value

a list containing the following objects:

ra item discrimination parameters, a $J \times K$ matrix rb item difficulty parameters, vector of length J rc item guessing parameters, vector of length J rs variational parameters s, a $N \times J$ matrix reta variational parameters $\eta(\xi)$, a $N \times J$ matrix reps variational parameters ξ , a $N \times J$ matrix

rsigma population variance-covariance matrix, a $K \times K$ matrix mu_i mean parameter for each person, a $K \times N$ matrix

sig_i covariance matrix for each person, a $K \times K \times N$ array

n the number of iterations for the EM cycle Q_mat factor loading structure, a $J \times K$ matrix

GIC model fit index
AIC model fit index
BIC model fit index

lbd numerical value of lasso penalty parameter λ

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References

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. *Psychometrika*. https://doi.org/10.1007/s11336-022-09874-6

Zou, H. (2006). The adaptive LASSO and its oracle properties. *Journal of the American Statistical Association*, 7, 1011418–1429.

See Also

```
sgvem_3PLEFA_rot, sgvem_3PLEFA_lasso, exampleIndic_efa3pl_c1, exampleIndic_efa3pl_c2
```

Examples

```
## Not run:
sgvem_3PLEFA_adaptlasso(exampleData_3pl, exampleIndic_efa3pl_c1,samp=50,
forgetrate=0.51,mu_b=0,sigma2_b=4,Alpha=10,Beta=40,max.iter=5000,
constrain="C1",non_pen=NULL,gamma=2)
sgvem_3PLEFA_adaptlasso(exampleData_3pl, exampleIndic_efa3pl_c2,samp=50,
forgetrate=0.51,mu_b=0,sigma2_b=4,Alpha=10,Beta=40,max.iter=5000,
constrain="C2",non_pen=19,gamma=2)
## End(Not run)
```

sgvem_3PLEFA_lasso

Stochastic GVEM with Lasso Penalty for Exploratory M3PL Analysis

Description

Stochastic GVEM with Lasso Penalty for Exploratory M3PL Analysis

Usage

```
sgvem_3PLEFA_lasso(
    u,
    indic,
    samp = 50,
    forgetrate = 0.51,
    mu_b,
    sigma2_b,
    Alpha,
    Beta,
    max.iter = 5000,
    constrain = "C1",
    non_pen = NULL
)
```

Arguments

u a $N \times J$ matrix or a data. frame that consists of binary responses of N individuals to J items. The missing values are coded as NA

indic

constrain

a $J \times K$ matrix or a data. frame that describes the factor loading structure of J items to K factors. It consists of binary values where 0 refers to the item is irrelevant with this factor, 1 otherwise. For exploratory factor analysis with lasso penalty, indic should be imposed certain constraints on the a $K \times K$ sub-matrix to ensure identifiability. The remaining parts do not assume any pre-specified zero structure but instead, the appropriate lasso penalty would recover the true zero structure. Also see constrain

a subsample for each iteration; default is 50 samp

forgetrate the forget rate for the stochastic algorithm. The value should be within the range

from 0.5 to 1. Default is 0.51

mu_b the mean parameter for the prior distribution of item difficulty parameters the variance parameter for the prior distribution of item difficulty parameters sigma2_b

Alpha the α parameter for the prior distribution of guessing parameters the β parameter for the prior distribution of guessing parameters Beta the maximum number of iterations for the EM cycle; default is 5000 max.iter

the constraint setting: "C1" or "C2". To ensure identifiablity, "C1" sets a $K \times K$ sub-matrix of indic to be an identity matrix. This constraint anchor K factors by designating K items that load solely on each factor respectively. Note that the $K \times K$ matrix does not have to appear at the top of the indic matrix. "C2" sets the $K \times K$ sub-matrix to be a lower triangular matrix with the diagonal being ones. That is, there are test items associated with each factor for sure and they may be associated with other factors as well. Nonzero entries (in the lower triangular part) except for the diagonal entries of the sub-matrix are penalized during the estimation procedure. For instance, assume K=3, then the "C2"

 $\begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$ constraint will imply the following submatrix: C2 =1 1 0 . As shown, 1 1 1

item 1 is allowed to only load on the first factor, item 2 will for sure load on the second factor but it may also load on the first factor (hence a penalty is added on the (2,1) element of "C1", i.e., $C2_{2,1}$). Item 3 will for sure load on the third factor but it may also load on the first two factors. However, note that for all remaining items their loading vector will all be (1, 1, 1) hence indistinguishable from the third anchor item. Therefore, we need to alert the algorithm that this third anchor item will for sure load on the third factor, and and whether or not it loads on the first two factors depends on the regularization results. Therefore, we need to specify "non_pen=" to identify the Kth anchor item. Although, "C2" is much weaker than "C1", it still ensures empirical identifiability. Default is "C1". During estimation, under both the "C1" and "C1" constraints, the population means and variances are constrained to be 0 and 1, respectively.

the index of an item which is associated with each factor to satisfy "C2". For non_pen C1, the input can be NULL

Value

a list containing the following objects:

item discrimination parameters, a $J \times K$ matrix ra item difficulty parameters, vector of length Jrb item guessing parameters, vector of length Jrc variational parameters s, a $N \times J$ matrix rs

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reta	variational parameters $\eta(\xi)$, a $N \times J$ matrix
reps	variational parameters ξ , a $N \times J$ matrix
rsigma	population variance-covariance matrix, a $K \times K$ matrix
mu_i	mean parameter for each person, a $K \times N$ matrix
sig_i	covariance matrix for each person, a $K\times K\times N$ array
n	the number of iterations for the EM cycle
Q_mat	factor loading structure, a $J \times K$ matrix
GIC	model fit index
AIC	model fit index
BIC	model fit index
1bd	numerical value of lasso penalty parameter λ

References

Cho, A. E., Xiao, J., Wang, C., & Xu, G. (2022). Regularized Variational Estimation for Exploratory Item Factor Analysis. *Psychometrika*. https://doi.org/10.1007/s11336-022-09874-6

See Also

```
sgvem_3PLEFA_rot, sgvem_3PLEFA_adaptlasso, exampleIndic_efa3pl_c1, exampleIndic_efa3pl_c2
```

Examples

```
## Not run:
sgvem_3PLEFA_lasso(exampleData_3pl, exampleIndic_efa3pl_c1,samp=50,
forgetrate=0.51,mu_b=0,sigma2_b=4,Alpha=10,Beta=40,max.iter=5000,
constrain="C1",non_pen=NULL)
sgvem_3PLEFA_lasso(exampleData_3pl, exampleIndic_efa3pl_c2,samp=50,
forgetrate=0.51,mu_b=0,sigma2_b=4,Alpha=10,Beta=40,max.iter=5000,
constrain="C2",non_pen=19)
## End(Not run)
```

sgvem_3PLEFA_rot

Stochastic GVEM for Exploratory M3PL Analysis

Description

Stochastic GVEM for Exploratory M3PL Analysis

Usage

```
sgvem_3PLEFA_rot(
    u,
    domain,
    samp = 50,
    forgetrate = 0.51,
    mu_b,
    sigma2_b,
    Alpha,
```

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```
Beta,
  max.iter = 5000,
  rot = "Promax"
)
```

Arguments

u a $N \times J$ matrix or a data. frame that consists of binary responses of N indi-

viduals to J items. The missing values are coded as NA

domain the number of factors

samp a subsample for each iteration; default is 50

forgetrate the forget rate for the stochastic algorithm. The value should be within the range

from 0.5 to 1. Default is 0.51

mu_b the mean parameter for the prior distribution of item difficulty parameters sigma2_b the variance parameter for the prior distribution of item difficulty parameters

Alpha the α parameter for the prior distribution of guessing parameters

Beta the β parameter for the prior distribution of guessing parameters

max.iter the maximum number of iterations for the EM cycle; default is 5000

rot the post-hoc rotation method: Promax or CF-Quartimax; default is "Promax",

but may also be "cfQ" for conducting the CF-Quartimax rotation

Value

a list containing the following objects:

ra item discrimination parameters, a $J \times K$ matrix rb item difficulty parameters, vector of length J rc item guessing parameters, vector of length J rs variational parameters s, a $N \times J$ matrix reta variational parameters $\eta(\xi)$, a $N \times J$ matrix variational parameters ξ , a $N \times J$ matrix

rsigma population variance-covariance matrix, a $K \times K$ matrix mu_i mean parameter for each person, a $K \times N$ matrix sig_i covariance matrix for each person, a $K \times K \times N$ array

n the number of iterations for the EM cycle Q_mat factor loading structure, a $J \times K$ matrix

rk factor loadings, a $J \times K$ matrix

GIC model fit index
AIC model fit index
BIC model fit index

ur_a item discrimination parameters before conducting the rotation, a $J \times K$ matrix

See Also

sgvem_3PLEFA_rot 29

Examples

```
## Not run:
sgvem_3PLEFA_rot(exampleData_3pl, 3,samp=50,forgetrate=0.51,
mu_b=0,sigma2_b=4,Alpha=10,Beta=40,max.iter=5000,rot="Promax")
## End(Not run)
```

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