Homework 7

Question 1

1. In class we derived the closed-loop system obtained with dynamic output feedback in $(x, \hat{x})$-coordinates:

$$\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} = \begin{pmatrix} A & -BK \\ LC & A - LC - BK \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}$$

and later rewrote it in (x, e)-coordinates. Rewrite the same system in $(\hat{x}, e)$-coordinates.

$$\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} \longrightarrow \begin{pmatrix} \dot{\hat{x}} \\ \dot{e} \end{pmatrix} = \begin{pmatrix} 0 \cdot \dot{x} + I \dot{\hat{x}} \\ I \dot{x} - I \dot{\hat{x}} \end{pmatrix} = \begin{pmatrix} 0 & I \\ I & -I \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix}$$

$$\begin{pmatrix} 0 & I \\ I & -I \end{pmatrix} \begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} = \begin{pmatrix} 0 & I \\ I & -I \end{pmatrix} \begin{pmatrix} A & -BK \\ LC & A-LC-BK \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}$$

$$\begin{pmatrix} \dot{\hat{x}} \\ \dot{e} \end{pmatrix} = \begin{pmatrix} LC & A-LC-BK \\ A-LC & LC-A \end{pmatrix} \begin{pmatrix} x \\ \hat{x} \end{pmatrix}$$

$$\Downarrow$$

$$\begin{cases} \dot{\hat{x}} = LCx + (A-LC-BK)\hat{x} = LC(e+\hat{x}) + (A-LC-BK)\hat{x} = (A-BK)\hat{x} + LCe \\ \dot{e} = (A-LC) \cdot x - (A-LC) \cdot \hat{x} = (A-LC)e \end{cases}$$

$$\Downarrow$$

$$\begin{pmatrix} \dot{\hat{x}} \\ \dot{e} \end{pmatrix} = \begin{pmatrix} A-BK & LC \\ 0 & A-LC \end{pmatrix} \begin{pmatrix} \hat{x} \\ e \end{pmatrix}$$

Question 2

Consider the system:

$$\dot{x} = \begin{pmatrix} 0 & -1 & 2/3 \\ -1 & -2 & 1 \\ 0 & -3 & 1 \end{pmatrix} x + \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} u, \qquad y = x_2$$

a) Write down the open-loop characteristic equation. (This involves computing a 3×3 determinant, which you can do either by hand or in MATLAB using a symbolic variable s.) Are all open-loop poles in the LHP?

b) Using the formula given in class, compute the transfer function of this system. (Use the general formula, do *not* take Laplace transform of individual differential equations. Look up the procedure for inverting a matrix by hand, or use the MATLAB command `inv`.)

c) Find another state-space realization of the same transfer function, in controller canonical form

$$\dot{x} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u, \qquad y = \begin{pmatrix} b_3 & b_2 & b_1 \end{pmatrix} x$$

Hint: you should see that, similarly to the 2×2 case discussed in class, there is a simple relationship between the entries in the above matrices and the coefficients in the transfer function.

(a) 开环特征方程 $A = \begin{pmatrix} 0 & -1 & \frac{2}{3} \\ -1 & -2 & 1 \\ 0 & -3 & 1 \end{pmatrix}$

$$\det(A - Is) = \det\begin{pmatrix} -s & -1 & \frac{2}{3} \\ -1 & -2-s & 1 \\ 0 & -3 & 1-s \end{pmatrix} = s^3 + s^2 - 1 = 0 \Rightarrow \begin{cases} s_1 \approx 0.7549 + 0i \\ s_2 \approx -0.8774 - 0.7449i \\ s_3 \approx -0.8774 + 0.7449i \end{cases} \Rightarrow \text{不全在 LHP}$$

(b). 计算传递函数 transfer function $G(s)$

$$\dot{x} = Ax + Bu \qquad A = \begin{pmatrix} 0 & -1 & 2/3 \\ -1 & -2 & 1 \\ 0 & -3 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \quad C = (0\ 1\ 0) \quad D = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$y = Cx + Du$$

$$G(s) = C(sI - A)^{-1}B + D = \frac{2s^2 - 1}{s^3 + s^2 - 1}$$

逆矩阵计算

c) Find another state-space realization of the same transfer function, in controller canonical form

$$\dot{x} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u, \qquad y = \begin{pmatrix} b_3 & b_2 & b_1 \end{pmatrix} x$$

Hint: you should see that, similarly to the 2×2 case discussed in class, there is a simple relationship between the entries in the above matrices and the coefficients in the transfer function.

(c). 将上述 state-space function 化为 CCF 能控标准型

① 计算能控性矩阵

$$C_{A,B} = [B \mid AB \mid A^2B] = \begin{pmatrix} 1 & 0 & 0 \\ 2 & -2 & 1 \\ 3 & -3 & 3 \end{pmatrix} \Rightarrow \det(C) = -3 \neq 0 \text{ 可逆}$$

② 计算 $\bar{A}, \bar{B}$

$$\bar{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{pmatrix} \qquad \bar{B} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\det(sI - A) = \det(sI - \bar{A}) \Rightarrow \begin{cases} a_1 = 1 \\ a_2 = 0 \\ a_3 = -1 \end{cases}$$

$$C_{\bar{A},\bar{B}} = [\bar{B} \mid \bar{A}\bar{B} \mid \bar{A}^2\bar{B}] = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$$

③ 计算映射矩阵

$$T = C_{\bar{A},\bar{B}} \cdot C_{A,B}^{-1} = \begin{pmatrix} 0 & -1 & 2/3 \\ 1 & 0 & -1/3 \\ 0 & 0 & 1/3 \end{pmatrix}$$

④ 应用映射矩阵

$$\left.\begin{array}{l} \bar{A} = TAT^{-1} \\ \bar{B} = TB \\ \bar{C} = CT^{-1} = (-1 \;\; 0 \;\; 2) \end{array}\right\} \Rightarrow \begin{cases} \dot{x} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & -1 \end{pmatrix} x + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} u \\ y = (-1 \;\; 0 \;\; 2)\, x \end{cases}$$