Lec02: Fourier's Law

Chapter Two

Section 2.1-2.2

Announcement

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- Look out for HW on BB
- ZJUI still ironing out the bugs for the Dingtalk sign-in

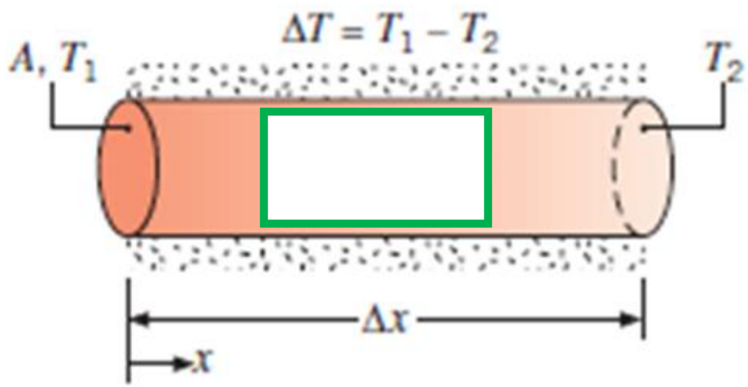
Content

1. Write down the main transport law in this lecture in 3D cartesian form. Define the variables.
2. What proportional constant relates the heat flux to the temperature gradient? What is its SI unit?
3. What is the expression for thermal conductivity from Kinetic theory? Define the variable used.
4. List two carriers that contribute to the thermal conductivity in solids.
5. Give a reason as to why the thermal conductivity of a bulk solid is different from a nanofilm of the same material. Which is smaller?
6. What is thermal diffusivity? What does a high thermal diffusivity tell you about the material?

Heat Conduction: Fourier Law

Fourier Law

Consider an insulated cylinder (no heat transfer along the sides),



At steady-state,

- If $T_1 > T_2$, q_x will flow from to
- By changing the cross-sectional area (A), temperatures at the end (T_1 and T_2), cylinder length (Δx), Fourier found his law:

$$q_x = -kA\frac{dT}{dx}$$

  - Note the negative sign as heat is transferred in the direction of decreasing temperature

- **Rate equation**
- **Phenomenological => from experimental observations**

Fourier Law

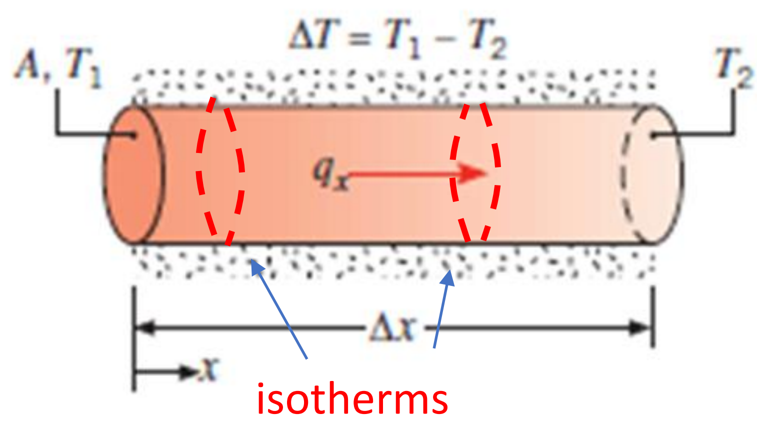
Most general (vector) form for multidimensional conduction is:

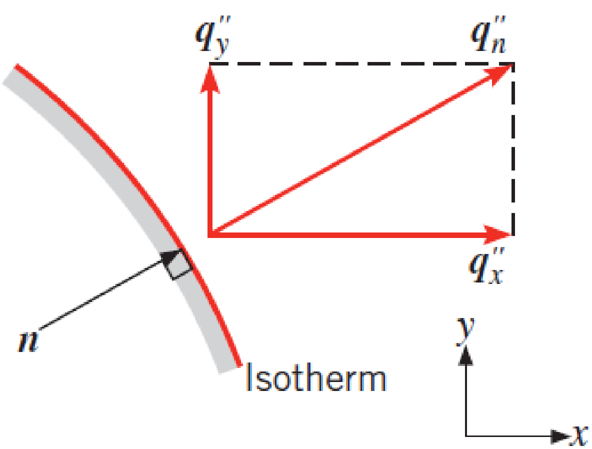
$$q = -kA\nabla T \quad \text{or} \quad q'' = -k\nabla T$$

Implications:

- Negative sign => Heat transfer is in the direction of decreasing temperature
- *k* is a proportional constant => thermal conductivity (units: W/mK)

Heat flows perpendicular to isotherms





Heat flux vector may be resolved into orthogonal components.

Heat Flux Components: Cartesian

For an **isotropic** material:

$T(x, y, z)$ $\qquad \vec{i}, \vec{j}, \vec{k}$ represent x, y, z

- Cartesian Coordinates:

$$\vec{q}'' = \underbrace{-k\frac{\partial T}{\partial x}\vec{i}}_{q''_x} - \underbrace{k\frac{\partial T}{\partial y}\vec{j}}_{q''_y} - \underbrace{k\frac{\partial T}{\partial z}\vec{k}}_{q''_z} \qquad (2.3)$$

For an **isotropic** material:

$T\left(r,\phi,z\right)$

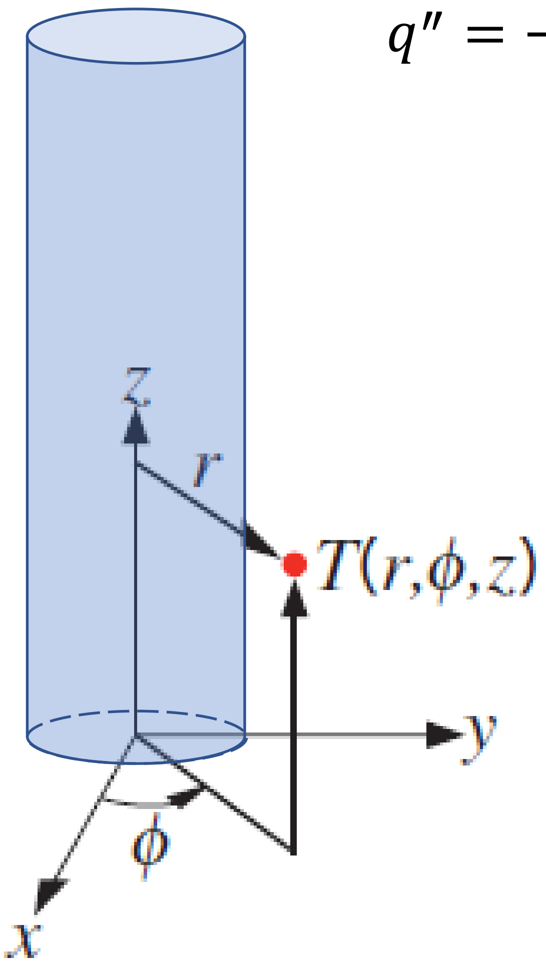
- Cylindrical Coordinates:

$$\vec{q}'' = \underbrace{-k\frac{\partial T}{\partial r}}_{q_r''}\vec{r} \underbrace{- k\frac{\partial T}{r\partial\phi}}_{q_\phi''}\vec{\phi} \underbrace{- k\frac{\partial T}{\partial z}}_{q_z''}\vec{z} \qquad (2.24)$$

$$q_r = A_r q_r'' = 2\pi r L q_r'' \qquad \text{[W]}$$

or

$$q_r' = A_r' q_r'' = 2\pi r q_r'' \qquad \text{[W/m]}$$



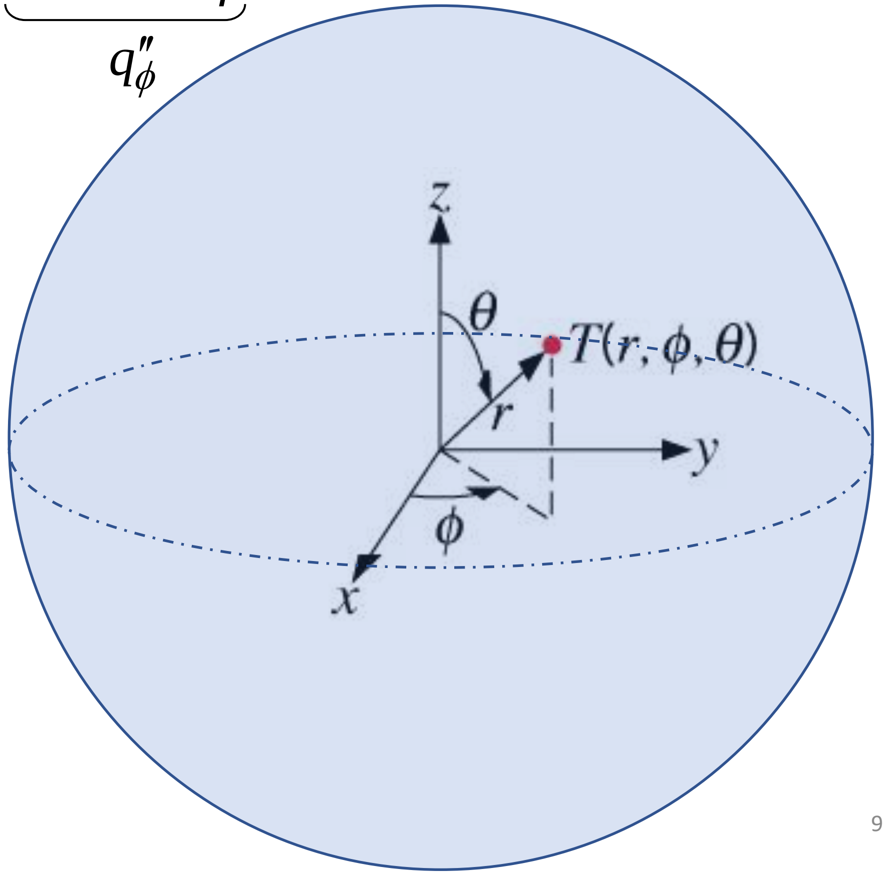
For an **isotropic** material:

$$T\left(r,\phi,\theta\right)$$

- Spherical Coordinates:

$$\overrightarrow{q''} = \underbrace{-k\frac{\partial T}{\partial r}\vec{r}}_{q''_r} - \underbrace{k\frac{\partial T}{r\partial \theta}\vec{\theta}}_{q''_\theta} - \underbrace{k\frac{\partial T}{r\sin\theta\,\partial\phi}\vec{\emptyset}}_{q''_\phi} \qquad (2.27)$$

$$q_r = A_r q''_r = 4\pi r^2 q''_r \qquad [\mathrm{W}]$$



Finer points

Note:

- In angular coordinates $(\phi \text{ or } \phi, \theta)$, the temperature gradient is still based on temperature change over a length scale and hence has units of °C/m and not °C/deg.

- For an anisotropic material, *k* is different for all three directions:

$$\vec{q}'' = -k_x \frac{\partial T}{\partial x} \vec{i} - k_y \frac{\partial T}{\partial y} \vec{j} - k_z \frac{\partial T}{\partial z} \vec{k}$$

Thermal conductivity (***k***)

Why study *k* for conduction?

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Thermal conductivity, ***k***

- transport property
- depends on atoms/molecules and structure of materials
- k_{solid} $\boxed{}$ k_{liquid} $\boxed{}$ k_{gas} (normally but there are many exceptions)

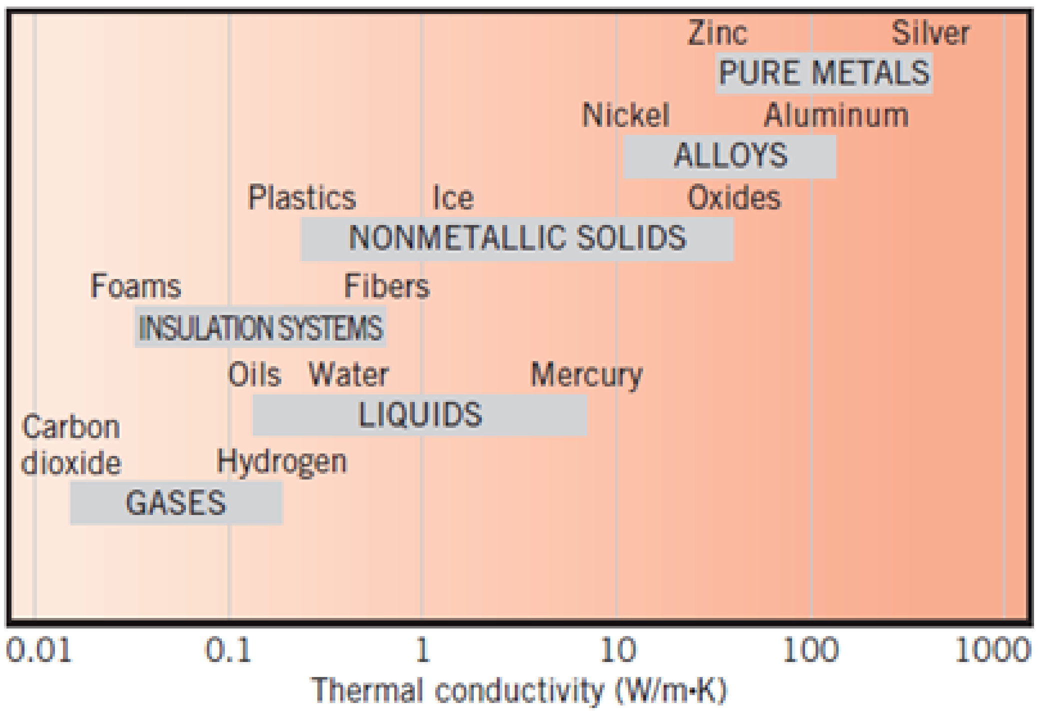


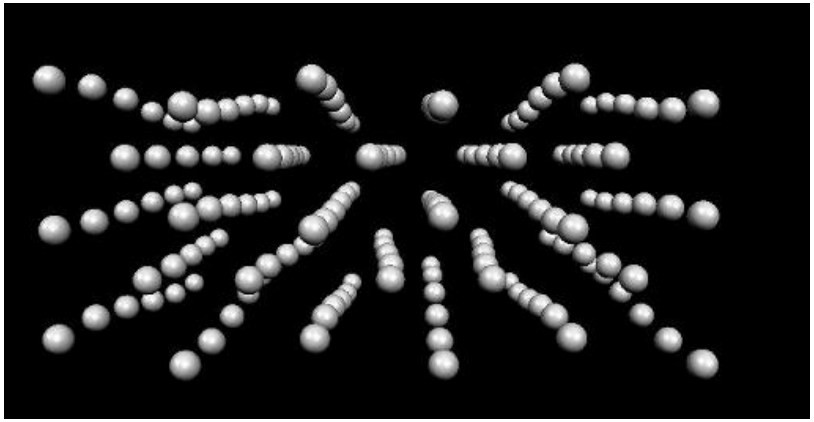
FIGURE 2.4 Range of thermal conductivity for various states of matter at normal temperatures and pressure.

Thermal conductivity of Solids

For **Solids**:

- Conduction:
  - energy carrier (electron or phonon or others) motion

- Phonon：
  - Quantum mechanic quasi-particle arising from periodic arrangement of atoms/molecules forming a lattice



Thermal conductivity of Solids

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For **Solids**:

- Its behavior is often described using ideal gas framework
- Kinetic theory gives:

$$k = \frac{1}{3} \underbrace{C}_{\text{energy carrier specific heat per unit volume.}} \underbrace{\overline{v}}_{\text{average energy carrier velocity, } \overline{v} < \infty.} \underbrace{\lambda_{\text{mfp}}}_{\text{mean free path} \rightarrow \text{average distance traveled by an energy carrier before a collision with something in solid.}} \qquad (2.7)$$

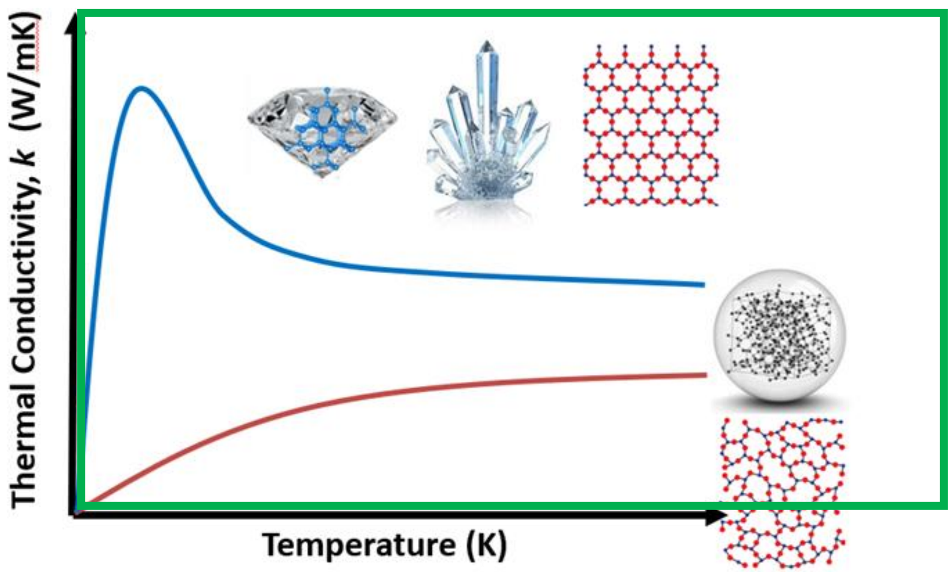
- Conductor: ***k*** **mostly by electrons (some are ions-based)**
- Non-conductor/dielectric crystals: ***k*** **are from phonons**
- Semi-conductor crystals: **mostly phonons**

k of a solid:

$$k = k_{ph} + k_e + \ k_{others} \qquad \text{(2.8 modified)}$$

- Normally, k_e **higher when electrical resistivity, ρ, is lower**
- **Metals: k_e $\boxed{}$ k_{ph}**
- **Other crystalline materials: k_{ph} dominates**
- **Amorphous materials: other carriers known as diffusons, propagons, and locons**
- Different temperature-dependent trends

Crystalline k versus Amorphous k



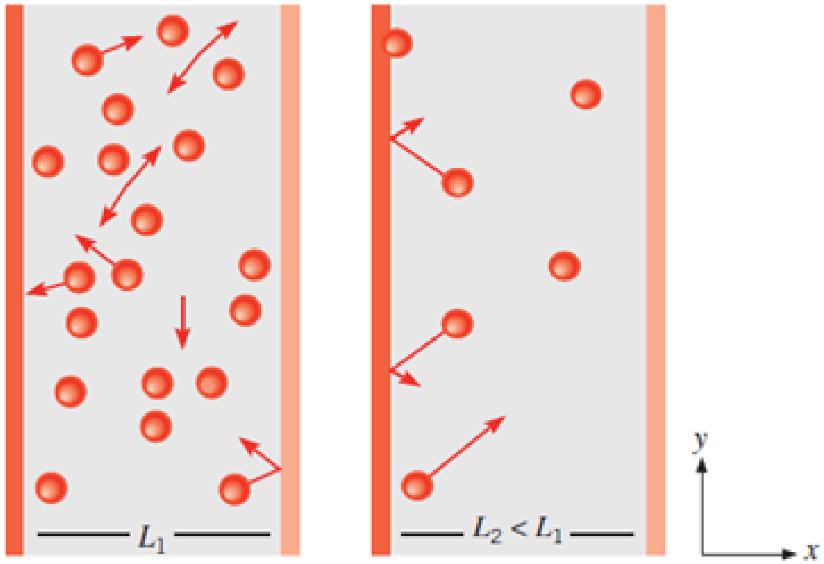
Nanoscale effect on Thermal conductivity (***k***)

Nanoscale Effects – size effects

Question:
Is the *k* of a block of gold same as a gold nanofilm?

- Collide with physical boundaries and **redirect** their propagation

From Kinetic Theory,
- **Scattering** reduces their mean free path, λ_{mfp} => smaller *k*



$k_x < k_y < k$ when the size effect is significant. *k* is the bulk thermal conductivity.

FIGURE 2.6 Electron or phonon trajectories in (*a*) a relatively thick film and (*b*) a relatively thin film with boundary effects.

Nanoscale Effects – More collision targets

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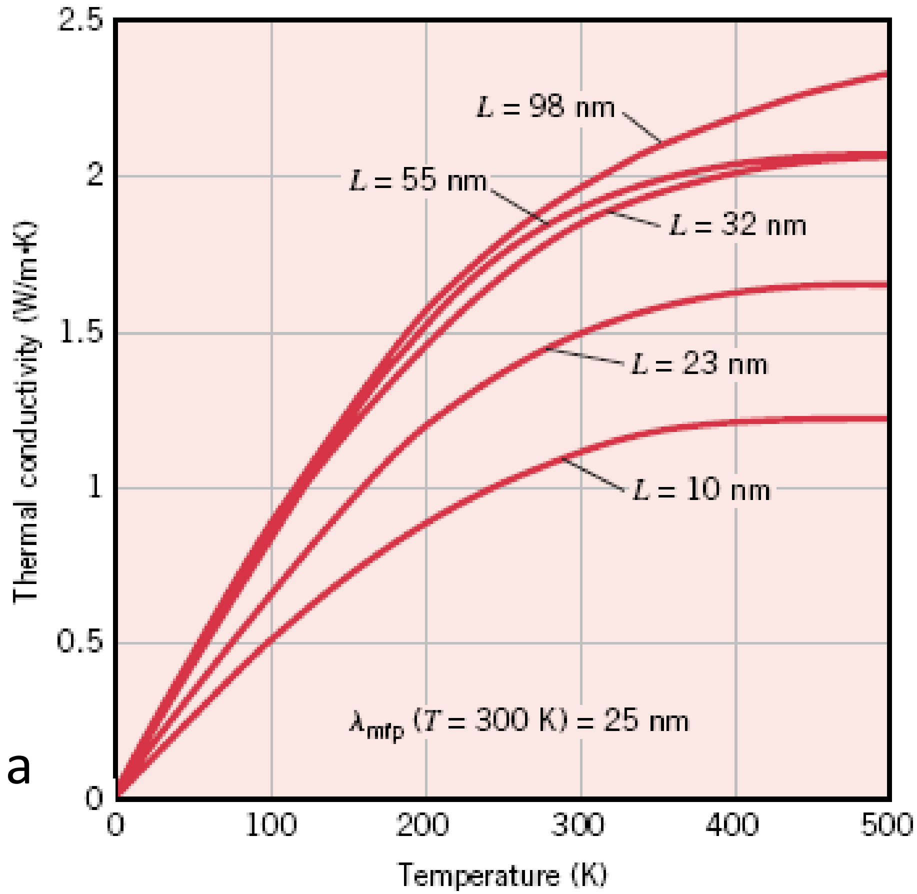
Other than the physical boundaries, energy carriers can collide with

- Grain boundaries in a solid
- Dopants, especially with they are in high concentrations
- Electrons with electrons
- Electrons with phonons
- Phonons with phonons



Measured thermal conductivity of a ceramic material vs. grain size, *L*

Is Fourier Law always true?

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Fourier's law fails in some situations!

- Ballistic Transport – no scattering during transport
- High temperature difference

A carrier with a mean free path of 1 m loses all its energy after moving 1m. Consider:

- What will be the surrounding temperature after 1 meter?
- What will be the surrounding temperature after 0.1 meter?

Thermal conductivity of Fluids

Fluid => Gas and Liquid

- Longer interatomic/intermolecular separation => more random motion => **lower *k***

- Kinetic theory can explain effects with different pressure/temperature
- Equation similar to (2.7)

$$k = \frac{1}{3}\rho c_v \overline{v} \lambda_{\mathrm{mfp}} \qquad (2.8)$$

Here, ρc_v = density * volumetric heat capacity at constant volume

- Temperature can change the ***k*** of fluids. How?

Thermal conductivity of Fluids

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Question:

***k* for gas is normally independent of pressure! Why?**

- Clue: equation 2.8

At a lower pressure,

- λ_{mfp} longer
- ρ smaller
- c_v relative unchanged*

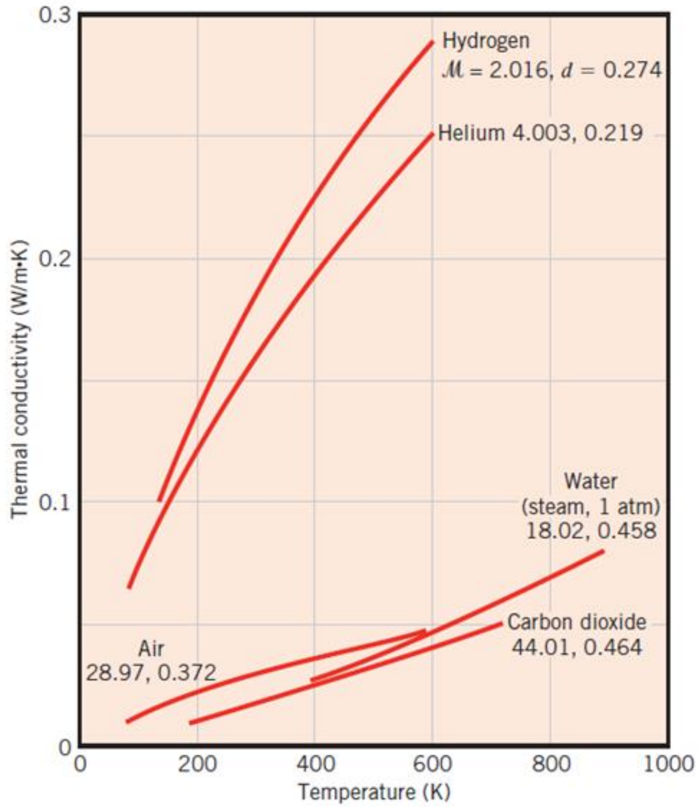
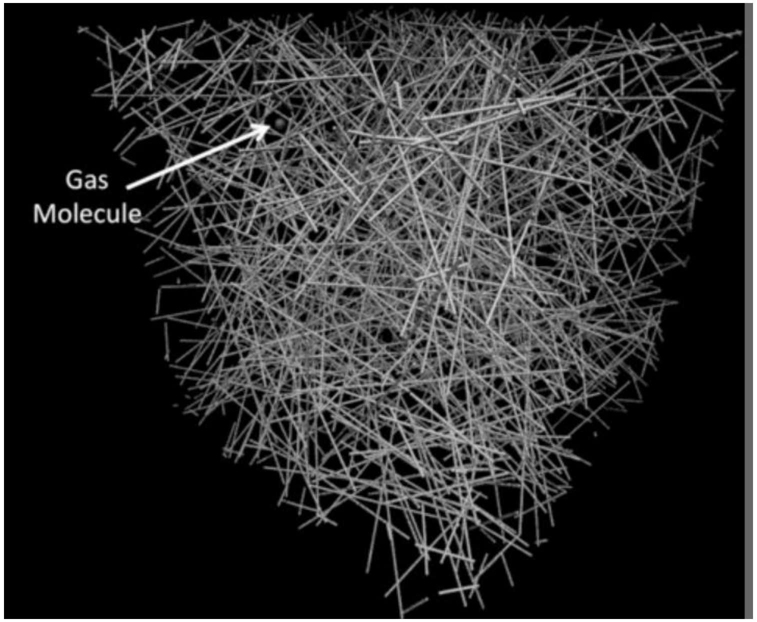


FIGURE 2.8 The temperature dependence of the thermal conductivity of selected gases at normal pressures. Molecular diameters (d) are in nm [10]. Molecular weights ($\mathcal{M}$) of the gases are also shown.

Example: Carbon Nanotube (CNT) aerogel (ρ ~ 8kg/m³)

At room temperature and pressure, experiments found:

- CNT k > 1000 W/mK for long tube about a few microns
- Air k ~ 0.03 W/mK
- CNT aerogel **effective *k*** ~ 0.025 W/mK
- ***Effective k*** increases with increasing pressure



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Transport vs Thermodynamics properties

- Transport – non-equilibrium in nature with carriers moving
  - Thermal conductivity
  - Diffusion
  - Kinematic viscosity

- Thermodynamics – equilibrium in nature
  - Volumetric heat capacity
  - Density

Another often used HT property is the Thermal Diffusivity

- $\alpha = \frac{k}{\rho c_p}$
- Material's ability to conduct heat relative to storing
- Large α will respond quickly to changes in thermal environment

Property Tables in text:

Solids:	Tables A.1 – A.3
Gases:	Table A.4
Liquids:	Tables A.5 – A.7

Example 1

EXAMPLE 2.1

The thermal diffusivity α is the controlling transport property for transient conduction. Using appropriate values of k, ρ, and c_p from Appendix A, calculate α for the following materials at the prescribed temperatures: pure aluminum, 300 and 700 K; silicon carbide, 1000 K; paraffin, 300 K.

Summary

- Fourier Law
  - 3D representations in different coordinate systems (more next lecture)
- Thermal conductivity
  - Solids
  - Fluids
  - Kinetic Theory Expression of *k*
  - Mixtures
- Nanoscale effects
- Thermal Diffusivity