University of Illinois

Stat 400 – Midterm 1

Instructors: Kelly Findley, Albert Yu

90 minutes: 7:00pm - 8:30pm

Name: _____ **NetID**: _____

Section(class meeting time): _____

INSTRUCTIONS:

- **You are not allowed** to give or receive any assistance to/from another person for this exam. **Please do not** discuss the contents of this exam comment until the weekend.
- **Allowable materials**: Graphing calculators, writing utensils, blank scratch paper, and a 2-sided **hand-written** cheat sheet 8.5 x 11 inch maximum. (No printed notes allowed).
- The word **evaluate** means you need to include a **numerical answer** (or reasonably simplified fraction). Feel free to leave it simplified and exact (fraction), or round your answer reasonably.
- Even if you are using a calculator that can do fancy and complex functions, you must show all reasonable work and steps as if you did those by hand with a simple calculator. If insufficient work is shown, **it is possible for a correct answer to receive no credit**.

This exam contains 12 pages and 6 questions. There are (100) points possible.
Please show your process (work) for full credit. Answers without supporting work may receive 0 points, even if correct. Good luck! :)

Question	Points	Score
1	32	
2	11	
3	10	
4	13	
5	12	
6	22	
Total:	100	

The space below may be used as scratch paper

1. Let $f(x) = \begin{cases} c(3x+1), & \text{if } 0 \leq x \leq 3 \\ 0, & \text{otherwise} \end{cases}$

Advice: If you get stuck on one part, write a note and make up values for what you are missing. If the work is correct, you can still receive credit for other parts.

(a) (5 points) Evaluate the **expected value** of X, $E[X]$.

(b) (7 points) Evaluate $P[X \leq 1 \cup X \geq 2]$.

(c) (6 points) Evaluate the median of X. Show as much work as possible for partial credit.

(d) (3 points) How would you find an expression for the moment generating function of X? *Just set up the integral or summation. No need to solve.*

Let Z be a linear transformation of X, such that

$$Z = 8 - 4X:$$

(e) (4 points) Find $E[Z]$.

(f) (3 points) Find $SD[Z]$.

(g) (4 points) Find $Var[-3Z]$.

2. Suppose that in any given month, a Komkast subscriber's internet goes out according to a Poisson Process with rate $\lambda = 20$ (times per month).

   (a) (3 points) What is the variance of the number of times that the internet will go out in a year?

   (b) (4 points) What is the probability that the internet will go out either 19 or 20 times (exactly) in any given month? (Just set up the equation.)

   (c) (4 points) What is the probability that the **3**rd internet outage happens before your first 24 hour period with Komkast? Assume that there are 30 days in every month. (Just set up the summation or integral; no need to evaluate.)

3. Vi is not sure if she will go to a particular party. If Jinx is going, the probability that Vi goes is 0.9. If Jinx is **not** going, the probability that Vi goes is 0.6. Assume that Jinx goes to 40% of all parties (and that they everyone stays the whole time).

   Evaluate the following probabilities:

   (a) (7 points) If Vi is going to this particular party, what the probability that Jinx is going? Please show your work.

   (b) (3 points) What is the probability that neither of them go to this party?

4. Albert's dog Chloe finds a secret stash of doggie treats with the following flavors: 30 beef, 30 chicken, 40 pork. She selects treats at random, licks them, and puts them back in the bag (she can select the same treat multiple times).

   (a) (7 points) Let X represent the number of trials it takes her to get a pork treat for the 10th time. What is the probability that she gets the 10th pork treat on (exactly) the 20th trial? **State the distribution of** X and write the equation you need to solve. (No need to evaluate).

   (b) (6 points) If Chloe selects 6 treats without replacement, evaluate the probability that she randomly selects exactly 2 beef and at least 3 chicken treats (in any order).

5. Let $X \sim N(\mu = 9, \sigma^2 = 16)$.

   Let $Y = 3X + 1$

   Show all your work, and show standardization step *by hand* for full credit. *If you are not sure how much work to show, pretend like you own a very* ***basic*** *calculator.*

   (a) (4 points) Evaluate $P[X > 10]$.

   (b) (8 points) Find the 10th percentile of Y (what value). *Hint: What is its distribution?*

6. Let $S = \{-1, 0, 1, 2, ...\}$.

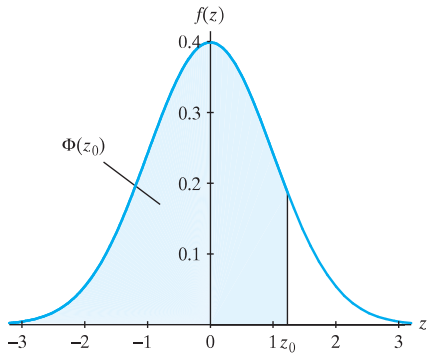
   Consider the pmf: $f(-1) = .5, \quad f(x) = c \cdot (\frac{1}{5})^x$, where $x = 0, 1, 2, ...$

   (a) (7 points) Find a value for the constant, c that would make this a valid pmf. Do by hand and show all of your work.

   (b) (8 points) Find $P[X = 2|\text{X is an even number}]$.
   Note: 0 is an even number.

   (c) (7 points) Evaluate $E[X]$. *If you were not able to solve part (a), evaluate it in terms of c.* Show your work for full credit!

Table Va The Standard Normal Distribution Function



$$P(Z \le z) = \Phi(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-w^2/2}\, dw$$

$$\Phi(-z) = 1 - \Phi(z)$$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

α	0.400	0.300	0.200	0.100	0.050	0.025	0.020	0.010	0.005	0.001
z_α	0.253	0.524	0.842	1.282	1.645	1.960	2.054	2.326	2.576	3.090
$z_{\alpha/2}$	0.842	1.036	1.282	1.645	1.960	2.240	2.326	2.576	2.807	3.291

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Note: If we need to grade something on this page, **do not detach**.