

# Computer Science G12

## Test 2 Term 3

Date: March 28, 2018

**Name:**

Each question emphasizes different skills: (K) Knowledge, (T) Thinking, (C) Communication and (A) Application.

The value of each problem is as follows: P1 20% , P2 40%, P3 30% and P4 10%.

1. (20% KTC) Prove that  $Q_n = \sum_{k=1}^n \frac{1}{2^k} = \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{2^n} < 1$  by way of finding a closed expression for  $Q_n$ . Make sure that your reasoning is clear.
2. (40% KTC) The gist of the **insertion sort** algorithm consists in building a sorted list of number by inserting each of them one by one in the right position. Say, the list of numbers  $R_1, R_2, R_3, \dots, R_n, R_{n+1}$  is such that the first  $n$  of them are already sorted. How can we sort the full list?
  1. Write a pseudo-code for this algorithm
  2. Explain the code's logic in your own words. Make sure to clearly show how PMI is used to guarantee the logic is correct.
3. (30% KCA) Calculate the closed form of the following sums and prove your result by mathematical induction. Hint: Use the method of shifting and the well-known result  $S_n = \sum_{k=1}^n k = n(n+1)/2$ 
  1.  $S_{2n} = \sum_{k=0}^n k^2$
  2.  $S_{3n} = \sum_{k=0}^n k^3$
4. (10% KTC) Consider the following recurrence relation

$$G_n = G_{n-1} + G_{n-2} + 1 \quad \text{with} \quad G_1 = 1 \quad G_2 = 1$$

and compare it to the Fibonacci one

$$F_n = F_{n-1} + F_{n-2} \quad F_1 = 1 \quad \text{with} \quad F_2 = 1$$

Because of the extra 1 it seems obvious that  $G_n > F_n$ . Yet the following *seems* a valid proof that  $G_n = F_n - 1$ , namely (by the strong PMI): Assume that  $G_k = F_k - 1 \forall k \leq n$ . We can then prove it holds for  $n+1$  as well:

$$G_{n+1} = G_n + G_{n-1} + 1 = F_n - 1 + F_{n-1} - 1 + 1 = F_{n+1} - 1$$

What is **wrong** with this proof?