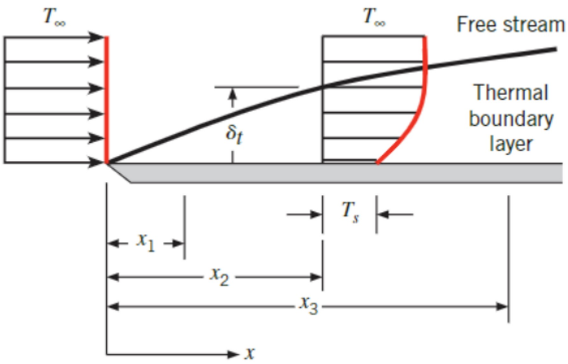
Homework 4 **边界层+雷诺相似**+Nu

The temperature distribution within a laminar thermal boundary layer associated with flow over an isothermal flat plate is shown in the sketch. The temperature distribution shown is located at $x = x_2$.



(a) Is the plate being heated or cooled by the fluid?

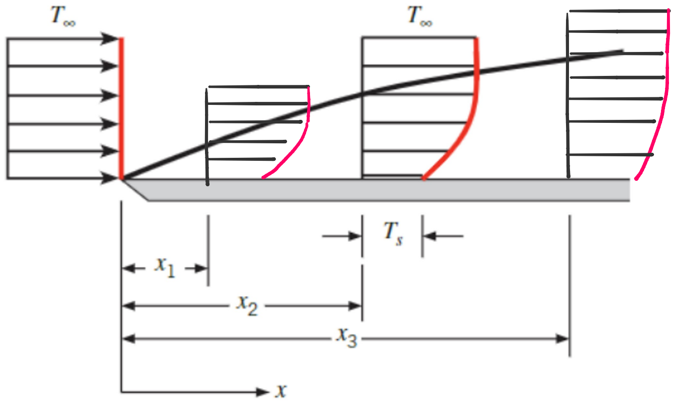
(b) Carefully sketch the temperature distributions at $x = x_1$ and $x = x_3$. Based on your sketch, at which of the three x-locations is the local heat flux largest? At which location is the local heat flux smallest?

(c) As the free stream velocity increases, the velocity and thermal boundary layers both become thinner. Carefully sketch the temperature distributions at $x = x_2$ for (i) a low free stream velocity and (ii) a high free stream velocity. Based on your sketch, which velocity condition will induce the larger local convective heat flux?

温度边界层

a)The plate is heated by the fluid

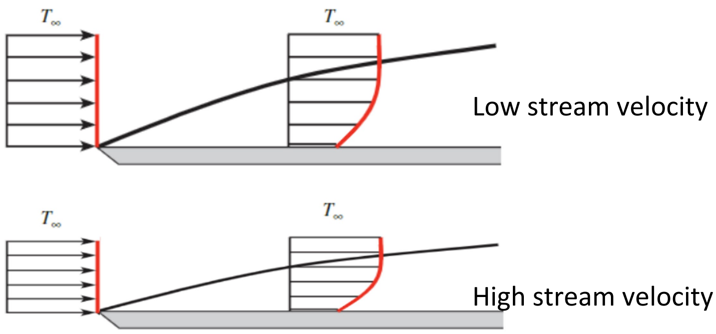
b)



The heat flux at x1 is the largest due to largest temperature gradient

The heat flux at x3 is the smallest

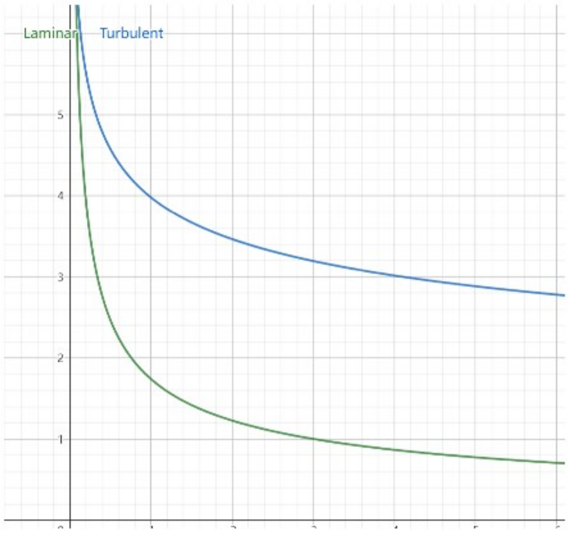
c)



I scale the plot in vertical direction

The high stream velocity will induce the larger local convection heat flux

Laminar flow normally persists on a smooth flat plate until a critical Reynolds number value is reached. However, the flow can be tripped to turbulent state by adding roughness to the leading edge of the plate. For a particular situation, experimental results show that the local heat transfer coefficients for laminar and turbulent conditions are



$$\text{Laminar: } h_x = 1.74\ x^{-0.5}$$
$$\text{Turbulent: } h_x = 3.98\ x^{-0.2}$$

Calculate the average heat transfer coefficients for laminar and turbulent conditions for plates of length L=0.1m and 1m.

$$\because\ \bar{h}_x = \frac{1}{x}\int_0^x h_x \cdot dx$$

$$\therefore\ \bar{h}_L = \frac{1}{x}\int_0^x 1.74\, x^{-0.5}\, dx = 3.48\, x^{-0.5} \Rightarrow \bar{h}_{L=0.1} = 5.5 \qquad \bar{h}_{L=1} = 3.48$$

$$\bar{h}_T = \frac{1}{x}\int_0^x 3.98\, x^{-0.2}\, dx = 4.975\, x^{-0.2} \Rightarrow \bar{h}_{T=0.1} = 7.89 \qquad \bar{h}_{T=1} = 4.975$$

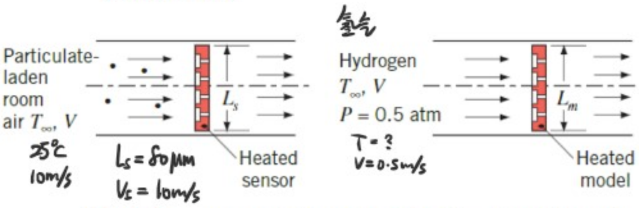
III. (Hint: The *Nu* number for both the model and the actual system must be the same so that we can get the required average heat transfer coefficient. For this to be true, what condition needs to be satisfied?)

A microscale detector monitors a steady flow (T_∞ =25 °C, V = 10 m/s) of air for the possible presence of small, hazardous particulate matter that may be suspended in the room. The sensor is heated to a slightly higher temperature to induce a chemical reaction associated with certain substances of interest that might impinge on the sensor's active surface. The active surface produces an electric current if such surface reactions occur; the electric current is then sent to an alarm. To maximize the sensor head's surface area and, in turn, the probability of capturing and detecting a particle, the sensor head is designed with a very complex shape. The value of the average heat transfer coefficient associated with the heated sensor must be known so that the required electrical power to the sensor can be determined.

Table A.4 *Continued*

T (K)	ρ (kg/m³)	c_p (kJ/kg·K)	$\mu \cdot 10^7$ (N·s/m²)	$\nu \cdot 10^6$ (m²/s)	$k \cdot 10^3$ (W/m·K)	$\alpha \cdot 10^6$ (m²/s)	Pr
Hydrogen (H_2), $\mathcal{M}$ = 2.016 kg/kmol							
100	0.24255	11.23	42.1	17.4	67.0	24.6	0.707
150	0.16156	12.60	56.0	34.7	101	49.6	0.699
200	0.12115	13.54	68.1	56.2	131	79.9	0.704
250	0.09693	14.06	78.9	81.4	157	115	0.707
300	0.08078	14.31	89.6	111	183	158	0.701
350	0.06924	14.43	98.8	143	204	204	0.700
400	0.06059	14.48	108.2	179	226	258	0.695
450	0.05386	14.50	117.2	218	247	316	0.689
500	0.04848	14.52	126.4	261	266	378	0.691
550	0.04407	14.53	134.3	305	285	445	0.685

T (K)	ρ (kg/m³)	c_p (kJ/kg·K)	$\mu \cdot 10^7$ (N·s/m²)	$\nu \cdot 10^6$ (m²/s)	$k \cdot 10^3$ (W/m·K)	$\alpha \cdot 10^6$ (m²/s)	Pr
Air, $\mathcal{M}$ = 28.97 kg/kmol							
100	3.5562	1.032	71.1	2.00	9.34	2.54	0.786
150	2.3364	1.012	103.4	4.426	13.8	5.84	0.758
200	1.7458	1.007	132.5	7.590	18.1	10.3	0.737
250	1.3947	1.006	159.6	11.44	22.3	15.9	0.720
300	1.1614	1.007	184.6	15.89	26.3	22.5	0.707



Consider a sensor with a characteristic dimension of L_s = 80 μm. A scale model of the sensor is placed in a recirculating (closed) wind tunnel using hydrogen as the working fluid. If the wind tunnel operates at a hydrogen absolute pressure of 0.5 atm and velocity of V = 0.5 m/s, find the required hydrogen temperature and characteristic dimension of the scale model, L_m.

Question: 计算模型 T_∞ 和 L_m 特征长度

根据 Table A-4,

对于空气 $Pr_s = 0.707$, $\nu_s = 1.571 \times 10^{-5}\ m^2/s$

对于氢气在 $Pr_m = 0.707$ 时, $\nu_{H_2(1atm)} = 81.4 \times 10^{-6}\ m^2/s$, $T = 250K$

$L_s = 80\ \mu m$

$V_s = 10\ m/s$

$\nu_s = 1.571 \times 10^{-5}\ m^2/s$

$V_m = 0.5\ m/s$

$\nu_m = 1.63 \times 10^{-4}\ m^2/s$

$\Rightarrow L_m = 16702\ \mu m = \boxed{16.7mm}$

$$\nu = \frac{\mu}{\rho} \Rightarrow \frac{\nu_{0.5atm}}{\nu_{1atm}} = \frac{\frac{1}{P_{0.5atm}}}{\frac{1}{P_{1atm}}} \Rightarrow \nu_{0.5} = 2\nu_{1.0} \Longrightarrow$$

$Nu_s = Nu_m$

$Nu = f(x^*, Re_L, Pr)$

x^*: 几何相似 (已满足)

$\Rightarrow$

$Re = \frac{V \cdot L}{\nu}$

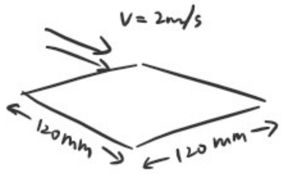
$Re_s = Re_m$ $\Rightarrow \frac{V_s \cdot L_s}{\nu_s} = \frac{V_m \cdot L_m}{\nu_m}$

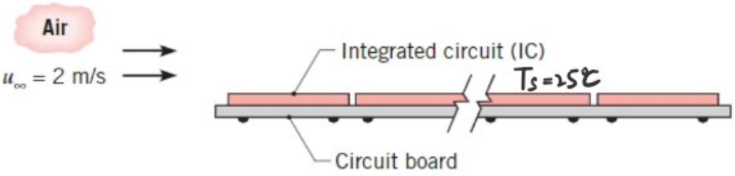
$Pr_s = Pr_m$

$Pr_s = 0.707$ $\Rightarrow Pr_m = 0.707 \Rightarrow \boxed{T = 250K}$

气压不同T不变 ②

A circuit board with a dense distribution of integrated circuits (ICs) and dimensions of 120 mm × 120 mm on a side is cooled by the parallel flow of atmospheric air with a velocity of 2 m/s.





$$\overline{\tau_f} = 0.0625\ N/m^2$$

From wind tunnel tests under the same flow conditions, the average frictional shear stress on the upper surface is determined to be 0.0625 N/m². What is the allowable power dissipation from the upper surface of the board if the average surface temperature of the ICs must not exceed the ambient air temperature by more than 25°C? Evaluate the thermophysical properties of air at 300 K.

根据 Table A-4， $C_{air} = 1007\ J/kg\cdot K$, $P_r = 0.707 \in (0.6, 60)$

根据 Modified Reynolds Analogy

$$\frac{C_f}{2} = St \cdot P_r^{\frac{2}{3}}$$

$$C_f = \frac{\overline{\tau_s}}{\frac{\rho_{air} \cdot V_{air}^2}{2}} \qquad St = \frac{\bar{h}}{\rho_{air} \cdot V_{air} \cdot C_{air}}$$

$$\therefore \frac{\overline{\tau_s}}{\cancel{\rho_{air}} \cdot V_{air}^{\cancel{2}}} = \frac{\bar{h}}{\cancel{\rho_{air}} \cdot \cancel{V_{air}} \cdot C_{air}} \cdot P_r^{\frac{2}{3}}$$

$$\therefore \bar{h} = \frac{C_{air} \cdot \overline{\tau_s}}{V_{air} \cdot P_r^{\frac{2}{3}}} = \frac{1007\ J/kg\cdot K \times 0.0625\ N/m^2}{2m/s \times 0.707^{\frac{2}{3}}} = 39.65\ W/m^2\cdot K$$

$$q_{conv} = \bar{h} \cdot A_s \cdot (\underbrace{T_s - T_\infty}_{=25°C}) = 39.65\ W/m^2\cdot K \times (120mm)^2 \times 25\ K = \boxed{14.27\ W}$$