Lec05: One-Dimensional, Steady-State Conduction with Heat Generation

Chapter Three

Section 3.5, Appendix C

Announcement

- HW 02 due Today
- Lab 02 on Thurs

TABLE 3.3 One-dimensional, steady-state solutions to the heat equation with no generation

	Plane Wall	Cylindrical Wall[a]	Spherical Wall[a]
Heat equation	$\frac{d^2T}{dx^2} = 0$	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$	$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$
Temperature distribution	$T_{s,1} - \Delta T\frac{x}{L}$	$T_{s,2} + \Delta T\frac{\ln(r/r_2)}{\ln(r_1/r_2)}$	$T_{s,1} - \Delta T\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right]$
Heat flux (q'')	$k\frac{\Delta T}{L}$	$\frac{k\,\Delta T}{r\ln(r_2/r_1)}$	$\frac{k\,\Delta T}{r^2[(1/r_1)-(1/r_2)]}$
Heat rate (q)	$kA\frac{\Delta T}{L}$	$\frac{2\pi Lk\,\Delta T}{\ln(r_2/r_1)}$	$\frac{4\pi k\,\Delta T}{(1/r_1)-(1/r_2)}$
Thermal resistance ($R_{t,\mathrm{cond}}$)	$\frac{L}{kA}$	$\frac{\ln(r_2/r_1)}{2\pi Lk}$	$\frac{(1/r_1)-(1/r_2)}{4\pi k}$

[a]The critical radius of insulation is $r_{cr} = k/h$ for the cylinder and $r_{cr} = 2k/h$ for the sphere.

1. What happens to the law of energy conservation when there is heat generation?
2. Name a few situations when heat generation will be non-zero in our heat equation?
3. How does the heat flux vary with *x* for a wall with internal heat generation?
4. What is a common boundary condition at the center of a solid radial system with heat generation? Does this boundary condition apply for a radial wall system?

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$$\dot{E}_g \neq 0$$

- Local (volumetric) source of thermal energy converted from another form of energy in the control volume.

Common sources:

- Can be uniformly distributed or non-uniform,
  a. Radioactive decay
  b. Electrical (Ohmic) heating:

$$\dot{q} = \frac{\dot{E}_g}{V} = \frac{I^2 R_e}{V} \qquad (3.43)$$

  c. absorption of radiation passing through a semi-transparent medium.

For a plane wall, $\dot{q} \propto e^{-\alpha x}$

  d. Chemical reaction (exothermic vs endothermic)

Heat generation in a medium affects

- **Temperature distribution**
- **Heat rate to change with location**
- **Cannot simply approximate as a thermal resistor**

Plane Wall

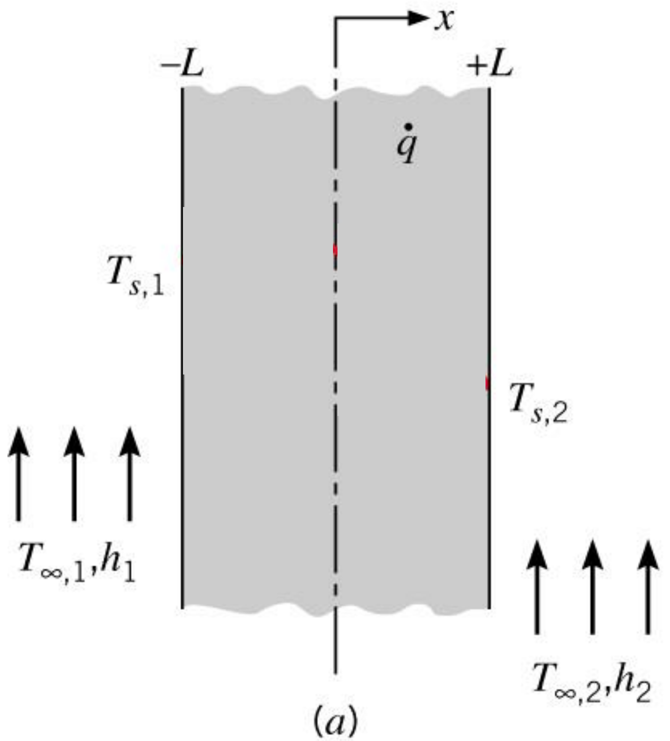
with Heat Generation

The Plane Wall

Consider one-dimensional, steady-state conduction in a plane wall of constant k, uniform generation , and asymmetric surface conditions:

- Heat Equation:

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$



$$\frac{d}{dx}\left(k\frac{dT}{dx}\right) + \dot{q} = 0 \qquad (3.44)$$

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

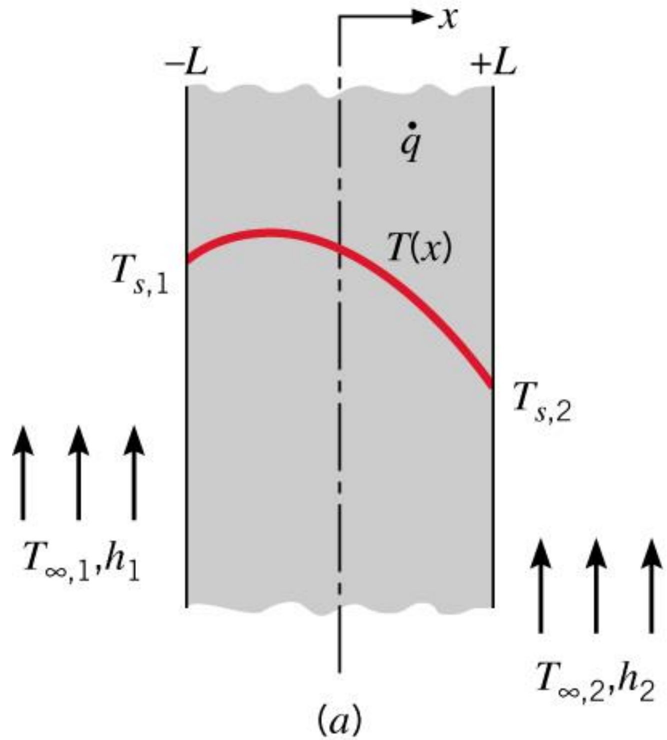
Is the heat flux q'' independent of x?

General Solution:

$$T(x) = -(\dot{q}/2k)x^2 + C_1 x + C_2 \quad (3.45)$$

What is the shape of the temperature distribution for

$\dot{q} = 0?$ $\dot{q} > 0?$ $\dot{q} < 0?$



How does the temperature distribution change with increasing $\dot{q}$?

For BCs:

- $x = -L$, $T = T_{s,1}$
- $x = L$, $T = T_{s,2}$

$$T(x) = \left(\frac{\dot{q}}{2k}L^2\right)\left(1 - \frac{x^2}{L^2}\right) + \left(\frac{T_{s,2} - T_{s,1}}{2}\right)\left(\frac{x}{L}\right) + \left(\frac{T_{s,1} + T_{s,2}}{2}\right) \quad (3.46)$$

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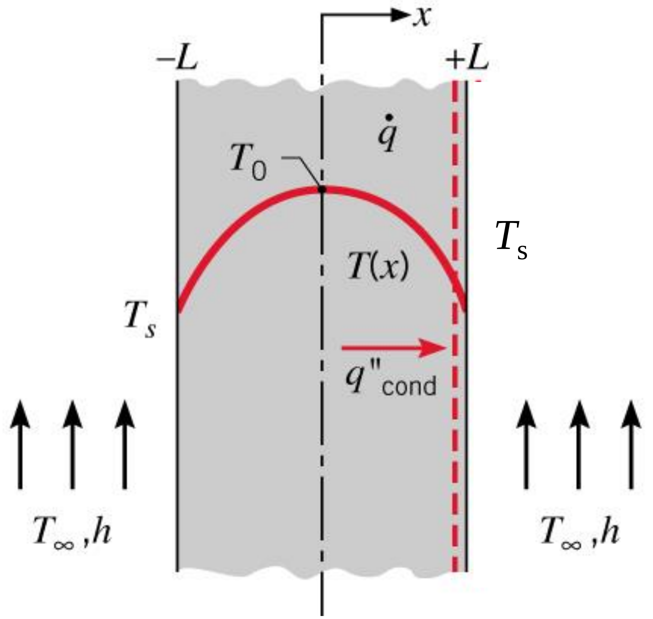
Special Case 1: Symmetric T_s with $T_s < T_0$



1. What is the temperature gradient at the centerline of the wall?

2. Why does the magnitude of the temperature gradient increase with increasing *x*?

3. Temperature Distribution (using 3.45):

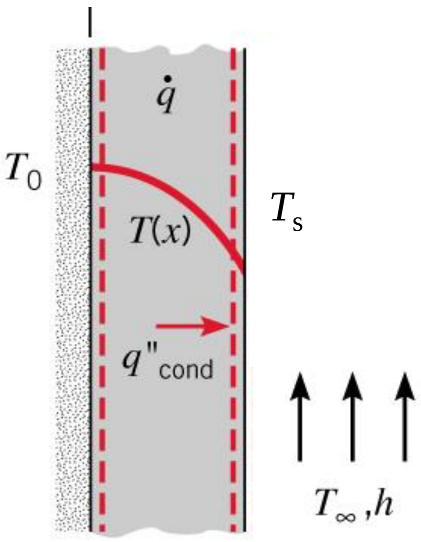
$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_s \qquad (3.47)$$

Plane Wall (One insulated surface)

Special Case 2: One insulated surface,
$dT/dx|_{x=0} = 0$

1. Same as Symmetric T_s, $dT/dx\,|_{\mathbf{x=0}} = 0$
2. The temperature profile will be ?

Same as the Symmetric T_s case!



- How do we determine ?

Overall energy balance on the wall surface,

$$\dot{E}_{in} - \dot{E}_{\text{out}} = 0$$

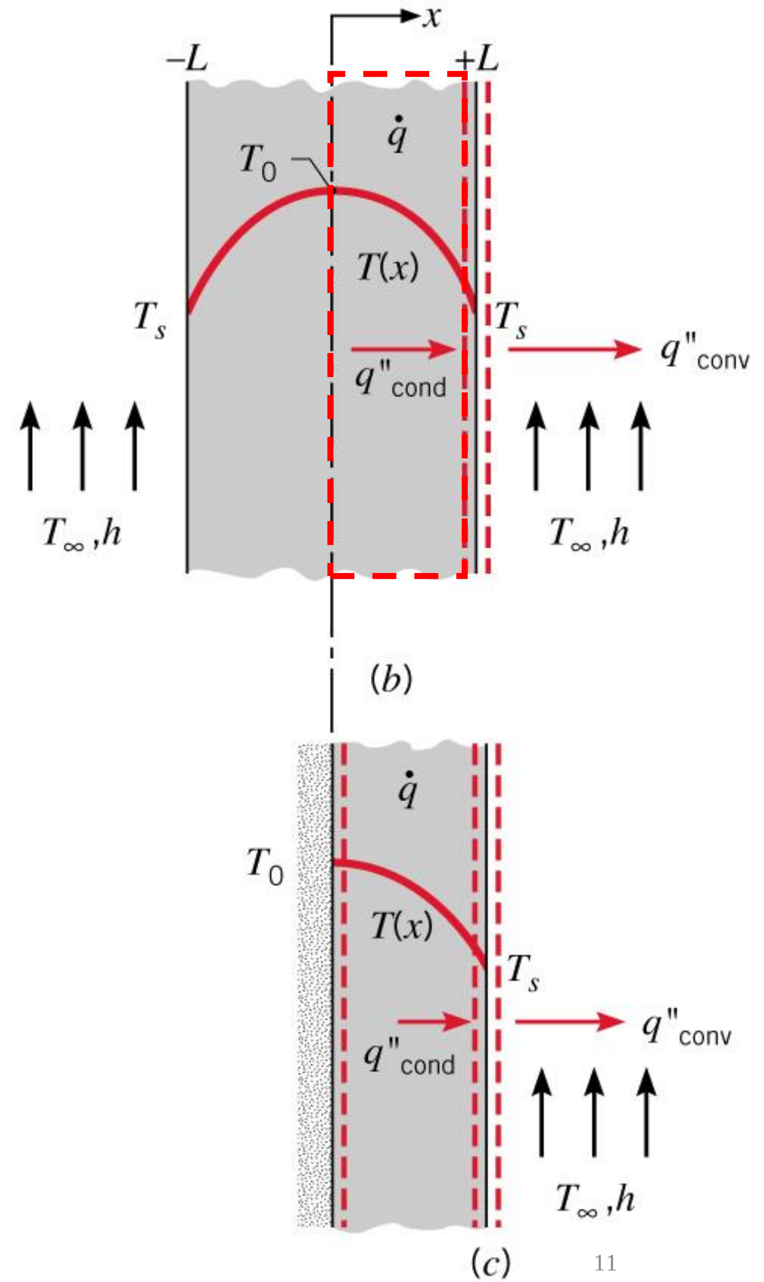
- What is $\dot{E}_{in}$?

$$\dot{E}_{in} = q''_{cond} = \dot{E}_g \ (why?)$$

So,

$$\dot{q}\, A_s L - hA_s(T_s - T_\infty) = 0$$

$$T_s = T_\infty + \frac{\dot{q}\, L}{h} \qquad (3.51)$$



Example 1

A plane wall is a composite of two materials, A and B. The wall of material A has uniform heat generation $\dot{q} = 1.5 \times 10^6\ \mathrm{W/m^3}$, $k_A = 75\ \mathrm{W/m \cdot K}$, and thickness $L_A = 50$ mm. The wall material B has no generation with $k_B = 150\ \mathrm{W/m \cdot K}$ and thickness $L_B = 20$ mm. The inner surface of material A is well insulated, while the outer surface of material B is cooled by a water stream with $T_\infty = 30°\mathrm{C}$ and $h = 1000\ \mathrm{W/m^2 \cdot K}$.

1. Sketch the temperature distribution that exists in the composite under steady-state conditions.
2. Determine the temperature T_0 of the insulated surface and the temperature T_2 of the cooled surface.

Example 1

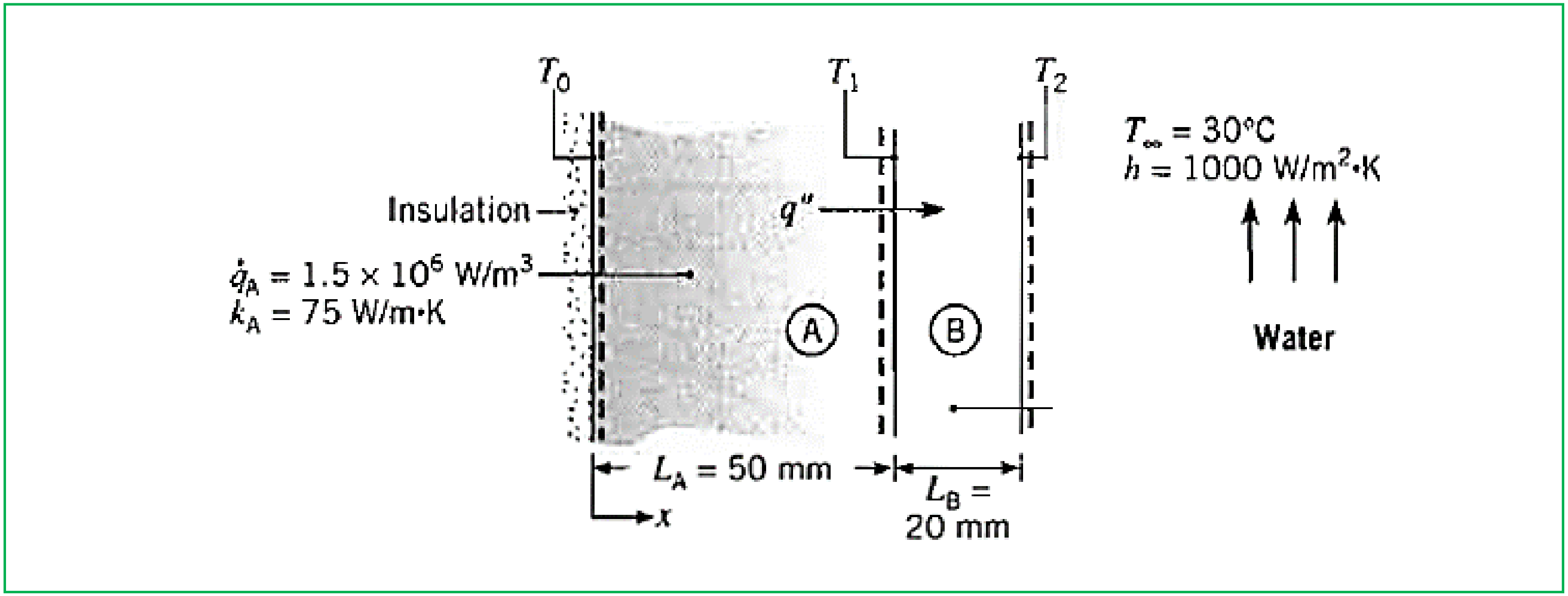
Known: Plane wall of material A with internal heat generation is insulated on one side and bounded by a second wall of material B, which is without heat generation and is subjected to convection cooling.

Find:

1. Sketch of steady-state temperature distribution in the composite.
2. Inner and outer surface temperatures of the composite.

Example 1

Schematic:

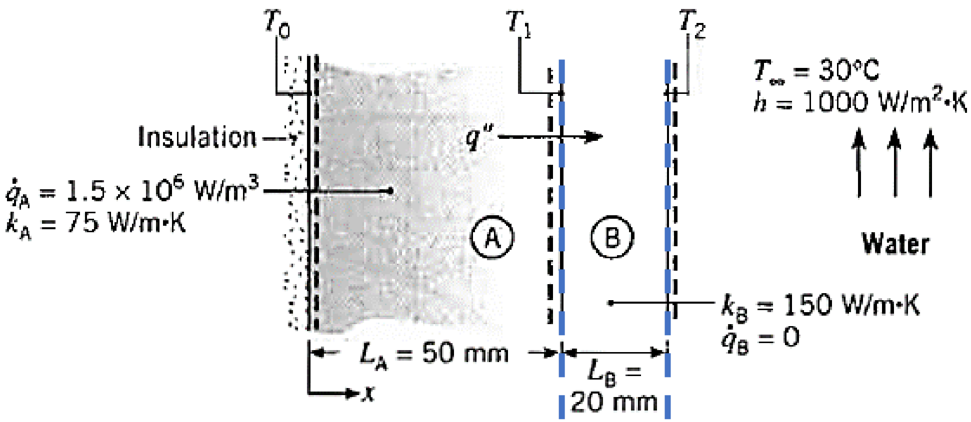


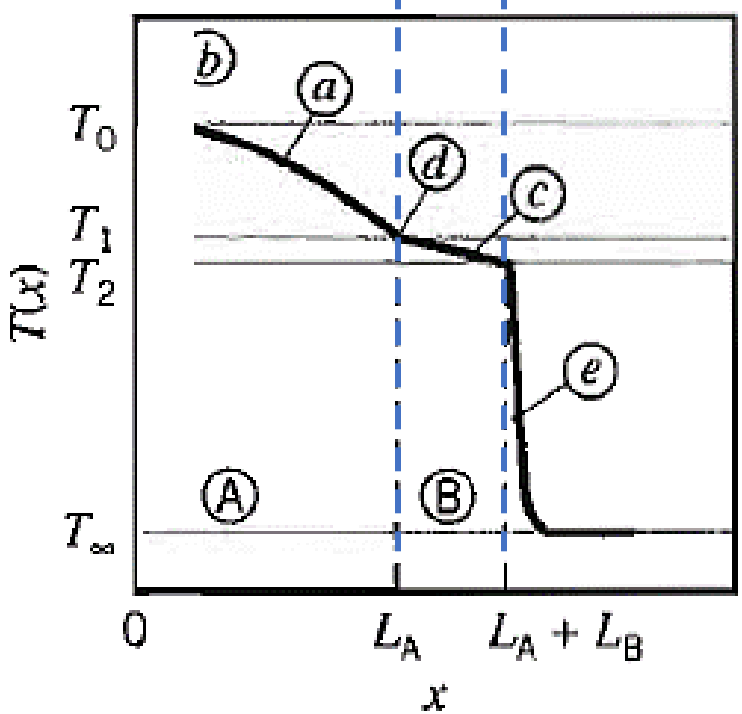
Assumptions:

1. Steady-state conditions.
2. One-dimensional conduction in x-direction.
3. Negligible contact resistance between walls.
4. Inner surface of A adiabatic.
5. Constant properties for materials A and B.

Example 1

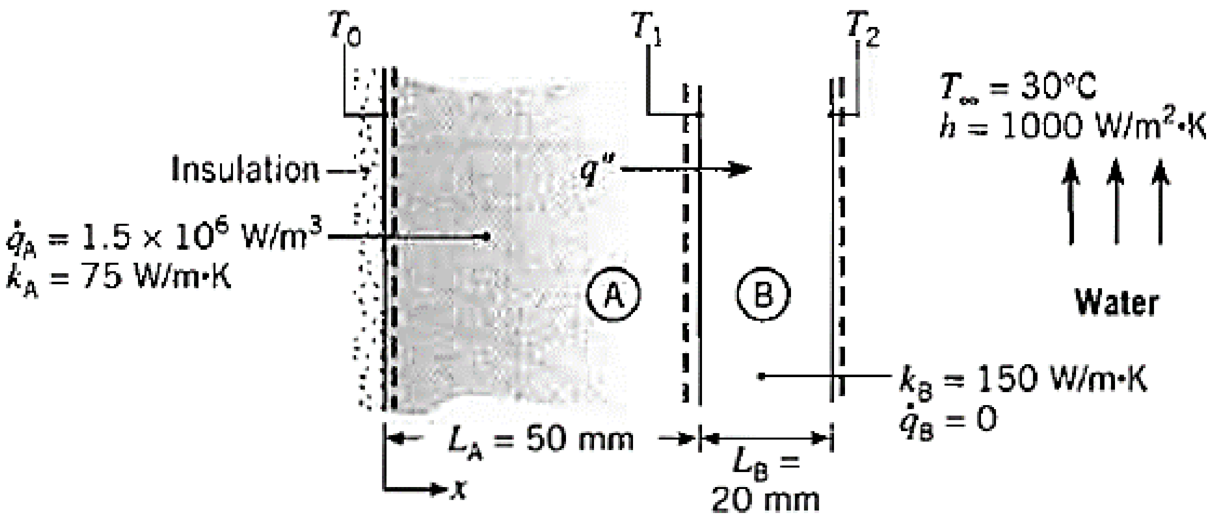
1. Sketch the temperature distribution that exists in the composite under steady-state conditions.





Example 1

2. Determine the temperature T_0 of the insulated surface and the temperature T_2 of the cooled surface.



T_2?

- Use energy balance at T_2 surface,

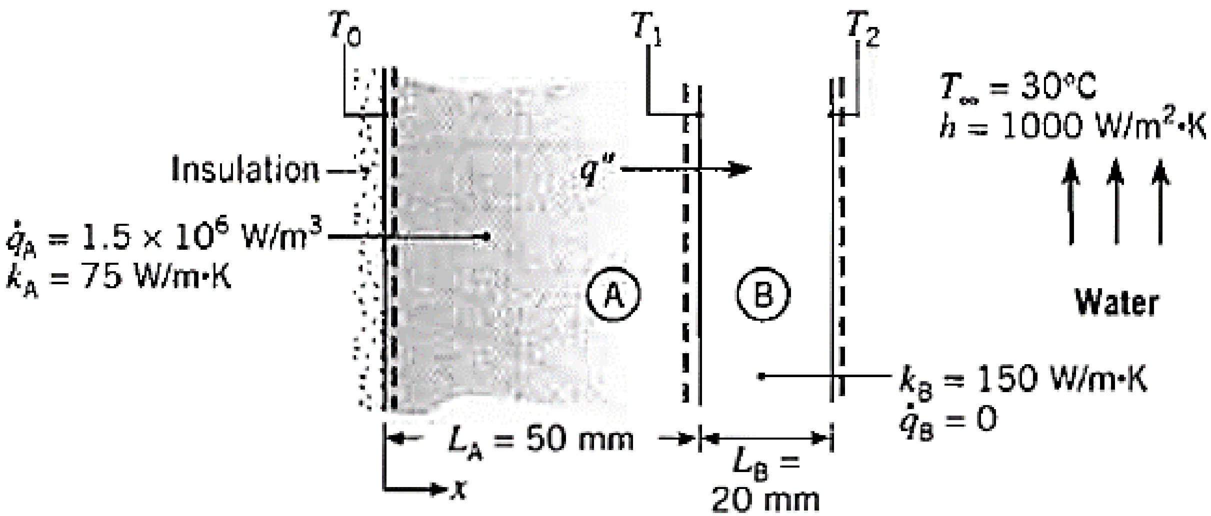
$$q_B'' = h(T_2 - T_\infty)$$

- But q_B" comes from A which is created from heat generation only,
- Use energy balance at T_1 surface,

$$\dot{q}'' L_A = q_B''$$

Example 1

2. Determine the temperature T_0 of the insulated surface and the temperature T_2 of the cooled surface.



T_0?

- Solve the heat equation to derive for wall A: $$T(0) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{0^2}{L^2}\right) + T_1 \qquad (3.47)$$

- To find T_1, energy balance at T_2 surface,

$$-k\frac{T_1 - T_2}{L_B} = h(T_2 - T_\infty)$$

What will happen to the temperature profile if there is a contact resistance between wall A and B?

Radial Systems

Heat Equations with Heat Generation

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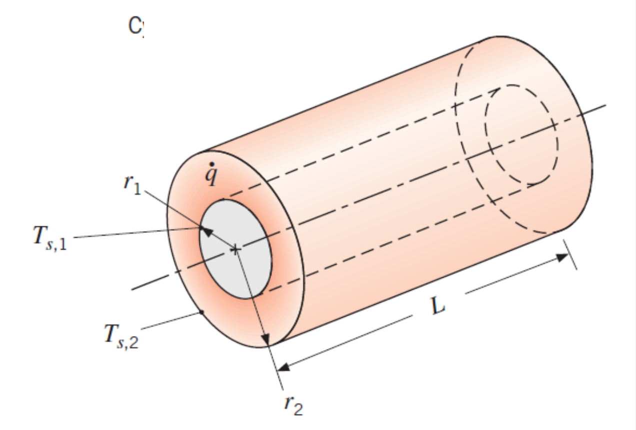
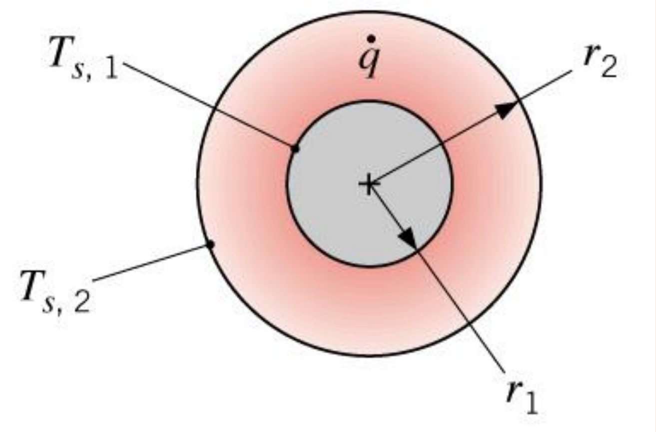
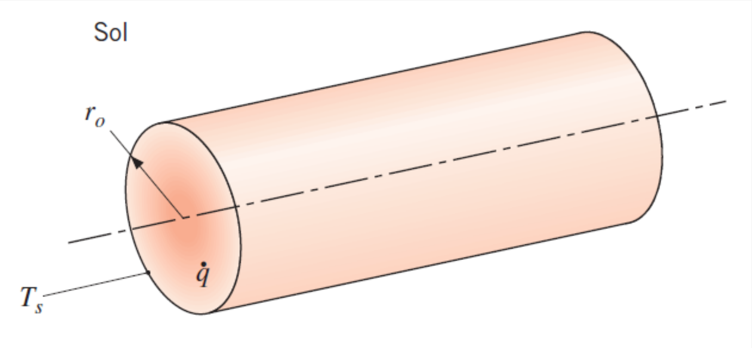
Cylindrical Tube Wall (hollow core)



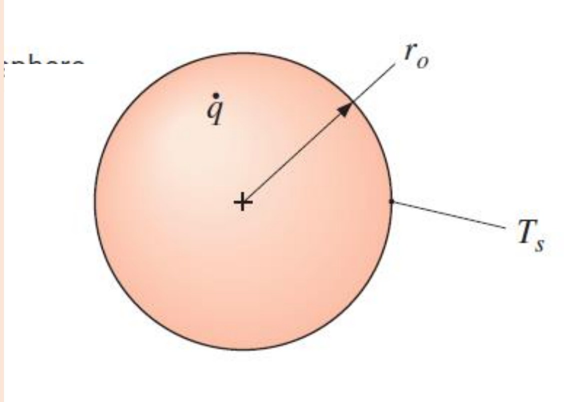
Spherical Wall (Shell)



Solid Cylinder (Circular Rod)



Solid Sphere



Heat Equations:

Cylindrical

$$\boxed{\frac{1}{r}\frac{d}{dr}\left(kr\frac{dT}{dr}\right)+\dot{q}=0}$$

Spherical

$$\boxed{\frac{1}{r^2}\frac{d}{dr}\left(kr^2\frac{dT}{dr}\right)+\dot{q}=0}$$

Solid Sphere: I. Constant Surf Temp

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Solution for Uniform Generation in a Solid Sphere of Constant k with constant surface temperature, T_s:

1. **Heat Equation**

$$\frac{1}{r^2}\frac{d}{dr}\left(kr^2\frac{dT}{dr}\right)+\dot{q}=0$$

2. **BCs:**

I. $r = 0, \quad dT/dr = 0$

II. $r = r_o, \quad T = T_s$

3. **Solving the Heat Equation,**

$$kr^2\frac{dT}{dr}=-\frac{\dot{q}r^3}{3}+C_1$$

$$T=-\frac{\dot{q}r^2}{6k}-\frac{C_1}{r}+C_2$$

Given ,

I. $\frac{dT}{dr}\Big|_{r=0}=0\rightarrow C_1=0$

II. $T(r_o)=T_s\rightarrow C_2=T_s+\frac{\dot{q}r_o{}^2}{6k}$

$$T(r)=\frac{\dot{q}r_o{}^2}{6k}\left(1-\frac{r^2}{r_o{}^2}\right)+T_s$$

Solid Sphere: II. Convection cooling at surface

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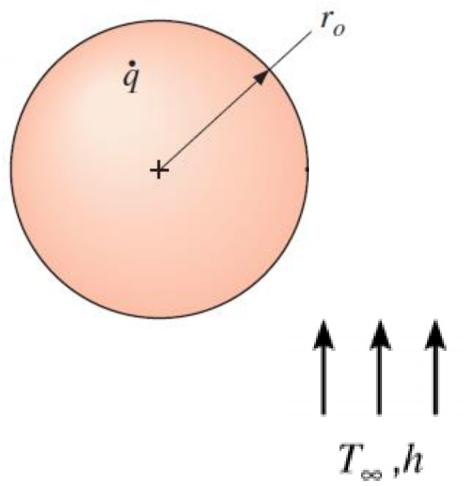
Instead of constant T_s at the surface, we have Convection Cooling:

$$T(r_o) = ? \rightarrow C_2 = ?$$



To find surface temperature, T_s

Overall energy balance:

$$\boxed{-\dot{E}_{\text{out}} + \dot{E}_g = 0} \rightarrow h(4\pi {r_o}^2)(T_s - T_\infty) = \dot{q}\left(\frac{4}{3}\pi {r_o}^3\right)$$

$$\rightarrow T_s = T_\infty + \frac{\dot{q}\, r_o}{3h}$$

Or from a surface energy balance (all E_{in} comes from E_g):

$$\boxed{\begin{array}{l} \dot{E}_{\text{in}} - \dot{E}_{\text{out}} = 0 \\ \rightarrow q_{\text{cond}}(r_o) = q_{\text{conv}} \quad \rightarrow T_s = T_\infty + \dfrac{\dot{q}\, r_o}{3h} \end{array}}$$

A summary of temperature distributions is provided in Appendix C for plane, cylindrical and spherical walls, as well as for solid cylinders and spheres. Note how different boundary conditions give rise to different $T(r)$, $q''(r)$, and $q(r)$. **You will be expected to derive them!**

Appendix C for quick reference

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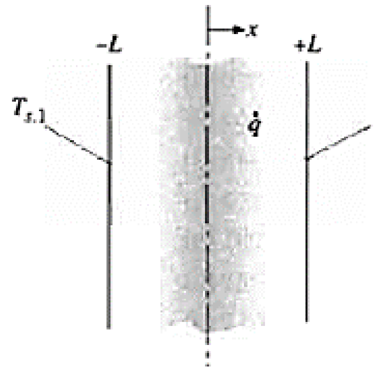
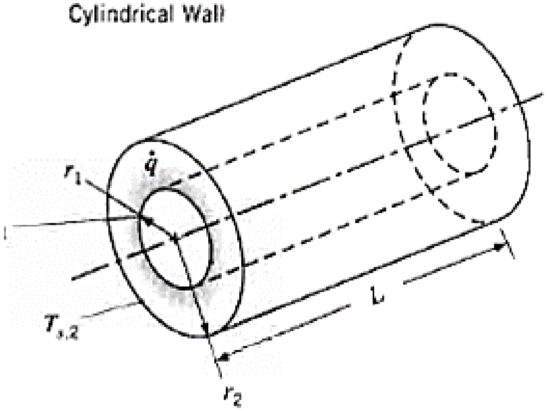
Plane Wall



Cylindrical Wall



Spherical Wall

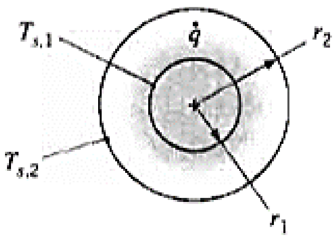


TABLE C.1 One-Dimensional, Steady-State Solutions to the Heat Equation for Plane, Cylindrical, and Spherical Walls with Uniform Generation and Asymmetrical Surface Conditions

Temperature Distribution

Plane Wall

$$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + \frac{T_{s,2} - T_{s,1}}{2}\frac{x}{L} + \frac{T_{s,1} + T_{s,2}}{2} \quad \text{(C.1)}$$

Cylindrical Wall

$$T(r) = T_{s,2} + \frac{\dot{q}r_2^2}{4k}\left(1 - \frac{r^2}{r_2^2}\right) - \left[\frac{\dot{q}r_2^2}{4k}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_{s,2} - T_{s,1})\right]\frac{\ln(r_2/r)}{\ln(r_2/r_1)} \quad \text{(C.2)}$$

Spherical Wall

$$T(r) = T_{s,2} + \frac{\dot{q}r_2^2}{6k}\left(1 - \frac{r^2}{r_2^2}\right) - \left[\frac{\dot{q}r_2^2}{6k}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_{s,2} - T_{s,1})\right]\frac{(1/r) - (1/r_2)}{(1/r_1) - (1/r_2)} \quad \text{(C.3)}$$

Heat Flux

Plane Wall

$$q''(x) = \dot{q}x - \frac{k}{2L}(T_{s,2} - T_{s,1}) \quad \text{(C.4)}$$

Cylindrical Wall

$$q''(r) = \frac{\dot{q}r}{2} - \frac{k\left[\frac{\dot{q}r_2^2}{4k}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_{s,2} - T_{s,1})\right]}{r\ln(r_2/r_1)} \quad \text{(C.5)}$$

Spherical Wall

$$q''(r) = \frac{\dot{q}r}{3} - \frac{k\left[\frac{\dot{q}r_2^2}{6k}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_{s,2} - T_{s,1})\right]}{r^2[(1/r_1) - (1/r_2)]} \quad \text{(C.6)}$$

Heat Rate

Plane Wall

$$q(x) = \left[\dot{q}x - \frac{k}{2L}(T_{s,2} - T_{s,1})\right]A_x \quad \text{(C.7)}$$

Cylindrical Wall

$$q(r) = \dot{q}\pi L r^2 - \frac{2\pi Lk}{\ln(r_2/r_1)} \cdot \left[\frac{\dot{q}r_2^2}{4k}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_{s,2} - T_{s,1})\right] \quad \text{(C.8)}$$

Spherical Wall

$$q(r) = \frac{\dot{q}4\pi r^3}{3} - \frac{4\pi k\left[\frac{\dot{q}r_2^2}{6k}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_{s,2} - T_{s,1})\right]}{(1/r_1) - (1/r_2)} \quad \text{(C.9)}$$

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Table C.2 Alternative Surface Conditions and Energy Balances for One-Dimensional, Steady-State Solutions to the Heat Equation for Plane, Cylindrical, and Spherical Walls with Uniform Generation

Plane Wall

Uniform Surface Heat Flux

$$x = -L: \quad q''_{s,1} = -\dot{q}L - \frac{k}{2L}(T_{s,2} - T_{s,1}) \tag{C.10}$$

$$x = +L: \quad q''_{s,2} = \dot{q}L - \frac{k}{2L}(T_{s,2} - T_{s,1}) \tag{C.11}$$

Prescribed Transport Coefficient and Ambient Temperature

$$x = -L: \quad U_1(T_{\infty,1} - T_{s,1}) = -\dot{q}L - \frac{k}{2L}(T_{s,2} - T_{s,1}) \tag{C.12}$$

$$x = +L: \quad U_2(T_{s,2} - T_{\infty,2}) = \dot{q}L - \frac{k}{2L}(T_{s,2} - T_{s,1}) \tag{C.13}$$

Cylindrical Wall

Uniform Surface Heat Flux

$$r = r_1: \quad q''_{s,1} = \frac{\dot{q}r_1}{2} - \frac{k\left[\frac{\dot{q}r_2^2}{4k}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_{s,2} - T_{s,1})\right]}{r_1 \ln(r_2/r_1)} \tag{C.14}$$

$$r = r_2: \quad q''_{s,2} = \frac{\dot{q}r_2}{2} - \frac{k\left[\frac{\dot{q}r_2^2}{4k}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_{s,2} - T_{s,1})\right]}{r_2 \ln(r_2/r_1)} \tag{C.15}$$

Prescribed Transport Coefficient and Ambient Temperature

$$r = r_1: \quad U_1(T_{\infty,1} - T_{s,1}) = \frac{\dot{q}r_1}{2} - \frac{k\left[\frac{\dot{q}r_2^2}{4k}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_{s,2} - T_{s,1})\right]}{r_1 \ln(r_2/r_1)} \tag{C.16}$$

$$r = r_2: \quad U_2(T_{s,2} - T_{\infty,2}) = \frac{\dot{q}r_2}{2} - \frac{k\left[\frac{\dot{q}r_2^2}{4k}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_{s,2} - T_{s,1})\right]}{r_2 \ln(r_2/r_1)} \tag{C.17}$$

Spherical Wall

Uniform Surface Heat Flux

$$r = r_1: \quad q''_{s,1} = \frac{\dot{q}r_1}{3} - \frac{k\left[\frac{\dot{q}r_2^2}{6k}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_{s,2} - T_{s,1})\right]}{r_1^2[(1/r_1) - (1/r_2)]} \tag{C.18}$$

$$r = r_2: \quad q''_{s,2} = \frac{\dot{q}r_2}{3} - \frac{k\left[\frac{\dot{q}r_2^2}{6k}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_{s,2} - T_{s,1})\right]}{r_2^2[(1/r_1) - (1/r_2)]} \tag{C.19}$$

$$r = r_1: \quad U_1(T_{\infty,1} - T_{s,1}) = \frac{\dot{q}r_1}{3} - \frac{k\left[\frac{\dot{q}r_2^2}{6k}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_{s,2} - T_{s,1})\right]}{r_1^2[(1/r_1) - (1/r_2)]} \tag{C.20}$$

$$r = r_2: \quad U_2(T_{s,2} - T_{\infty,2}) = \frac{\dot{q}r_2}{3} - \frac{k\left[\frac{\dot{q}r_2^2}{6k}\left(1 - \frac{r_1^2}{r_2^2}\right) + (T_{s,2} - T_{s,1})\right]}{r_2^2[(1/r_1) - (1/r_2)]} \tag{C.21}$$

Appendix C – Solid shapes (reference)

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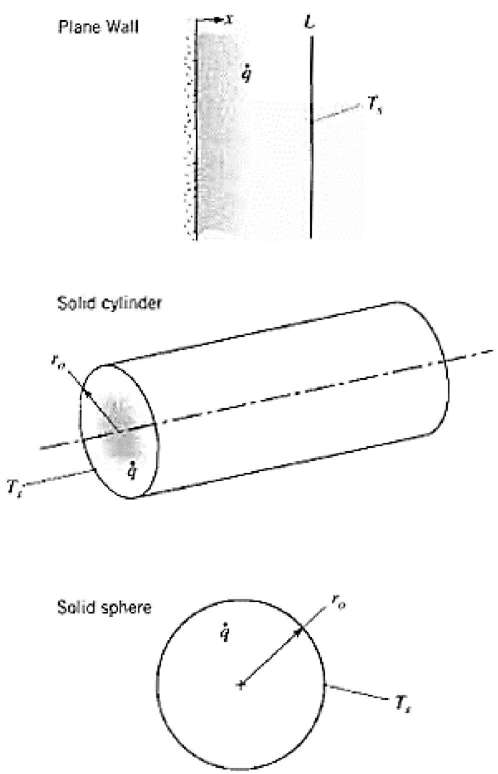

FIGURE C.2
One-dimensional conduction systems with uniform thermal energy generation: a plane wall with one adiabatic surface, a cylindrical rod, and a sphere.

TABLE C.3 One-Dimensional, Steady-State Solutions to the Heat Equation for Uniform Generation in a Plane Wall with One Adiabatic Surface, a Solid Cylinder, and a Solid Sphere

	Temperature Distribution	
Plane Wall	$T(x) = \frac{\dot{q}L^2}{2k}\left(1 - \frac{x^2}{L^2}\right) + T_s$	(C.22)
Circular Rod	$T(r) = \frac{\dot{q}r_o^2}{4k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s$	(C.23)
Sphere	$T(r) = \frac{\dot{q}r_o^2}{6k}\left(1 - \frac{r^2}{r_o^2}\right) + T_s$	(C.24)
	Heat Flux	
Plane Wall	$q''(x) = \dot{q}x$	(C.25)
Circular Rod	$q''(r) = \frac{\dot{q}r}{2}$	(C.26)
Sphere	$q''(r) = \frac{\dot{q}r}{3}$	(C.27)
	Heat Rate	
Plane Wall	$q(x) = \dot{q}xA_x$	(C.28)
Circular Rod	$q(r) = \dot{q}\pi Lr^2$	(C.29)
Sphere	$q(r) = \frac{\dot{q}4\pi r^3}{3}$	(C.30)

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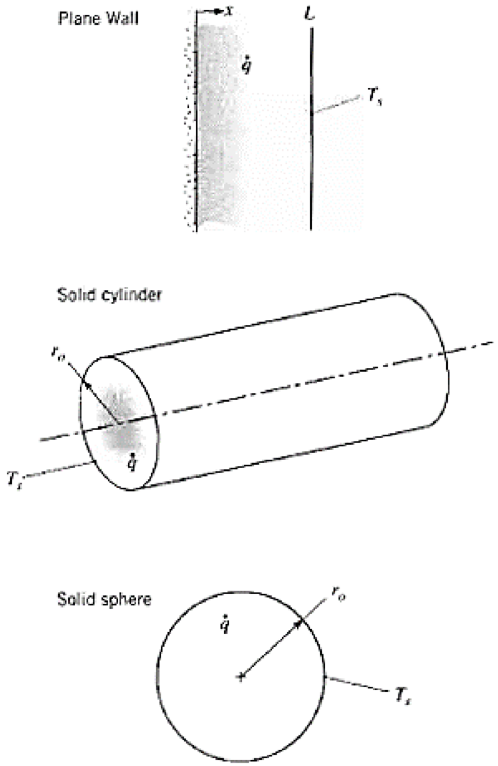


FIGURE C.2
One-dimensional conduction systems with uniform thermal energy generation: a plane wall with one adiabatic surface, a cylindrical rod, and a sphere.

TABLE C.4 Alternative Surface Conditions and Energy Balances for One-Dimensional, Steady-State Solutions to the Heat Equation for Uniform Generation in a Plane Wall with One Adiabatic Surface, a Solid Cylinder, and a Solid Sphere

Prescribed Transport Coefficient and Ambient Temperature

Plane Wall

$x = L$: $$\dot{q}L = U(T_s - T_\infty) \tag{C.31}$$

Circular Rod

$r = r_o$: $$\frac{\dot{q}r_o}{2} = U(T_s - T_\infty) \tag{C.32}$$

Sphere

$r = r_o$: $$\frac{\dot{q}r_o}{3} = U(T_s - T_\infty) \tag{C.33}$$

Example 2: Nuclear Fuel Rod

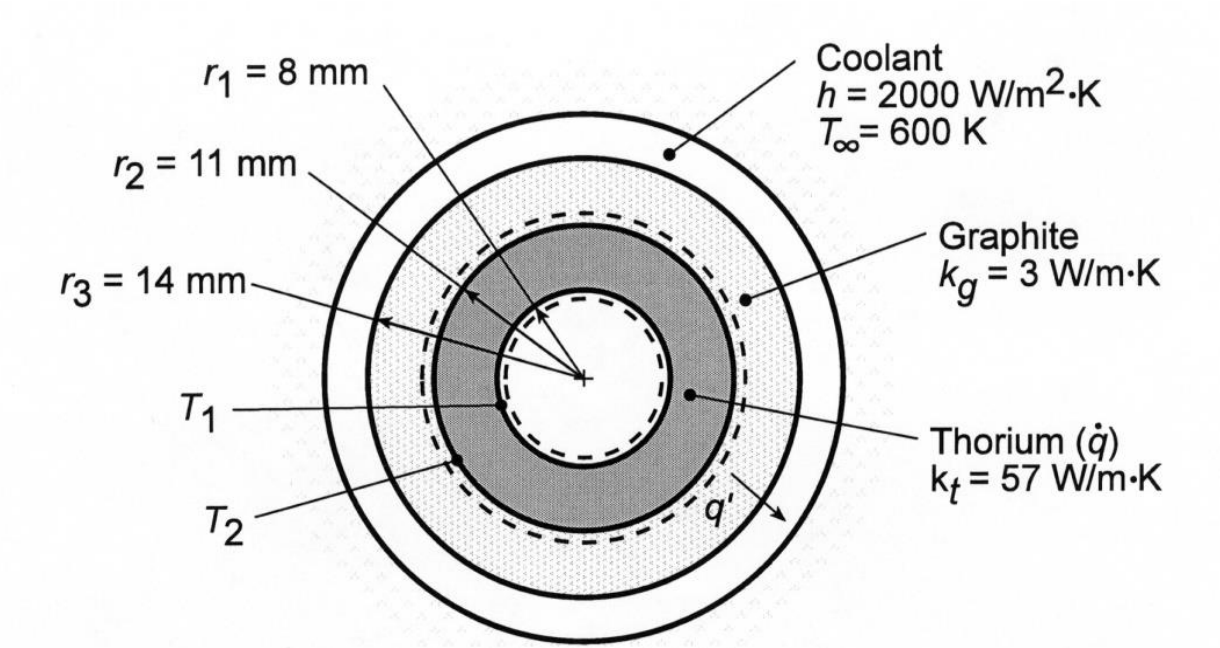
Thermal conditions in a gas-cooled nuclear reactor with a tubular composite cylindrical shell with thorium fuel rod (with an insulated core) and a concentric graphite sheath. A gaseous coolant flows to cool the graphite sheath :

(a) Assessment **of thermal integrity** for a generation rate of $\dot{q} = 10^8\,\mathrm{W/m^3}$

(b) Evaluation of temperature distributions in the thorium and graphite for generation rates in the range $10^8 \leq \dot{q} \leq 5\times10^8 \quad \mathrm{W/m^3}$.

Example 2: Nuclear Fuel Rod

SCHEMATIC:



ASSUMPTIONS: (1) Steady-state conditions, (2) One-dimensional conduction, (3) Constant properties, (4) Negligible contact resistance, (5) Negligible radiation, (6) **Assume surface at T_1 is adiabatic.**

PROPERTIES: Table A.1, Thorium: $T_{mp} = 2023$ K; Table A.2, Graphite: $T_{mp} = 2273$ K.

Example 2: Nuclear Fuel Rod (cont.)

ANALYSYS: (a) The outer surface temperature of the fuel, T_2 , may be determined from the rate equation:

$$q' = \frac{T_2 - T_\infty}{R'_{\text{tot}}} \quad \text{(Eq 1)} \qquad \text{where} \quad R'_{\text{tot}} = \frac{\ln(r_3/r_2)}{2\pi k_g} + \frac{1}{2\pi r_3 h} = 0.0185\ \text{m} \cdot \text{K/W}$$

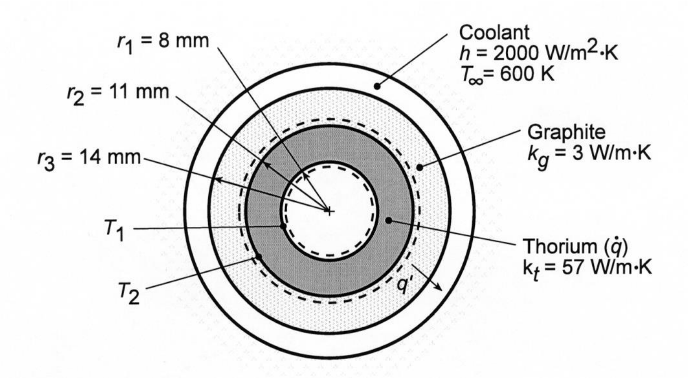
Why can you apply thermal resistance when there is heat generation in this question?

The heat rate (where does the heat come from?) may be determined by applying an energy balance to a control surface about the fuel element,

$$\dot{E}_{\text{out}} = \dot{E}_g$$

or, per unit length,

$$\dot{E}'_{\text{out}} = \dot{E}'_g$$



Since the interior surface of the thorium is essentially adiabatic, it follows that

$$q' = \dot{q}\pi\left(r_2^2 - r_1^2\right) = 17{,}907\ \text{W/m}$$

Example 2: Nuclear Fuel Rod (cont.)

ZJUI

Hence from the earlier Eq 1,

$$T_2 = q'R'_{\text{tot}} + T_\infty = 17{,}907\ \text{W/m}\left(0.0185\ \text{m}\cdot\text{K/W}\right) + 600\ \text{K} = 931\ \text{K}$$

With zero heat flux at the inner surface of the fuel element (i.e., at T_1 surface), Eq. C.14 yields

$$T_1 = T_2 + \frac{\dot{q}\, r_2^2}{4k_t}\left(1 - \frac{r_1^2}{r_2^2}\right) - \frac{\dot{q}\, r_1^2}{2k_t}\ln\left(\frac{r_2}{r_1}\right) = 931\ \text{K} + 25\ \text{K} - 18\ \text{K} = 938\ \text{K}$$

Since T_1 and T_2 are well below the melting points of thorium and graphite, the prescribed operating condition is acceptable.

$<$

Example 2: Nuclear Fuel Rod (cont.)

(b) The solution for the temperature distribution in a cylindrical wall with generation is

$$T_t(r) = T_2 + \frac{\dot{q}\, r_2^2}{4k_t}\left(1 - \frac{r^2}{r_2^2}\right) - \left[\frac{\dot{q}\, r_2^2}{4k_t}\left(1 - \frac{r_1^2}{r_2^2}\right) + \left(T_2 - T_1\right)\right]\frac{\ln\left(r_2/r\right)}{\ln\left(r_2/r_1\right)} \qquad \text{(C.2)}$$

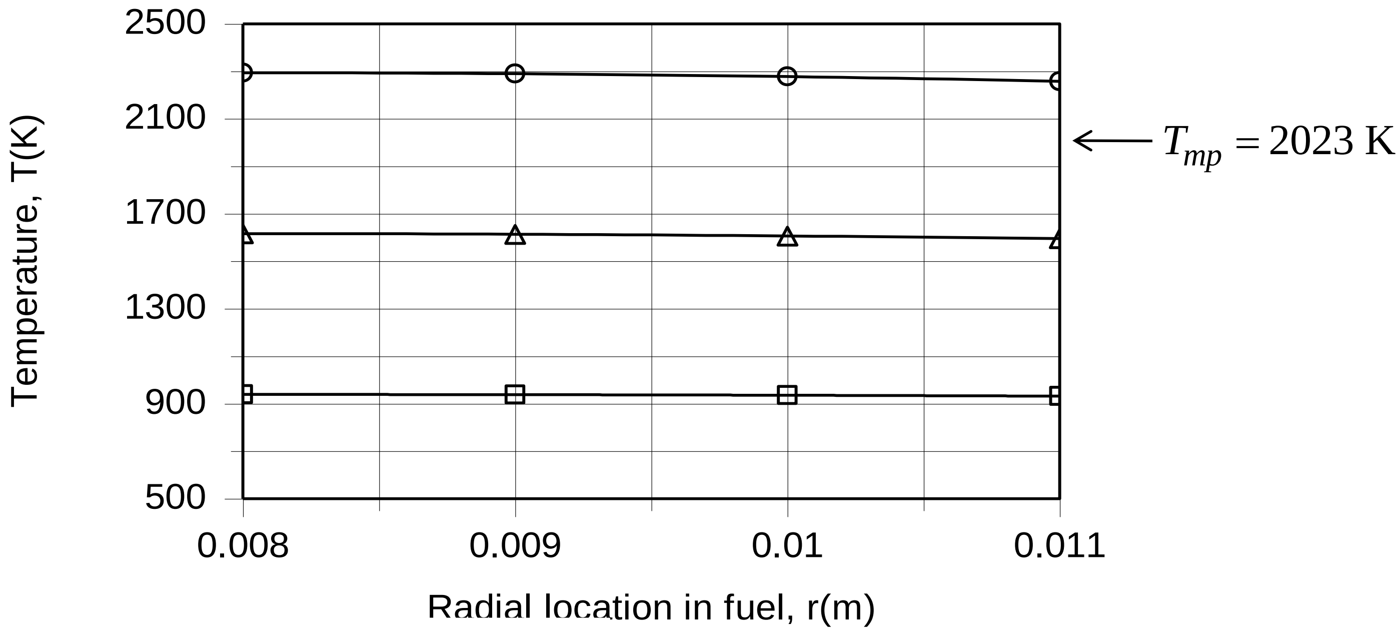
Boundary conditions at $r1$ and $r2$ are used to determine T_1 and T_2.

$$r = r_1: \quad \boxed{q_1'' = 0} = \frac{\dot{q} r_1}{2} - \frac{k\left[\frac{\dot{q}\, r_2^2}{4k_t}\left(1 - \frac{r_1^2}{r_2^2}\right) + \left(T_2 - T_1\right)\right]}{r_1 \ln\left(r_2 / r_1\right)} \qquad \text{(C.14)}$$

$$r = r_2: \quad \boxed{U_2\left(T_2 - T_\infty\right)} = \frac{\dot{q}\, r_2}{2} - \frac{k\left[\frac{\dot{q} r_2^2}{4k_t}\left(1 - \frac{r_1^2}{r_2^2}\right) + \left(T_2 - T_1\right)\right]}{r_2 \ln\left(r_2 / r_1\right)} \qquad \text{(C.17)}$$

where $U_2 = \left(A_2' R_{\text{tot}}'\right)^{-1} = \left(2\pi r_2 R_{\text{tot}}'\right)^{-1}$ (3.37)

The following results are obtained for temperature distributions in the thorium.



$\dot{q} = 5\times10^{8}\,\mathrm{W/m^3}$

$\dot{q} = 3\times10^{8}\,\mathrm{W/m^3}$

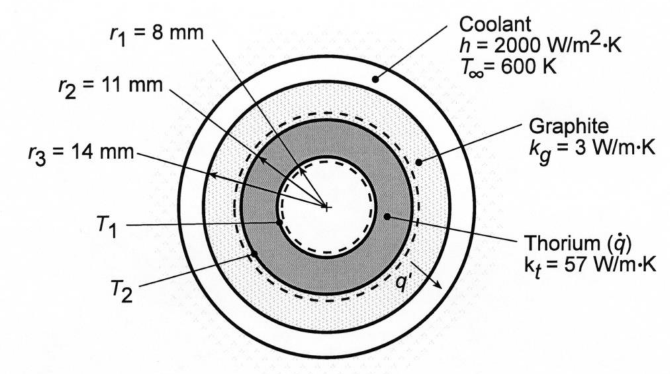
$\dot{q} = 1\times10^{8}\,\mathrm{W/m^3}$

Operation at $\dot{q} = 5\times10^{8}\,\mathrm{W/m^3}$ is clearly unacceptable since the melting point of thorium would be exceeded. The reactor should not be operated much above $\dot{q} = 3\times10^{8}\,\mathrm{W/m^3}$. The small radial temperature gradients are attributable to the large value of k_t.

Example 2: Nuclear Fuel Rod (cont.)

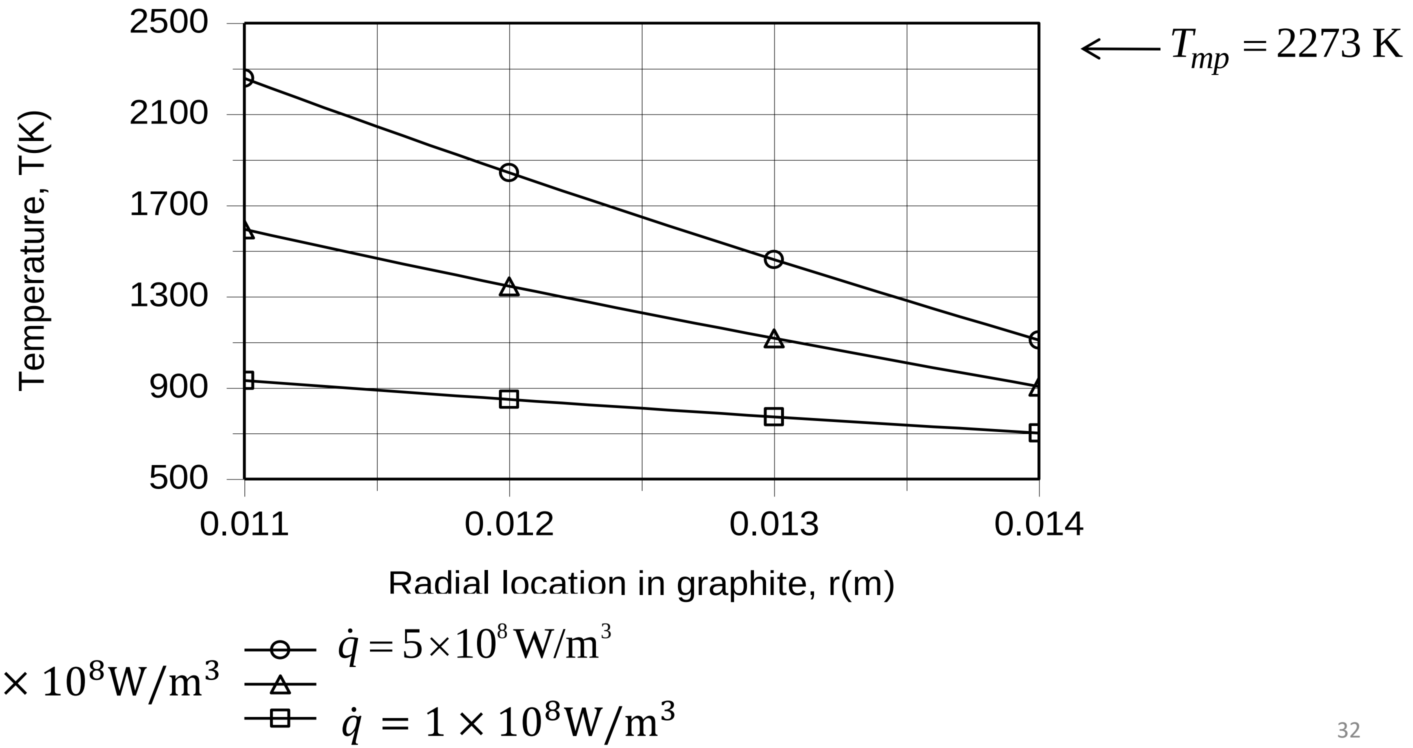
Using the value of T_2 from the foregoing solution and computing T_3 (outer temp of Graphite) from the surface condition,

$$q' = \frac{2\pi k_g\left(T_2 - T_3\right)}{\ln\left(r_3 / r_2\right)}$$



the temperature distribution in the graphite is

$$T_g(r) = \frac{T_2 - T_3}{\ln\left(r_2 / r_3\right)} \ln\left(\frac{r}{r_3}\right) + T_3 \qquad (3.31)$$



Operation at $\dot{q} = 5 \times 10^{8}\,\mathrm{W/m^3}$ is problematic for the graphite. Larger temperature gradients are due to the small value of k_g.

COMMENTS

(i) Referring to the schematic, where might radiation effects be significant? What would be the influence of such effect on temperatures in the fuel element and the maximum allowable value of $\dot{q}$? How do you include radiation effect here?

Summary

- 1D Steady State heat transfer with heat generation
  - Plane wall
  - Radial systems
  - No thermal resistance inside material with heat generation
  - Outside of that, it is ok if the required conditions still hold (See previous lectures).
  - Different BCs will change the form for $T(r)$, $q(r)$, and $q''(r)$.