

Computer Science G11 at The Dragon Academy

Test II Term 1

November 1, 2018

Note:

- All questions have the same value towards your mark.
 - Make sure to strategize your time: Pick the easiest questions first; move on to the next one if you get stuck.
 - Duration: 1h
1. (KtiCa) Express the following statements in propositional logic
 - (a) If I'm sleepy, then I'm slow and inefficient
 - (b) If I'm slow and inefficient it is because I'm sleepy.
 - (c) I'm sleepy if I did not go to bed at a reasonable time
 2. (kTICA) Express the following reasoning in propositions logic. How is this type of reasoning called? *If I didn't go to bed at a reasonable time, I'm sleepy. If I'm sleepy, I'm slow and inefficient. Hence, If I didn't go to bed early enough, I'm slow and inefficient.*
 3. (Ktica) What does it mean that two logical expressions A and B are equivalent? Give an example.
 4. (Ktica) State which of the following are propositions:
 - (a) Try to build a routine
 - (b) Do not lie
 - (c) It's cold out there
 - (d) What do you mean?
 5. (KtiCa) Simplify the following expression and choose the right answer: $[\neg(p \vee q) \wedge \neg(r \vee s \vee t)] \vee \neg(p \vee q)$
 - (a) $p \vee q$
 - (b) $\neg p \wedge \neg q$
 - (c) $r \vee s \vee t$
 - (d) $\neg r \wedge \neg s \wedge \neg t$
 - (e) $\neg p \wedge \neg q \wedge \neg r \wedge \neg s \wedge \neg t$
 6. (KtiCa) Express the following function $f(p, q, r) = [(p \vee q) \wedge r] \vee (p \wedge q \wedge r)$ in **Disjunctive Normal Form (DNF)**, that is, as a **disjunction** of terms, each of which consisting on a *conjunction* of atomic literals or negation of atomic literals.
 7. (KtiCa) Simplify and determine whether each of the following sentences is (a) contingent, (b) contradictory or (c) tautological:
 - (a) $(p \rightarrow q) \rightarrow \neg q$
 - (b) $p \rightarrow \neg p$
 - (c) $[(p \rightarrow r) \rightarrow (p \wedge q)] \wedge r \wedge \neg q$

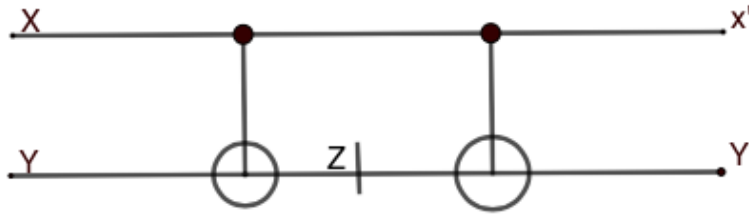


Figure 1: Two CNOT in series

8. (KTICa) Consider a gate consisting in two CNOT gates coupled in series, i.e., the output of the first is fed into the input of the second (see figure 1).
- Write the algebraic expression of Z in terms of the inputs x and y
 - Determine an algebraic expressions for x' and y' in terms of x and y
 - A CNOT gate is reversible. This means we can build a gate, called the *inverse gate* of the CNOT, that *undoes* what the CNOT does. What is that gate?
9. (kTICa) Consider the CCNOT or Toffoli circuit of figure 2 and determine the expressions for a , b , c in the following cases. What function do we obtain in each case?
- $x = 1$
 - $y = 1$
 - $z = 0$
 - $z = 0$ and before x , y enter the gate we pass each through a NOT gate. What's then a, b, c ?

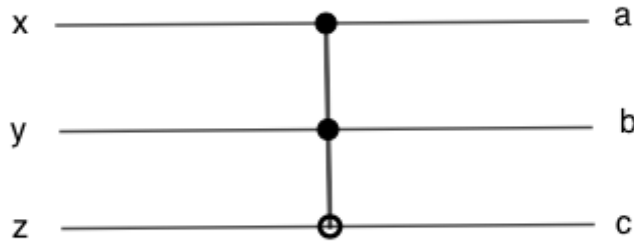


Figure 2: Toffoli gate

10. (kTICa) From your answers to exercise 9, figure out how to write the OR gate using only NOT's and a CCNOT.