Chapter 2
Essential Dictionary I- Part 1

Writing About Sets

Writing About Sets

The vocabulary on sets developed so far is sufficient for our purpose. We begin to use it in short phrases which define sets.

1. *The set of ordered pairs of complex numbers.*
2. *The set of rational points on the unit circle.*

Note that we haven't used any symbols. The set in item 1 is $\mathbb{C}^2$.

In item 2, among the infinitely many points of the unit circle, we consider those having rational coordinates. There is no difficulty in writing this set symbolically:

$$\{(x, y) \in \mathbb{Q}^2 : \ x^2 + y^2 = 1\}$$

Writing About Sets

It is possible to specify a *type* of set, without revealing its precise identity. In each of the following sets there is at least one unspecified quantity.

The set of fractions representing a rational number.

The set of divisors of an odd integer.

A proper infinite subset of the unit circle.

The cartesian product of two finite sets of complex numbers.

A finite set of consecutive integers.

Writing About Sets

Next we define sets in two ways, first with a combination of words and symbols, and then with words only. One should consider the relative merits of the two Formulations

Let $X = \{3\}$.

The set whose only element is the integer 3.

Let $X = \{m\}$, *with* $m \in \mathbb{Z}$.

A set whose only element is an integer.

Let $m \in \mathbb{Z}$, *and let* X *be a set such that* $m \in X$.

A set which contains a given integer.

Let X *be a set such that* $X \cap \mathbb{Z} \neq \emptyset$.

A set which contains at least one integer.

Let X *be a set such that* $\#(X \cap \mathbb{Z}) = 1$.

A set which contains precisely one integer.

Writing About Sets

In the first two examples the combination of ‘let’ and ‘=’ replaces an assignment operator. An expression such as ‘*Let* $\stackrel{\text{def}}{=}$’ would be overloaded.

The distinction between definite and indefinite articles is essential, the former describing a unique object, the latter an unspecified representative of a class of objects. In the following phrases, a change in one article, highlighted in boldface, has resulted in a logical mistake.

Bad: *A proper infinite subset of* **a** *unit circle.*

Bad: **A** *set whose only element is the integer 3.*

Bad: **The** *set whose only element is an integer.*

Bad: **The** *set which contains precisely one integer.*

Writing About Sets

As a final exercise, we express some geometric facts using set terminology.

The intersection of a line and a conic section has at most two points.

The set of rational points in any open interval is infinite.

A cylinder is the cartesian product of a segment and a circle.

The complement of the unit circle consists of two disjoint components.

Grammar Part 2

DEFINITE ARTICLE (The)

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- 1. Meaning “mentioned earlier”, “that”:

  Example: Let $A \subset X$. If $aB = 0$ for every B intersecting the set A, then .....

- 2. In front of a noun (possibly preceded by an adjective) referring to a single, uniquely determined object (e.g. in definitions):

  Example: Let f be the linear form defined by (2).

  So $u = 1$ in the compact set K of all points at distance 1 from L.

- 3. In the construction: the + property (or another characteristic) + of + object:

  Example: The continuity of f follows from .....

  The existence of test functions is not evident.

  The intersection of a decreasing family of such sets is convex.

  But : Every nonempty open set in $\mathbb{R}^k$ is a union of disjoint boxes.

DEFINITE ARTICLE (The)

- In front of a cardinal number if it embraces all objects considered:

  Example: The two groups have been shown to have the same number of generators.

  Each of the three products on the right of (4) satisfies .....

- In front of an ordinal number:

  Example: The first Poisson integral in (4) converges to g.

  The second statement follows immediately from the first.

- In front of surnames used attributively:

  Example: the Dirichlet problem

  the Taylor expansion

  But : Taylor's formula

- In front of a noun in the plural if you are referring to a class of objects as a whole, and not to particular members of the class:

  Example: The real measures form a subclass of the complex ones.

  This class includes the Helson sets.

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