






## External Flow

**TABLE 7.7** Summary of convection heat transfer correlations for external flow<sup>a,b</sup>

Correlation	Geometry	Conditions <sup>c</sup>
$\delta = 5x Re_x^{-1/2}$ (7.19)	Flat plate	Laminar, $T_f$
$C_{f,x} = 0.664 Re_x^{-1/2}$ (7.20)	Flat plate	Laminar, local, $T_f$
$Nu_x = 0.332 Re_x^{1/2} Pr^{1/3}$ (7.23)	Flat plate	Laminar, local, $T_f$ , $Pr \gtrsim 0.6$
$\delta_t = \delta Pr^{-1/3}$ (7.24)	Flat plate	Laminar, $T_f$
$\bar{C}_{f,x} = 1.328 Re_x^{-1/2}$ (7.29)	Flat plate	Laminar, average, $T_f$
$\bar{Nu}_x = 0.664 Re_x^{1/2} Pr^{1/3}$ (7.30)	Flat plate	Laminar, average, $T_f$ , $Pr \gtrsim 0.6$
$Nu_x = 0.564 Pe_x^{1/2}$ (7.32)	Flat plate	Laminar, local, $T_f$ , $Pr \leq 0.05$ , $Pe_x \gtrsim 100$
$C_{f,x} = 0.0592 Re_x^{-1/5}$ (7.34)	Flat plate	Turbulent, local, $T_f$ , $Re_x \leq 10^8$
$\delta = 0.37x Re_x^{-1/5}$ (7.35)	Flat plate	Turbulent, $T_f$ , $Re_x \leq 10^8$
$Nu_x = 0.0296 Re_x^{4/5} Pr^{1/3}$ (7.36)	Flat plate	Turbulent, local, $T_f$ , $Re_x \leq 10^8$ , $0.6 \leq Pr \leq 60$
$\bar{C}_{f,L} = 0.074 Re_L^{-1/5} - 1742 Re_L^{-1}$ (7.40)	Flat plate	Mixed, average, $T_f$ , $Re_{x,c} = 5 \times 10^5$ , $Re_L \leq 10^8$
$\bar{Nu}_L = (0.037 Re_L^{4/5} - 871) Pr^{1/3}$ (7.38)	Flat plate	Mixed, average, $T_f$ , $Re_{x,c} = 5 \times 10^5$ , $Re_L \leq 10^8$ , $0.6 \leq Pr \leq 60$
$\bar{Nu}_D = C Re_D^m Pr^{1/3}$ (Table 7.2)	Cylinder	Average, $T_f$ , $0.4 \leq Re_D \leq 4 \times 10^5$ , $Pr \gtrsim 0.7$
$\bar{Nu}_D = C Re_D^m Pr^n (Pr/Pr_s)^{1/4}$ (Table 7.4)	Cylinder	Average, $T_m$ , $1 \leq Re_D \leq 10^6$ , $0.7 \leq Pr \leq 500$
$\bar{Nu}_D = 0.3 + [0.62 Re_D^{1/2} Pr^{1/3} \times [1 + (0.4/Pr)^{2/3}]^{-1/4}] \times [1 + (Re_D/282,000)^{5/8}]^{4/5}$ (7.54)	Cylinder	Average, $T_f$ , $Re_D Pr \gtrsim 0.2$
$\bar{Nu}_D = 2 + (0.4 Re_D^{1/2} + 0.06 Re_D^{2/3}) Pr^{1/4} \times (\mu/\mu_s)^{1/4}$ (7.56)	Sphere	Average, $T_m$ , $3.5 \leq Re_D \leq 7.6 \times 10^4$ , $0.71 \leq Pr \leq 380$ , $1.0 \leq (\mu/\mu_s) \leq 3.2$
$\bar{Nu}_D = 2 + 0.6 Re_D^{1/2} Pr^{1/3}$ (7.57)	Falling drop	Average, $T_m$
$\bar{Nu}_D = C_1 C_2 Re_{D,max} Pr^{\beta/36} (Pr/Pr_s)^{1/2}$ (7.58), (7.59)	Tube bank <sup>d</sup>	Average, $\bar{T}$ , $10 \leq Re_D \leq 2 \times 10^6$ , $0.7 \leq Pr \leq 500$

<sup>d</sup>For tube banks and packed beds, properties are evaluated at the average fluid temperature,  $\bar{T} = (T_i + T_o)/2$ .

**TABLE 7.3** Constants of Equation 7.52 for noncircular cylinders in cross flow of a gas [13, 14]<sup>a</sup>

Geometry	$Re_D$	$C$	$m$
Square $V \rightarrow$ 	$\frac{T}{D}$ 6000–60,000	0.304	0.59
$V \rightarrow$ 	$\frac{T}{D}$ 5000–60,000	0.158	0.66
Hexagon $V \rightarrow$ 	$\frac{T}{D}$ 5200–20,400 20,400–105,000	0.164 0.039	0.638 0.78
$V \rightarrow$ 	$\frac{T}{D}$ 4500–90,700	0.150	0.638
Thin plate perpendicular to flow $V \rightarrow$ 	$\frac{h}{L}$ 10,000–50,000 7000–80,000	0.667 0.191	0.500 0.667

<sup>a</sup>These tabular values are based on the recommendations of Sparrow et al. [14] for air, with extension to other fluids through the  $Pr^{1/3}$  dependence of Equation 7.52. A Prandtl number of  $Pr = 0.7$  was assumed for the experimental results for air that are described in [14].

## 计算流程

$$Re_{\text{长度}} = \frac{\rho V L}{\mu} = \frac{V L}{\nu}$$
$$Re_{D\text{直径}} = \frac{\rho V D}{\mu} = \frac{V D}{\nu}$$
$$\bar{h} = \frac{k}{L} \cdot \bar{Nu}$$
$$q = h \cdot A_s \cdot (T_s - T_{\infty})$$

## Internal Flow (Tube)

**TABLE 8.4** Summary of convection correlations for flow in a circular tube<sup>a,b,e</sup>

Correlation	Conditions
$f = 64/Re_D$ (8.19)	Laminar, fully developed
$Nu_D = 4.36$ (8.53)	Laminar, fully developed, uniform $q''_s$ 恒定热通量
$Nu_D = 3.66$ (8.55)	Laminar, fully developed, uniform $T_s$ 恒定表面温度
$\bar{Nu}_D = 3.66 + \frac{0.0668 Gz_D}{1 + 0.04 Gz_D^{1/3}}$ (8.57)	Laminar, thermal entry (or combined entry with $Pr \gtrsim 5$ ), uniform $T_s$ , $Gz_D = (D/x) Re_D Pr$
$\bar{Nu}_D = \frac{3.66}{\tanh[2.264 Gz_D^{1/3} + 1.7 Gz_D^{2/3}]} + 0.0499 Gz_D \tanh(Gz_D^{-1})$ (8.58)	Laminar, combined entry, $Pr \gtrsim 0.1$ , uniform $T_s$ , $Gz_D = (D/x) Re_D Pr$
$Nu_D = 0.023 Re_D^{4/5} Pr^n$ 湍流混合比较均匀, 所以 Nu 局部=平均 (8.60) <sup>f</sup>	Turbulent, fully developed, $0.6 \leq Pr \leq 160$ , $Re_D \gtrsim 10,000$ , $(L/D) \gtrsim 10$ , $n = 0.4$ for $T_s > T_m$ and $n = 0.3$ for $T_s < T_m$
$Nu_D = 0.027 Re_D^{4/5} Pr^{1/3} \left(\frac{\mu}{\mu_s}\right)^{0.14}$ (8.61) <sup>f</sup>	Turbulent, fully developed, $0.7 \leq Pr \leq 16,700$ , $Re_D \gtrsim 10,000$ , $L/D \gtrsim 10$
$Nu_D = \frac{(f/8)(Re_D - 1000) Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/5} - 1)}$ (8.62) <sup>f</sup>	Turbulent, fully developed, $0.5 \leq Pr \leq 2000$ , $3000 \leq Re_D \leq 5 \times 10^6$ , $(L/D) \gtrsim 10$
$Nu_D = 4.82 + 0.0185(Re_D Pr)^{0.827}$ (8.64)	Liquid metals, turbulent, fully developed, uniform $q''_s$ , $3.6 \times 10^3 \leq Re_D \leq 9.05 \times 10^3$ , $3 \times 10^{-3} \leq Pr \leq 5 \times 10^{-2}$ , $10^6 \leq Re_D Pr \leq 10^7$
$Nu_D = 5.0 + 0.025(Re_D Pr)^{0.8}$ (8.65)	Liquid metals, turbulent, fully developed, uniform $T_s$ , $Re_D Pr \gtrsim 100$
$\frac{1}{\sqrt{f}} = -2.0 \log \left[ \frac{e/D}{3.7} + \frac{2.51}{Re_D \sqrt{f}} \right]$ (8.20) <sup>f</sup>	Turbulent, fully developed
$f = (0.790 \ln Re_D - 1.64)^{-2}$ (8.21) <sup>f</sup>	Turbulent, fully developed, smooth walls, $3000 \leq Re_D \leq 5 \times 10^6$











<sup>f</sup>Properties in Equations 8.53, 8.55, 8.60, 8.61, 8.62, 8.64, and 8.65 are based on  $T_{s,c}$  properties in Equations 8.19, 8.20, and 8.21 are based on  $T_f = (T_i + T_o)/2$ ; properties in Equations 8.57 and 8.58 are based on  $T_m = (T_{m,i} + T_{m,o})/2$ .

<sup>g</sup>Equation 8.20 pertains to smooth or rough tubes. Equation 8.21 pertains to smooth tubes.

<sup>h</sup>As a first approximation, Equations 8.60, 8.61, or 8.62 may be used to evaluate the average Nusselt number  $\bar{Nu}_D$  over the entire tube length, if  $(L/D) \gtrsim 10$ . The properties should then be evaluated at the average of the mean temperature,  $\bar{T}_m = (T_{m,i} + T_{m,o})/2$ .

<sup>i</sup>For tubes of noncircular cross section,  $Re_D = D_h \mu_m / \nu$ ,  $D_h = 4A_c/P$ , and  $u_m = \dot{m}/\rho A_c$ . Results for fully developed laminar flow are provided in Table 8.1. For turbulent flow, Equation 8.60 may be used as a first approximation.

**TABLE 8.1** Nusselt numbers and friction factors for fully developed laminar flow in tubes of differing cross section

Cross Section	$\frac{b}{a}$	$Nu_D = \frac{hD_h}{k}$		$f Re_{D_h}$
		(Uniform $q''_s$ )	(Uniform $T_s$ )	
	—	4.36	3.66	64
$a$  $b$	1.0	3.61	2.98	57
$a$  $b$	1.43	3.73	3.08	59
$a$  $b$	2.0	4.12	3.39	62
$a$  $b$	3.0	4.79	3.96	69
$a$  $b$	4.0	5.33	4.44	73
$a$  $b$	8.0	6.49	5.60	82
	$\infty$	8.23	7.54	96
	$\infty$	5.39	4.86	96
	—	3.11	2.49	53

## NTU Method

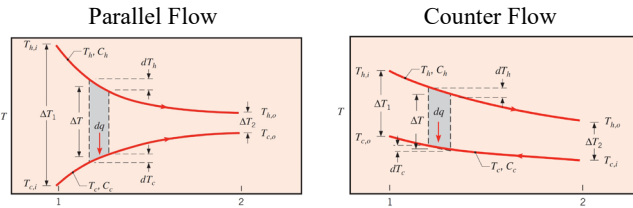
Flow Arrangement	Relation
<b>Parallel flow</b>	$\varepsilon = \frac{1 - \exp[-NTU(1 + C_r)]}{1 + C_r}$ (11.28a)
<b>Counterflow</b>	$\varepsilon = \frac{1 - \exp[-NTU(1 - C_r)]}{1 - C_r \exp[-NTU(1 - C_r)]}$ ( $C_r < 1$ ) $\varepsilon = \frac{NTU}{1 + NTU}$ ( $C_r = 1$ ) (11.29a)
<b>Shell-and-tube</b>	
One shell pass (2, 4, . . . tube passes)	$\varepsilon_1 = 2 \left\{ 1 + C_r + (1 + C_r^2)^{1/2} \times \frac{1 + \exp[-(NTU)_1(1 + C_r^2)^{1/2}]}{1 - \exp[-(NTU)_1(1 + C_r^2)^{1/2}]} \right\}^{-1}$ (11.30a)
$n$ shell passes (2n, 4n, . . . tube passes)	$\varepsilon = \left[ \left( \frac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1} \right)^n - 1 \right] \left[ \left( \frac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1} \right)^n - C_r \right]^{-1}$ (11.31a)
<b>Cross-flow (single pass)</b>	
Both fluids unmixed	$\varepsilon = 1 - \exp \left[ \left( \frac{1}{C_r} \right) (NTU)^{0.22} \{ \exp[-C_r(NTU)^{0.78}] - 1 \} \right]$ (11.32)
$C_{max}$ (mixed), $C_{min}$ (unmixed)	$\varepsilon = \left( \frac{1}{C_r} \right) (1 - \exp[-C_r(1 - \exp[-(NTU)])])$ (11.33a)
$C_{min}$ (mixed), $C_{max}$ (unmixed)	$\varepsilon = 1 - \exp(-C_r^{-1} [1 - \exp[-C_r(NTU)])])$ (11.34a)
<b>All exchangers (<math>C_r = 0</math>)</b>	$\varepsilon = 1 - \exp(-NTU)$ (11.35a)
<b>Parallel flow</b>	$NTU = -\frac{\ln[1 - \varepsilon(1 + C_r)]}{1 + C_r}$ (11.28b)
<b>Counterflow</b>	$NTU = \frac{1}{C_r - 1} \ln \left( \frac{\varepsilon - 1}{\varepsilon C_r - 1} \right)$ ( $C_r < 1$ ) $NTU = \frac{\varepsilon}{1 - \varepsilon}$ ( $C_r = 1$ ) (11.29b)
<b>Shell-and-tube</b>	
One shell pass (2, 4, . . . tube passes)	$(NTU)_1 = -(1 + C_r^2)^{-1/2} \ln \left( \frac{\varepsilon - 1}{\varepsilon + 1} \right)$ (11.30b) $E = \frac{2\varepsilon e_1 - (1 + C_r)}{(1 + C_r^2)^{1/2}}$ (11.30c)
$n$ shell passes (2n, 4n, . . . tube passes)	Use Equations 11.30b and 11.30c with $\varepsilon_1 = \frac{E - 1}{F - C_r}$ $F = \left( \frac{\varepsilon C_r - 1}{\varepsilon - 1} \right)^{1/n}$ $NTU = n(NTU)_1$ (11.31b, c, d)
<b>Cross-flow (single pass)</b>	
$C_{max}$ (mixed), $C_{min}$ (unmixed)	$NTU = -\ln \left[ 1 + \left( \frac{1}{C_r} \right) \ln(1 - \varepsilon C_r) \right]$ (11.33b)
$C_{min}$ (mixed), $C_{max}$ (unmixed)	$NTU = -\left( \frac{1}{C_r} \right) \ln[C_r \ln(1 - \varepsilon) + 1]$ (11.34b)
<b>All exchangers (<math>C_r = 0</math>)</b>	$NTU = -\ln(1 - \varepsilon)$ (11.35b)

$$NTU = \frac{UA}{C_{min}}$$
$$NTU = f(\varepsilon, C_r)$$
$$\varepsilon = \frac{q}{q_{max}} \quad C_r = \frac{C_{min}}{C_{max}}$$
$$q_{max} = C_{min} \Delta T_{max}$$

## LMTD Method

$$q = UA \Delta T_{lm} \quad q_h = \dot{m}_h c_{p,h} \Delta T_h \quad q_c = \dot{m}_c c_{p,c} \Delta T_c$$

$$\Delta T_{lm} = \frac{\Delta T_1 - \Delta T_2}{\ln(\Delta T_1/\Delta T_2)}$$



## Boundary Layer

速度边界层	温度边界层
$\delta_{\text{边界层厚度}} = \frac{0 - u(y)}{0 - u_{\infty}} = 0.99$	$\delta_{\text{边界层厚度}} = \frac{T_s - T(y)}{T_s - T_{\infty}} = 0.99$
$\tau_s = \mu \left. \frac{\partial u}{\partial y} \right _{y=0}$ 修改雷诺相似 Modified Reynold Analogy $F_D = \int_{A_s} \tau_s dA_s$ 扩展到 Pr 不需要 = 1 使用条件 $C_f = \frac{\tau_s}{\frac{1}{2} \rho u_{\infty}^2}$ 使用公式 $\frac{C_f}{2} = St \cdot Pr^{\frac{2}{3}} \quad 0.6 < Pr < 60$	局部 对流传热系数 h, 因为 $\frac{\partial T}{\partial y}$ 在变化 conduction $q_s'' = -k_{\text{流体}} \left. \frac{\partial T}{\partial y} \right _{y=0} \Rightarrow h_{\text{局部}} = \frac{-k_{\text{流体}} \left. \frac{\partial T}{\partial y} \right _{y=0}}{T_s - T_{\infty}}$ convection $q_s'' = h(T_s - T_{\infty})$ 平均 对流传热系数 $\bar{h}_{\text{平均}} = \frac{1}{A_s} \int_{A_s} h_{\text{局部}} dA_s$

## Free Convection

竖直平面	水平平面
Laminar Flow $\overline{Nu}_L = 0.68 + \frac{0.67 Ra_L^{\frac{1}{4}}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{4}{9}}}$	
All Condition $\overline{Nu}_L = \left\{ 0.825 + \frac{0.387 Ra_L^{\frac{1}{4}}}{\left[1 + \left(\frac{0.492}{Pr}\right)^{\frac{9}{16}}\right]^{\frac{8}{27}}} \right\}^2$	$T_s > T_{\infty}$ $\overline{Nu}_L = 0.54 Ra_L^{\frac{1}{4}} \quad (10^4 < Ra_L < 10^7)$ $\overline{Nu}_L = 0.15 Ra_L^{\frac{1}{3}} \quad (10^7 < Ra_L < 10^{11}; \text{ all } Pr)$ where $L = A/P$ .
All these properties are evaluated at film temperature 	$T_s < T_{\infty}$ $\overline{Nu}_L = 0.52 Ra_L^{\frac{1}{5}} \quad (10^4 < Ra_L < 10^9; Pr > 0.7)$

## Mixed Convection

- Heat Transfer Correlations for Mixed Convection:

$$Nu^n \approx Nu_{FC}^n \pm Nu_{NC}^n$$

$Nu_{FC} \rightarrow$  Nusselt number for forced convection

$Nu_{NC} \rightarrow$  Nusselt number for natural (free) convection

- $\rightarrow$  assisting and transverse flows
- $\rightarrow$  opposing flows

$n \approx 3$  normally but there are more values of possible n depending on the configurations

## Internal Flow (Tube)

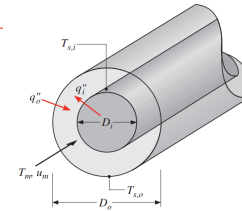
速度边界层 Hydrodynamic	温度边界层 Thermal
完全发展流 $\rightarrow$ 入口长度 Fully developed Flow $\rightarrow$ Entry Length $\left(\frac{x_{fd,h}}{D}\right)_{\text{层流}} \approx 0.05 Re_D$ $10 \lesssim \left(\frac{x_{fd,h}}{D}\right)_{\text{湍流}} \lesssim 60$	完全发展流 $\rightarrow$ 入口长度 Fully developed Flow $\rightarrow$ Entry Length $\left(\frac{x_{fd,t}}{D}\right)_{\text{层流}} \approx 0.05 Re_D Pr$ $10 \lesssim \left(\frac{x_{fd,t}}{D}\right)_{\text{湍流}}$
管道表面 恒定热通量 $q_s'' = \text{定值}$ $T_m(x) = T_{m,i} + \frac{q_s'' P}{\dot{m} c_p} x$ $P = \text{截面周长}$ $q_s'' = \text{管道表面热通量}$ $q_{\text{对流总}} = q_s'' \cdot PL$	管道表面 恒定温度 $T_s = \text{定值}$ "External" Flow $\frac{T_s - T_m(x)}{T_s - T_{m,i}} = \exp\left(-\frac{Px}{\dot{m} c_p \bar{h}}\right)$ $\frac{\Delta T_{\text{出}} - T_{\infty}}{\Delta T_{\text{进}} - T_{\infty}} = \exp\left(-\frac{\bar{U} A_s}{\dot{m} c_p}\right)$ $\Delta T_{\text{lm}} = \frac{\Delta T_{\text{出}} - \Delta T_{\text{进}}}{\ln(\Delta T_{\text{出}} / \Delta T_{\text{进}})}$ $q_{\text{conv}} = \bar{h} A_s \Delta T_{\text{lm}}$ $q = \bar{U} A_s \Delta T_{\text{lm}}$

**TABLE 8.2** Nusselt number for fully developed laminar flow in a circular tube annulus with one surface insulated and the other at constant temperature

$D_i/D_o$	$Nu_{ti}$	$Nu_{to}$	Comments
0	—	3.66	See Equation 8.55
0.05	17.46	4.06	
0.10	11.56	4.11	
0.25	7.37	4.23	
0.50	5.74	4.43	
$\approx 1.00$	4.86	4.86	See Table 8.1, $b/a \rightarrow \infty$

**TABLE 8.3** Influence coefficients for fully developed laminar flow in a circular tube annulus with uniform heat flux maintained at both surfaces

$D_i/D_o$	$Nu_{ti}$	$Nu_{to}$	$\theta_i^*$	$\theta_o^*$
0	—	4.364 <sup>a</sup>	$\infty$	0
0.05	17.81	4.792	2.18	0.0294
0.10	11.91	4.834	1.383	0.0562
0.20	8.499	4.833	0.905	0.1041
0.40	6.583	4.979	0.603	0.1823
0.60	5.912	5.099	0.473	0.2455
0.80	5.58	5.24	0.401	0.299
1.00	5.385	5.385 <sup>b</sup>	0.346	0.346



Case 1 普通热交换器 Conduction 热传递

$$\frac{1}{UA} = \frac{1}{(hA)_{\text{冷端}}} + R_{\text{管壁}} + \frac{1}{(hA)_{\text{热端}}}$$

Convection 热对流

Case 2 污染

$$\frac{1}{UA} = \frac{1}{(hA)_{\text{冷端}}} + \frac{R_f''_{\text{冷端}}}{A_{\text{冷端}}} + R_{\text{管壁}} + \frac{R_f''_{\text{热端}}}{A_{\text{热端}}} + \frac{1}{(hA)_{\text{热端}}}$$

Case 3 污染+鳍片

$$\frac{1}{UA} = \frac{1}{(\eta_o hA)_{\text{冷端}}} + \frac{R_f''_{\text{冷端}}}{(\eta_o A)_{\text{冷端}}} + R_{\text{管壁}} + \frac{R_f''_{\text{热端}}}{(\eta_o A)_{\text{热端}}} + \frac{1}{(\eta_o hA)_{\text{热端}}}$$

$\eta_o = 1 - \frac{A_{fin}}{A} (1 - \eta_{fin})$   
 $R_{\text{管壁}} = \frac{\ln(D_o/D_i)}{2\pi Lk}$   
Overall surface efficiency of fin array

$A = A_t = \text{总表面积 (fin + base)}$

$A_{fin} = \text{鳍片面积}$

$$\eta_{fin} = \frac{\tanh mL}{mL} \quad m = \sqrt{\frac{2h}{kt}}$$

Case 4 污染+鳍片 (整合版)

$$\frac{1}{UA} = \frac{1}{(\eta_o U_p A)_{\text{冷端}}} + R_{\text{管壁}} + \frac{1}{(\eta_o U_p A)_{\text{热端}}}$$

$$U_p = \frac{h}{1 + h R_f''} \quad \text{Partial overall coefficient 部分整合系数}$$

## Dimensionless Number

$St = \text{Stanton number 斯坦顿数}$

$$St = \frac{h}{\rho V c_p} = \frac{Nu}{Re \cdot Pr}$$

$Gr = \text{Grashof number 格拉晓夫数}$

$$Gr_L = Re_L^2$$
$$Gr_L = \frac{g \beta (T_s - T_{\infty}) L^3}{\nu^2}$$

$Pr = \text{Prandtl number 普朗特数}$

$$Pr = \frac{c_p \mu}{k} = \frac{\nu_{\text{粘度}}}{\alpha_{\text{热扩散率}}}$$

$Ra = \text{Rayleigh number 瑞利数}$

$$Ra_L = Gr_L \cdot Pr = \left(\frac{UL}{\nu}\right)^2 \left(\frac{\nu}{\alpha}\right) = \frac{U^2 L^2}{\nu \alpha} = \frac{g \beta (T_s - T_{\infty}) L^3}{\nu \alpha}$$

$\alpha = \text{热扩散率}$

$$\beta = \frac{1}{T_f}$$