1.4 Independence

Independence

Two events are **independent** if the occurrence of one does not affect the probability of another occurring (and vice versa).

$P[A|B] = P[A]$
$P[B|A] = P[B]$

Independence

Definition 1.4-1

Events A and B are **independent** if and only if $P(A \cap B) = P(A)P(B)$. Otherwise, A and B are called **dependent** events.

Theorem 1.4-1 If A and B are independent events, then the following pairs of events are also independent:

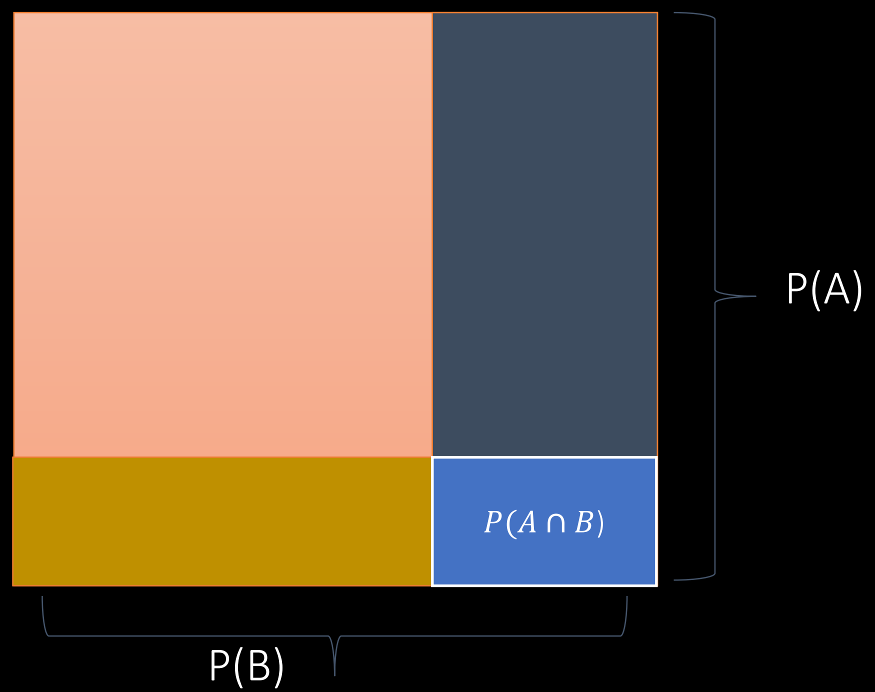
(a) A and B';

(b) A' and B;

(c) A' and B'.

Independence

Suppose A and B are Independent Events:



Independence

Let A be the event of drawing a queen from a standard deck of cards.
Let B be the event of drawing a spade.

Then (by *definition 1.4-1*), A and B are independent because the probability of their intersection (drawing the queen of spades) is equal to $P(A)P(B)$.

$$P(A \cap B) = \frac{1}{52} = \frac{1}{13} \cdot \frac{1}{4} = P(A) \cdot P(B)$$

Note: there are 52 cards total.
4 of each value (4 Queens) and 13 of each suit (spades)

Independence for more than 2 events:

- Pairwise Independence
- Mutual Independence

- Multiple events are considered to be **Pairwise Independent** if every pair of events is independent.

Example We throw two dice. Let A be the event "the sum of the points is 7", B the event "die #1 came up 3", and C the event "die #2 came up 4". Now, $P[A] = P[B] = P[C] = \frac{1}{6}$. Also,

$$P[A \cap B] = P[A \cap C] = P[B \cap C] = \frac{1}{36}$$

Are events A, B, and C pairwise independent?

Mutual Independence

Definition 1.4-2

Events A, B, and C are **mutually independent** if and only if the following two conditions hold:

(a) A, B, and C are pairwise independent; that is,

$$P(A \cap B) = P(A)P(B), \qquad P(A \cap C) = P(A)P(C),$$

and

$$P(B \cap C) = P(B)P(C).$$

(b) $P(A \cap B \cap C) = P(A)P(B)P(C)$.

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Are events A, B, and C mutually independent?

Examples

Event	Probability
Alligator (A)	0.5
Hammer (H)	0.3
Poison (P)	0.2

Each time a contestant pulls on Yzma's lever, a single event from this table occurs randomly. Assume 5 contestants pull on the lever, and that the results are independent.

1. What is the probability that all 5 will get Hammered?

Event	Probability
Alligator (A)	0.5
Hammer (H)	0.3
Poison (P)	0.2

2. What is the probability of selecting {A,H,H,P,A}?

3. What is the probability that at least one will get an alligator?

4. What is the probability that exactly 2 will be poisoned?

5. Suppose a Christmas tree has 100 bulbs in a series circuit (all bulbs must work for the tree to turn on). If each bulb has a 99% chance of working independent of the other bulbs, what is the probability that the tree will not turn on?

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6. Let P[A] = 0.2, P[B] = 0.5.

If A and B are independent, evaluate P[A U B].