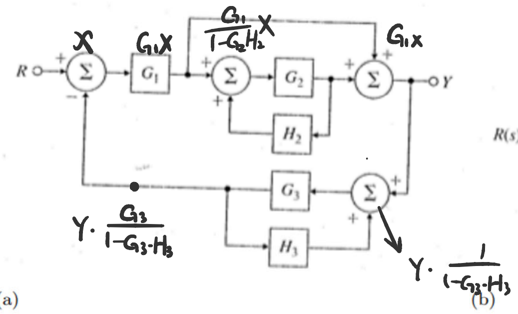
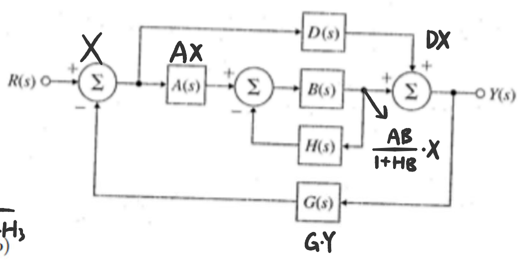
Homework 2

Problems:

1. Using techniques for block diagram reduction discussed in class, find the transfer functions of the systems shown below (p156 from the textbook, 3rd edition)



(a)



(b)

(a)

$$\begin{cases} X = R - Y\cdot\dfrac{G_3}{1-G_3H_3} \\ Y = G_1X + \dfrac{G_1G_2}{1-G_2H_2}X \end{cases} \Rightarrow Y = G_1\left(1+\frac{G_2}{1-G_2H_2}\right)\cdot\left(R - Y\frac{G_3}{1-G_3H_3}\right)$$

$$Y = G_1R\left(1+\frac{G_2}{1-G_2H_2}\right) - G_1\left(1+\frac{G_2}{1-G_2H_2}\right)\cdot\frac{G_3}{1-G_3H_3}\cdot Y$$

$$Y\left(1+\frac{G_1G_3}{1-G_3H_3}\left(1+\frac{G_2}{1-G_2H_2}\right)\right) = G_1R\left(1+\frac{G_2}{1-G_2H_2}\right)$$

$$\boxed{\frac{Y}{R} = \frac{G_1\left(1+\frac{G_2}{1-G_2H_2}\right)}{1+\frac{G_1G_3}{1-G_3H_3}\left(1+\frac{G_2}{1-G_2H_2}\right)}}$$

(b)

$$\begin{cases} Y = DX + \dfrac{AB}{1+HB}X \\ X = R - GY \end{cases} \Rightarrow Y = \left(D+\frac{AB}{1+HB}\right)(R-GY)$$

$$Y = R\left(D+\frac{AB}{1+HB}\right) + \left(D+\frac{AB}{1+HB}\right)GY$$

$$\boxed{\frac{Y}{R} = \frac{\left(D+\frac{AB}{1+HB}\right)}{1+\left(D+\frac{AB}{1+HB}\right)G}}$$

2. Consider the following state-space model (so-called "observer canonical form"):

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & -a_0 \\ 1 & -a_1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} b_0 \\ b_1 \end{pmatrix} u, \qquad y = \begin{pmatrix} 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

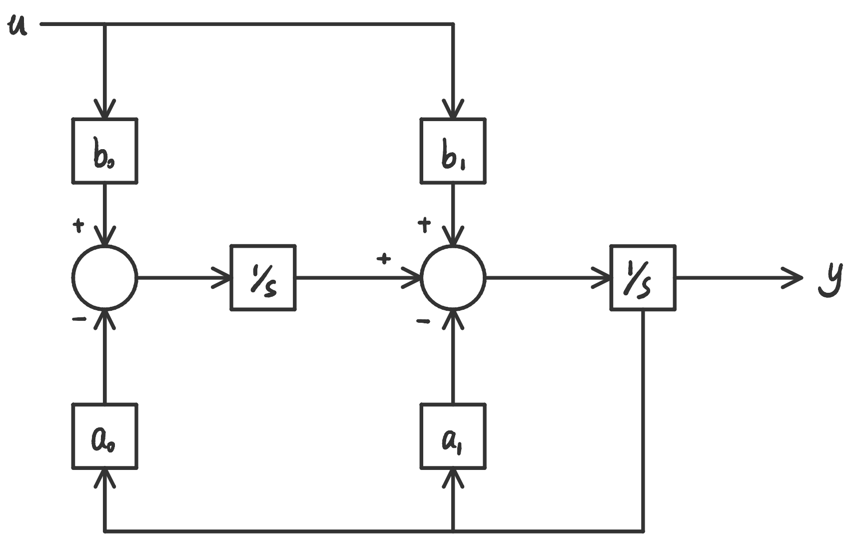
Build an all-integrator diagram for this system.

$$\dot{x}_1 = -a_0 x_2 + b_0 u$$

$$\dot{x}_2 = x_1 - a_1 x_2 + b_1 u$$

$$\begin{cases} x_1 = x \\ x_2 = \dot{x} \end{cases} \Rightarrow \ddot{x} = x - \left(\frac{a_1 b_0}{1+a_0} + b_1\right) \cdot u$$

$$y = x_2 = \dot{x}$$



3. Consider the plant with transfer function $L(s) = \dfrac{1}{s^2 + 2s + \underset{>0}{K}}$ where K is a positive parameter you can tune.

a) Consider the settling time spec $t_s \leq 4$. Give some value (or range of values) of K for which the system meets this spec. Justify your choice.

b) Consider the rise time spec $t_r \leq 1$. Give some value (or range of values) of K for which the system meets this spec.

c) Consider the overshoot spec $M_p \leq 0.1$. Give some value (or range of values) of K for which the system meets this spec. Justify your choice.

(a). $$L(s) = \frac{1}{s^2 + 2s + K} \Rightarrow \begin{cases} 2 = 2\xi W_n \\ K = W_n^2 \end{cases} \Rightarrow \begin{cases} W_n = \sqrt{K} \\ \xi = \frac{1}{\sqrt{K}} \\ W_d = \sqrt{K-1} = W_n \cdot \sqrt{1-\xi^2} \\ \sigma = \xi \cdot W_n = 1 \end{cases}$$ (双虚部情况)

$$t_s = \frac{3}{\sigma} \leqslant 4$$

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对实数解 $\sigma = -(\pm\sqrt{1-K} - 1) \geqslant \frac{3}{4} \Rightarrow K \geqslant \frac{15}{16} = 0.9375$

(b). $$t_r \approx \frac{1.8}{W_n} \leq 1 \Rightarrow W_n \geqslant 1.8 \Rightarrow \sqrt{K} \geqslant 1.8 \Rightarrow K \geqslant 3.24$$

(c). $$M_p = e^{\left(-\frac{\pi\xi}{\sqrt{1-\xi^2}}\right)} = e^{-\frac{\pi}{\sqrt{K-1}}} \leqslant 0.1 \Rightarrow \frac{\pi}{\sqrt{K-1}} \geqslant \ln(10) \Rightarrow K \leqslant 2.86$$