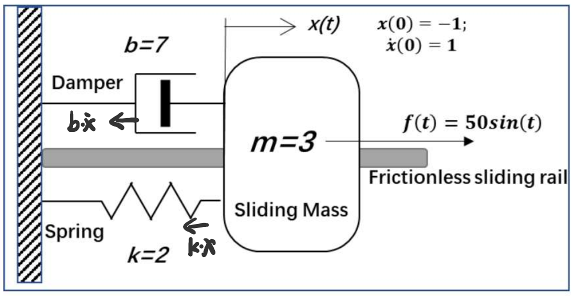
Homework 1

Question 1



a) Write down the dynamic equation in the form of a 2nd order differential equation. (2 points)

b) Write down the state-space equation of the system. (2 points)

c) Express the equation in the s-domain. (4 points)

d) Obtain the system response for the input $f(t)$=50sin(t). (7 points)

(a). $\sum F = m \cdot a$ 基本公式

$\downarrow$

$$f(t) - kx - b\dot{x} = m \cdot \ddot{x}$$

$$m\ddot{x} + b\dot{x} + kx = f(t)$$

$$\boxed{3\ddot{x} + 7\dot{x} + 2x = 50\sin(t)}$$

(b) 令 $\begin{cases} x_1 = x \\ x_2 = \dot{x} \end{cases} \Rightarrow \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = \frac{1}{3}(50\sin(t) - 7\dot{x} - 2x) = \frac{1}{3}(50\sin(t) - 7x_2 - 2x_1) \end{cases}$

$$\therefore \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}' = \begin{pmatrix} x_2 \\ \frac{1}{3}(50\sin(t) - 7x_2 - 2x_1) \end{pmatrix} = \boxed{\begin{pmatrix} 0 & 1 \\ -\frac{7}{3} & -\frac{2}{3} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{3} \end{pmatrix} f(t)}$$

(c) $3\ddot{x} + 7\dot{x} + 2x = f(t)$

$$\mathcal{L}\{x(t)\} = X(s)$$

$$\mathcal{L}\{\dot{x}(t)\} = s \cdot \mathcal{L}\{x(t)\} - x(0) = sX(s) + 1$$

$$\mathcal{L}\{\ddot{x}(t)\} = s \cdot \mathcal{L}\{\dot{x}(t)\} - \dot{x}(0) = s^2X(s) + s - 1$$

$$\therefore \boxed{3s^2X(s) + 7s \cdot X(s) + 2X(s) + 3s + 4 = F(s)}$$

(d). $3s^2X(s) + 7s \cdot X(s) + 2X(s) + 3s + 4 = \frac{50}{s^2+1}$

$$(3s+1)(s+2) \cdot X(s) = \frac{50}{s^2+1} - 3s - 4$$

$$X(s) = \frac{50}{(s^2+1)(3s+1)(s+2)} - \frac{3s+4}{(3s+1)(s+2)}$$

$$\therefore \boxed{x(t) = \mathcal{L}^{-1}\{X(s)\} = x(t) = \frac{42}{5}e^{-\frac{1}{3}t} - \frac{12}{5}e^{-2t} - \sin(t) - 7\cos(t)}$$

Question 2

a) Obtain the impulse response, *x(t)*, of the system with a transfer function (2 points)

$$X(s) = \frac{8s}{4s^2+1}$$

b) Show that FVT is not applicable and briefly explain why. (3 points)

$$\text{(a)} \because X(s) = \frac{8s}{4s^2+1} = 2\cdot\frac{s}{s^2+\left(\frac{1}{2}\right)^2} = 2\cdot\mathcal{L}\left\{\cos\left(\tfrac{1}{2}t\right)\cdot u(t)\right\}$$

$$\therefore x(t) = 2\cdot\cos\left(\tfrac{1}{2}t\right)\cdot u(t)$$

$\mathcal{L}[1] = \frac{1}{s}$. $\mathcal{L}[e^{at}] = \frac{1}{s-a}$.

$\mathcal{L}[t^m] = \frac{m!}{s^{m+1}}$. $\mathcal{L}[e^{at}t^m] = \frac{m!}{(s-a)^{m+1}}$.

$\mathcal{L}[\cos bt] = \frac{s}{s^2+b^2}$. $\mathcal{L}[e^{at}\cos bt] = \frac{s-a}{(s-a)^2+b^2}$.

$\mathcal{L}[\sin bt] = \frac{b}{s^2+b^2}$. $\mathcal{L}[e^{at}\sin bt] = \frac{b}{(s-a)^2+b^2}$.

$\mathcal{L}[\delta(t)] = 1$.

(b) $x(t\to\infty) \neq \lim_{s\to 0} s\cdot X(s)$

分析 $s\cdot X(s)$ 的 poles

$X(s) = \frac{8s^2}{4s^2+1} \Rightarrow \lambda_{1,2} = \pm\frac{1}{2}j$ 不在 OLHP上

所以 FVT 终值定理不适用

The Final Value Theorem

We can now deduce the Final Value Theorem (FVT):

If all poles of $sY(s)$ are *strictly stable* or lie in the *open left half-plane* (OLHP), i.e., have $\mathrm{Re}(s) < 0$, then

$$y(\infty) = \lim_{s\to 0} sY(s).$$

In our examples, multiply $Y(s)$ by s, check poles:

- $Y(s) = \frac{1}{s+a}$ $\quad sY(s) = \frac{s}{s+a}$
  if $a > 0$, then $y(\infty) = 0$; if $a < 0$, FVT does not give correct answer
- $Y(s) = \frac{1}{s^2+\omega^2}$ $\quad sY(s) = \frac{s}{s^2+\omega^2}$
  poles are purely imaginary (not in OLHP), FVT does not give correct answer
- $Y(s) = \frac{c}{s}$ $\quad sY(s) = c$
  poles at infinity, so $y(\infty) = c$ – FVT gives correct answer