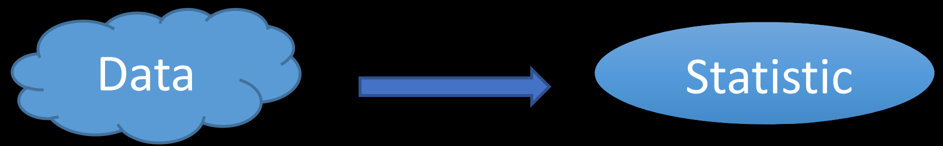
1.1

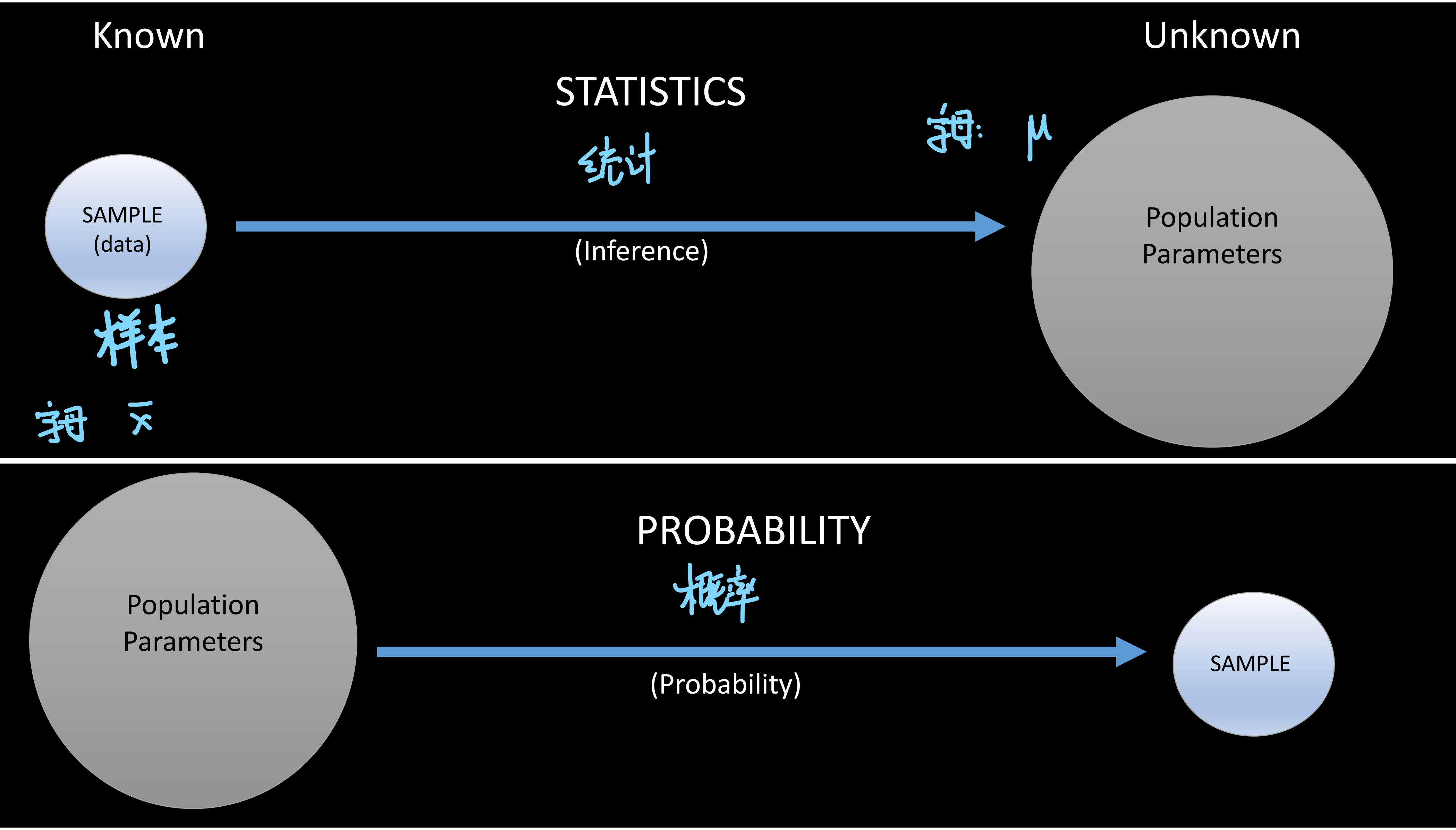
Properties of Probability
Probability and Set Notation
Infinite Series



What is Statistics?

What is a statistic? A function of **data**

Statistics: study of the collection, analysis, interpretation, presentation, and organization of **data.**


Known
Unknown
STATISTICS
统计
SAMPLE
(data)
(Inference)
Population
Parameters
样本
μ
PROBABILITY
概率
Population
Parameters
(Probability)
SAMPLE

What is Probability?

Probability is a real-valued set function P that assigns, to each event A in the sample space S, a number $P(A)$, called the probability of the event A, such that the following properties are satisfied: $P \to$ Probability

性质

(a) $P(A) \geq 0$; ① 不能为负数

(b) $P(S) = 1$; ② $P(A_1) + P(A_2) + \cdots P(A_n) = P(S) = 1$

$\hookrightarrow$ sample space $\to$

(c) if $A_1, A_2, A_3, \ldots$ are events and $A_i \cap A_j = \emptyset$, $i \neq j$, then

$$P(A_1 \cup A_2 \cup \cdots \cup A_k) = P(A_1) + P(A_2) + \cdots + P(A_k)$$

for each positive integer k, and

$$P(A_1 \cup A_2 \cup A_3 \cup \cdots) = P(A_1) + P(A_2) + P(A_3) + \cdots$$

for an infinite, but countable, number of events.

What is Probability?

sample　　　　　　　　　　real number

$S \rightarrow \boxed{P} \rightarrow R \in [0,1]$

结果一定为[0,1]的实数

Real valued function:

A function that assigns a real number to each member of its domain.

Set function:

A function whose domain is a family of subsets of some given set.

1.1 Properties of Probability

set A = 奇数 = $\{1,3,5\}$

$S = \{1,2,3,4,5,6\}$

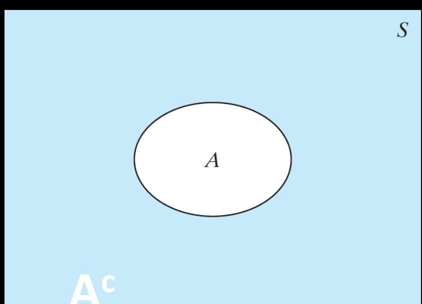
interval
( ), [ ]

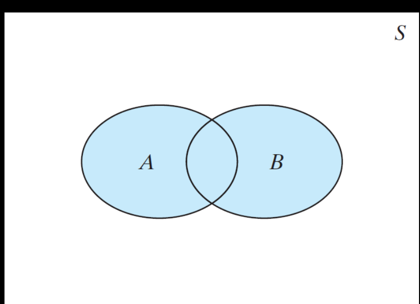
Random Experiments 随机测试

In Statistics, we consider experiments where the outcome can not be predicted with certainty.

- **Outcome space** or **Sample space**, *S* – collection of all possible outcomes 结果的全集
- An **Event** is a collection of outcomes in *S*.
- If a random experiment is performed and the outcome of the experiment is in A, we say **event** *A* **has occurred.**

Set notation and operations





	Notation	**Meaning**	
空集	$\emptyset$, {}	**Null** or empty set	
属于	$x \in A$	x is an element of A	
并集	$A \cup B$	the union of A and B	or
交集	$A \cap B$	the intersection of A and B	and
子集	$A \subseteq B$	A is a subset of B	
真子集	$A \subset B$	A is a proper subset of B	$A \neq B$
补集	A', A^c	the complement of A	

6 sided die

- S = {1, 2, 3, 4, 5, 6}

- P[S] = 1

Theorem 1.1-1 For each event A,

$$P(A) = 1 - P(A').$$

证明

Proof [See Figure 1.1-1(a).] We have

$$S = A \cup A' \qquad \text{and} \qquad A \cap A' = \emptyset.$$

Thus, from properties (b) and (c), it follows that

$$1 = P(A) + P(A').$$

Hence

$$P(A) = 1 - P(A'). \qquad \square$$

定理

Probability Theorems

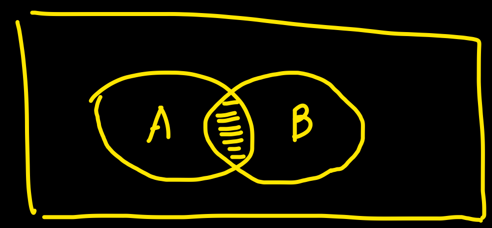
- **Theorem 1**
  - P[A’] = 1 – P[A]
- **Theorem 2**
  - P[Ø]=0
- **Theorem3**
  - If $A \subset B$, then $P[A] \leq P[B]$.

Probability Theorems

- **Theorem 4**
  - For any event A, $P[A] \leq 1$
- **Theorem 5**
  - If A and B are any two events, then

    $P[A \cup B] = P[A] + P[B] - P[A \cap B]$
- **Theorem 6**
  - $P[A \cup B \cup C] = P[A] + P[B] + P[C] - P[A \cap B] - P[A \cap C] - P[B \cap C] + P[A \cap B \cap C]$

Von map



独立事件 $A \cap B = \emptyset$?

1. Suppose a 6-sided die is rolled:

$\{1, 2, 3, 4, 5, 6\}$

- **Let event A = {The outcome is even}** 偶数 $A = \{2, 4, 6\}$
- **Let event B = {The outcome is greater than 3}** $B = \{4, 5, 6\}$
  - a) What are the outcomes in [A ∩ B]?

    ii) *What is P[A ∩ B]?*

$\because A \cap B = \{4, 6\}$

$\therefore P(A \cap B) = \frac{2}{6} = \frac{1}{3}$

1. Suppose a 6-sided die is rolled:

- **Let event A = {The outcome is even}**
- **Let event B = {The outcome is greater than 3}**
  - b) What are the outcomes in [A U B]? $\because A \cup B = \{2, 4, 5, 6\}$

    ii) *What is P[A U B]?*

$$\therefore P(A \cup B) = \frac{4}{6} = \frac{2}{3}$$

1... Suppose the die is loaded so that the probability of an outcome is proportional to the outcome, i.e.

P[1] = *p*, P[2] = *2p*, P[3] = *3p*, P[4] = *4p*, P[5] = *5p*, P[6] = *6p*,

$21p = p + 2p + 3p + 4p + 5p + 6p = 1$ $\quad 1 \;\; 21 \;\; 6 \quad \Rightarrow p = \frac{1}{21}$

- c) Find the value of *p* that would make this a valid probability model

- d) Find the following probabilities:
  - i) P[A], ii) P[A'], iii) P[A U B]

$= P(2)+P(4)+P(6)$ $\qquad = 1 - P(A)$ $\qquad = P(A)+P(B)-P(A\cap B)$

Let event A = {The outcome is even}

Let event B = {The outcome is greater than 3}

2. The probability that a randomly selected student at Zoom University owns a bicycle is 0.55, the probability that a student owns a car is 0.30, and the probability that a student owns both is 0.10.

- A) What is the probability that a student selected at random does not own a bicycle?
- B) What is the probability that a selected student at random owns either a car or a bicycle (or both)?

设A为骑车

B为开车

$P(A) = 0.55$

$P(B) = 0.3$

$P(A \cap B) = 0.1$

a) $P(A') = 1 - P(A) = 0.45$

b) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.75$

2…

The probability that a randomly selected student at Zoom University owns a bicycle is 0.55, the probability that a student owns a car is 0.30, and the probability that a student owns both is 0.10.

- C) What is the probability that a student selected at random neither has a car nor a bicycle?

$P(A' \cap B') = P(A \cup B)' = 1 - 0.75 = 0.25$

3. Let a > 2. Suppose S = {0, 1, 2, 3, …. } and

$P[0] = c, \quad P[k] = \frac{1}{a^k}, k = 1, 2, 3\ldots$

A) Find the value of c that will make this a valid probability distribution.
B) Find the probability of an odd outcome

a).

$$P[\bar{0}] = \frac{1}{a^1} + \frac{1}{a^2} + \frac{1}{a^3} + \cdots \frac{1}{a^n} \cdots = \frac{\frac{1}{a_1(1-q^n)}}{1-\frac{1}{a}} = \frac{\frac{1}{a}}{1-\frac{1}{a}} = \frac{1}{a-1}$$

$$P[0] = 1 - \frac{1}{a-1} = c$$

$$P = a_1 \cdot q^0 + a_1 q^1 + \cdots a_n \cdot q^n$$

$$P \cdot q = \quad a_1 q^1 + \cdots a_n \cdot q^n + a_1 \cdot q^{n+1}$$

b).

$$\frac{1}{a^1} + \frac{1}{a^3} + \cdots \frac{1}{a^{2n-1}} \cdots$$

$$= a\left(\left(\frac{1}{a^2}\right)^1 + \left(\frac{1}{a^2}\right)^2 + \cdots \left(\frac{1}{a^2}\right)^n \cdots\right)$$

$$= \frac{\frac{1}{a}}{1-\frac{1}{a^2}} = \frac{a}{a^2-1}$$

4. Suppose S = {0, 1, 2, 3, …. }, P[0] = p, and P[k] = $\frac{1}{2^k k!}$, k = 1, 2, 3…

Find the value of p that will make this a valid probability distribution.

$$e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$$

5

1.1-16. Let p_n, $n = 0, 1, 2, \ldots$, be the probability that an automobile policyholder will file for n claims in a five-year period. The actuary involved makes the assumption that $p_{n+1} = (1/4)p_n$. What is the probability that the holder will file two or more claims during this period?

- R resource http://www.peterhaschke.com/files/IntroToR.pdf