

$$m_i \ddot{\mathbf{r}}_i = F_i(\mathbf{r}_1, \mathbf{r}_2, \dots, \mathbf{r}_N)$$

$$m_i \ddot{\mathbf{r}}_i = G \sum_{j \neq i} \frac{m_i m_j (\mathbf{r}_j - \mathbf{r}_i)}{(\|\mathbf{r}_j - \mathbf{r}_i\|^2 + \epsilon^2)^{3/2}}$$

$$\dot{r}_i(t^n) \approx \frac{r_i^{n+1} - 2r_i^n + r_i^{n-1}}{\Delta t^2}$$

$$\frac{r_i^{n+1} - 2r_i^n + r_i^{n-1}}{\Delta t^2} = a_i(r^n)$$

$$r_i^{n+1} = 2r_i^n - r_i^{n-1} + \Delta t^2 a_i(r_i^n)$$

$$r_i^1 = r_i^0 + \Delta t v_i^0 + \frac{1}{2} \Delta t^2 a_i(r^0)$$

$$= - \Delta t$$

$$v_i^n \approx \frac{r_i^{n+1} - r_i^{n-1}}{2\Delta t}$$

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