Lec08: Transient Conduction: The Lumped Capacitance Method

Chapter Five

5.1 - 5.3

Announcement

- HW 3 will be up today. Look out for it.

- Exam will be on 1 Nov during class (1hr 30 mins).
  - Lec01 – 08 until today's lecture
  - Everything on Heat Conduction
  - Bring 1-sided Cheat Side
  - I will give your Annex A in your textbook for material's properties

Content

1. For the heat diffusion equation, which term will definitely not go to zero when there is a sudden change in the operating conditions?

2. What has to be true before the lumped capacitance method can be used?

3. How is Biot number defined? What does it mean physically?

4. How is Fourier number defined? What does it mean physically?

5. How is the thermal time constant defined? What does it mean physically?

6. What are some of the different transient HT cases?

ZJUI

What to solve the heat transfer for a hot iron just out of a furnace?

$$\dot{E}_{in} - \dot{E}_{\mathrm{out}} + \dot{E}_{\mathrm{g}} = \dot{E}_{st}$$

?

- 1D or 2D or 3D?
- Steady state or time-dependent?
- Any heat generation or loss?
- Any mass transport or advection?

3D time-dependent process

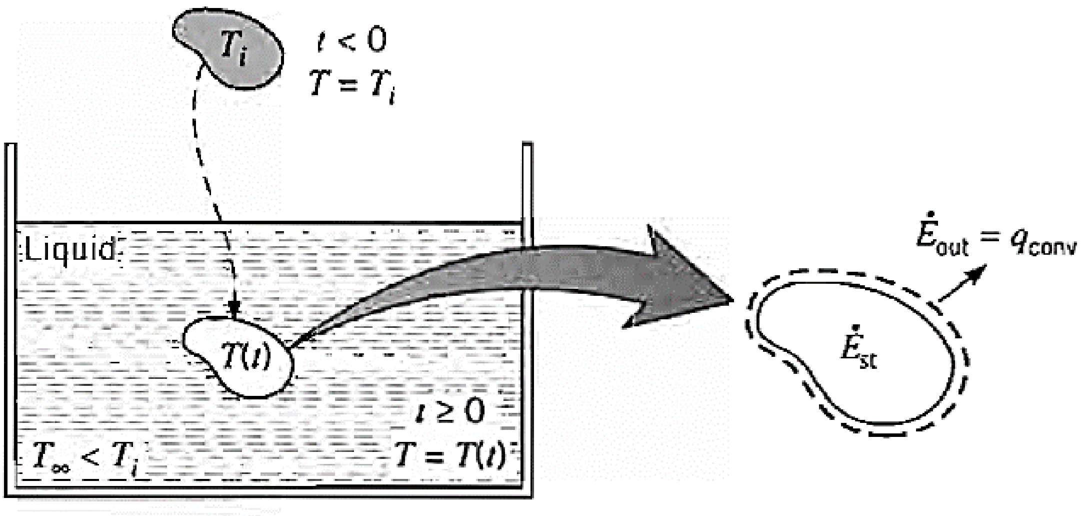
- Heat transfer process for which
  - Temperature varies with time (due to a change in operating conditions)
  - Temperature varies within a solid

- Due to changes in:
  - surface convection conditions (h, T_∞),
  - surface radiation conditions (h_r, T_{sur}),
  - a surface temperature or heat flux, and/or
  - internal energy generation.

- Solution Techniques
  - Lumped Capacitance Method
  - Exact Solutions
  - The Finite-Difference Method

- **BIG** assumption: A spatially uniform temperature distribution in solid throughout the transient process.

Hence, $T(\vec{r},t) \approx T(t)$



T – Temperature, t – Time,
r – Spatial coordinates

- Assumption means NO temperature gradient inside solid!
- Why is assumption can never be fully realized in practice?

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

$$\boxed{\dot{E}_{out} = -\dot{E}_{st}}$$

$$\boxed{-hA_s(T - T_\infty) = \rho V c \frac{dT}{dt}} \quad (5.2)$$

A_s – surface area
h – heat transfer coefficient
ρ – density
V – volume
c – heat capacity

Lumped Capacitance Approach

Section 5.1

ZJUI

$$-hA_s(T - T_\infty) = \rho V c \frac{dT}{dt}$$

- How to solve?

- Variable separable using, $\theta = T - T_\infty$

$$\frac{\rho V c}{hA_s} \int_{\theta_i}^{\theta} \frac{d\theta}{\theta} = -\int_0^t dt$$

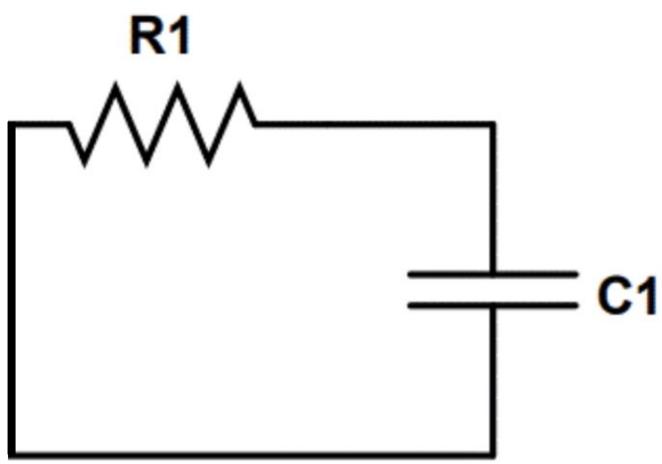
- Initial condition, $t = 0 \Rightarrow \theta = \theta_i$

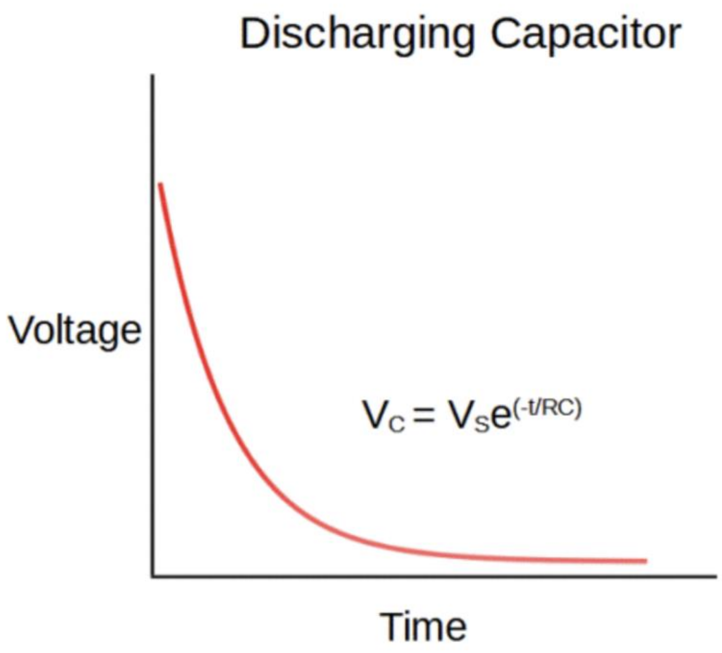
$$\boxed{\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho V c}\right)t\right]} \qquad (5.6)$$

- Why do we not need any boundary conditions????

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho Vc}\right)t\right]$$

- Do you know what is the time constant term in a Resistance-Capacitance circuit? $\tau = ?$





- See any similarities?

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho Vc}\right)t\right]$$

- Do you know what is the time constant (τ) term in a Resistance-Capacitance circuit?

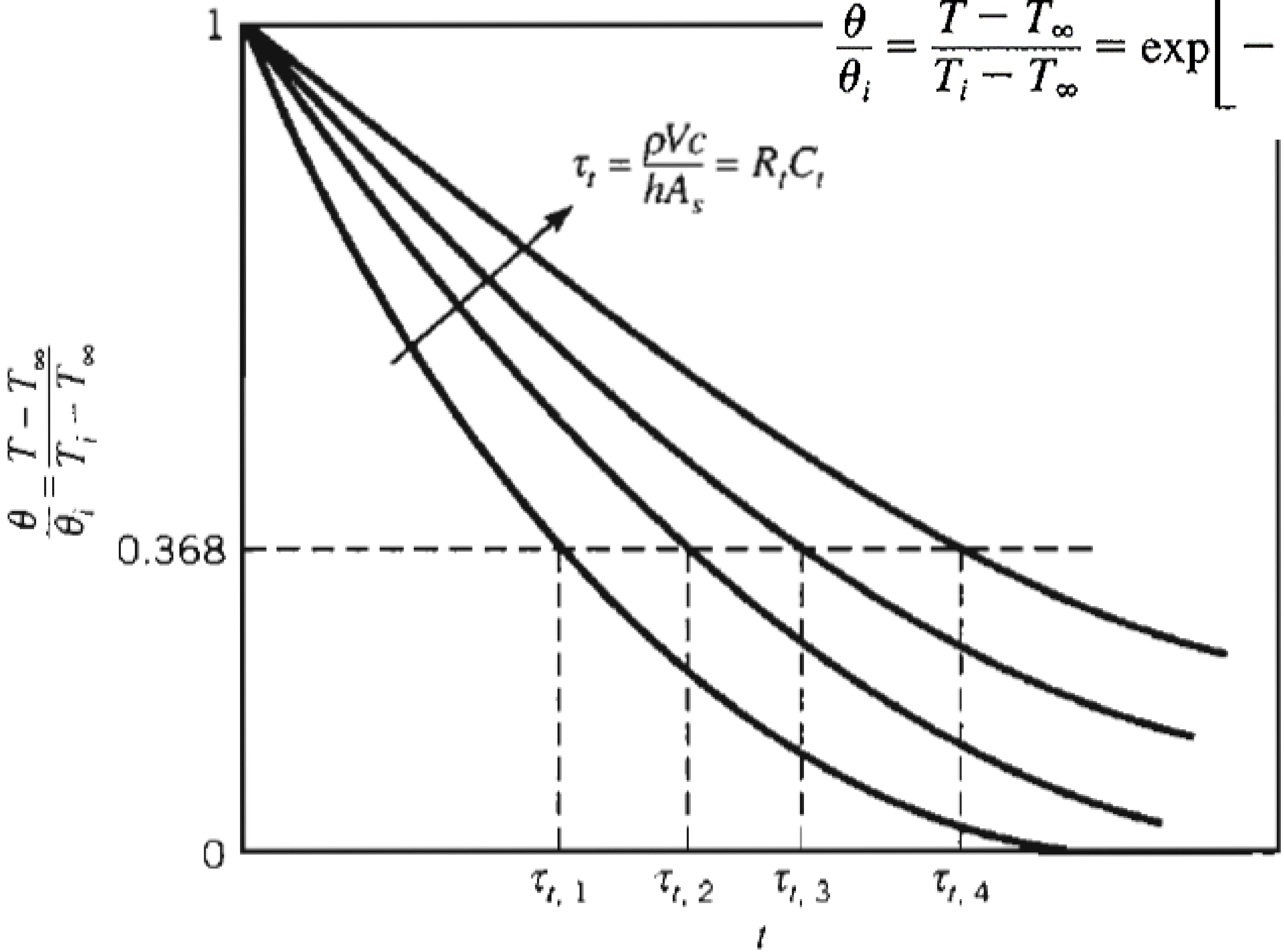
- Thermal time constant

$$\boxed{\tau_t = \left(\frac{1}{hA_s}\right)(\rho Vc) = R_t C_t} \qquad (5.7)$$

- Why $\boldsymbol{R_t}$ **is 1/(*hA*$_s$)?** Why $\boldsymbol{C_t = \rho Vc}$ **?**

5.1 Lumped Capacitance Method

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho V c}\right)t\right]$$



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To find total energy transfer out (Q),

- From (5.2)

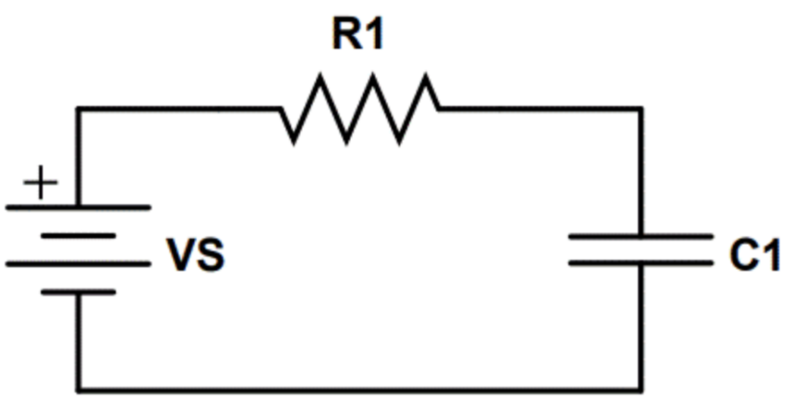
$$-hA_s(T - T_\infty) = \rho V c \frac{dT}{dt} = Q$$

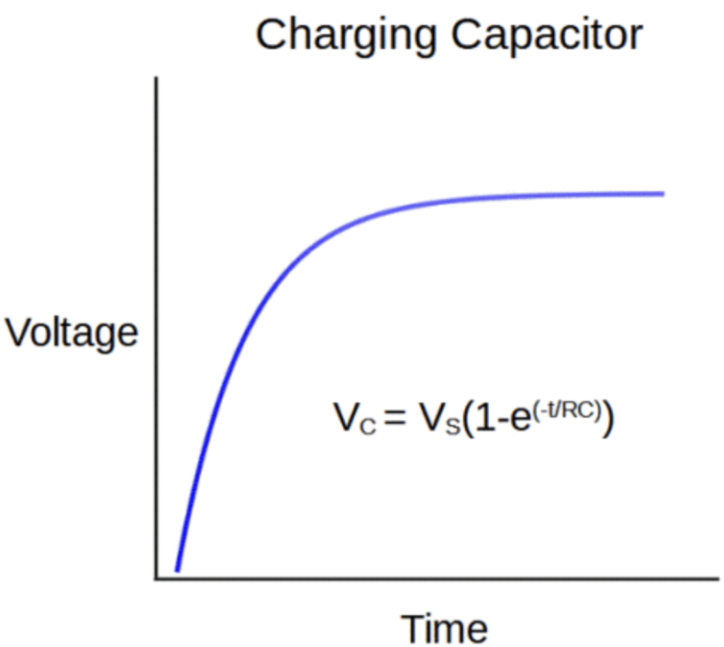
- Therefore, the total heat energy is given,

$$\boxed{Q = \int_0^t q\, dt = \boxed{hA_s \int_0^t \theta\, dt} = \boxed{(\rho V c)\theta_i \left[1 - \exp\left(-\frac{t}{\tau_t}\right)\right]} = -\Delta \boldsymbol{E_{st}}} \quad (5.8)$$

5.1 Lumped Capacitance Method

- What about when charging a capacitor?





- Does this look similar to something being heated up?
- If yes, should the temperature distribution equation (5.6) change?

- Read Text Section 5.1 (Last 2 sentences)

Biot and Fourier Number

Section 5.2

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The Biot Number:

- First of the many dimensionless parameters to be considered.

➢ Definition:

$$\boxed{Bi \equiv \frac{hL_c}{k}}$$

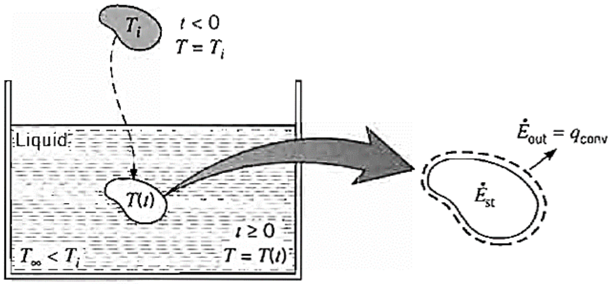
$h \rightarrow$ convection or radiation coefficient

$k \rightarrow$ thermal conductivity of the solid

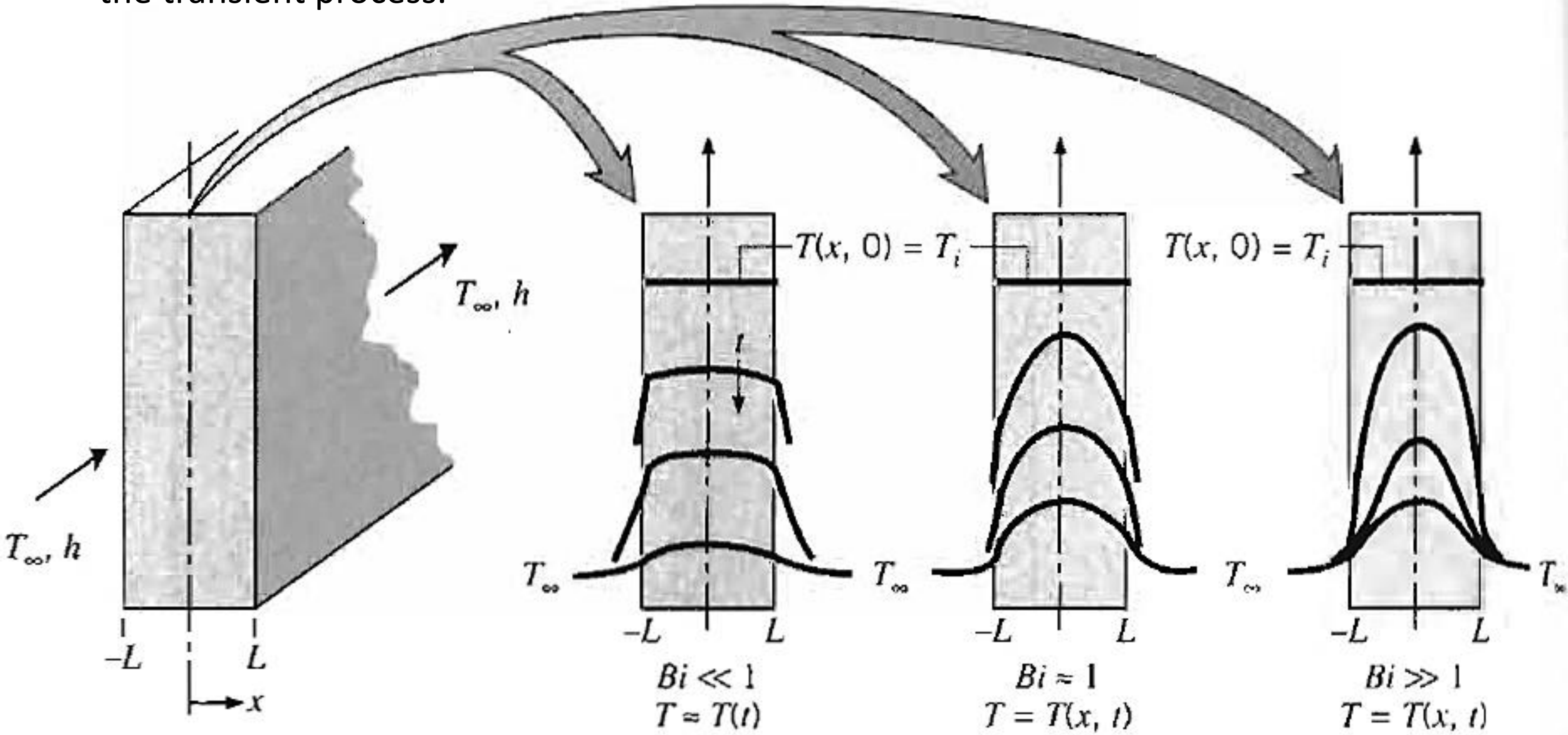
$L_c \rightarrow$ characteristic length of the solid
(V/A_s or **distance associated with maximum spatial temperature difference**)
Normally:
***L*/2 for wall, r_o for sphere (sometimes r_o/3) and cylinder (sometimes r_o/2). But when is this "sometimes"?**

➢ Physical Interpretation:

$$Bi = \frac{L_c / kA_s}{1/hA_s} \approx \boxed{\frac{R_{cond}}{R_{conv}}} \approx \frac{\Delta T_{solid}}{\Delta T_{solid/fluid}}$$



BIG assumption: A spatially uniform temperature distribution in solid throughout the transient process.



➢ Criterion for Applicability of Lumped Capacitance Method:

$$\boxed{\begin{array}{c} Bi << 1 \\ \text{But normally, } \mathbf{Bi < 0.1} \end{array}}$$

ALWAYS check this before applying Lumped Capacitance Method

The Fourier Number:

- From (5.6)

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho Vc}\right)t\right]$$

- From the exponential coefficient

$$\frac{hA_s t}{\rho Vc} = \frac{ht}{\rho c L_c} = \frac{hL_c}{k}\frac{k}{\rho c}\frac{t}{L_c^2} = \frac{hL_c}{k}\frac{\alpha t}{L_c^2} = \boxed{Bi \cdot Fo}$$

➢ Definition:

$$\boxed{Fo \equiv \frac{\alpha t}{L_c^2}}$$

$\alpha \rightarrow$ thermal diffusivity $\rightarrow \dfrac{k}{\rho c}$

$L_c \rightarrow$ characteristic length of the solid
(V/A_s or distance associated with **maximum spatial temperature difference**)

➢ Physical Interpretation:

Dimensionless time; How fast heat is transported compared to being stored. If it is high what does it mean? How will the temperature profile look like?

General Lumped Capacitance Method (more terms...)

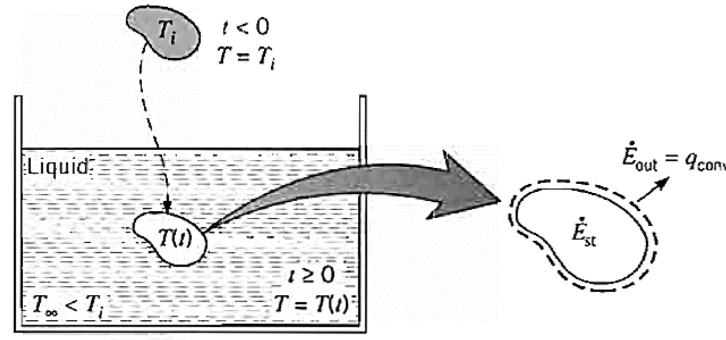
Section 5.3

5.3 General Lumped Capacitance Analysis

Originally,

$$\dot{E}_{in} - \dot{E}_{\text{out}} + \dot{E}_{\text{g}} = \dot{E}_{st}$$

$$\dot{E}_{\text{out}} = -\dot{E}_{st} \qquad (5.2)$$



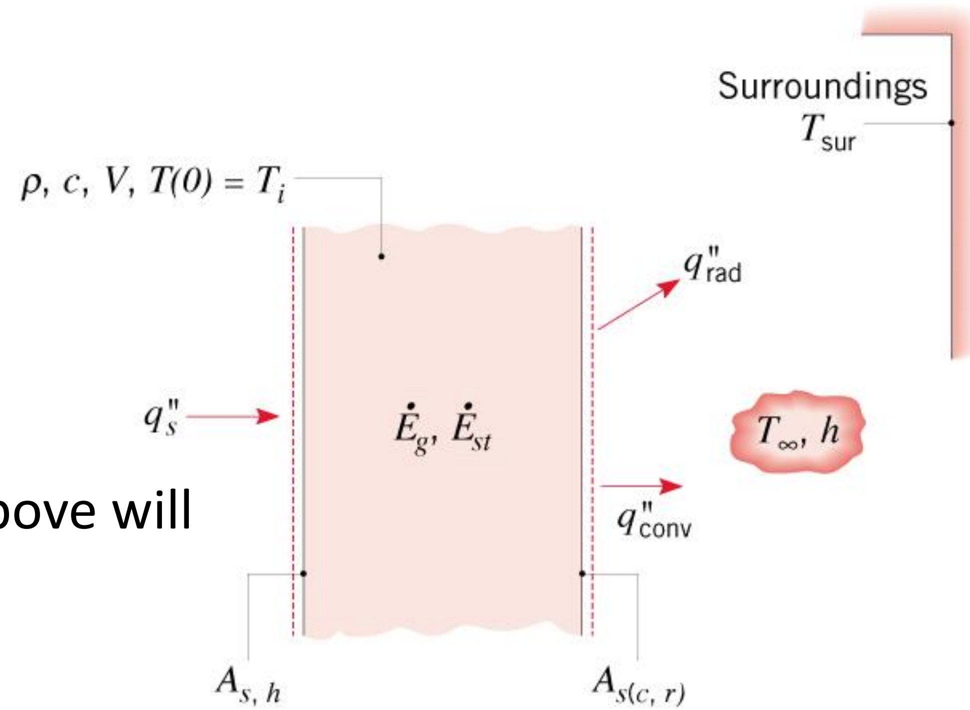
But no heat in, heat out or generation!

Consider a general case which has

- Convection (c)
- Radiation (r)
- An applied heat flux at specified surfaces
- Internal energy generation

$$(A_{s,c}, A_{s,r}, A_{s,h})$$

- A sudden change in any of the above will change the heat transfer



How to analyze?

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➢ First Law:

$$\frac{dE_{st}}{dt} = \rho V c \frac{dT}{dt} = \dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_g$$

- Assuming energy
  - outflow due to convection and radiation
  - inflow q_s'', due to an applied heat flux

$$\boxed{\rho V c \frac{dT}{dt} = q_s'' A_{s,h} - h A_{s,c}(T - T_\infty) - h_r A_{s,r}(T - T_{\text{sur}}) + \dot{E}_g} \quad (5.15)$$

- h and h_r are normally assumed to be constant throughout the transient process

- How to solve?
  - (5.15) cannot be integrated to get an exact solution
  - Want to get an close-form solution? Try simplifying once!

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- Negligible Convection and Source Terms $\left(h_r >> h,\ \dot{E}_g = 0,\ q_s'' = 0\right)$:

(5.15) $$\rho V c \frac{dT}{dt} = q_s'' A_{s,h} - h A_{s,c}(T - T_\infty) - h_r A_{s,r}(T - T_{\text{sur}}) + \dot{E}_g$$

Assuming radiation exchange with large surroundings and $\alpha = \varepsilon$,

$$\rho V c \frac{dT}{dt} = -\varepsilon A_{s,r} \sigma \left(T^4 - T_{\text{sur}}^4\right)$$

How?

$$\frac{\varepsilon A_{s,r} \sigma}{\rho V c} \int_0^t dt = \int_{T_i}^{T} \frac{dT}{T_{\text{sur}}^4 - T^4}$$

$$t = \frac{\rho V c}{4 \varepsilon A_{s,r} \sigma T_{sur}^3} \left\{ \ln \left| \frac{T_{\text{sur}} + T}{T_{\text{sur}} - T} \right| - \ln \left| \frac{T_{\text{sur}} + T_i}{T_{\text{sur}} - T_i} \right| + 2 \left[\tan^{-1}\left(\frac{T}{T_{\text{sur}}} \right) - \tan^{-1}\left(\frac{T_i}{T_{\text{sur}}} \right) \right] \right\} \quad (5.18)$$

- here, $T(0) \equiv T_i$

This result will require implicit evaluation of T.

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- Negligible Radiation $(Define\ \theta \equiv T - T_\infty, \Rightarrow\ \theta' \equiv \theta - b/a)$:

$$\rho V c \frac{dT}{dt} = q_s'' A_{s,h} - h A_{s,c}(T - T_\infty) - h_r A_{s,r}(T - T_{\text{sur}}) + \dot{E}_g$$

The non-homogeneous differential equation is transformed into a homogeneous equation of the form:

$$\frac{d\theta'}{dt} = -a\theta + b$$

With $a \equiv \mathrm{h} A_{s,c}/\rho V c$ $\quad b \equiv \left(q_s'' A_{s,h} + \dot{E}_g\right)/\rho V c$

Integrating from t = 0 to any t and rearranging,

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp(-at) + \frac{b/a}{T_i - T_\infty}\left[1 - \exp(-at)\right] \qquad (5.25)$$

What form does (5.25) reduce to as steady state is approached?

ZJUI

- Negligible Radiation and Source Terms $\left(h >> h_r,\ \dot{E}_g = 0,\ q_s'' = 0 \right)$:

$$\rho V c \frac{dT}{dt} = q_s'' A_{s,h} - h A_{s,c}(T - T_\infty) - h_r A_{s,r}(T - T_{\mathrm{sur}}) + \dot{E}_g$$

$$\rho V c \frac{dT}{dt} = -\mathrm{h} A_{s,c}(T - T_\infty) \qquad (5.2)$$

- If h changes with temperature (normally it does), often it can be approximated as:

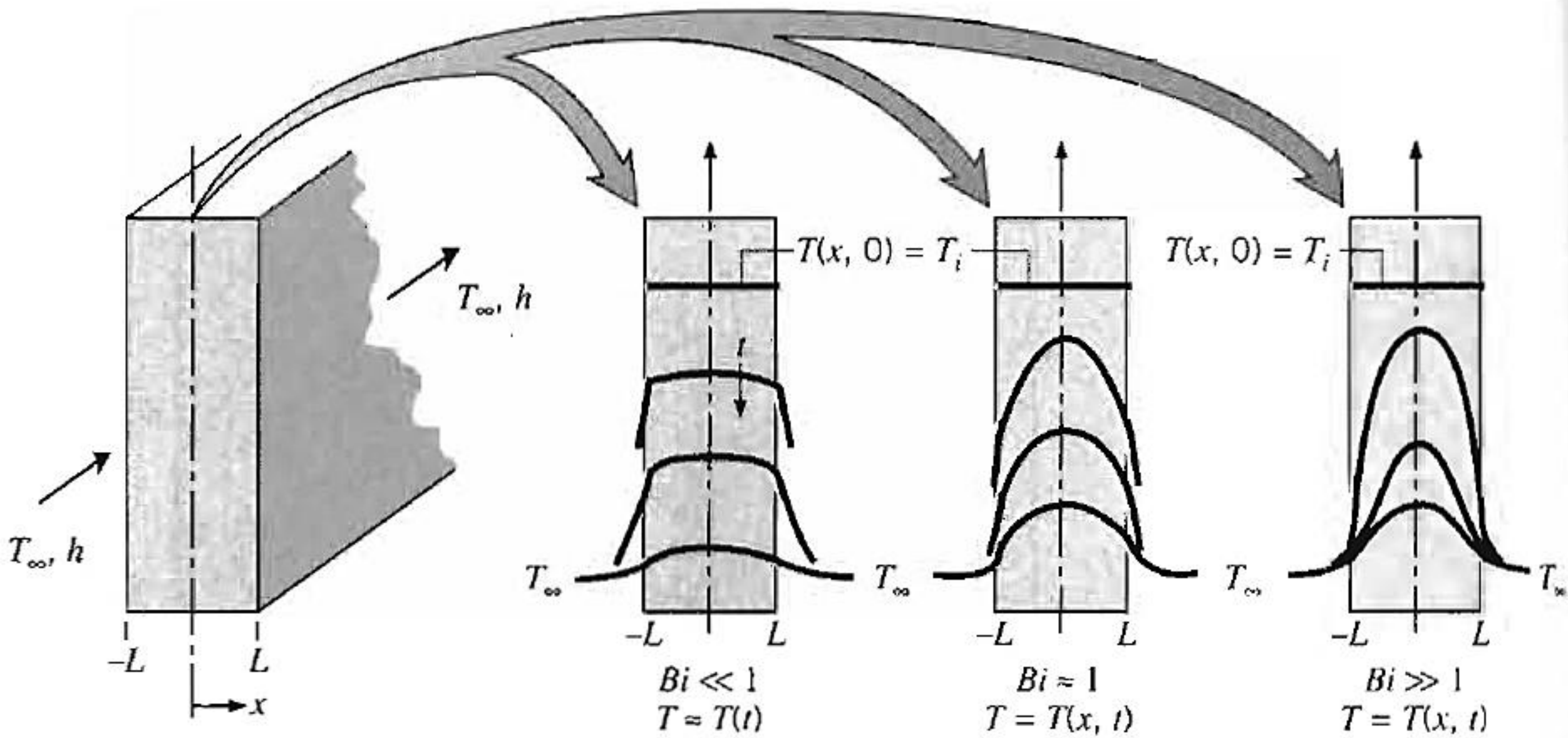
$$h = C(T - T_\infty)^{\mathrm{n}} \qquad (5.26)$$

$$\Rightarrow \frac{\theta}{\theta_i} = \left[\frac{nCA_{s,c}\theta_i^n}{\rho V c} t + 1 \right]^{-1/n} \qquad (5.28)$$

Note: $\theta \equiv T - T_\infty$; $d\theta = dT$

When Biot >> 0.1 => Spatial temperature variations !

Section 5.4



T_∞, h
T_∞, h
$-L$
L
x
$T(x, 0) = T_i$
$T(x, 0) = T_i$
t
T_∞
T_∞
T_∞
T_∞
$-L$
L
$Bi \ll 1$
$T \approx T(t)$
$-L$
L
$Bi \approx 1$
$T = T(x, t)$
$-L$
L
$Bi \gg 1$
$T = T(x, t)$

If $Bi > 0.1$ => How?

What if Biot >> 0.1?

Lumped capacitance approximation cannot be made

- Must consider spatial and temporal temperature variations during the transient process.
- Solution will have to start from,

$$\dot{E}_{in} - \dot{E}_{\mathrm{out}} + \dot{E}_{\mathrm{g}} = \dot{E}_{st}$$

- Solve the governing equation analytically or numerically
- Can non-dimensionalize so that the final solution form is independent of the exact values of the different variables involved in the HT situation.

- Exact Solution is difficult
  - See Section 5.4 of Text

Optional:
Approximate Solution for Plane Wall with convection (easier way)

Section 5.5

Plane Wall Example – One-Term Approximation

The One-Term Approximation $(\textbf{Valid for } \boldsymbol{Fo > 0.2})$:
Error is about 2%

- Variation of temperature with location (x^*) and time (Fo) :

$$\theta^* = C_1 \exp\left(-\zeta_1^2 Fo\right) cos\left(\zeta_1 \; x^*\right) \qquad (5.43a)$$

- But, variation of midplane temperature (θ_o^* *at* $x^* = 0$) with time (Fo):

$$\theta_o^* \equiv \frac{(T_o - T_\infty)}{(T_i - T_\infty)} \approx C_1 \exp\left(-\zeta_1^2 Fo\right) \qquad (5.44)$$

See Table 5.1 $\rightarrow$ C_1 and ζ_1 as a function of Bi

- So, (5.43a) can be written as :

$$\theta^* = \theta_o^* \cos(\zeta_1 x^*) \qquad (5.43b)$$

So what does the θ^* depend on?

Plane Wall Example – One-Term Approximation

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- Change in thermal energy storage with time:
  - Start with conservation of energy,

$$\Delta E_{st} = -Q \Rightarrow Q \text{ is the net energy until that time} \qquad (5.46a)$$

$$Q = -\int \rho c\,[\mathrm{T}(x,t) - T_i]\,dV$$

$$\boxed{\Rightarrow Q = Q_o\left(1 - \frac{\sin\zeta_1}{\zeta_1}\theta_o^*\right) \qquad (5.49)}$$

where $Q_o = \rho c V(T_i - T_\infty)$ (5.47)

- So, what is the meaning of Q_o?

Plane Wall Example – One-Term Approximation

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TABLE 5.1 Coefficients used in the one-term approximation to the series solutions for transient one-dimensional conduction

	Plane Wall		Infinite Cylinder		Sphere	
Bi^a	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1	ζ_1 (rad)	C_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.03	0.1723	1.0049	0.2440	1.0075	0.2991	1.0090
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.05	0.2218	1.0082	0.3143	1.0124	0.3854	1.0149
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.07	0.2615	1.0114	0.3709	1.0173	0.4551	1.0209
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.09	0.2956	1.0145	0.4195	1.0222	0.5150	1.0268
0.10	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.15	0.3779	1.0237	0.5376	1.0365	0.6609	1.0445
0.20	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.25	0.4801	1.0382	0.6856	1.0598	0.8447	1.0737
0.30	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0932	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0919	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5994	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2402	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	1.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2881	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2733	2.4050	1.6018	3.1415	2.0000

[a] $Bi = hL/k$ for the plane wall and hr_o/k for the infinite cylinder and sphere. See Figure 5.6.

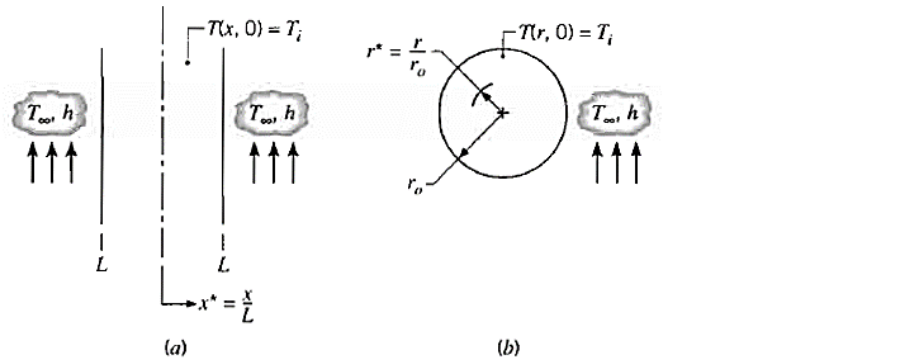

FIGURE 5.6 One-dimensional systems with an initial uniform temperature subjected to sudden convection conditions. (*a*) Plane wall. (*b*) Infinite cylinder or sphere.

$$\theta^* = C_1 \exp\left(-\zeta_1^2 Fo\right) cos\left(\zeta_1 \; x^*\right)$$

Heisler Chart

Graphical form for one-term approximation

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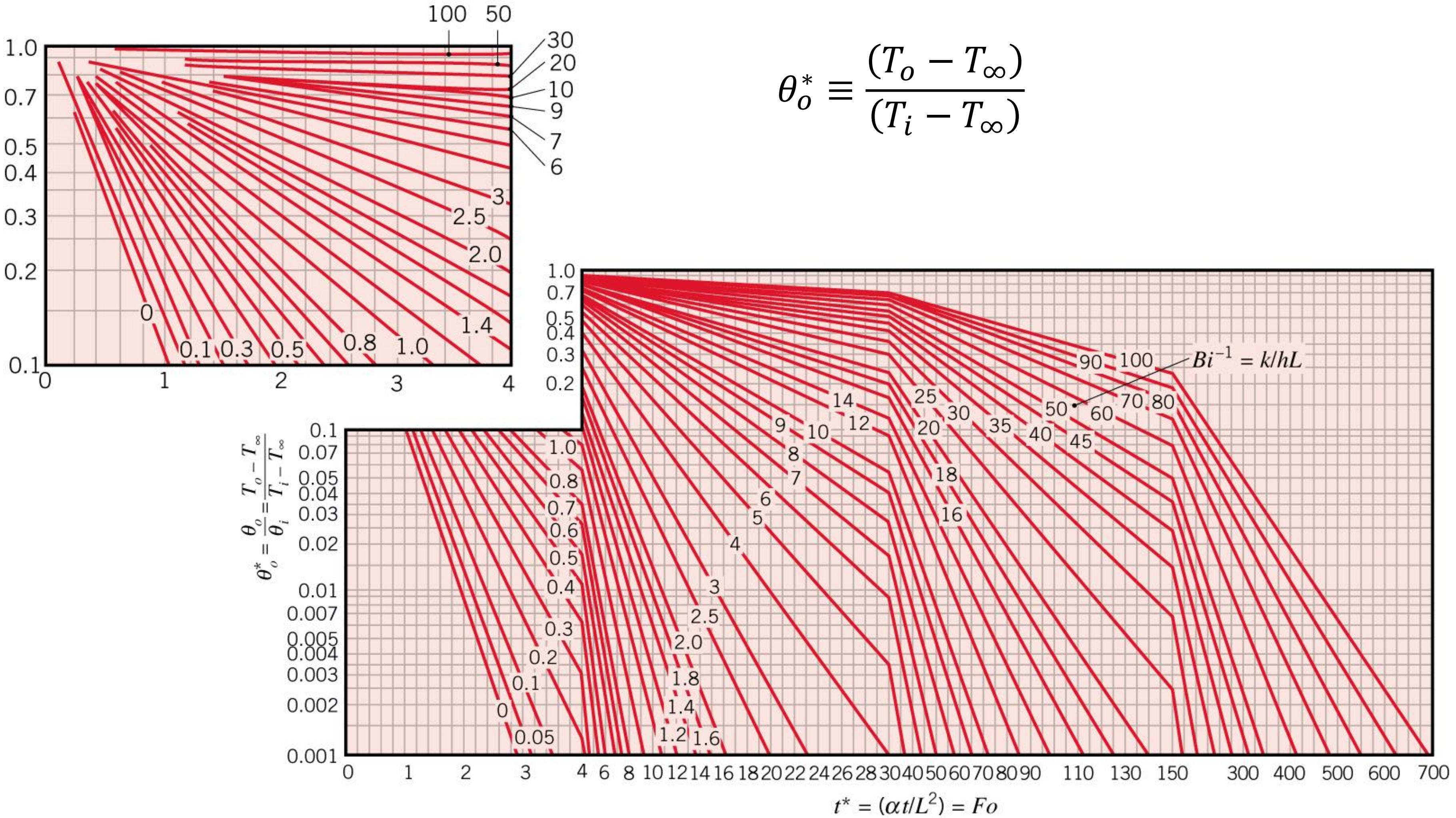
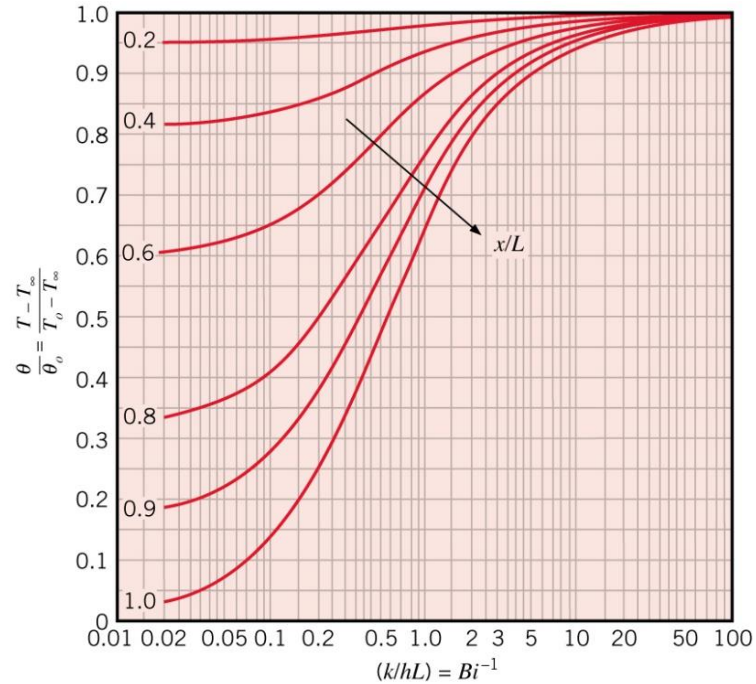
The Heisler Charts for Plane Wall, Section 5 S.1

- Midplane Temperature:

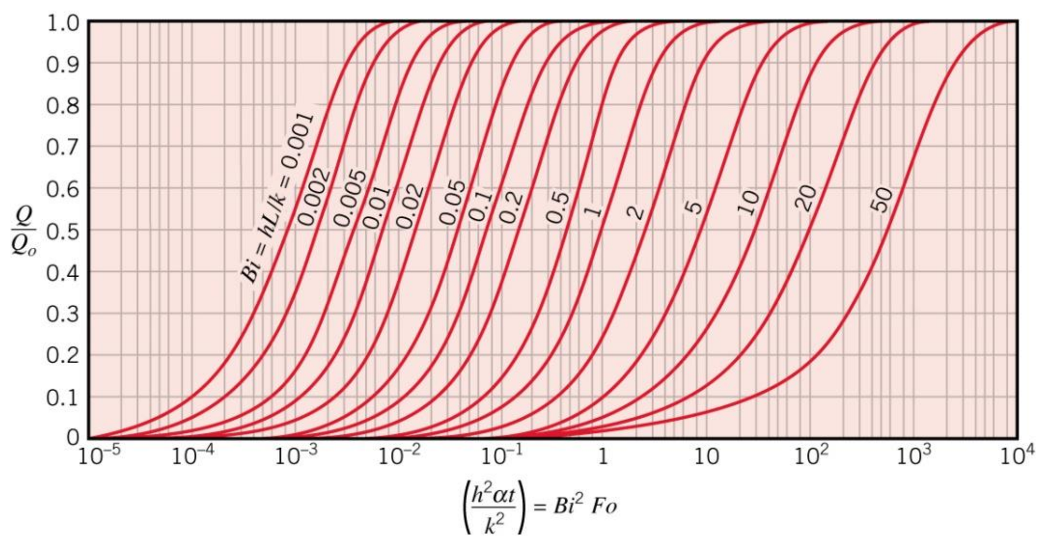
$$\theta_o^* \equiv \frac{(T_o - T_\infty)}{(T_i - T_\infty)}$$



- Temperature Distribution at other part of the wall:



- Change in Thermal Energy Storage:



1. Plug in values of α, t, and characteristic length into the Fourier's number equation. This will be your x-axis value.

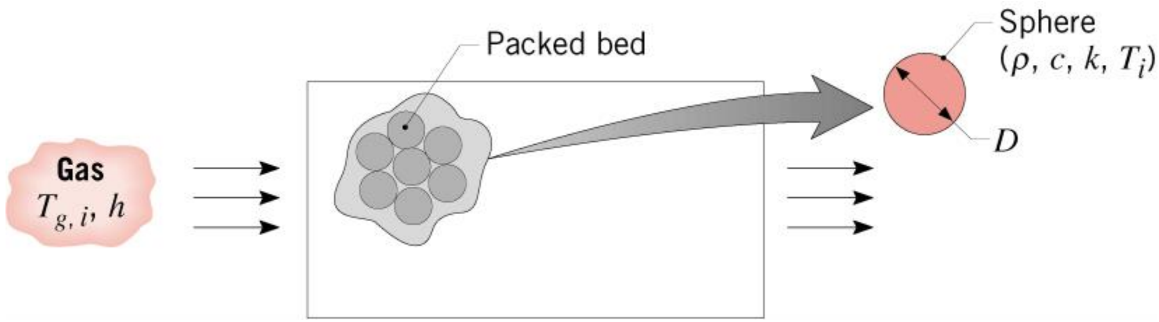
2. Plug in values of k, h, and characteristic length into the inverse Biot number equation.

3. Get the temperature needed

Summary

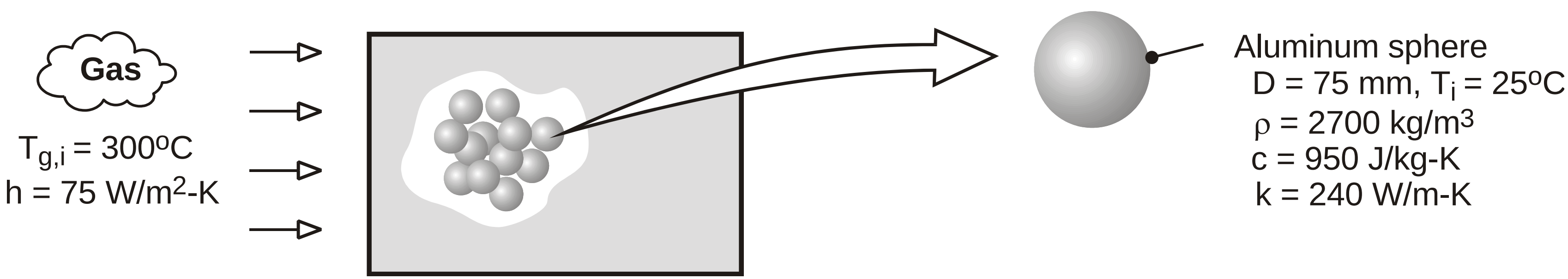
- Factors that can cause Transient HT
- Lumped Capacitance Method
  - Biot number
  - Fourier number
- General Lumped Capacitance Method

Problem 5.13: Charging a thermal energy storage system consisting of a packed bed of aluminum spheres.



FIND: Time required for sphere at inlet to acquire 90% of maximum possible thermal energy and the corresponding center temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Negligible heat transfer to or from a sphere by radiation or conduction due to contact with other spheres, (2) Constant properties.

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ANALYSIS: $$-hA_s(T - T_\infty) = \rho Vc\frac{dT}{dt}$$

To determine whether a lumped capacitance analysis can be used, first compute $\boxed{Bi = h(r_o/3)/k}$ = 75 W/m²·K (0.0125 m)/240 W/m·K = 0.004 << 0.1.

Hence, the lumped capacitance approximation may be made, and a uniform temperature may be assumed to exist in the sphere at any time.

From Eq. 5.8, achievement of 90% of the maximum possible thermal energy storage corresponds to

$$\boxed{Q = (\rho Vc)\theta_i\left[1 - \exp\left(-\frac{t}{\tau_t}\right)\right] = -\Delta \boldsymbol{E}_{\boldsymbol{st}}}$$

$$-\frac{\Delta E_{\mathrm{st}}}{\rho c V\theta_i} = 0.90 = 1 - \exp(-t/\tau_t)$$

$$\tau_t = \rho Vc/hA_s = \rho Dc/6h = \frac{2700\mathrm{kg/m^3} \times 0.075\ \mathrm{m} \times 950\mathrm{J/kg \cdot K}}{6 \times 75\mathrm{W/m^2 \cdot K}} = 427\ \mathrm{s}$$

$$t = -\tau_t \ln(0.1) = 427\ \mathrm{s}\ \times 2.30 = 984\ \mathrm{s}$$

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From Eq. (5.6), the corresponding temperature at any location in the sphere is

$$\boxed{\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\left(\frac{hA_s}{\rho V c} \right) t \right]}$$

$$T(984\ \text{s}) = T_{\infty,i} + \left(T_i - T_{\infty,i}\right) \exp(-6ht/\rho Dc)$$

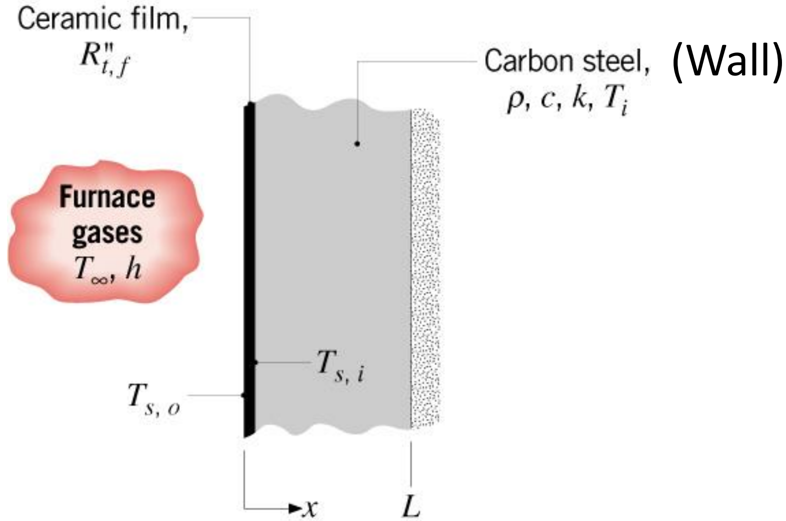
$$T\left(984\ \text{s}\right) = 300°\text{C} - 275°\text{C}\ \exp\left(-6 \times 75\,\text{W/m}^2 \cdot \text{K} \times 984\ \text{s} / \left(2700\,\text{kg/m}^3 \times 0.075\ \text{m} \times 950\,\text{J/kg} \cdot \text{K}\right)\right)$$

$$T(984\ \text{s}) = 272.5°\text{C}$$

If the product of the density and specific heat of copper is $(\rho c)_{\text{Cu}} \approx 8900\ \text{kg/m}^3 \times 400\ \text{J/kg·K} = 3.56 \times 10^6\ \text{J/m}^3\text{·K}$, is there any advantage to using copper spheres of equivalent diameter in lieu of aluminum spheres?

Does the time required for a sphere to reach a prescribed state of thermal energy storage change with increasing distance from the bed inlet? If so, how and why?

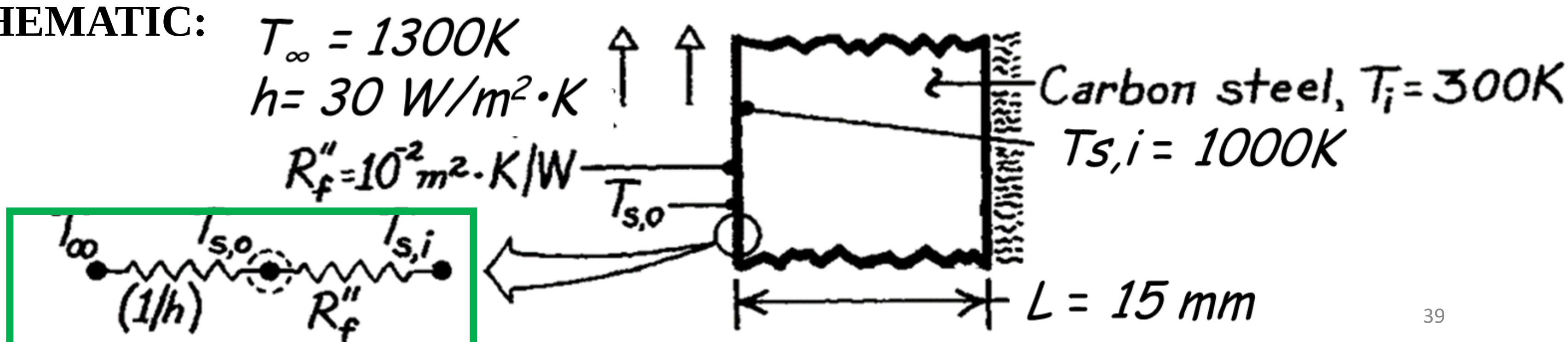
Problem 5.19: Heating of coated furnace wall during start-up.



KNOWN: Thickness and properties of furnace wall. Thermal resistance of ceramic coating on surface of wall exposed to furnace gases. Initial wall temperature.

FIND: (a) Time required for surface of wall to reach a prescribed temperature,
(b) Corresponding value of coating surface temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Constant properties, (2) Negligible coating thermal capacitance, (3) Negligible radiation.

PROPERTIES: Carbon steel: $\rho = 7850\ \mathrm{kg/m^3}$, $c = 430\ \mathrm{J/kg \cdot K}$, $k = 60\ \mathrm{W/m \cdot K}$.

ANALYSIS:

$$\boxed{Bi = \frac{UL}{k} \qquad \text{Check if Lumped Capacitance can be used.}}$$

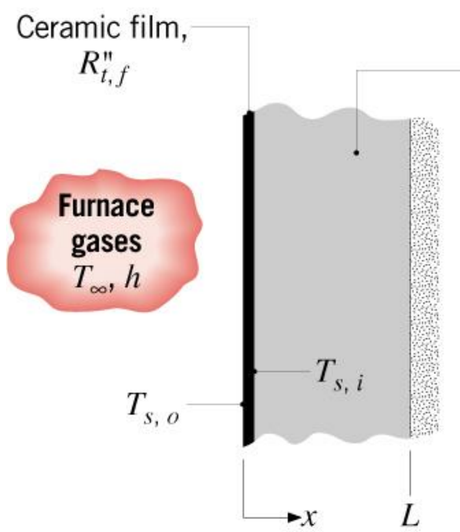
$$= \frac{23.1\ \mathrm{W/m^2 \cdot K} \times 0.015\ \mathrm{m}}{60\ \mathrm{W/m \cdot K}}$$

$$= 0.0058 << 0.1$$

where $\boxed{U = \left(R''_{\mathrm{tot}}\right)^{-1} = \left(\frac{1}{h} + R''_f\right)^{-1}}$

$$= \left(\frac{1}{30\ \mathrm{W/m^2 \cdot K}} + 10^{-2}\mathrm{m^2} \cdot \mathrm{K/W}\right)^{-1}$$

$$= 23.1\ \mathrm{W/m^2} \cdot \mathrm{K}.$$



Why *U* not *h*?

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(a) From Eqs. (5.6) and (5.7),

$$\frac{T - T_\infty}{T_i - T_\infty} = \exp(-t/\tau_t) = \exp(-t/R_t C_t) = \exp(-Ut/\rho L c)$$

$$t = -\frac{\rho L c}{U} \ln \frac{T - T_\infty}{T_i - T_\infty}$$

$$= -\frac{7850\ \mathrm{kg/m^3} * (0.015\ \mathrm{m}) * 430\ \mathrm{J/kg{\cdot}K}}{23.1\ \mathrm{W/m^2{\cdot}K}} \ln \frac{1000-1300}{300-1300}$$

$$t = 2642\ \mathrm{s} = 0.734\ \mathrm{h}.$$

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Example 2: Furnace Start-up

(b) Performing an energy balance at the outer surface (*s*,*o*),

$$h\left(T_\infty - T_{s,o}\right) = \left(T_{s,o} - T_{s,i}\right)/R_f''$$

$$T_{s,o} = \frac{hT_\infty + T_{s,i}/R_f''}{h + \left(1/R_f''\right)}$$

$$= \frac{30\ \mathrm{W/m^2 \cdot K} \times 1300\ \mathrm{K} + 1000\ \mathrm{K}/10^{-2}\mathrm{m^2 \cdot K/W}}{(30 + 100)\mathrm{W/m^2 \cdot K}}$$

$$T_{s,o} = 1069\ \mathrm{K}.$$

How does the coating affect the thermal time constant?