R 旋转矩阵

2D 旋转矩阵 ⟶ 2D 旋转矩阵性质

$$R(\theta) = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$$

$$\boldsymbol{R}^T = \boldsymbol{R}^{-1}$$

$$\boldsymbol{R}_\alpha \boldsymbol{R}_\beta = \boldsymbol{R}_\beta \boldsymbol{R}_\alpha \quad \text{(交换律)}$$

3D 旋转矩阵 ⟶ 3D 旋转矩阵性质

$$\boldsymbol{R}^T = \boldsymbol{R}^{-1}$$

无交换律

x 轴逆时针旋转 θ

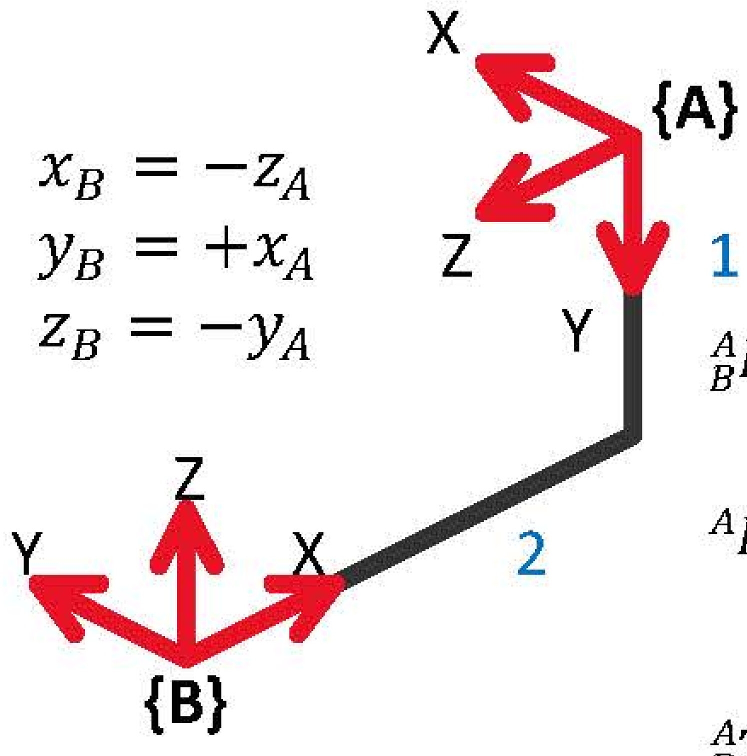
$$ {}_B^A\boldsymbol{R}_x = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

y 轴逆时针旋转 θ

$$ {}_B^A\boldsymbol{R}_y = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

z 轴逆时针旋转 θ

$$ {}_B^A\boldsymbol{R}_z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$x_B = -z_A$$
$$y_B = +x_A$$
$$z_B = -y_A$$

$$ {}_B^A\boldsymbol{R} = \begin{bmatrix} 0 & +1 & 0 \\ 0 & 0 & -1 \\ -1 & 0 & 0 \end{bmatrix}$$

$$ {}^A P_{B,Org} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$$ {}_B^A\boldsymbol{T} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ -1 & 0 & 0 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

表示说明:
$^A\vec{P}$ 代表 *Frame A* 坐标系下点 *P* 的坐标

旋转矩阵作用

1.描述 *Frame B* 相对于 *Frame A* 的姿态

$$ {}_B^A R = \begin{bmatrix} | & | & | \\ {}^A\hat{X}_B & {}^A\hat{Y}_B & {}^A\hat{Z}_B \\ | & | & | \end{bmatrix}$$

2.[Mapping] 将一个点由 *Frame B* 表示变为 *Frame A* 表示

$$ {}^A\vec{P} = {}_B^A R\, {}^B\vec{P}$$

3.[Operator] 将一个点在 *Frame A* 下进行旋转

$$ {}^A\vec{P} = R(\theta)\, {}^A\vec{P}$$

T 变换矩阵

$$ {}_B^A\boldsymbol{T} = \begin{bmatrix} {}_B^A\boldsymbol{R} & {}^A P_{B,Org} \\ 0\,0\,0 & 1 \end{bmatrix} = \begin{bmatrix} R & p \\ 0 & 1 \end{bmatrix}$$

$$ {}_A^B\boldsymbol{T} = {}_B^A\boldsymbol{T}^{-1} = \begin{bmatrix} R^T & -R^T p \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} {}^A P \\ 1 \end{bmatrix} = {}_B^A T \begin{bmatrix} {}^B P \\ 1 \end{bmatrix}$$ 将 *P* 点由 *Frame B* 改为由 *Frame A* 表达

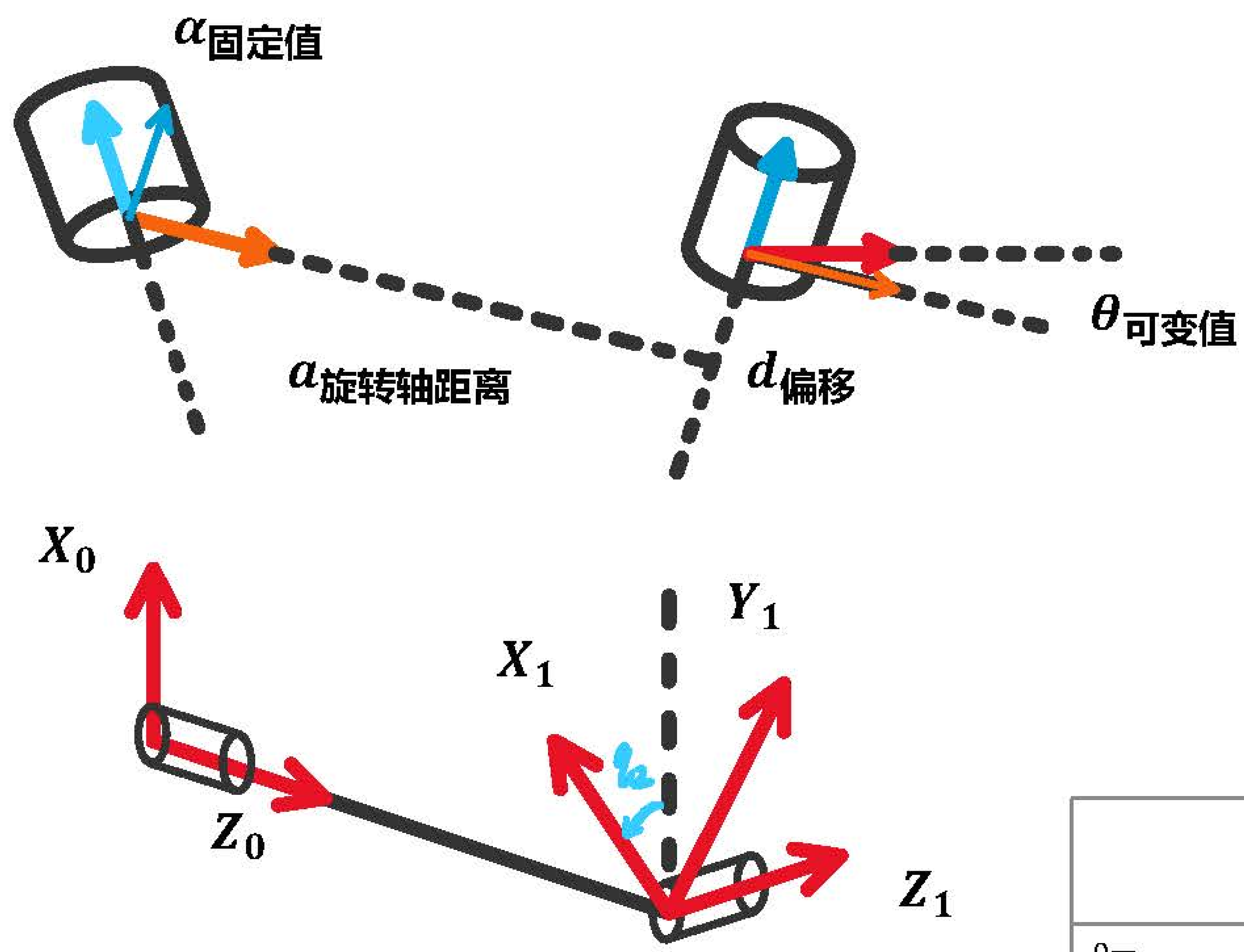
拉格朗日运动学

$$L = K_{\text{动能}} - P_{\text{势能}}$$

$$T_1 = \frac{d}{dt}\left(\frac{\partial \boldsymbol{L}}{\partial \dot{q}_1}\right) - \frac{\partial \boldsymbol{L}}{\partial q_1}$$

$$T_2 = \frac{d}{dt}\left(\frac{\partial \boldsymbol{L}}{\partial \dot{q}_2}\right) - \frac{\partial \boldsymbol{L}}{\partial q_2}$$

DH 变换



$$ {}_i^{i-1}\mathrm{T} = R_x(\alpha_{i-1}) D_x(a_{i-1})\ R_z(\theta_i) D_z(d_i)$$

$$ {}_i^{i-1}\mathrm{T}(\theta_i) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\alpha_{i-1} & -\sin\alpha_{i-1} & 0 \\ 0 & \sin\alpha_{i-1} & \cos\alpha_{i-1} & 0 \\ & \mathbf{0} & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I} & a_{i-1} \\ & 0 \\ & 0 \\ \mathbf{0} & 1 \end{bmatrix} \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I} & 0 \\ & 0 \\ & d_i \\ \mathbf{0} & 1 \end{bmatrix}$$

$$ {}_1^0\mathrm{T}(\theta_1) = [\mathrm{I}][\mathrm{I}] \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix} \begin{bmatrix} \mathbf{I} & 0 \\ & 0 \\ & l_0 \\ \mathbf{0} & 1 \end{bmatrix} = \begin{bmatrix} c1 & -s1 & 0 & 0 \\ s1 & c1 & 0 & 0 \\ 0 & 0 & 1 & l_0 \\ & \mathbf{0} & & 1 \end{bmatrix}$$

$$ {}_2^1\mathrm{T}(\theta_2) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix} [\mathrm{I}] \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ \sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix} [\mathrm{I}] = \begin{bmatrix} c\,\theta_2 & -s\,\theta_2 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s\,\theta_2 & c\,\theta_2 & 0 & 0 \\ & \mathbf{0} & & 1 \end{bmatrix}$$

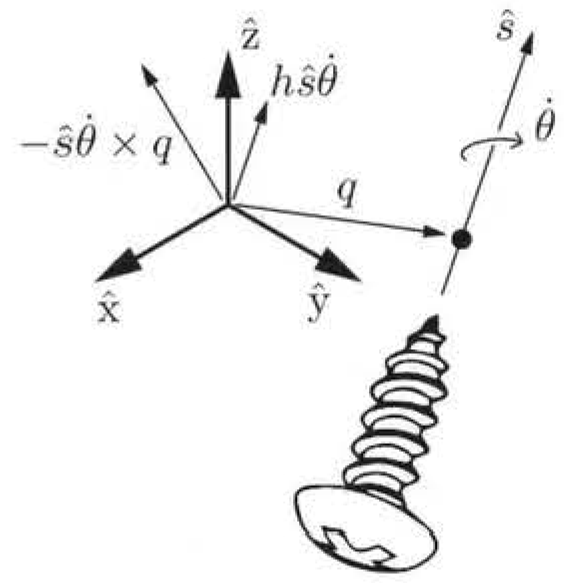
	Link Twist α_{i-1}	Link Length a_{i-1}	Joint Angle θ_i	Link Offset d_i
${}_1^0\boldsymbol{T}$	0	0	0	q_1
${}_2^1\boldsymbol{T}$	90	0	$-q_2$	0

$\alpha_{\text{固定值}}$ = 从X_0的看Z_1旋转了多少度

$\theta_{\text{可变值}}$ = 从Z_1的看X_1旋转了多少度

$$ {}_i^{i-1}\boldsymbol{T} = R_x(\alpha_{i-1}) D_x(a_{i-1}) R_z(\theta_i) D_z(d_i)$$

螺旋运动 Skrew Motion



逆对称矩阵

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \xrightarrow{\text{逆对称矩阵}} \begin{bmatrix} 0 & -z & +y \\ +z & 0 & -x \\ -y & +x & 0 \end{bmatrix}$$

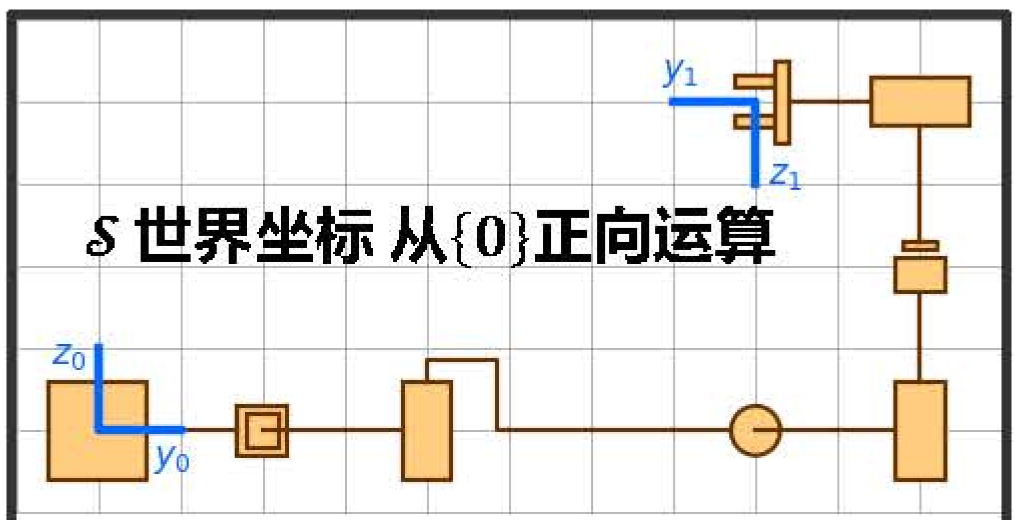
$$\vec{a}\times\vec{b} = [\vec{a}]_{\text{逆}}\cdot\vec{b}$$

螺旋运动

$$v = -\omega \times q$$

$$\mathcal{V} = \begin{bmatrix} \vec{\omega} \\ \vec{v} \end{bmatrix}$$

$[\mathcal{V}_b] = \begin{bmatrix} [\vec{\boldsymbol{\omega}}_b] & \vec{\boldsymbol{v}}_b \\ \vec{0} & 0 \end{bmatrix}$	$[\mathcal{V}_s] = \begin{bmatrix} [\vec{\boldsymbol{\omega}}_s] & \vec{\boldsymbol{v}}_s \\ \vec{0} & 0 \end{bmatrix}$
$T(\theta) = e^{[S_1]\theta_1}\dots e^{[S_n]\theta_n}M$	$T(\theta) = Me^{[\mathcal{B}_1]\theta_1}\dots e^{[\mathcal{B}_n]\theta_n}$



S 世界坐标 从{0}正向运算

$$x_1 = +x_0 \quad y_1 = -y_0 \quad z_1 = -z_0$$

$$M = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & -2 \\ 0 & 0 & -1 & -2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\omega_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad q_2 = \begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix} \quad v_2 = \begin{bmatrix} 4 \\ 0 \\ 0 \end{bmatrix}$$

$$S = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 4 & 0 & 10 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -8 & 0 & 1 & 0 \end{bmatrix}$$

$$T_{01} = \boxed{e^{[S_1]\theta_1}}\dots e^{[S_6]\theta_6}M$$

$$S_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \Rightarrow [S_1] = \begin{bmatrix} [\vec{\boldsymbol{\omega}}_s] & \vec{\boldsymbol{v}}_s \\ \vec{0} & 0 \end{bmatrix}$$

矩阵指数

若 $|\omega| = 1$

$$e^{[S]\theta} = \begin{bmatrix} e^{[\omega]\theta} & (I\theta + (1-\cos\theta)[\omega] + (\theta - \sin\theta)[\omega]^2)v \\ 0 & 1 \end{bmatrix}$$

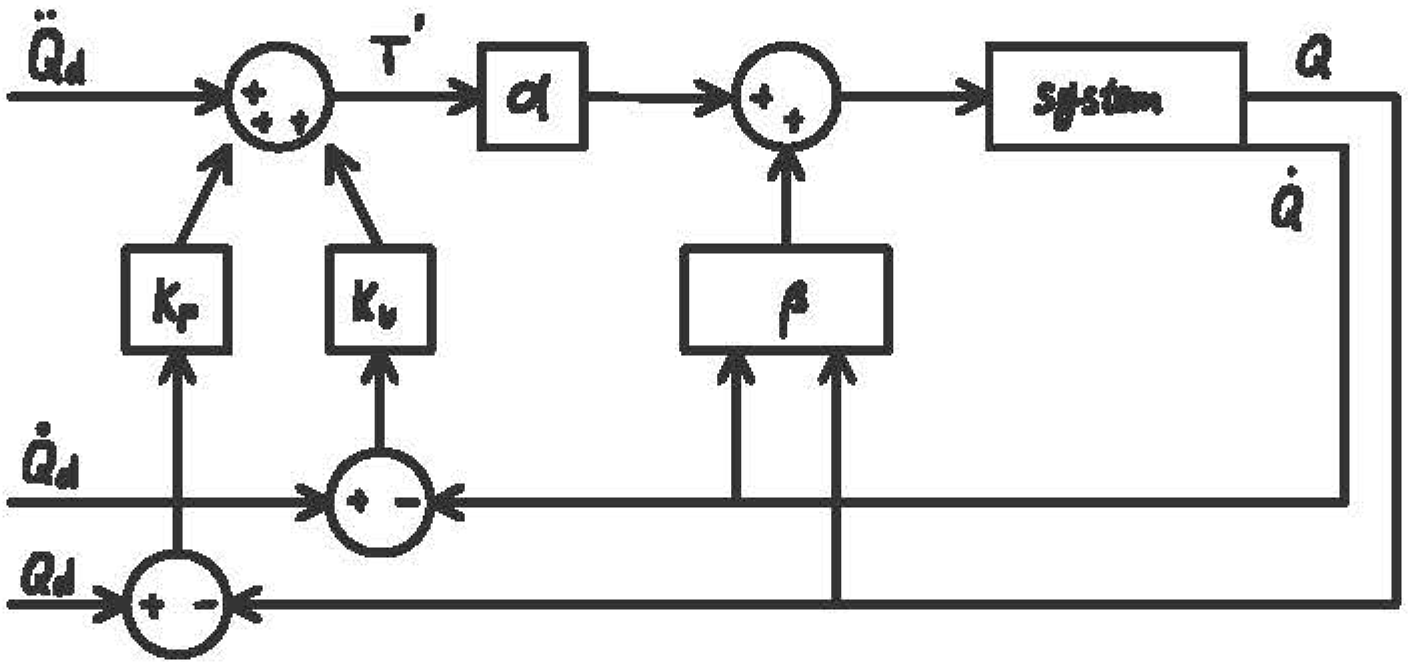
$$e^{[\omega]\theta} = I + [\omega]\theta + [\omega]^2\frac{\theta^2}{2!} + [\omega]^3\frac{\theta^3}{3!} + \cdots$$

$$e^{[\omega]\theta} = I + \sin\theta[\omega] + (1-\cos\theta)[\omega]^2$$

若 $|\omega| = 0$ 并 $|v| = 1$ (平移关节)

$$e^{[S]\theta} = \begin{bmatrix} \boldsymbol{I} & v\theta \\ 0 & 1 \end{bmatrix}$$

框图



$$Q = \begin{bmatrix} q_1 \\ q_2 \end{bmatrix} \quad Q_d = \begin{bmatrix} q_{1d} \\ q_{2d} \end{bmatrix}$$

$$\dot{Q} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \end{bmatrix} \quad \dot{Q}_d = \begin{bmatrix} \dot{q}_{1d} \\ \dot{q}_{2d} \end{bmatrix}$$

d=desire

$$T = \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} (m_1+m_2)L_1^2 & m_2L_1L_2\cos(q_2-q_1) \\ m_2L_1L_2\cos(q_2-q_1) & m_2L_2^2 \end{bmatrix}\begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix} + \begin{bmatrix} -m_2L_1L_2\sin(q_2-q_1)\dot{q}_2^2 + (m_1+m_2)gL_1\cos(q_1) \\ m_2L_1L_2\sin(q_2-q_1)\cdot\dot{q}_1^2 + m_2gL_2\cos(q_2) \end{bmatrix}$$

$$T = \alpha T' + \beta$$

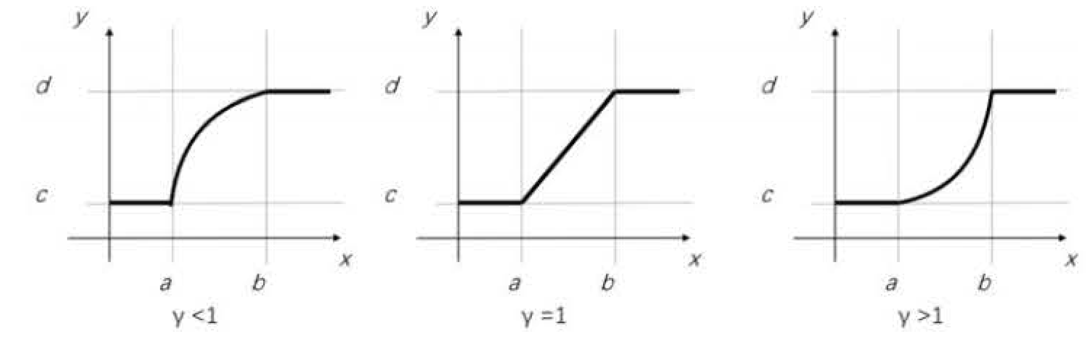
$$\alpha = \begin{bmatrix} (m_1+m_2)L_1^2 & m_2L_1L_2\cos(q_2-q_1) \\ m_2L_1L_2\cos(q_2-q_1) & m_2L_2^2 \end{bmatrix}$$

$$\beta = \begin{bmatrix} -m_2L_1L_2\sin(q_2-q_1)\dot{q}_2^2 + (m_1+m_2)gL_1\cos(q_1) \\ m_2L_1L_2\sin(q_2-q_1)\cdot\dot{q}_1^2 + m_2gL_2\cos(q_2) \end{bmatrix}$$

$$T' = \ddot{Q}_d + K_v\dot{E} + K_pE = \begin{bmatrix} \ddot{q}_{1d} \\ \ddot{q}_{2d} \end{bmatrix} + K_v\cdot\begin{bmatrix} \dot{q}_{1d} - \dot{q}_1 \\ \dot{q}_{2d} - \dot{q}_2 \end{bmatrix} + K_p\cdot\begin{bmatrix} q_{1d} - q_1 \\ q_{2d} - q_2 \end{bmatrix}$$

$$\tau_{\text{输出}} = \eta_{\text{减速比}}\tau_{m,\text{电机}}$$

直方图

拉伸 Stretch	均衡化 Equalization
$$y=\left(\frac{x-a}{b-a}\right)^{\gamma}(d-c)+c$$	$$S_k=\left(\frac{n_1+n_2+\cdots n_k}{n_{\text{区域像素总数}}}\right)(L_{\text{总灰度}}-1)$$
	(see table below)

n	1	2	1	1	2	1	2	2	2	1	1	3	1	1	3	1
原灰度	124	125	129	130	131	133	134	136	138	139	140	141	142	143	145	149
均衡灰度	$10\approx\frac{1}{25}255$	$30\approx\frac{1+2}{25}255$	$30\approx\frac{4}{25}255$	51	71	81	102	122	122	142	163	193	204	214	244	255

滤波

- 均值滤波 Mean Filter
- 高斯滤波 Gaussian Filter

Mean filter $\frac{1}{n}\cdot I_{a\times a}$	Gaussian filter with σ=1	Gaussian filter with σ=3
		

边缘检测

- Sobel 算子

x方向算子	y方向算子
$\begin{matrix}-1 & 0 & 1\\ -2 & 0 & 2\\ -1 & 2 & 1\end{matrix}$	$\begin{matrix}1 & 2 & 1\\ 0 & 0 & 0\\ -1 & -2 & -1\end{matrix}$

- Canny 算子

  1.使用高斯滤波对图像消噪

  2.计算图片梯度

  3.非极大值抑制 + 双阈值

  超过阈值就保留，小于阈值就变0

  阈值中间实行非极大值抑制

  简单来说就是查看周围的梯度，仅保留最大值，其他都变成0

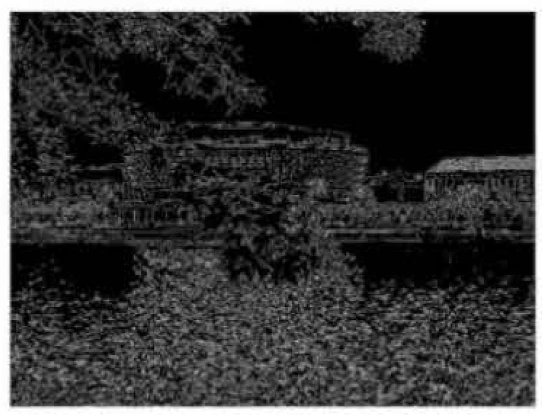
Image 8

[0.1,0.2]

Image 9

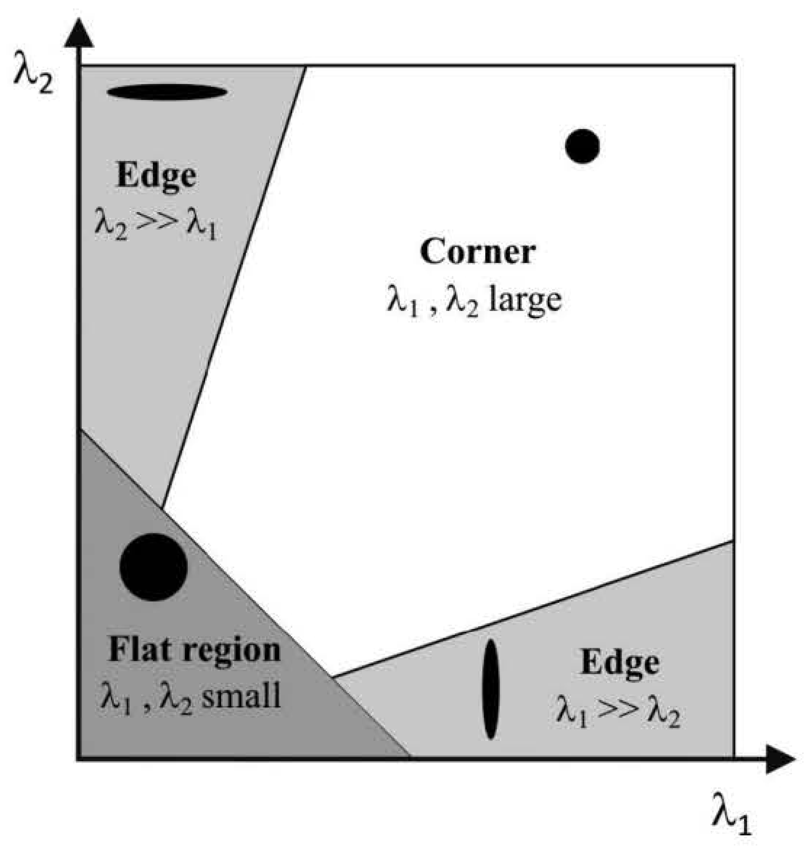
[0.4,0.6]

Image 10



[0.01,0.1]

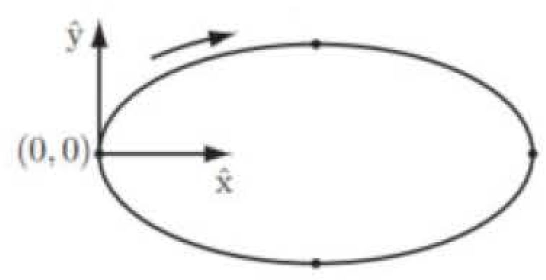
角点检测



路径规划

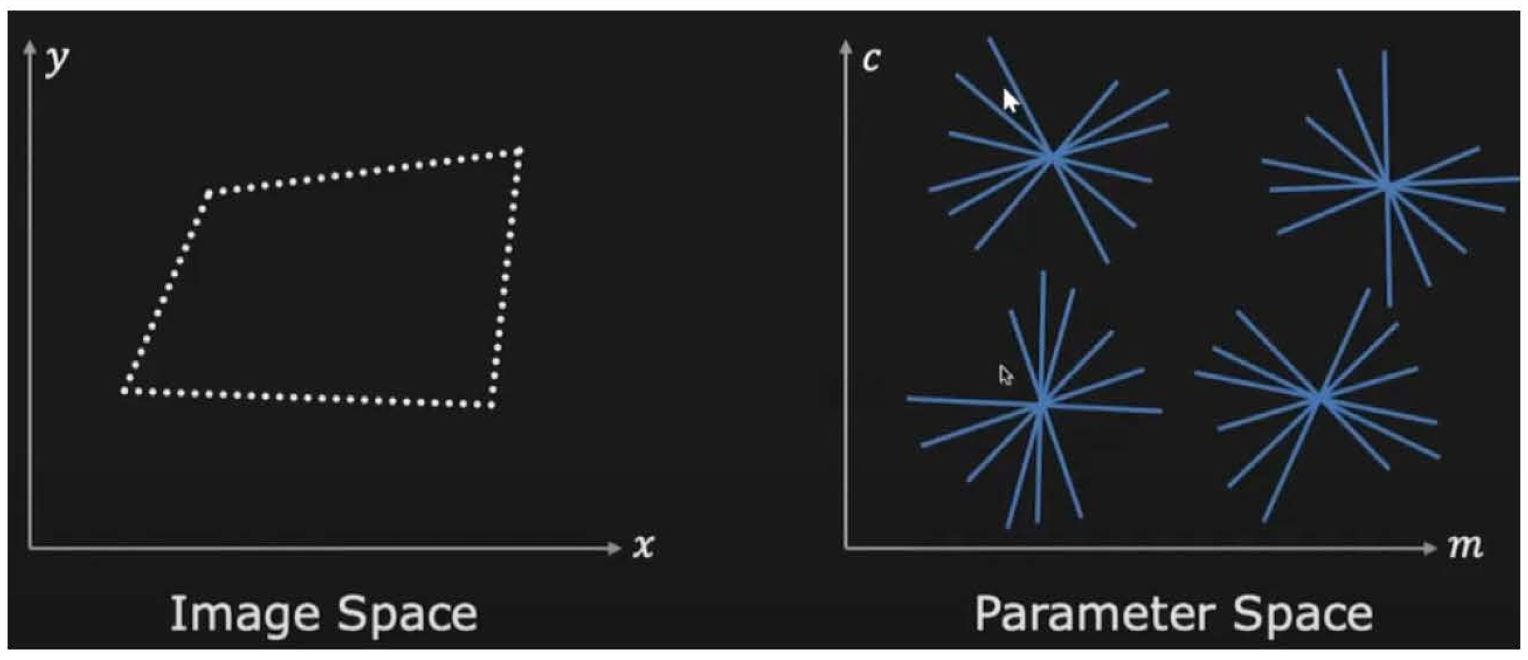
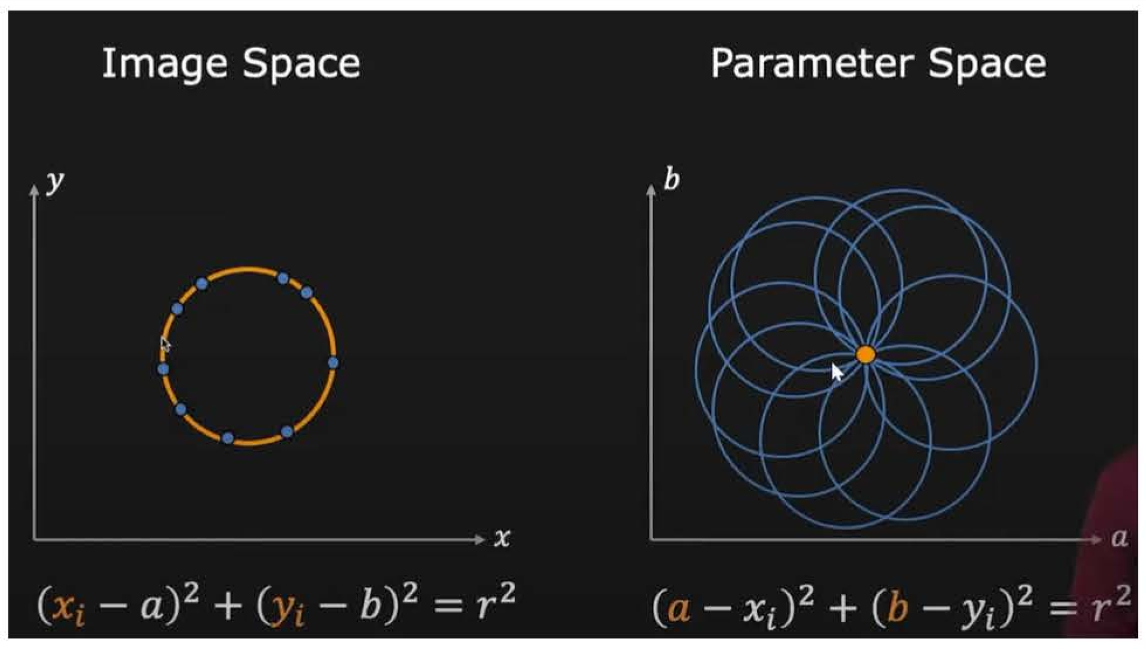
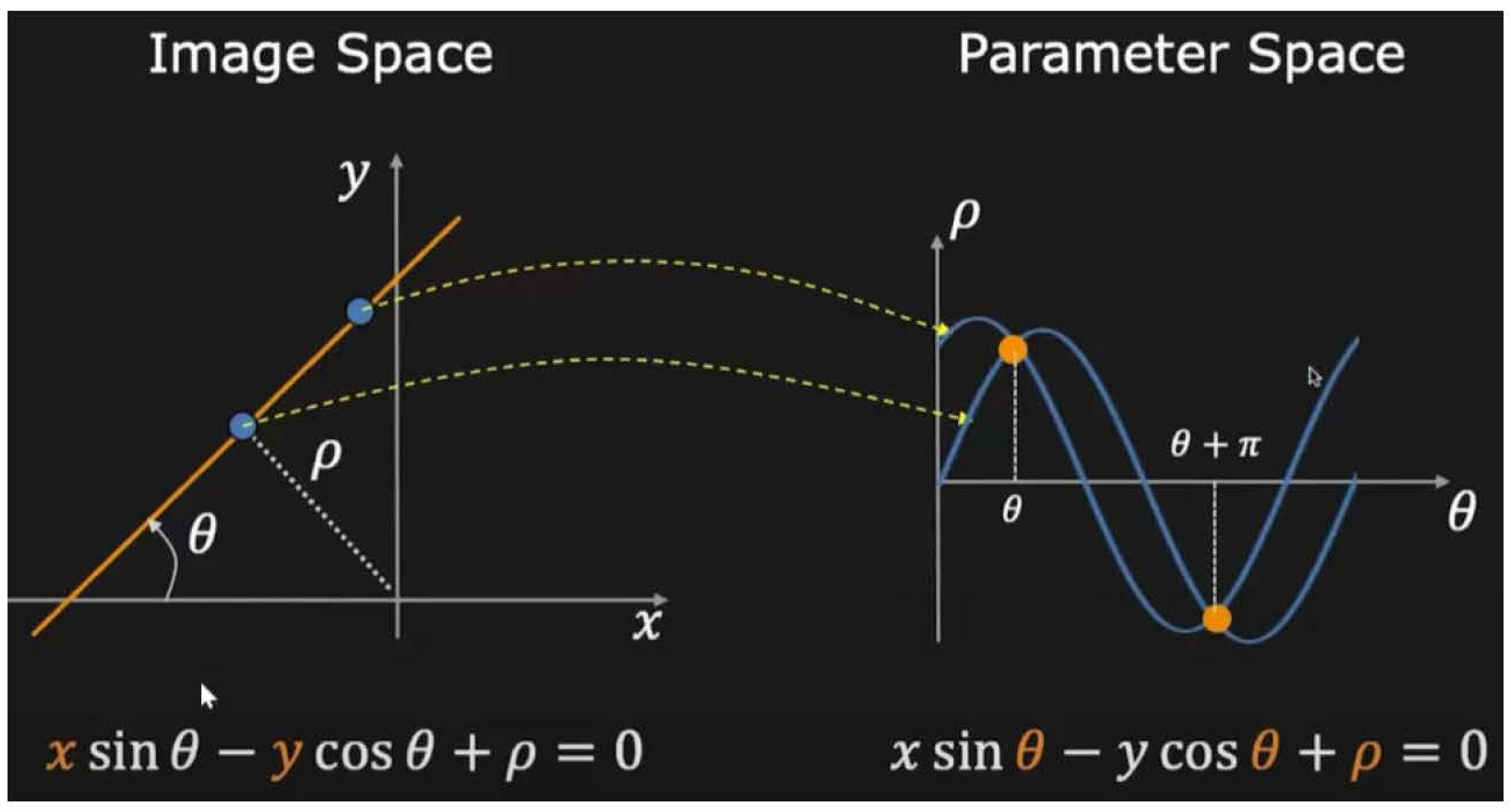
$$s(t)=a_0+a_1t+a_2t^2+a_3t^3$$

$$\left.\begin{matrix}s(0)=0\\ \dot{s}(0)=0\\ s(1)=1\\ \dot{s}(1)=0\end{matrix}\right\}\Rightarrow a=[0,0,3,-2]$$



$$(8,2),(16,0),(8,-2),s=0.8\Rightarrow\begin{cases}x_1=5.527\\ y_1=-1.90\end{cases}$$

霍夫变换 Hough Transformation

<table>
<tr><th>直线拟合</th><th>圆形拟合</th></tr>
<tr><td>
</td><td>
</td></tr>
<tr><td>
</td><td>对于未知半径
以R为纵轴
将会形成圆锥</td></tr>
</table>

相机模型

$$\boldsymbol{u}_{\text{像素}} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \xLeftarrow[M_{\text{内参矩阵}}]{} \boldsymbol{x}_{\text{相机}} = \begin{bmatrix} x_C \\ y_C \\ z_C \\ 1 \end{bmatrix} \xLeftarrow[M_{\text{外参矩阵}}]{} \boldsymbol{x}_{\text{世界}} = \begin{bmatrix} x_W \\ y_W \\ z_W \\ 1 \end{bmatrix}$$

内参矩阵 Intrinsic Matrix

$$\boldsymbol{M_{In}} = [\boldsymbol{K} \mid \boldsymbol{0}] = \begin{bmatrix} \boldsymbol{f_x} & \boldsymbol{0} & \boldsymbol{o_x} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{f_y} & \boldsymbol{o_y} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{1} & \boldsymbol{0} \end{bmatrix}$$

外参矩阵 Extrinsic Matrix (T变换矩阵)

$$\boldsymbol{M}_{Ex} = {}^{C}\boldsymbol{T}_{W} = \begin{bmatrix} {}^{C}\boldsymbol{R}_{W} & {}^{C}\boldsymbol{t}_{W,org} \\ 0\ 0\ 0 & 1 \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^{C}\boldsymbol{P} = {}^{C}\boldsymbol{T}_{W}\,{}^{W}\boldsymbol{P}$$

$$s_{\text{常数}} \cdot \boldsymbol{u}_{\text{像素}} = \boldsymbol{M}_{In} \cdot {}^{C}\boldsymbol{P}$$
$$s_{\text{常数}} \cdot \boldsymbol{u}_{\text{像素}} = \boldsymbol{M}_{In} \cdot \boldsymbol{M}_{Ex}\,{}^{W}\boldsymbol{P}$$