# Lec16: Heat Exchangers: Design Considerations

Chapter 11

Sections 11.1 through 11.3

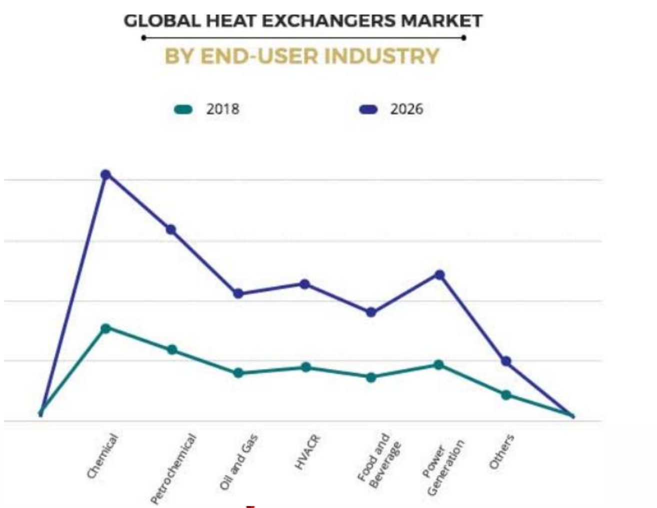
- Exam02 coming on 6 Dec
  - Lec 9 – 17
    - External Flow
    - Internal Flow
    - Heat Exchanger
    - Natural Convection

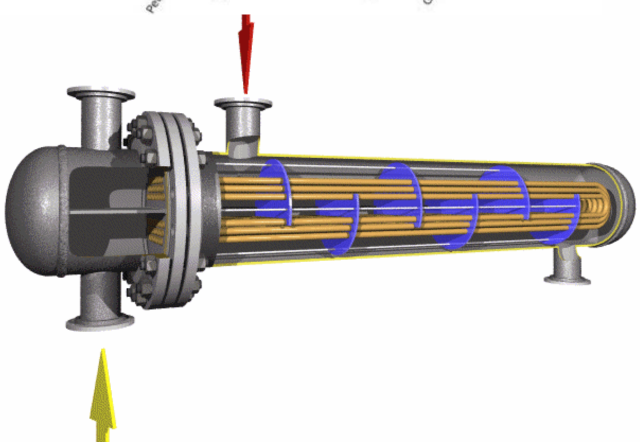
# Content

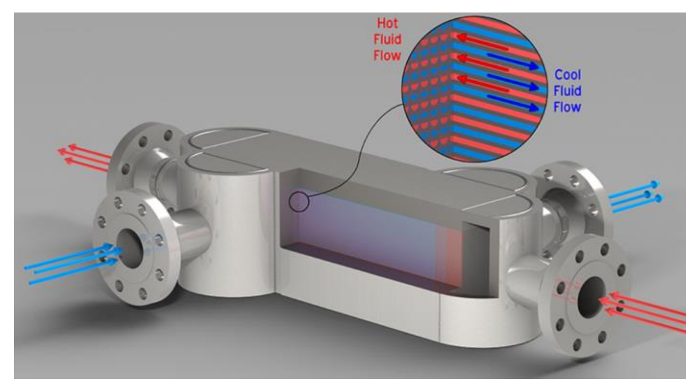
1. What are the common forms of HXs?
2. What are the common flow configurations in HXs?
3. What are the three different but equivalent heat rates in a HX?
4. How do fins and fouling affect HXs?
5. What is the LMTD ?
6. What is the LMTD method for analysing HXs performance?

## Heat Exchangers Market Outlook-2026

The global heat exchangers market was valued at $16,624.0 million in 2018, and is expected to reach $29,316.0 million by 2026, registering a CAGR of 7.2% from 2019 to 2026.



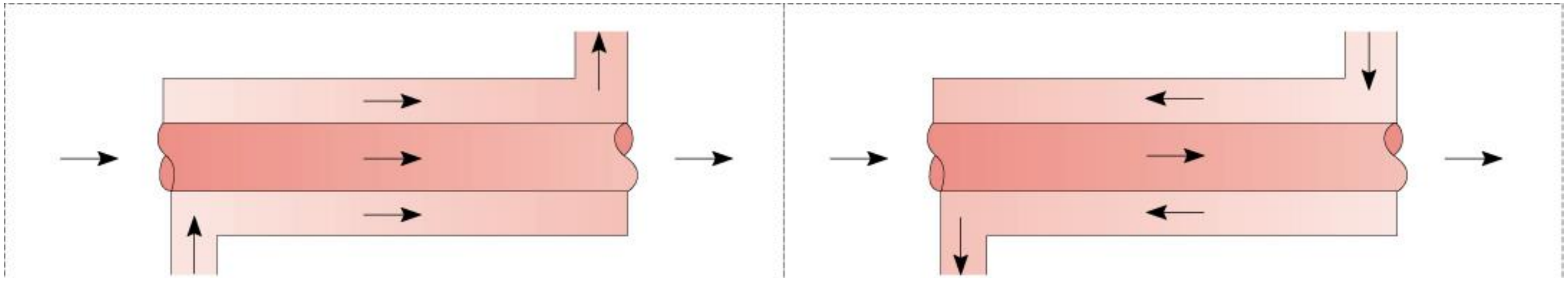


# 11.1 Heat Exchanger Types

Heat exchangers

- Common in energy conversion and utilization
- **Heat exchange between two fluids** (one hot, one cold) separated by a solid
- Classified by
  - **Flow arrangement**
    - ➢ Parallel flow
    - ➢ Counterflow
    - ➢ Crossflow
  - **Type of construction**
    - ➢ Concentric
    - ➢ Finned / unfnned tubular => unmixed or mixed
    - ➢ Shell-tube
    - ➢ Compact heat exchangers

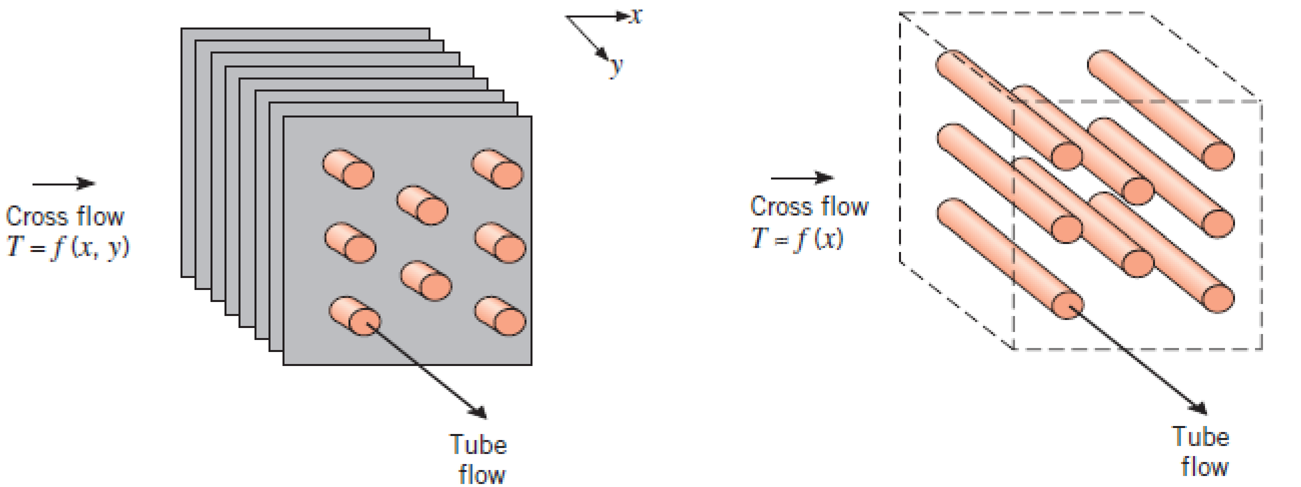
Concentric-Tube Heat Exchangers (simplest, superior performance with counter flow)



Parallel Flow

Counterflow

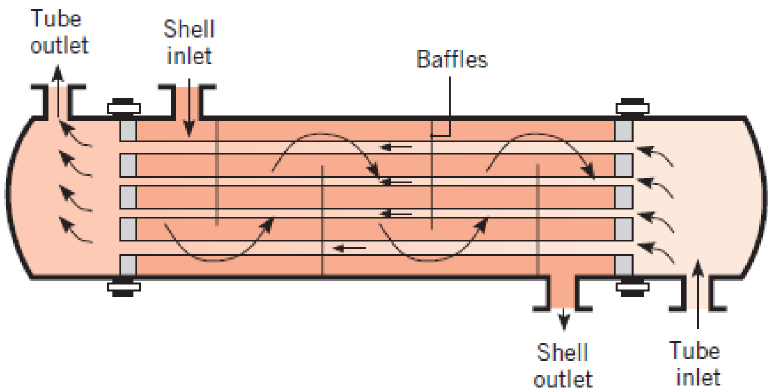
Cross-flow Heat Exchangers (performance influenced by fluid mixing)



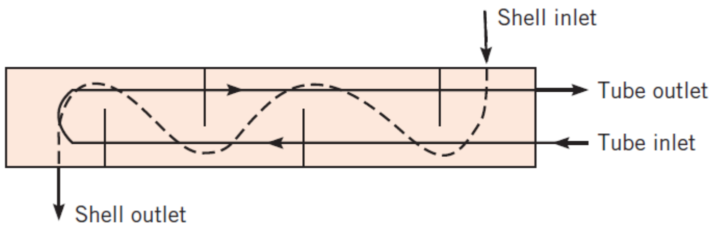

Finned-Both Fluids Unmixed

Unfinned-One Fluid Mixed the Other Unmixed

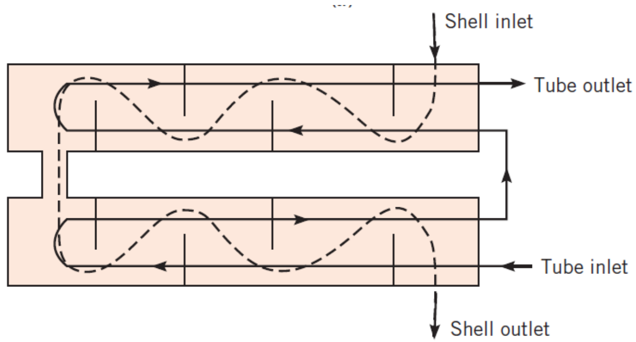
## Shell-and-Tube Heat Exchangers




One Shell Pass and One Tube Pass

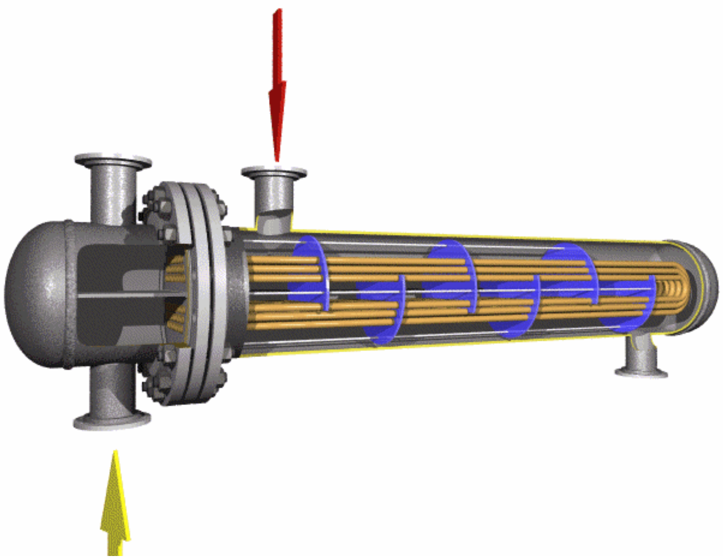



One Shell Pass, Two Tube Passes



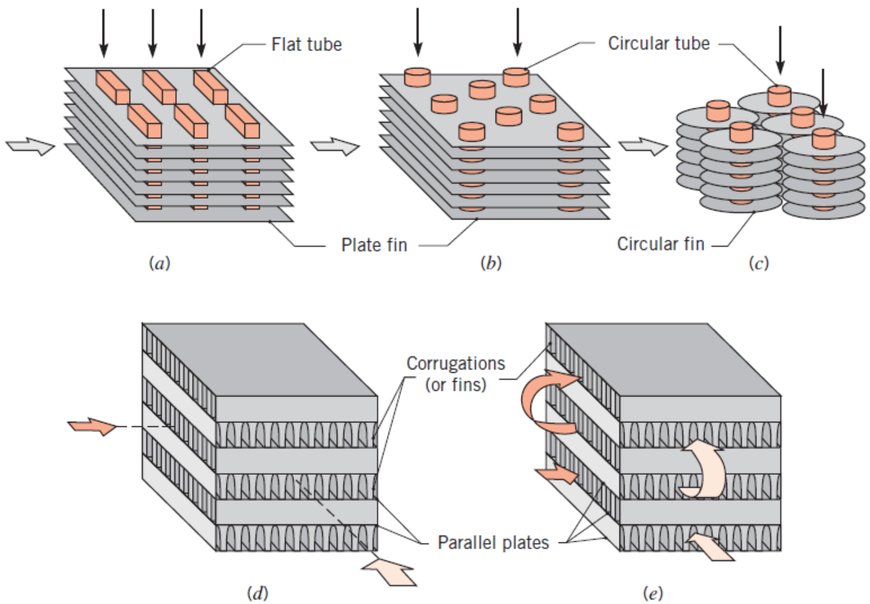

Two Shell Passes, Four Tube Passes

- Baffles are used to establish a cross-flow and to induce turbulent mixing of the shell-side fluid, both of which enhance convection.



## Compact Heat Exchangers

- Widely used to achieve large heat rates per unit volume, particularly when one or both fluids is a gas.
- Characterized by large heat transfer surface areas per unit volume, small flow passages, and laminar flow.




(a) Fin-tube (flat tubes, continuous plate fins)
(b) Fin-tube (circular tubes, continuous plate fins)
(c) Fin-tube (circular tubes, circular fins)
(d) Plate-fin (single pass)
(e) Plate-fin (multipass)

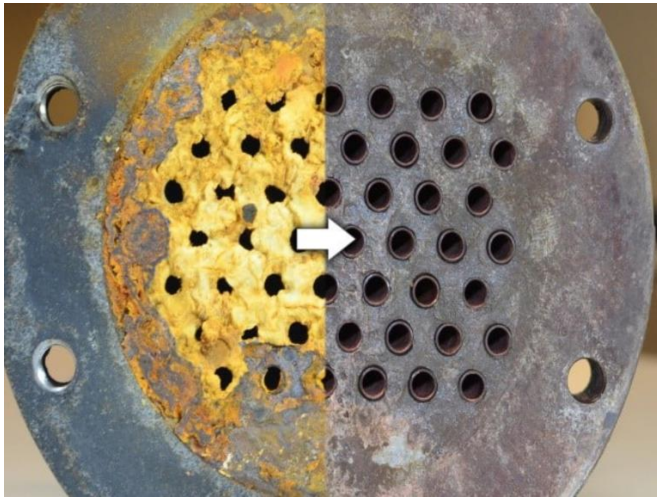
# Overall HT coefficient

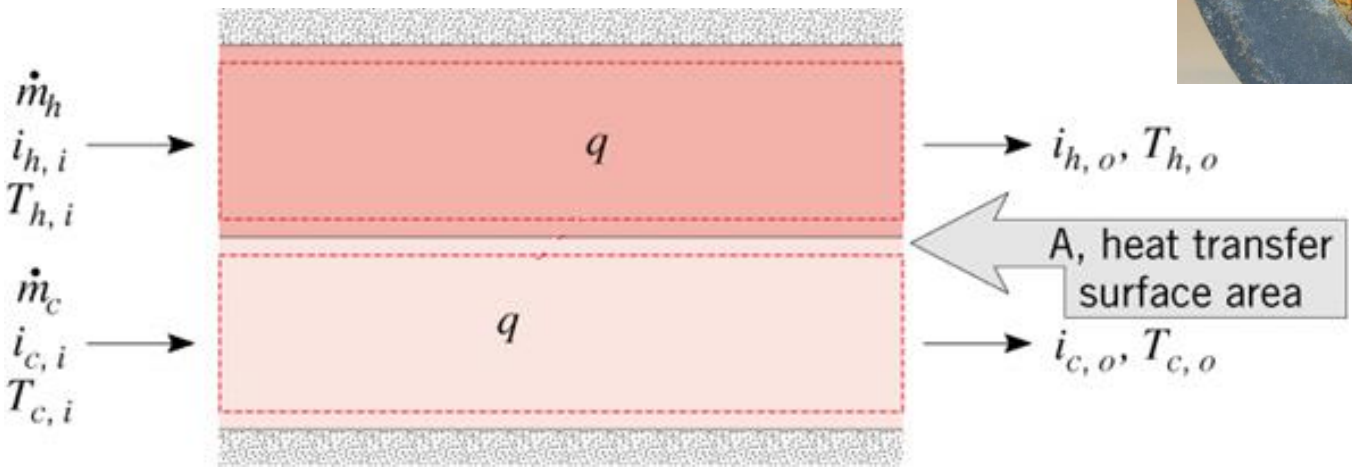
Section 11.2

# 11.2 Overall HT Coefficient

Convection heat transfer, $\mathbf{q} = \boldsymbol{hA(T_1 - T_2)}$

- Need $h$ !
- But which fluid's $h$ ?
- Use $U$ instead to include effects from convection or/and conduction of
  - Two fluids
  - The solid wall
  - Fins on both sides
  - Time-dependent surface fouling.

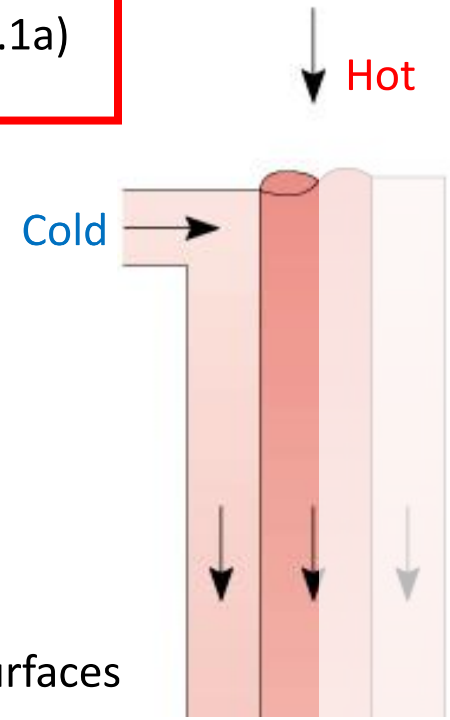




- Subscripts *c* and *h* *for* the *cold* and *hot* fluids (does not matter which fluid is which), and *w* the wall.

The simplest expression for the overall coefficient across a wall is (case 1):

$$\boxed{\frac{1}{UA} = \frac{1}{(hA)_c} + R_w + \frac{1}{(hA)_h} \qquad (11.1a)}$$




Clean and unfinned surfaces

- Case 2. With *fouling*:

$$\frac{1}{UA} = \frac{1}{(hA)_c} + \frac{R''_{f,c}}{(A)_c} + R_w + \frac{R''_{f,h}}{(A)_h} + \frac{1}{(hA)_h}$$

- Case 3. With *fins* and *fouling* (no fouling on fins) for the overall coefficient is:

$$\boxed{\frac{1}{UA} = \frac{1}{(\eta_o hA)_c} + \frac{R''_{f,c}}{(\eta_o A)_c} + R_w + \frac{R''_{f,h}}{(\eta_o A)_h} + \frac{1}{(\eta_o hA)_h}} \quad (11.1b)$$

Fin on cold side

Fouling on cold side

Tube wall

Fouling on hot side

Fin on hot side

- $R''_f \rightarrow$ Fouling factor for a unit surface area ($\mathrm{m}^2 \cdot \mathrm{K/W}$)
  $\rightarrow$ Table 11.1
- $R_w \rightarrow$ Wall conduction resistance (K/W)
- $\eta_o \rightarrow$ Overall surface efficiency of fin array (textbook section 3.6.5/ Lec 8 *amd* 9)
  $\eta_o = 1 - \frac{A_{fin}}{A}(1 - \eta_{fin})$, hot and cold side normally has different values

$A = A_t \rightarrow$ total surface area (fins and exposed base)
$A_{fin} \rightarrow$ surface area of fins only

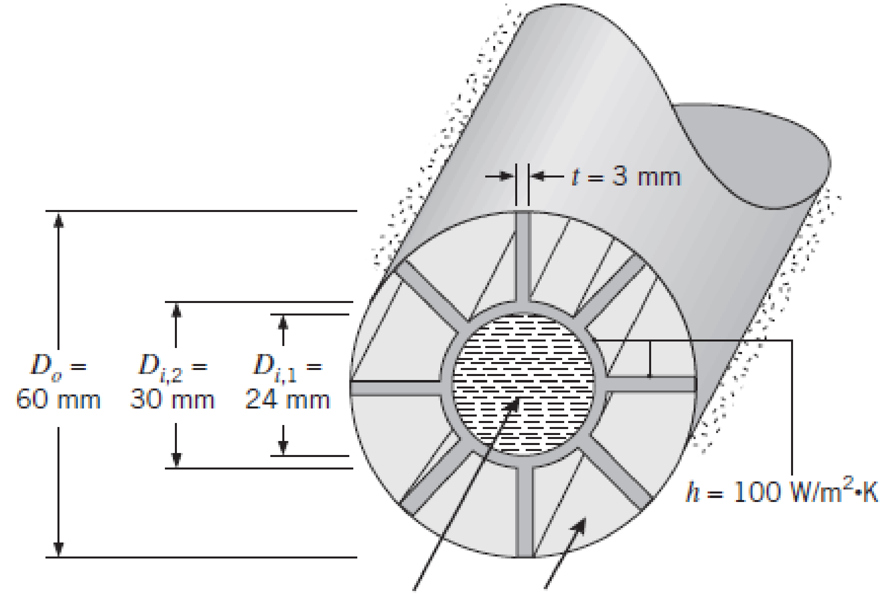
Case 4. With *fins* and *fouling* (on fins and all surfaces) for the overall coefficient is:

$$\boxed{\frac{1}{UA} = \frac{1}{(\eta_o U_p A)_c} + R_w + \frac{1}{(\eta_o U_p A)_h}}$$

$$U_p = \left(\frac{h}{1+hR''_f}\right)$$

→ partial overall coefficient can be different for *c* and *h* side

**Problem 11.5:** Determination of initial heat transfer per unit length from the hot flue gases flowing outside to the pressurized water flowing in the center.




$T_{w,i}$ = 300K
$m_w$ = 0.161 kg/s

$T_{g,i}$ = 800K
$h_g$ = 100 W/m$^2$K

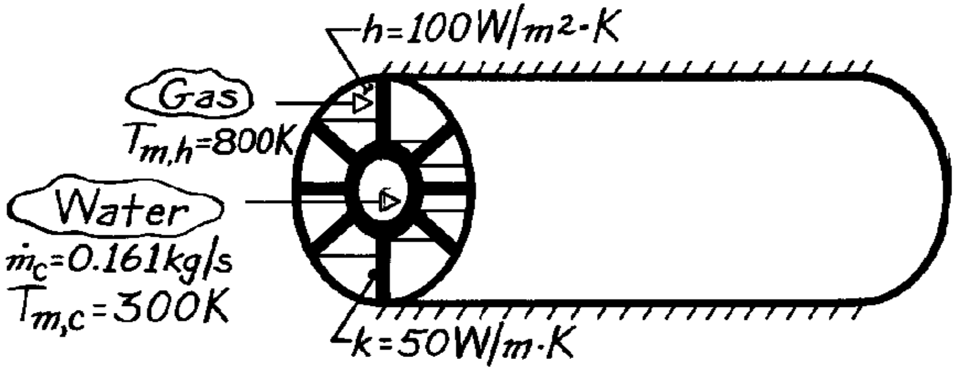
**KNOWN:** Geometry of finned, annular heat exchanger. Gas-side temperature and convection coefficient. Water-side flowrate and temperature. Insulated outside.

**FIND:** Heat rate per unit length.

# Example 1:

**FIND:** Heat rate per unit length.

**SCHEMATIC:**




$D_o$ = 60 mm
$D_{i,1}$ = 24 mm
$D_{i,2}$ = 30 mm
$t$ = 3 mm = 0.003m
$L$ = (60-30)/2 mm = 0.015m

- Which *h* to use? *h* of water or gas? How to get them?
- What equation to use to calculate heat rate?
- Is there a fin?

# Example 1: Solution

**ASSUMPTIONS:** (1) Steady-state conditions, (2) Constant properties, (3) One-dimensional conduction in strut, (4) Adiabatic outer surface conditions, (5) Negligible gas-side radiation, (6) Fully-developed internal flow, (7) Negligible fouling.

**PROPERTIES:** *Table A-6*, Water (300 K): $k = 0.613$ W/m·K, $Pr = 5.83$, $\mu = 855 \times 10^{-6}$ N·s/m$^2$.

**ANALYSIS:** The heat rate is $q = (UA)_c \left(T_{m,h} - T_{m,c}\right)$

where

Tube

$$1/(UA)_c = 1/(hA)_c + R_w + 1/(\eta_o hA)_h$$

Internal flow    Fin

- Tube: the center tube is a cylinder, so

$$R_w = \frac{\ln\left(D_{i,2}/D_{i,1}\right)}{2\pi kL} = \frac{\ln(30/24)}{2\pi(50\ \text{W/m}\cdot\text{K})\ 1\ \text{m}} = 7.10\times10^{-4}\ \text{K/W}.$$

# Example 1: Solution

- Internal flow in the tube,

$$Re_D = \frac{4\dot{m}}{\pi D_{i,1}\mu} = \frac{4\times 0.161\ \text{kg/s}}{\pi(0.024m)855\times 10^{-6}\ \text{N}\cdot\text{s/m}^2} = 9990$$

Hence, the internal flow is turbulent and the Dittus-Boelter correlation from Chapter 8 gives

$$h_c = (k / D_{i,1})0.023 Re_D^{4/5} Pr^{0.4} = \left(\frac{0.613\ \text{W/m}\cdot\text{K}}{0.024\ \text{m}}\right)0.023(9990)^{4/5}(5.83)^{0.4} = 1883\ \text{W/m}^2\cdot\text{K}$$

$$(hA)_c^{-1} = \left(1883\ \text{W/m}^2\cdot\text{K}\times\pi\times 0.024\text{m}\ \times 1\text{m}\right)^{-1} = 7.043\times 10^{-3}\ \text{K/W}.$$

# Example 1: Solution

- Since the outside is insulated, it is a find with adiabatic tip.
  The Fin efficiency is

$$\eta_o = 1 - \left(A_f / A\right)\left(1 - \eta_f\right)$$

$$A_f = 8 \times 2\left(L \cdot w\right) = 8 \times 2\left(0.015\text{m} \times 1\text{m}\right) = 0.24\text{m}^2$$

$$A = A_f + \left(\pi D_{i,2} - 8t\right) w = 0.24\text{m}^2 + \left(\pi \times 0.03\text{m} - 8 \times 0.003\text{m}\right) = 0.31\text{m}^2.$$

From Eq. 11.4,

$$\eta_f = \frac{\tanh\left(mL\right)}{mL}$$

where

$$m = \left[2h / kt\right]^{1/2} = \left[2 \times 100\ \text{W/m}^2 \cdot \text{K} / 50\ \text{W/m} \cdot \text{K}\left(0.003\text{m}\right)\right]^{1/2} = 36.5\,\text{m}^{-1}$$

$$mL = \left(2h / kt\right)^{1/2} L = 36.5\,\text{m}^{-1} \times 0.015\text{m} = 0.55$$

$$\tanh\left[\left(2h / kt\right)^{1/2} L\right] = 0.499.$$

# Example 1: Solution

Hence

$$\eta_f = 0.499 / 0.55 = 0.911$$

$$\eta_o = 1 - \left(A_f / A\right)\left(1 - \eta_f\right) = 1 - (0.24 / 0.31)(1 - 0.911) = 0.931$$

$$(\eta_o hA)_h^{-1} = \left(0.931 \times 100\ \text{W/m}^2 \cdot \text{K} \times 0.31\text{m}^2\right)^{-1} = 0.0347\ \text{K/W}.$$

It follows that

$$(UA)_c^{-1} = \left(7.043 \times 10^{-3} + 7.1 \times 10^{-4} + 0.0347\right) \text{K/W}$$

$$(UA)_c = 23.6\ \text{W/K}$$

and

$$q = 23.6\ \text{W/K}\left(800 - 300\right)\text{K} = 11{,}800\ \text{W}$$

for a 1m long section.

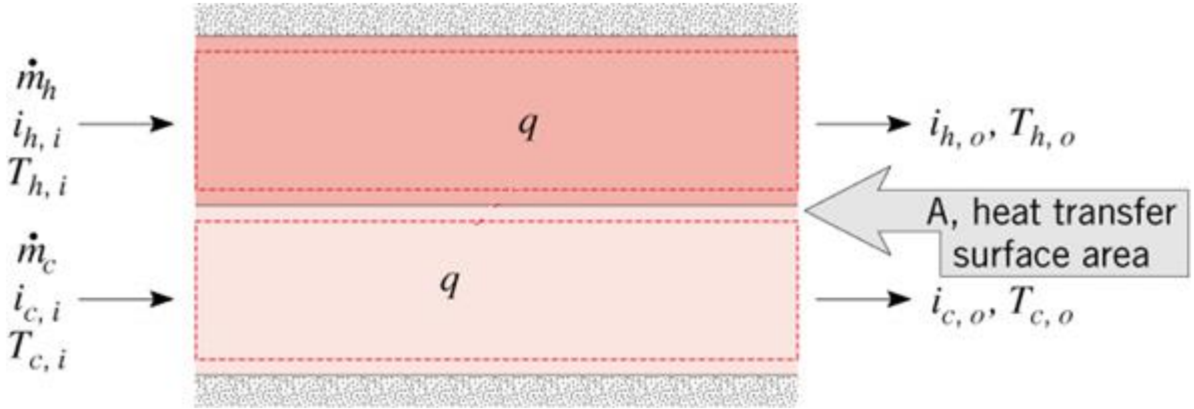
# LMTD Method for Heat Exchanger

Section 11.3

Design and analyse Heat Exchangers using two methods:

- Log Mean Temperature Difference
- NTU-E

- Heat is exchanged between the *hot* and *cold* fluids
- Apply to the *hot (h)* and *cold (c)* fluids regardless of what kind of flow configuration:




Assume negligible heat transfer between the exchanger and its surroundings and negligible potential and kinetic energy changes for each fluid =>

$$q \text{ loss at hot} = q \text{ gain at cold}$$

$$q = \dot{m}_h\left(i_{h,i} - i_{h,o}\right) \qquad q = \dot{m}_c\left(i_{c,o} - i_{c,i}\right) \qquad \text{(11.6a, 11.7a)}$$

$i \rightarrow$ fluid enthalpy

# 11.3 LMTD Method – energy balance

- Assuming no liquid-vapor phase change and constant properties like specific heats etc,

$$q = \dot{m}_h c_{p,h} \left(T_{h,i} - T_{h,o}\right) \quad = C_h \left(T_{h,i} - T_{h,o}\right) \qquad (11.6b)$$

$$q = \dot{m}_c c_{p,c} \left(T_{c,o} - T_{c,i}\right) \quad = C_c \left(T_{c,o} - T_{c,i}\right) \qquad (11.7b)$$

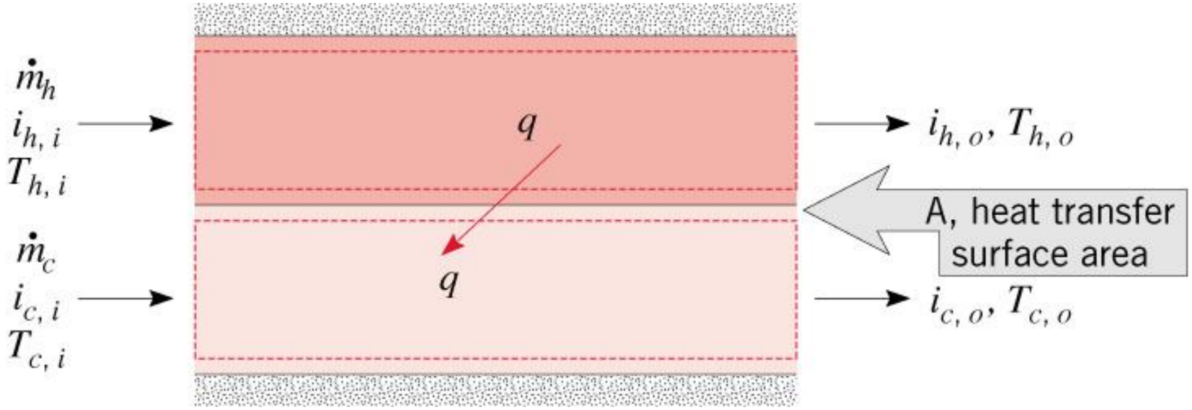
$C_h, C_c \rightarrow$ Heat capacity rates

- Note *T* are the mean temperature of specific locations

But where does this *q* come from?

# 11.3 LMTD Method – energy balance

- How does the heat move from the hot to cold fluid?
- So, there must be a heat transfer between the hot and cold sides which is of $q$,




Define $\Delta T = T_h - T_c$ => $dq = h\Delta T dA$

- And $d(\Delta T) = dT_h - dT_c$
- **But what is *h* ?**
- **Which $T_h$ and $T_c$ along the tube?**

**Solution:**

- We can use =>

$$\boxed{q = U\,A\,\Delta T_{\ell m}}$$

where *UA* is from the section 11.2

- Derivation in Textbook Pg 661.

## The Log Mean Temperature Difference (LMTD) Method

may be applied to find heat exchangers by using a log-mean temperature difference between the two fluids:

$$q = U\,A\,\Delta T_{\ell m} \qquad (11.14)$$

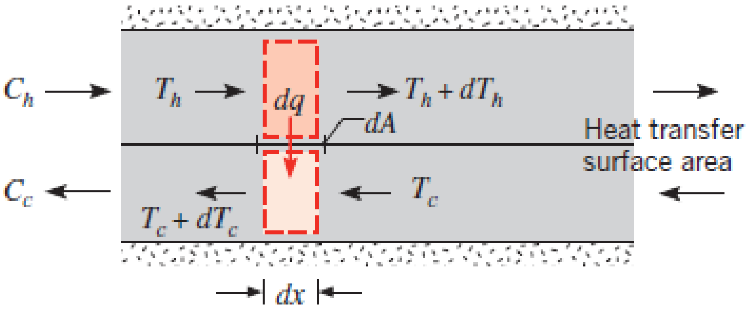
$$\Delta T_{\ell m} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\Delta T_1 / \Delta T_2\right)} \qquad (11.15)$$

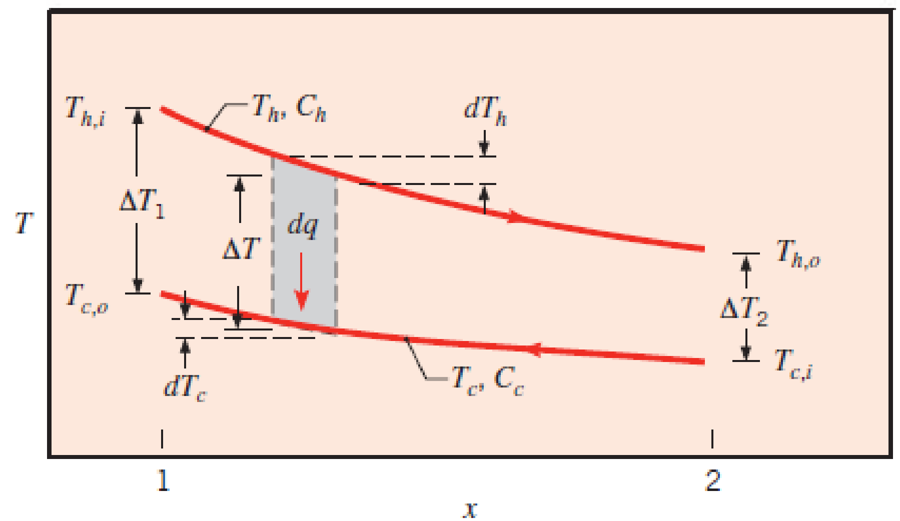
- What are $\Delta T_1$ and $\Delta T_2$ ?
- Depends on the heat exchanger type!

Case 1: Counter-Flow (CF) Heat Exchanger:

$$\Delta T_1 \equiv T_{h,1} - T_{c,1} = T_{h,i} - T_{c,o}$$

$$\Delta T_2 \equiv T_{h,2} - T_{c,2} = T_{h,o} - T_{c,i}$$



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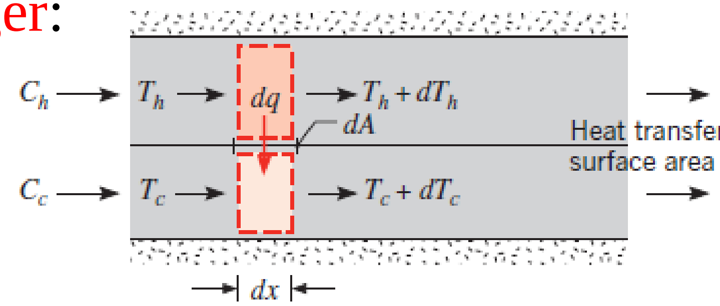
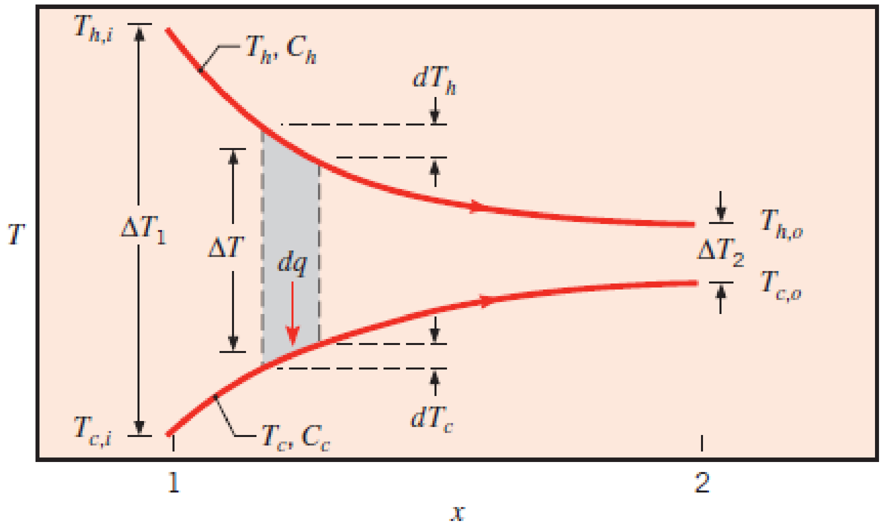
## Case 2: Parallel-Flow (PF) Heat Exchanger:




$$\Delta T_1 \equiv T_{h,1} - T_{c,1} = T_{h,i} - T_{c,i}$$

$$\Delta T_2 \equiv T_{h,2} - T_{c,2} = T_{h,o} - T_{c,o}$$




- Note that $T_{c,o}$ cannot exceed $T_{h,o}$ for a PF HX, but can do so for a CF HX. Why?

- For equivalent values of $UA$ and inlet temperatures,

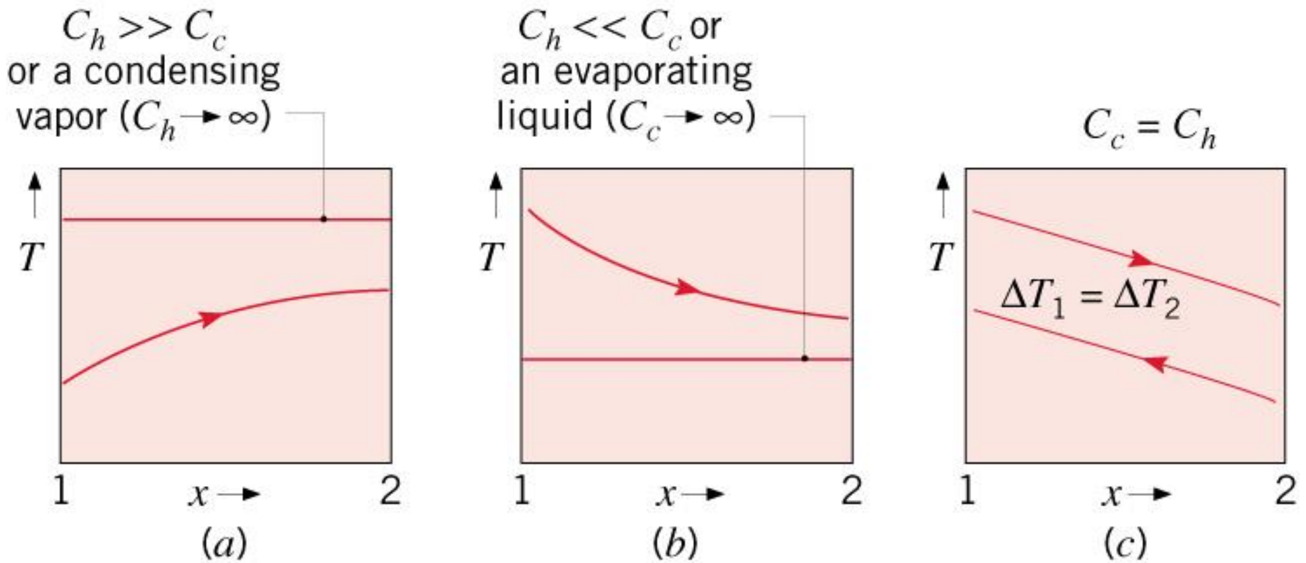
$$\Delta T_{\ell m,\mathrm{CF}} > \Delta T_{\ell m,\mathrm{PF}}$$

## Case 3: Shell-and-Tube and Cross-Flow Heat Exchangers:

$$\Delta T_{\ell m} = F \;\Delta T_{\ell m,\mathrm{CF}}$$

$F \rightarrow$ Figures 11S.1 - 11S.4

# Special Conditions



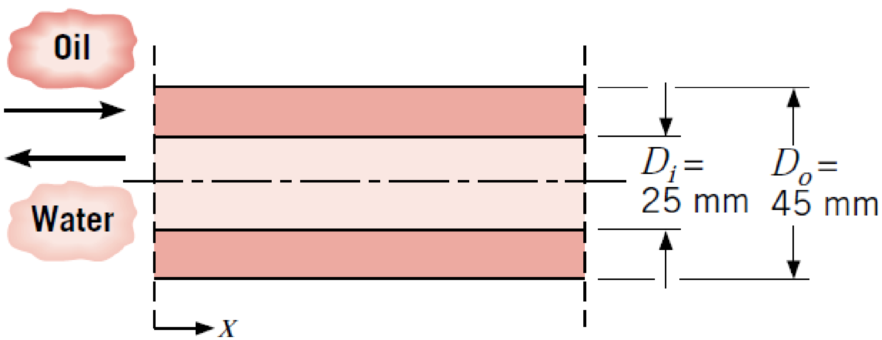

Why condensing vapor is $C_h$, and very big in value?

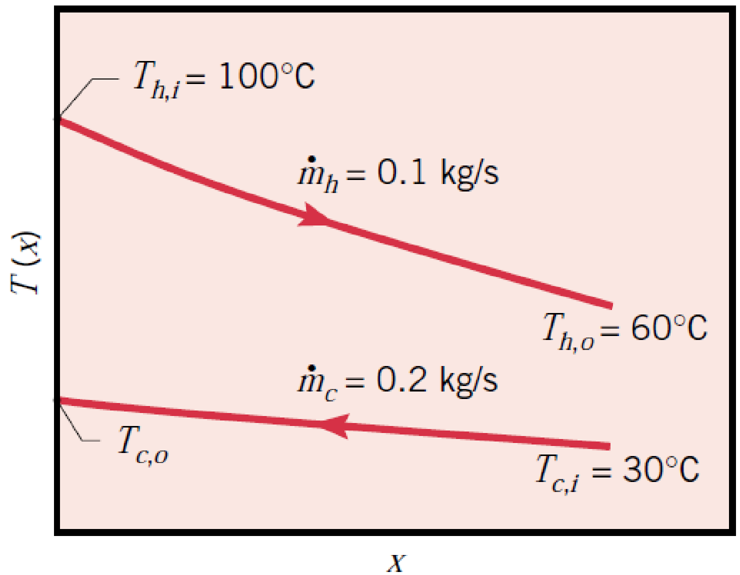
Why evaporating liq is $C_c$, and very big in value?

- Case (a): $C_h >> C_c$ or $h$ is a condensing vapor $(C_h \to \infty)$.
  - Negligible or no change in $T_h$ $\left(T_{h,o} = T_{h,i}\right)$.
- Case (b): $C_c >> C_h$ or $c$ is an evaporating liquid $(C_c \to \infty)$.
  - Negligible or no change in $T_c$ $\left(T_{c,o} = T_{c,i}\right)$.
- Case (c): $C_h$=$C_c$. *(Counter-flow)*
  - $\Delta T_1 = \Delta T_2 = \Delta T_{\ell m}$

# Example 2: HX using LMTD

**Text Example 11.1:** For the HX with a thin wall separating the hot and cold flow (shown below), what is the length needed for outlet temperature of oil to be 60 °C?



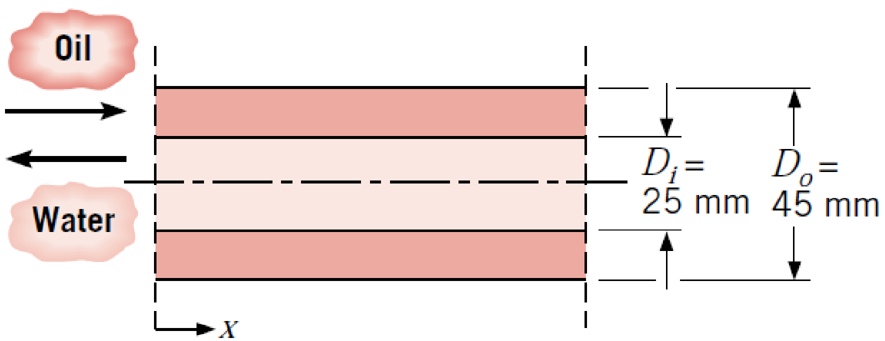



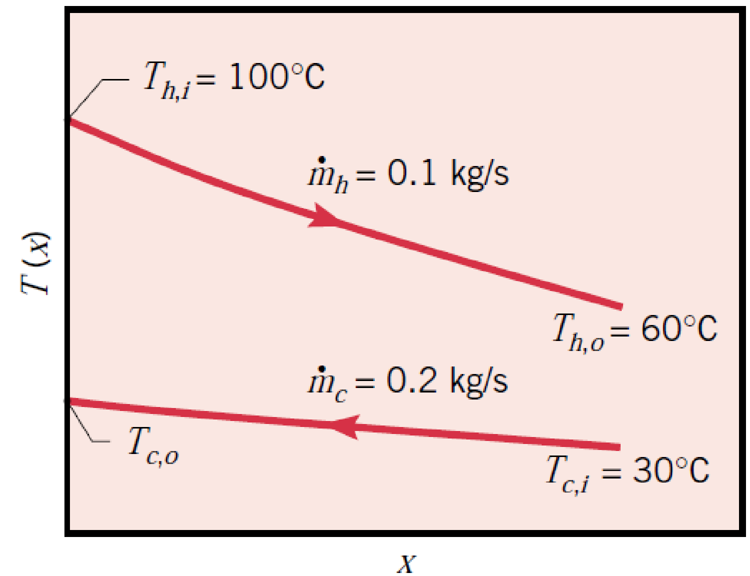

**KNOWN:** Fluid flow rates, inlet temperatures

**FIND:** Length needed for outlet temperature of oil to be 60 °C?

# Example 2:

**FIND:** Length needed for outlet temperature of oil to be 60 °C?




- What HX type? What flow type?
- What is the $q$ to be exchanged?
- Any missing temperatures?
- Which LMTD formula to use?
- What is $UA$ ? What is the configuration for the water part? What is the configuration for the oil part? Both are circular tubes?

$$q = U\,A\,\Delta T_{\ell m}$$

$$\Delta T_{\ell m} = \frac{\Delta T_1 - \Delta T_2}{\ln\left(\Delta T_1 / \Delta T_2\right)}$$

# Example 2: Solution

***Analysis:*** The required heat transfer rate may be obtained from the overall energy balance for the hot fluid, Equation 11.6b.

- What is the $q$ to be exchanged?

$$q = \dot{m}_h c_{p,h}(T_{h,i} - T_{h,o})$$

$$q = 0.1 \text{ kg/s} \times 2131 \text{ J/kg} \cdot \text{K } (100 - 60)°\text{C} = 8524 \text{ W}$$

Applying Equation 11.7b, the water outlet temperature is

- Any missing temperatures?

$$T_{c,o} = \frac{q}{\dot{m}_c c_{p,c}} + T_{c,i}$$

$$T_{c,o} = \frac{8524 \text{ W}}{0.2 \text{ kg/s} \times 4178 \text{ J/kg} \cdot \text{K}} + 30°\text{C} = 40.2°\text{C}$$

Accordingly, use of $\overline{T}_c = 35°\text{C}$ to evaluate the water properties was a good choice. The required heat exchanger length may now be obtained from Equation 11.14,

$$q = UA\,\Delta T_{\text{lm}}$$

where $A = \pi D_i L$ and from Equations 11.15 and 11.17

- Which LMTD formula to use?

$$\Delta T_{\text{lm}} = \frac{(T_{h,i} - T_{c,o}) - (T_{h,o} - T_{c,i})}{\ln[(T_{h,i} - T_{c,o})/(T_{h,o} - T_{c,i})]} = \frac{59.8 - 30}{\ln(59.8/30)} = 43.2°\text{C}$$

From Equation 11.5 the overall heat transfer coefficient is

- What is $UA$ ?

$$U = \frac{1}{(1/h_i) + (1/h_o)}$$

For water flow through the tube,

$$Re_D = \frac{4\dot{m}_c}{\pi D_i \mu} = \frac{4 \times 0.2 \text{ kg/s}}{\pi(0.025 \text{ m})725 \times 10^{-6} \text{ N} \cdot \text{s/m}^2} = 14{,}050$$

Accordingly, the flow is turbulent and the convection coefficient may be computed from Equation 8.60

$$Nu_D = 0.023\, Re_D^{4/5}\, Pr^{0.4}$$
$$Nu_D = 0.023\, (14{,}050)^{4/5}\, (4.85)^{0.4} = 90$$

Hence

$$h_i = Nu_D \frac{k}{D_i} = \frac{90 \times 0.625 \text{ W/m} \cdot \text{K}}{0.025 \text{ m}} = 2250 \text{ W/m}^2 \cdot \text{K}$$

For the flow of oil through the annulus, the hydraulic diameter is, from Equation 8.71, $D_h = D_o - D_i = 0.02$ m, and the Reynolds number is

$$Re_D = \frac{\rho u_m D_h}{\mu} = \frac{\rho(D_o - D_i)}{\mu} \times \frac{\dot{m}_h}{\rho\pi(D_o^2 - D_i^2)/4}$$
$$Re_D = \frac{4\dot{m}_h}{\pi\,(D_o + D_i)\,\mu} = \frac{4 \times 0.1 \text{ kg/s}}{\pi(0.045 + 0.025) \text{ m} \times 3.25 \times 10^{-2} \text{ kg/s} \cdot \text{m}} = 56.0$$

The annular flow is therefore laminar. Assuming uniform temperature along the inner surface of the annulus and a perfectly insulated outer surface, the convection coefficient at the inner surface may be obtained from Table 8.2. With $(D_i/D_o) = 0.56$, linear interpolation provides

$$Nu_i = \frac{h_o D_h}{k} = 5.63$$

# Example 2: Solution

and

$$h_o = 5.63 \frac{0.138 \text{ W/m} \cdot \text{K}}{0.020 \text{ m}} = 38.8 \text{ W/m}^2 \cdot \text{K}$$

The overall convection coefficient is then

$$U = \frac{1}{(1/2250 \text{ W/m}^2 \cdot \text{K}) + (1/38.8 \text{ W/m}^2 \cdot \text{K})} = 38.1 \text{ W/m}^2 \cdot \text{K}$$

and from the rate equation it follows that

$$L = \frac{q}{U \pi D_i \Delta T_{\text{lm}}} = \frac{8524 \text{ W}}{38.1 \text{ W/m}^2 \cdot \text{K} \, \pi \, (0.025 \text{ m}) \, (43.2^\circ\text{C})} = 65.9 \text{ m}$$

# Heat Exchangers: The Effectiveness – NTU Method

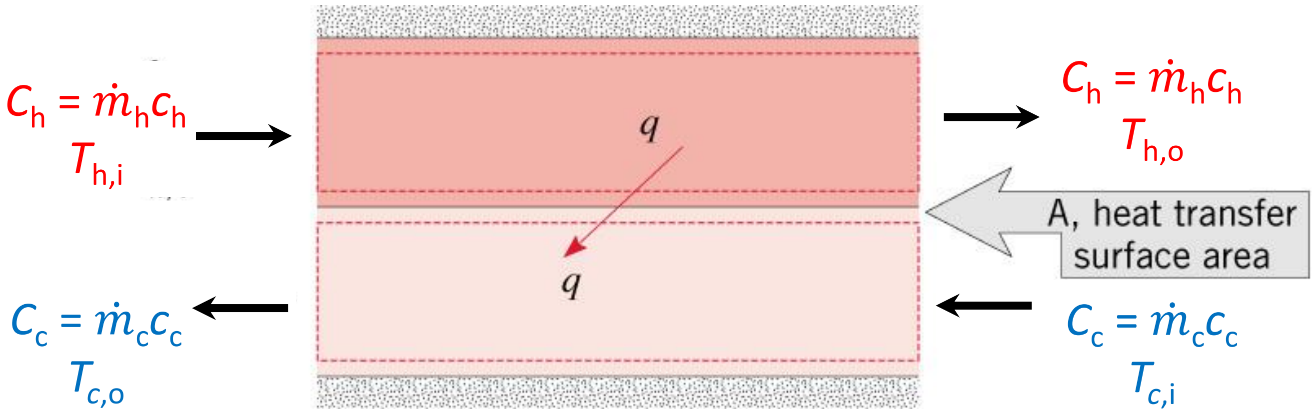
**Chapter 11**

**Sections 11.4**

1. What two methods are used to analyze the performance of a HX?
2. Which of the two fluids in a HX will experience the largest temperature change?
3. Why is the maximum possible heat rate for a heat exchanger not equal to $C_{max}(T_{h,i} - T_{c,i})$? Can the outlet temperature of the cold fluid ever exceed the inlet temperature of the hot fluid?
4. What is the effectiveness of a heat exchanger? What is its range of possible values?
5. What is the number of transfer units (NTU)?
6. Generally, how does the effectiveness change if the size (surface area) of a heat exchanger is increased? If the overall heat transfer coefficient is increased? If the ratio of heat capacity rates is decreased? Any penalty is associated with increasing the size of a heat exchanger?

# Definitions for NTU-E






- What is the maximum possible temperature change through this HX in the ideal case?
  - $T_{h,i} - T_{c,I}$
- Which fluid will reach this temperature difference first?
  - Whichever has a smaller heat capacity, $C$
- What is the maximum possible heat transfer rate through this HX in the ideal case?
  - $C_{min}(T_{h,i} - T_{c,i})$ where $C_{min}$ is the smaller of $C_h$ and $C_c$. Why?
- How to achieve this maximum possible heat transfer rate?
  - Counter-flow

ZJUI

- Maximum possible heat rate:

$$q_{\max} = C_{\min}\left(T_{h,i} - T_{c,i}\right) \qquad (11.18)$$

$$C_{\min} = \begin{cases} C_h \text{ if } C_h < C_c \\ \text{or} \\ C_c \text{ if } C_c < C_h \end{cases}$$

E in NTU-E = = effectiveness, $\varepsilon$

- Heat exchanger effectiveness, $\varepsilon$ :

$$\varepsilon = \frac{q}{q_{\max}} \qquad 0 \le \varepsilon \le 1 \quad (11.19)$$

$$\Rightarrow q = \varepsilon C_{min}\left(T_{h,i} - T_{c,i}\right)$$

- Why is $C_{\min}$ and not $C_{\max}$ used in the definition of $q_{\max}$?
  - Answered in front

ZJUI

- Number of Transfer Units, NTU

$$\boxed{\mathrm{NTU} \equiv \frac{UA}{C_{\min}} \qquad (11.24)}$$

➢ A dimensionless parameter whose magnitude influences HX performance:

$$q \uparrow \text{ with } \uparrow \mathrm{NTU}$$

- Performance Calculations for different HX configurations:

➢ $\varepsilon = f\left(\mathrm{NTU}, \underbrace{C_{\min} / C_{\max}}_{C_r}\right)$ Relations $\rightarrow$ Table 11.3 or Figs. 11.10 - 11.15

Or

➢ $\mathrm{NTU} = f\left(\varepsilon, C_{\min} / C_{\max}\right)$ Relations $\rightarrow$ Table 11.4 or Figs. 11.10 - 11.15

**TABLE 11.3 Heat Exchanger Effectiveness Relations [5]**

| Flow Arrangement | Relation | | |
|---|---|---|---|
| **Concentric tube** | | | |
| Parallel flow | $\varepsilon = \dfrac{1 - \exp[-\mathrm{NTU}(1 + C_r)]}{1 + C_r}$ | | (11.28a) |
| Counterflow | $\varepsilon = \dfrac{1 - \exp[-\mathrm{NTU}(1 - C_r)]}{1 - C_r \exp[-\mathrm{NTU}(1 - C_r)]}$ | $(C_r < 1)$ | |
| | $\varepsilon = \dfrac{\mathrm{NTU}}{1 + \mathrm{NTU}}$ | $(C_r = 1)$ | (11.29a) |
| **Shell-and-tube** | | | |
| One shell pass (2, 4, . . . tube passes) | $\varepsilon_1 = 2\left\{1 + C_r + (1 + C_r^2)^{1/2} \times \dfrac{1 + \exp[-(\mathrm{NTU})_1(1 + C_r^2)^{1/2}]}{1 - \exp[-(\mathrm{NTU})_1(1 + C_r^2)^{1/2}]}\right\}^{-1}$ | | (11.30a) |
| $n$ Shell passes ($2n$, $4n$, . . . tube passes) | $\varepsilon = \left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - 1\right]\left[\left(\dfrac{1 - \varepsilon_1 C_r}{1 - \varepsilon_1}\right)^n - C_r\right]^{-1}$ | | (11.31a) |
| **Cross-flow (single pass)** | | | |
| Both fluids unmixed | $\varepsilon = 1 - \exp\left[\left(\dfrac{1}{C_r}\right)(\mathrm{NTU})^{0.22}\{\exp[-C_r(\mathrm{NTU})^{0.78}] - 1\}\right]$ | | (11.32) |
| $C_{\max}$ (mixed), $C_{\min}$ (unmixed) | $\varepsilon = \left(\dfrac{1}{C_r}\right)(1 - \exp\{-C_r[1 - \exp(-\mathrm{NTU})]\})$ | | (11.33a) |
| $C_{\min}$ (mixed), $C_{\max}$ (unmixed) | $\varepsilon = 1 - \exp(-C_r^{-1}\{1 - \exp[-C_r(\mathrm{NTU})]\})$ | | (11.34a) |
| **All exchangers ($C_r = 0$)** | $\varepsilon = 1 - \exp(-\mathrm{NTU})$ | | (11.35a) |

**Table 11.4** Heat Exchanger NTU Relations

| Flow Arrangement | Relation | |
|---|---|---|
| **Concentric tube** | | |
| Parallel flow | $\mathrm{NTU} = -\dfrac{\ln[1-\varepsilon(1+C_r)]}{1+C_r}$ | (11.28b) |
| Counterflow | $\mathrm{NTU} = \dfrac{1}{C_r - 1}\ln\left(\dfrac{\varepsilon - 1}{\varepsilon C_r - 1}\right) \quad (C_r < 1)$ | |
| | $\mathrm{NTU} = \dfrac{\varepsilon}{1-\varepsilon} \quad (C_r = 1)$ | (11.29b) |
| **Shell-and-tube** | | |
| One shell pass (2, 4, . . . tube passes) | $(\mathrm{NTU})_1 = -(1 + C_r^2)^{-1/2}\ln\left(\dfrac{E-1}{E+1}\right)$ | (11.30b) |
| | $E = \dfrac{2/\varepsilon_1 - (1+C_r)}{(1+C_r^2)^{1/2}}$ | (11.30c) |
| $n$ Shell passes ($2n$, $4n$, . . . tube passes) | Use Equations 11.30b and 11.30c with $\varepsilon_1 = \dfrac{F-1}{F-C_r} \quad F = \left(\dfrac{\varepsilon C_r - 1}{\varepsilon - 1}\right)^{1/n} \quad \mathrm{NTU} = n(\mathrm{NTU})_1$ | (11.31b, c, d) |
| **Cross-flow (single pass)** | | |
| $C_{\max}$ (mixed), $C_{\min}$ (unmixed) | $\mathrm{NTU} = -\ln\left[1 + \left(\dfrac{1}{C_r}\right)\ln(1-\varepsilon C_r)\right]$ | (11.33b) |
| $C_{\min}$ (mixed), $C_{\max}$ (unmixed) | $\mathrm{NTU} = -\left(\dfrac{1}{C_r}\right)\ln[C_r \ln(1-\varepsilon) + 1]$ | (11.34b) |
| **All exchangers ($C_r = 0$)** | $\mathrm{NTU} = -\ln(1-\varepsilon)$ | (11.35b) |

# Summary

Heat Exchanger

- Flow configuration
- Construction type
- Determine *U* to find the heat transfer
  - Fouling
  - Fins
  - Resistances
- Log Mean Temperature Method

# Summary

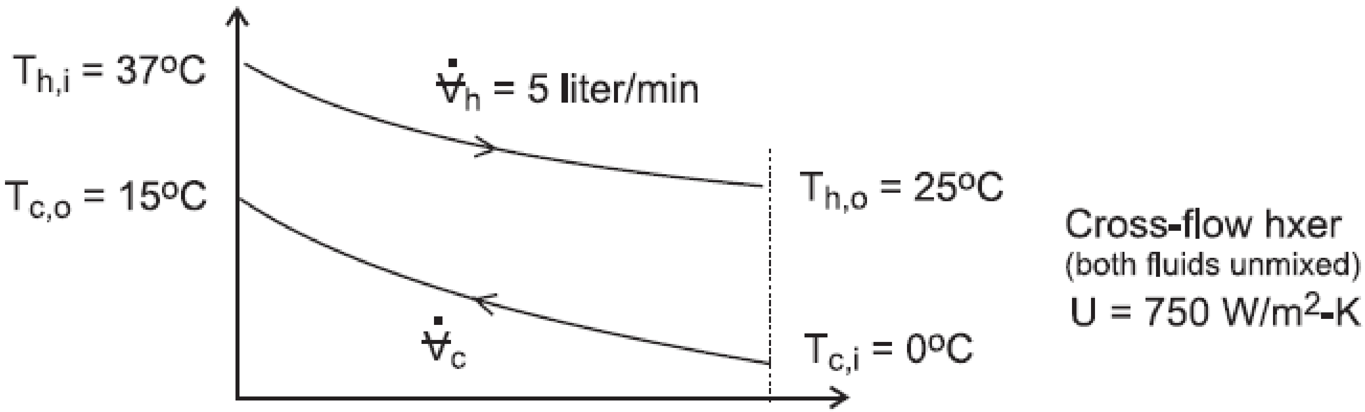
Heat Exchanger

- Log Mean Temperature Method
- NTU-E method
  - First calculating E and (Cmin /Cmax)
  - Use appropriate equation (or chart) to obtain the NTU value
  - Can be used to determine Area

  OR

  - NTU and (Cmin /Cmax) values may be first computed
  - E may then be determined from the appropriate equation (or chart) for a particular exchanger type.
- Actual heat transfer rate, $q = \varepsilon q_{max}$
- Fluid temperatures from q = $\dot{m}c(T_o - T_i)$

In open heart surgery under hypothermic conditions, the patient's blood is cooled before the surgery and rewarmed afterward. Use of a cross-flow heat exchanger for this as shown below.




**KNOWN:** Cross-flow heat exchanger (both fluids unmixed) cools blood to induce body hypothermia using ice-water as the coolant.

**PROPERTIES:** *Table A-6*, Water $\left(\overline{T}_c = 280\,\text{K}\right)$, $\rho = 1000\ \text{kg/m}^3$, $c = 4198\ \text{J/kg}\cdot\text{K}$.

Blood (given): $\rho = 1050\ \text{kg/m}^3$, $c = 3740\ \text{J/kg}\cdot\text{K}$.

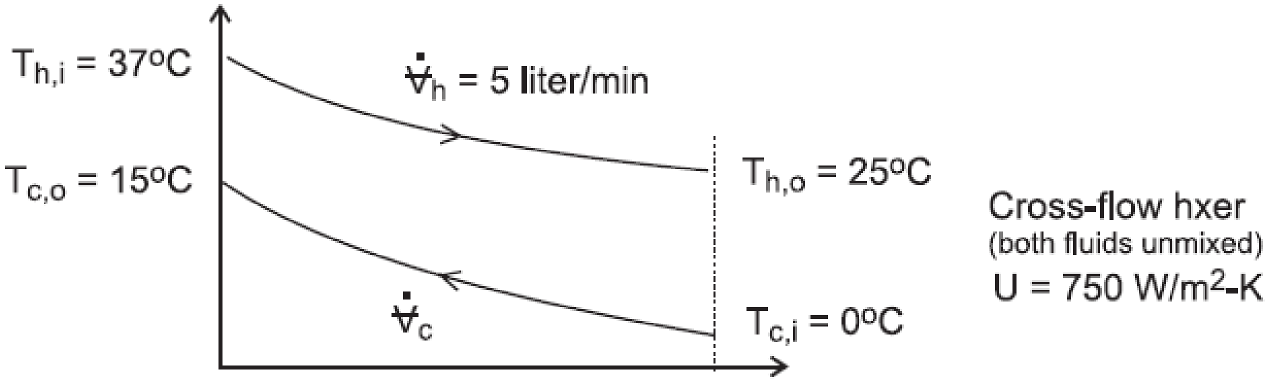
**FIND:**

(a) Heat transfer rate from the blood

(b) Water flow rate, $\dot{\forall}_c$ (liter/min)

(c) Surface area of the exchanger

# Example 1: Blood Cooling




**FIND:**

(a) Heat transfer rate from the blood

(b) Water flow rate, $\dot{\forall}_c$ (liter/min), (c) Surface area of the exchanger

- What is the HT from blood to water? What is this magnitude equal to?
- Is the change in thermal energy in blood = the thermal change in energy in water?
- What kind of HX? Do we know its efficiency?

**ASSUMPTIONS:** (1) Steady-state conditions, (2) Negligible losses to the surroundings, (3) Overall heat transfer coefficient remains constant with water flow rate changes, and (4) Constant properties.

**ANALYSIS:** (a) The heat transfer rate from the blood is calculated from an energy balance on the hot fluid,

$$\dot{m}_h = \rho_h \dot{\forall}_h = 1050\ \text{kg/m}^3 \times \left(5\ \text{liter/min} \times 1\ \text{min/60 s}\right) \times 10^{-3}\ \text{m}^3\ /\ \text{liter} = 0.0875\ \text{kg/s}$$

$$q = \dot{m}_h c_h (T_{h,i} - T_{h,o}) = 0.0875\ \text{kg/s} \times 3740\ \text{J/kg} \cdot \text{K} \left(37 - 25\right) \text{K} = 3927\ \text{W} \qquad < (1)$$

(b) From an energy balance on the cold fluid, find the coolant water flow rate,

$$q = \dot{m}_c\, c_c \left(T_{c,o} - T_{c,i}\right) \qquad (2)$$

$$3927\ \text{W} = \dot{m}_c \times 4198\ \text{J/kg}\cdot\text{K}\left(15-0\right)\text{K} \qquad \dot{m}_c = 0.0624\ \text{kg/s}$$

$$\dot{\forall}_c = \dot{m}_c / \rho_c = 0.0624\ \text{kg/s}/1000\ \text{kg/m}^3 \times 10^3\text{liter/m}^3 \times 60\ \text{s/min} = 3.74\ \text{liter/min}$$

<

(c) The surface area can be determined using the effectiveness - NTU method. The capacity rates for the exchanger are

$$C_h = \dot{m}_h c_h = 327\ \text{W/K} \qquad C_c = \dot{m}_c c_c = 262\ \text{W/K} \qquad C_{\min} = C_c \qquad (3, 4, 5)$$

From Eqs. 11.18 and 11.19, the maximum heat rate and effectiveness are

$$q_{\max} = C_{\min}\left(T_{h,i} - T_{c,i}\right) = 262\ \text{W/K}\ \left(37-0\right)\text{K} = 9694\ \text{W} \qquad (6)$$

$$\varepsilon = q / q_{\max} = 3927 / 9694 = 0.405 \qquad (7)$$

For the cross flow exchanger, with both fluids unmixed, substitute numerical values into Eq. 11.32 to find the number of transfer units, NTU, where $C_r = C_{\min} / C_{\max}$.

$$\varepsilon = 1 - \exp\left[\left(1/C_r\right)\text{NTU}^{0.22}\left\{\exp\left[-C_r\text{NTU}^{0.78}\right] - 1\right\}\right] \qquad (8,9)$$

$$\text{NTU} = 0.691$$

From Eq. 11.24, find the surface area, *A*.

$$\mathrm{NTU} = UA / C_{\min}$$

$$A = 0.691 \times 262\ \mathrm{W/K}/750\ \mathrm{W/m^2 \cdot K} = 0.241\ \mathrm{m^2}$$