Revisiting Skip-Gram Negative Sampling Model With Regularization

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Abstract

We revisit skip-gram negative sampling (SGNS), a popular neural-network based approach to learning distributed word representation. We first point out the ambiguity issue undermining the SGNS model, in the sense that the word vectors can be entirely distorted without changing the objective value. To resolve this issue, we rectify the SGNS model with quadratic regularization. A theoretical justification, which provides a novel insight into quadratic regularization, is presented. Preliminary experiments are also conducted on Google's analytical reasoning task to support the modified SGNS model.

1 Introduction

Distributed word representations, a.k.a. word embeddings, represents each word with a real-valued vector as an approximation to its linguistic meaning. Different from the traditional discrete and sparse one-hot encoding, such continuous and dense representations are shown to better capture syntactic and semantic regularities in language, and have been successfully applied in various natural language processing tasks, such as document classification [Kim14], information retrieval [GDR+15, NMCC16], question answering [IBGC+14, SSH16], named entity recognition [Sie15], and parsing [SBMN13].

One of the main approaches to learning distributed word representation is the neural-network based one ([BDVJ03, MB05, BSS+06, CW08, MH09, CWB+11, LOA+11, PSM14]), in which word vectors are trained to maximize the likelihood of word-context occurrences observed from large text corpus (e.g., news collections, Wikipedia and Web Crawl) based on probabilistic models. In particular, a series of recent papers by Mikolov et al. [MKB+10, MYZ13, MCCD13, MSC+13, MGB+17] culminated in and popularized the skip-gram model with negative-sampling training scheme (a.k.a. the SGNS model), which was shown to achieve state-of-the-art results on a variety of linguistic tasks [BDK14].

Despite the empirical success of the SGNS model, in this paper, we will first point out that the optimization problem induced by the SGNS model is essentially an *ill-posed* one. In specific, we can easily distort the output solution without changing its objective value. As a remedy, we propose a simple way to rectify the SGNS model by appending quadratic regularization terms to the

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original objective of SGNS. A theoretical justification, which provides novel insight into quadratic regularization, is also established. Preliminary experiments are conducted to evaluate word vectors on Google's analytical reasoning task, which shows the modified SGNS model outperforms the original SGNS model in a consistent manner.

2 SGNS Model

The SGNS model is essentially the skip-gram word neural embedding model introduced in [MCCD13] trained using the negative-sampling procedure proposed in [MSC⁺13]. In this section, we will briefly review the SGNS model together with related notation. Although the SGNS model is initially proposed and described in the the language of neural network, we find the explanation provide by Goldberg and Levy [GL14] is more transparent and better disclose the rationale behind the model. Therefore, in the following, we adopt their approach in formulating the SGNS model.

Let $\mathcal W$ be the word vocabulary of our interest with $n:=|\mathcal W|$. The training data $\mathcal D$, normally collected based on some text corpus, consists of word-context pairs $(w,c)\in\mathcal W\times\mathcal W$ in both positive and negative sense. For a word w, its positive context word c is often sampled from the neighborhood centering around the locations where w shows up in the text corpus, while its negative context word c is normally sampled from $\mathcal W$ randomly according to certain predefined distribution [LGD15]. For each word $w\in\mathcal W$, its *center-word embedding* and *context-word embedding* are assumed to exist and represented as $\mathcal U[w]$ and $\mathcal V[w]$, where

$$\mathcal{U}: \mathcal{W} \to \mathbb{R}^d \quad \text{and} \quad \mathcal{V}: \mathcal{W} \to \mathbb{R}^d.$$
 (2.1)

The center-word embedding $\mathcal{U}[\cdot]$ is normally outputted as word representation, which will be used either by itself or as an important ingredient in subsequent natural language processing and machine learning applications.

The SGNS model learns the embeddings by solving the following optimization problem,

$$\max_{\mathcal{U}:\mathcal{W}\to\mathbb{R}^d,\ \mathcal{V}:\mathcal{W}\to\mathbb{R}^d} \quad \sum_{(w,c)\in\mathcal{D}^+} \log \sigma(\mathcal{U}[w]^\top \mathcal{V}[c]) + \sum_{(w,c)\in\mathcal{D}^-} \log \sigma(-\mathcal{U}[w]^\top \mathcal{V}[c]), \tag{2.2}$$

where \mathcal{D}^+ and \mathcal{D}^- denotes the positive and negative pairs in \mathcal{D} , and $\sigma(\cdot)$ denotes the usual sigmoid function, i.e. $\sigma(x) = 1/(1 + \exp(-x))$. For simplicity, we denote the center-word embedding matrix U (resp. context-word embedding matrix V) as the matrix in $\mathbb{R}^{n \times d}$ whose row vectors are stacked by the center-word embeddings (resp. context-word embedding) of all words from the vocabulary. We will use $u_i, v_i \in \mathbb{R}^d$ to denote the i-th row of U and V. With a slight abuse of notation, we will also use interchangeably U[w] and $\mathcal{U}[w]$, V[w] and $\mathcal{V}[w]$, i.e.

$$U[w] := \mathcal{U}[w], \quad \text{and} \quad V[w] := \mathcal{V}[w],$$
 (2.3)

to represent the center-word and the context-word embeddings of the word $w \in \mathcal{W}$. Then clearly we can rewrite (2.2) equivalently as a maximization problem over the matrices U and V in $\mathbb{R}^{n \times d}$,

$$\max_{\boldsymbol{U},\boldsymbol{V}\in\mathbb{R}^{n\times d}} \quad \mathcal{L}(\boldsymbol{U},\boldsymbol{V}) := \sum_{(\boldsymbol{w},\boldsymbol{c})\in\mathcal{D}^+} \log \sigma(\boldsymbol{U}[\boldsymbol{w}]^\top \boldsymbol{V}[\boldsymbol{c}]) + \sum_{(\boldsymbol{w},\boldsymbol{c})\in\mathcal{D}^-} \log \sigma(-\boldsymbol{U}[\boldsymbol{w}]^\top \boldsymbol{V}[\boldsymbol{c}]). \tag{2.4}$$

The SGNS model models how words are interacted with their contexts, which is rooted deeply in the distributional hypothesis of Harris [Har54], stating that words sharing similar contexts possess similar meanings. Intuitively, the SGNS model attempts to find embeddings $\{U[w]\}_{w\in\mathcal{W}}$ and $\{V[c]\}_{c\in\mathcal{W}}$ in a way such that their inner-products are encouraged to be large for good context pairs, but to be small for bad ones. Several insightful interpretations—e.g., implicit matrix factorization [LG14b], representation learning [LXT+15], weighted logistic PCA [LB17], to just name a few—have been further proposed to better understand the underlying principles of the model. However, as we will point out in the next section, the SGNS model is essentially an ill-posed problem from the perspective of optimization.

3 Ambiguity in the SGNS Model

In this section, we will address a fundamental ambiguity issue undermining the SGNS model (2.4). Specifically, we will show that the solution from SGNS can be easily distorted without affecting the objective value.¹

Suppose (U^*, V^*) is one optimal solution to (2.4). Then for any invertible matrix $M \in \mathbb{R}^{d \times d}$, $(U^*M, V^*M^{-\top})$ is another optimal solution to SGNS as the objective value remains the same:

$$\mathcal{L}(\boldsymbol{U}^{\star}\boldsymbol{M}, \boldsymbol{V}^{\star}\boldsymbol{M}^{-\top})$$

$$= \sum_{(w,c)\in\mathcal{D}^{+}} \log \sigma \left(\left\langle \boldsymbol{M}^{\top}\boldsymbol{U}^{\star}[w], \boldsymbol{M}^{-1}\boldsymbol{V}^{\star}[c] \right\rangle \right) + \sum_{(w,c)\in\mathcal{D}^{-}} \log \sigma \left(-\left\langle \boldsymbol{M}^{\top}\boldsymbol{U}^{\star}[w], \boldsymbol{M}^{-1}\boldsymbol{V}^{\star}[c] \right\rangle \right)$$

$$= \sum_{(w,c)\in\mathcal{D}^{+}} \log \sigma (\left\langle \boldsymbol{U}^{\star}[w], \boldsymbol{V}^{\star}[c] \right\rangle) + \sum_{(w,c)\in\mathcal{D}^{-}} \log \sigma (-\left\langle \boldsymbol{U}^{\star}[w], \boldsymbol{V}^{\star}[c] \right\rangle)$$

$$= \mathcal{L}(\boldsymbol{U}^{\star}, \boldsymbol{V}^{\star}).$$

$$(3.1)$$

Therefore, there is an extremely large amount of freedom to manipulate (U^*, V^*) without affecting the optimality, which could lead to entirely different embeddings in terms of encoded semantic properties (i.e., vector lengths and angles). To better understand the severity of this ambiguity, let us think about the following toy example.

Example 1. Suppose we have $W = \{w_1, w_2, w_3\}$, and

$$\boldsymbol{U}^{\star} = \begin{bmatrix} \boldsymbol{U}^{\star}[w_1] \\ \boldsymbol{U}^{\star}[w_2] \\ \boldsymbol{U}^{\star}[w_3] \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1/2 & 1/2 \end{bmatrix},$$

whose row vectors are pretty spread out in \mathbb{R}^2 . However, by choosing

$$M = \begin{bmatrix} 1/2 + \varepsilon & 1/2 \\ 1/2 & 1/2 - \varepsilon \end{bmatrix},$$

where $0 \neq \varepsilon \in \mathbb{R}$, as argued above, U^*M is also an optimal solution to (2.4) with

$$\boldsymbol{U}^{\star}\boldsymbol{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1/2 & 1/2 \end{bmatrix} \begin{bmatrix} 1/2 + \varepsilon & 1/2 \\ 1/2 & 1/2 - \varepsilon \end{bmatrix} = \begin{bmatrix} 1/2 + \varepsilon & 1/2 \\ 1/2 & 1/2 - \varepsilon \\ 1/2 + \varepsilon/2 & 1/2 - \varepsilon/2 \end{bmatrix},$$

¹In addition to the SGNS model, following the same logic, the fundamental ambiguity issue is shared by many other prevailing word embedding models (e.g., the CBOW model with negative sampling [MCCD13, MSC⁺13, MGB⁺17], and the GloVe model [PSM14].

whose row vectors now become almost parallel as ε approaches 0.

To sum up, even though U^* and U^*M have entirely different word representations in essence, the SGNS model makes no differentiation among them. In order to ensure intrinsic embeddings being learned, we have to avoid those M's that distort the semantic properties of the word vectors. Since for any $u, v \in \mathbb{R}^d$, $\|Mu\| = \|Mu\|$ and $\langle Mu, Mv \rangle = \langle u, v \rangle$ if and only if $M \in \mathbb{R}^{d \times d}$ is orthogonal, i.e., $M^\top M = I$, the innocuous ambiguities are the ones resulting from rotation and reflection. Therefore, an ideal word embedding model should be expected to have unique optimal solutions up to orthogonal transformation, i.e.,

[*] $(U^*M, V^*M^{-\top})$ is optimal if and only if M is orthogonal.

We will elaborate how we are able to achieve this in the next section.

4 SGNS Model with Quadratic Regularization

In this section, we will work towards the goal stated in [*] by modifying the SGNS model. Let us consider the extended SGNS model with regularization,

$$\max_{\boldsymbol{U},\boldsymbol{V}\in\mathbb{R}^{n\times d}} \quad \mathcal{L}(\boldsymbol{U},\boldsymbol{V}) - \mathcal{R}(\boldsymbol{U},\boldsymbol{V}), \tag{4.1}$$

where $\mathcal{R}:(\mathbb{R}^{n\times d},\mathbb{R}^{n\times d})\to\mathbb{R}\cup\{+\infty\}$ is some regularizer. The aim is to leverage the regularization term \mathcal{R} to enforce the solution to be unique up to orthogonal transformation without (on the other hand) making the model too hard to be optimized. In the following, we will choose \mathcal{R} to be a simple quadratic form, and show this slight modification is sufficient to achieve the goal stated in [*] and thus resolve the ambiguity issues undermining the SGNS model (2.2).

Consider the following SGNS model with quadratic regularization (named as the SGNS-qr model thereafter)

$$\max_{\boldsymbol{U},\boldsymbol{V}\in\mathbb{R}^{n\times d}} f(\boldsymbol{U},\boldsymbol{V}) := \sum_{(w,c)\in\mathcal{D}^+} \log \sigma(\boldsymbol{U}[w]^\top \boldsymbol{V}[c]) + \sum_{(w,c)\in\mathcal{D}^-} \log \sigma(-\boldsymbol{U}[w]^\top \boldsymbol{V}[c]) - \frac{\lambda}{2} \|\boldsymbol{U}\|_F^2 - \frac{\lambda}{2} \|\boldsymbol{V}\|_F^2,$$
(4.2)

where $\lambda>0$ is the regularization parameter and $\|\cdot\|_F$ denotes the matrix Frobenius norm. A similar model has been proposed in [Joh14] in the context of collaborative filtering, which falls into the general framework of low-rank models [UHZB16] with the logistic loss function and the quadratic regularization. The quadratic regularizer $\mathcal{R}(U, V) = \frac{\lambda}{2} \|U\|_F^2 + \frac{\lambda}{2} \|V\|_F^2$ explicitly encourages entries in both U and V to be small in magnitude, which (perhaps surprisingly) has the effect of penalizing the non-orthogonal transformation. We will state this novel insight regarding quadratic regularization in the following theorem.

Theorem 1. Let (U^*, V^*) be an optimal solution to (4.2). Suppose U^* and V^* are both full rank. Then $(\hat{U}, \hat{V}) := (U^*M, V^*M^{-\top})$ is an optimal solution if and only if M is orthogonal.

Proof. Let us first prove the *if* direction. Since M is orthogonal,

$$\|U^{\star}\|_{F} = \|U^{\star}M\|_{F}, \quad \|V^{\star}\|_{F} = \|V^{\star}M\|_{F} = \|V^{\star}M^{-\top}\|_{F},$$
 (4.3)

and therefore

$$f(\boldsymbol{U}^{\star}, \boldsymbol{V}^{\star}) = \mathcal{L}(\boldsymbol{U}^{\star}, \boldsymbol{V}^{\star}) + \frac{\lambda}{2} \|\boldsymbol{U}^{\star}\|_{F}^{2} + \frac{\lambda}{2} \|\boldsymbol{V}^{\star}\|_{F}^{2}$$

$$= \mathcal{L}(\boldsymbol{U}^{\star}\boldsymbol{M}, \boldsymbol{V}^{\star}\boldsymbol{M}^{-\top}) + \frac{\lambda}{2} \|\boldsymbol{U}^{\star}\boldsymbol{M}\|_{F}^{2} + \frac{\lambda}{2} \|\boldsymbol{V}^{\star}\boldsymbol{M}^{-\top}\|_{F}^{2}$$

$$= f(\hat{\boldsymbol{U}}, \hat{\boldsymbol{V}}),$$

which implies the optimality of (\hat{U}, \hat{V}) .

In the rest of the proof, we will establish the *only if* direction.

Let $U\Sigma V^{\top}$ be the reduced singular value decomposition (SVD) [TBI97] of $U^{\star}(V^{\star})^{\top}$, i.e., $U^{\star}(V^{\star})^{\top} = U\Sigma V^{\top}$ where $U \in \mathbb{R}^{n \times d}$ and $V \in \mathbb{R}^{n \times d}$ have orthonormal columns, and $\Sigma = \operatorname{diag}(\sigma_1, \sigma_2, \dots, \sigma_d)$ with $\sigma_1 \geq \sigma_2 \geq \dots \geq \sigma_d > 0$. Here we write $\sigma_d > 0$ since U^{\star} and V^{\star} are full rank, and by Sylvester inequality [HJ90]

$$d = \operatorname{rank}(\boldsymbol{U}^{\star}) + \operatorname{rank}(\boldsymbol{V}^{\star}) - d \leq \operatorname{rank}(\boldsymbol{U}^{\star}(\boldsymbol{V}^{\star})^{\top}) \leq \min\{\operatorname{rank}(\boldsymbol{U}^{\star}), \operatorname{rank}(\boldsymbol{V}^{\star})\} = d.$$

Now we will first derive a upper bound for $f(U^*, V^*)$:

$$f(\boldsymbol{U}^{\star}, \boldsymbol{V}^{\star}) = \mathcal{L}(\boldsymbol{U}^{\star}, \boldsymbol{V}^{\star}) - \mathcal{R}(\boldsymbol{U}^{\star}, \boldsymbol{V}^{\star})$$

$$= \mathcal{L}(\boldsymbol{U}^{\star}, \boldsymbol{V}^{\star}) - \frac{\lambda}{2} \|\boldsymbol{U}^{\star}\|_{F}^{2} - \frac{\lambda}{2} \|\boldsymbol{V}^{\star}\|_{F}^{2}$$

$$\leq \mathcal{L}(\boldsymbol{U}^{\star}, \boldsymbol{V}^{\star}) - \lambda \cdot \|\boldsymbol{U}^{\star}\|_{F} \cdot \|\boldsymbol{V}^{\star}\|_{F}$$

$$\leq \mathcal{L}(\boldsymbol{U}^{\star}, \boldsymbol{V}^{\star}) - \lambda \cdot \|\boldsymbol{U}^{\top}\boldsymbol{U}^{\star}\|_{F} \cdot \|\boldsymbol{V}^{\top}\boldsymbol{V}^{\star}\|_{F}$$

$$\leq \mathcal{L}(\boldsymbol{U}^{\star}, \boldsymbol{V}^{\star}) - \lambda \cdot \operatorname{trace}(\boldsymbol{U}^{\top}\boldsymbol{U}^{\star}(\boldsymbol{V}^{\star})^{\top}\boldsymbol{V})$$

$$= \mathcal{L}(\boldsymbol{U}^{\star}, \boldsymbol{V}^{\star}) - \lambda \cdot \|\boldsymbol{\sigma}\|_{1}, \qquad (4.4)$$

where the third and the fifth lines uses Cauchy-Schwartz inequality, the fourth line holds as the operator norms $\|U\| \le 1$, $\|V\| \le 1$, and $\|AB\|_F \le \|A\| \|B\|_F$ for any compatible matrices A and B, and the last line follows directly from the definition of SVD.

But on the other hand, we can also derive the following lower bound for $f(U^*, V^*)$:

$$f(\boldsymbol{U}^{\star}, \boldsymbol{V}^{\star}) \geq f(\boldsymbol{U}\boldsymbol{\Sigma}^{\frac{1}{2}}, \boldsymbol{V}\boldsymbol{\Sigma}^{\frac{1}{2}})$$

$$= \mathcal{L}(\boldsymbol{U}^{\star}, \boldsymbol{V}^{\star}) - \frac{\lambda}{2} \left\| \boldsymbol{U}\boldsymbol{\Sigma}^{\frac{1}{2}} \right\|_{F}^{2} - \frac{\lambda}{2} \left\| \boldsymbol{V}\boldsymbol{\Sigma}^{\frac{1}{2}} \right\|_{F}^{2}$$

$$= \mathcal{L}(\boldsymbol{U}^{\star}, \boldsymbol{V}^{\star}) - \frac{\lambda}{2} \left\| \boldsymbol{\Sigma}^{\frac{1}{2}} \right\|_{F}^{2} - \frac{\lambda}{2} \left\| \boldsymbol{\Sigma}^{\frac{1}{2}} \right\|_{F}^{2}$$

$$= \mathcal{L}(\boldsymbol{U}^{\star}, \boldsymbol{V}^{\star}) - \lambda \left\| \boldsymbol{\sigma} \right\|_{1}, \qquad (4.5)$$

where $\Sigma^{\frac{1}{2}} := \operatorname{diag}(\sqrt{\sigma_1}, \sqrt{\sigma_2}, \dots, \sqrt{\sigma_d}).$

Combining (4.4) and (4.5), one can easily derive that

$$f(U^{\star}, V^{\star}) = \mathcal{L}(U^{\star}, V^{\star}) - \lambda \|\sigma\|_{1}, \quad \text{and}$$
 (4.6)

$$\frac{1}{2} \| \boldsymbol{U}^{\star} \|_{F}^{2} + \frac{1}{2} \| \boldsymbol{V}^{\star} \|_{F}^{2} = \| \boldsymbol{U}^{\star} \|_{F} \| \boldsymbol{V}^{\star} \|_{F} = \| \boldsymbol{U}^{\star} \|_{F}^{2} = \| \boldsymbol{V}^{\star} \|_{F}^{2} = \| \boldsymbol{\sigma} \|_{1}.$$

$$(4.7)$$

Now we are ready to show that $U^* = U\Sigma^{\frac{1}{2}}Q$ for some orthogonal matrix $Q \in \mathbb{R}^{d \times d}$.

As $U\Sigma V^{\top}$ is the SVD of $U^{\star}(V^{\star})^{\top}$, there exist full rank matrices $S\in\mathbb{R}^{d\times d}$ and $T\in\mathbb{R}^{d\times d}$ such that $U^{\star}=US$, $V^{\star}=VT$ and $ST^{\top}=\Sigma=\mathrm{diag}\left(\sigma\right)$. Then from (4.7), one has

$$\|U^{\star}\|_{F} = \|US\|_{F} = \|S\|_{F} = \|\sigma\|_{1}^{1/2},$$
 (4.8)

$$\|V^{\star}\|_{F} = \|VT\|_{F} = \|T\|_{F} = \|\sigma\|_{1}^{1/2}. \tag{4.9}$$

Now let us write

$$X := \begin{bmatrix} S \\ T \end{bmatrix} \begin{bmatrix} S^{\top} T^{\top} \end{bmatrix} = \begin{bmatrix} SS^{\top} & ST^{\top} \\ TS^{\top} & TT^{\top} \end{bmatrix} = \begin{bmatrix} SS^{\top} & \Sigma \\ \Sigma^{\top} & TT^{\top} \end{bmatrix} \succeq 0.$$
 (4.10)

Define

$$s^* \in \arg\min_{i \in [d]} \left\{ (SS^\top)_{ii} - \sigma_i \right\} \quad \text{and} \quad t^* \in \arg\min_{i \in [d]} \left\{ (TT^\top)_{ii} - \sigma_i \right\}.$$
 (4.11)

Due to the facts that

$$\sum_{ii} (SS^{\top})_{ii} = ||S||_F^2 = \sum_{i \in [d]} \sigma_i \quad \text{and} \quad \sum_{ii} (TT^{\top})_{ii} = ||T||_F^2 = \sum_{i \in [d]} \sigma_i,$$
(4.12)

we must have

$$(SS^{\top})_{s^{\star}s^{\star}} - \sigma_{s^{\star}} \le 0 \quad \text{and} \quad (TT^{\top})_{t^{\star}t^{\star}} - \sigma_{t^{\star}} \le 0.$$
 (4.13)

Since X is positive semidefinite [HJ90],

$$(e_{s^*} - e_{t^*})^\top X (e_{s^*} - e_{t^*}) = (SS^\top)_{s^*s^*} + (TT^\top)_{t^*t^*} - \sigma_{s^*} - \sigma_{t^*} \ge 0.$$
(4.14)

which together with (4.13) leads to

$$(SS^{\top})_{s^{\star}s^{\star}} = \sigma_{s^{\star}} \quad \text{and} \quad (TT^{\top})_{t^{\star}t^{\star}} = \sigma_{t^{\star}}.$$
 (4.15)

Combining (4.11) and (4.13), it can be easily verified that

$$\operatorname{diag}\left(\boldsymbol{S}\boldsymbol{S}^{\top}\right) = \boldsymbol{\sigma} = \operatorname{diag}\left(\boldsymbol{T}\boldsymbol{T}^{\top}\right),\tag{4.16}$$

which implies that for any $i \in [d]$, s_i and t_i (the i-th row of S and T) satisfies $\|s_i\|^2 = \|t_i\|^2 = \sigma_i$. In addition, since $ST^\top = \Sigma$, the inner-product $\langle s_i, t_i \rangle = \sigma_i$. Due to Cauchy-Schwartz inequality, $s_i = t_i$. Therefore, S = T, $\Sigma = ST^\top = SS^\top = TT^\top$. Then it can be easily verified that $S = T = \Sigma^{\frac{1}{2}}Q$ for some orthogonal matrix Q. Therefore, we have proved that $U^\star = U\Sigma^{\frac{1}{2}}Q$ for some orthogonal matrix $Q \in \mathbb{R}^{d \times d}$.

Next, as (\hat{U}, \hat{V}) is also optimal and $\hat{U}\hat{V}^{\top} = U^{\star}(V^{\star})^{\top}$, we can follow exactly the same argument to show that $\hat{U} = U\Sigma^{\frac{1}{2}}\hat{Q}$ for anther orthogonal matrix $\hat{Q} \in \mathbb{R}^{d \times d}$. Therefore, in order to achieve

$$\hat{\boldsymbol{U}} = \boldsymbol{U}\boldsymbol{\Sigma}^{\frac{1}{2}}\hat{\boldsymbol{Q}} = \underbrace{\boldsymbol{U}\boldsymbol{\Sigma}^{\frac{1}{2}}\boldsymbol{Q}}_{\boldsymbol{U}^{\star}}\boldsymbol{M},\tag{4.17}$$

one must have $M=Q^{ op}\hat{Q}$, which is also orthogonal. That completes our proof. $\hfill\Box$

Theorem 1 states that optimal solutions to (4.2) are not unique, but are essentially all equivalent in terms of their encoded linguistic properties, as a result of the quadratic regularization removing all the adversarial ambiguities (e.g. the one described in Example 1) undermining the original SGNS model (2.4).

d	λ						
	0	10	50	100	250	500	1000
100	0.5642	0.5652	0.5666	0.5665	0.5645	0.5570	0.5397
200	0.6618	0.6617	0.6640	0.6656	0.6668	0.6605	0.6355
300	0.6768	0.6772	0.6798	0.6848	0.6909	0.6869	0.6593
400	0.6851	0.6860	0.6902	0.6938	0.7005	0.6952	0.6658
500	0.6909	0.6920	0.6947	0.6971	0.7035	0.6965	0.6554

Table 1: Evaluation of SGNS ($\lambda=0$) and SGNS-qr models on Google's analytical reasoning task. The SGNS-qr model consistently outperforms the SGNS model and the improvement is increasingly enlarged with the growth in the embedding dimension d.

5 Experiment

In this section, we will conduct some preliminary experiments to compare the SGNS model with and without quadratic regularization.

Algorithm. We use the popular toolbox word2vec [MCCD13, MSC⁺13] with its default parameter setting to solve the SGNS model, which leverages the standard *stochastic gradient method* (SGM) [Ber11] to optimize the objective. We solve the SGNS-qr model by modifying the SGM in word2vec to accommodate the additional quadratic terms.

Dataset. We use a publicly accessible dataset Enwik9² as our text corpus, which contains about 128 million tokens collected from English Wikipedia articles. The vocabulary \mathcal{W} is constructed by filtering out words that appear less than 200 times. The positive and negative word-context pairs are generated in exactly the same manner with the one implemented in word2vec using its default setting. We adopt Google's analogy dataset to evaluate word embeddings on analytical reasoning task.

Evaluation. In Google's analogy dataset, 19,544 questions are presented with the form "a is to a^* as b is to b^* ", where b^* is hidden and to be inferred from the whole vocabulary \mathcal{W} based on the input (a, a^*, b) . Among all these analogy questions, around half of them are syntactic ones (e.g., "think is to thinking as code is to coding"), and the other half are semantic ones (e.g., "man is to women as king is to queen"). The questions are answered using the 3CosMul scheme [LG14a]:

$$\mathcal{B}^{\star} = \arg \max_{x \in \mathcal{W}/\{a, a^{\star}, b\}} \frac{\cos(\mathcal{U}[x], \mathcal{U}[a^{\star}]) \cdot \cos(\mathcal{U}[x], \mathcal{U}[b])}{\cos(\mathcal{U}[x], \mathcal{U}[a]) + \varepsilon}$$
(5.1)

where $\mathcal{U}: \mathcal{W} \to \mathbb{R}^d$ is the word embedding to evaluate and $\varepsilon = 1\text{e-}3$ is set to avoid zero-division. The performance is measured as the percentage of questions answered correctly, i.e., $b^* \in \mathcal{B}^*$.

Experiment result. We evaluate the SNGS model (2.4) and the SGNS-qr model (4.2) with different choices of λ . The performance of each model is reported in Table 1 in terms of the analytical

²http://mattmahoney.net/dc/textdata.html

reasoning accuracy. As presented in Table 1, with a wide range of choices in λ , the SGNS-qr model outperforms the SGNS model ($\lambda=0$) in a consistent manner, and the improvement becomes more and more non-trivial with the growth in the embedding dimension d. In particular, when $\lambda=250$, a boost as large as 1.5% in the prediction accuracy is achieved.

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