# Homework 9 假设测试

**Exercise 1**

Let p be the proportion of cadets who believe there is a Dairy Queen in Altgeld Hall.

Boyle says it is believed that 20% of cadets believe in the Dairy Queen Altgeld. Santiago thinks the true proportion is actually lower.

In a random poll that their captain put together, 32 cadets out of 200 claimed to believe in the Dairy Queen in Altgeld.

(a) (1 point) Perform an appropriate test at significance level $\alpha = 0.05$ to test Santiago's claim.

(b) (0.5 points) Create a 95% two-sided confidence interval for p.

假设验证 Proportion

a). $H_0: p = 20\%$ $\quad n = 200 \quad Y = 32 \quad \hat{p} = \frac{Y}{n} = 0.16$

$H_A: p < 20\%$

$$Z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.16 - 0.2}{\sqrt{\frac{0.2 \times 0.8}{200}}} = -1.41421$$

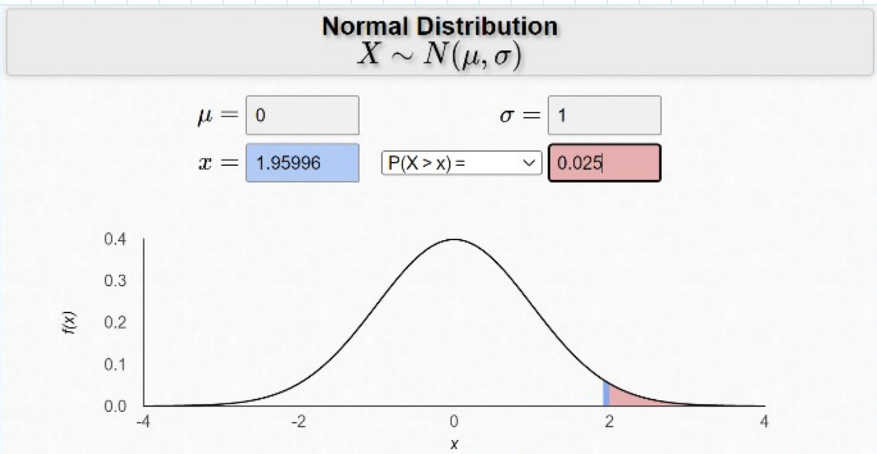
$$\text{pnorm}(-1.41421) = 0.07865 > \alpha = 0.05$$

置信区间 Proportion

b). $95\% \Rightarrow \frac{\alpha}{2} = 0.025 \Longrightarrow Z_{\frac{\alpha}{2}} = 1.96$

$$\Rightarrow \left[\hat{p} - Z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + Z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$$

$$\Rightarrow [0.109,\ 0.211]$$



**Exercise 2**

Wario is looking at go-karts with the hope that he might get an edge on Waluigi at the next big race. Through extensive study, he has determined that his current kart hits an average top highway speed of 63.5kph.

Wario test drives a new kart on some open roadways and tracks his top speed on 8 straight-line stretches. He lists the following top speeds: {63.2, 63.4, 63.4, 63.7, 63.8, 64.1, 64.1, 64.2}

You may calculate $s^2$ by hand or with R.

(a) (1 point) Perform a hypothesis test at $\alpha = 0.02$ to test whether there is sufficient evidence to suggest that the new kart has **faster** top speeds on average than his current kart. (*Note*: You may calculate $s^2$ by hand or with R).

(b) (0.5 points) Create a 98% Confidence interval for $\mu_{new\ kart}$.

假设验证 (μ)(σ未知)

a). $H_0: \mu = 63.5$

$H_A: \mu > 63.5$

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var(c(63.2, 63.4, 63.4, 63.7, 63.8, 64.1, 64.1, 64.2))
```

$\bar{x} = 63.7375 \quad s^2 = 0.1427 \quad s = 0.3777 \quad n = 8 \Rightarrow df = 7$

$$t = \frac{\bar{x} - p}{\frac{s}{\sqrt{n}}} = \frac{63.7375 - 63.5}{\frac{0.3777}{\sqrt{8}}} = 1.77853$$

$p = 1 - pt(1.77853, 7) = 0.0593 > \alpha = 0.02$

∴ There is no enough evidence to reject the hypothesis

置信区间 (μ)

b). $98\% \Rightarrow \frac{\alpha}{2} = 0.01$

$$CI: \left[\bar{x} - t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}},\ \bar{x} + t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}}\right]$$

$$= [63.33717,\ 64.1378]$$

**Exercise 3**

Daenerys' advisors tell her that that less than half of her army is plotting to stage a coup but she is suspicious that it may be more than half. She has a spy interview 100 of them randomly and finds that 58 of them said they planned to stage a coup. Let $p$ represent the proportion of her army planning to stage a coup (overthrow her).

a) (1 point) Perform an appropriate test at significance level $\alpha = 0.01$ to test her hypothesis.

b) (0.5 point) Create a 95% confidence interval for p **by hand**.

c) (0.5 point) Create a 99% confidence upper bound for p **by hand**.

d) (1 point) Daenerys hopes that she is significantly more popular than Greyjoy. Suppose a random sample of people in Greyjoy's navy revealed that 30 of the 50 respondents were plotting to overthrow him. Test to see if Daenerys is significantly more popular than Greyjoy at $\alpha = 0.05$. *i.e. whether a smaller proportion of people want to overthrow Daenerys.*

假设验证 Proportion

a). $H_0: P \leqslant 50\%$ $\quad n = 100 \quad Y = 58 \quad \hat{p} = 0.58$

$H_A: P > 50\%$

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.58 - 0.5}{\sqrt{\frac{0.5^2}{100}}} = 1.6$$

$$\text{pnorm}(1.6) = 0.0548 > \alpha = 0.01$$

$\therefore$ No enough evidence to reject the hypothesis

置信区间 Proportion

b). $95\% \Rightarrow \frac{\alpha}{2} = 0.025$ From the table we know $Z_{\frac{\alpha}{2}} = 1.96$

$$CL: \left[\hat{p} - Z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}},\ \hat{p} + Z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$$

$$= \left[0.58 - 1.96 \times \sqrt{\frac{0.58 \times 0.42}{100}},\ 0.58 + 1.96 \times \sqrt{\frac{0.58 \times 0.42}{100}}\right]$$

$$= [0.483,\ 0.6767]$$

置信区间 Proportion

c). $99\% \Rightarrow \alpha = 0.01$ From the table we know $Z_\alpha = 2.326$

$$CL: \left[0,\ \hat{p} + Z_\alpha \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right]$$

$$= \left[0,\ 0.58 + 2.326 \times \sqrt{\frac{0.58 \times 0.42}{100}}\right]$$

$$= [0,\ 0.6948]$$

**Exercise 4**

Assume Albert is testing whether students who come to lecture do better on exams (on average). Suppose we collected two samples of exam scores for midterm 2: students who are coming to lecture (Group 1 = $x_1, x_2, \ldots x_{25}$) and students who are not coming to lecture (Group 2 = $y_1, y_2, \ldots y_{16}$). The results are shown below. You may copy and paste this code into R to get the same data.

(a) (1 points) Perform the appropriate test to determine if this difference is significant at $\alpha = 0.01$ using Welch's formula for degrees of freedom **by hand**. Please show all work. Identify your test statistic, calculate its value and distribution under $H_0$ (show your work for the df calculation), p-value, and decision.

(b) (0.5 points) Perform the same test from part (a) using R.

(c) (0.5 points) Assuming there is equal variance between the two groups, calculate the value of the pooled variance by hand, and use this updated value to perform your test (by hand).

(d) (0.5 point) Calculate a 95% confidence interval **by hand** for the true difference in exam score means (assume un-equal variances).

双元假设验证

a). $H_0: \mu_X = \mu_Y$

$H_A: \mu_X > \mu_Y$

Test statistic:

From the question we know X and Y are not pooled and not paired

$$\therefore df = \left\lfloor \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\left(\frac{s_2^2}{n_2}\right)^2} \right\rfloor = \left\lfloor \frac{\left(\frac{3.74^2}{25} + \frac{6.0937^2}{16}\right)^2}{\frac{1}{24}\cdot\left(\frac{3.74^2}{25}\right)^2 + \frac{1}{15}\left(\frac{6.094^2}{16}\right)^2} \right\rfloor = \lfloor 22.29 \rfloor = 22$$

$$t = \frac{\bar{X} - \bar{Y} - 0}{\sqrt{\frac{S_X^2}{25} + \frac{S_Y^2}{16}}} = \frac{93.64 - 81.75}{\sqrt{\frac{3.74^2}{25} + \frac{6.094^2}{16}}} \approx 7 \sim t_{22}$$

$\Rightarrow P \approx 0 < \alpha = 0.01$ $\therefore$ There is enough evidence to reject $H_0$

b).

```r
rm(list=ls()) #remove all data

x=c(90,94,90,99,94,90,95,96,95,92,99,95,91,84,97,93
y=c(82,81,70,78,85,93,81,85,82,71,79,79,82,91,88,81

n1=length(x)
n2=length(y)

s1=sd(x)
s2=sd(y)

df=floor((s1^2/n1+s2^2/n2)^2/(s1^4/n1^2/(n1-1)+s2^4


t=(mean(x)-mean(y)-0)/sqrt(s1^2/n1+s2^2/n2)


ans=1-pt(t,df)
ans
```

```
> ans
[1] 2.482062e-07
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Pooled

c). $$S_{pooled}^2 = \frac{(n_1-1)\cdot S_1^2 + (n_2-1)\cdot S_2^2}{n_1+n_2-2} = \frac{24\cdot 3.74^2 + 15\cdot 6.094^2}{25+16-2} \approx 22.89$$

$$df = n_1+n_2-2 = 25+16-2 = 39$$

$$t = \frac{\bar{X}-\bar{Y}-0}{S_{pooled}\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{93.64 - 81.75}{\sqrt{22.89}\cdot\sqrt{\frac{1}{25}+\frac{1}{16}}} = 7.76 \sim t_{39}$$

$\therefore P \approx 0 < \alpha = 0.01$

双元置信区间

d). $\alpha = 0.05 \Rightarrow \frac{\alpha}{2} = 0.025$ $df = 22$ $\Rightarrow t_{\frac{\alpha}{2},(22)} = 2.074$

$$\therefore CI: \left[\bar{X}-\bar{Y} - t_{\frac{\alpha}{2}}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}},\ \bar{X}-\bar{Y} + t_{\frac{\alpha}{2}}\sqrt{\frac{S_1^2}{n_1}+\frac{S_2^2}{n_2}}\right]$$

$$= [8.37,\ 15.41]$$

**Exercise 5**

Spider Man is trying out a new web-making formula and wants to know if it will yield more stable results. His supervisor, Happy, claims that the old process creates webs with normally distributed tensile strengths with $\sigma = 0.3\ GPa$. Spiderman collects 16 measurements from webs using the new formula and finds a sample standard deviation of $s = 0.21\ GPa$.

Parts (a) - (b), state the following, showing work where appropriate:

- null and alternative hypothesis
- distribution and value of the test statistic
- p value
- conclusion (reject or fail to reject $H_0$).

a) (1 point) Perform a hypothesis test at $\alpha = 0.01$ to test whether there is sufficient evidence to suggest that the new formula is better (has less variability) than the original.

b) (0.5 points) Suppose that Spiderman now takes 100 measurements (instead of 16) and still finds a sample standard deviation of 0.21GPa Perform a hypothesis test at $\alpha = 0.01$ to test whether there is sufficient evidence to suggest that the new suit is better (more accurate) than the original.

假设验证 ($\sigma$)

a). Let $\sigma_1$ denote the old version, $\sigma_2$ denote the new version.

$H_0: \sigma_2 = 0.3$

$H_A: \sigma_2 < 0.3$

$df = 16 - 1 = 15$

$$\chi^2 = \frac{(n-1)\cdot s^2}{\sigma^2} = \frac{(16-1)\cdot 0.21^2}{0.3^2} = 7.35 \sim \chi^2_{(15)}$$

From Rstudio we know

$p = \text{pchisq}(7.35, 15) = 0.0528 > \alpha = 0.01$

Therefore, there is no enough evidence to reject $H_0$

b). $H_0: \sigma_2 = 0.3$

$H_A: \sigma_2 < 0.3$

$df = 100 - 1 = 99$

$$\chi^2 = \frac{(n-1)\cdot s^2}{\sigma^2} = \frac{99 \cdot 0.21^2}{0.3^2} \approx 48.51 \sim \chi^2_{99}$$

From Rstudio we know

$p = \text{pchisq}(48.51, 99) \approx 4.56\times10^{-6} < \alpha = 0.01$

Therefore, there is enough evidence to reject $H_0$