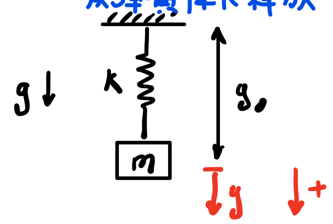
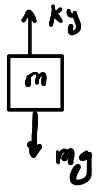
Static equilibrium position and Coordinate frame :

从弹簧原长释放



y_0 belongs to relaxed spring



质量恒定

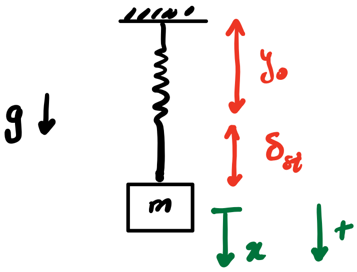
$$\sum \vec{F} = \frac{d}{dt}(m\dot{y}) = m\ddot{y}$$

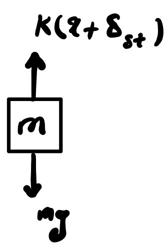
$$\sum \vec{F} = mg - ky = m\ddot{y}$$

$$m\ddot{y} + ky = mg$$

从静态平衡释放

Static equilibrium

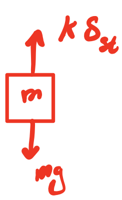




$$\sum \vec{F} = mg - k(x+\delta_{st}) = m\ddot{x}$$

$$m\ddot{x} + k(x+\delta_{st}) = mg \quad ①$$

FBD of static equilibrium

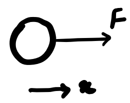


$\rightsquigarrow$ $mg = k\delta_{st}$ ②

From ① and ②, we have: $m\ddot{x} + kx = 0$

纯平移

- pure translation



position : $x\,[m]$

velocity : $\dot{x}\,[m/s]$

acceleration : $\ddot{x}\,[m/s^2]$

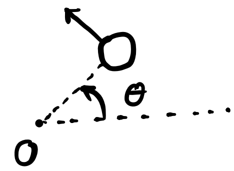
Linear momentum : $p = m\dot{x}$

Forces : $\frac{d}{dt}(p) = \Sigma F$

if m is const : $\Sigma F = m\ddot{x}$

纯旋转

- pure rotation



angle : $\theta\,[rad]$

angular velocity : $\dot{\theta} = \omega\,[rad/s]$

angular acceleration : $\ddot{\theta} = \alpha\,[rad/s^2]$

angular momentum : $L = r\,(m\dot{x})$

$= r\,(m r\dot{\theta})$

$= \underbrace{m r^2}_{I}\dot{\theta}$

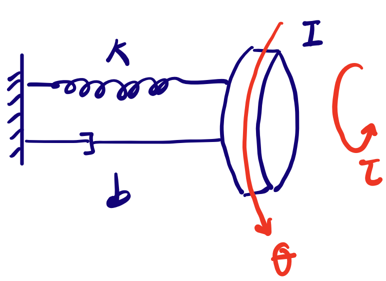
mass moment of inertia

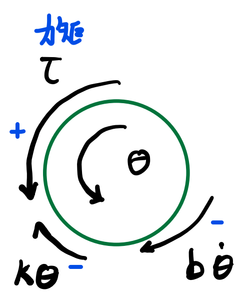
$\Sigma \tau = \frac{d}{dt}(L) = \frac{d}{dt}(I\dot{\theta})$

If m is const : $\Sigma\tau = I\ddot{\theta}$

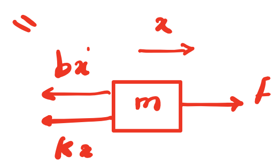
旋转系统

Rotational systems





Free-body diagram



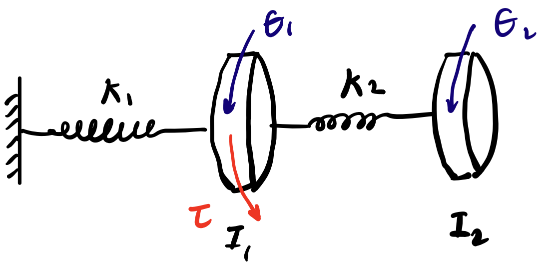
$\Sigma\tau = I\ddot{\theta} \sim \tau - k\theta - b\dot{\theta} = I\ddot{\theta}$

$I\ddot{\theta} + b\dot{\theta} + k\theta = \tau$

Assume IC = 0 , take laplace transform : $G(s) = \frac{\Theta(s)}{T(s)} = \frac{1}{Is^2+bs+k}$

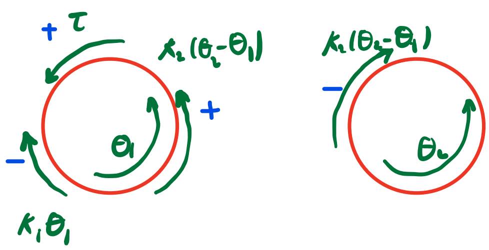
串联旋转

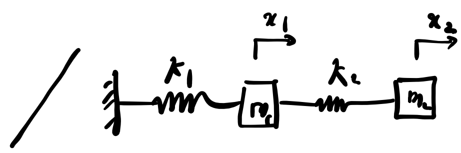
ex :



view from this side ←

Free body diagram 受力分析





Assume $\theta_2 > \theta_1$

I_1

Disk 1: $\tau - k_1\theta_1 + k_2(\theta_2-\theta_1) = I\ddot{\theta}_1$

$$I\ddot{\theta}_1 + (k_1+k_2)\theta_1 - k_2\theta_2 = \tau$$

I_2

Disk 2: $-k_2(\theta_2-\theta_1) = I\ddot{\theta}_2$

$$I\ddot{\theta}_2 + k_2\theta_2 - k_2\theta_1 = 0$$