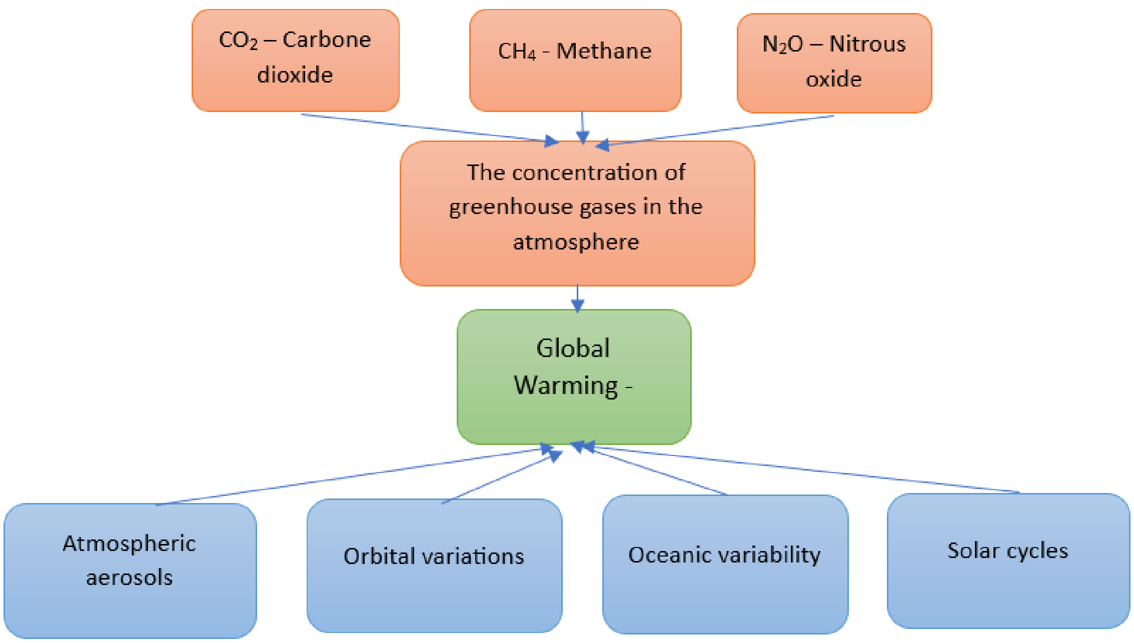
Problem formulation

The problem we focused on is predicting the global temperature anomaly based on concentration of gases in the atmosphere. Data is collected from year 2001 to 2022. The result of our research can show directly how the global warming is going to change in the future years. This information can be useful for preparing society for the impact of temperature changes. It can also show how our population influence global weather changes because of gases emission. Data used in our project come from two different sources. Data about global temperature anomaly comes from NASA Global Climate Change https://climate.nasa.gov/vital-signs/global-temperature/ and the gases concentration comes from Global Monitoring Laboratory https://gml.noaa.gov/. The first one contains yearly temperature anomaly around the globe. Original data was from years 1880-2023. The second data set has information about three gases concentration in the atmosphere. It not only has yearly data but also monthly. Before starting the project we created the dag graph that shows what data impacts global warming.



. The parameters we have taken into consideration are carbon dioxide(CO2), methan(CH4), nitrous oxide(N2O). Although there were more factors than only gases the ones we have chosen had the most significant impact on temperature changes.

Data preprocessing

As the data were clear without any NaN or Null values we didn't change much while cleaning it. We had to compute the mean value for every year when it comes to gases because the data had values for every month which we didn't need to use.

Imports

```python
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import numpy as np
from scipy import stats
import seaborn as sns
from cmdstanpy import CmdStanModel
from sklearn.preprocessing import StandardScaler
import arviz as az

df = pd.read_csv("/home/data.csv", index_col=0)
```

Data standarization

```python
df.head()
```

	year	CO2	CH4	N2O	Temperature
0	2001	371.319167	1771.269167	316.364167	0.54
1	2002	373.452500	1772.731667	316.942500	0.63
2	2003	375.983333	1777.334167	317.631667	0.62
3	2004	377.698333	1776.995833	318.262500	0.53
4	2005	379.983333	1774.180000	318.920000	0.68

```python
df.describe()
```

	year	CO2	CH4	N2O	Temperature
count	22.000000	22.000000	22.000000	22.000000	22.000000
mean	2011.500000	394.154091	1817.905000	325.018220	0.744091
std	6.493587	14.618055	43.282048	5.969065	0.158284
min	2001.000000	371.319167	1771.269167	316.364167	0.530000
25%	2006.250000	382.574375	1778.368750	319.980625	0.632500
50%	2011.500000	392.953333	1805.632917	324.638333	0.680000
75%	2016.750000	406.171875	1848.098542	329.547083	0.887500
max	2022.000000	418.564167	1911.968333	335.662500	1.020000

Data was standarized using StandardScaler class form sklearn. We wanted to have our parameters in the similar range and in not so big scale as they were before. The standard score of a sample x is calculated as: z = (x - u) / s where u is the mean of the training sample, and s is the standard deviation of the training samples.

```python
scaler = StandardScaler()
df[['CO2', 'CH4', 'N2O']] = scaler.fit_transform(df[['CO2', 'CH4',
'N2O']])

print(df)
```

	year	CO2	CH4	N2O	Temperature
0	2001	-1.598865	-1.102843	-1.483935	0.54
1	2002	-1.449492	-1.068258	-1.384767	0.63

2	2003	-1.272287	-0.959418	-1.266593	0.62
3	2004	-1.152206	-0.967419	-1.158423	0.53
4	2005	-0.992214	-1.034008	-1.045679	0.68
5	2006	-0.844650	-1.015326	-0.890782	0.64
6	2007	-0.709223	-0.861555	-0.782897	0.67
7	2008	-0.582723	-0.728771	-0.602136	0.54
8	2009	-0.455931	-0.575395	-0.469245	0.66
9	2010	-0.283744	-0.448149	-0.312204	0.73
10	2011	-0.161270	-0.348670	-0.137302	0.61
11	2012	-0.006880	-0.231750	0.007021	0.65
12	2013	0.180886	-0.105253	0.159489	0.68
13	2014	0.326174	0.114358	0.355969	0.75
14	2015	0.480156	0.389186	0.541731	0.90
15	2016	0.718277	0.598510	0.674336	1.02
16	2017	0.882529	0.752518	0.810657	0.92
17	2018	1.019531	0.934489	1.010995	0.85
18	2019	1.225327	1.153114	1.177752	0.98
19	2020	1.406266	1.447059	1.375375	1.02
20	2021	1.561182	1.833171	1.595432	0.85
21	2022	1.709154	2.224407	1.825205	0.90

```python
#TODO Alternative standarization way
df['CO2'] /= 100
df['CH4'] /= 1000
df['N2O'] /= 100

df['CO2'] = df['CO2'] - df['CO2'].mean()
df['CH4'] = df['CH4'] - df['CH4'].mean()
df['N2O'] = df['N2O'] - df['N2O'].mean()
df
```

	year	CO2	CH4	N2O	Temperature
0	2001	-0.015989	-0.001103	-0.014839	0.54
1	2002	-0.014495	-0.001068	-0.013848	0.63
2	2003	-0.012723	-0.000959	-0.012666	0.62
3	2004	-0.011522	-0.000967	-0.011584	0.53
4	2005	-0.009922	-0.001034	-0.010457	0.68
5	2006	-0.008446	-0.001015	-0.008908	0.64
6	2007	-0.007092	-0.000862	-0.007829	0.67
7	2008	-0.005827	-0.000729	-0.006021	0.54
8	2009	-0.004559	-0.000575	-0.004692	0.66
9	2010	-0.002837	-0.000448	-0.003122	0.73
10	2011	-0.001613	-0.000349	-0.001373	0.61
11	2012	-0.000069	-0.000232	0.000070	0.65
12	2013	0.001809	-0.000105	0.001595	0.68
13	2014	0.003262	0.000114	0.003560	0.75
14	2015	0.004802	0.000389	0.005417	0.90
15	2016	0.007183	0.000599	0.006743	1.02
16	2017	0.008825	0.000753	0.008107	0.92
17	2018	0.010195	0.000934	0.010110	0.85

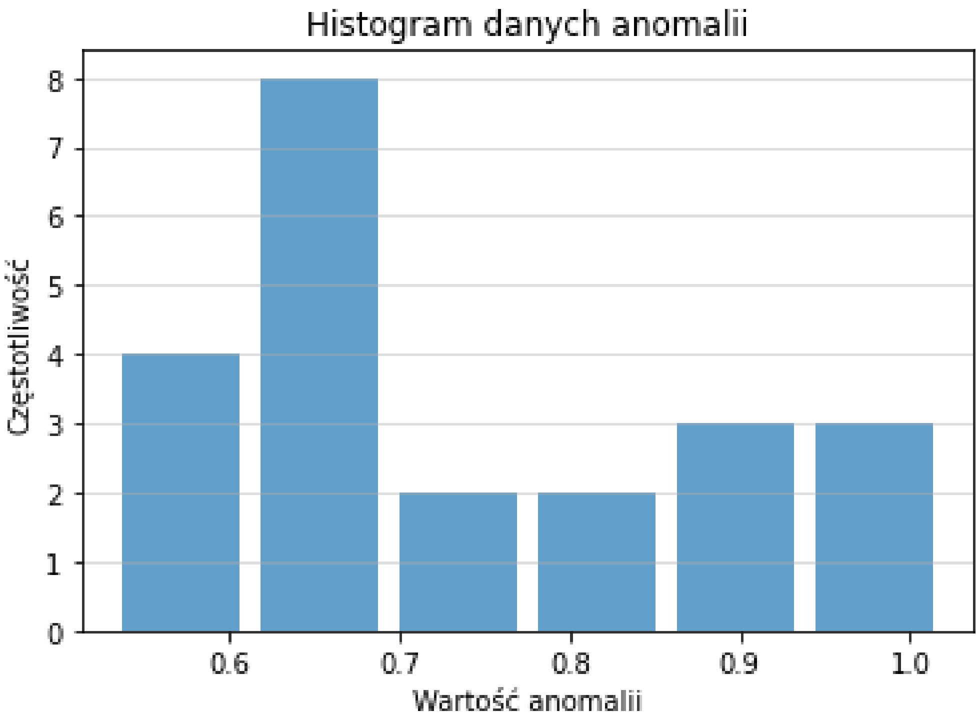
```
18  2019  0.012253  0.001153  0.011778         0.98
19  2020  0.014063  0.001447  0.013754         1.02
20  2021  0.015612  0.001833  0.015954         0.85
21  2022  0.017092  0.002224  0.018252         0.90
```

Checking id data is suitable for normal model

```
normality_test = stats.normaltest(np.array(df['Temperature']))

# Wyświetlanie wyników testu
print("Statystyka testu: ", normality_test.statistic)
print("Wartość p-wartości: ", normality_test.pvalue)

# Sprawdzenie interpretacji wyników testu
alpha = 0.05
if normality_test.pvalue < alpha:
    print("Dane nie pochodzą z rozkładu normalnego")
else:
    print("Dane są zgodne z rozkładem normalnym")
```

```
Statystyka testu:  3.3440751699526974
Wartość p-wartości:  0.18786388675829657
Dane są zgodne z rozkładem normalnym
```

```
plt.hist(np.array(df['Temperature']), bins='auto', alpha=0.7,
rwidth=0.85)
plt.grid(axis='y', alpha=0.5)
plt.xlabel('Anomaly value')
plt.ylabel('Frequency')
plt.title('Histogram of anomaly data')
plt.show()
```



From histogram above we can see that the data doesn't have strong resemblance for any of distributions. At first we thought about Gamma distribution but after some actions the results weren't satisfying so we decided to go with Normal Distribution

---

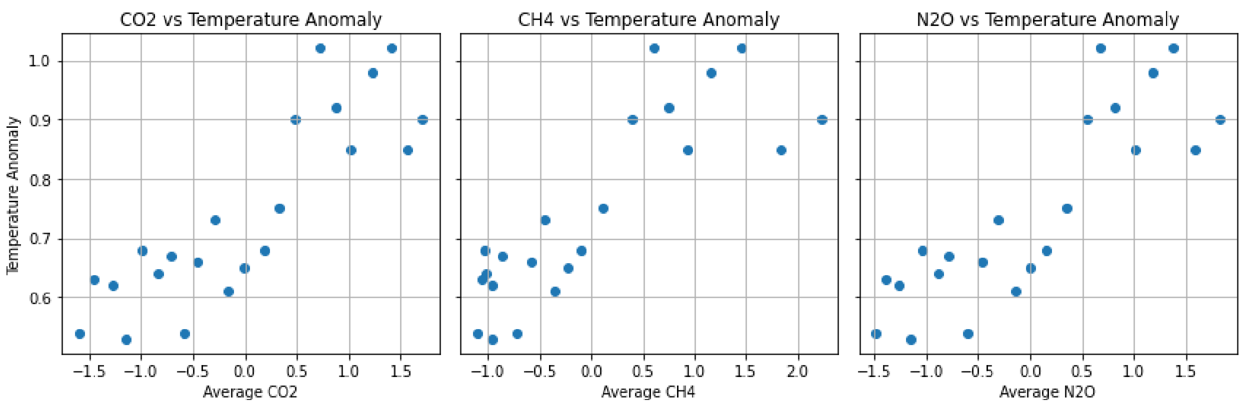
Model 1 - Normal Distribution

Our first approach was to create model with Normal Distribution. It is characterized by its symmetric bell shaped curve. It is defined by two parameters: mean and standard deviation. It is symmetric around mean value so it represents center of distribution while the standard deviation determines the spread of the data. Below plots shows that correlation between our parameters and temperature data can be considered to be linear and that is why we have chosen this approach in the model. Standard Bayesian model: $outcome_i \sim$ Normal(μ_i, σ) $\mu_i = \alpha + \beta * predictor_i$ $\alpha \sim$ Normal(a,b) $\beta \sim$ Normal(c,d) $\sigma \sim$ Normal(f,g)

The sum of multiplications beta and predictors is added two more times to the equation for every single predictor.

```
fig, axs = plt.subplots(1, 3, sharey=True, figsize=(12, 4))

axs[0].scatter(df['CO2'], df['Temperature'])
axs[0].set_xlabel('Average CO2')
axs[0].set_ylabel('Temperature Anomaly')
axs[0].set_title('CO2 vs Temperature Anomaly')
```

```
axs[0].grid()

axs[1].scatter(df['CH4'], df['Temperature'])
axs[1].set_xlabel('Average CH4')
axs[1].set_title('CH4 vs Temperature Anomaly')
axs[1].grid()

axs[2].scatter(df['N2O'], df['Temperature'])
axs[2].set_xlabel('Average N2O')
axs[2].set_title('N2O vs Temperature Anomaly')
axs[2].grid()

plt.tight_layout()
plt.show()
```



Prior

Alpha normal distribution is based on the mean value of 'Temperature Anomaly' in the data set and sigma is based on standard deviation of the same data. Beta for every predictor and sigma is also normally distributed. The parameters for distribution of beta where chosen considering the output of this prior model. It was the most difficult challenge to fit beta parameters properly.

```
%%writefile root/stan_files/temp3_ppc.stan
data {
  int<lower=0> N;
  vector[N] CO2;
  vector[N] CH4;
  vector[N] N2O;
}

generated quantities {
  real alpha = normal_rng(0.7, 0.1);
  real beta_CO2 = normal_rng(0, 0.1);
  real beta_CH4 = normal_rng(0, 0.1);
  real beta_N2O = normal_rng(0, 0.1);
  real sigma = normal_rng(0.1, 0.02);
```

```
  vector[N] temperature;

  for (i in 1:N) {
    temperature[i] = normal_rng(alpha + beta_CO2 * CO2[i] + beta_CH4 *
CH4[i] + beta_N2O * N2O[i], sigma);
  }
}
```

```
Overwriting root/stan_files/temp3_ppc.stan
```

```
data_sim={'N':len(df),
'CO2':np.linspace(df.CO2.min(),df.CO2.max(),len(df)),'CH4':np.linspace
(df.CH4.min(),df.CH4.max(),len(df)),'N2O':np.linspace(df.N2O.min(),df.
N2O.max(),len(df))}
model_ppc1=CmdStanModel(stan_file='root/stan_files/temp3_ppc.stan')
R = 1000
sim=model_ppc1.sample(data=data_sim,
                     iter_sampling=R,
                     iter_warmup=0,
                     chains=1,
                     refresh=R,
                     fixed_param=True,
                     seed=29042020)
```

```
INFO:cmdstanpy:compiling stan file /root/stan_files/temp3_ppc.stan to
exe file /root/stan_files/temp3_ppc
INFO:cmdstanpy:compiled model executable: /root/stan_files/temp3_ppc
INFO:cmdstanpy:CmdStan start processing
chain 1 |██████████| 00:00 Sampling completed
```

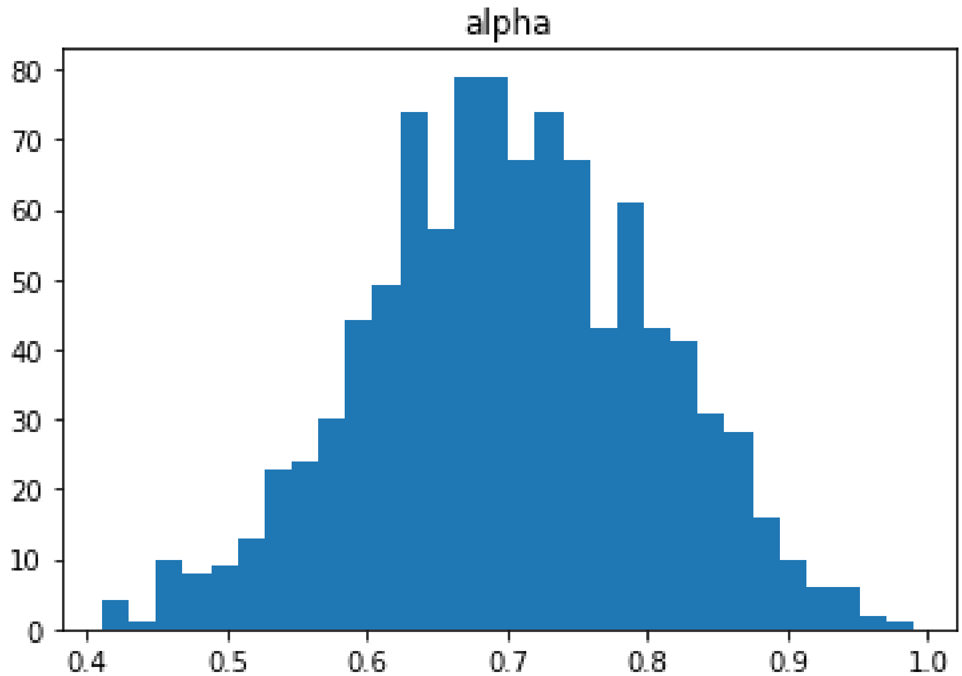
```
INFO:cmdstanpy:CmdStan done processing.
```

```
ppc_df = sim.draws_pd()
ppc_df.head()
```

```
   lp__  accept_stat__     alpha  beta_CO2  beta_CH4  beta_N2O
sigma  \
0   0.0            0.0  0.970817  0.077718 -0.127227  0.012759
0.072212
1   0.0            0.0  0.635055  0.118083 -0.014296 -0.015998
0.112775
2   0.0            0.0  0.699322 -0.000371  0.121486  0.075957
0.114794
3   0.0            0.0  0.606081  0.106421 -0.002303  0.037027
0.088234
4   0.0            0.0  0.577813  0.141649 -0.018822 -0.016031
```

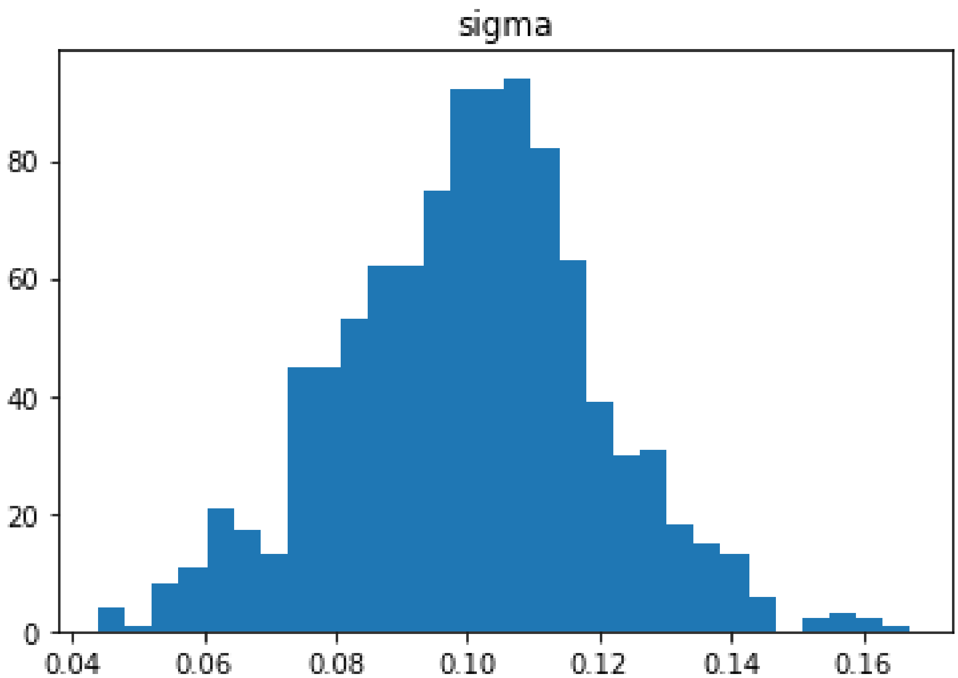
```
0.112103

   temperature[1]  temperature[2]  temperature[3]  ...
temperature[13]  \
0        0.985710        1.048590        1.119710  ...
0.854489
1        0.534230        0.179542        0.630123  ...
0.863881
2        0.588463        0.623650        0.606261  ...
0.792784
3        0.462742        0.446727        0.355780  ...
0.570904
4        0.440043        0.511638        0.191112  ...
0.558346

   temperature[14]  temperature[15]  temperature[16]  temperature[17]
\
0         0.911704         0.880068         0.887731         0.919615

1         0.838968         0.708483         0.537071         0.646070

2         0.899589         0.881236         0.767561         1.034040

3         0.658725         0.610738         0.762513         0.647393

4         0.491609         0.602761         0.621372         0.716101


   temperature[18]  temperature[19]  temperature[20]  temperature[21]
\
0         0.931942         0.844500         0.862722         0.726557

1         0.689926         0.820564         0.780421         0.834648

2         1.184980         0.975659         0.824738         1.118990

3         0.752692         0.898824         0.799267         0.790523

4         0.585402         0.747984         0.789003         0.786724


   temperature[22]
0         0.851031
1         0.640667
2         1.120850
3         0.749964
4         0.607919

[5 rows x 29 columns]
```

```
plt.hist(ppc_df['alpha'], bins=30)
plt.title('alpha')
plt.show()
```



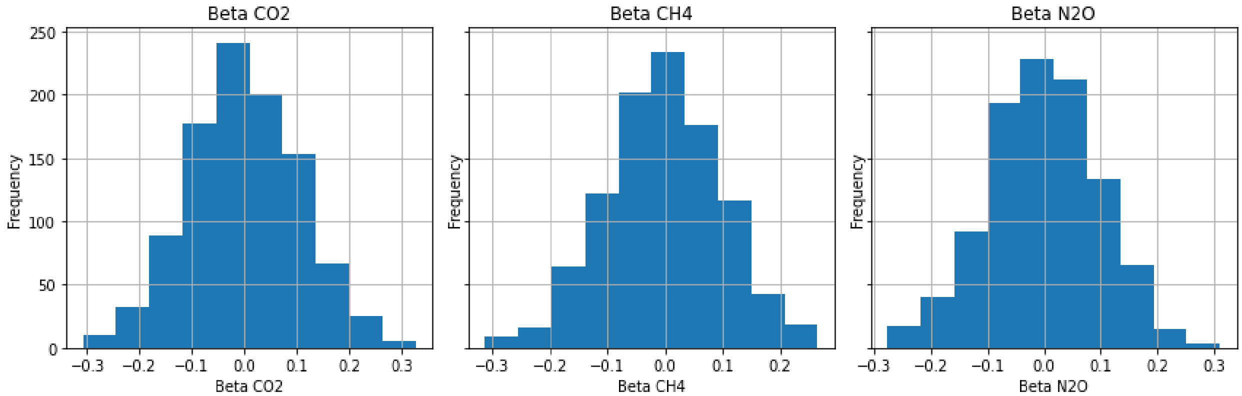
Parameter alpha is in the right range and is wider on the center which is good.

```
plt.hist(ppc_df['sigma'], bins=30)
plt.title('sigma')
plt.show()
```



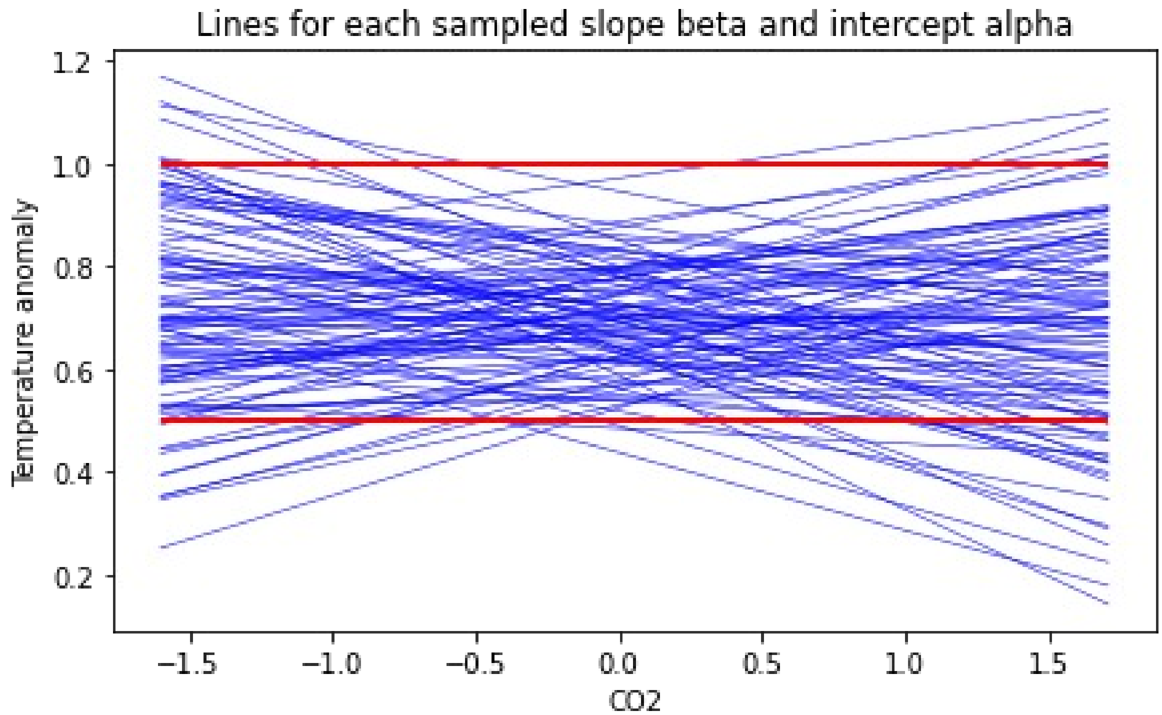
Parameter sigma looks also okay with tha values at the meant range

```
fig, axs = plt.subplots(1, 3, sharey=True, figsize=(12, 4))

axs[0].hist(ppc_df['beta_CO2'])
axs[0].set_xlabel('Beta CO2')
axs[0].set_ylabel('Frequency')
axs[0].set_title('Beta CO2')
axs[0].grid()

axs[1].hist(ppc_df['beta_CH4'])
axs[1].set_xlabel('Beta CH4')
axs[1].set_ylabel('Frequency')
axs[1].set_title('Beta CH4')
axs[1].grid()

axs[2].hist(ppc_df['beta_N2O'])
axs[2].set_xlabel('Beta N2O')
axs[2].set_ylabel('Frequency')
axs[2].set_title('Beta N2O')
axs[2].grid()

plt.tight_layout()
plt.show()
```



As we put the same values for beta distribution parameters for predictors their histograms look almost the same. We wanted for all the predictors to have the same impact on our model.

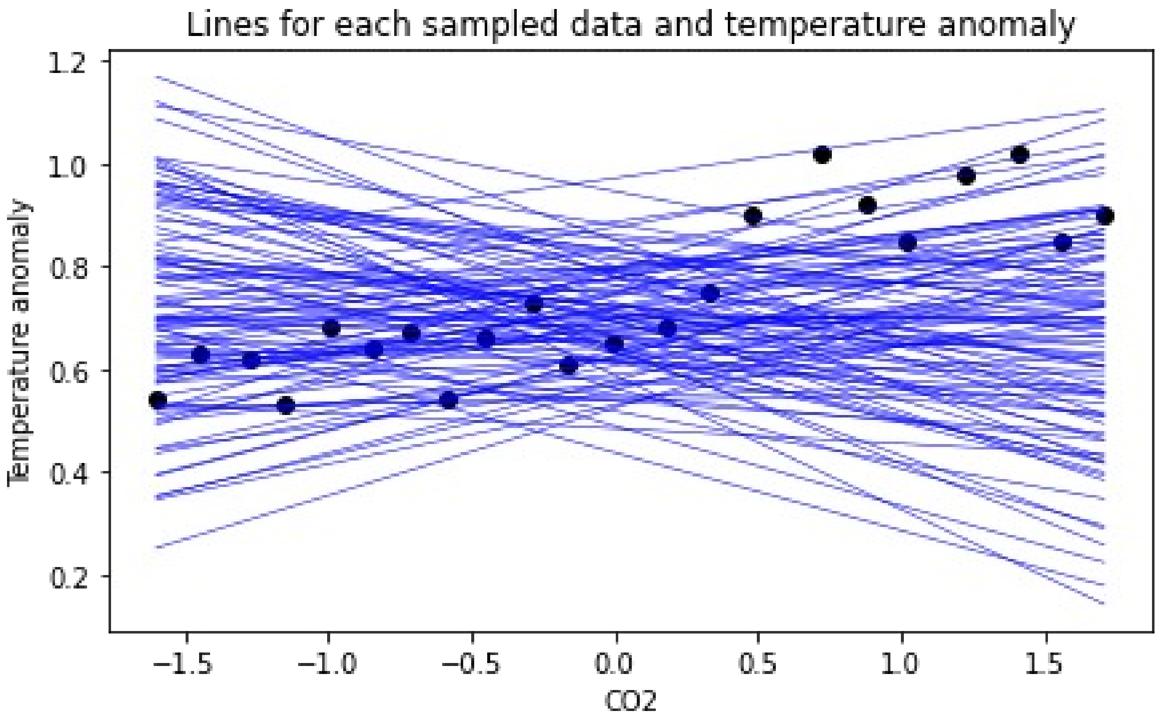
```
fig, axes = plt.subplots(1,1,figsize=(7,4))

beta_humid = sim.stan_variable('beta_CO2')
alpha_humid = sim.stan_variable('alpha')
for i in range(100):
    axes.plot(df['CO2'], alpha_humid[i]
+beta_humid[i]*np.array(df['CO2']), linewidth = 0.5, color='b')
plt.title("Lines for each sampled slope beta and intercept alpha")
axes.set_xlabel('CO2')
axes.set_ylabel('Temperature anomaly')
axes.hlines([0.5, 1],xmin = df['CO2'].min(), xmax = df['CO2'].max(),
linestyles = '-',linewidth = 2, color = 'r')
axes.annotate(text='max',xy=(80,320), weight = 'bold', color = 'r',
fontsize = 15)
axes.annotate(text='min',xy=(80,20), weight = 'bold', color = 'r',
fontsize = 15)

Text(80, 20, 'min')
```



This model fits just fine. Those lines that are under or above min and max values on the plot are acceptable because temperature anomaly can go below 0.5 (even below 0) and above 1. After fitting the data to the model everything should be between those lines perfectly.

Lets see how the actual temperature data corresponds with the data from prior.

```
fig, axes = plt.subplots(1,1,figsize=(7,4))

beta_humid = sim.stan_variable('beta_CO2')
alpha_humid = sim.stan_variable('alpha')
for i in range(100):
    axes.plot(df['CO2'], alpha_humid[i]
+beta_humid[i]*np.array(df['CO2']), linewidth = 0.5, color='b')
plt.title("Lines for each sampled data and temperature anomaly")
axes.scatter(df['CO2'], df['Temperature'], color= 'black')
axes.set_xlabel('CO2')
axes.set_ylabel('Temperature anomaly')
axes.annotate(text='max',xy=(80,320), weight = 'bold', color = 'r',
fontsize = 15)
axes.annotate(text='min',xy=(80,20), weight = 'bold', color = 'r',
fontsize = 15)

Text(80, 20, 'min')
```



Posterior predictive check

Fitting model to data

```
%%writefile root/stan_files/temp4_ppc.stan

data {
    int<lower=0> N;
    vector[N] temp;
    vector[N] CO2;
    vector[N] CH4;
    vector[N] N2O;
}

parameters {
    real<lower=0> alpha;
    real<lower=0> sigma;
    real<lower=0> beta_CO2;
    real<lower=0> beta_CH4;
    real<lower=0> beta_N2O;
}

transformed parameters {
    vector[N] mean;
    for (i in 1:N) {
        mean[i] = alpha + beta_CO2 * CO2[i] + beta_CH4 * CH4[i] +
beta_N2O * N2O[i];
    }
```

```
}

model {
    alpha ~ normal(0.7, 0.1);
    sigma ~ normal(0.1, 0.02);
    beta_CO2 ~ normal(0, 0.1);
    beta_CH4 ~ normal(0, 0.1);
    beta_N2O ~ normal(0, 0.1);
    for (i in 1:N) {
        temp[i] ~ normal(mean[i], sigma);
    }
}

generated quantities {
    vector[N] temp_;
    vector[N] log_lik;
    for (i in 1:N) {
        temp_[i] = normal_rng(mean[i], sigma);
        log_lik[i] = normal_lpdf(temp_[i]|mean[i], sigma);
    }
}
```

```
Overwriting root/stan_files/temp4_ppc.stan
```

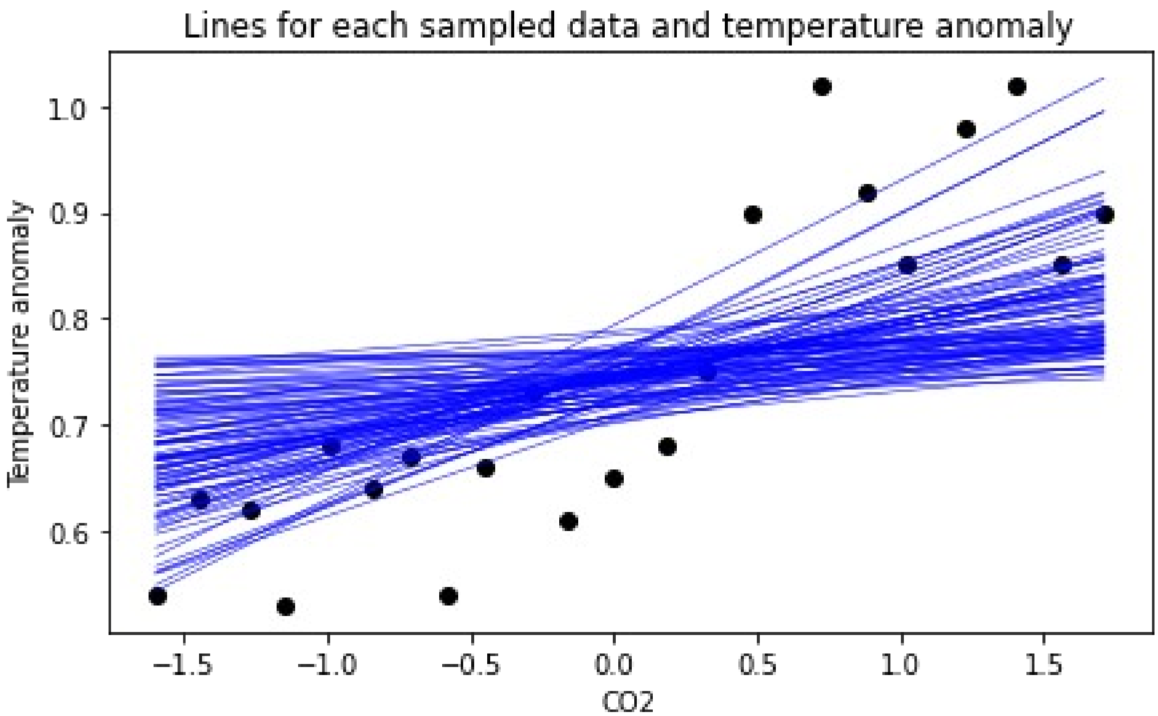
```
model_1_fit=CmdStanModel(stan_file='root/stan_files/temp4_ppc.stan')
N = len(df)
data_fit = {'N': N, 'CO2': df.CO2.values[:N], 'temp':
df.Temperature.values[:N], 'CH4': df.CH4.values[:N], 'N2O':
df.N2O.values[:N]}
fit=model_1_fit.sample(data=data_fit,seed=28052020)
```

```
INFO:cmdstanpy:found newer exe file, not recompiling
INFO:cmdstanpy:CmdStan start processing
chain 1 |          | 00:00 Status
█         | 00:00 Iteration:  100 / 2000 [  5%]  (Warmup)
██████    | 00:00 Iteration: 1200 / 2000 [ 60%]  (Sampling)
██████████| 00:00 Sampling completed
chain 2 |██████████| 00:00 Sampling completed
chain 3 |██████████| 00:00 Sampling completed
chain 4 |██████████| 00:00 Sampling completed




INFO:cmdstanpy:CmdStan done processing.
```

There were no issues with the sampling

```
df_ = fit.draws_pd()
df_.head()

       lp__  accept_stat__  stepsize__  treedepth__  n_leapfrog__
divergent__  \
0  27.9625       0.985636    0.167017          5.0          31.0
0.0
1  30.4195       0.999059    0.167017          5.0          31.0
0.0
2  31.0292       0.994093    0.167017          5.0          31.0
0.0
3  29.5738       0.978468    0.167017          4.0          15.0
0.0
4  28.9220       0.907824    0.167017          4.0          15.0
0.0

   energy__     alpha     sigma  beta_CO2  ...  log_lik[13]
log_lik[14]  \
0  -27.1543  0.742672  0.098222  0.037101  ...     0.418371
1.389580
1  -27.0724  0.747539  0.082064  0.046266  ...     1.489310
0.153962
2  -29.3316  0.736152  0.099784  0.070291  ...     0.176925    -
0.812033
3  -28.5692  0.707534  0.084646  0.040462  ...     1.460780
1.083030
4  -26.5229  0.718149  0.100418  0.032382  ...     0.689437
1.013680

   log_lik[15]  log_lik[16]  log_lik[17]  log_lik[18]  log_lik[19]  \
0     0.909479     0.610863     0.854060     1.389200     0.446135
1     1.581290     0.519966    -0.033177     1.436920     1.380760
2    -2.591740     1.385520     1.025260     1.371490     1.347590
3     1.498680    -0.623255     0.146163     1.412070     1.408160
4     0.833834     1.349390    -0.252048     0.725727     1.340430

   log_lik[20]  log_lik[21]  log_lik[22]
0     1.284320    -0.643606      1.26768
1     1.513080    -3.353400      1.54954
2     1.221020     1.383010      1.37259
3     0.949363     1.061360      1.41005
4     1.205930    -1.550260      1.28491

[5 rows x 78 columns]

fig, axes = plt.subplots(1,1,figsize=(7,4))

beta_humid = fit.stan_variable('beta_CO2')
alpha_humid = fit.stan_variable('alpha')
for i in range(100):
```

```
    axes.plot(df['CO2'], alpha_humid[i]
+beta_humid[i]*np.array(df['CO2']), linewidth = 0.5, color='b')
plt.title("Lines for each sampled data and temperature anomaly")
axes.scatter(df['CO2'], df['Temperature'], color= 'black')
axes.set_xlabel('CO2')
axes.set_ylabel('Temperature anomaly')
axes.annotate(text='max',xy=(80,320), weight = 'bold', color = 'r',
fontsize = 15)
axes.annotate(text='min',xy=(80,20), weight = 'bold', color = 'r',
fontsize = 15)

Text(80, 20, 'min')
```

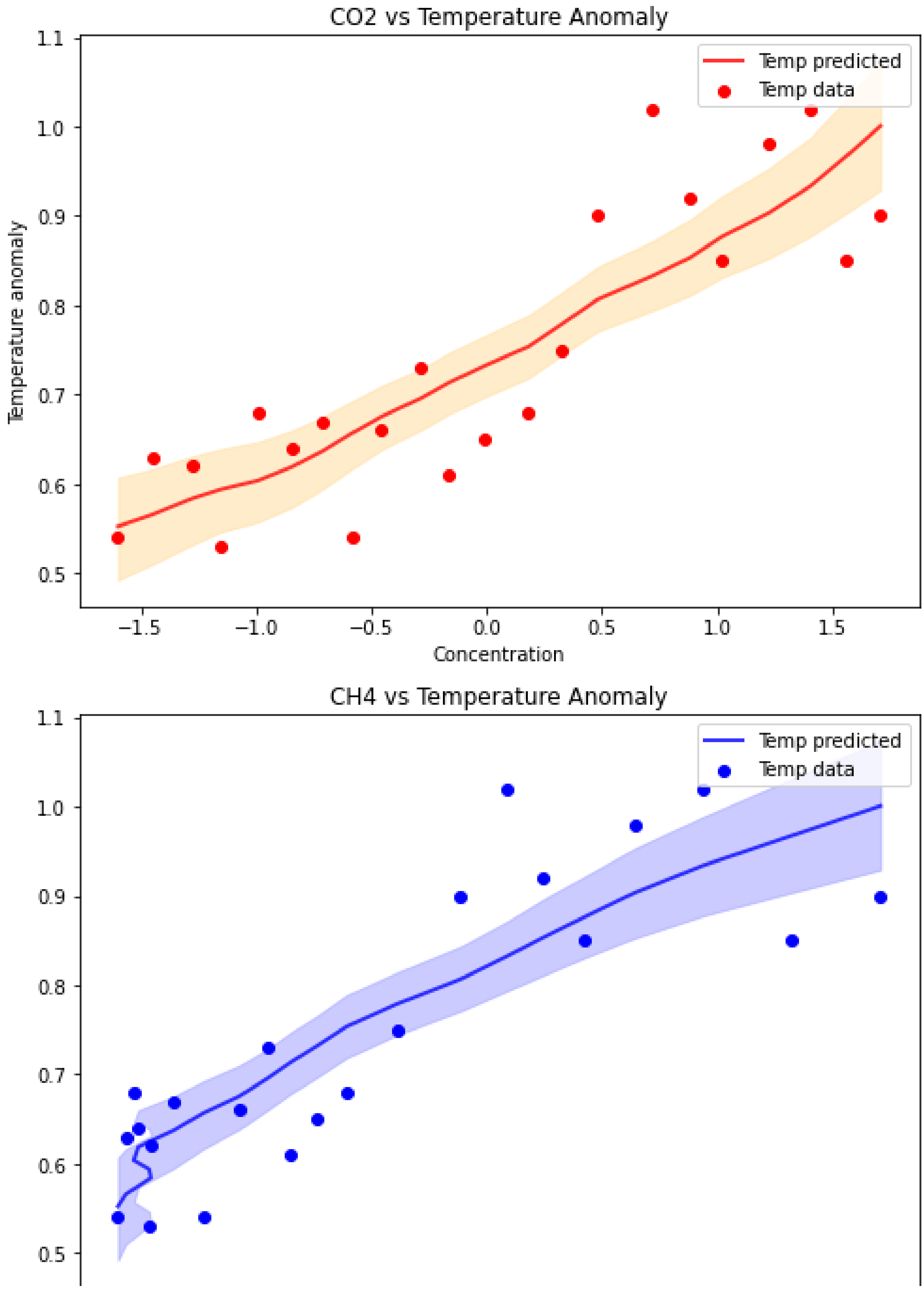


Now the lines are more adjusted to actual data. There are less data points on the center and so there are less wide lines there. As the data is spread at the ends so are the lines

```
import matplotlib.pyplot as plt
import matplotlib as mpl
import numpy as np

CO2 = np.array(df['CO2'])
CH4 = np.array(df['CH4'])
N2O = np.array(df['N2O'])
Temperature = np.array(df['Temperature'])
mu_CO2 = fit.stan_variable('mean')
mu_CH4 = fit.stan_variable('mean')
mu_N2O = fit.stan_variable('mean')
```

```
fig, ax = plt.subplots(3, 1, figsize=(7 ,15))

ax[0].fill_between(
    CO2,
    np.percentile(mu_CO2, 5, axis=0),
    np.percentile(mu_CO2, 95, axis=0),
    color=1 - 0.4 * (1 - np.array(mpl.colors.to_rgb('orange'))),
    alpha=0.5
)

ax[1].fill_between(
    CH4,
    np.percentile(mu_CH4, 5, axis=0),
    np.percentile(mu_CH4, 95, axis=0),
    color=1 - 0.4 * (1 - np.array(mpl.colors.to_rgb('blue'))),
    alpha=0.5
)

ax[2].fill_between(
    N2O,
    np.percentile(mu_N2O, 5, axis=0),
    np.percentile(mu_N2O, 95, axis=0),
    color=1 - 0.4 * (1 - np.array(mpl.colors.to_rgb('green'))),
    alpha=0.5
)

ax[0].plot(
    CO2,
    np.percentile(mu_CO2, 50, axis=0),
    color='red',
    linewidth=2,
    alpha=0.8,
    label='Temp predicted'
)
ax[1].plot(
    CH4,
    np.percentile(mu_CH4, 50, axis=0),
    color='blue',
    linewidth=2,
    alpha=0.8,
    label='Temp predicted'
)
ax[2].plot(
    N2O,
    np.percentile(mu_N2O, 50, axis=0),
    color='green',
    linewidth=2,
    alpha=0.8,
```

```
    label='Temp predicted'
)

ax[0].scatter(CO2, Temperature, color='red', label='Temp data')
ax[1].scatter(CH4, Temperature, color='blue', label='Temp data')
ax[2].scatter(N2O, Temperature, color='green', label='Temp data')

ax[0].set_xlabel('Concentration')
ax[0].set_ylabel('Temperature anomaly')


ax[0].legend()
ax[1].legend()
ax[2].legend()

ax[0].set_title('CO2 vs Temperature Anomaly')
ax[1].set_title('CH4 vs Temperature Anomaly')
ax[2].set_title('N2O vs Temperature Anomaly')
plt.tight_layout()
plt.show()
```

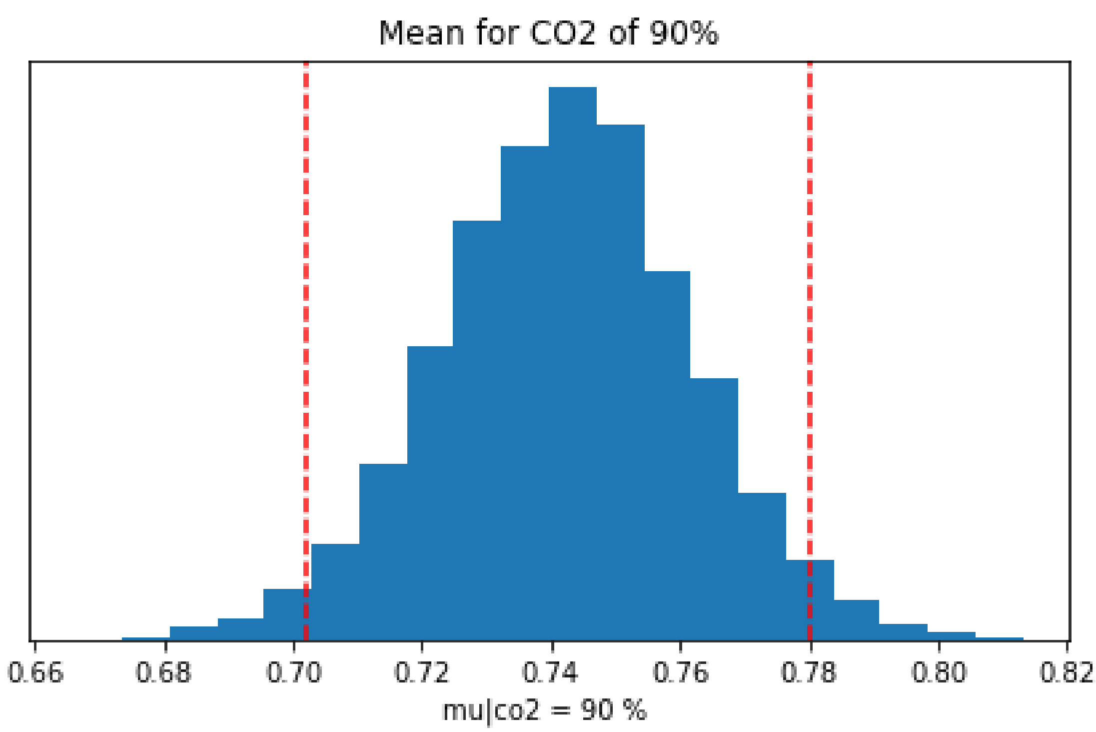

CO2 vs Temperature Anomaly
Temp predicted
Temp data
Temperature anomaly
Concentration
1.1
1.0
0.9
0.8
0.7
0.6
0.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
CH4 vs Temperature Anomaly
Temp predicted
Temp data
1.1
1.0
0.9
0.8
0.7
0.6
0.5

Shown output data is consistent with the provided data of temperatures anomaly. For each gases the output mean fits great.

Marginal Distribution

```
alpha_post =  fit.stan_variable('alpha')
beta_post = fit.stan_variable('beta_CO2')
mu_post = fit.stan_variable('mean')

mu90 = alpha_post+beta_post*(np.mean(df['CO2']))
mu_95p = az.hdi(mu90,.95)
fig, ax = plt.subplots(1, 1, figsize=(7, 4))
ax.hist(mu90,bins=20,density=True)
plt.axvline(mu_95p[0], linestyle = '--', color = 'r')
plt.axvline(mu_95p[1], linestyle = '--', color = 'r')
ax.set_title('Mean for CO2 of 90%')
ax.set_yticks(())
ax.set_xlabel('mu|co2 = 90 % ')
plt.show()
print('Mean: {:4.2f}'.format(np.mean(mu90)))
print('95% confidence interval: ',['{:4.2f}'.format(k) for k in
az.hdi(mu90,.95)])
```

```
0.7803419999999998
```




```
Mean: 0.74
95% confidence interval:  ['0.70', '0.78']
```

From above histogram we can see that on 90% propability the temperature anomaly will be in range 0.7 and 0.78. The other gases are not gonna be checked due to the similar data with CO2

Model 2 - Student_t distribution

Our second model approach will be to apply a Student-t distribution   The Student-t distribution is similar in shape to the Gaussian but has heavier tails. It is symmetric and bell-shaped, with single peak. This model have more probability mass and allows to better represnt data that may have outliers.   The t-distribution is characterized by called degrees of freedom ν (nu) which determines the shape of the distribution. The more degrees of freedom the closer it is to the Gaussian distribution. The model have two other parameters: μ (mu) – location, σ (sigma)- scale witch are similar to the Gaussian model.   For this model we used the linear relation too.

Prior

Alpha, beta and sigma data were selected similarly as in the first model. The main difference is new nu parameter which we choose based on internet: https://statmodeling.stat.columbia.edu/2015/05/17/do-we-have-any-recommendations-for-priors-for-student_ts-degrees-of-freedom-parameter/

```
%%writefile /home/temp7_ppc_prior.stan
data {
  int<lower=0> N;
  vector[N] CO2;
  vector[N] CH4;
  vector[N] N2O;
}

generated quantities {
  real sigma = normal_rng(0.1,0.05);
  real nu = gamma_rng(2, 0.1);
  real alpha = normal_rng(0.7, 0.1);
  real beta_CO2 = normal_rng(0, 0.1);
  real beta_CH4 = normal_rng(0, 0.1);
  real beta_N2O = normal_rng(0, 0.1);
  vector[N] temperature;

  for (i in 1:N) {
    temperature[i] = student_t_rng(nu, alpha + beta_CO2 * CO2[i] +
beta_CH4 * CH4[i] + beta_N2O * N2O[i], sigma);
  }
}
```

```
Overwriting /home/temp7_ppc_prior.stan
```

```
data_sim={'N':len(df),
'CO2':np.linspace(df.CO2.min(),df.CO2.max(),len(df)),'CH4':np.linspace
```

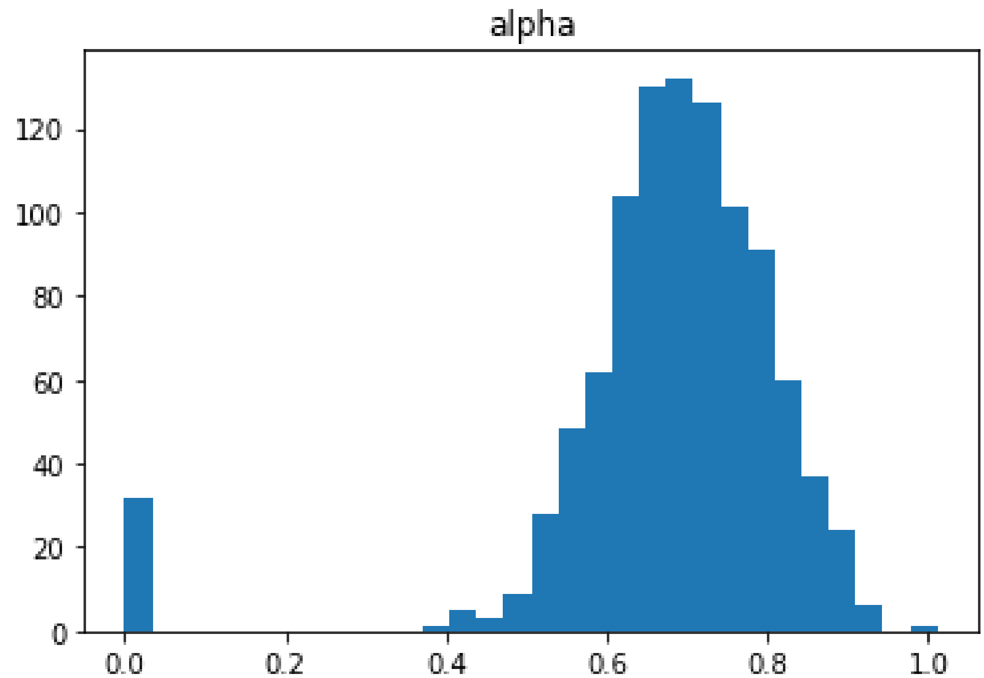
```
(df.CH4.min(),df.CH4.max(),len(df)),'N2O':np.linspace(df.N2O.min(),df.
N2O.max(),len(df))}

model_ppc7_p=CmdStanModel(stan_file='/home/temp7_ppc_prior.stan')
R = 1000
sim7=model_ppc7_p.sample(data=data_sim,
                     iter_sampling=R,
                     iter_warmup=0,
                     chains=1,
                     refresh=R,
                     fixed_param=True,
                     seed=29042020)
df_7_p = sim7.draws_pd()
df_7_p.head()
```

```
INFO:cmdstanpy:found newer exe file, not recompiling
INFO:cmdstanpy:CmdStan start processing
chain 1 |██████████| 00:00 Sampling completed



INFO:cmdstanpy:CmdStan done processing.
```

```
   lp__  accept_stat__     sigma       nu     alpha  beta_CO2
beta_CH4  \
0   0.0            0.0  0.235409  10.8336  0.693634 -0.127227
0.012759
1   0.0            0.0  0.122755  20.6335  0.643012  0.104238
0.116276
2   0.0            0.0  0.048386  26.1987  0.650050  0.024688 -
0.058932
3   0.0            0.0  0.000000   0.0000  0.000000  0.000000
0.000000
4   0.0            0.0  0.101699   9.8558  0.743006  0.110379 -
0.049400

   beta_N2O  temperature[1]  temperature[2]  ...  temperature[13]  \
0 -0.138942        1.146710        1.216910  ...         0.541558
1  0.188285        0.299489        0.171137  ...         0.850172
2 -0.001650        0.727714        0.671177  ...         0.603377
3  0.000000        0.000000        0.000000  ...         0.000000
4 -0.065600        0.685745        0.779902  ...         0.881137

   temperature[14]  temperature[15]  temperature[16]  temperature[17]
\
0         0.803151         0.689611         0.268663         0.122831

1         1.016260         1.171410         1.099780         0.851386
```

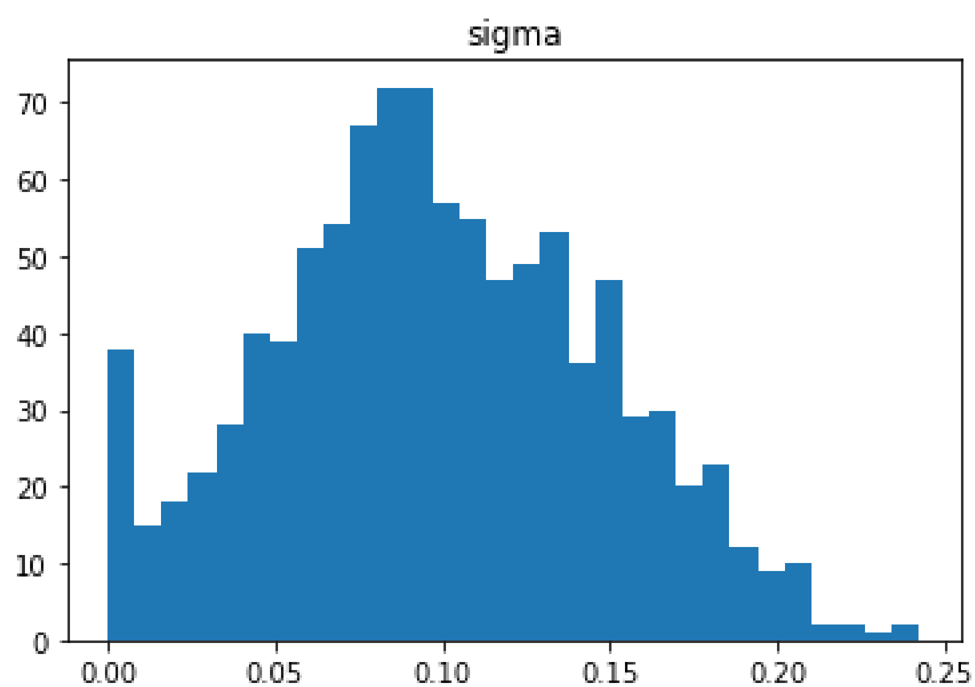
```
2         0.573427         0.637035         0.582179         0.608737

3         0.000000         0.000000         0.000000         0.000000

4         0.738824         0.655149         0.699817         0.662181


   temperature[18]  temperature[19]  temperature[20]  temperature[21]
\
0         0.573486        -0.109956         0.486857         0.529211

1         0.944927         1.277530         1.283710         1.525910

2         0.525994         0.634293         0.627904         0.529219

3         0.000000         0.000000         0.000000         0.000000

4         0.733661         0.765332         0.944646         0.704042


   temperature[22]
0         0.217541
1         1.516970
2         0.455498
3         0.000000
4         0.814588

[5 rows x 30 columns]
```

```
plt.hist(df_7_p['alpha'], bins=30)
plt.title('alpha')
plt.show()
```



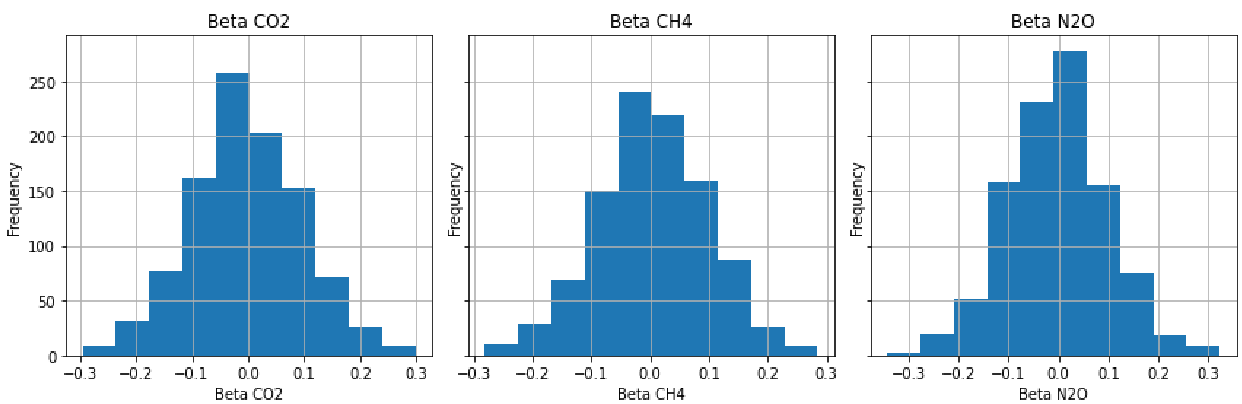
Parameter alpha is in right range and most of the values are in the middle but some stick out.

```
plt.hist(df_7_p['sigma'], bins=30)
plt.title('sigma')
plt.show()
```



Parameter sigma have values in the middle and this is okay

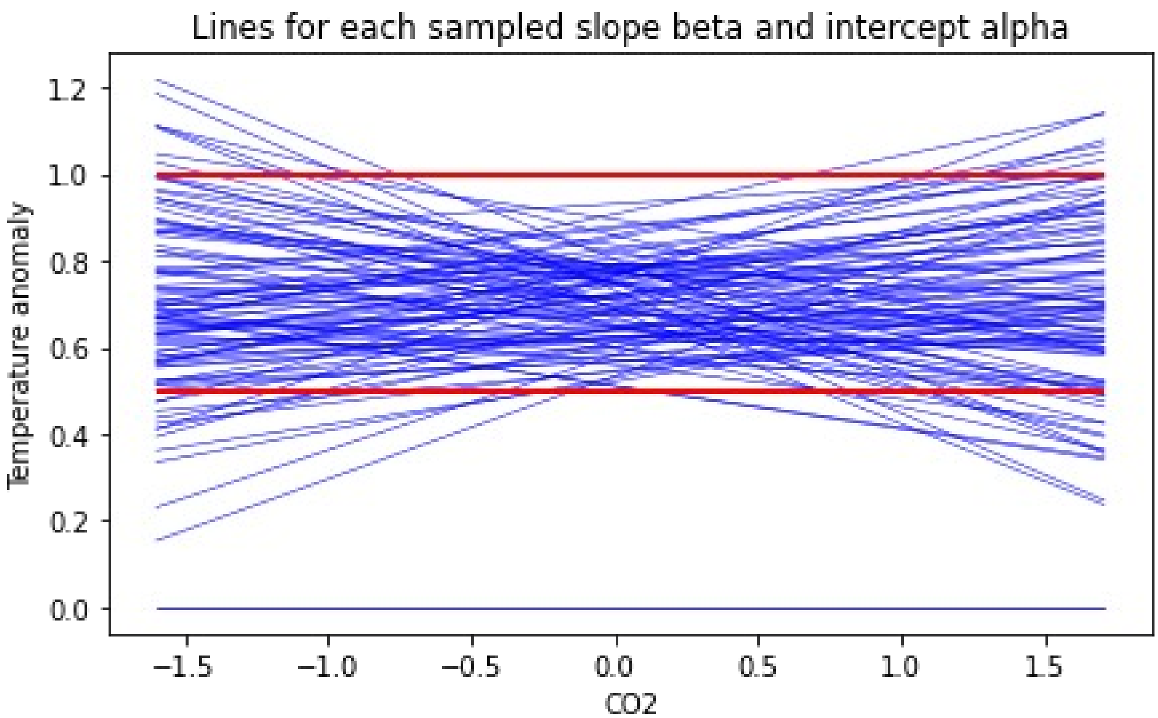
```
fig, axs = plt.subplots(1, 3, sharey=True, figsize=(12, 4))

axs[0].hist(df_7_p['beta_CO2'])
axs[0].set_xlabel('Beta CO2')
axs[0].set_ylabel('Frequency')
axs[0].set_title('Beta CO2')
axs[0].grid()

axs[1].hist(df_7_p['beta_CH4'])
axs[1].set_xlabel('Beta CH4')
axs[1].set_ylabel('Frequency')
axs[1].set_title('Beta CH4')
axs[1].grid()

axs[2].hist(df_7_p['beta_N2O'])
axs[2].set_xlabel('Beta N2O')
axs[2].set_ylabel('Frequency')
axs[2].set_title('Beta N2O')
axs[2].grid()

plt.tight_layout()
plt.show()
```



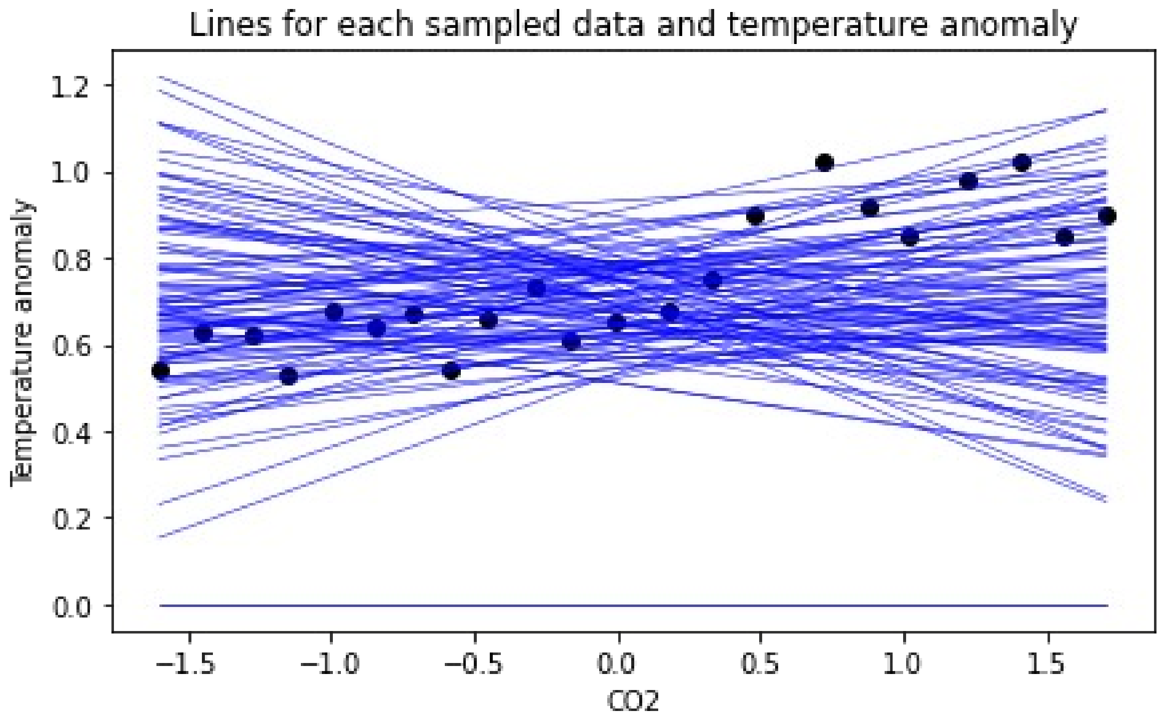
In this model we put the same values for beta distribution like in the first model and the historams look almost the same

```
fig, axes = plt.subplots(1,1,figsize=(7,4))

beta_humid = sim7.stan_variable('beta_CO2')
alpha_humid = sim7.stan_variable('alpha')
for i in range(100):
    axes.plot(df['CO2'], alpha_humid[i]
+beta_humid[i]*np.array(df['CO2']), linewidth = 0.5, color='b')
plt.title("Lines for each sampled slope beta and intercept alpha")
axes.set_xlabel('CO2')
axes.set_ylabel('Temperature anomaly')
axes.hlines([0.5, 1],xmin = df['CO2'].min(), xmax = df['CO2'].max(),
```

```
linestyles = '-',linewidth = 2, color = 'r')
axes.annotate(text='max',xy=(80,320), weight = 'bold', color = 'r',
fontsize = 15)
axes.annotate(text='min',xy=(80,20), weight = 'bold', color = 'r',
fontsize = 15)

Text(80, 20, 'min')
```




```
fig, axes = plt.subplots(1,1,figsize=(7,4))

beta_humid = sim7.stan_variable('beta_CO2')
alpha_humid = sim7.stan_variable('alpha')
for i in range(100):
    axes.plot(df['CO2'], alpha_humid[i]
+beta_humid[i]*np.array(df['CO2']), linewidth = 0.5, color='b')
plt.title("Lines for each sampled data and temperature anomaly")
axes.scatter(df['CO2'], df['Temperature'], color= 'black')
axes.set_xlabel('CO2')
axes.set_ylabel('Temperature anomaly')
axes.annotate(text='max',xy=(80,320), weight = 'bold', color = 'r',
fontsize = 15)
axes.annotate(text='min',xy=(80,20), weight = 'bold', color = 'r',
fontsize = 15)

Text(80, 20, 'min')
```



In this two plots we can see that the actual temperature data correspods with data from priors and the most of samples fits.

Prosterior predictive check

Fitting model to data

```
%%writefile /home/temp7_ppc.stan
data {
  int<lower=0> N; // number of data points
  vector[N] CO2;
  vector[N] CH4;
  vector[N] N2O;
  vector[N] temp;
}
parameters {
  real<lower=0> alpha;
  real<lower=0> beta_CO2;
  real<lower=0> beta_CH4;
  real<lower=0> beta_N2O;
  real<lower=0> sigma;
  real<lower=1, upper=80> nu;
}
transformed parameters {
  vector[N] mu;
```

```
  mu = alpha + beta_CO2 * CO2 + beta_CH4 * CH4 + beta_N2O * N2O;
}
model {
  nu ~ gamma(2, 0.1); // found this online: Juarez and Steel(2010)
  temp ~ student_t(nu, mu, sigma);
  alpha ~ normal(0.7, 0.1);
  beta_CO2 ~ normal(0, 0.1);
  beta_CH4 ~ normal(0, 0.1);
  beta_N2O ~ normal(0, 0.1);
  sigma ~ normal(0.1,0.05);

}
generated quantities {
    vector[N] temp_i;
    vector[N] log_lik;
    for (i in 1:N) {
        temp_i[i] = student_t_rng(nu,mu[i],sigma);
        log_lik[i] = student_t_lpdf(temp[i] | nu, mu[i], sigma);
    }
}
```

```
Overwriting /home/temp7_ppc.stan
```

```
N = len(df)
data_fit = {'N':N, 'CO2': df.CO2.values[:N], 'temp':
df.Temperature.values[:N], 'CH4': df.CH4.values[:N], 'N2O':
df.N2O.values[:N]}

model_ppc7=CmdStanModel(stan_file='/home/temp7_ppc.stan')
fit7=model_ppc7.sample(data=data_fit,seed=28052020)
```

```
INFO:cmdstanpy:compiling stan file /home/temp7_ppc.stan to exe file
/home/temp7_ppc
INFO:cmdstanpy:compiled model executable: /home/temp7_ppc
INFO:cmdstanpy:CmdStan start processing
chain 1 |          | 00:00 Status
▌          | 00:00 Status

█          | 00:01 Iteration:    1 / 2000 [  0%]  (Warmup)
██         | 00:01 Iteration:  200 / 2000 [ 10%]  (Warmup)

chain 1 |███       | 00:01 Iteration:  400 / 2000 [ 20%]  (Warmup)

████       | 00:01 Iteration:  600 / 2000 [ 30%]  (Warmup)

chain 1 |█████     | 00:01 Iteration:  800 / 2000 [ 40%]  (Warmup)
██████▌    | 00:01 Iteration: 1001 / 2000 [ 50%]  (Sampling)
███████▌   | 00:02 Iteration: 1200 / 2000 [ 60%]  (Sampling)
████████▌  | 00:02 Iteration: 1400 / 2000 [ 70%]  (Sampling)

chain 1 |█████████ | 00:02 Iteration: 1500 / 2000 [ 75%]  (Sampling)
```

```
chain 1 |█████████  | 00:02 Iteration: 1600 / 2000 [ 80%]  (Sampling)
██████████ | 00:03 Iteration: 1800 / 2000 [ 90%]  (Sampling)
chain 1 |██████████| 00:03 Sampling completed
chain 2 |██████████| 00:03 Sampling completed

chain 3 |██████████| 00:03 Sampling completed
chain 4 |██████████| 00:03 Sampling completed




INFO:cmdstanpy:CmdStan done processing.
```

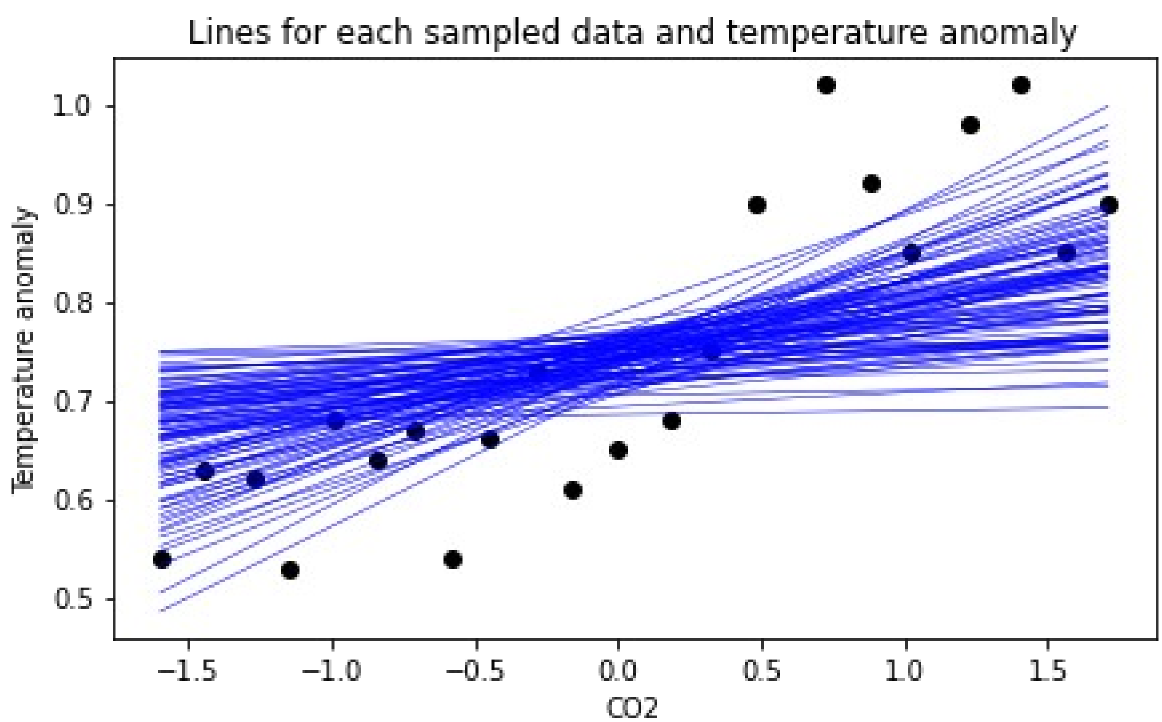
```
df_7 = fit7.draws_pd()
df_7.head()
```

```
      lp__  accept_stat__  stepsize__  treedepth__  n_leapfrog__
divergent__  \
0  25.7689       0.998325    0.161263          5.0          31.0
0.0
1  25.7824       0.915042    0.161263          4.0          31.0
0.0
2  26.5245       0.846326    0.161263          4.0          15.0
0.0
3  25.5080       0.970196    0.161263          4.0          31.0
0.0
4  26.0037       0.991388    0.161263          5.0          31.0
0.0

   energy__     alpha  beta_CO2  beta_CH4  ...  log_lik[13]
log_lik[14]  \
0  -24.0963  0.728749  0.093085  0.020597  ...      1.19597
1.44717
1  -24.4012  0.739005  0.076416  0.029738  ...      1.17832
1.51840
2  -23.1515  0.740603  0.012135  0.044226  ...      1.19051
1.51555
3  -24.3903  0.720865  0.011273  0.081586  ...      1.35722
1.47353
4  -23.2490  0.713015  0.029480  0.059188  ...      1.38129
1.48738

   log_lik[15]  log_lik[16]  log_lik[17]  log_lik[18]  log_lik[19]  \
0     0.695641    -0.875695     1.024390      1.46124     0.853017
1     0.706552    -1.155270     1.028280      1.54840     0.778901
2     0.916877    -0.983429     1.192180      1.52808     1.092050
3     0.687008    -0.970646     1.029450      1.46343     0.990059
4     0.557741    -1.232300     0.922152      1.48755     0.836874
```

```
   log_lik[20]  log_lik[21]  log_lik[22]
0     0.666402     1.059290     1.271890
1     0.536973     1.125190     1.357240
2     0.978348     0.534046     0.737766
3     0.953195     0.533380     0.591975
4     0.746938     0.839755     0.964892

[5 rows x 79 columns]
```

```
fig, axes = plt.subplots(1,1,figsize=(7,4))

beta_humid = fit7.stan_variable('beta_CO2')
alpha_humid = fit7.stan_variable('alpha')
for i in range(100):
    axes.plot(df['CO2'], alpha_humid[i]
+beta_humid[i]*np.array(df['CO2']), linewidth = 0.5, color='b')
plt.title("Lines for each sampled data and temperature anomaly")
axes.scatter(df['CO2'], df['Temperature'], color= 'black')
axes.set_xlabel('CO2')
axes.set_ylabel('Temperature anomaly')
axes.annotate(text='max',xy=(80,320), weight = 'bold', color = 'r',
fontsize = 15)
axes.annotate(text='min',xy=(80,20), weight = 'bold', color = 'r',
fontsize = 15)
```

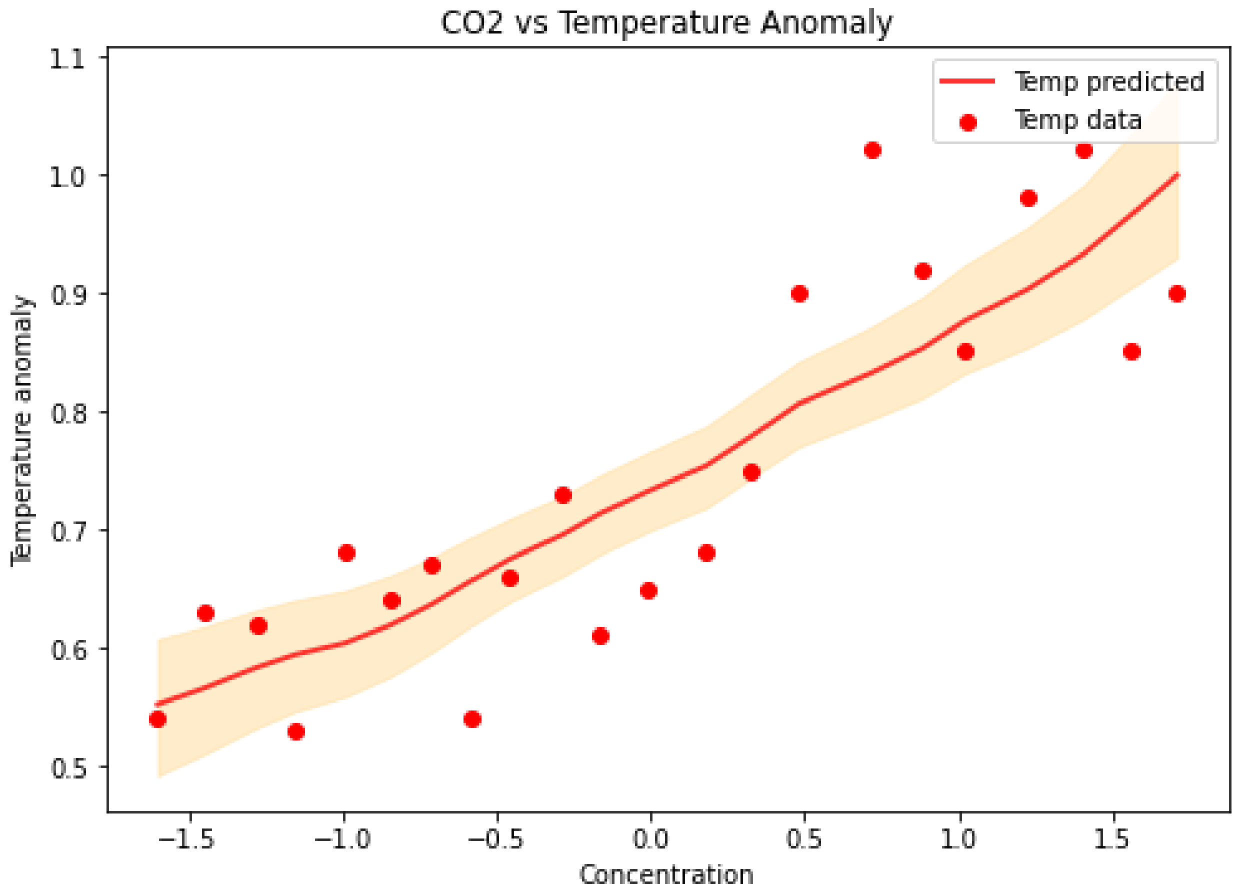
```
Text(80, 20, 'min')
```

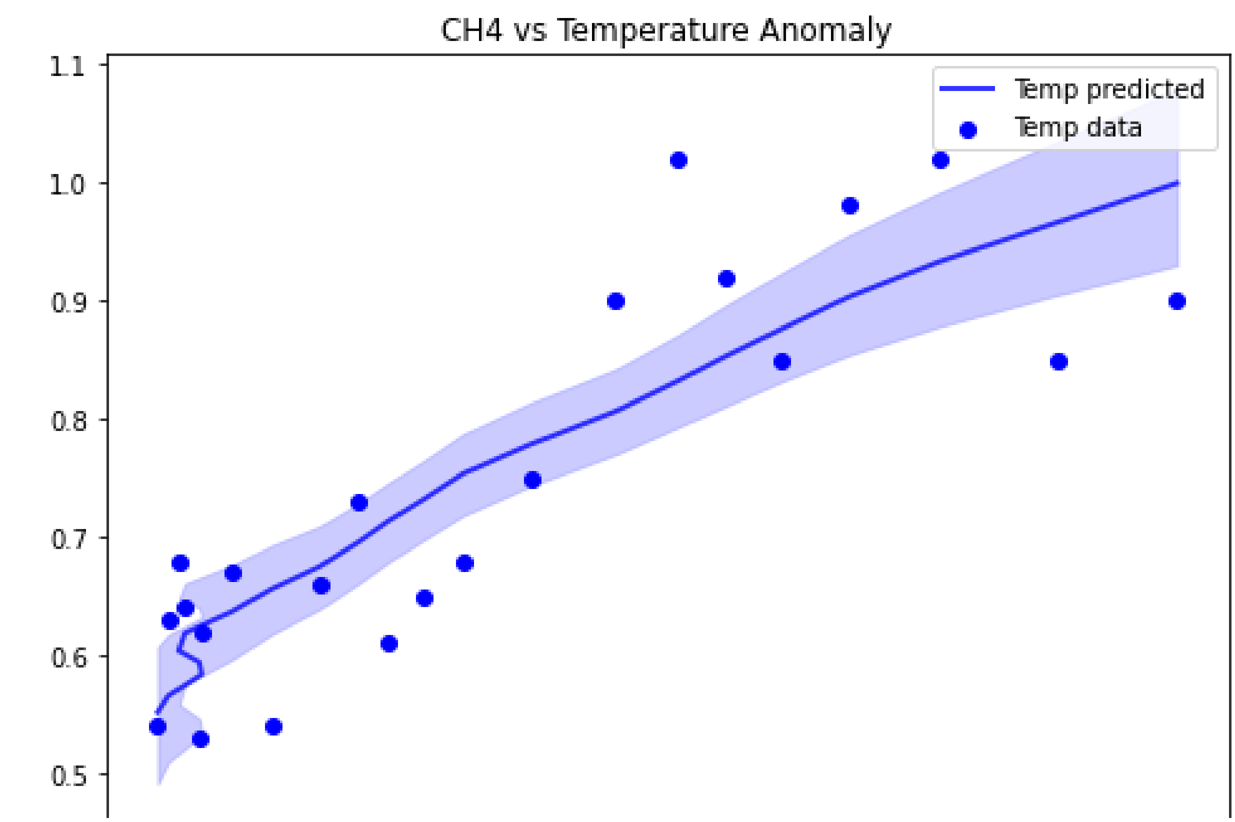


In this plot we can see lines adjusted to actual data. In the model we can see more spread at the end of the x-axis than in Gaussian model and model fits better.

```
import matplotlib as mpl
CO2 = np.array(df['CO2'])
CH4 = np.array(df['CH4'])
N2O = np.array(df['N2O'])
Temperature = np.array(df['Temperature'])
mu_CO2 = fit7.stan_variable('mu')
mu_CH4 = fit7.stan_variable('mu')
mu_N2O = fit7.stan_variable('mu')


fig, ax = plt.subplots(3, 1, figsize=(7 ,15))

ax[0].fill_between(
    CO2,
    np.percentile(mu_CO2, 5, axis=0),
    np.percentile(mu_CO2, 95, axis=0),
    color=1 - 0.4 * (1 - np.array(mpl.colors.to_rgb('orange'))),
    alpha=0.5
)

ax[1].fill_between(
    CH4,
    np.percentile(mu_CH4, 5, axis=0),
    np.percentile(mu_CH4, 95, axis=0),
    color=1 - 0.4 * (1 - np.array(mpl.colors.to_rgb('blue'))),
    alpha=0.5
)

ax[2].fill_between(
    N2O,
    np.percentile(mu_N2O, 5, axis=0),
    np.percentile(mu_N2O, 95, axis=0),
    color=1 - 0.4 * (1 - np.array(mpl.colors.to_rgb('green'))),
    alpha=0.5
)

ax[0].plot(
    CO2,
    np.percentile(mu_CO2, 50, axis=0),
    color='red',
    linewidth=2,
    alpha=0.8,
    label='Temp predicted'
)
ax[1].plot(
    CH4,
    np.percentile(mu_CH4, 50, axis=0),
```

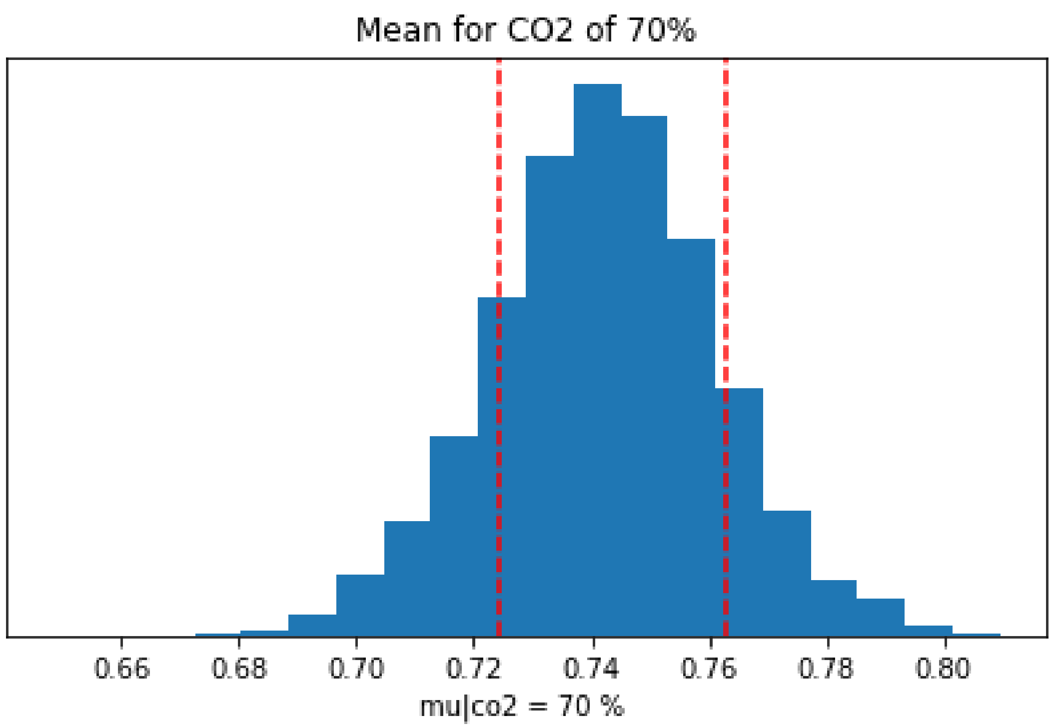
```
    color='blue',
    linewidth=2,
    alpha=0.8,
    label='Temp predicted'
)
ax[2].plot(
    N2O,
    np.percentile(mu_N2O, 50, axis=0),
    color='green',
    linewidth=2,
    alpha=0.8,
    label='Temp predicted'
)

ax[0].scatter(CO2, Temperature, color='red', label='Temp data')
ax[1].scatter(CH4, Temperature, color='blue', label='Temp data')
ax[2].scatter(N2O, Temperature, color='green', label='Temp data')

ax[0].set_xlabel('Concentration')
ax[0].set_ylabel('Temperature anomaly')


ax[0].legend()
ax[1].legend()
ax[2].legend()

ax[0].set_title('CO2 vs Temperature Anomaly')
ax[1].set_title('CH4 vs Temperature Anomaly')
ax[2].set_title('N2O vs Temperature Anomaly')
plt.tight_layout()
plt.show()
```


CO2 vs Temperature Anomaly
Temp predicted
Temp data
Temperature anomaly
Concentration
1.1
1.0
0.9
0.8
0.7
0.6
0.5
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5


CH4 vs Temperature Anomaly
Temp predicted
Temp data
1.1
1.0
0.9
0.8
0.7
0.6
0.5

In the output data we can see that provided data of temperatures anomaly fits great.

```
alpha_post =  fit7.stan_variable('alpha')
beta_post = fit7.stan_variable('beta_CO2')
mu_post = fit7.stan_variable('mu')

mu70 = alpha_post+beta_post*(np.mean(df['CO2']))
mu_70p = az.hdi(mu70,.70)
fig, ax = plt.subplots(1, 1, figsize=(7, 4))
ax.hist(mu70,bins=20,density=True)
plt.axvline(mu_70p[0], linestyle = '--', color = 'r')
plt.axvline(mu_70p[1], linestyle = '--', color = 'r')
ax.set_title('Mean for CO2 of 70%')
ax.set_yticks(())
ax.set_xlabel('mu|co2 = 70 % ')
plt.show()
print('Mean: {:4.2f}'.format(np.mean(mu70)))
print('70% confidence interval: ',['{:4.2f}'.format(k) for k in
az.hdi(mu70,.70)])
```




```
Mean: 0.74
70% confidence interval:  ['0.72', '0.76']
```

Histogram shows that on 70% probability will be in range 0.72 and 0.76.

Model comparison

```
print("Summary - Normal model:")
fit.summary()
```

```
Summary - Normal model:

                Mean      MCSE  StdDev        5%      50%      95%   N_Eff  \
name

lp__          29.000  0.05200   1.800  25.0000  29.000  31.000  1200.0

alpha          0.740  0.00041   0.020   0.7100   0.740   0.770  2400.0

sigma          0.092  0.00024   0.012   0.0730   0.091   0.110  2600.0

beta_CO2       0.049  0.00064   0.032   0.0046   0.045   0.110  2500.0

beta_CH4       0.042  0.00054   0.030   0.0035   0.037   0.097  3000.0

...              ...      ...     ...      ...     ...     ...     ...

log_lik[18]    0.980  0.01100   0.710  -0.4700   1.200   1.600  3910.0

log_lik[19]    0.970  0.01100   0.690  -0.4500   1.200   1.600  3980.0

log_lik[20]    0.970  0.01200   0.740  -0.4800   1.200   1.600  3759.0

log_lik[21]    0.990  0.01100   0.690  -0.3700   1.200   1.600  3864.0

log_lik[22]    0.980  0.01200   0.760  -0.4600   1.200   1.600  4136.0


             N_Eff/s  R_hat
name
lp__          2600.0    1.0
alpha         5200.0    1.0
sigma         5700.0    1.0
beta_CO2      5500.0    1.0
beta_CH4      6600.0    1.0
...              ...    ...
log_lik[18]   8613.0    1.0
log_lik[19]   8766.0    1.0
log_lik[20]   8279.0    1.0
log_lik[21]   8512.0    1.0
log_lik[22]   9109.0    1.0

[72 rows x 9 columns]
```

```python
print("Summary - Student model:")
fit7.summary()
```

```
Summary - Student model:

                  Mean     MCSE  StdDev        5%      50%      95%   N_Eff  \
name
lp__            24.000  0.05500   2.000  21.0000   25.000   27.000  1300.0
alpha            0.740  0.00038   0.019   0.7100    0.740    0.770  2600.0
beta_CO2         0.049  0.00060   0.032   0.0053    0.046    0.110  2800.0
beta_CH4         0.043  0.00053   0.030   0.0040    0.038    0.099  3200.0
beta_N2O         0.043  0.00059   0.031   0.0040    0.037    0.100  2800.0
...                ...      ...     ...      ...      ...      ...     ...
log_lik[18]      1.400  0.00380   0.200   1.1000    1.400    1.700  2688.0
log_lik[19]      1.000  0.00580   0.340   0.3900    1.100    1.500  3362.0
log_lik[20]      0.900  0.00700   0.410   0.1000    0.970    1.400  3511.0
log_lik[21]      0.480  0.00950   0.590  -0.6300    0.570    1.300  3858.0
log_lik[22]      0.690  0.00960   0.600  -0.4800    0.800    1.500  3945.0

             N_Eff/s  R_hat
name
lp__          1800.0    1.0
alpha         3700.0    1.0
beta_CO2      3900.0    1.0
beta_CH4      4500.0    1.0
beta_N2O      3900.0    1.0
...              ...    ...
log_lik[18]   3765.0    1.0
log_lik[19]   4708.0    1.0
log_lik[20]   4918.0    1.0
log_lik[21]   5403.0    1.0
log_lik[22]   5525.0    1.0

[73 rows x 9 columns]
```

Values for returned parameters are quite similar. We assumed that in both models that may look alike

```
fitStudent_ = az.from_cmdstanpy(posterior=fit7,
                           log_likelihood='log_lik',
                           posterior_predictive='temp_i',

observed_data={'temperature':df['Temperature']})
fitStudent_
```

```
Inference data with groups:
      > posterior
      > posterior_predictive
      > log_likelihood
      > sample_stats
      > observed_data
```

```
fitNormal_ = az.from_cmdstanpy(posterior=fit,
                           log_likelihood='log_lik',
                           posterior_predictive='temp_',

observed_data={'temperature':df['Temperature']})

az.loo(fitStudent_)
```

```
Computed from 4000 by 22 log-likelihood matrix

         Estimate       SE
elpd_loo    20.87     2.62
p_loo        2.55        -
------

Pareto k diagnostic values:
                         Count   Pct.
(-Inf, 0.5]   (good)       22  100.0%
 (0.5, 0.7]   (ok)          0    0.0%
   (0.7, 1]   (bad)         0    0.0%
   (1, Inf)   (very bad)    0    0.0%
```

```
az.waic(fitStudent_)
```

```
/usr/local/lib/python3.9/site-packages/arviz/stats/stats.py:1635:
UserWarning: For one or more samples the posterior variance of the log
predictive densities exceeds 0.4. This could be indication of WAIC
starting to fail.
See http://arxiv.org/abs/1507.04544 for details
  warnings.warn(

Computed from 4000 by 22 log-likelihood matrix

          Estimate       SE
elpd_waic    20.92     2.60
p_waic        2.51        -
```

```
There has been a warning during the calculation. Please check the
results.
```

The model with students distribution gives very similar result for WAIC and LOO

```
az.loo(fitNormal_)

/usr/local/lib/python3.9/site-packages/arviz/stats/stats.py:811:
UserWarning: Estimated shape parameter of Pareto distribution is
greater than 0.7 for one or more samples. You should consider using a
more robust model, this is because importance sampling is less likely
to work well if the marginal posterior and LOO posterior are very
different. This is more likely to happen with a non-robust model and
highly influential observations.
  warnings.warn(

Computed from 4000 by 22 log-likelihood matrix

         Estimate       SE
elpd_loo     6.01     0.73
p_loo       18.99        -

There has been a warning during the calculation. Please check the
results.
------

Pareto k diagnostic values:
                         Count   Pct.
(-Inf, 0.5]   (good)        0    0.0%
 (0.5, 0.7]   (ok)          1    4.5%
   (0.7, 1]   (bad)        16   72.7%
   (1, Inf)   (very bad)    5   22.7%

az.waic(fitStudent_)

/usr/local/lib/python3.9/site-packages/arviz/stats/stats.py:1635:
UserWarning: For one or more samples the posterior variance of the log
predictive densities exceeds 0.4. This could be indication of WAIC
starting to fail.
See http://arxiv.org/abs/1507.04544 for details
  warnings.warn(

Computed from 4000 by 22 log-likelihood matrix

          Estimate       SE
elpd_waic    20.92     2.60
p_waic        2.51        -

There has been a warning during the calculation. Please check the
results.
```

In the model with Normal distribution only WAIC is the same as WAIC nad LOO for Student-t model. The reason why LOO and WAIC varies here it is because they have different evaluation strategies. WAIC focus on entire dataset while LOO on every point of data. When it comes to LOO it shows that model is not so good but focusing on whole dataset the evaluation is much better.

LOO

```
LOO_compare = az.compare({'Student model':fitStudent_, 'Normal
model':fitNormal_}, ic='loo')
LOO_compare
```

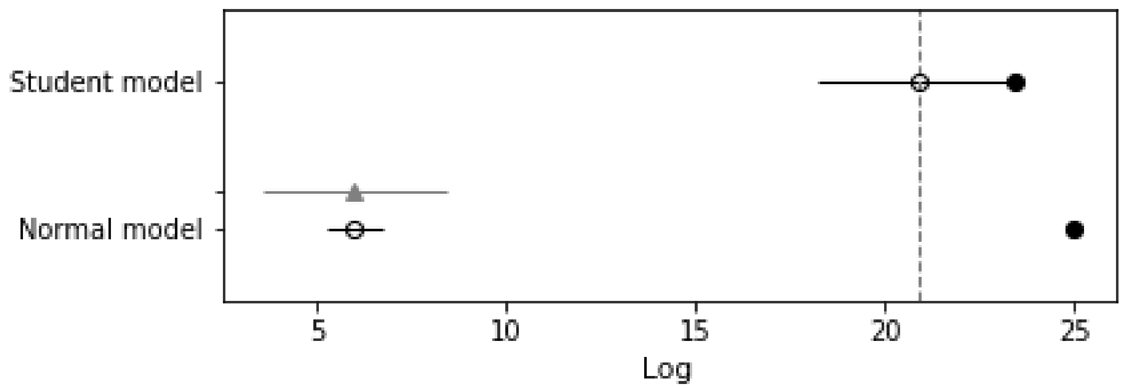
```
/usr/local/lib/python3.9/site-packages/arviz/stats/stats.py:811:
UserWarning: Estimated shape parameter of Pareto distribution is
greater than 0.7 for one or more samples. You should consider using a
more robust model, this is because importance sampling is less likely
to work well if the marginal posterior and LOO posterior are very
different. This is more likely to happen with a non-robust model and
highly influential observations.
  warnings.warn(
```

```
               rank        loo      p_loo      d_loo  weight        se
\
Student model     0  20.873971   2.553913   0.000000     1.0  2.616999

Normal model      1   6.009034  18.990402  14.864938     0.0  0.731370


                    dse  warning loo_scale
Student model  0.000000    False       log
Normal model   2.418414     True       log
```

Smaller rank indicates which model is better. Here we can see that Student model is much better from the other one. But lets see the output when it comes to whole dataset -WAIC.

```
az.plot_compare(LOO_compare)
```

```
<AxesSubplot:xlabel='Log'>
```



WAIC

```
WAIC_compare = az.compare({'Student model':fitStudent_, 'Gaussian
model':fitNormal_}, ic='waic')
WAIC_compare
```

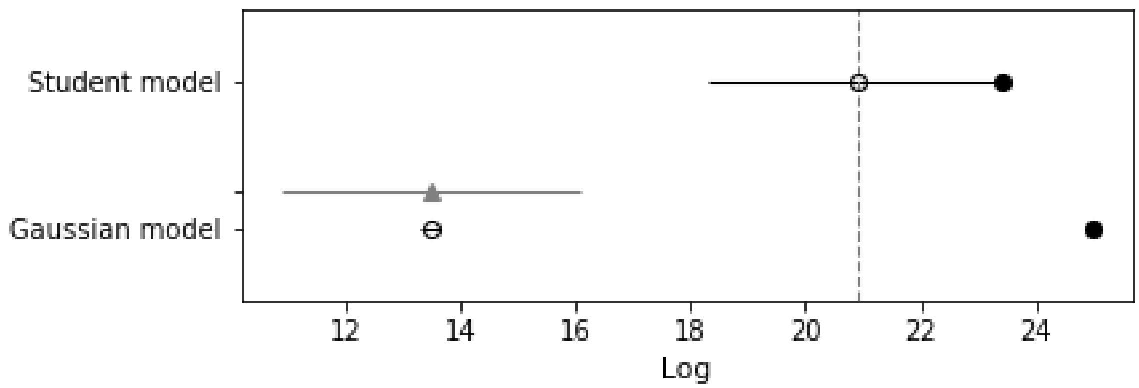
```
/usr/local/lib/python3.9/site-packages/arviz/stats/stats.py:1635:
UserWarning: For one or more samples the posterior variance of the log
predictive densities exceeds 0.4. This could be indication of WAIC
starting to fail.
See http://arxiv.org/abs/1507.04544 for details
  warnings.warn(
/usr/local/lib/python3.9/site-packages/arviz/stats/stats.py:1635:
UserWarning: For one or more samples the posterior variance of the log
predictive densities exceeds 0.4. This could be indication of WAIC
starting to fail.
See http://arxiv.org/abs/1507.04544 for details
  warnings.warn(
```

```
                rank       waic     p_waic    d_waic  weight        se
\
Student model      0  20.916747   2.511138  0.000000     1.0  2.602951

Gaussian model     1  13.469474  11.529962  7.447273     0.0  0.163882


                     dse  warning waic_scale
Student model   0.000000     True        log
Gaussian model  2.600035     True        log
```

Here also the output says that the Student model is better than the Gaussian Model.

```
az.plot_compare(WAIC_compare)
```

```
<AxesSubplot:xlabel='Log'>
```



With WAIC evaluation the difference is much smaller than with LOO but the better model is still the same.

To sum up both models adjusted to the data pretty well. The secound approach turned out to be better than the first one - Gaussian.