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Definition and Measurement of Crimp of Textile Fibers

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Abstract

Two fundamental concepts are defined, the effective crimp diameter and the effective wave number. The corresponding figures for these concepts are calculated from the measurements of the width values of the plane projection of a rotating fiber and from the measurements of the crimp ratio. These figures are obtained for a fiber under a certain load, and take into account the spatial nature and the statistical aspect of crimp. An effective curvature is determined as well. Results are obtained for 90's Merino and 52's Scotch-Blackface wool fibers. It is shown that the higher quality wool fibers are associated with lower effective crimp diameters and higher effective wave numbers.

Introduction

The natural crimp of the wool fiber is one of the main reasons for wool's pre-eminence in high quality fabrics, and artificially crimped fibers are made with the aim of matching the natural crimp of wool.

Although crimp seems to be one of the important properties characterizing the quality of wool and other fibers, no valid definition of crimp seems to have been worked out, and no method for measuring this property seems to have been established.

Several investigations are reported in the literature, in which attempts have been made to define and measure crimp. Methods of measuring crimp in wool fiber and fleece have recently been surveyed [2]. Owing to the complexity of the shape of the crimped fibers, investigators are by no means agreed as to what properties should be used to define the crimp characteristics.

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Duerden in 1929 [3] and Norris [11] were concerned only with measuring the crimp in the stapled wool. They defined crimp as the number of waves per unit length appearing on the staple. Swart and Kotze [14] showed, by using the same method of measurement, that the wave length thus measured on a staple is not constant along the direction of the fibers forming the staple. In these methods for the measurement of crimp, no attempt was made to measure directly the amplitude or depth of the waves in the staple. Their approach was not concerned with the crimp of the individual fibers and dealt mainly with a property of a bulk of fibers in the original staple. However, in this state the fibers are under considerable strains (see Woods [16], Goldsworthy and Lang [6]); therefore the above method cannot present reliable information and may not give the necessary data on the fiber crimp as related to the evaluation of the wool quality. This method of measuring the number of waves per staple length seems to be also of a very limited scope, since it is applicable only to wool with well-defined staple waves, e.g., Merino, Corriedale.

O'Connell and Yeiser [12] were concerned with the effect of crimp on mechanical properties of wet wool, but did not define this property; they only "estimated the amount of crimp in individual fibers" by "rough counts of crimps and ranked fibers in their order of relative crimpiness," differentiating only qualitatively between "low, medium, and high crimp."

The fibers in the staple are subject to deformation owing to their closeness of packing. Therefore when a single fiber is withdrawn from the staple, it departs from its form in the staple [16]. Goldsworthy and Lang [6] have studied the relationship of fiber form and staple crimp in Merino wool by qualitative observations of the behavior of strands formed from individual fibers. Woods [16] and others [2] dealt with the crimp in the individual fibers after they were separated one from another. Woods measured the radius of curvature and the tortuosity at individual points along a single fiber. Since he was concerned only with the relation between crimp and the natural twist of the fiber, he did not attempt to define figures for characterizing the crimp of the fiber as a whole.

Wildman [15] seems to have suggested a method for determination of crimp, based on microscopic measurements of the mean crimp depth, details of which have not been published.

Rainard and Abbot [13] define crimp in terms of what they call "single-fiber bulking capacity." In this method the fiber was stretched inside a slot of a given width, released, and flattened into a plane. A grid of lines parallel to the slot was placed on top of the slot, and the number of intersections of the fiber with the grid lines was counted. This number was used in the definition of bulking capacity. From the description of this method it is evident that during measurements uncontrolled and unknown strains of considerable magnitude must have been present.

Another method of measurement of crimp on a single fiber was suggested by Zart [17] and subsequently simplified by Ecochard [4]. In this method the number of waves per unit length, and the crimp ratio, defined as a percentage shortening of the fiber due to crimp, were measured on a flattened fiber. The crimp ratio was also used by Human and Speakman [8] who, in their short communication dealing with the crimping and heterogeneity of keratin fibers, compared the reduction in crimp after different oxidation treatments.

The main drawbacks of the method of Ecochard [4] have already been pointed out by Millard [10] who stated that the fiber cannot be assumed to be crimped in one distinct plane, and that no uniform shape of crimp exists. He also points out that this method of determination of crimp in textile fibers cannot even be applied to man-made fibers, which are sometimes manufactured with a uniplanar crimp or even a sinusoidal crimp, but which do not remain in this form in later technical processes. Further, it would appear that in view of the marked changes in crimp caused at each stage of handling of the fiber, such handling as suggested by Ecochard cannot assure uniform treatment of each fiber either.

The use of the properties of the force-extension curve of the fibers at low loads and low rates of extension in relation to crimp, as given by Evans and Montgomery [5], is a more promising development. They have defined the following parameters to characterize the crimp.

1. The energy to uncrimp, or the integral of the "crimping force" over the region of uncrimping.
2. The "stiffness of crimp," or the force necessary to "uncrimp" the fiber.

3. The "contraction due to crimp," which is based on a more exact measurement of what has been called the "crimp ratio" by previous authors [4, 8, 17].

These parameters were successfully used by the authors for studying changes in sets of fibers. This method seems to be more accurate than the former ones. But in addition to certain dynamic features it deals with only one aspect of the geometry of the fiber, without paying enough attention to the actual shape of the fiber. Unfortunately no further details of this method have been published.

Zilahi and Dobozy [18] investigated the role of heterogeneous swelling in the formation and origin of certain wool properties, with a view of determining some relation between the internal fiber characteristics governing the shape of the crimp and the swelling potential of the fiber. The authors based a definition of crimp on the following characteristics: the number of waves per axial unit length, the length of an individual wave, and the length of straight line portions and radii of curvature. They conclude that further research is necessary in order to answer many questions connected with crimp.

Recently some papers have been published in which a correlation between the crimp and the chemical and morphological structure of the wool fiber cortex is described [7, 9]. These investigations dealing with single fibers are mainly concerned with the causes of crimp and not with the measurement or the definition of the crimp for a fiber as a whole.

The present paper deals with the geometrical shape of the separate fibers.

Although the definitions and methods of measurements described in the literature differ widely, it is obvious that the concept of crimp was used as a synonym for the general geometrical shape of non-straight fibers. In most fibers this geometrical shape is of a complex statistical nature.

At least two fundamental parameters are necessary for the characterization of crimp, one giving a measure of the amplitude, which is generally called in the literature the "depth" of the crimp; and a second factor which describes the number of crimps per unit length. In the present paper it is proposed to characterize these two factors by defining the effective crimp diameter (E.C.D.) and the effective wave number (E.W.N.). Both figures are defined in such a way as to account for the three-dimensional nature of crimp and for its statistical aspect. The E.C.D. covers the "depth of the crimp" together with what has been called "the bulking capacity" [13]. The E.W.N. measures the number of waves per unit length of the fiber, and may also be related to the number of waves on the staple, as measured by Duerden [3] and Norris [11]. It will be shown (see Appendix II) that an effective radius of curvature can be calculated by using the two parameters given in the present paper; this effective radius of curvature may be related to an average of the radii of curvature as measured by Woods [16] at individual points along single fibers. It will be seen from the methods of calculation that the crimp ratio as used by Zart [17], Ecochard [4], and Human and Speakman [8] is incorporated in the two parameters defined in the present paper.

The measuring methods and instrumentation described in this paper take into account that it is impossible to carry out consistent measurements in the complete absence of strain. All measurements were therefore carried out under well-defined tensions. These tensions were systematically varied, and the changes obtained at different tensions seem to be related to the changes observed by Evans and Montgomery [5].

The Effective Crimp Diameter

In a previous paper [1], measurements of the "effective crimp diameter" have been described. The measurements were performed on vertically suspended wool fibers which were loaded with small weights w, ranging from 50 mg. up to the weight necessary to "uncrimp" the fiber. For each load the horizontal coordinates y_i of a projection of the fiber on a vertical plane were measured at constant vertical intervals with the aid of a vertically moving comparator fitted with an ocular-micrometer. From these the effective crimp diameters were calculated in a way similar to the effective voltage of an alternating electric current. In the equations and tables, values of E.C.D. are represented by D_{eff}.

$$D_{eff} = 2 \cdot \sqrt{\left(\sum_{i=1}^{N} y_i^2 \right) \Big/ N} \qquad (1)$$

where N is the number of measurements.

This method seems to be incomplete for the following reason. For the same load w, different values of D_{eff} are obtained when carrying out the measurements at different planes of projection of the fiber.

It is seen from the few measurements listed in Table I that the values of D_{eff} may increase by up to 60% on passing from one projection plane to another. This is high as compared with the accuracy of the measurements.

TABLE I. Change of Effective-Crimp-Diameter (D_{eff}) with Change of Plane of Projection of the Fibers. (Fibers under Load of 0.2 g.)

Fiber number	D_{eff} (mm.) First plane	Second plane	Third plane	Mean	Standard deviation	Coefficient of variation
1	0.32	0.29	0.20	0.27	0.05	18.5
2	0.41	0.30	0.44	0.36	0.06	16.6
3	0.22	0.25	0.23	0.23	0.01	4.3
4	0.25	0.24	0.26	0.25	0.01	4.0
5	0.27	0.26	0.25	0.26	0.01	3.8
6	0.29	0.26	0.27	0.27	0.01	3.7

Although these differences do not invalidate the method described previously [1] as a simple method of measurement of E.C.D., it was felt that a more accurate approach is desirable. Among the fibers studied by the present authors no fiber exhibiting a well-defined crimp plane was encountered; no such uniplanarity was found on fibers withdrawn from the staple, even in fine Merino wool fibers which exhibit staple crimp of straight waves, suggesting uniplanar crimp of the individual fibers. It was realized, nevertheless, that a method of measurement of the E.C.D. should be independent of the existence or nonexistence of preferred crimp planes.

In order to take into account the three-dimensional distribution of the crimp of the fiber it is proposed to redefine the E.C.P. in the following way:

$$D_{eff} = 2 \cdot \sqrt{\left(\int r^2 \cdot dl \right) \Big/ l_w} \qquad (2)$$

where r is the distance between each point of the fiber and the straight line connecting the endpoints of the section of the fiber under study; l_w is the distance between the endpoints of the fiber section; the integral is to be extended over the whole of l_w.

In practice the measurements are carried out only at a finite number of points N and the formula used is therefore

$$D_{eff} = 2 \cdot \sqrt{\left(\sum_{i=1}^{N} r_i^2 \right) \Big/ N} \qquad (3)$$

It can be readily seen that this definition of E.C.D. is identical with that given previously (Equation 1), provided the crimp of the fiber is uniplanar and the measurements are carried out on a projection plane parallel to this particular plane. In all other cases this new definition gives a higher value of D_{eff} than the previous one, since

$$y_i \leq r_i$$

The measurements of r_i are carried out by rotating the fiber around an axis passing through its end points. The fiber then describes a surface of revolution, and on its projection on a plane parallel to its axis (see Figure 1), the values of $2r_i$ can readily be measured.

The value of D_{eff} obtained in this way for a fiber under a certain load presents an unequivocal measure of a fundamental property of the fiber.

Experimental

The measuring apparatus consisted of a special fastening and rotating device and a projection sys-

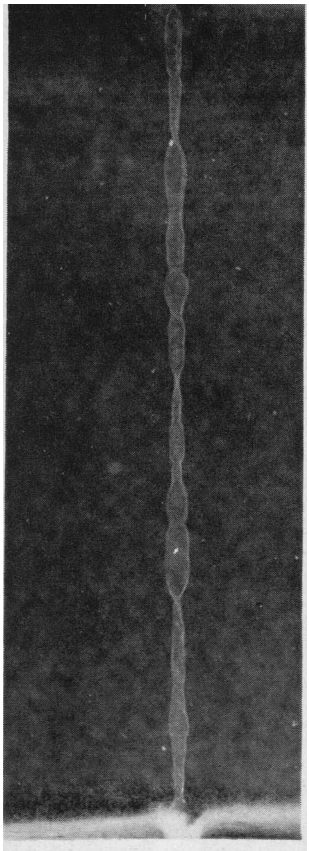

Fig. 1. A 52's fiber suspended in a frame and rotated 6 t.p.s. through an axis passing through its endpoints.

tem. The fastening and rotating device is shown in Figure 2. It consists of a frame A fitted with two bearings, B_1 and B_2. Each bearing carries an adjustable jaw, C_1 and C_2. With the help of the axle D and two identical pairs of cogwheels, E_1 and E_2, E_3 and E_4, the two jaws C_1 and C_2 are rotated simultaneously by a suitable motor.

The projection system (see Figure 3) contains a lamp G which illuminates the fiber H in its fastening frame, and a lens I arranged so as to give an enlargement of the rotating fiber by a factor of approximately six. The image of the rotating fiber is formed on a millimetric square scale J of 20×20 cm. on an opaque glass plate.

For the measurements an S-shaped hook weighing 10 mg. was tied to one end of the fiber. The other end of the fiber was fixed on the upper jaw (C_1 in Figure 2), while the end carrying the hook hung freely through the opened lower jaw (C_2 in Figure 2). After applying the appropriate load to the hook,

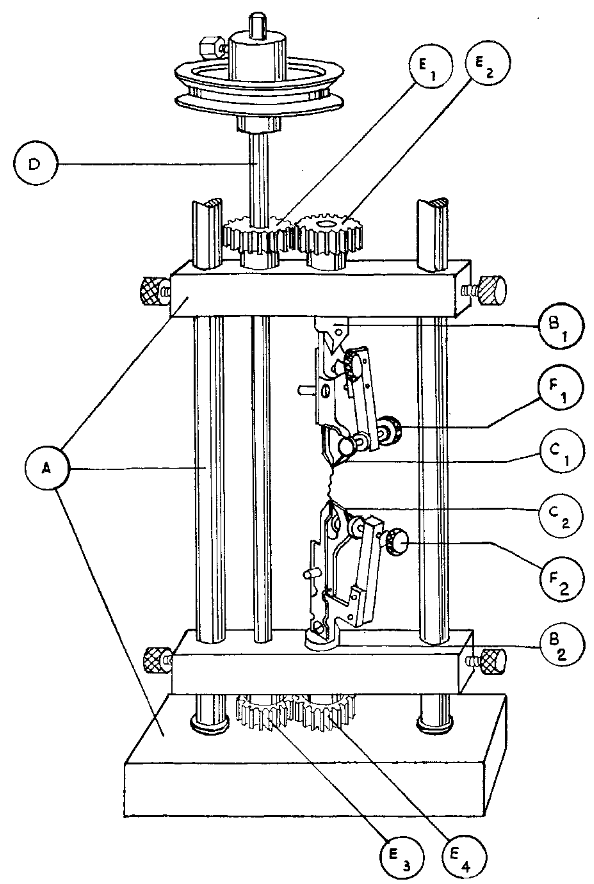

Fig. 2. The fastening and rotating frame.

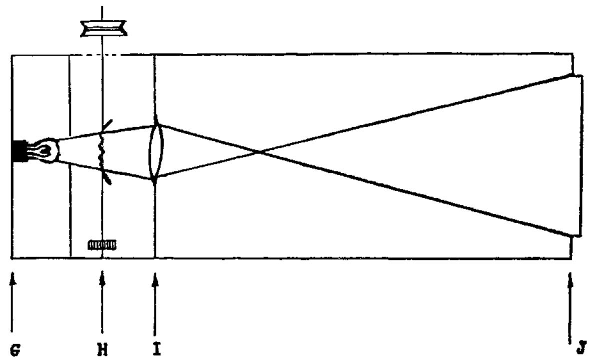

Fig. 3. The projection system.

the lower jaw C_2 could be closed without disturbing the equilibrium of the fiber. Before rotation, the end points of the fiber were brought into coincidence with the axis of rotation by slightly adjusting the jaws with the help of two screws (F_1 and F_2 in Figure 2). This adjustment was necessary as it proved very difficult to clamp the fiber at the exact center of the jaws.

The fiber is then rotated and measurements of $2r_i$ taken along the section of the axis of rotation between two centered points, at constant intervals of about 1 mm.

Several series of measurements of the E.C.D. of the same fiber were performed at various rates of revolution, up to 6 t.p.s., and it was found that no significant deformation of the image occurred (see Figure 1), i.e., the same values were obtained in all cases. The actual measurements were performed at 1 to 2 t.p.s., since it was found that these rates were the most convenient ones.

The values of $2r_i$ were measured between the outside boundaries of the image formed on the opaque glass plate, and therefore, include the image of the fiber itself. This inclusion is of no practical consequence as long as the E.C.D. is large compared with the diameter of the fiber. With increase of load, however, the E.C.D. as measured here decreases to the limiting value of the fiber diameter. If the E.C.D. were defined as a geometrical figure related to the shape of the fiber axis, the diameter of the fiber d_f should have been subtracted from each of the above measured values of $2r_i$. In this case also the values of E.C.D. would be reduced to

$$D'_{eff} = D_{eff} - d_f.$$

The limit of D'_{eff} for a straight fiber will be zero, whereas the limit of D_{eff} will be d_f.

TABLE II. Change in E.C.D. with Load
(The Mean of 50 Fibers)

	52's Scotch-Blackface scoured wool fibers mean diameter: 38μ				90's Merino scoured wool fibers mean diameter: 22μ			
Load, mg.	20	40	60	80	20	40	60	80
Mean D_{eff} mm.	1.4	0.8	0.5	0.4	0.37	0.26	0.21	0.17
Standard error	0.09	0.05	0.03	0.02	0.02	0.011	0.007	0.006
Coefficient of variation	46	44	41	44	35	31	25	26

Results

Since varying the moisture content of wool fibers brings about considerable changes in the fibers [16], all the measurements were performed at 65% R.H. and 20° C. on fibers previously kept at these conditions for 48 hr.

Results obtained for 52's Scotch-Blackface and 90's Merino scoured wool fibers are given in Table II. In both cases measurements were taken on 50 fibers from each quality, loaded with 20, 40, 60, and 80 mg.

The comparison of the E.C.D. of various fibers from the same lot, at any given load, shows that this figure varies within rather large limits. At all loads there are some fibers which show relatively low values of D_{eff}. Among the sample of 50 fibers from 52's Scotch-Blackface fibers, D_{eff} varied around an average of 1.4 mm. from 0.4 to 3.0; and in a sample of the same size from 90's Merino wool fibers, it varied about an average of 0.37 mm. from 0.20 to 0.82. At the higher load of 60 mg., variations from 0.1 to 1.1, around an average of 0.5 mm., were observed for the former; and from 0.14 to 0.40, with an average of 0.21 mm., for the latter. At a load of about 40 to 60 mg., erratically high values of D_{eff} rarely occur, and the distribution of D_{eff} within the sample approaches normality. The coefficient of variation then approaches about 40% for 52's Scotch-Blackface and 20% for 90's Merino.

It has still to be investigated whether the variability of D_{eff} within a sample of wool has any significance for the technical qualities of the wool concerned. However, it might be mentioned that the coefficient of variation of D_{eff} was found to be smaller with respect to 90's Merino than with respect to 52's Scotch-Blackface.

In view of the fairly low variability of D_{eff} (20 to 46% variation) from one sufficiently loaded fiber to another, a small sample of about 15 fibers taken from the same lot will often be sufficient to estimate the average E.C.D. of a lot with a satisfactory degree of reliability. With a sample of this size, it can be determined whether the mean E.C.D. of a lot deviates from a standard value by more than about 20%. Similar samples from each of two lots permits differentiation between them, if the actual difference between the average E.C.D.'s is about 30%.

The present results, incidentally, indicate certain interesting relationships between the E.C.D., the load, and characteristics of the fiber itself, including the fiber diameter, such as an association of high fiber diameters with higher values of E.C.D. Additional relationships which can be deducted from the data will form the subject of subsequent publications.

In Figure 4 the change of the mean E.C.D. values with stress for the 52's Scotch-Blackface and 90's Merino wool fibers is represented.

Measurements of E.C.D. of the same fiber show that repeated loading and unloading cause a reduction in the E.C.D. Partial or full restoration of crimp occurs after a certain time, and this time differs

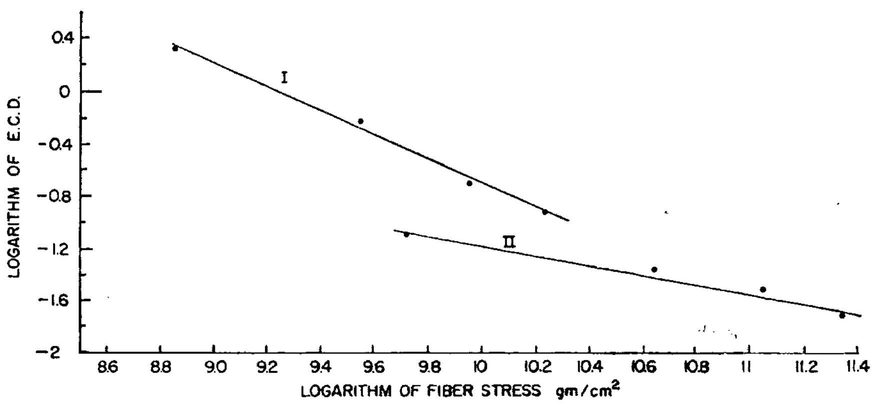

Fig. 4. Change in E.C.D. with stress. I, mean of 50 fibers of 52's quality; II, mean of 50 fibers of 90's quality.

for individual fibers, i.e., the fibers show hysteresis of crimp under the loads applied. Figure 5 shows the relation between the E.C.D. values and the loads applied to a fiber:[2] I, when the fiber was loaded for the first time with increasing weights; II, when the fiber was gradually unloaded in the opposite direction after having remained under the maximum load of I for 14 days; and III, when the fiber was again loaded with the same weights after having been held for another 14 days under the minimum load of II.

It follows that the history of the fiber influences crimp, and should be considered carefully in each set of measurements.

The Effective Wave Number

From the effective crimp diameter an "effective volume" can be estimated, which may be related to the volume actually occupied by a loose fiber assembly, e.g., webs, slivers, felts, etc.

On the other hand the E.C.D. of a fiber represents only one feature of the geometrical shape influencing the physical behavior of the fibers. It is obvious that very differently shaped fibers may yield the same E.C.D., e.g., Figure 6 shows two fibers of the same length and the same E.C.D. with very different curvatures.

Since the E.C.D. can characterize only one aspect of the geometrical shape of a crimped fiber, an additional geometrically independent term is necessary for describing this shape.

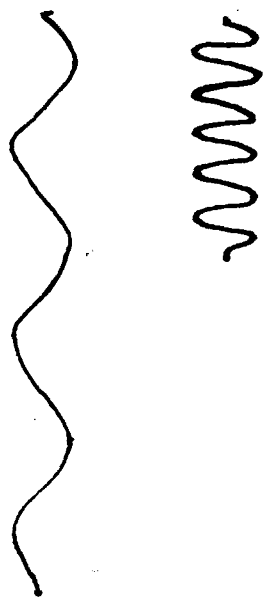

Fig. 6. Two fibers with the same E.C.D. and different curvature and wave length.

It is generally impossible to define and measure directly the wave length of a crimped fiber. However, an effective wave length or wave number can be calculated. It is proposed to use effective wave number (E.W.N.), defined as the number of waves per unit of fiber length, as the second term. However, it is recognized that E.W.N. alone would not be sufficient to describe crimp.

In the following, a method of deriving E.W.N. is proposed which makes use of the E.C.D. as defined previously, together with the "crimp ratio" (C_r), or "contraction" as used by previous authors [4, 5, 8, 17].

Although it would be possible to use these two measures (E.C.D. and C_r) for characterizing the shape of the fiber, it is desirable and possible to deduce from these two measures an E.W.N. which seems to be a more meaningful measure than the crimp ratio.

In order to arrive at an estimate of the E.W.N., it is sufficient to assume any regular fiber shape, and to calculate the wave number n from the E.C.D. and C_r, with the help of the model of the assumed shape.

The E.W.N. thus estimated gives a characteristic of every fiber, whether regularly or irregularly shaped; in the latter case it can be interpreted as an *average* wave number of the fiber. From this it

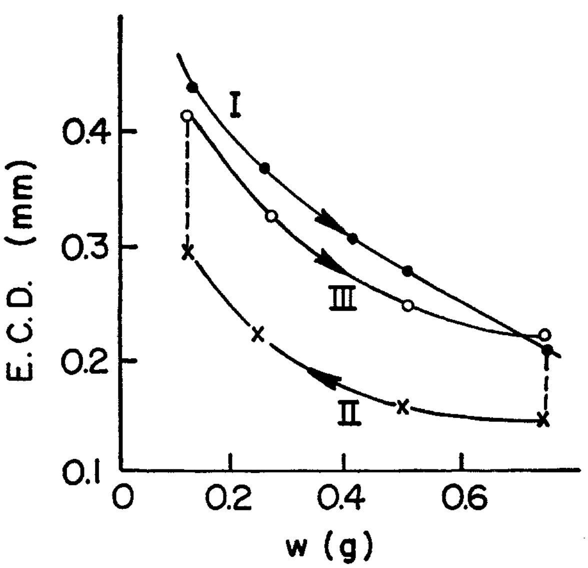

Fig. 5. Hysteresis curve of a single fiber showing change in E.C.D. with load w. I, loading; II, unloading; III, loading.

[2] These measurements relate to E.C.D. as defined in Equation 1.

will be seen that the thus computed wave number depends only slightly on the particular shape used in the computations.

The regular shapes assumed in the following calculations are: (1) a circular helix and (2) a sine curve. It is assumed that the fiber's D_{eff} was measured, as well as its crimp ratio C_r:

$$C_r = (L - l_w)/L$$

where L is the length of the stretched fiber,[3] and l_w is the distance between the fiber ends under the load w.

In the following calculations, the wave number n is obtained as a function of D_{eff} and C_r.

As the wave-number is given by

$$n = \nu/L = \nu/\nu s = 1/s$$

ν being the total number of waves on the fiber of length L, and s the curve length of one complete wave, it is necessary to find s for any given values of C_r and D_{eff}.

The effective wave number computed from the helix model is called n_1 and that computed from the sine curve model, n_2. The method of calculating these two figures is given in Appendix I.

Results

These two regular shapes were used with the following experimental results, which are presented here as an illustration of the actual estimation of the E.W.N.

Table III shows the values of D_{eff} and C_r as measured on three fibers of different grades. Each fiber was loaded with 10, 20, and 30 mg., respectively.

[3] The stretched length L of the fiber should be defined in a way similar to that given by Evans and Montgomery [5] (see Introduction). However, its actual measurements were taken after loading the fiber to the extent where the E.C.D. approaches the diameter of the fiber.

For these fibers, the distance l_w between the fiber ends and the stretched length L (see footnote 3) were measured, and C_r computed. Then the wave numbers n_1 and n_2 were calculated from the helix and from the sine curve respectively (Equations 4 and 8, Appendix I).

It is seen from Table III that in all cases the wave number n_2 computed from the sine curve are slightly higher than those obtained from the helix n_1; but in view of the considerable difference observed here between various fibers and under different loads, these smaller differences between n_1 and n_2 seem to be of no practical importance. It therefore seems permissible to use either model for the actual calculations, and no preference can be stated.

Under different loads, the wave number of a regularly shaped fiber, homogeneous along its whole length, should remain constant. However, the few empirical data listed in Table III show, in general, a tendency of the average wave number to increase with load. This suggests that the longer waves tend often to disappear before the shorter ones. A lack of homogeneity of the fiber along its length may explain those cases (like the 46's fiber under a weight increase of from 20 to 30 mg.), where this tendency is reversed.

It is seen from Table III that the 90's Merino fiber has approximately a ten-times higher wave number than the 46's and 52's fibers. A high wave number may therefore serve as a good indication of the quality of the wool.

While the E.C.D. and the E.W.N. seem to be highly characteristic for different kinds of wool fibers, it can be seen from Table III that the crimp ratio alone is not significant for such a differentiation.

TABLE III. Estimation of E.W.N.

Fiber grade	w (mg.)	D_{eff} (mm.)	l_w (mm.)	L (mm.)	C_r $\times$ 100	n_1 (mm.$^{-1}$)	n_2 (mm.$^{-1}$)
Scotch-Blackface							
46's	10	2.92	40.5	49	17.3	0.061	0.063
	20	1.95	41	49	16.3	0.089	0.092
	30	1.89	42.5	49	13.3	0.084	0.086
52's	10	2.61	50.5	56	9.8	0.053	0.054
	20	1.98	51	56	8.9	0.066	0.068
	30	1.44	51.5	56	8.0	0.087	0.091
Merino							
90's	10	0.21	42	48.5	13.4	0.76	0.78
	20	0.17	42.5	48.5	12.4	0.90	0.93
	30	0.16	43	48.5	11.3	0.92	0.94

APPENDIX I

Computation of the Effective Wave Number

1. *The Circular Helix*

In this case one turn of the helix represents a complete wave. The curve length s of one turn of a helix of radius r and pitch h is

$$s = \sqrt{(2\pi r)^2 + h^2}$$

In the present case,

$$D_{eff} = 2r \quad \text{and} \quad C_r = (s - h)/s$$
$$h = s(1 - C_r)$$

therefore

$$s^2 = (\pi D_{eff})^2 + s^2(1 - C_r)^2$$

and

$$s = \pi D_{eff}/\sqrt{C_r(2 - C_r)}$$

The wave number n_1 of a helix is therefore

$$n_1 = 1/s = \sqrt{C_r(2 - C_r)}/\pi D_{eff} \tag{4}$$

2. *The Sine Curve*

The curve length s of one wave of a sine curve $y = y_0 \sin (2\pi x/x_0)$, of wave length x_0 and extreme amplitude y_0, is given by

$$s = 4y_0 E(k)/k \tag{5}$$

where $E(k)$ is the complete elliptic integral of the second kind.[4]

$$E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \varphi} \cdot d\varphi$$
$$(\varphi = 2\pi x/x_0)$$

and

$$k = 2\pi y_0/\sqrt{x_0^2 + (2\pi y_0)^2} \tag{6}$$

This can be easily seen in the following:

$$s = \int_0^s ds = 4 \cdot \int_0^{x_0/4} \sqrt{1 + (dy/dx)^2} \cdot dx$$
$$= 4 \cdot \int_0^{x_0/4} \sqrt{1 + (2\pi y_0/x_0)^2 \cos^2 (2\pi x/x_0)} \cdot dx$$

By introducing

$$\varphi = 2\pi x/x_0$$
$$d\varphi = (2\pi/x_0) \cdot dx$$

we get

$$s = (4x_0/2\pi) \cdot \int_0^{x_0/4} \sqrt{1 + (2\pi y_0/x_0)^2 - (2\pi y_0/x_0)^2 \sin^2 \varphi} \cdot d\varphi$$
$$= (4/2\pi) \cdot \sqrt{x_0^2 + (2\pi y_0)^2} \cdot \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 \varphi} \cdot d\varphi$$
$$= (4/2\pi) \sqrt{x_0^2 + (2\pi y_0)^2} \cdot E(k)$$

Taking k as given in (6),

$$\sqrt{x_0^2 + (2\pi y_0)^2} = (2\pi y_0)/k$$

it follows that

$$s = 4y_0(E(k)/k)$$

In the case of the sine curve

$$D_{eff} = y_0\sqrt{2}$$

as given in Equation 2 under "The Effective Crimp Diameter,"

$$D_{eff}^2 = \frac{1}{l_w} \int_0^{l_w} y^2 \cdot dx$$
$$= (y_0^2/x_0) \cdot \int_0^{x_0} \sin^2 (2\pi x/x_0) \cdot dx = 2y_0^2$$

s, from (5), will be

$$s = 2\sqrt{2}\, D_{eff}(E(k)/k) \tag{7}$$

and the wave number n_2 will be

$$n_2 = 1/s = \frac{1}{2\sqrt{2}\, D_{eff}} \cdot (k/E(k)) \tag{8}$$

The crimp ratio C_r is given by

$$C_r = (s - x_0)/s$$

therefore

$$s = x_0/(1 - C_r)$$

From (6) we get

$$x_0 = (2\pi y_0) \cdot \frac{\sqrt{1 - k^2}}{k}$$

so from the last two equations it follows that

$$s = (2\pi y_0) \cdot \left(\frac{\sqrt{1 - k^2}}{k(1 - C_r)} \right) \tag{9}$$

from (9) and (5),

$$\frac{E(k)}{\sqrt{1 - k^2}} = \left(\frac{\pi}{2} \right) \cdot \frac{1}{1 - C_r}$$

or

$$C_r = 1 - (\pi/2) \sqrt{1 - k^2}/E(k) \tag{10}$$

[4] Tables for this integral as a function of $\sin^{-1} k$ can be found in the 34th edition of the Handbook of Chemistry and Physics, pp. 234–236 (and in other handbooks).

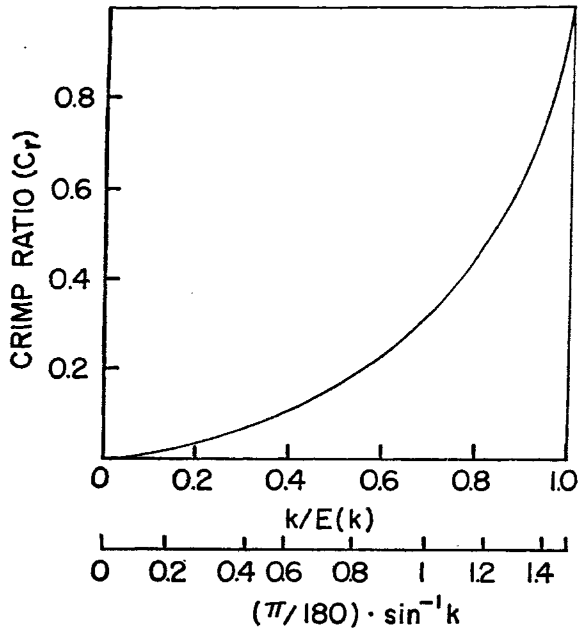

Fig. 7. Graphical representation of the function $C_r(k) = 1 - (\pi/2)\sqrt{1-k^2}/E(k)$.

It is useful to define the function

$$C_r(k) = 1 - (\pi/2)\sqrt{1-k^2}/E(k)$$

This function can be calculated from the tabulated values of $E(k)$, and the values of $C_r(k)$ can be plotted graphically against the similarly calculated values of another function: $k/E(k)$. This is represented graphically in Figure 7.

The wave number n_2 can now be calculated from the measured values of C_r and D_{eff} in the following way. From Figure 7 we get for C_r the corresponding value of $k/E(k)$ directly from the abscissa. By introducing this value into Equation 8, n_2 is obtained.

APPENDIX II

The Effective Curvature of Fibers

In a way similar to the computation of the wave number, it is also possible to obtain from the measured values of D_{eff} and C_r a figure representing the "effective curvature" of the fiber (or its reciprocal, the "effective radius of curvature"), which could also be used as a parameter which serves to illustrate the geometrical shape of a fiber.

The effective curvature (E.K.) can be estimated by computing the curvature of the two regular fiber shapes also used for the E.W.N.: 1. a circular helix, 2. a sine curve.

Computation of the Effective Curvature

1. *The circular helix.* The curvature K_1 of a three-dimensional curve is given by

$$K_1^2 = (d^2x/dS^2)^2 + (d^2y/dS^2)^2 + (dz^2/dS^2)^2$$

where S is the curve length of the helix.

The circular helix is given by

$$x = r\cos\theta \qquad y = r\sin\theta \qquad z = r\theta\tan\alpha$$

where r is the radius and α the slope of the helix.

$$(dS/d\theta)^2 = (dx/d\theta)^2 + (dy/d\theta)^2 + (dz/d\theta)^2 = \frac{r^2}{\cos^2\alpha}$$

therefore

$$S = r\theta/\cos\alpha \tag{11}$$

From (11) it follows that

$$\theta = S\cdot\cos\alpha/r$$

so that

$$(d^2x/dS^2) = (d^2x/d\theta^2)\cdot\cos^2\alpha/r^2 = -\cos^2\alpha\cdot\cos\theta/r$$
$$(d^2y/dS^2) = -\cos^2\alpha\cdot\sin\theta/r \qquad (d^2z/dS^2) = 0$$

$$K_1^2 = \cos^4\alpha\cdot\cos^2\theta/r^2 + \cos^4\alpha\cdot\sin^2\theta/r^2 = \cos^4\alpha/r^2$$

and the curvature

$$K_1 = \cos^2\alpha/r$$

which is constant along the helix.

It is now necessary to calculate α in terms of the known values of D_{eff} and C_r. In the present case,

$$D_{eff} = 2r$$

The crimp ratio is

$$C_r = (S - z)/S$$

By introducing $z = r\theta\tan\alpha$ and S from (11), we obtain

$$C_r = \left(\frac{r\theta}{\cos\alpha} - r\theta\tan\alpha\right)\Big/\left(\frac{r\theta}{\cos\alpha}\right) = 1 - \sin\alpha$$

so that

$$\cos^2\alpha = 1 - (1 - C_r)^2 = C_r(2 - C_r)$$

and from (4) (Appendix I),

$$n_1^2 = C_r(2 - C_r)/\pi^2 D_{eff}^2$$

and

$$\cos^2\alpha = n_1^2\pi^2 D_{eff}^2$$

therefore

$$K_1 = 2n_1^2\pi^2 \cdot D_{eff} \tag{12}$$

2. *The sine curve.* The curvature K_2 of a plane curve $y(x)$ is given by

$$K_2 = \frac{y''}{(1 + y'^2)^{3/2}}$$

For

$$y = y_0 \sin (2\pi x/x_0)$$
$$y' = (2\pi/x_0)y_0 \cos (2\pi x/x_0)$$
$$y'' = -(2\pi/x_0)^2 y_0 \sin (2\pi x/x_0)$$

and

$$K_2 = \frac{-(2\pi/x_0)^2 y_0 \sin (2\pi x/x_0)}{[1 + (2\pi/x_0)^2 y_0^2 \cos^2 (2\pi x/x_0)]^{3/2}}$$

In order to obtain an average value of $K_2(x)$, we calculate

$$\bar{K}_2 = \frac{1}{s} \cdot \int_0^s K_2 \cdot ds$$

with $s = 1/n_2$ (see Appendix I) (s being the curve length of one complete wave).

$$\bar{K}_2 = n_2 \cdot \int_0^s K_2 \cdot ds$$
$$= 4n_2 \cdot \int_0^{x_0/4} \left[\frac{-(2\pi/x_0)^2 y_0 \sin (2\pi x/x_0)}{1 + (2\pi/x_0)^2 y_0^2 \cos^2 (2\pi x/x_0)} \right] \cdot dx$$

Introducing

$$u = (2\pi y_0/x_0) \cos (2\pi x/x_0)$$

gives

$$\bar{K}_2 = 4n_2 \cdot \int_{(2\pi y_0/x_0)}^{0} \frac{du}{1 + u^2} = -4n_2 \cdot \tan^{-1} (2\pi y_0/x_0)$$

when introducing x_0 from (6) (Appendix I), we obtain

$$\bar{K}_2 = -4n_2 \cdot \tan^{-1} (k/\sqrt{1 - k^2}) = -4n_2 \cdot \sin^{-1} k$$

When disregarding the sign of the curvature, the average of the absolute value of the curvature is consequently given by

$$|\bar{K}_2| = 4n_2 \cdot \sin^{-1} k \tag{13}$$

In Figure 7 the $C_r(k)$ values were plotted against the $k/E(k)$ values on a linear scale, from which the wave numbers n_2 can be read. An additional nonlinear scale of $\sin^{-1} k$ (in radians, so as to get K_2 in mm.$^{-1}$) is plotted in the same figure, and with the aid of this scale, the average curvatures of the sine curve can be obtained.

Results

The following values of curvature (in mm.$^{-1}$) were obtained (see Table IV) for the same fibers listed in Table III.

TABLE IV. Estimation of the Effective Curvature

Fiber grade	w (mg.)	$K_{1_{eff}}$ (mm.$^{-1}$)	$K_{2_{eff}}$ (mm.$^{-1}$)
Scotch-Blackface			
46's	10	0.21	0.20
	20	0.31	0.28
	30	0.26	0.24
52's	10	0.14	0.13
	20	0.17	0.16
	30	0.22	0.20
Merino			
90's	10	2.40	2.17
	20	2.55	2.49
	30	2.68	2.43

It is seen from Table IV that there are slight differences between the curvatures obtained from the two representations, and the helix model is associated with higher curvatures.

It seems that the general picture of these figures resembles those obtained from the E.W.N.

Summary

A new approach to the problem of defining and measuring crimp, or the geometrical shape of fibers, is reported in the present paper.

In previous methods the stresses under which measurements were performed were not controlled, and the spatial arrangement of crimp, as well as its irregular, statistical aspect was not considered sufficiently. The present paper attempts to account for these factors.

Two fundamental concepts are defined, the effective crimp diameter (E.C.D.) and the effective wave number (E.W.N.).

The E.C.D. gives the average distance between the nonlinear fiber and the straight line connecting its end points. It therefore covers the "depth of the crimp" and the "bulking capacity." It is obtained from measurements of the width of the plane projection of the rotating fiber. For this purpose a special apparatus containing a fastening and rotating device was designed.

The E.W.N. is defined as an average number of waves per unit length of the fiber. A figure for the E.W.N. for a given fiber can be obtained from the

E.C.D. (D_{eff}) and the crimp ratio C_r which is defined as the relative shortening due to crimp of the distance between the fiber ends. This can be done only with the help of a geometrical model. Two models, a circular helix and a sine curve, were used. From the helix model the E.W.N. (n_1) is found to be

$$n_1 = \frac{1}{\pi D_{eff}} \cdot \sqrt{C_r(2 - C_r)} \tag{4}$$

and from the sine curve model,

$$n_2 = \frac{1}{2\sqrt{2}\, D_{eff}} \cdot [k/E(k)] \tag{8}$$

where $k/E(k)$ is derived from C_r.

The E.W.N. is related to the length of the waves by which the irregular crimp can be approximately represented.

Each measurement for determining the E.C.D. and E.W.N. is performed under a well-defined stress, by application of weights to the suspended fiber. The spatial nature of crimp is accounted for by the rotation of the fiber. At the same time, the E.C.D. and the E.W.N., being statistical averages, take the variability of the amplitude and the wave length of crimp waves into consideration.

It is shown that an effective radius of curvature or an effective curvature K is determined with the help of the same models. For the helical model,

$$K_1 = 2n_1^2\pi^2 D_{eff} \tag{12}$$

and for the sine curve model,

$$K_2 = 4n_2 \sin^{-1} k \tag{13}$$

It is found that both the E.C.D. and E.W.N. assume different values for different types of wool. For a single wool fiber the E.C.D. decreases and the E.W.N. increases slightly with increasing loads. Fibers of the same batch do not differ considerably in E.C.D., the coefficient of variation being fairly low. A small sample is therefore sufficient to determine the average E.C.D. of a lot.

The E.C.D. shows hysteresis. Repeated loading of a fiber causes a reduction in the E.C.D. values, with partial or full restoration after rest.

Merino 90's fibers show smaller E.C.D. values (about 0.1 mm.), than those of Scotch-Blackface 52's and 46's (about 1 mm.), when compared under the same loads; while the E.W.N. values obtained for the 90's fibers are higher than those of the other grades (about 1 mm.$^{-1}$ for the 90's fibers, and 0.1 mm.$^{-1}$ for the 52's and 46's). A similar trend is observed for the values of the curvature.

It is possible, therefore, to differentiate between wools of different qualities by the E.C.D. and E.W.N.

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