Chapter 2
Essential Dictionary I- Part 1

Sets of Numbers

Sets of Numbers

The ‘open face’ symbols $\mathbb{N}, \mathbb{Z}, \mathbb{Q}$ were introduced in Sect. 2.1.1 to represent the natural numbers, the integers, and the rationals, respectively. Likewise, we denote by $\mathbb{R}$ the set of **real** numbers (its symbolic definition is left as Exercise 2.13.

The set of **complex** numbers is denoted by $\mathbb{C}$. The set $\mathbb{C}$ may be written as

$$\mathbb{C} \stackrel{\text{def}}{=} \{x + iy : i^2 = -1, \qquad x, y \in \mathbb{R}\}.$$

The symbol i is called the **imaginary unit**, while x and y are, respectively, the **real part** $Re(z)$ and the **imaginary part** $Im(z)$ of the complex number $z = x + iy$.

The sets $\mathbb{R}$ and $\mathbb{C}$ are represented geometrically as the **real line** and the **complex plane** (or **Argand plane**), respectively.

A plot of complex numbers in the Argand plane is called an **Argand diagram**.

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An **interval** is a subset of $\mathbb{R}$ of the type

$$[a, b] := \{x \in \mathbb{R} : a \leq x \leq b\}$$

where a, b are real numbers, with $a < b$. This interval is **closed**, that is, it contains its end points.

**A point is sometimes regarded as a degenerate closed interval, by allowing $a = b$ in the definition.

We also have **open** intervals:

$$(a, b) := \{x \in \mathbb{R} : a < x < b\}$$

as well as **half-open** intervals $[a, b)$ $(a, b]$.

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The interval with end-points $a = 0$ and $b = 1$ is the (open, closed, half-open) **unit interval**.

A semi-infinite interval

$$\{x \in \mathbb{R} : a < x\} \qquad \text{or} \qquad \{x \in \mathbb{R} : x > b\}$$

is called a **ray**.

The rays consisting of all positive real or rational numbers are particularly important, and have a dedicated notation

$$\mathbb{R}+ := \{x \in \mathbb{R}, x > 0\}, \qquad \mathbb{Q}+ := \{x \in \mathbb{Q}, x > 0\}.$$

Some authors extend the meaning of interval to include also rays and lines, and use expressions such as

$$(-\infty,\infty) \quad [a,\infty) \quad (-\infty, b].$$

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As infinity does not belong to the set of real numbers, the notation $[1,\infty]$ is incorrect.

The set $\mathbb{R}^2$ of all ordered pairs of real numbers is called the **cartesian plane**, which is the cartesian product of the real line with itself.

If $(x, y) \in \mathbb{R}^2$, then the first component x is called the **abscissa** and the second component y the **ordinate**.

The set $\mathbb{Q}^2 \subset \mathbb{R}^2$, the collection of points of the plane having rational coordinates, is called the set of **rational points** in $\mathbb{R}^2$.

The set $[0,1]^2 \subset \mathbb{R}^2$ is called the **unit square**.

In $\mathbb{R}^3$ we have the **unit cube** $[0,1]^3$, and for $n > 3$ we have the **unit hypercube** $[0,1]^n \subset \mathbb{R}^n$.

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The following subsets of the cartesian plane are related to the geometrical figure of the circle:

$\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ **unit circle**

$\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$ **closed unit disc**

$\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 < 1\}$ **open unit disc**.

Thus the closed unit disc is the union of the open unit disc and the unit circle. The (unit) circle is denoted by the symbol s^1.

For $n > 0$, the n**-dimensional unit sphere** s^n is defined as follows:

$$s^n = \{(x_0, \ldots, x_n) \in \mathbb{R}^{n+1} : {x_0}^2 + \ldots + {x_n}^2 = 1\}$$