GRAMAZIO KOHLER RESEARCH

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ETH zürich

MAS Digital Fabrication

Vectors and Matrices

October 7th, 2020

ETH zürich DARCH Departement Architektur

Institute of Technology in Architecture
Faculty of Architecture / ETH Zurich

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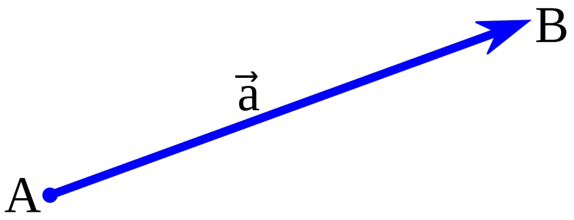
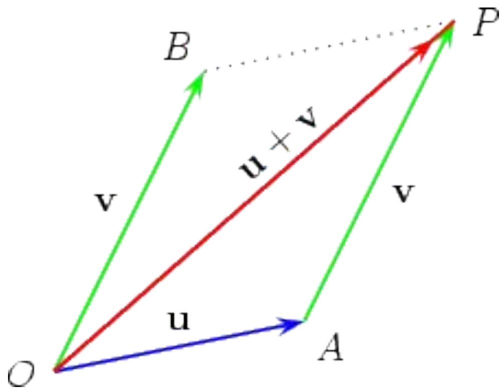
Vectors

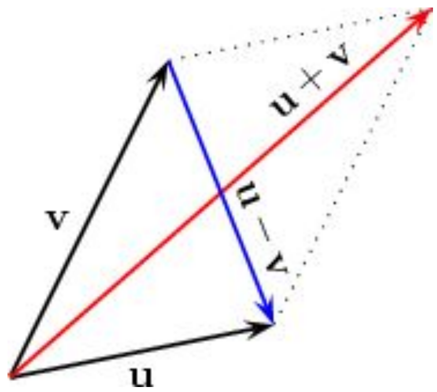


a = [1.9 , 4.3]

b = [2.2 , 1.3 , 9.8]

Vector addition and subtraction

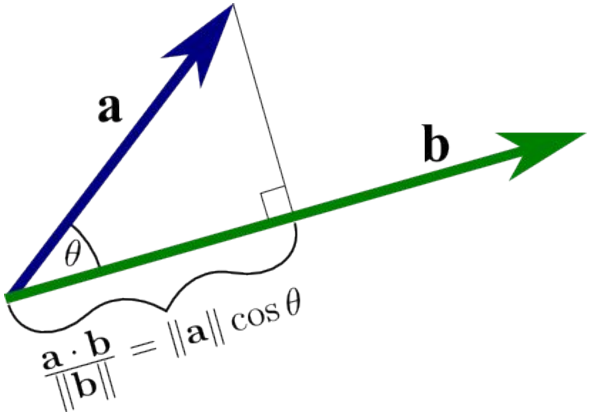


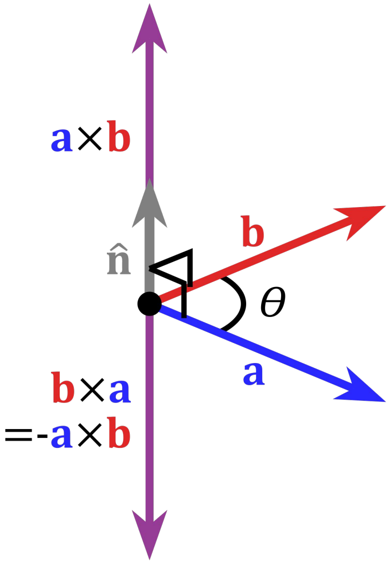


u + v= [ux + vx , uy + vy, uz + vz]

u - v= [ux - vx , uy - vy, uz - vz]

Vector multiplication

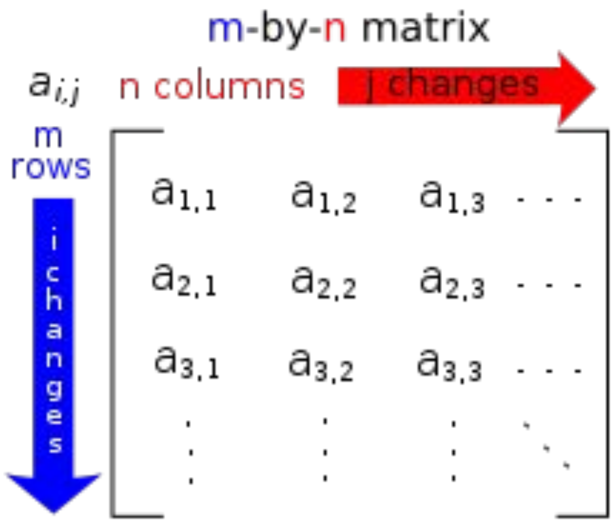




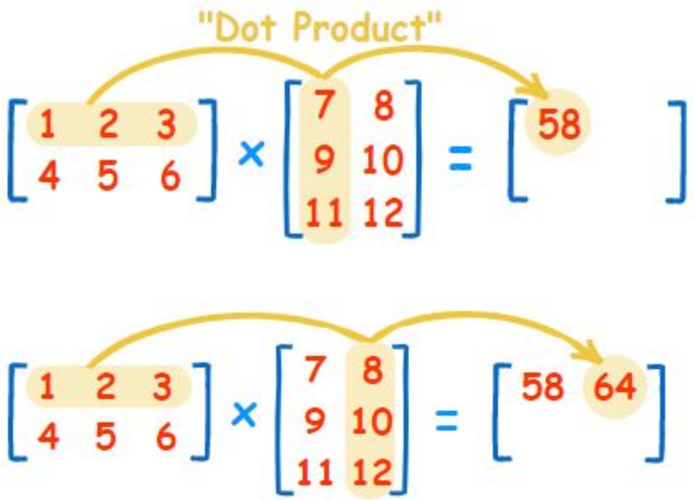
inner product

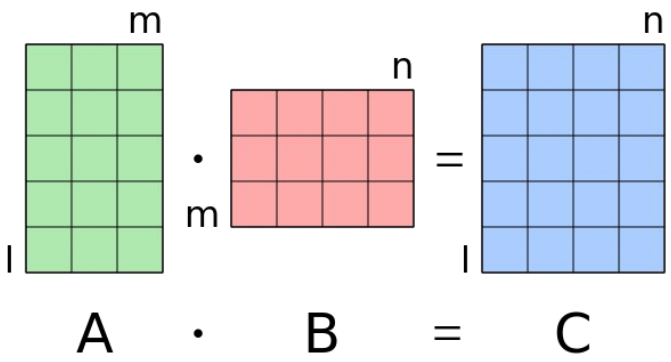
outer product

Matrices


m-by-n matrix
$a_{i,j}$
n columns
j changes
m
rows
i changes
$a_{1,1}$ $a_{1,2}$ $a_{1,3}$ ...
$a_{2,1}$ $a_{2,2}$ $a_{2,3}$ ...
$a_{3,1}$ $a_{3,2}$ $a_{3,3}$ ...

Matrix multiplication





(l,m) x (m,n) = (l,n)

Transformation Matrices

- **Rotation**

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- **Scaling**

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \bullet \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Transformation Matrices

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

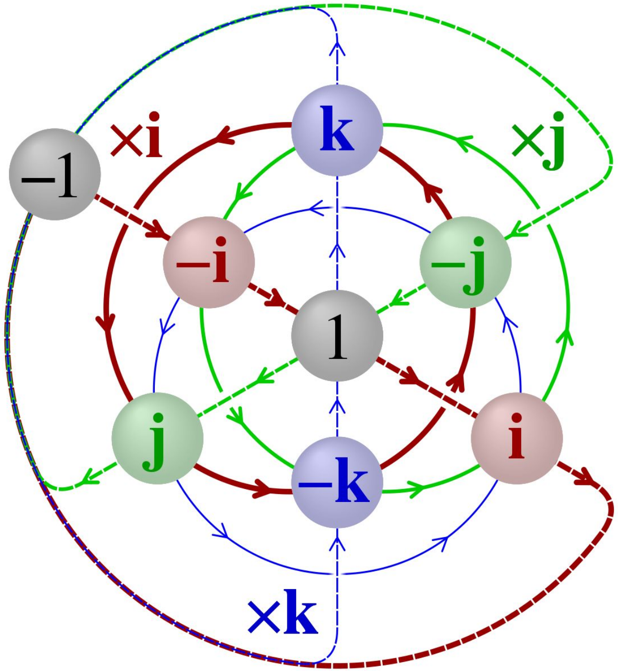
A * B * v != B * A * v

Homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Awesome lecture on transformations

Quaternions



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