$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

$$f(x \pm k \cdot \Delta x) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n f(x)}{dx^n} (\pm k \cdot \Delta x)^n$$

$$f(x \pm k \cdot \Delta x) = f(x) + \frac{df(x)}{dx} (\pm k \cdot \Delta x) + \frac{1}{2!} \frac{d^2 f(x)}{dx^2} (\pm k \cdot \Delta x)^2 + \cdots + \frac{1}{n!} \frac{d^n f(x)}{dx^n} (\pm k \cdot \Delta x)^n$$

$$f(x \pm k \cdot \Delta x) = f(x) + f'(x) (\pm k \cdot \Delta x) + \frac{1}{2!} f''(x) (\pm k \cdot \Delta x)^2 + \cdots + \frac{1}{n!} f^{(n)}(x) (\pm k \cdot \Delta x)^n$$

$$f(x \pm k \cdot \Delta x) = f(x) + f'(x)\,(\pm k \cdot \Delta x) + \frac{1}{2!} f''(x)\,(\pm k \cdot \Delta x)^2 + \cdots + \frac{1}{n!} f^{(n)}(x)\,(\pm k \cdot \Delta x)^n$$

$$
\begin{aligned}
f(x+\Delta x) &= f(x) + f'(x)\Delta x + \frac{1}{2!}f''(x)\left(\Delta x\right)^2 + \cdots + \frac{1}{n!}f^{(n)}(x)\left(\Delta x\right)^n \\
f'(x) &= \frac{f(x+\Delta x)-f(x)}{\Delta x} - \frac{1}{2!}f''(x)\left(\Delta x\right) - \cdots - \frac{1}{n!}f^{(n)}(x)\left(\Delta x\right)^{n-1}
\end{aligned}
$$

$$
\begin{aligned}
f(x-\Delta x) &= f(x) + f'(x)\,(-\Delta x) + \frac{1}{2!}f''(x)\,(-\Delta x)^2 + \cdots + \frac{1}{n!}f^{(n)}(x)\,(-\Delta x)^n \\
f(x-\Delta x) &= f(x) - f'(x)\Delta x + \frac{1}{2!}f''(x)\,(\Delta x)^2 + \cdots + \frac{1}{n!}f^{(n)}(x)\,(-\Delta x)^n \\
f'(x) &= \frac{f(x) - f(x-\Delta x)}{\Delta x} + \frac{1}{2!}f''(x)\,(\Delta x) - \cdots - \frac{(-1)^{n-1}}{n!}f^{(n)}(x)\,(\Delta x)^{n-1}
\end{aligned}
$$

$$f'(x) = \frac{f(x + \Delta x) - f(x)}{\Delta x} - \frac{1}{2!} f''(x)\,(\Delta x) - \frac{1}{3!} f'''(x)(\Delta x)^2 - \cdots - \frac{1}{n!} f^{(n)}(x)\,(\Delta x)^{n-1}$$

$$f'(x) = \frac{f(x) - f(x - \Delta x)}{\Delta x} + \frac{1}{2!} f''(x)\,(\Delta x) - \frac{1}{3!} f'''(x)(\Delta x)^2 - \cdots - \frac{(-1)^{n-1}}{n!} f^{(n)}(x)\,(\Delta x)^{n-1}$$

$$2f'(x) = \frac{f(x + \Delta x) - f(x)}{\Delta x} + \frac{f(x) - f(x - \Delta x)}{\Delta x} - \frac{2}{3!} f'''(x)(\Delta x)^2 - \cdots - \frac{1}{n!} \left[1 + (-1)^{n-1}\right] f^{(n)}(x)(-\Delta x)^{n-1}$$

$$f'(x) = \frac{f(x + \Delta x) - f(x - \Delta x)}{2\Delta x} - \frac{1}{3!} f'''(x)(\Delta x)^2 - \cdots - \frac{1}{2n!} \left[1 + (-1)^{n-1}\right] f^{(n)}(x)(-\Delta x)^{n-1}$$

$$f'(x) \sim \frac{f(x+\Delta x) - f(x-\Delta x)}{2\Delta x}$$

$$f(x+\Delta x) = f(x) + f'(x)\Delta x + \frac{1}{2!}f''(x)\left(\Delta x\right)^2 + \cdots + \frac{1}{n!}f^{(n)}(x)\left(\Delta x\right)^n$$

$$f(x+2\Delta x) = f(x) + 2f'(x)\Delta x + \frac{1}{2!}f''(x)\left(2\Delta x\right)^2 + \cdots + \frac{1}{n!}f^{(n)}(x)\left(2\Delta x\right)^n$$
$$f(x+2\Delta x) = f(x) + 2f'(x)\Delta x + \frac{4}{2!}f''(x)\left(\Delta x\right)^2 + \cdots + \frac{2^n}{n!}f^{(n)}(x)\left(\Delta x\right)^n$$

$$f'(x) \sim \frac{a_0 f(x) + a_1 f(x + \Delta x) + a_2 f(x + 2\Delta x)}{\Delta x}$$

$$f'(x) = a_0\,[f(x)] + a_1\left[f(x) + f'(x)\Delta x + \frac{1}{2!}f''(x)\,(\Delta x)^2 + \frac{1}{3!}f'''(x)\,(\Delta x)^3 + \cdots + \frac{1}{n!}f^{(n)}(x)\,(\Delta x)^n\right] +$$
$$a_2\left[f(x) + 2f'(x)\Delta x + \frac{4}{2!}f''(x)\,(\Delta x)^2 + \frac{8}{3!}f'''(x)\,(\Delta x)^3 + \cdots + \frac{2^n}{n!}f^{(n)}(x)\,(\Delta x)^n\right]$$
$$f'(x) = (a_0 + a_1 + a_2)f(x) + (a_1 + 2a_2)f'(x)\Delta x + \left(\frac{1}{2!}a_1 + \frac{4}{2!}a_2\right)f''(x)(\Delta x)^2 + \left(\frac{1}{3!}a_1 + \frac{8}{3!}a_2\right)f'''(x)(\Delta x)^3$$
$$+\ \mathscr{O}(\Delta x^4)$$

$$\left.\begin{aligned} f(x):&\quad 0 = a_0 + a_1 + a_2 \\ f'(x):&\quad 1 = a_1 + 2a_2 \\ f''(x):&\quad 0 = \frac{1}{2}a_1 + 2a_2 \end{aligned}\right\} \quad \begin{aligned} a_0 &= -(a_1 + a_2) \\ 1 &= a_1 + 2a_2 \\ a_1 &= -4a_2 \end{aligned} \quad \left.\vphantom{\begin{aligned} a \\ a \\ a \end{aligned}}\right\} \quad \begin{aligned} a_0 &= -3/2 \\ a_1 &= 2 \\ a_2 &= -1/2 \end{aligned}$$

$$\left.\begin{aligned} f'(x) &\sim \frac{a_0 f(x) + a_1 f(x+\Delta x) + a_2 f(x+2\Delta x)}{\Delta x} \\ f'(x) &\sim \frac{-\frac{3}{2} f(x) + 2f(x+\Delta x) - \frac{1}{2} f(x+2\Delta x)}{\Delta x} \end{aligned}\right\} \quad f'(x) \sim \frac{-3f(x) + 4f(x+\Delta x) - f(x+2\Delta x)}{2\Delta x}$$

$$\left.\begin{array}{l} E(\Delta x) = \left(\frac{1}{3!}a_1 + \frac{8}{3!}a_2\right) f'''(x)(\Delta x)^3 \\ E(\Delta x) = \left(\frac{1}{3!}[2] + \frac{8}{3!}\left[-\frac{1}{2}\right]\right) f'''(x)(\Delta x)^3 \end{array}\right\} \quad \left.\begin{array}{l} E(\Delta x) = \left(\frac{1}{3} - \frac{2}{3}\right) f'''(x)(\Delta x)^3 \\ E(\Delta x) = -\frac{1}{3} f'''(x)(\Delta x)^3 \end{array}\right\} \quad E(\Delta x) \sim -\frac{\Delta x^2}{3} f'''(\xi)$$