Computer Science at The Dragon Academy
Assignment II
Propositional Logic
Due date: Tue. Oct. 2 2018

September 21, 2018

40% Exercises and 60% Problems.

1 Exercises

1. (Ktica) State which of the following are propositions:
   - (a) *Try to build a routine*
   - (b) *Do not lie*
   - (c) *It's cold out there*
   - (d) *What do you mean?*
2. (KtiCa) Simplify the following expression and choose the right answer: $[\neg(p \vee q) \wedge \neg(r \vee s \vee t)] \vee \neg(p \vee q)$
   - (a) $p \vee q$
   - (b) $\neg p \wedge \neg q$
   - (c) $r \vee s \vee t$
   - (d) $\neg r \wedge \neg s \wedge \neg t$
   - (e) $\neg p \wedge \neg q \wedge \neg r \wedge \neg s \wedge \neg t$
3. (KtiCa) Given $F = (\neg p \wedge \neg q) \vee (\neg r \wedge \neg s \wedge \neg t)$, which of the following represents the only correct expression for $\neg F$ (write down the derivation that justifies your answer):
   - (a) $\neg F = \neg p \vee \neg q \vee \neg r \vee \neg s \vee \neg t$
   - (b) $\neg F = \neg p \wedge \neg q \wedge \neg r \wedge \neg s \wedge \neg t$
   - (c) $\neg F = (\neg p \wedge \neg q) \wedge (\neg r \wedge \neg s \wedge \neg t)$

(d) $\neg F = (p \wedge q) \vee (\neg r \wedge \neg s \wedge \neg t)$

(e) $\neg F = (p \vee q) \wedge r \wedge s \wedge t$

4. (KtiCa) Express the following function $f(p, q, r, s) = (q \vee r \vee s) \wedge (p \vee r \vee s) \wedge (p \vee q \vee s)$ as a *disjunction* of terms, each of which consisting on a *conjunction* of atomic literals or negation of atomic literals.

5. (KtiCa) Express the following function $f(p, q, r) = [(p \vee q) \wedge r] \vee (p \wedge q \wedge r)$ as a *conjunction* of terms, each of which consisting on a *disjunction* of atomic literals or negation of atomic literals.

2 Problems

1. (kTICa) Prove algebraically and by truth table that $p \wedge (p \vee q) \leftrightarrow p$.

2. (kTICa) Given $p \rightarrow q$ and p, conclude, both via truth table and algebraically, $p \rightarrow (p \wedge q)$ (Hint: Modus Ponens).

3. (kTICa) From the premises $p \rightarrow q$ and $q \rightarrow p$, conclude, algebraically and via truth table, $\neg q \vee (p \wedge q)$ (Hint: Modus Ponens).

4. (KTICa) Found an equivalent expression for $(p \leftrightarrow q) \vee (p \rightarrow q)$ that has not conditionals nor bi-conditionals.

5. (KTICa) The following is a list of conditionals, i.e., logical statements with the pattern $p \rightarrow q$. For each of them, write the sentences corresponding to (I) $\neg q \rightarrow \neg p$, (II) $p \wedge \neg q$ and (III) $\neg p \rightarrow \neg q$:

   (a) *If it is January, then it is cold*

   (b) If $y + 5 \neq 7$ then $y < 0$