Lec16: Free Convection - General Considerations and Results for Vertical and Horizontal Plates

Chapter 9

Announcement

ZJUI

- Exam on 6 Dec
  - Lec 9 – 17
  - Convention
    - External Flow
    - Internal Flow
    - Heat Exchanger
    - Natural Convection

TABLE 8.4 Summary of convection correlations for flow in a circular tube[a,b,e]

Correlation		Conditions
$f = 64/Re_D$	(8.19)	Laminar, fully developed
$Nu_D = 4.36$	(8.53)	Laminar, fully developed, uniform q''_s
$Nu_D = 3.66$	(8.55)	Laminar, fully developed, uniform T_s
$\overline{Nu}_D = 3.66 + \dfrac{0.0668(D/L)Re_D Pr}{1 + 0.04[(D/L)Re_D Pr]^{2/3}}$	(8.56)	Laminar, thermal entry (or combined entry with $Pr \gtrsim 5$), uniform T_s
or		
$\overline{Nu}_D = 1.86\left(\dfrac{Re_D Pr}{L/D}\right)^{1/3}\left(\dfrac{\mu}{\mu_s}\right)^{0.14}$	(8.57)	Laminar, combined entry, $0.6 \lesssim Pr \lesssim 5$, $0.0044 \lesssim (\mu/\mu_s) \lesssim 9.75$, uniform T_s
$f = 0.316\, Re_D^{-1/4}$	(8.20a)[c]	Turbulent, fully developed, $Re_D \lesssim 2 \times 10^4$
$f = 0.184\, Re_D^{-1/5}$	(8.20b)[c]	Turbulent, fully developed, $Re_D \gtrsim 2 \times 10^4$
or		
$f = (0.790 \ln Re_D - 1.64)^{-2}$	(8.21)[c]	Turbulent, fully developed, $3000 \lesssim Re_D \lesssim 5 \times 10^6$
$Nu_D = 0.023 Re_D^{4/5}\, Pr^n$	(8.60)[d]	Turbulent, fully developed, $0.6 \lesssim Pr \lesssim 160$, $Re_D \gtrsim 10{,}000$, $(L/D) \gtrsim 10$, $n = 0.4$ for $T_s > T_m$ and $n = 0.3$ for $T_s < T_m$
or		
$Nu_D = 0.027 Re_D^{4/5}\, Pr^{1/3}\left(\dfrac{\mu}{\mu_s}\right)^{0.14}$	(8.61)[d]	Turbulent, fully developed, $0.7 \lesssim Pr \lesssim 16{,}700$, $Re_D \gtrsim 10{,}000$, $L/D \gtrsim 10$
or		
$Nu_D = \dfrac{(f/8)(Re_D - 1000)Pr}{1 + 12.7(f/8)^{1/2}(Pr^{2/3} - 1)}$	(8.62)[d]	Turbulent, fully developed, $0.5 \lesssim Pr \lesssim 2000$, $3000 \lesssim Re_D \lesssim 5 \times 10^6$, $(L/D) \gtrsim 10$
$Nu_D = 4.82 + 0.0185(Re_D\, Pr)^{0.827}$	(8.64)	Liquid metals, turbulent, fully developed, uniform q''_s, $3.6 \times 10^3 \lesssim Re_D \lesssim 9.05 \times 10^5$, $10^2 \lesssim Pe_D \lesssim 10^4$
$Nu_D = 5.0 + 0.025(Re_D\, Pr)^{0.8}$	(8.65)	Liquid metals, turbulent, fully developed, uniform T_s, $Pe_D \gtrsim 100$

Content

1. What is the Grashof number? What is the Rayleigh number?

2. What creates Buoyancy-driven flow?

3. How the velocity profile in the free convection boundary layer on a heated vertical plate differ from the velocity profile in the boundary layer associated with forced flow over a parallel plate?

4. What is the general buoyancy term for a free convection boundary? How may it be approximated if the flow is due to temperature variations? What is the name of the approximation?

5. For a heated horizontal plate in quiescent air, do you expect heat transfer to be larger for the top or bottom surface?

6. What is meant by the term mixed convection? Under what conditions is heat transfer enhanced by mixed convection? Under what conditions is it reduced?

General Considerations

Free convection == no external pumping/blowing force

- Which is bigger? *h* from free convection and *h* from forced convection?
- Is it less important?
  - If smaller *h* => large HT resistance
  - Study to reduce operating cost
  - Minimize HT in building/winter clothes

Free convection induced by buoyancy forces.

- Buoyancy forces in a fluid from
  - density gradients
  - body force that is proportional to density (like in atmosphere, rotating machines).

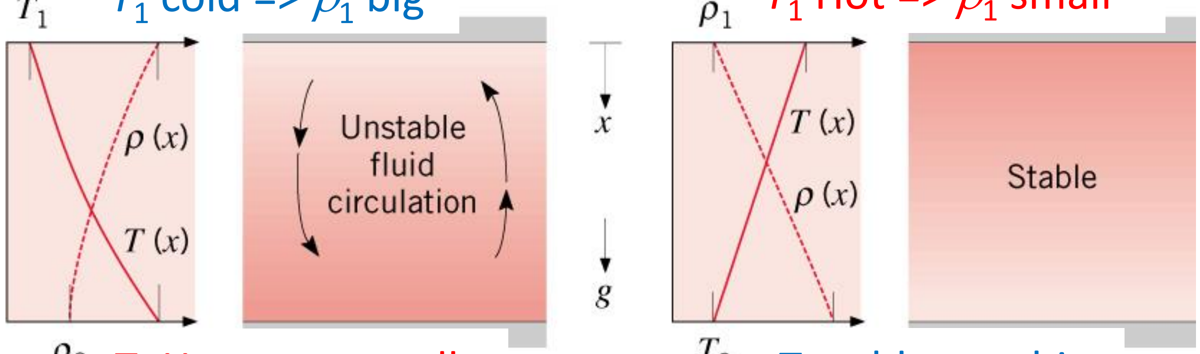
- In heat transfer,
  - density gradients are due to temperature gradients
  - body force is gravitational.

Will free convection always occur when there is density gradient/temperature gradient?

Unstable and Stable Temperature Gradients

T_1 cold => ρ_1 big

T_1 Hot => ρ_1 small



T_2 Hot => ρ_2 small

T_2 cold => ρ_2 big

$$\frac{dT}{dx} > 0, \frac{d\rho}{dx} < 0$$

$$\frac{dT}{dx} < 0, \frac{d\rho}{dx} > 0$$

Free Convection type:

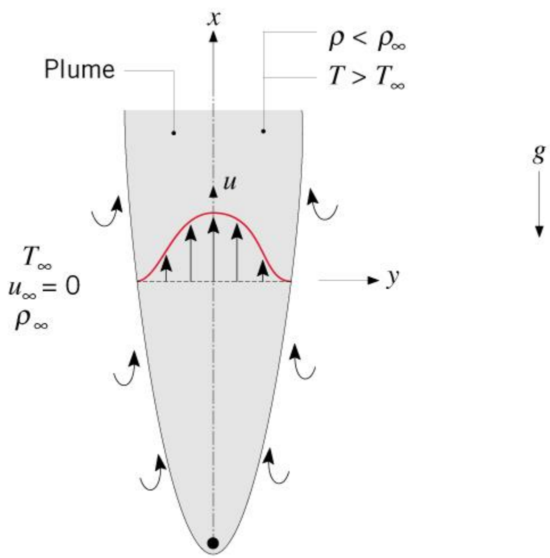
- Free Boundary Flows
- Bounded Boundary Flows

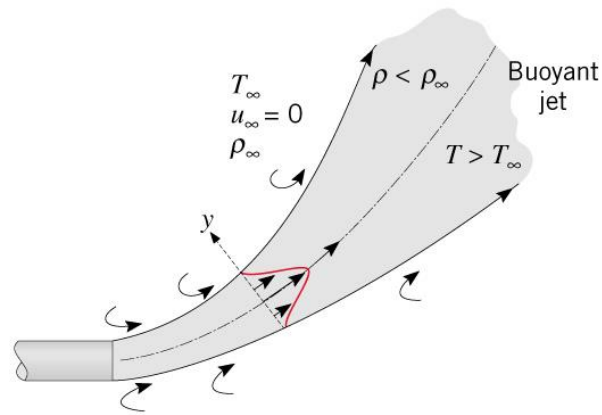
General Considerations (cont.)

Free Boundary Flows

- Not near objects
- In an extensive (or infinite), quiescent fluid (motionless surrounding at locations far from the hot/cold/disturbance source).

- Plumes and Buoyant Jets:



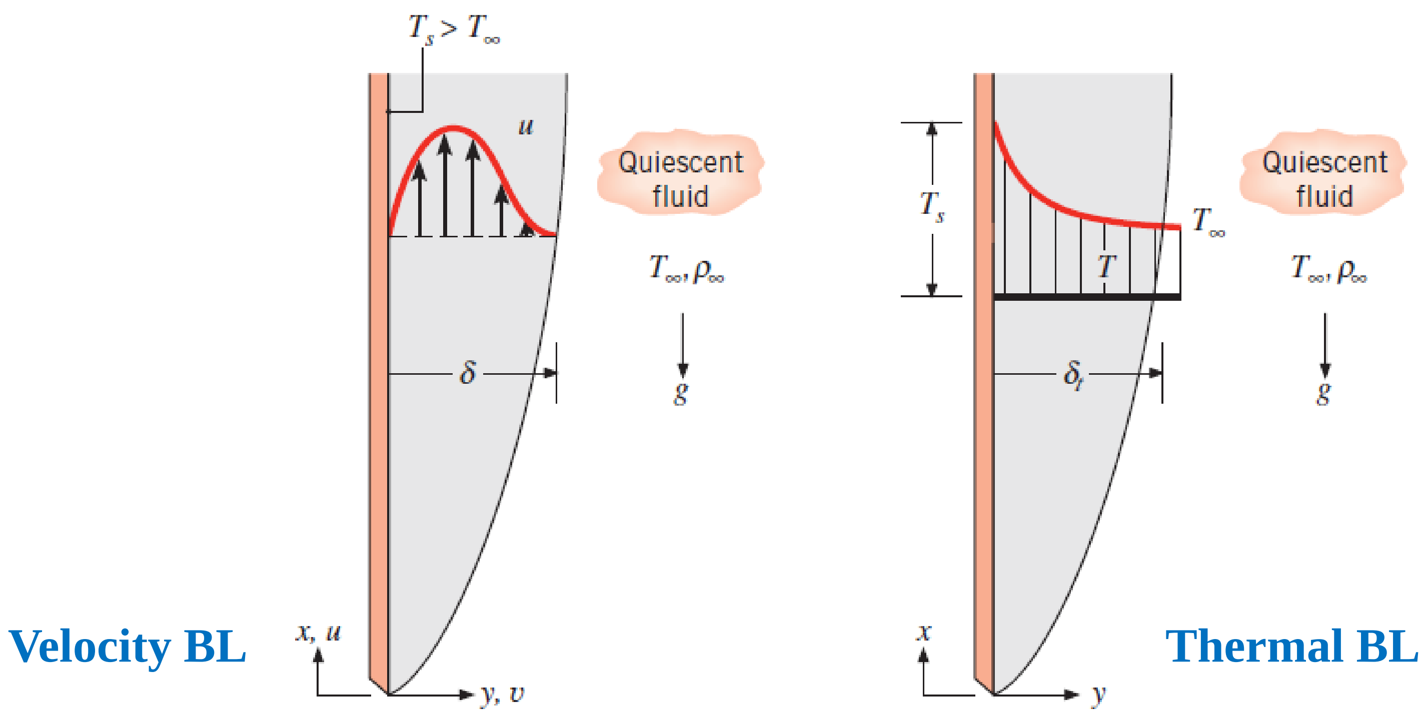


- Hot sphere in a cold still fluid
- Wire heating in still air

- Hot fluid jets discharged from a tube

Bounded Boundary Flow

- Course focuses on this



Example:

- Hot vertical plate => natural/free convection
- How does the velocity BL develop?
- Does the velocity distribution look similar but not the same as from forced convection over a flat plate?
  - $V_y \to \infty = 0$

Governing Equation

Section 9.2

The *x*-Momentum Equation for Laminar Flow

Starting, $$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp_\infty}{dx}-g+\nu\frac{\partial^2 u}{\partial y^2}$$

Defining, $$\beta\approx-\frac{1}{\rho}\frac{\Delta\rho}{\Delta T}=-\frac{1}{\rho}\frac{\rho_\infty-\rho}{T_\infty-T}$$

Boussinesq Approx.

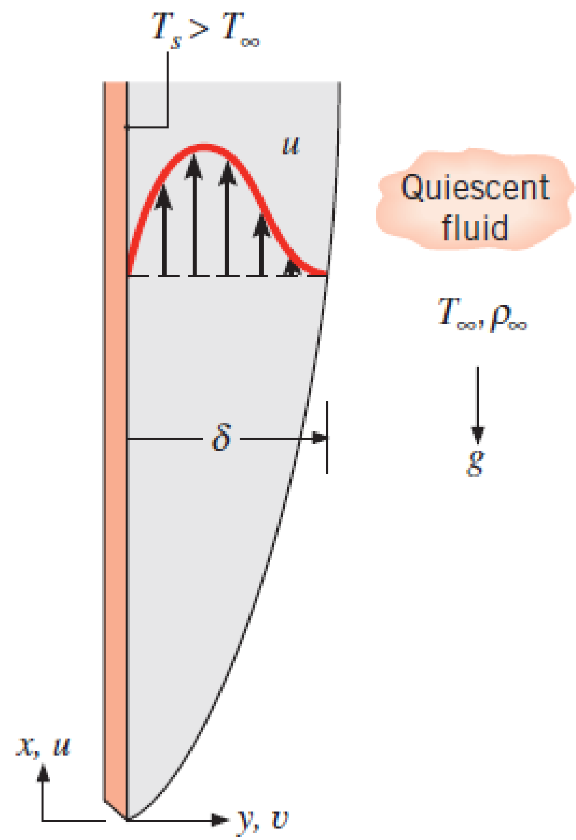
β -- the volumetric thermal expansion coefficient

and

$$\frac{dp_\infty}{dx}=-\rho_\infty g$$

as u = 0 in the Quiescent fluid

So, $$\underbrace{u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}}_{\text{Net Momentum Fluxes (Inertia Forces)}}=\underbrace{g\beta(T-T_\infty)}_{\text{Buoyancy Force}}+\underbrace{\nu\frac{\partial^2 u}{\partial y^2}}_{\text{Viscous Force}}$$

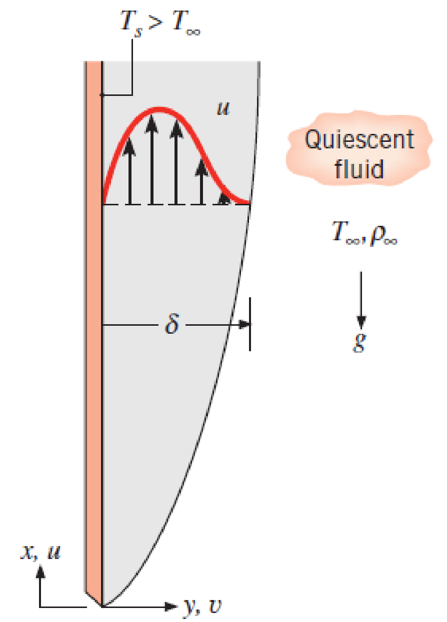


- The *x*-Momentum Equation for Laminar Flow

$$\underbrace{u\frac{\partial u}{\partial x}+\upsilon\frac{\partial u}{\partial y}}_{\substack{\text{Net Momentum Fluxes}\\ \text{(Inertia Forces)}}}=\underbrace{g\beta(T-T_\infty)}_{\text{Buoyancy Force}}+\underbrace{\nu\frac{\partial^2 u}{\partial y^2}}_{\text{Viscous Force}} \qquad (9.7)$$

$$\beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty-\rho}{T_\infty-T}$$

  - What β is this for ideal gas? Hint: ρ = P/(RT)

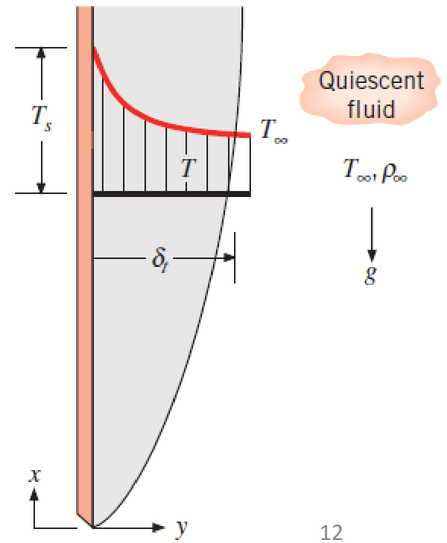


- Temperature dependence of the bouyancy force requires that solution for $u\,(x,y)$ be obtained concurrently with solution of the boundary layer energy equation for $T\,(x,y)$.

$$u\frac{\partial T}{\partial x}+\upsilon\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2} \qquad (9.8)$$

– The solutions of velocity and thermal BLs are said to be coupled. Where are these equations coupled?

– How to solve?



Dimensionless Analysis

Section 9.3

Let us get lazy and use similarity to solve complex coupled PDEs

With BL, it is easiest to look at these equations through dimensionless variables:

$$u^* \equiv \frac{u}{u_0} \qquad v^* \equiv \frac{v}{u_0} \qquad T^* \equiv \frac{T - T_\infty}{T_s - T_\infty}$$

- Therefore, (9.7) and (9.8) become:,

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \boxed{\frac{g\beta(T_s - T_\infty)L}{u_0^2}} T^* + \frac{1}{Re_L} \frac{\partial^2 u^*}{\partial y^{*2}} \tag{9.10}$$

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{1}{Re_L Pr} \frac{\partial^2 T^*}{\partial y^{*2}} \tag{9.11}$$

- What is u_o?
  - Easy! Reference velocity
  - What is this value for free/natural convection?
  - Actually, we do not have one!
  - How? Choose one which can make life easy for us!
  - What will you choose that can simplify the equations above? Hint: Red box

- Choose $u_o^2 = g\beta(T_s - T_\infty)L$,
  - so the coefficient of T^* in (9.10) will be 1.
  - $\boxed{Re_L = \frac{uL}{v} \Rightarrow \left(\frac{g\beta(T_s - T_\infty)L^3}{v^2}\right)^{0.5}}$

- Define Grashof Number:

$$\boxed{Gr_L = Re_L^2 = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} \approx \frac{\text{Buoyancy Force}}{\text{Viscous Force}}}$$

$L \rightarrow$ characteristic length of surface

$\beta \rightarrow$ thermal expansion coefficient (a thermodynamic property of the fluid)

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

Units: K^{-1}

Liquids: $\beta \rightarrow$ Tables A.5, A.6

$\boxed{\text{Ideal Gas: } \beta = 1/T\,(\mathrm{K})}$

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Since $Gr_L = Re_L^2$,

- In forced convection, $Nu = f\,(Re, Pr)$

=> In free convection, $Nu = f\,(Gr, Pr)$

➢ Rayleigh Number:

$$\boxed{Ra_L = Gr_L Pr = \frac{g\beta(T_s - T_\infty)L^3}{\nu\alpha}}$$

Nu takes a general form of

$$\overline{Nu}_L = \frac{\bar{h}L}{k} = C\,Ra_L^n \qquad (9.24)$$

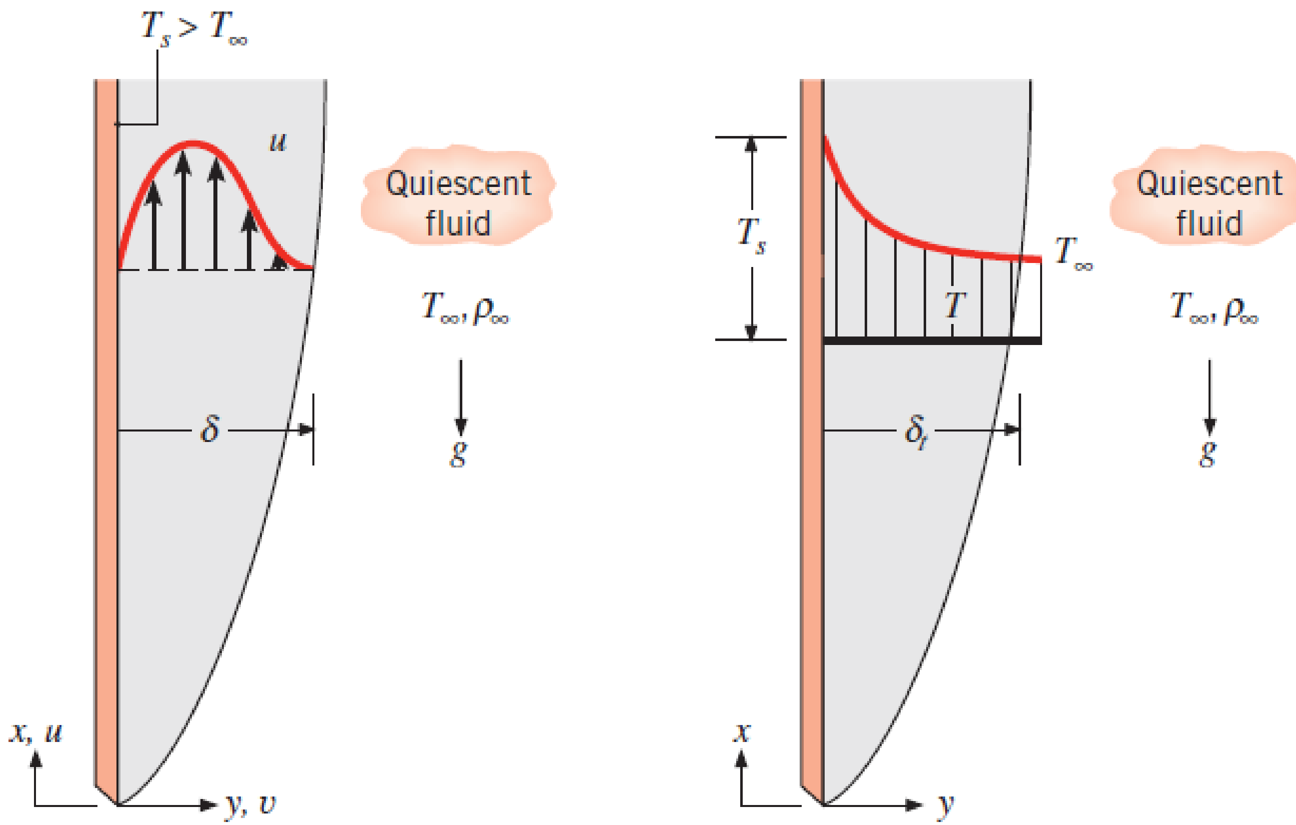
where n = 1/4 for laminar and 1/3 for turbulent flow

Vertical Plate

Section 9.4 and 9.5

We have the general Nu form, what is the actual form?

- Free Convection Boundary Layer Development on a Hot Plate (isothermal):



x-component velocity

temperature

➢ Ascending flow with the maximum velocity occurring in the boundary layer and zero velocity at both the surface and outer edge.

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Nusselt Numbers for laminar flow $\left(Nu_x \text{ and } \overline{Nu}_L\right)$:

$$Nu_x = \frac{hx}{k} = -\left(\frac{Gr_x}{4}\right)^{1/4} \frac{dT^*}{d\eta}\bigg|_{\eta=0} = \left(\frac{Gr_x}{4}\right)^{1/4} g\left(Pr\right) \qquad (9.19)$$

with $$g\left(Pr\right) = \frac{0.75\; Pr^{1/2}}{\left(0.609 + 1.221\; Pr^{1/2} + 1.238\; Pr\right)^{1/4}} \qquad \left(0 < Pr < \infty\right) \qquad (9.20)$$

$$\boxed{\bar{h} = \frac{1}{L}\int_o^L h\,dx \rightarrow \overline{Nu}_L = \frac{4}{3} Nu_L} \qquad (9.22)$$

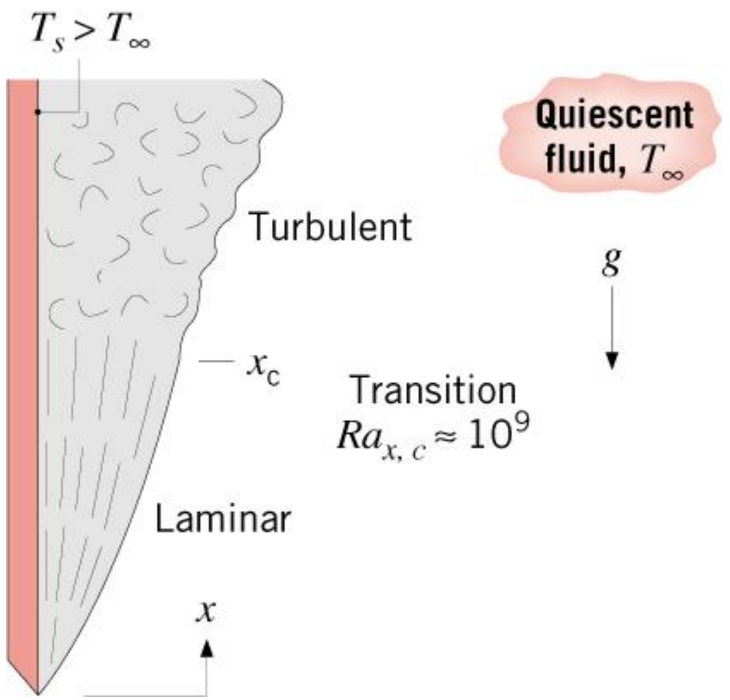
Nu number at x = *L*

Transition to Turbulence

- Transition depends on amplification of disturbances

- Transition occurs at a critical Rayleigh Number.

$$\boxed{Ra_{x,c} = Gr_{x,c}Pr = \frac{g\beta(T_s - T_\infty)x^3}{\nu\alpha} \approx 10^9} \quad (9.23)$$



Empirical Heat Transfer Correlations (9.19 and 9.22 can be used as well)

- Laminar Flow $\left(Ra_L < 10^9\right)$:

$$\boxed{\overline{Nu}_L = 0.68 + \frac{0.670\ Ra_L^{1/4}}{\left[1 + \left(0.492 / Pr\right)^{9/16}\right]^{4/9}}}$$

(9.27 more accurate than 9.26 when laminar)

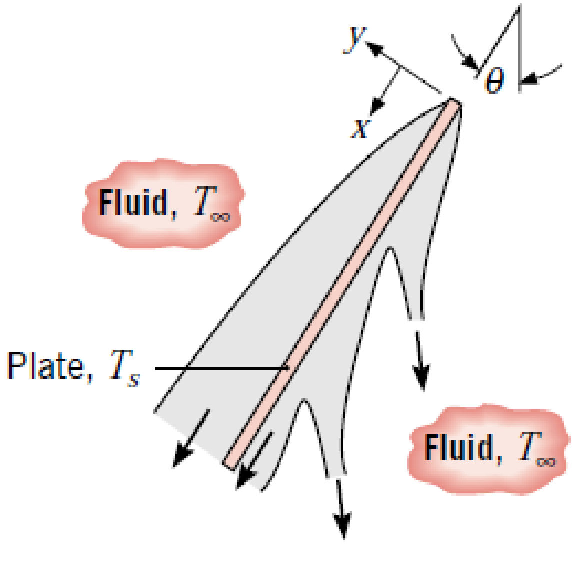
- All Conditions:

$$\boxed{\overline{Nu}_L = \left\{0.825 + \frac{0.387\ Ra_L^{1/6}}{\left[1 + \left(0.492 / Pr\right)^{9/16}\right]^{8/27}}\right\}^2} \qquad (9.26)$$

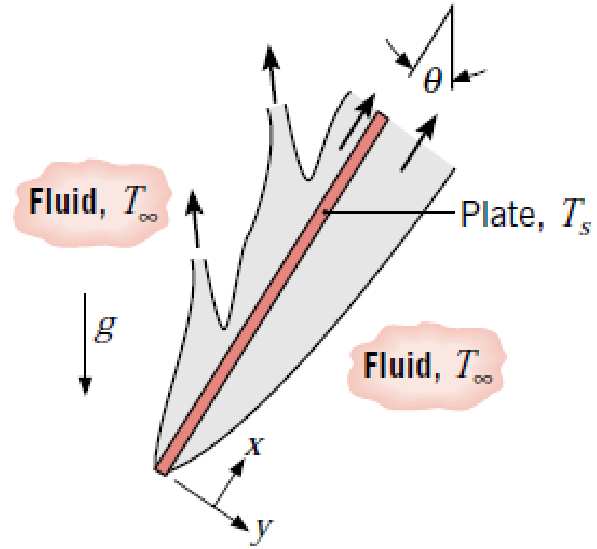
- As with forced convection, all these properties are evaluated at film temperature, $T_f = (T_s + T_\infty)/2$, unless stated otherwise

Inclined and Horizontal Plate

Section 9.6



(a)



(c)

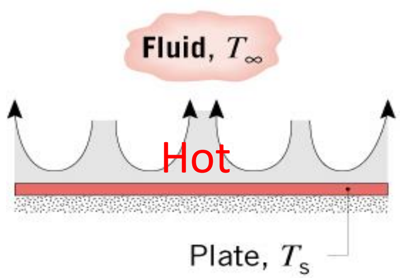
- Is T_s bigger or smaller than T_∞ in (a) and (c)?

In (a) plate,

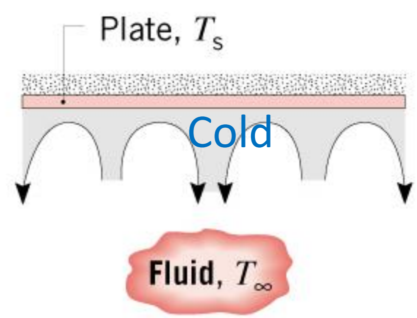
- At the top surface, fluid velocities along the plate slower than vertical case, resulting in lower convection heat transfer.
- At the bottom surface, the *y*-component of the buoyancy force moves the fluid from the surface with discharge of cool fluid that is replaced by the warmer ambient fluid.
- This increases convection heat transfer to the bottom surface.
- Heat transfer is typically enhanced for the whole plate

- Buoyancy force is normal, instead of parallel, to the plate.
- Flow and heat transfer depend on whether the plate is hot or cold and whether it is facing upward or downward.

Hot Surface Facing Upward or Cold Surface Facing Downward



$T_s > T_\infty$



$T_s < T_\infty$

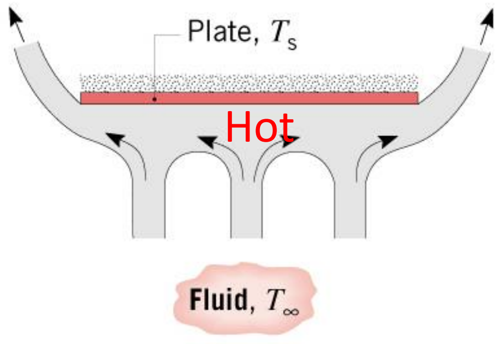
$$\overline{Nu}_L = 0.54\, Ra_L^{1/4} \qquad \left(10^4 < Ra_L < 10^7\right) \qquad (9.30)$$

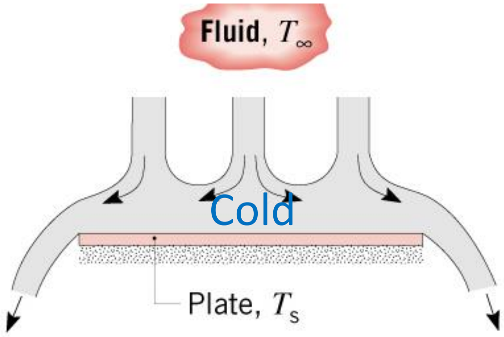
$$\overline{Nu}_L = 0.15\, Ra_L^{1/3} \qquad \left(10^7 < Ra_L < 10^{11};\ \text{all } Pr\right) \qquad (9.31)$$

where $L = A_s/P$. A_s surface area of one side; P is perimeter

How does $\bar{h}$ depend on L when $\overline{Nu}_L \propto Ra_L^{1/3}$?

Hot Surface Facing Downward or Cold Surface Facing Upward





$T_s > T_\infty$ $\qquad$ $T_s < T_\infty$

$$\boxed{\overline{Nu}_L = 0.52 Ra_L^{1/5} \qquad \left(10^4 < Ra_L < 10^9;\ Pr > 0.7\right)} \tag{9.32}$$

- Why do these conditions yield smaller h than those for a hot upper surface or cold lower surface?

More shapes and correlations

- There are more free convection correlation for different shapes and configurations in the textbook.

- If you cannot find the correct correlation to use here, please refer to the textbook.

Mixed Convection

Section 9.9

Mixing Force and Natural Convection is as easy as 1+1… for undergrad

Mixed Convection

- A condition where forced and free convection are present and comparable.
- It is easier in these cases to choose reference u_o as u_∞ instead of the free convection case.
- The coefficient in front of T^* in (9.10) will not be 1 but equals to Gr_L/Re_L^2

Therefore,

- Mixed convection important when $\rightarrow \left(Gr_L/Re_L^2\right) \approx 1$
- Free convection important when $\rightarrow \left(Gr_L/Re_L^2\right) >> 1$
- Forced convection important when $\rightarrow \left(Gr_L/Re_L^2\right) << 1$

Mixed Convection

- Heat Transfer Correlations for Mixed Convection:

$$Nu^n \approx Nu_{FC}^n \pm Nu_{NC}^n$$

$Nu_{FC} \rightarrow$ Nusselt number for forced convection
$Nu_{NC} \rightarrow$ Nusselt number for natural (free) convection

$+ \rightarrow$ assisting and transverse flows
$- \rightarrow$ opposing flows

$n \approx 3$ normally but there are more values of possible n depending on the configurations

- Also, this is a first approximation approach. More accurate method can be found in the literature.

- If forced flow is turbulent => free convection can be safely neglected. Why?

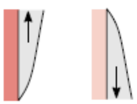
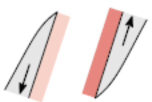
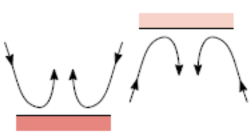
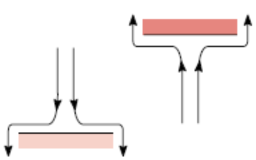
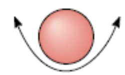
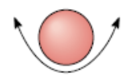
Summary

Free convection HT

- Velocity profile in the free convection boundary
- General form of the buoyancy term in x-momentum equation, Boussinesq Approx.
- Grashof number and Rayleigh number
- Free convection on a vertical, inclined, horizontal plate
- Mixed convection

Summary

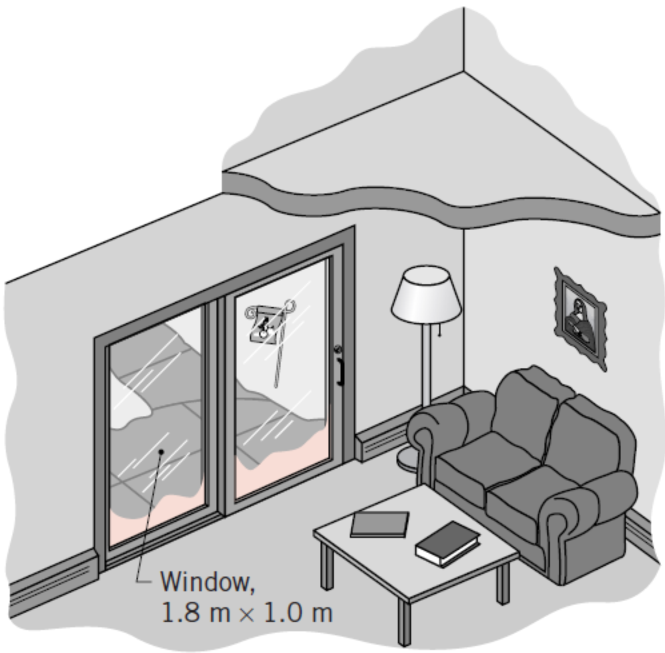
Table 9.2 Summary of free convection empirical correlations for immersed geometries

Geometry	Recommended Correlation	Restrictions
1. Vertical plates[a] 	Equation 9.26	None
2. Inclined plates Cold surface up or hot surface down 	Equation 9.26 $g \longrightarrow g\cos\theta$	$0 \le \theta \lesssim 60°$
3. Horizontal plates (a) Hot surface up or cold surface down 	Equation 9.30 Equation 9.31	$10^4 \lesssim Ra_L \lesssim 10^7$ $10^7 \lesssim Ra_L \lesssim 10^{11}$
(b) Cold surface up or hot surface down 	Equation 9.32	$10^5 \lesssim Ra_L \lesssim 10^{10}$
4. Horizontal cylinder 	Equation 9.34	$Ra_D \lesssim 10^{12}$
5. Sphere 	Equation 9.35	$Ra_D \lesssim 10^{11}$ $Pr \gtrsim 0.7$

[a] The correlation may be applied to a vertical cylinder if $(D/L) \gtrsim (35/Gr_L^{1/4})$.

Example 3: Winter Room

Problem 9.17: During winter, a frost line appears near the base of the window. The room wall and air temperature is 15°C



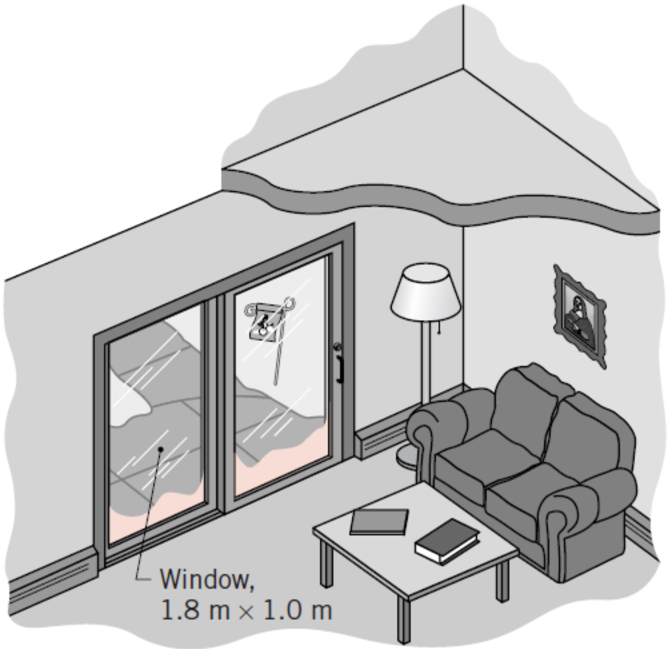
(a) Explain why the frost layer appears at the base rather than the top.
(b) Estimate the heat loss through the window due to free convection and radiation. Assume window at 0°C and the emissivity of the glass is 0.94.

Example 3: Winter Room

(a) Explain why the frost layer appears at the base rather than the top.

(b) Estimate the heat loss though the window due to free convection and radiation. Assume window at 0°C and the emissivity of the glass is 0.94.

- Is the frost outside or inside the room?
- Which part of the room is hotter?
- How does the thermal BL look like along the window? Free or Forced?
- What are the different heat loss mechanisms
- What are the formulas?
- What correlations to used?



ASSUMPTIONS: (1) Steady-state conditions, (2) Window has a uniform temperature, (3) Ambient air is quiescent, and (4) Room walls are isothermal and large compared to the window.

PROPERTIES: *Table A-4*, Air ($T_f = (T_s + T_\infty)/2 = 280$ K, 1 atm): $\nu = 14.11 \times 10^{-6}$ m^2/s, k = 0.0247 W/m·K, $\alpha = 1.986 \times 10^{-5}$ m^2/s, Pr = 0.710.

ANALYSIS: (a) For these winter conditions, a frost line could appear and it would be at the bottom of the window. The boundary layer is thinnest at the top of the window, and hence the heat flux from the warmer room is greater than compared to that at the bottom portion of the window where the boundary layer is thicker. Also, the air in the room may be stratified and cooler near the floor compared to near the ceiling.

(b) The rate of heat loss from the room to the window having a uniform temperature $T_s = 0°C$ by convection and radiation is

$$q_{loss} = q_{cv} + q_{rad} \tag{1}$$

$$q_{loss} = A_s\left[\bar{h}_L\left(T_\infty - T_s\right) + \varepsilon\sigma\left(T_{sur}^4 - T_s^4\right)\right] \tag{2}$$

The average convection coefficient is estimated from the Churchill-Chu correlation, Eq. 9.26, using properties evaluated at $\boxed{T_f = (T_s + T_\infty)/2.}$

$$\overline{Nu}_L = \frac{\bar{h}_L L}{k} = \left\{ 0.825 + \frac{0.387\, Ra_L^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}} \right\}^2 \qquad (3)$$

$$Ra_L = g\beta T\left(T_\infty - T_s\right)L^3 / \nu\alpha \qquad (4)$$

Substituting numerical values in the correlation expressions, find

$$Ra_L = 1.084 \times 10^{10} \qquad \overline{Nu}_L = 258.9 \qquad \bar{h}_L = 3.6\ W/m^2 \cdot K$$

Using Eq. (2), the rate of heat loss with $\sigma = 5.67 \times 10^{-8}\ W/m^2 \cdot K^4$ is

$$q_{loss} = (1 \times 1.8)m^2\left[3.6\ W/m^2 \cdot K(15\ K) + 0.940\,\sigma\left(288^4 - 273^4\right)K^4\right]$$

$$q_{loss} = (96.1 + 127.1)W = 223\ W$$

COMMENTS: Note that the heat loss by radiation is 30% larger than by free convection.

Vertical Plates – Laminar (optional)

- Similar to forced convection, there is a similarity variable η, that transforms a partial differential equation with two-independent variables (x and y) to an ordinary differential equation expressed exclusively in terms of η .

$$\eta \equiv \frac{y}{x}\left(\frac{Gr_x}{4}\right)^{1/4}$$

- The momentum and energy ODEs are (from 9.7 to 9.8):

$$f''' + 3ff'' - 2(f')^2 + T^* = 0$$

$$T^{*''} + 3\,Pr\,fT^{*'} = 0$$

with $\quad f'(\eta) \equiv \frac{df}{d\eta} = \frac{x}{2\nu}\left(Gr_x^{-1/2}\right)u \qquad T^* \equiv \frac{T - T_\infty}{T_s - T_\infty}$

- Solutions are in Figure 9.4 in textbook