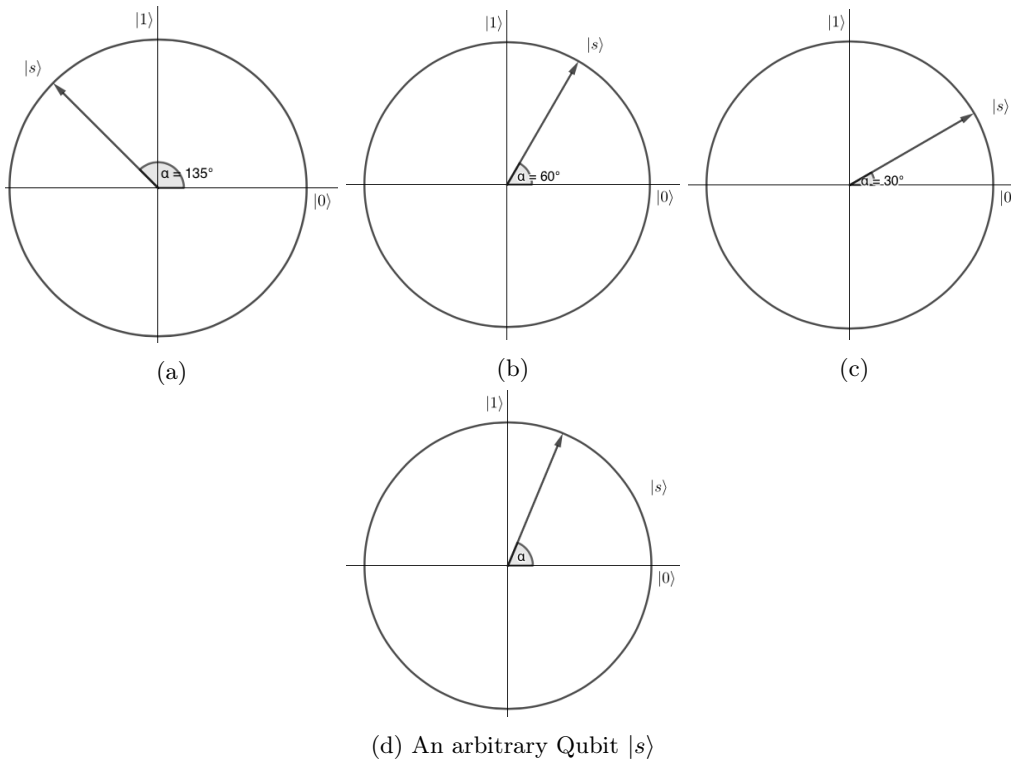


Computer Science G11 at The Dragon Academy

Assignment 4

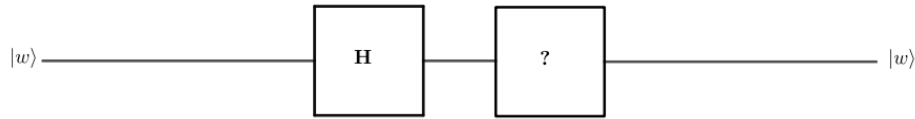
November 5, 2018

1. (KtiCa) A qubit is in the state given by $|s\rangle$ in figure 1a.
 - (a) Determine the expression of $|s\rangle$ in terms of the fundamental states $|0\rangle$, $|1\rangle$, that is, write $|s\rangle$ in the form $a|0\rangle + b|1\rangle$.
 - (b) What is the probability to find the qubit in state $|0\rangle$?
 - (c) What is the probability to find the qubit in state $|1\rangle$?
2. (KtiCa) Repeat exercise 1 for figures 1b and 1c.

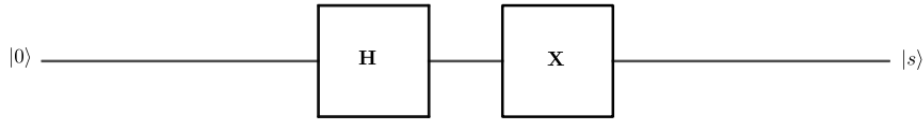


3. (kTiCa) What is the expression of the qubit in figure 1d?
4. (kticA) If $\alpha = 45^\circ$, what is the expression of the qubit in figure 1d?
5. (KtiCa) What is the *action* of the gate **Z** on the states $|+\rangle$ and $|-\rangle$?
6. (KtiCa) Let's name by $|+\rangle$ the combination $(|0\rangle + |1\rangle)/\sqrt{2}$. What is the *action* of the gate **X** on the state $|+\rangle$?
7. (KtiCa) Let's name by $|-\rangle$ the combination $(|0\rangle - |1\rangle)/\sqrt{2}$. What is the *action* of the gate **X** on the state $|-\rangle$?

8. (kticA) Consider the Hadamard gate \mathbf{H}
- (a) Evaluate $\mathbf{H}|0\rangle$
 - (b) Evaluate $\mathbf{H}|1\rangle$
 - (c) Evaluate $\mathbf{H}|+\rangle$
 - (d) Evaluate $\mathbf{H}|-\rangle$
9. (kTICA) From your answers to exercise 8, what is the inverse of the gate \mathbf{H} ? See figure 2a.
10. (KticA) Determine the output state $|s\rangle$ from the circuit of figure 2b.



- (a) What's the gate that *undoes* what \mathbf{H} does so that the output is the same as the input $|w\rangle$?



- (b) What's the output $|s\rangle$? Write it in terms of the fundamental states $|0\rangle$, $|1\rangle$

11. (KticA) Consider the quantum circuit of figure 3 that operates on 2-qubits.
- (a) Determine the state $|w_0\rangle$ in terms of the fundamental states $|0\rangle$ $|1\rangle$.
 - (b) Determine the state $|w_1\rangle$ in terms of the fundamental states $|0\rangle$ $|1\rangle$.
 - (c) Determine the state $|w_2\rangle$ in terms of the fundamental states $|0\rangle$ $|1\rangle$.
 - (d) Determine the state $|w_3\rangle$ in terms of the fundamental states $|0\rangle$ $|1\rangle$.

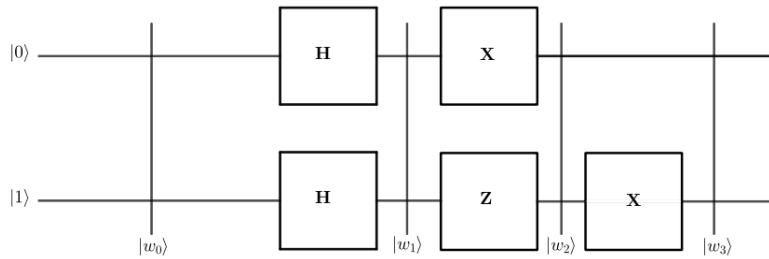


Figure 3: 2-qubits Quantum circuit

12. (KTICA) At the end of the previous circuit (see Fig.3), what is the probability of the system of 2-qubits to be in the states
- a) $|00\rangle$
 - b) $|01\rangle$
 - c) $|10\rangle$
 - d) $|11\rangle$