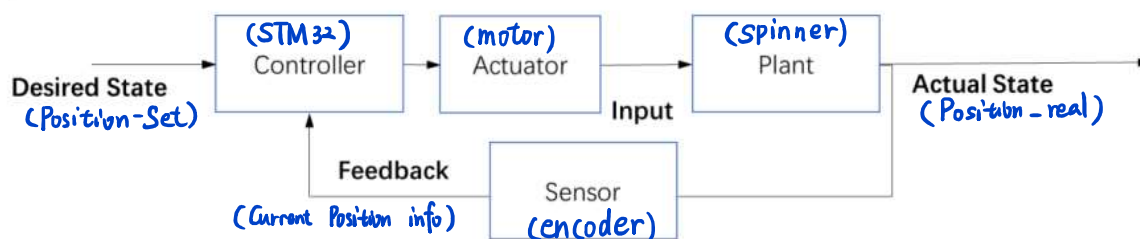


# Homework 0

## Question 1 (6 points)

Give an example of a closed loop control system. Using your example, explain the following terms associated with the control system represented by Figure 1:

- a) Plant
- b) Sensors
- c) Actuator
- d) Desired State
- e) Actual State
- f) Feedback



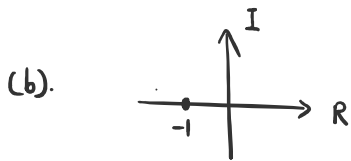
- (a) Plant: Spinner in the motor [The system being controlled]
- (b) Sensor: Encoder in the motor for position detection
- (c) Actuator: The coil in the motor for spinner control
- (d) Desired State: The motor position that we set
- (e) Actual State: The actual motor position that motor outputs
- (f) Feedback: The current motor spinner position data that the encoder gets

## Question 2 (9 points)

Given  $z = \frac{1}{j} \left( \frac{1-j}{2+2j} - \frac{1+j}{2-2j} \right)$

- Write  $z$  in the form  $\alpha + \beta j$
- Sketch  $z$  in the complex plane
- Obtain the inverse of  $z$  in polar form
- Given  $x^3 = -8$ , find the complex values of  $x$  that satisfy the equation.

(a). 
$$z = \frac{1}{j} \left( \frac{1-j}{2+2j} - \frac{1+j}{2-2j} \right) = -j \left( \frac{(1-j)(2-2j)}{4+4} - \frac{(1+j)(2+2j)}{4+4} \right) = -j \cdot \frac{-4j}{4} = -1 + 0j$$



(c). 
$$\frac{1}{z} = \frac{1}{-1} = -1 = 1 \cdot e^{j\pi}$$

(d) Assume  $x = re^{j\theta}$

$$\therefore r^3 e^{3j\theta} = 2^3 \cdot e^{j(\pi + 2k\pi)}$$

$$\therefore \begin{cases} r=2 \\ \theta = \frac{\pi + 2k\pi}{3} \end{cases} \quad \therefore x = 2 \cdot e^{j \cdot \frac{\pi + 2k\pi}{3}} \quad \therefore x = -2, 1 + \sqrt{3}j, 1 - \sqrt{3}j$$

### Question 3 (5 points)

Consider the following differential equation:

$\ddot{x}(t) + 5\dot{x}(t) + 2x(t) = 0$ . Find all values of  $\lambda$  such that  $x(t) = e^{\lambda t}$  satisfies the above differential equation.

$$\therefore \lambda^2 + 5\lambda + 2 = 0$$

$$\therefore \left(\lambda + \frac{5}{2}\right)^2 = \frac{17}{4}$$

$$\therefore \lambda = -\frac{5 \pm \sqrt{17}}{2}$$