6 现代控制理论

Question 1

<u>With an example,</u> discuss the benefits of using state-space control design. (4 Points)

Because using state-space control we can achieve multiple input and multiple output control (MIMO), so we can establish more sophisticated system like controlling both the angle and position of reverse pendulum.

Question 2

Consider a system $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u$; $\mathbf{y} = \mathbf{C}\mathbf{x}$

where, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -2 \end{bmatrix}; B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}; C = [1 \quad 1 \quad 0]$

Write down a transfer function representation of the system. (4 Points)

$$\left.\begin{aligned} X &= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \\ \ddot{x} \end{bmatrix} \\ \dot{X} &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -1 & -1 & -2 \end{bmatrix} X + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u \end{aligned}\right\} \Rightarrow \begin{cases} \dot{x} = \dot{x} \\ \ddot{x} = \ddot{x} \\ \dddot{x} = -x - \dot{x} - 2\ddot{x} + u \end{cases} \Rightarrow \dddot{x} + 2\ddot{x} + \dot{x} + x = u$$

$$\Downarrow$$

$$\frac{X(s)}{U(s)} = \frac{1}{s^3 + 2s^2 + s + 1}$$

$$Y = [1 \quad 1 \quad 0] \cdot X \Rightarrow y = x + \dot{x} \Rightarrow Y(s) = X(s) + sX(s) \Rightarrow \frac{Y(s)}{X(s)} = s+1$$

$$\frac{Y(s)}{U(s)} = \frac{Y(s)}{X(s)} \cdot \frac{X(s)}{U(s)} = \frac{s+1}{s^3 + 2s^2 + s + 1}$$

Question 3

Consider the system:

$$\dot{x} = \begin{pmatrix} 0 & 0 & a_3 \\ 1 & 0 & a_2 \\ 0 & 1 & a_1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u \ ; \ y = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} x$$

(a) Are there real values for a_1, a_2, a_3 that make the system non controllable? (6 Points)

(b) Are there real values for a_1, a_2, a_3 that make the system non observable? (6 Points)

(a) 判断是否能够成 能控标准型 CCF

step1 计算能控性矩阵 $\mathcal{C}$

$$\mathcal{C} = [B \mid AB \mid A^2B] = \begin{bmatrix} 0 & a_3 & a_1a_3 \\ 0 & a_2 & a_3 + a_1a_2 \\ 1 & a_1 & a_1^2 + a_2 \end{bmatrix}$$

$\Downarrow$

$\det(\mathcal{C}) = a_3^2 \implies$ 当 $a_3 = 0$ 时不可逆

$\therefore$ $a_3 = 0$ 时不能变成 CCF

$\mathcal{C}(A,B) = [B \mid AB \mid A^2B \mid \cdots A^{n-1}B]$

矩阵

$[A^0B \mid A^1B \mid A^2B \cdots A^{n-1}B]$

A 为 $n \times n$

B 为 $n \times 1$

$\Downarrow$

$\mathcal{C}(A,B)$ 为 $n \times n$

(b) 判断是否能构成 能观标准型 OCF

step1 计算能观性矩阵 $\mathcal{O}$

能观性矩阵 Observability Matrix

$$\mathcal{O}(A,C) = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}$$

$$\mathcal{O} = \begin{bmatrix} CA^0 \\ CA^1 \\ CA^2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & a_1 \\ 1 & a_1 & a_1^2 + a_2 \end{bmatrix}$$

$\Downarrow$

$\det(\mathcal{O}) = -1 \implies$ 可逆

$\therefore$ a_1, a_2, a_3 无论如何取值，都能变成 OCF