# MAST-1D model description

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# Contents

# 1 Introduction

MAST-1D (morphodynamic and sediment tracers in 1-D) is a bed evolution model where the channel and floodplain are coupled. Details can be found in the original publications (Lauer and Parker, 2008a,b; Lauer et al., 2016).

MAST-1D is designed to model long spatial (10s to 100s km) and temporal (decades-millenia) timescales where bank erosion and channel migration allow for channel sediment to be sourced and stored within the floodplain. The channel (active layer), substrate, and floodplain are treated as a set of reservoirs, each with a characteristic geometry, volume, and grainsize distribution. Mass is conserved within each reservoir on a size-specific basis. Channel exchange occurs between reservoirs via longitudinal sediment transport, bank erosion, channel narrowing, avulsion, and bed elevation change, all of which are functions of an imposed water discharge.

A model schematic is presented in Figure 1. The model space is structured into a series of nodes aligned in the longitudinal direction. During each time step, the outgoing sediment load in the upstream node is calculated. It is a function of the sediment transport capacity of the active layer reservoir and the depositional properties of the floodplain. When the flow is high enough to overtop the banks, some sediment is deposited in the floodplain reservoir as overbank material. The rest is transported downstream and becomes the incoming flux to the next node. Once transport is calculated, lateral exchanges of sediment between reservoirs are characterized for each node. When flow strength is low, we assume that the bed is stable and that vegetation is able to grow on channel surfaces. This leads to transfer of sediment into the floodplain from a point bar deposit, which is composed of material from the active layer reservoir and the sediment load. The point bar is assumed to have a single constant height. Floodplain sediment is transferred to the active layer reservoir via bank erosion when flow is strong enough to mobilize bank material. Net fluxes to and from the active layer, from the incoming and outgoing sediment load and from bank erosion, are

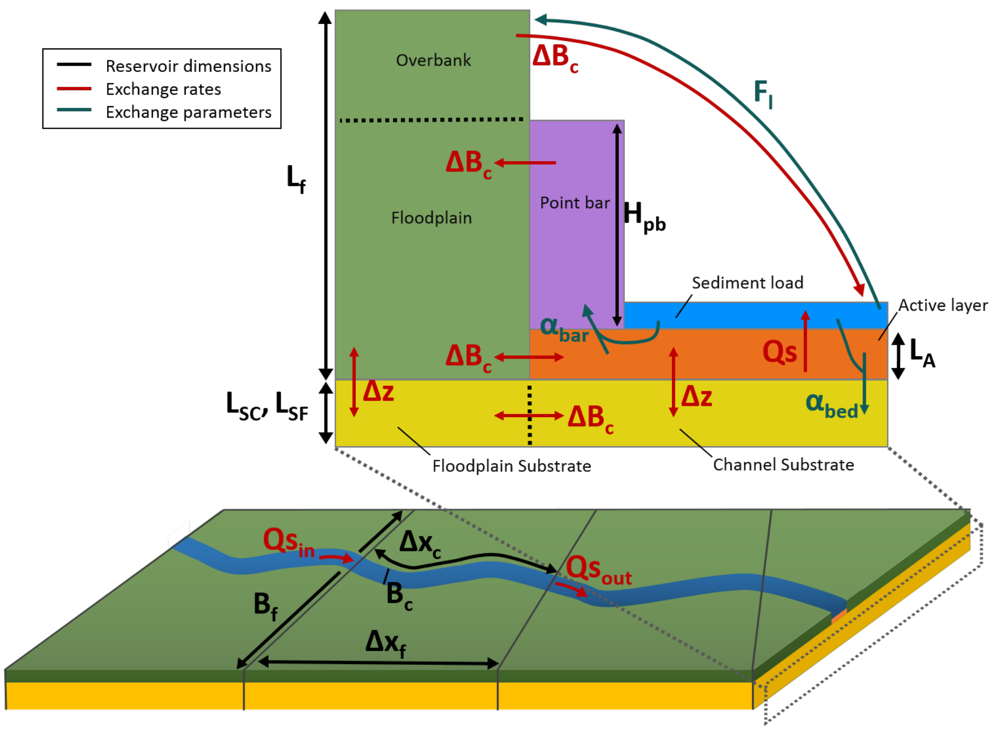


Figure 1: Model schematic

calculated to determine changes in bed elevation ($z$) and channel width ($B_c$). When input to the active layer exceeds output, aggradation occurs and the underlying channed substrate increases in thickness. When sediment transport capacity is greater than supply to the active layer, the channel degrades and substrate material is incorporated into the active layer.

In MAST-1D, flow strength is calculated from an imposed discharge, which can be represented either by a stepped flow duration curve or a hydrograph. When using a the latter, flow strength, sediment transport, and reservoir exchanges are determined using each discharge value in turn. Each flow is imposed over a number of timesteps that cumulatively equal the temporal resolution of the discharge record (e.g. if a daily discharge record is used and each model time step is 0.25 days, then 4 time steps will be performed for each discharge). When a flow duration curve is used, the discharge record is divided into bins that are assigned a time-averaged duration. Flow strength, sediment transport capacity, and reservoir exchanges are determined for each discharge in turn, and the total flux for each exchange is the duration-weighted average of all imposed flows. While the flow duration curve does not account for temporal variability in the hydrologic regime, it is more numerically stable and allows MAST-1D to run much faster. For both flow algorithms, we assume that the channel is rectangular and that sediment distribution within each reservoir is spatially homogenous.

Full details of the model are divided into three sections. First, the model steps are listed in order with references to the equations used. Then, the governing equations for flow, sediment transport, and reservoir exchange are presented in detail. Finally, methods for determining initial model conditions are highlighted.

# 2 Model procedure

The model steps proceed as follows:

1. Boundary conditions are set using Equations 4.1-4.3. The upstream boundary is an imposed size-specific sediment load. The downstream water surface elevation is imposed, either as a constant (i.e. in the case of a downstream control such as a dam) or as a function of discharge.

2. The floodplain number (Lauer and Parker, 2008a), which is a parameter that determines the ability of the floodplain to trap overbank material, is either calculated with Equations 4.4-4.5 or set by the user. The initial floodplain grainsize distribution is set to reflect long-term steady-state conditions with Equations 4.6-4.7.

3. Hydraulics are calculated for the entire reach using the standard step backwater method (Equations 3.1-3.16).

4. Bedload transport for the upstream-most node is determined with Equations 3.17-3.25.

5. Equations 3.43-3.47 are used to calculate rates of channel widening and narrowing.

6. Lateral reservoir exchange rates are calculated with Equations 3.48-3.49 and 3.51-3.58. These include exchanges between the floodplain and active layer due to bank erosion and channel narrowing.

7. The Exner equation (Equation 3.59) is solved to determine whether the channel is aggrading or degrading. Vertical exchange rates between the substrate, active layer, and floodplain reservoirs are determined using Equations 3.60-3.64.

8. The suspended sediment concentration and deposition rate is determined with Equations 3.26-3.28 and 3.40.

9. Volumes and grainsize distributions of the reservoirs are updated using Equations 3.70-3.72.

10. Equations 3.74-3.78 are used to update channel geometry. If the conditions for avulsion are met, Equations 3.83-3.87 are used to adjust reservoir dimensions and grainsizes.

11. Equations 3.79-3.82 are used to split or combine substrate layers. If the thickness of the uppermost substrate layer becomes thicker than a threshold, it is split into two stratigraphic units. If the thickness dips below a threshold, it is combined with the stratigraphic unit below it.

12. If the net amount of channel degradation drops below a critical value and the node designated as bedrock-influenced, then the node becomes partly alluvial. In future steps, partly alluvial transport is calculated as a function of the bed that is alluvial. When the volume of sediment

in the active layer of the partly-alluvial node reaches its capacity, then the node becomes fully alluvial.

13. The sediment transport rates $Q_{s,i}$ and $Q_{s,w}$ are set as the feed for the next node in the downstream direction. Steps 4-12 are repeated for all nodes.

14. The boundary conditions are changed if needed by the user. Steps 3-13 are repeated for the specified number of timesteps or discharges.

# 3 Governing equations

## 3.1 Hydraulics

Hydraulics are calculated using the standard-step method applied to the backwater equation, assuming steady, gradually-varied, subcritical flow. The water surface elevation (WSE) of the downstream-most node is provided as a boundary condition (see Section 4.1.2), and conservation of energy is used to determine the WSE of the next node upstream. The WSE of the second node is then used to calculate the third node upstream, and so on. The procedure was adapted from that used in the HEC-RAS model (Brunner, 2016).

### 3.1.1 Calculation of flow depth and velocity

Hydraulics between each node are calculated using the 1-dimensional form of the Bernoulli Equation:

$$z_2 + y_2 + \frac{\alpha_{v,2}\bar{v}_2^2}{2g} = z_1 + y_1 + \frac{\alpha_{v,1}\bar{v}_1^2}{2g} + h_e \tag{3.1}$$

The subscript 1 denotes the downstream node, while 2 represents the upstream node. $z$ and $y$ are bed elevation and flow depth, respectively, and WSE $= z + y$. $\bar{v}$ is average velocity over the node cross-section, $g$ represents gravitational acceleration, $\alpha_v$ is a weighting coefficient that accounts for the partitioning of average velocity between the channel and floodplain, and $h_e$ denotes the energy loss between the upstream and downstream nodes.

The channel cross-section is divided into two parts: the channel and the floodplain, each with a characteristic roughness. The mean kinetic energy head ($\alpha_v \frac{\bar{v}^2}{2g}$) is defined as the discharge-weighted average between the velocity heads of the two sections:

$$\alpha_v \frac{\bar{v}^2}{2g} = \frac{Q_c \frac{v_c^2}{2g} + Q_f \frac{v_f^2}{2g}}{Q_c + Q_f} \tag{3.2}$$

where $Q_c$ and $Q_f$ are the discharges over the channel and floodplain zones, respectively, and $v_c$ and $v_f$ are the corresponding velocities. If we rewrite $v$ in terms of $Q$ and flow area $A$:

$$v = \frac{Q}{A} \tag{3.3}$$

and

$$\bar{v} = \frac{Q}{A_c + A_f}, \tag{3.4}$$

then we can use the Manning Equation:

$$Q = K S_f^{1/2} \tag{3.5}$$

to reformulate equation 3.2 in terms of area and conveyance $K$, which is defined as

$$K = \frac{1}{n} A R^{2/3} \tag{3.6}$$

$R$ is the hydraulic radius, estimated as the flow depth $y$ (assuming a wide channel) and $n$ is the Manning coefficient. For the channel, it is calculated using a modified form of the Strickler relation:

$$n_c = C_{n,m}(0.0146D_{65}^{1/6} + C_{n,a}) \tag{3.7}$$

where $D_{65}$ is the 65th percentile of the bed material grainsize distribution and $C_{n,a}$ and $C_{n,m}$ are user-defined constants that account for roughness due to form drag and sinuosity. The Manning coefficient for the floodplain in each node is a user-defined constant.

The energy slope ($S_f$) is assumed to be constant between the two nodes and is defined as:

$$S_f = \Big(\frac{Q_1 + Q_2}{K_{c,1} + K_{f,1} + K_{c,2} + K_{f,2}}\Big)^2 \tag{3.8}$$

Using values of $A$ and $K$ calculated for the channel and floodplain, we can solve for $\alpha$:

$$\alpha_v = \frac{(A_c + A_f)^2[\frac{K_c^3}{A_c^2} + \frac{K_f^3}{A_f^2}]}{(K_c + K_f)^2} \tag{3.9}$$

The energy loss ($h_e$) between the upstream and downstream nodes is a function of both friction and expansion/contraction. We are ignoring the effects of expansion and contraction so that $h_e$ depends on friction only:

$$h_e = \frac{\Delta x(Q_{c,1} + Q_{c,2}) + \Delta x\chi(Q_{f,1} + Q_{f,2})}{Q_{c,1} + Q_{f,1} + Q_{c,2} + Q_{f,2}} \tag{3.10}$$

where $\Delta x$ is the length of channel in the node and $\chi$ is the channel sinuosity. Finally, Equations 3.4, 3.9, and 3.10 can be substituted into Equation 3.1 to solve for the WSE of the upstream node, $z_2 + y_2$.

### 3.1.2 Iteration procedure

Equation 3.1 cannot be solved using direct methods, so iteration must be used to converge on the proper WSE. An initial guess is made for $y_2$ (usually as the flow depth from the previous timestep). Then the respective channel area for both the upstream and downstream nodes are calculated as

$$A_c = y * B_c \tag{3.11}$$

and the floodplain areas are

$$A_f = B_f[y - (L_f - L_a)] \tag{3.12}$$

where $B_c$ is the channel width, $B_f$ is the width of the floodplain, $L_f$ is the height of the floodplain, and $L_a$ is the thickness of the active layer.

Equation 3.1 is solved for $y_2$, and the error is calculated as the difference between the input and output $y_2$ values. The error divided by a user-defined stabilizing term is subtracted from the input $y_2$ term to create the $y_2$ for the next iteration. This continues until the error is less than .001 m.

Finally, the values from the final iteration are input into the Manning Equation, resulting in the friction slope and discharge for the node:

$$S_f = (\frac{Q}{K_c + K_f})^2, \tag{3.13}$$

$$Q_c = Q(\frac{K_c}{K_c + K_f}), \tag{3.14}$$

$$Q_f = Q - Q_c, \tag{3.15}$$

and

$$y_f = y_c - (L_f + L_a) \tag{3.16}$$

where $y_f$ is the flow depth on the floodplain. Channel velocity ($V_c$) is calculated using Equation 3.3, with the $Q_c$ and $A_c$.

## 3.2 Sediment transport

There are two forms of sediment transport in MAST-1D, bedload transport and suspended load. In this implementation, it is assumed that all silt and clay (termed washload here) travels as suspension, regardless of discharge. Sand and gravel may also travel in suspension and be deposited on the floodplain depending on the flow conditions.

### 3.2.1 Bedload

Bedload transport is calculated using the Gaeuman et al. (2009) equations, which are a form of the Wilcock and Crowe equations that are suitable for large, cobble-bed streams. The sediment transport rate is a function of the excess channel-wide shear stress over a grainsize-dependent threshold. The shear stress exerted on the sediment grains (the skin friction, or $\tau'$) is calculated following the method in the BAGS Primer (Wilcock et al., 2009) as

$$\tau' = 0.00148\rho * g * (2S_f * D_{65})^{0.25} V_c^{1.5} \tag{3.17}$$

where $\rho$ is the density of water, set at 1000 kg/m$^3$ and $V_c$ is channel velocity. The dimensionless reference shear stress for the mean particle size $\tau^*_{rm}$ is

$$\tau^*_{rm} = 0.03 + \frac{0.022}{1 + e^{7.1\sigma_{SG} - 1.66}} \tag{3.18}$$

$\sigma_{SG}$ is the standard deviation of the sediment grainsize on the psi scale. This is converted into a dimensional reference shear stress ($\tau_{rm}$) using the Shields equation:

$$\tau_{rm} = \tau^*_{rm}(\rho_s - \rho)gD_g \tag{3.19}$$

where $\rho_s$ is sediment density and $D_g$ is the mean particle size. A hiding function is used to calculate the reference shear stress for each particle size:

$$\tau_{ri} = \tau_{rm}\left(\frac{D_i}{D_{50}}\right)^b \tag{3.20}$$

where $\tau_{ri}$ is the reference shear stress and $D_i$ is the grain diameter for size class $i$ and

$$b = \frac{0.7}{1 + e^{1.9 - D_i/3D_g}} \tag{3.21}$$

where $D_g$ is the mean grain size. The dimensionless transport rate for each size class $i$ depends on $\phi_i$, the ratio between the shear stress and the reference shear stress for that size, where

$$\phi_i = \frac{\tau'}{\tau_{ri}} \tag{3.22}$$

The equation is

$$w^*_i = \begin{cases} 0.002\phi_i^{7.5}, & \phi_i < 1.35 \\ 14(1 - \frac{0.894}{\phi_i^{0.5}})^{4.5}, & \phi_i \geq 1.35 \end{cases} \tag{3.23}$$

The fractional transport rate is then put into dimensional form and multiplied by its fraction in the bed:

$$q_{s,i} = f_i \frac{w^*_i u^{*3} B_c \rho}{g(\rho_s - \rho)} \tag{3.24}$$

where $q_{s,i}$ is the sediment transport rate for size class $i$ and $u^*$ is the shear velocity, which is

$$u^* = \left(\frac{\tau}{\rho}\right)^{0.5} \tag{3.25}$$

### 3.2.2 Washload

Sediment can enter the suspended load in two ways: 1) via bank erosion, and 2) as incoming load from upstream. It is assumed that washload sediment is neither entrained from nor deposited on the channel bed. It may be deposited on the point bar and thus transferred to the floodplain through lateral migration, or it may be deposited directly on the floodplain through overbank deposition (see Section 3.4.1). Washload sediment is not entrained from the floodplain; the only mechanism for moving washload from the floodplain to the channel is through bank erosion. The amount of deposition on the floodplain is a function of the sediment concentration in the overbank flow scaled by a constant floodplain number (Section 4.2):

$$d_w = \frac{FCQ_f}{B_f} \tag{3.26}$$

where $d_w$ is the average amount of washload sediment deposited on the floodplain per unit channel length, $F$ is the floodplain number and $C$ is the suspended sediment concentration,

$$C = \frac{q_{w,in}}{Q_c} \tag{3.27}$$

where $q_{w,in}$ is the incoming suspended sediment from the upstream node or boundary feed. The suspended sediment transport rate ($q_w$) is calculated via conservation of mass:

$$q_w = (f_{SAL,w} I_{v,SAL,w} + f_{FP,w}\frac{E}{\Delta t} + f_{PB,w}\frac{N}{\Delta t} + q_{w,in} - d_w \Delta x) \tag{3.28}$$

where $I_{v,SAL,w}$ is the incoming substrate sediment due to vertical channel change (Section 3.4.2), $\frac{E}{\Delta t}$ and $\frac{N}{\Delta t}$ are rates of widening and narrowing, respectively, and $\Delta x$ is the length of the node. ($f_{SAL,w}$, $f_{FP,w}$, and $f_{PB,w}$ refer to the fractions of mud in the active layer, floodplain, and point bar, respectively). Note that $\frac{N}{\Delta t}$ is negative.

### 3.2.3 Suspended sand and gravel

If flooding occurs and turbulence is strong enough so that the diffusive forces lifting particles exceed gravitational forces, some sediment travels in suspension and is deposited on the floodplain. The gravitational forces acting on the grain are characterized by its settling velocity, which is calculated using the emperical formulation derived by Dietrich et al. (1982). The velocity is determined via a dimensionless parameter, $D*$, which quantifies the ratio between the gravitational force acting on the particle and the viscous properties of the flow:

$$D* = \frac{(\rho_s - \rho)(D * 10^{-3})^3 g}{\rho \nu^2} \tag{3.29}$$

where $\nu$ is the kinematic viscosity. The dimensionless velocity, $W_i^*$, is

$$W_i^* = \begin{cases} 10^a, & D*^2 > 0.5 \\ \frac{D*^2}{5832}, & D*^2 \leq 0.05 \end{cases} \tag{3.30}$$

where $a$ is

$$a = 10^{-3.76715+1.92944log(D*)-0.09815[log(D*)]^2-0.00575[log(D*)]^3+0.00056[log(D*)]^4} \tag{3.31}$$

The settling velocity $v_{b,i}$ is

$$v_{b,i} = \left(\frac{(\rho_s - \rho)}{\rho} W_i^* g \nu\right)^{1/3} \tag{3.32}$$

To determine the amount of sediment that deposits on the floodplain, the proportion of total suspended sediment that is transported overbank must be calculated. To do so, a Rouse profile is created. We assume that sediment in the bottom 5% of the profile travels as bedload. By this definition, the suspended sediment transport rate within the channel that occurs below the top of the bank is

$$q_{s,b,i} = \int_{0.05}^{(L_F - L_{AL})/y} \frac{0.05(1-z)}{0.95z} Z dz \tag{3.33}$$

where $y$ is the flow depth in the channel and Z is the Rouse number,

$$Z = \frac{v_{b,i}}{\kappa u^*} \tag{3.34}$$

where $\kappa$ is the von Karman constant (0.4). The sediment transport rate above the level of the floodplain is

$$q_{s,o,i} = \int_{(L_F - L_{AL})/y}^{1} \frac{0.05(1-z)}{0.95z} Z dz \tag{3.35}$$

The total proportion of overbank suspended sediment that is transported above the level of the banks ($P_o$) is

$$P_o = \frac{0.95}{1 - (L_F - L_{AL})/y} * \frac{q_{s,o,i}}{q_{s,o,i} + q_{s,b,i}} \tag{3.36}$$

Equations 3.33 and 3.35 are discretized into 20 segments.

Only a portion of sand and gravel in any given size class travels in suspension. The rest saltates along the bed. We define the former portion with a constant, $\alpha_{FS}$, which ranges between 0 and 1. The sediment concentration of size class $i$ in the overbank water column is

$$C_i = \frac{q_{s,i,in} \alpha_{FS} P_o}{Q_c} \tag{3.37}$$

where $q_{s,i,in}$ is the incoming sediment feed in size class $i$. The fraction of overbank sediment that deposits on the floodplain per unit channel length is

$$d_i = \frac{F_{bed} C_i Q_f}{B_f} \tag{3.38}$$

where $F_{bed}$ is the floodplain number for bed material, a constant.

### 3.2.4 Calculation of total transport and floodplain deposition

For each timestep, the total bedload sediment transport rates per size class, $Q_{s,i}$, is the weighted sum of the rates for each flow in the duration curve, so that

$$Q_{s,i} = \sum_{j=1}^{n} q_{i,j} p_j \tag{3.39}$$

where $q_{i,j}$ is the size-specific transport rate and $p_j$ is the flow frequency for flow $j$, and $n$ is the number of discharges in the flow duration curve. When running MAST-1D with a hydrograph, $n = 1$. $Q_{s,i}$ is calculated using Equation 3.24. The total suspended sediment transport rate ($Q_w$) is

$$Q_{s,w} = \sum_{j=1}^{n} q_{w,j} \tag{3.40}$$

Equation 3.28 is used to calculate $q_{w,j}$.

The overbank deposition rates $d_w$ and $d_i$ are also duration-averaged sums:

$$d_w = \sum_{j=1}^{n} d_{w,j} p_j \tag{3.41}$$

and

$$d_i = \sum_{j=1}^{n} d_{i,j} p_j \tag{3.42}$$

where $d_{w,j}$ and $d_{i,j}$ are solved for using Equations 3.26 and3.38, respectively.

## 3.3 Width change

There are two components to width change that result in sediment exchanges: channel widening via erosion and narrowing from vegetation encroachment onto bars. When rates of erosion and vegetation growth are not equal, width change occurs. When they are equal, there is migration but no net change in width. The governing equations are briefly described here. Full details on the theory and rationale can be found in Chapter **??**.

### 3.3.1 Channel widening

Our simple model of channel widening only relates bank erosion to sediment transport capacity. Channel widening occurs when a supply-normalized unit transport rate of the upper tail of the grainsize distribution, $qs_{Cmax}$, exceeds a threshold, $qs_{cr}$. We define the supply-normalized unit coarse transport rate as

$$qs_{Cmax} = qs_C / f_C \tag{3.43}$$

where $qs_C$ is the unit sediment transport rate of the coarse end of the surface sediment mixture and $f_C$ is the fraction of that group of sizes present in the bed. $qs_C$ is calculated via the equations in Section 3.2.1. $qs_{Cmax}$ represents the transport rate expected with an unlimited supply of coarse sediment. There is currently no straightforward way to determine the threshold unit transport rate $qs_{cr}$, and for now it is a user-defined constant.

Once bank erosion is initiated ($qs_{Cmax} > qs_{cr}$, floodplain sediment mixes with the active layer adjacent to the bank, and the magnitude of bank erosion depends on the ability of the flow to transport this near-bank sediment. When coarse sediment supply from the bank exceeds the transport capacity, it will build up along the bank toe and protect it from further erosion. The near-bank sediment transport capacity, $qs_{NB}$, is a function of the grainsize distribution of the near bank region, which is defined by

$$f_{i,NB} = \alpha_f f_i + (1 - \alpha_f) f_{i,FP} \tag{3.44}$$

where $f_{i,NB}$ is the near-bank fraction of size class $i$, $f_i$ is the fraction in the active layer, $f_{i,FP}$ is the fraction in the floodplain, and $\alpha_f$ is a mixing constant that ranges between 0 and 1. $qs_{NB}$ is calculated using the bedload relations outlined in Section 3.2.1, with $f_{i,NB}$ as the grainsize distribution. The portion of $qs_{i,NB}$ that transports coarse floodplain material, $qs_{C,FP}$, is

$$qs_{C,FP} = \frac{qs_{C,NB}}{f_{C,NB}} f_{C,FP}(1 - \alpha_f) \tag{3.45}$$

where $qs_{C,NB}$ is the unit coarse sediment transport rate of the near-bank mixture and $f_{C,FP}$ is the fraction of coarse material in the floodplain. The bank erosion rate ($E/\Delta t$) is

$$\frac{E}{\Delta t} = \begin{cases} 0, & qs_{Cmax} \leq qs_{cr} \\ (qs_{C,FP})/(L_F * f_{C,FP}), & qs_{Cmax} > qs_{cr} \end{cases} \tag{3.46}$$

### 3.3.2 Channel narrowing

The narrowing function is: Channel narrowing results from multiple interrelated processes, including deposition on bars, degradation leading to the development of benches, and encroachment of vegetation onto exposed surfaces. We assume that channel narrowing only occurs during relatively low flows. The rate of vegetation enroachment is treated as a constant, $\alpha_n$:

$$\frac{N}{\Delta t} = \begin{cases} -\alpha_n * (B_c - B_{min}), & \tau < \tau_r \\ 0, & \tau \geq \tau_r \end{cases} \tag{3.47}$$

$B_{min}$ is a constant user-defined minimum width and $B_c - B_{min}$ represents the unvegetated point bar. $\tau_r$ represents a reference shear stress, below which flow is low enough to leave surfaces exposed for colonization.

## 3.4 Sediment reservoir exchanges

There are five sediment reservoir types in MAST-1D: the load, active layer, floodplain, channel substrate, and floodplain substrate. There are multiple layers of substrate, and layers may be added, removed, and combined, depending on the evolution of the bed. Substrate is accounted for in two zones: one under the channel region and the other beneath the floodplain. The size-specific amount of sediment for all reservoirs except the load is determined by a conservation of mass equation:

$$\frac{\Delta V_r, i}{\Delta t} = (1 - \lambda)\frac{\Delta S_{r,i}}{\Delta t} \tag{3.48}$$

where $V_r$ is the new volume of material size class $i$ in reservoir $r$, $\lambda$ is porosity, and $\Delta S_{r,i}$ is the change in storage of sediment in a given size class. $\Delta S_{r,i}$ is calculated as the difference between the inputs ($I$) and the outputs ($O$) during each timestep. For the floodplain and substrate types,

$$\Delta S_{r,i} = (I_{m,r,i} + I_{v,r,i}) - (O_{m,r,i} + O_{v,r,i}) \tag{3.49}$$

where $I_{m,r,i}$ and $O_{m,r,i}$ are inputs and outputs due to net erosion and $I_{v,r,i}$ and $O_{v,r,i}$ are inputs and outputs due to the vertical change in the position of the bed. The active layer has additional terms because it exchanges material with the sediment load:

$$\Delta S_{AL} = (I_{m,r,i} + I_{v,r,i} + Q_{s,in,i}) - (O_{m,r,i} + O_{v,r,i} + Q_{s,i}) \tag{3.50}$$

$Q_{s,in,i}$ is the bedload feed and $Q_{s,i}$ is the bedload for size $i$.

The mass balance for the sediment load is described in terms of suspended sediment discharge, Equations 3.28 and 3.40. $I$ and $O$ terms for each reservoir are described in more detail below.

### 3.4.1 Lateral exchanges

Lateral reservoir exchanges are driven by channel migration. The size distribution of substrate underlying the channel may be different from that below the floodplain because of selective deposition onto the channel and point bar and the size-specific supply of sediment from upstream nodes. Therefore, there are two substrate reservoirs, one each for the channel and floodplain. As the channel moves across the floodplain, it lay above old floodplain substrate, which becomes incorporated into the channel substrate. It also abandons a portion of both its underlying substrate, which mixes into the floodplain substrate reservoir. For each grainsize class $i$ in the substrate,

$$I_{m,SF,i} = O_{m,SC,i} = -(1 - \lambda)\frac{N}{\Delta t} L_S f_{SC,i} \Delta x \tag{3.51}$$

and

$$O_{m,SF,i} = I_{m,SC,i} = (1 - \lambda)\frac{E}{\Delta t} L_S f_{SF,i} \Delta x \tag{3.52}$$

$L_S$ is the height of the substrate. The subscript $SF$ represents the floodplain portion of the substrate and $SC$ denotes the channel substrate. Washload from the floodplain (subscript $FP$) goes straight into the sediment load (subscript $L$) and does not interact with the active layer:

$$O_{m,FP,w} = I_{m,L,w} = (1-\lambda)\frac{E}{\Delta t}L_F f_w \Delta x \tag{3.53}$$

where the subscript $w$ denotes the size class traveling as suspended load. Bed material-sized sediment that erodes from the floodplain is exchanged directly with the active layer:

$$O_{m,FP,i} = I_{m,AL,i} = (1-\lambda)\frac{E}{\Delta t}L_S f_{FP,i} \Delta x \tag{3.54}$$

The subscript $AL$ refers to the active layer.

Inputs to the floodplain from the active layer and load occur in two ways: from overbank deposition and from channel narrowing. The latter is modulated by a point bar reservoir, which has a grainsize distribution and height, $L_{PB}$. The fraction of pointbar that is composed of washload sediment is

$$f_{PB,w} = 1 - \left(1 + \frac{\bar{k}Q_{s,w}}{Q_{s,b}}\right)^{-1} \tag{3.55}$$

where $Q_{s,b}$ is the duration-averaged bed material load ($\sum Q_{s,i}$) and $\bar{k}$ is a user-defined relationship between the proportion between suspended and bedload in the load and that proportion on the point bar. The export of suspended sediment from the load and the input into the floodplain via vegetation encroachment and overbank deposition becomes

$$O_{m,AL,w} = I_{m,FP,w} = (1-\lambda)f_{PB,w}L_{PB}\frac{N}{\Delta t}\Delta x + d_w \Delta x \tag{3.56}$$

The input of each size class of bed material into the floodplain due to vegetation encroachment and overbank deposition is described as

$$O_{m,AL,i} = I_{m,FP,i} = (1-\lambda)f_{PB,i}L_{PB}\frac{N}{\Delta t}\Delta x + d_i \Delta x \tag{3.57}$$

where

$$f_{PB,i} = (1 - f_{PB,w})[\alpha_{bar} f_{AL,i} + (1 - \alpha_{bar}) f_{Qs,i}] \tag{3.58}$$

and $\alpha_{bar}$ is the proportion of point bar bed material sediment that is sourced from the active layer as opposed to the load.

### 3.4.2 Vertical exchanges

Vertical reservoir exchanges (subscript $v$) are driven by the Exner equation, where

$$\frac{\Delta z}{\Delta t} = \frac{1}{B_c(1-\lambda)} * \frac{(\sum(I_{m,AL,i} + Q_{s,f}) - Q_s}{\Delta x} \tag{3.59}$$

$\frac{\Delta z}{\Delta t}$ is the rate of bed elevation change and $Q_{s,f}$ is the total incoming bedload feed and $Q_s$ is the computed load, with is exported to the next downstream node. If the channel is aggrading ($\frac{\Delta z}{\Delta t} > 0$), then the uppermost substrate channel and floodplain layers receive bed material sediment from the active layer and floodplain, respectively:

$$I_{v,SC,i} = O_{v,AL,i} = \begin{cases} \frac{\Delta z}{\Delta t} B_c \Delta x (1-\lambda)(\alpha_{bed} f_{AL,i} + (1 - \alpha_{bed} f_{L,i}), & z > 0 \\ 0, & z \leq 0 \end{cases} \tag{3.60}$$

where $\alpha_{bed}$ is the proportion of sediment entering the substrate from the bed vs. the bedload, the subscript $v$ refers to vertical exchange, and

$$I_{v,SF,i} = O_{v,FP,i} = \begin{cases} \frac{\Delta z}{\Delta t} \frac{\Delta x}{\chi} B_f (1-\lambda) f_{FP,i}, & z > 0 \\ 0, & z \leq 0 \end{cases} \tag{3.61}$$

If the bed is degrading ($\frac{\Delta z}{\Delta t} < 0$), then the uppermost substrate layers provides sediment to the active layer and active floodplain:

$$O_{v,SC,i} = I_{v,AL,i} = \begin{cases} -\frac{\Delta z}{\Delta t} B_c \Delta x (1-\lambda) f_{SC,i}, & z < 0 \\ 0, & z \geq 0 \end{cases} \tag{3.62}$$

and

$$O_{v,SF,i} = I_{v,F,i} = \begin{cases} -\frac{\Delta z}{\Delta t} \frac{\Delta x}{\chi} B_c (1-\lambda) f_{SF,i}, & z < 0 \\ 0, & z \geq 0 \end{cases} \tag{3.63}$$

It is assumed that washload sediment does not infiltrate into the bed during aggradation (i.e. $I_{v,SC,w} = O_{v,AL,w} = 0$). However, during degradation, fine sediment from the uppermost substrate layer is entrained and enters the load:

$$O_{v,SC,w} = I_{v,L,w} = \begin{cases} -\frac{\Delta z}{\Delta t} B_c \Delta x (1-\lambda) f_{SC,w}, & z < 0 \\ 0, & z \geq 0 \end{cases} \tag{3.64}$$

Fine sediment in the floodplain is exchanged with the uppermost floodplain substrate in the same way as bed material, using Equations 3.61 and 3.63, but replacing $i$ with $w$.

### 3.4.3 Exchanges in bedrock channels

The user may specify nodes that are underlain by non-erodable material such as bedrock. The channel is only allowed to degrade to a user-defined threshold, after which $\frac{\Delta z}{\Delta t}$ is set at 0 and the channel becomes 'partly alluvial.' Conservation of mass is maintained by adjusting the total volume of the active layer instead of sourcing material from the substrate. In partly-alluvial nodes, washload sediment may be evacuated from the active layer when total sediment inputs exceed outputs:

$$Q_{s,adj,w} = Q_{s,in,w} - f_{w,AL} * (Q_{s,in} - Q_s) \tag{3.65}$$

Equation 3.65 ensures that the active layer grainsize distribution of a partly-alluvial node does not become dominated by fine sediment. The change in washload volume in the active layer ($\Delta S_{AL,w}$) is

$$\Delta S_{AL,w} = (I_{m,r,w} + Q_{s,in,w}) - (O_{m,r,w} + Q_{s,adj,w}) \tag{3.66}$$

When the inputs to a partly-alluvial node exceed outputs, the size-specific bedload exiting the node is adjusted to fill the active layer with bed material sediment:

$$\Delta S_{AL,i} = (I_{m,r,i} + Q_{s,in,i}) - (O_{m,r,i} + Q_{s,adj,i}) \tag{3.67}$$

where

$$Q_{s,adj,i} = Q_{s,in,i} - (Q_{s,in} - Q_s)[f_{i,AL}\alpha_{pa} + f_{i,L}(1 - \alpha_{pa})] \tag{3.68}$$

$Q_{s,adj,i}$ is the adjusted sediment load for size class $i$, $Q_{s,in}$ and $Q_{s,out}$ are the total sediment feed and load, respectively, and $\alpha_{pa}$ is the ratio between the volume of a fully alluvial active layer and the current volume:

$$\alpha_{pa} = \frac{V_{AL}}{L_{AL} Bc\Delta x} \tag{3.69}$$

Equations 3.67 and 3.66 replace Equation 3.50 in partly alluvial nodes. When $\alpha_{pa}$ is greater than or equal to 1, the bed is no longer partially alluvial, and bed elevation changes may occur again.

### 3.4.4 Reservoir geometry and grainsize distributions

For each timestep, the volume for each reservoir size class (including the suspended load) is calculated by multiplying the result from Equation 3.48 by the length of the timestep and adding it to the initial volume:

$$V_{r,i} = V_{0,r,i} + \frac{\Delta V_{r,i}}{\Delta t}\Delta t \tag{3.70}$$

where $V_{0,r,i}$ is the volume after the previous timestep of length $t$ (and, in the case of the fine sediment fraction, $i$ is replaced with $w$). The total volume $V_r$ is

$$V_r = \sum_{i=1}^{n} V_{r,i} + V_{r,w} \tag{3.71}$$

and the size fractions are

$$f_{r,i} = \frac{V_{r,i}}{V_r} \tag{3.72}$$

for the bed material load and

$$f_{r,w} = \frac{V_{r,w}}{V_r} \tag{3.73}$$

for the washload. Reservoir volumes and size fractions are updated during each timestep for the active layer (AL), floodplain (F), and substrate layers (S). Channel width is calculated based on the encroachment and erosion rates:

$$B_c = B_{c,0} + \left(\frac{N}{\Delta t} + \frac{E}{\Delta t}\right)\Delta t \tag{3.74}$$

where $B_{c,0}$ is the previous channel width. The floodplain width is

$$B_f = B_{f,0} - \left( \frac{N}{\Delta t} + \frac{E}{\Delta t} \right) \chi \Delta t \tag{3.75}$$

where $B_{f,0}$ is the previous floodplain width. The floodplain height then becomes

$$L_f = V_F \left( B_f \frac{\Delta x}{\chi} \right)^{-1} \tag{3.76}$$

The height of the uppermost substrate layer is a function of the vertical bed change:

$$L_s = L_{s,0} + \frac{\Delta z}{\Delta t} \Delta t \tag{3.77}$$

where $L_{s,0}$ is the previous upper substrate height. The heights of deeper substrate layers do not change. The new bed elevation $z$ is

$$z = z_0 + \frac{\Delta z}{\Delta t} \Delta t \tag{3.78}$$

where $z_0$ is the previous bed elevation.

## 3.5 Substrate maintenance

All substrate layers are initially set at a uniform thickness, $L_{S,0}$. Substrate layers are split or combined when the thickness of the uppermost layer, $L_S$, exceeds or drops below thickness thresholds or when aggradation begins to approach the height of the floodplain.

### 3.5.1 Stratigraphy

In order to preserve the stratigraphy of subsurface deposits, the substrate reservoirs are modified as the river aggrades and degrades. If the river aggrades over a defined threshold ($L_{sp}$), the uppermost substrate layer is split in two, creating a new stratigraphic layer. The thickness of this new layer is

$$L_{s,new} = L_s - L_{s,0} \tag{3.79}$$

and the thickness of the old layer becomes $L_{s,0}$. New volumes are calculated for the layers as

$$V_r = L_s B_c \Delta x \tag{3.80}$$

where $r$ represents the channel ($SC$) and floodplain (SF) substrate reservoirs. The grainsize distribution of the new layers is the same as in the respective parent layers.

If the uppermost substrate layer thickness is reduced below a threshold due to degradation, that layer is combined with the layer below it. The thickness of the new layer becomes

$$L_{s,new} = L_s + L_{s,-1} \tag{3.81}$$

where $L_{s,-1}$ is the thickness of the lower layer. The grainsize distribution of the combined layer is a weighted average of the two layers

$$f_{S,i} = f_{S,i} L_s + f_{S,-1,i} L_{S,-1} \tag{3.82}$$

where $f_{S,i}$ and $f_{S,-1,i}$ are the fractions for size $i$ of the upper and lower layers, respectively. The volume of the reservoir is solved using Equation 3.80.

### 3.5.2 Avulsion

Avulsion (the rapid shift of the dominant channel to a new location) is common in alluvial rivers when the channel is blocked by sediment, large wood, or other obstructions. In MAST-1D, avulsion is triggered in model nodes experiencing high levels of aggradation where the bed elevation approaches that of the floodplain. The implication is that avulsions occur in areas of persistent sediment deposition (such as deltas). When the floodplain height ($L_f$) dips below a user-defined threshold value, the bed elevation lowers by a spacing constant ($L_{av}$):

$$z_{new} = z_{old} - L_{av} \tag{3.83}$$

where $z_{old}$ is the pre-avulsion bed elevation and $z_{new}$ is the resulting elevation. The surface of the new channel becomes active, so that the volume of floodplain material added to the active layer for size class $i$ ($AL_{in,i}$) in the avulsed node is

$$AL_{in,i} = \alpha_a * B_c * L_{AL} * f_{i,FP} * \Delta x \tag{3.84}$$

where $\alpha_a$ is the fraction of channel that avulses, $L_{AL}$ is the thickness of the active layer, and $f_{i,FP}$ is the fraction of size class $i$ in the floodplain. We make the simplification that the abandoned portion of channel becomes vegetated immediately. Channel substrate and the avulsed portion of the active layer are incorporated into the floodplain. Floodplain material is mixed into the active layer to represent the surface material forming the base of the new channel. The volume of channel sediment sequestered into the floodplain reservoir ($FP_{in,i}$) is

$$FP_{in,i} = [\alpha_a * B_c * L_{AL} * f_{i,AL} + B_c * L_a * f_{i,SC}] * \Delta x \tag{3.85}$$

The grainsize distribution of the active layer is adjusted to incorporate floodplain material under the new channel:

$$f_{AL,i} = \alpha_a f_{FP,i} + (1 - \alpha_a) f_{AL,i} \tag{3.86}$$

Sediment from the substrate and the old active layer are incorporated into the floodplain to conserve mass and a new floodplain volume is calculated:

$$V_{FP,i} = V_{FP,old,i} + V_{SC,i} + V_{SF,i} + V_{AL,old,i} - V_{AL,new,i} \quad (3.87)$$

where $V_{AL,old,i}$ and $V_{AL,new,i}$ are the old and new active layer size-specific volumes, respectively and $V_{FPold,i}$ is the old floodplain volume. If the uppermost substrate layer is lower or higher than $L_{av}$, then it is either split or combined using the methodology in Section 3.5.1 before performing Equation3.87 so that $L_{FP} = L_{av}$.

# 4 Initial and boundary conditions

## 4.1 Boundary conditions

As a 1-dimensional model, MAST-1D requires two boundary conditions: an upstream sediment supply, and a downstream hydraulic boundary.

### 4.1.1 Sediment supply

By default, the upstream sediment feed is a user-defined proportion of the bedload sediment capacity, where

$$Q_{s,i,in} = C_{f,b} Q_{s,i,cap} \quad (4.1)$$

and

$$Q_{s,w,in} = C_{f,w} \sum_{j=1}^{n} Q_{s,i,in} \quad (4.2)$$

$Q_{s,i,in}$ and $Q_{s,w,in}$ are the total size-specific feed rates for the bedload and suspended load, respectively, $Q_{s,i,cap}$ is the size-specific bedload transport capacity, calculated using Equations 3.17-3.25 and a user-specified size distribution, and $C_{f,b}$ is proportion of capacity that is input as bedload feed.

Suspended feed is assumed to be a set multiple of initial beload capacity, modulated by the constant $C_{f,w}$.

The user may allow $C_{f,b}$ and $C_{f,w}$ to change over time, either directly or via a function. When a flow duration curve is used, $Q_{s,i,cap}$ is determined using the duration-weighted sum of sediment transport rate for all flows in the curve. When hydraulics are calculated using a hydrograph, $Q_{s,i,cap}$ is recalculated for each new flow, using the initial geometric and sediment conditions. In other words, a sediment capacity rating curve is used.

### 4.1.2 Hydraulics

The water surface elevation is set at the downstream boundary. The user has the option of setting the boundary as a constant or changing it manually (for example, if the modeled reach ends at a reservoir or shoreline). If the water surface elevation is not known, it is calculated assuming normal flow conditions and a wide channel with the Manning equation:

$$z + y = z + \left( \frac{n_c Q}{B_c S_c^{0.5}} \right)^{3/5} \tag{4.3}$$

where $S_c$ is the channel bed slope. If $y$ is greater than the channel depth $(L_f - L_a)$, then the flow depth is solved for iteratively using Equations 3.1-3.10. $\Delta x$ is set at 100 m, and iterations continue until the upper and lower channel depths ($y_1$ and $y_2$) converge.

When a flow duration curve is known, the boundary water surface elevation is solved for each flow at the beginning of the run. If a hydrograph is used, then the boundary condition is solved for each discharge using the initial channel geometry and sediment conditions.

## 4.2 Initial conditions

The initial channel geometry and grainsize distribution of the active layer are supplied by the user. Given these conditions, as well as the upstream sediment boundary and sediment mixing parameters, the floodplain grainsize distribution and floodplain number are calculated so that the model river would be in equilibrium if channel width and migration rate were constant.

### 4.2.1 Floodplain number

The floodplain number $F$ determines the proportion of the suspended load that is deposited during each timestep. The depth of fine sediment on the floodplain during the initial condition ($L_w$) is assumed to be equivalent to the depth during equilibrium and is calculated as

$$L_{w,0} = L_f - (L_a + L_{pb}) \tag{4.4}$$

The floodplain number then is the proportion that the sediment concentration would have to be reduced to reproduce $L_w$ within an average floodplain reworking time ($B_f/m$), assuming that mud is being transported at the capacity determined in Equation 4.2 and given a constant user-defined migration rate $m$:

$$F = \frac{L_{w,0} m}{d_{full}} \tag{4.5}$$

where $d_{full}$ is the average suspended sediment deposition rate per unit floodplain if the entire sediment load were deposited. The floodplain number can also be set manually by the user.

### 4.2.2 Initial floodplain grainsize distribution

The initial grainsize distribution for the floodplain and floodplain substrate reservoirs is a combiniation of the distributions of the active layer and point bar:

$$f_{F,i,0} = \frac{f_{AL,i,0}L_{al} + f_{PB,i,0}L_{pb}}{L_f} \tag{4.6}$$

and

$$f_{F,i,0} = \frac{f_{AL,w,0}L_{al} + f_{PB,w,0}L_{pb} + L_{w,0}}{L_f} \tag{4.7}$$

The subscript 0 refers to the values of the variables at the initial condition.

# 5 Variable list

Table 1: MAST-1D list of variables

| Variable | Unit | Description |
|---|---|---|
| $A$ | $m^2$ | total area of flow |
| $A_c$ | $m^2$ | area of flow for channel |
| $A_f$ | $m^2$ | area of flow for floodplain |
| $AL$ | - | active layer |
| $B_c$ | $m$ | channel width |
| $B_f$ | $m$ | floodplain width |
| $B_{min}$ | $m$ | minimum channel width |
| $C$ | - | washload sediment concentration |
| $C_{f,b}$ | - | bedload sediment feed boundary capacity parameter |
| $C_{n,a}$ | - | addition constant for Manning's n |
| $C_{n,m}$ | - | multiplier for Manning's n |
| $C_{f,w}$ | - | suspended sediment feed boundary capacity parameter |

Table 1: MAST-1D list of variables

| **Variable** | **Unit** | **Description** |
|---|---|---|
| $C_i$ | - | concentration of size class $i$ in overbank flow |
| $C_{max}$ | - | size classes of 'coarse' particles, roughly greater than the $D_{90}$ |
| $d_w$ | $m^2/s$ | average washload deposition rate per unit length on floodplain |
| $d_i$ | $m^2/s$ | average suspended sand/gravel deposition rate per unit length on floodplain for size class $i$ |
| $d_{full}$ | $m^2/s$ | average suspended sediment deposition rate per unit length on floodplain if entire sediment load were deposited |
| $D_g$ | $m$ | mean grain size |
| $D_i$ | $m$, $mm$ | grain size at percentile $i$ |
| $E$ | $m$ | bank erosion |
| $F$ | - | floodplain number |
| $F_{bed}$ | - | floodplain number for bed material |
| $FP$ | - | floodplain |
| $f_i$ | - | fraction of size $i$ in sediment mixture |
| $f_{i,NB}$ | - | fraction of size $i$ in near-bank sediment mixture |
| $f_w$ | - | fraction of suspended-size sediment in mixture |
| $g$ | $m/s^2$ | gravitational acceleration |
| $h_e$ | $m$ | energy head loss |
| $I$ | - | reservoir inputs |

Table 1: MAST-1D list of variables

| **Variable** | **Unit** | **Description** |
|---|---|---|
| $\bar{k}$ | - | coefficient of suspended and bedload sediment in the load and on the point bar |
| $K$ | - | conveyance |
| $K_c$ | - | conveyance for channel |
| $K_f$ | - | conveyance for floodplain |
| $L_a$ | $m$ | thickness of active layer |
| $L_{av}$ | $m$ | bed lowering during avulsion |
| $L_f$ | $m$ | floodplain height |
| $L_{pb}$ | $m$ | thickness of point bar |
| $L_s$ | $m$ | substrate thickness |
| $L_w$ | $m$ | thickness of fine sediment on floodplain during initial condition |
| $N$ | $m$ | vegetation encroachment |
| $n_c$ | - | Manning's n for channel |
| $n_f$ | - | Manning's n for floodplain |
| $O$ | - | outputs |
| $PB$ | - | point bar |
| $p_j$ | - | flow frequency for flow $j$ |
| $P_o$ | - | proportion of suspended load for size class $i$ that is overbank |
| $q_{s,b,i}$ | - | Rouse integral for in-channel portion of flow |
| $q_{s,b,i}$ | - | Rouse integral for overbank portion of flow |
| $qs_{Cmax}$ | $m^3/s$ | transport rate for the coarse fraction |
| $qs_{cr}$ | - | mobility threshold for bank erosion |
| $Q$ | $m^3/s$ | total discharge |
| $Q_c$ | $m^3/s$ | channel discharge |

Table 1: MAST-1D list of variables

| **Variable** | **Unit** | **Description** |
|---|---|---|
| $Q_f$ | $m^3/s$ | floodplain discharge |
| $Q_{s,adj,i}$ | $m^3/s$ | adjusted fractional sediment load for partially-alluvial channel |
| $Q_{s,b,i}$ | $m^3/s$ | proportion of suspended sand/gravel traveling within the banks |
| $Q_{s,f}$ | $m^3/s$ | total bedload sediment feed |
| $Q_{s,i}$ | $m^3/s$ | total sediment load for size $i$ over duration curve |
| $Q_{s,in,i}$ | $m^3/s$ | total sediment feed for size $i$ over duration curve |
| $Q_{s,o,i}$ | $m^3/s$ | proportion of suspended sand/gravel traveling above the banks |
| $Q_{s,w}$ | $m^3/s$ | total suspended sediment load over duration curve |
| $q_{s,i}$ | $m^3/s$ | channel-wide sediment load for size class i |
| $q_w$ | $m^3/s$ | suspended sediment load |
| $q_{w,in}$ | $m^3/s$ | suspended sediment feed |
| $S$ | $m^3$ | storage of sediment in reservoir |
| $SC$ | - | channel substrate |
| $S_c$ | - | channel bed slope |
| $S_f$ | - | energy slope |
| $SF$ | - | floodplain substrate |
| $u^*$ | $m/s$ | shear velocity |
| $\bar{v}$ | $m/s$ | cross-sectional average velocity |
| $v_c$ | $m/s$ | flow velocity in channel |
| $v_f$ | $m/s$ | flow velocity on floodplain |

Table 1: MAST-1D list of variables

| **Variable** | **Unit** | **Description** |
|---|---|---|
| $V$ | $m^3$ | volume |
| $w_i^*$ | - | dimensionless transport for size $i$ |
| $x$ | $m$ | channel-wise coordinate |
| $y$ | $m$ | flow depth in the channel |
| $z$ | $m$ | bed elevation |
| $Z$ | $-$ | Rouse number |
| $\alpha_a$ | - | portion of channel that avulses |
| $\alpha_{bar}$ | - | fraction of point bar bed material sediment sourced from active layer |
| $\alpha_{bed}$ | - | fraction of sediment entering substrate from bed vs. bedload |
| $\alpha_e$ | - | channel widening coefficient |
| $\alpha_f$ | - | proportion of active layer transport in near-bank region |
| $\alpha_n$ | - | channel narrowing coefficient |
| $\alpha_{pa}$ | - | fraction between volume of partly-alluvial fully alluvial active layer |
| $\alpha_v$ | - | weighting coefficient for average velocity |
| $\lambda$ | - | porosity |
| $\nu$ | $m^2/s$ | kinematic viscosity |
| $\rho$ | $kg/m^3$ | density of water |
| $\rho_s$ | $kg/m^3$ | sediment density |
| $\sigma_{SG}$ | - | standard deviation of sediment mixture on psi scale |
| $\tau'$ | | skin friction (shear stress on grains) |
| $\tau_{cr}$ | $N/m^2$ | shear stress needed to entrain reference D84 |

Table 1: MAST-1D list of variables

| Variable | Unit | Description |
|---|---|---|
| $\tau_r$ | $N/m^2$ | reference shear stress below which vegetation encroachment occurs |
| $\tau^*_{rm}$ | - | dimensionless reference shear stress for the mean particle size |
| $\tau_{rm}$ | $N/m^2$ | reference shear stress for the mean particle size |
| $\tau_{ri}$ | $N/m^2$ | reference shear stress for size class $i$ |
| $\phi_i$ | - | ratio between skin friction and reference shear stress of size class |
| $\chi$ | - | channel sinuosity |

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