Computer Science G12

Test 2 Term 3

Date: March 29, 2018

Name:

Each question emphasizes on different skills: (K) Knowledge, (T) Thinking, (C) Communication and (A) Application.

The value of each problem is as follows: P1 20% , P2 40%, P3 30% and P4 10%.

1. (20% KTC) Prove that $Q_n = \sum_{k=1}^{n} \frac{1}{2^k} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots + \frac{1}{2^n} < 1$ by way of finding a closed expression for Q_n. Make sure that your reasoning is clear.

2. (40% KTC) The gist of the **insertion sort** algorithm consists in building a sorted list of number by inserting each of them one by one in the right position. Say, the list of numbers R_1, R_2, R_3, ... R_n R_{n+1} is such that the first n of them are already sorted. How can we sort the full list?

    1. Sketch a reasoning of how to sort the full list using PMI and proving it is correct.
    2. Write a pseudo-code for this algorithm

3. (30% KCA) Calculate the closed form of the following sums and prove your result by mathematical induction. Hint: Use the method of shifting and the well-known result $S_n = \sum_{k=1}^{n} k = n(n+1)/2$

    1. $S2_n = \sum_{k=0}^{n} k^2$
    2. $S3_n = \sum_{k=0}^{n} k^3$

4. (10% KTC) Consider the following recurrence relation

    $$G_n = G_{n-1} + G_{n-2} + 1 \quad \text{with} \quad G_1 = 1\, G_2 = 1$$

    and compare it to the Fibonacci one

    $$F_n = F_{n-1} + F_{n-2} \quad F_1 = 1 \quad \text{with} \quad F_2 = 1$$

    Because of the extra 1 it seems obvious that $G_n > F_n$. Yet the following *seems* a valid proof that $G_n = F_n - 1$, namely (by the strong PMI): Assume that $G_k = F_k - 1 \,\forall\, k \leq n$. We can then prove it holds for $n+1$ as well:

    $$G_{n+1} = G_n + G_{n-1} + 1 = F_n - 1 + F_{n-1} - 1 + 1 = F_{n+1} - 1$$

    What is **wrong** with this proof?