Optimization Model for Leftover Component Utilization in Bike Production

Objective

This document explains the formulation, implementation, and reasoning behind the Mixed-Integer Linear Programming (MILP) model for optimizing the utilization of leftover components in bike production. The model maximizes profit while minimizing production time and unused inventory and ensures production alignment with inventory constraints.

Mathematical Formulation

Decision Variables

1. x_{bt}: Number of bikes of type bt to produce (integer variable).
2. u_c: Unused inventory of component c (continuous variable).

Parameters

- Pbt: Weighted Average Selling Price of bike type bt.
- C_{bt}: Unit cost for producing bike type bt.
- Q_{bt} : Quantity of component c required for one bike of type bt.
- I_c : Available inventory of component c.
- T_{bt} : Production time (in hours) required for one bike of type bt.
- W_{bt}: Priority weight for bike type bt to emphasize its production.

Objective Function

The objective is to:

1. Maximize total profit: $\sum_{bt} [(P_{bt} - C_{bt}) * x_{bt}]$

2. Minimize unused inventory: $-\sum_c u_c$

3. Minimize total production time: $-\sum_{bt} [T_{bt} * x_{bt}]$

4. Optionally, maximize bike production weighted by priority: $\sum_{bt} [W_{bt} * x_{bt}]$

The objective function:

Maximize Z = 0.01 * $\sum_{bt}$ [(P_{bt}- C_{bt}) * x_{bt}]- 2.0 * -$\sum_{c}$ u_c - 3.0 * -$\sum_{bt}$ [T_{bt} * x_{bt}] + 1.0 *$\sum_{bt}$ [W_{bt} * x_{bt}]

Constraints

1. Inventory Balance Constraint

Code Snippet:

```
# Add Inventory Constraints

for component in components:

    total_required = quicksum(required_qty[(bike_type, component)] *
x[bike_type] for bike_type in bike_types)

    model.addConstr(total_required + unused_inventory[component] ==
available_inventory[component])
```

For each component c:

$$\sum_{bt} [Q_{bt,c} * x_{bt}] + u_c = I_c$$

Explanation:

- This constraint ensures that for each component:

$$\sum_{bt} Q_{bt,c} \cdot x_{bt} + u_c = I_c$$

  - $Q_{bt,c}$: `required_qty[(bike_type, component)]` – Quantity of component c needed for bike type bt.
  - x_{bt}: `x[bike_type]` – Number of bikes of type bt to produce.
  - u_c: `unused_inventory[component]` – Unused inventory for component c.
  - I_c: `available_inventory[component]` – Available inventory for component c.

This ensures that the sum of components used in production and leftover components equals the initially available inventory.

2. Non-Negativity Constraint

Implicit in the Variable Declarations:

```
x = {bike_type: model.addVar(vtype=GRB.INTEGER,
name=f"Produce_{bike_type}") for bike_type in bike_types}

unused_inventory = {component: model.addVar(vtype=GRB.CONTINUOUS,
name=f"Unused_{component}") for component in components}
```

$$x_{bt} \geq 0, u_c \geq 0$$

Explanation:

- The `GRB.INTEGER` and `GRB.CONTINUOUS` variable types in Gurobi automatically enforce non-negativity unless explicitly stated otherwise.
  - $x_{bt} \geq 0$: Number of bikes produced must be non-negative.
  - $u_c \geq 0$: Unused inventory must be non-negative.

This ensures that negative production or negative unused inventory does not occur.

Model Enhancements

1. Crossover Variants

New crossover bike models (e.g., Hybrid_Crossover) leverage parts from multiple quality types (Type A, B, C). These variants expand the solution space, balancing the production of premium and economical bike types.

2. Sales Probabilities

To reflect market demand:

- Assign fixed probabilities for full-price and discounted sales.
- Calculate WASP using normalized probabilities for each pricing category:

$$P_{bt} = \sum_k p_k . SP_k$$

where p_k is the probability and SP_k is the selling price in category k.

WASP represents the expected revenue per bike, calculated by weighting the selling price of each pricing category (e.g., Full Price, 10% Discount, 15% Discount) by its corresponding probability. This ensures a balanced estimation of revenue by considering different market scenarios. The purpose of using this is that it reflects the **realistic expected selling price** by integrating potential discounts and customer behavior into the model. It ensures the optimization considers practical market dynamics rather than just ideal scenarios.

Expanded Equation:

Using the probabilities and selling prices defined in the code:

$$\text{WASP}_i = (\text{Full Price} \times 0.1) + (10\%\ \text{Discount Price} \times 0.7) + (15\%\ \text{Discount Price} \times 0.2)$$

Where:

- Full Price $S_{i,\text{Full}} = \text{Base Price}.$
- 10% Discount Price $S_{i,10\%} = \text{Base Price} \times 0.9.$
- 15% Discount Price $S_{i,15\%} = \text{Base Price} \times 0.85.$

3. Multi-Objective Optimization

To address other priorities:

- Balance profit with production efficiency.
- Minimize leftover components post-production.

Implementation

The optimization problem is solved using **Gurobi**, leveraging Python for data preprocessing and visualization.

Steps:

1. **Input Data Preparation:**
    1. Load bike production data (components, costs, probabilities).
    2. Add crossover variants and compute WASP.
2. **MILP Model Definition**
    1. Declare decision variables and parameters.
    2. Define the objective function and constraints.
3. **Model Execution:**
    1. Solve the model using Gurobi.
    2. Extract optimal production plans and inventory utilization.

4. **Visualization:**

    1. Use Plotly to display results in tabular format.

Benefits

- **Maximized Profitability:** The WASP-based objective ensures focus on high-revenue bike types while avoiding over production of low-margin bikes.

- **Efficient Resource Allocation:**

    - Ensures minimal leftover inventory.
    - Introduces versatile bike models, catering to diverse market demands.

- **Data-Driven Decision Making:**

    - Incorporates fixed sales probabilities for consistency.
    - Uses advanced algorithms for actionable insights.

Key Challenges and Mitigations

Gravitating Towards Premium Models:

1. Introduced crossover variants with multi-type components.
2. Added priority weights to emphasize diverse production.

Dynamic Sales Probabilities:

1. Fixed probabilities for model consistency.

Inventory Utilization Constraints:

1. Explicit constraints ensure feasibility, avoiding overuse of components.

Future Extensions

- Integrate predictive algorithms for demand forecasting.
- Automate dynamic pricing adjustments based on market conditions.

This detailed formulation and rationale provide a robust foundation to understand the methodology, ensuring confidence in its implementation and results.

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2. Key Observations from Results

Production Results

Optimal Production Results

Bike Type	Produced	WASP (€)	Revenue (€)	Profit (€)	Production Time (hours)
City_Basic	0	37.8	0	0	0
City_Medium	0	86.4	0	0	0
City_Premium	0	172.8	0	0	0
Mountain_Basic	102	70.2	7160.4	530.4	40.8
Mountain_Medium	7	151.2	1058.4	78.4	3.5
Mountain_Premium	0	302.4	0	0	0
Gravel_Basic	27	70.2	1895.4	140.4	10.8
Gravel_Medium	0	151.2	0	0	0
Gravel_Premium	26	302.4	7862.4	582.4	26
Electric_Basic	0	135	0	0	0
Electric_Medium	0	280.8	0	0	0
Electric_Premium	89	561.6	49982.4	3702.4	89
Hybrid_Basic	0	59.4	0	0	0
Hybrid_Premium	0	118.8	0	0	0
Hybrid_Crossover	0	95.04	0	0	0
Gravel_Crossover	21	81	1701	126	13.92
Hybrid_Crossover	0	95.04	0	0	0
Gravel_Crossover	21	81	1701	126	13.92

- **Mountain Basic** bikes were the most produced (102 units), contributing significantly to profit (€530.4) with a reasonable production time (40.8 hours). This aligns with its lower WASP (€70.2) but efficient use of components and high demand.
- **Electric Premium** bikes were also prioritized (89 units), generating the highest profit (€3702.4) due to their high WASP (€561.6) and premium pricing potential.
- **Gravel Premium** and **Gravel Crossover** bikes were produced in moderate quantities (26 and 21 units, respectively), leveraging the flexibility introduced by crossover variants.
- **City Bikes** and **Hybrid Bikes** were not produced, indicating these types were not competitive in terms of WASP, revenue, or profit compared to other bike categories.

Inventory Utilization Results

Inventory Utilization Results

Component	Initial Inventory	Utilized Inventory	Remaining Inventory
Frame	1251	1251	0
Wheels	1191	1191	0
Saddle	1385	1385	0
Suspension	298	298	0
Motor	158	89	69
Battery	89	89	0

- All components except **Motor** were fully utilized, demonstrating efficient use of available inventory.
- **Motors** had remaining inventory (69 units), likely because production focused on premium electric bikes, which only partially used this component.

3. Interpretation of Results

Focus on Profitability:

- The model prioritized bike types with higher WASP and profit margins, such as Electric Premium and Gravel Premium bikes.
- Mountain Basic bikes, despite their lower WASP, were produced in large numbers due to high demand and efficient use of available components.

Inventory Efficiency:

- By introducing crossover variants, the model effectively reduced unused inventory, as shown by the full utilization of components like Frames, Saddles, and Suspension.

Production Constraints:

- Production time limits and inventory constraints likely prevented the production of City and Hybrid bikes, which were less competitive under the given WASP and profit conditions.

Impact of Crossover Variants:

- Gravel Crossover bikes were produced in significant quantities, validating the logic of introducing crossover variants to better utilize inventory and diversify production.

I have implemented **sensitivity analysis** to assess how variations in sales probabilities (mean ± standard deviation) impact key performance indicators (KPIs) such as revenue, profit, and production time for different bike types. This analysis provides valuable insights into how inherent uncertainty in sales probabilities affects the Weighted Average Selling Price (WASP) and the overall performance of the optimization model.

- **Purpose**:
  - To understand the robustness of the model under different scenarios: **pessimistic (-1 std)**, **expected (mean)**, and **optimistic (+1 std)** conditions.
  - To quantify the impact of variability in probabilities on **WASP, revenue, profit, and production time**.

- **How It Works**:
  - **Baseline Probabilities**: Use the default mean values for sales probabilities (e.g., 0.1 for Full Price, 0.7 for 10% Discount, and 0.2 for 15% Discount).
  - **Adjustments**: Vary probabilities using standard deviations (e.g., 0.02 for Full Price) to simulate pessimistic, expected, and optimistic scenarios.
  - **Normalization**: Ensure adjusted probabilities sum to 1.
  - **Recalculate WASP**: For each scenario, compute WASP using adjusted probabilities and selling prices.
  - **Impact on KPIs**: Evaluate how changes in WASP affect revenue, profit, and production decisions.

- **Why This is Important**:
  - Helps identify which bike types and pricing strategies are sensitive to changes in market conditions.
  - Ensures that the production plan is resilient to uncertainty in sales probabilities.
  - Provides actionable insights for decision-making under different market scenarios.

Output:

When you run the sensitivity_analysis_variate_std.py you will get this table:

Sensitivity Analysis Results with Varying Standard Deviations

Bike Type	Produced	Baseline WASP (€)	Variation (std)	Adjusted WASP (€)	Adjusted Revenue (€)	Adjusted Profit (€)	Production Time (hours)
City_Basic	1	37.8	-1.0	37.78	37.78	2.78	0.4
City_Basic	1	37.8	+0.0	37.8	37.8	2.8	0.4
City_Basic	1	37.8	+1.0	37.82	37.82	2.82	0.4
City_Medium	0	86.4	-1.0	86.35	0	0	0
City_Medium	0	86.4	+0.0	86.4	0	0	0
City_Medium	0	86.4	+1.0	86.44	0	0	0
City_Premium	0	172.8	-1.0	172.69	0	0	0
City_Premium	0	172.8	+0.0	172.8	0	0	0
City_Premium	0	172.8	+1.0	172.89	0	0	0
Mountain_Basic	142	70.2	-1.0	70.16	9962.25	732.25	56.8
Mountain_Basic	142	70.2	+0.0	70.2	9968.4	738.4	56.8
Mountain_Basic	142	70.2	+1.0	70.24	9973.43	743.43	56.8
Mountain_Medium	1	151.2	-1.0	151.11	151.11	11.11	0.5
Mountain_Medium	1	151.2	+0.0	151.2	151.2	11.2	0.5
Mountain_Medium	1	151.2	+1.0	151.28	151.28	11.28	0.5
Mountain_Premium	0	302.4	-1.0	302.21	0	0	0
Mountain_Premium	0	302.4	+0.0	302.4	0	0	0
Mountain_Premium	0	302.4	+1.0	302.55	0	0	0
Gravel_Basic	0	70.2	-1.0	70.16	0	0	0
Gravel_Basic	0	70.2	+0.0	70.2	0	0	0
Gravel_Basic	0	70.2	+1.0	70.24	0	0	0
Gravel_Medium	2	151.2	-1.0	151.11	302.21	22.21	1
Gravel_Medium	2	151.2	+0.0	151.2	302.4	22.4	1

Gravel_Medium	2	151.2	+1.0	151.28	302.55	22.55	1
Gravel_Premium	10	302.4	-1.0	302.21	3022.13	222.13	10
Gravel_Premium	10	302.4	+0.0	302.4	3024	224	10
Gravel_Premium	10	302.4	+1.0	302.55	3025.53	225.53	10
Electric_Basic	0	135	-1.0	134.92	0	0	0
Electric_Basic	0	135	+0.0	135	0	0	0
Electric_Basic	0	135	+1.0	135.07	0	0	0
Electric_Medium	0	280.8	-1.0	280.63	0	0	0
Electric_Medium	0	280.8	+0.0	280.8	0	0	0
Electric_Medium	0	280.8	+1.0	280.94	0	0	0
Electric_Premium	89	561.6	-1.0	561.25	49951.55	3671.55	89
Electric_Premium	89	561.6	+0.0	561.6	49982.4	3702.4	89
Electric_Premium	89	561.6	+1.0	561.88	50007.64	3727.64	89
Hybrid_Basic	0	59.4	-1.0	59.36	0	0	0
Hybrid_Basic	0	59.4	+0.0	59.4	0	0	0
Hybrid_Basic	0	59.4	+1.0	59.43	0	0	0
Hybrid_Premium	0	118.8	-1.0	118.73	0	0	0
Hybrid_Premium	0	118.8	+0.0	118.8	0	0	0
Hybrid_Premium	0	118.8	+1.0	118.86	0	0	0
Hybrid_Crossover	37	95.04	-1.0	94.98	3514.31	258.31	24.53
Hybrid_Crossover	37	95.04	+0.0	95.04	3516.48	260.48	24.53
Hybrid_Crossover	37	95.04	+1.0	95.09	3518.26	262.26	24.53
Gravel_Crossover	73	81	-1.0	80.95	5909.35	434.35	48.4
Gravel_Crossover	73	81	+0.0	81	5913	438	48.4
Gravel_Crossover	73	81	+1.0	81.04	5915.99	440.99	48.4

The analysis highlights how the Weighted Average Selling Price (WASP) adjusts with pessimistic (-1 std), expected (0 std), and optimistic (+1 std) variations in probabilities. This adjustment is directly linked to pricing probabilities across Full Price, 10% Discount, and 15% Discount categories, showcasing how price expectations impact the valuation of each bike type.As WASP changes, the corresponding adjusted revenue and profit shift. For instance, bike types like "Electric_Premium" and "Gravel_Crossover" show significant changes in profit when WASP increases, suggesting these types benefit most from favorable pricing scenarios. On the other hand, some bike types like "City_Basic" maintain stable performance, even under pessimistic conditions, indicating lower sensitivity.The production quantities for each bike type also vary with WASP adjustments. "Mountain_Basic" shows consistent production across scenarios, suggesting stable demand, while "Gravel_Crossover" sees increased production in optimistic scenarios, reflecting higher profitability potential.

We also observed the small differences in adjusted WASP these are primarily because the standard deviation values used in the model (e.g., **0.02 for full price**, **0.05 for 10% discount**, and **0.03 for 15% discount**) represent relatively low variability in the probabilities. Here's why:

Low Standard Deviations Mean Low Variability: A standard deviation of 0.02 or 0.05 implies that the probability adjustments for pessimistic (-1 std) or optimistic (+1 std) scenarios are small. For instance:

1. For the **full price** (mean = 0.1, std = 0.02):
    1. **Pessimistic (-1 std):** 0.1 - 0.02 = 0.08
    2. **Optimistic (+1 std):** 0.1 + 0.02 = 0.12
2. This variation is only a small shift, so the weighted average selling price (WASP) remains fairly consistent.

Dominance of Probabilities in 10% Discount: The **10% discount tier** has the highest mean probability (0.7), and even with a standard deviation of 0.05, the adjustments are relatively small:

1. **Pessimistic (-1 std):** 0.7 - 0.05 = 0.65
2. **Optimistic (+1 std):** 0.7 + 0.05 = 0.75
3. Because this tier has the highest weight, small shifts in probabilities here result in minor WASP changes.

Balanced Probability Distributions: Since all probabilities are normalized after adjustments, the total sum remains 1. This normalization dampens the overall impact of changes in individual price tiers, leading to even smaller changes in WASP.

Conclusion

The optimization successfully prioritized profitable and feasible bike types while minimizing unused inventory. Mountain Basic and Electric Premium bikes emerged as the most viable products, with crossover variants playing a key role in enhancing inventory efficiency. Future efforts should focus on improving the competitiveness of low-priority bike types and leveraging unused inventory for greater profitability.

The relatively small changes in adjusted WASP are indeed due to the **low standard deviation values** used, which ensure stability in the pricing model. If larger standard deviations (e.g., 0.1 or higher) were used, the variability in adjusted WASP would increase significantly, leading to greater differences between the baseline and adjusted values and their impact on KPIs.