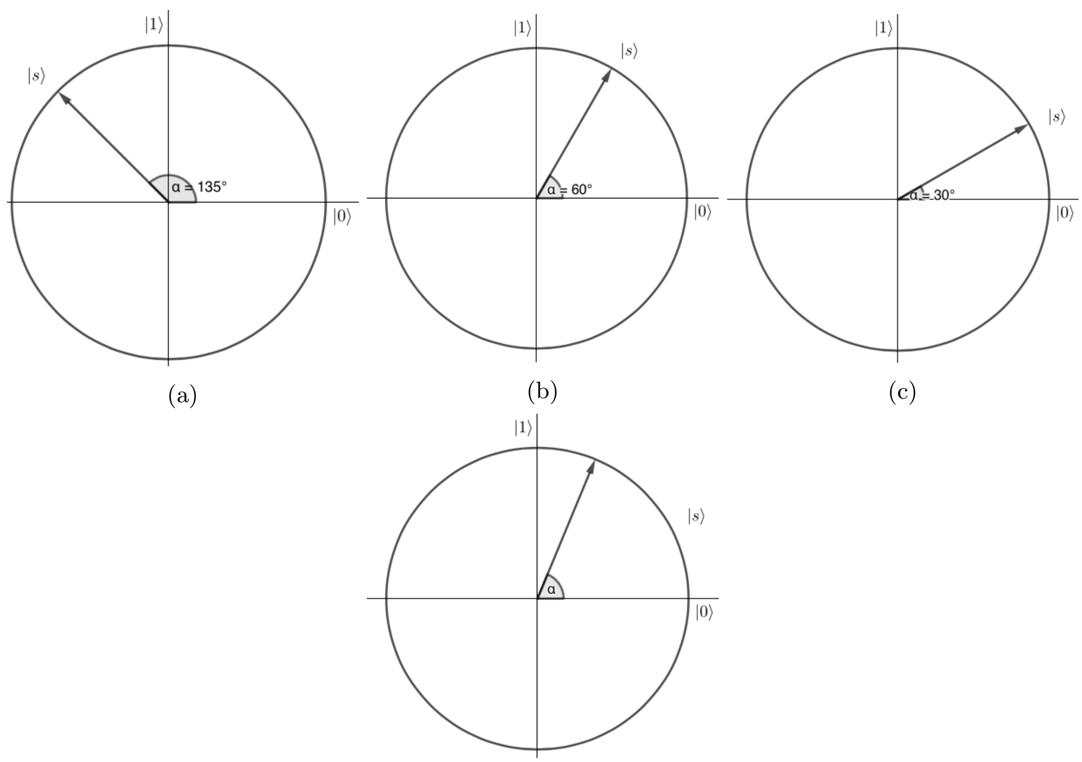
Computer Science G11 at The Dragon Academy
Assignment 4

November 5, 2018

Assume in all questions that the *relative phase* of any state is 0, i.e., we can treat the components of the state vectors as numbers. All questions weigh the same.

1. (KtiCa) A qubit is in the state given by $|s\rangle$ in figure 1a.
   (a) Determine the expression of $|s\rangle$ in terms of the fundamental states $|0\rangle$, $|1\rangle$, that is, write $|s\rangle$ in the form $a|0\rangle + b|1\rangle$.
   (b) What is the probability to find the qubit in state $|0\rangle$?
   (c) What is the probability to find the qubit in state $|1\rangle$?
2. (KtiCa) Repeat exercise 1 for figures 1b and 1c.

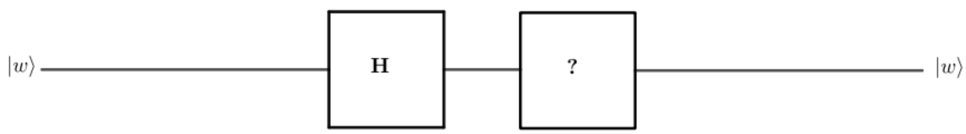


(a) (b) (c)

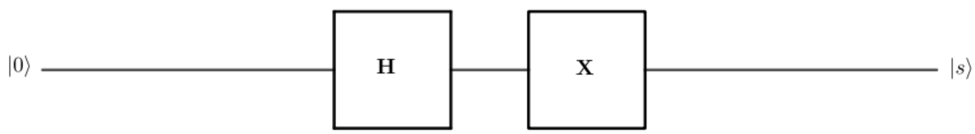
(d) An arbitrary Qubit $|s\rangle$

3. (kTICa) What is the expression of the qubit in figure 1d?
4. (kticA) If $\alpha = 45°$, what is the expression of the qubit in figure 1d?
5. (KtiCa) What is the *action* of the gate **Z** on the states $|+\rangle$ and $|-\rangle$?
6. (KtiCa) Let's name by $|+\rangle$ the combination $(|0\rangle + |1\rangle)/\sqrt(2)$. What is the *action* of the gate **X** on the state $|+\rangle$?
7. (KtiCa) Let's name by $|-\rangle$ the combination $(|0\rangle - |1\rangle)/\sqrt(2)$. What is the *action* of the gate **X** on the state $|-\rangle$?

8. (kticA) Consider the Hadamard gate $\mathbf{H}$
   (a) Evaluate $\mathbf{H}\,|0\rangle$
   (b) Evaluate $\mathbf{H}\,|1\rangle$
   (c) Evaluate $\mathbf{H}\,|+\rangle$
   (d) Evaluate $\mathbf{H}\,|-\rangle$
9. (kTIca) From your answers to exercise 8, what is the inverse of the gate $\mathbf{H}$? See figure 2a.
10. (KticA) Determine the output state $|s\rangle$ from the circuit of figure 2b.



(a) What's the gate that *undoes* what $\mathbf{H}$ does so that the output is the same as the input $|w\rangle$?



(b) What's the output $|s\rangle$? Write it in terms of the fundamental states $|o\rangle$, $|1\rangle$

11. (KticA) Consider the quantum circuit of figure 3 that operates on 2-qubits.
    (a) Determine the state $|w_0\rangle$ in terms of the fundamental states $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$.
    (b) Determine the state $|w_1\rangle$ in terms of the fundamental states $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$.
    (c) Determine the state $|w_2\rangle$ in terms of the fundamental states $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$.
    (d) Determine the state $|w_3\rangle$ in terms of the fundamental states $|00\rangle$, $|01\rangle$, $|10\rangle$, $|11\rangle$.

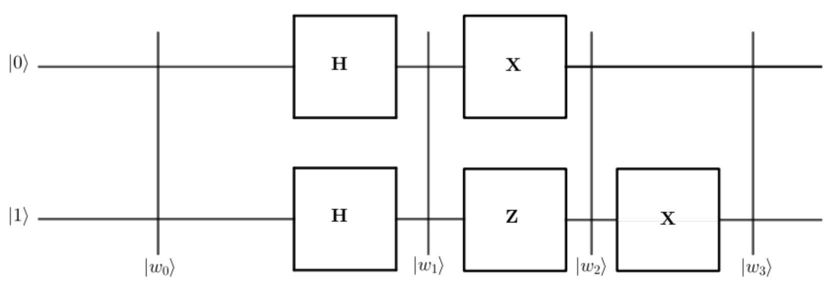


Figure 3: 2-qubits Quantum circuit

12. (KTIca) At the end of the previous circuit (see Fig.3), what is the probability of the system of 2-qubits to be in the states
    a) $|00\rangle$ b) $|01\rangle$ c) $|10\rangle$ d) $|11\rangle$
13. (KTIca) Are the two qubits *entangled* at any of the following four states ?
    a) $|w_0\rangle$ b) $|w_1\rangle$ c) $|w_2\rangle$ d) $|w_3\rangle$