Computer Science at The Dragon Academy
Assignment III
Review of Propositional Logic, Logic Gates & Circuits
Due date: Wed. Oct. 31st 2018

October 27, 2018

50% questions and 50% problems.

1 Exercises

1. (KtiCa) Given $p: x + y = 5$ and $q: x \cdot y \geq 6$, translate to English/math the following expressions (use math for expressing identities or inequalities):
   (a) $\neg q \rightarrow \neg p$
   (b) $p \oplus q$
   (c) $p \vee q$
   (d) $\neg q \rightarrow (p \vee \neg p)$
2. (KtiCa) Identify p and q from $p \wedge q \equiv$ *"It's raining and cold"* and find a natural translation to English of the following sentences:
   (a) $\neg(p \wedge q)$
   (b) $\neg q \rightarrow (p \vee \neg p)$
3. (KtiCa) Prove that $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$
4. (KtiCa) Prove that $[(a \rightarrow (b \rightarrow c)) \wedge (a \wedge b)] \rightarrow c$
5. (KtiCa) Simplify and determine whether each of the following sentences is (a) contingent, (b) contradictory or (c) tautological:
   (a) $(p \rightarrow q) \rightarrow \neg q$
   (b) $p \rightarrow \neg p$
   (c) $[(p \rightarrow r) \rightarrow (p \wedge q)] \wedge r \wedge \neg q$
6. (KTIca) Consider a gate consisting in two CNOT gates coupled in series, i.e., the output of the first is fed into the input of the second.
   (a) Write the truth table of such a gate
   (b) If we call the top and bottom inputs x, y. respectively, and the top and bottom outputs x', y', respectively, determine an algebraic expressions for x' and y' in terms of x and y
   (c) A CNOT gate is reversible. This means we can build a gate, the *inverse gate* of the CNOT, that *undoes* what the CNOT does. What is that gate?

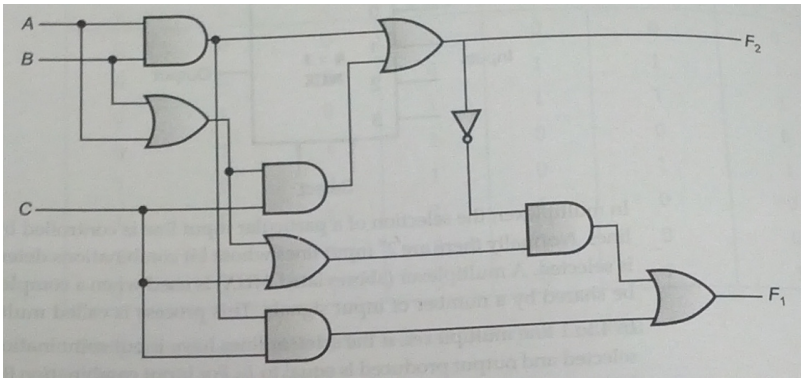

Figure 1: Logic circuit 1

2 Problems

7. (ktiCA) Determine the expressions for F_1 and F_2 as given by the circuit of figure 1 and simplify as much as possible.

8. (kTICa) Consider the circuit of figure 2 and determine the expressions for a, b, c in the following cases. What function do we obtain in each case?

   (a) $y = 1$

   (b) $z = 0$

   (c) $z = 0$ and before x, y enter the gate we pass each through a NOT gate.

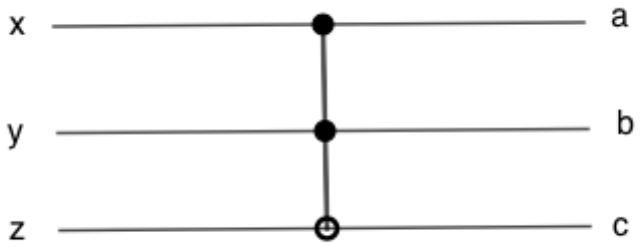

Figure 2: Toffoli gate

9. (kTIca) From your answers to exercise 8, figure out how to write the OR gate using only NOT's and a CCNOT.