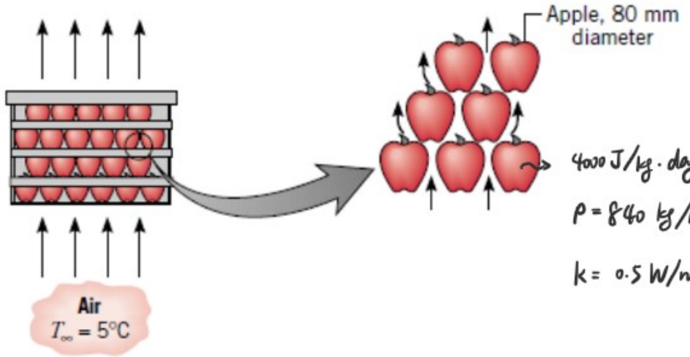
ME 320 Homework 3

Unique characteristics of biologically active materials such as fruits, vegetables, and other products require special care in handling. Following harvest and separation from producing plants, glucose is catabolized to produce carbon dioxide, water vapor, and heat, with attendant internal energy generation. Consider a carton of apples, each of 80-mm diameter, which is ventilated with air at 5°C and a velocity of 0.5 m/s. The corresponding value of the heat transfer coefficient is 7.5 W/m$^2\cdot$K. Within each apple thermal energy is uniformly generated at a total rate of 4000 J/kg$\cdot$day. The density and thermal conductivity of the apple are 840 kg/m^3 and 0.5 W/m$\cdot$K, respectively.

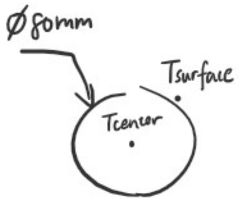


4000 J/kg·day

$\rho = 840$ kg/m^3

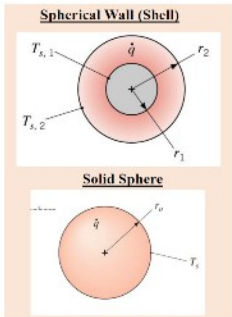
$k = 0.5$ W/m·k

(a) Determine the apple center and surface temperatures.



空气 5°C 0.5m/s $h = 7.5$ W/m$^2\cdot$k

Question: 计算苹果中心和表面温度



Spherical

$$\boxed{\frac{1}{r^2}\frac{d}{dr}\left(kr^2\frac{dT}{dr}\right)+\dot{q}=0}$$

1. Heat Equation

$$\frac{1}{r^2}\frac{d}{dr}\left(kr^2\frac{dT}{dr}\right)+\dot{q}=0$$

2. BCs:

I. $r=0,\quad dT/dr=0$

II. $r=r_o,\quad T=T_s$

3. Solving the Heat Equation,

$$kr^2\frac{dT}{dr}=-\frac{\dot{q}r^3}{3}+C_1$$

$$T=-\frac{\dot{q}r^2}{6k}-\frac{C_1}{r}+C_2$$

Given,

I. $\frac{dT}{dr}\Big|_{r=0}=0\rightarrow C_1=0$

II. $T(r_o)=T_s\rightarrow C_2=T_s-\frac{\dot{q}r_o^2}{6k}$

$$T(r)=\frac{\dot{q}r_o^2}{6k}\left(1-\frac{r^2}{r_o^2}\right)+T_s$$

$\dot{q}$ = 4000J/kg·day = 0.0463 J/kg·s

ρ = 840 kg/m^3

根据能量守恒公式

$$\dot{E}_{in}-\dot{E}_{out}+\dot{E}_g=0$$

$$-\dot{q}_{conv}+\dot{q}\cdot\rho\cdot V=0$$

$$-h\cdot A_s\cdot(T_s-T_\infty)+\dot{q}\cdot\rho\cdot V=0$$

$$-h\cdot(4\pi r_o^2)(T_s-T_\infty)+\dot{q}\cdot\rho\cdot\frac{4}{3}\pi r_o^3=0$$

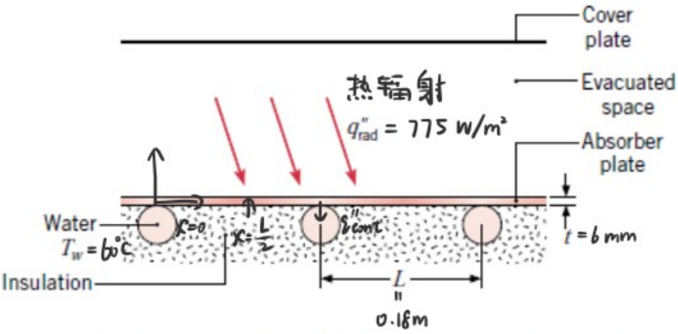
$$\therefore T_s=\frac{\dot{q}\cdot\rho\cdot r_o}{3h}+T_\infty=5.069\,°C$$

根据球形温度分布公式

$$T(r)=\frac{(\dot{q}\rho)r_o^2}{6k}\left(1-\frac{r^2}{r_o^2}\right)+T_{surface}$$

$$\therefore T_c=T(r=0)=\frac{0.0463\times 840\times(0.04)^2}{6\times 0.5}+5.069=5.0897\,°C$$

Copper tubing is joined to the absorber of a flat-plate solar collector as shown.



The aluminum alloy (2024-T6) absorber plate is 6 mm thick and well insulated on its bottom. The top surface of the plate is separated from a transparent cover plate by an evacuated space. The tubes are spaced a distance L of 0.18 m from each other, and water is circulated through the tubes to remove the collected energy. The water may be assumed to be at a uniform temperature of $T_w = 60°C$. Under steady-state operating conditions for which the *net* radiation heat flux to the surface is $q''_{rad} = 775\ W/m^2$, what is the maximum temperature on the plate and the heat transfer rate per unit length of tube? Note that q''_{rad} represents the net effect of solar radiation absorption by the absorber plate and radiation exchange between the absorber and cover plates. You may assume the temperature of the absorber plate directly above a tube to be equal to that of the water.

Question: 计算铝板表面最高温度和单位长度管道热交换速率

根据能量守恒公式

$$\dot{E}_{in} - \dot{E}_{out} + 0 = 0$$

$$\dot{E}_{in} = q''_x + q''_{rad} \qquad q''_x = -k\frac{dT}{dx}$$

$$\dot{E}_{out} = q''_{x+dx} = q''_x + \frac{dq''_x}{dx} \cdot dx$$

$$\therefore \quad q''_{rad} = \frac{dq''_x}{dx} \cdot dx$$

③

$$q''_{rad} = \frac{d}{dx}\left(-k\frac{dT}{dx}\right) \cdot t$$

$$\frac{d^2T}{dx^2} + \frac{q''_{rad}}{kt} = 0$$

$\downarrow$ 积分

$$T(x) = \frac{q''_{rad}}{2kt} \cdot x^2 + C_1 x + C_2$$

边界条件

$$\begin{cases} T(x=0) = T_w = 60°C \\ \dfrac{dT(x=\frac{L}{2})}{dx} = 0 \end{cases} \Rightarrow T(x) = \frac{q''_{rad}}{2kt}(Lx - x^2) + T_w$$

根据表格 A-1 铝合金 (2024-T6) 导热系数为 $k = 180\ W/m \cdot K$

$$\therefore T_{max} = T(x=\tfrac{L}{2}) = \frac{q''_{rad} \cdot L^2}{8kt} + T_w$$

$$= \frac{775 \times 0.18^2}{8 \times 180 \times 0.006} + 60°C$$

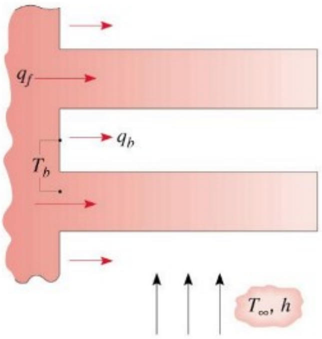
$$= 62.9°C$$

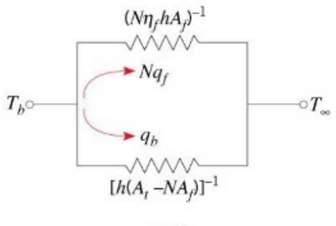
Fin Array:

III. Assume an array of N fins arranged perpendicular to the airflow as shown in the figure below. Starting from a single fin with an efficiency of η_f, derive the two expressions for the equivalent resistor network shown in the figure.

Question 推导公式

A_f is the surface area of a fin and A_t is the total surface area of the fin array.





$$q_f = \eta_f h A_f \theta_b \cdot N$$

$$q_b = h A_b \theta_b$$

$$\because q = \frac{\theta}{R_t}$$

$$\therefore R_t = \frac{\theta}{q}$$

$$\therefore \begin{cases} R_{t,f} = \dfrac{1}{N\eta_f h A_f} \\ R_{t,b} = \dfrac{1}{hA_b} = \dfrac{1}{h(A_t - NA_f)} \end{cases}$$

Transient Heat

IV. A hot ladle made of stainless steel is allowed to hang in a kitchen to cool off. The surface area is 20 cm2, the volume is 2 cm3 and the heat transfer coefficient for convection by air is 10 W/m2K.

$A_s = 20\,cm^2$

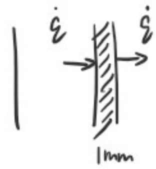
$V = 2\,cm^3$

$h = 10\,W/m^2K$

(a) Find the time taken for it to cool off by 50% of its excess temperature. Assume properties at 300 K for steel. Justify any assumptions.

$k = 0.5\,W/mK$

(b) The same ladle gets coated by a 1 mm thick layer of oil, with thermal conductivity 0.5 W/mK. What is the ratio of the new thermal time constant to the old?



(a) 根据瞬态传热公式

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = e^{-\frac{hA_s}{\rho V c}\cdot t}$$

不锈钢密度 $\rho \approx 8055\ kg/m^3$

不锈钢比热容 $c \approx 480\ J/kg\cdot K$

$$\therefore\ 0.5 = e^{-\frac{h\cdot A_s}{\rho V c}\cdot t}$$

$$\therefore\ \boxed{t} = -\frac{\rho V c}{h A_s}\cdot \ln\frac{1}{2} = \frac{8055\times 2\times(0.01)^3\times 480}{10\times 20\times(0.01)^2}\ln 2 = \boxed{268\ s}$$

(b) $$\tau = \left(\frac{1}{h\cdot A_s}\right)\cdot \rho V c = R_t\cdot C_t$$

$$R_{t,1} = \frac{1}{h\cdot A_s} = \frac{1}{10\times 20\times(0.01)^2} = 50\ K/W$$

$$R_{t,2} = \frac{1}{h\cdot A_s} + \frac{L}{kA_s} = \frac{1}{10\times 20\times(0.01)^2} + \frac{0.001}{0.5\times 20\times(0.01)^2} = 51\ K/W$$

$$\therefore\ \boxed{\frac{\tau_1}{\tau_2}} = \frac{R_{t,1}}{R_{t,2}} = \boxed{\frac{50}{51}}$$

Fin Array:

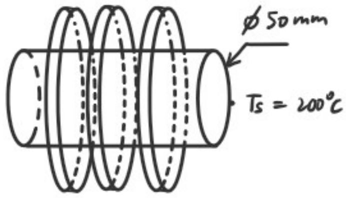
V. **环形散热鳍片**

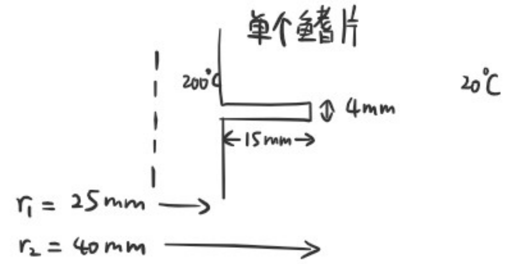
Annular aluminum fins of rectangular profile are attached to a circular tube having an outside diameter of 50 mm and an outer surface temperature of 200°C. The fins are 4 mm thick and 15 mm long. The system is in ambient air at a temperature of 20°C, and the surface convection coefficient is 40 W/m^2·K.

(a) What are the fin efficiency and effectiveness?

(b) If there are 125 such fins per meter of tube length, what is the rate of heat transfer per unit length of tube?

(Note this is an annular fin array. You need to use some graphs)





(a) 根据表格 A-1，纯铝导热系数 $k = 240\ W/m\cdot K$

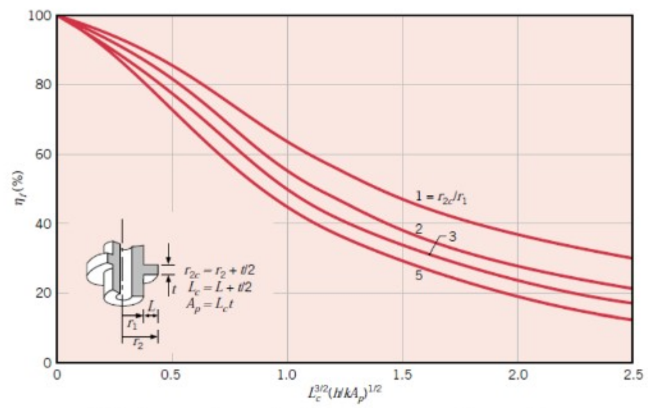
$$r_{2c} = r_1 + L + \frac{t}{2} = 42\ mm$$

$$\frac{r_{2c}}{r_1} = \frac{42mm}{25mm} = 1.68$$

$$L_c = L + \frac{t}{2} = 17\ mm$$

$$A_p = L_c \cdot t = 17mm \times 4mm = 68\ mm^2$$

$$L_c^{1.5}\sqrt{\frac{h}{k\cdot A_p}} = (17mm)^{1.5}\cdot\sqrt{\frac{40\ W/m^2\cdot K}{240\ W/m\cdot K \cdot 68\ mm^2}} = 0.11$$



Efficiency of annular fins of rectangular profile

根据右侧图表对照得

$$\boxed{\eta_f \approx 97\%}$$

$$\because\ \eta_f = \frac{q_f}{q_{f,max}} = \frac{q_f}{h A_f \theta_b}$$

$$\therefore\ q_f = \eta_f \cdot h \cdot A_f \cdot \theta_b$$

$$\varepsilon_f = \frac{q_f}{h\cdot A_{c,b}\cdot\theta_b} = \frac{\eta_f\cdot h\cdot A_f\cdot\theta_b}{h\cdot A_{c,b}\cdot\theta_b} = \frac{\eta_f\cdot A_f}{A_{c,b}} = \frac{97\%\times 2\pi(r_{2c}^2 - r_1^2)}{2\pi(r_1\cdot t)} = 11$$

(b)

$$q_t' = N\cdot q_f' + q_b'$$

$$= N\cdot 2\cdot\pi(r_{2c}^2 - r_1^2)\cdot h\cdot\theta_b + (1 - N\cdot t)\cdot h\cdot(2\pi r_1)\cdot\theta_b$$

$$= 6815\ W/m$$