Lecture 9: Introduction to Convection: Flow and Thermal Considerations

Chapter Six

Sections 6.1 – 6.3

Announcement

- Quiz07 up

Content

1. What is a boundary layer (BL)?

2. How does the velocity (temperature) gradient at the surface change along the flow direction?

3. How does surface shear stress (heat transfer rate) vary along the flow direction in a velocity (thermal) BL? Why?

4. How does the velocity (thermal) BL thickness change with along the flow direction? Why?

5. What is the difference between a local and average heat transfer coefficient?

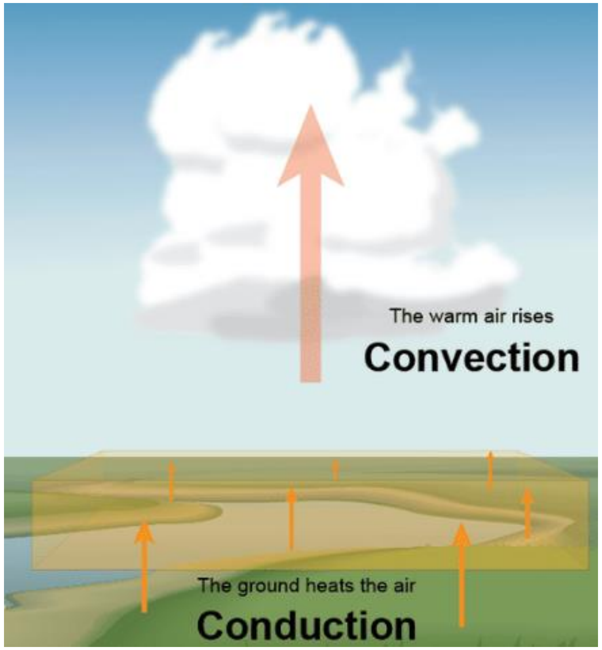
6. What is the Reynolds number? What is the critical value for judging laminar or turbulent flow over a flat plate?

7. How and why does h change in the laminar, laminar-turbulent, turbulent flow?

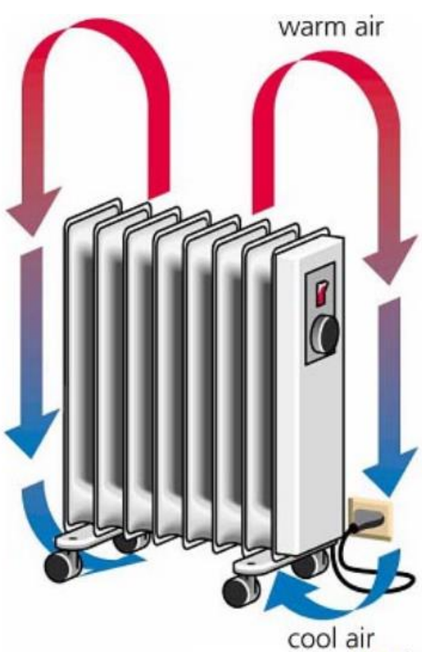
Convection == Heat transfer from surface by bulk fluid motion (advection) and diffusion/conduction

Section 6.1

Convection???


The warm air rises
Convection
The ground heats the air
Conduction


CONVECTION OVEN
CONVENTIONAL OVEN


warm air
cool air


Convection
Conduction
Radiation

Convection Heat Transfer Equation

$$q_s = hA_{SA}(T - T_\infty)$$

q_s - heat flow
h – heat transfer coefficient
A_{SA} - surface area of convection
T – Temperature of surface
T_∞ - Temperature of surrounding at ∞

What do we know about this equation?

So, what do we not know?

Fluid moving on Surface ➔ Boundary Layers (Velocity and Thermal)

Section 6.1

ZJUI

- Velocity Boundary Layer (velocity)

  - What is happening to the particles in the fluid near to the wall?

    Zero velocity. Why?

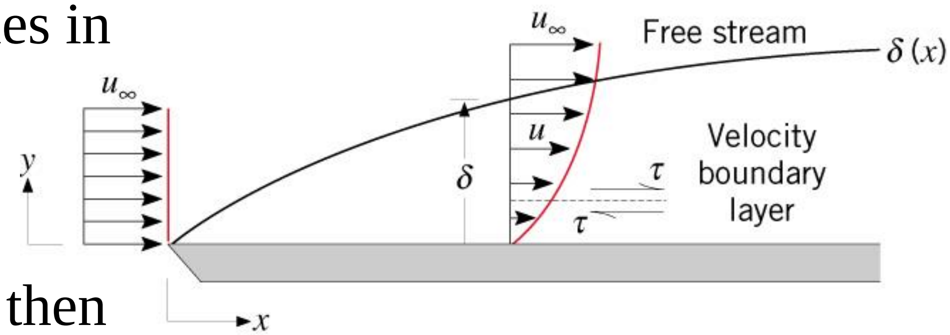
  - What happens to the next layer, then the next layer…

  - What is slowing them down?

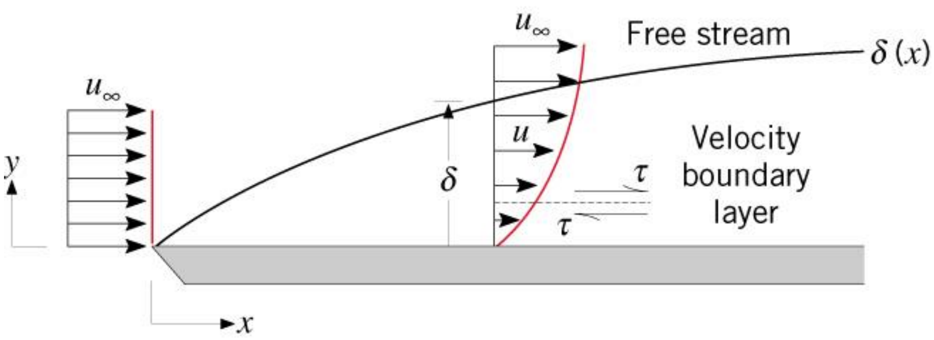
    Shear stress

  - What happens to u_x at any x in the BL?

    Increasing



- Velocity Boundary Layer (velocity)



  - A region
    - ➢ between the surface and the free stream whose thickness δ increases in the flow direction.
    - ➢ the flow characterized by shear stresses and velocity gradients.

  - Why does δ increase in the flow direction?

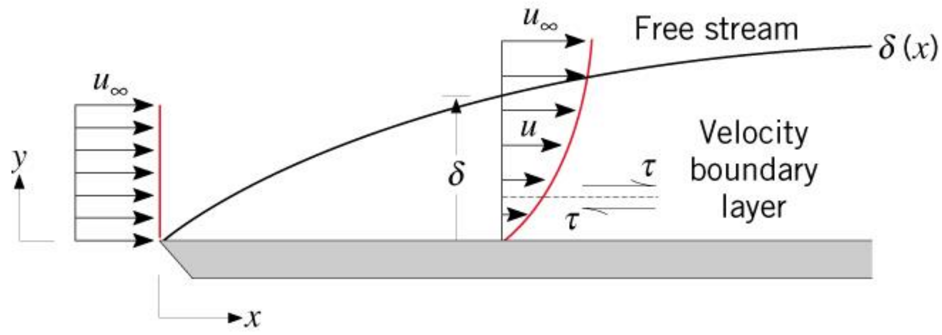
    Increasing number of layers slowing down

$$\delta \to \frac{u(y)}{u_\infty} = 0.99$$

ZJUI

- Velocity Boundary Layer (Shear)

  - Arises due to viscous effects associated with relative motion between a fluid and a surface.



  - A surface shear stress τ_s provides a drag force, F_D with μ as the dynamic viscosity [N s/m^2].

$$\tau_s = \mu \frac{\partial u}{\partial y}\bigg|_{y=0}$$

[N/m^2]

$$F_D = \int_{A_s} \tau_s \, dA_s$$

[N]

$$C_f = \frac{\tau_s}{\rho u_\infty^2/2}$$

[dimensionless]

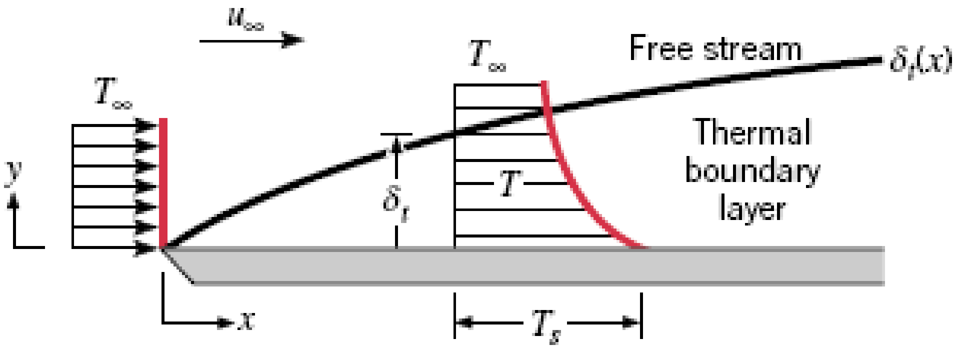
  - How does τ_s vary in the flow direction? Why?

Smaller as difference in velocity is getting smaller

ZJUI

- Thermal Boundary Layer (temperature)
  - Heat transfer between surface and fluid. => $T_s \neq T_\infty$
  - In this picture, which is bigger? T_s or T_∞?
  - Thickness δ_t increases in the flow direction:

$$\delta_t \rightarrow \frac{T_s - T(y)}{T_s - T_\infty} = 0.99$$



  - Why does the temperature along the surface increase with increasing x? Will the temperature stop increasing?

Heat transfer from surface to fluid

  - Why does δ_t increase in the flow direction?

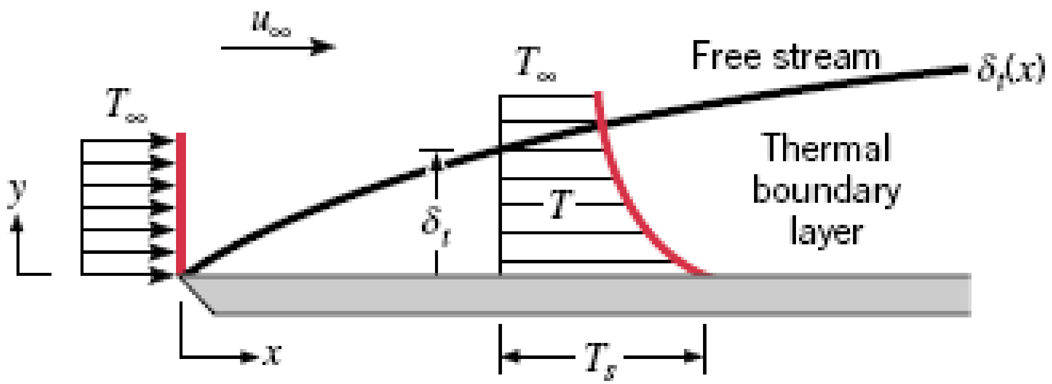
More HT to the fluid along x through time

ZJUI

- Thermal Boundary Layer (heat)
  - How does the surface heat flux q_s'' transfer to the fluid?

$$\boxed{q_s'' = -k_f \frac{\partial T}{\partial y}\bigg|_{y=0}}$$ [W/m²]

$$\boxed{q_s'' = h(T_s - T_\infty)}$$ [W/m²·K]



Why there is a Fourier's Law at the fluid surface???

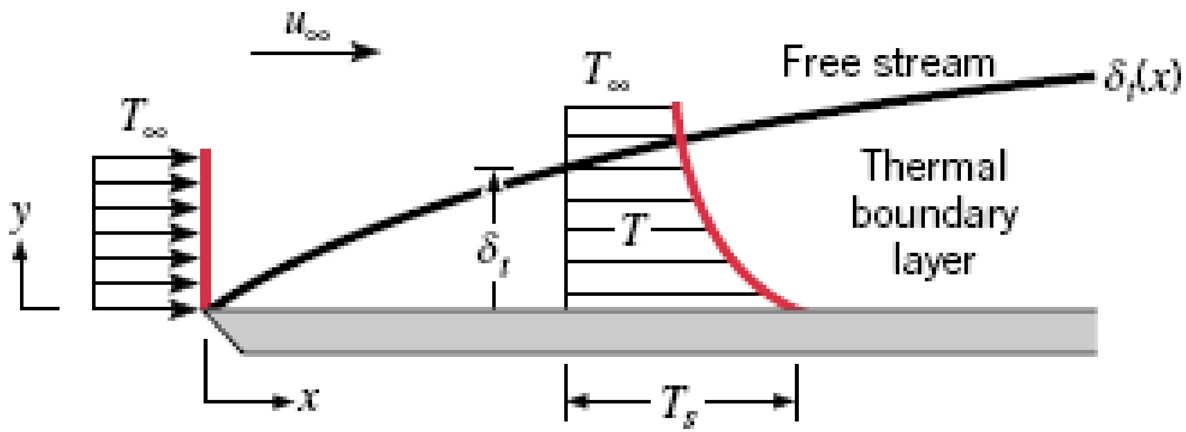
Heat has to transfer to the fluid from the solid. With surface fluid at 0 velocity and in contact with surface, heat is transferred through conduction

$$\Rightarrow h \equiv \frac{-k_f \frac{\partial T}{\partial y}\Big|_{y=0}}{T_s - T_\infty}$$

ZJUI

- Thermal Boundary Layer (heat)
  - Region with temperature gradient is the Thermal Boundary Layer

$$\boxed{h \equiv \frac{-k_f \partial T / \partial y \big|_{y=0}}{T_s - T_\infty}}$$



– Does $\partial T/\partial y$ at the surface change with increasing x-direction? Increase or decrease? How do q_s'' and local h change along the flow direction?

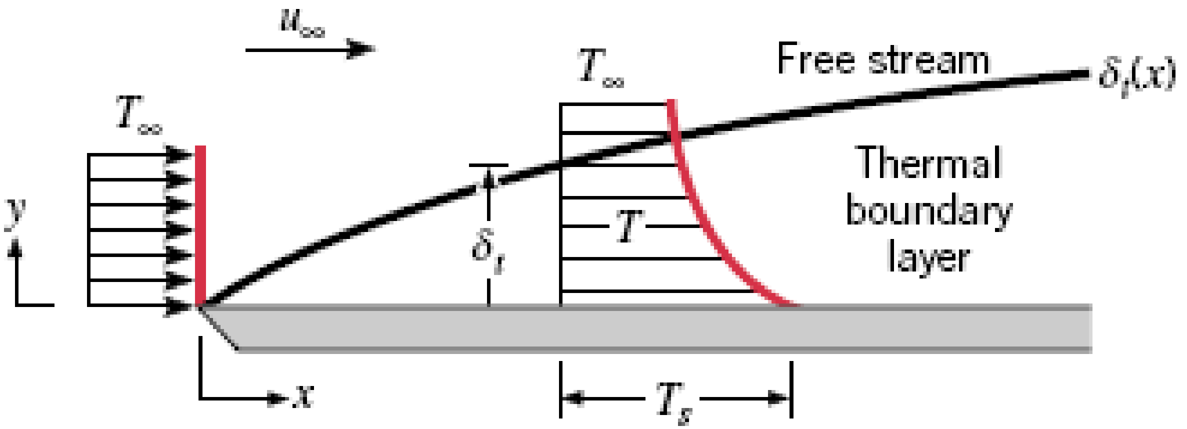
– Are the thickness of δ and δ_t the same?

Local and Average Convection Coefficients (h_{local} or h_{ave})

Section 6.2

$$\boxed{q_s'' = -k_f \frac{\partial T}{\partial y}\bigg|_{y=0}}$$

$$\boxed{h \equiv \frac{-k_f \partial T / \partial y\big|_{y=0}}{T_s - T_\infty}}$$



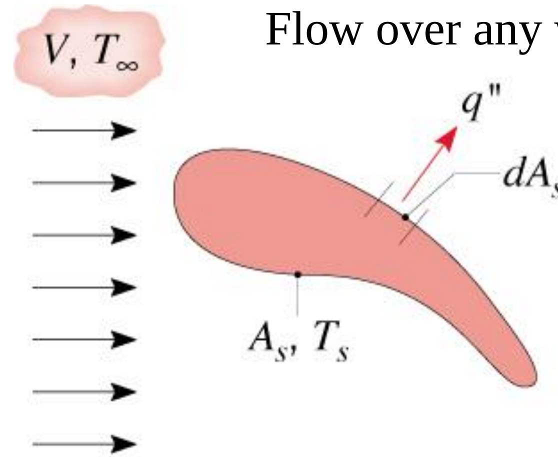
q_s'' and local h along x drops with increasing x.

So, how to find the total heat transfer across the whole surface?
– find average h !

Local Heat Flux and Coefficient:

$$q'' = h(T_s - T_\infty)$$

Flow over any weird shape



- Total Heat Rate:

$$q = \int_{A_s} q'' dA_s = (T_s - T_\infty)\int_{A_s} h dA_s$$

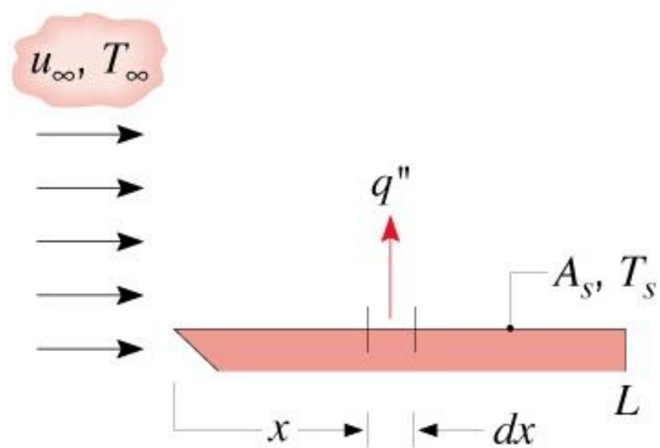
Average Heat Flux and Coefficient for a Uniform Surface Temperature:

Define as $q = \bar{h} A_s (T_s - T_\infty)$

So, $\bar{h} = \dfrac{1}{A_s}\int_{A_s} h dA_s$

- For a flat plate in parallel flow:

$$\bar{h} = \frac{1}{L}\int_o^L h dx$$



Example: 6.1 from Text

Experimental results for the local heat transfer coefficient h_x for flow over a flat plate with an extremely rough surface were found to fit the relation

$$h_x(x) = ax^{-0.1}$$

where a is a coefficient ($\mathrm{W/m^{1.9} \cdot K}$) and x (m) is the distance from the leading edge of the plate.

1. Develop an expression for the ratio of the average heat transfer coefficient $\overline{h}_x$ for a plate of length x to the local heat transfer coefficient h_x at x.
2. Show, in a qualitative manner, the variation of h_x and $\overline{h}_x$ as a function of x.

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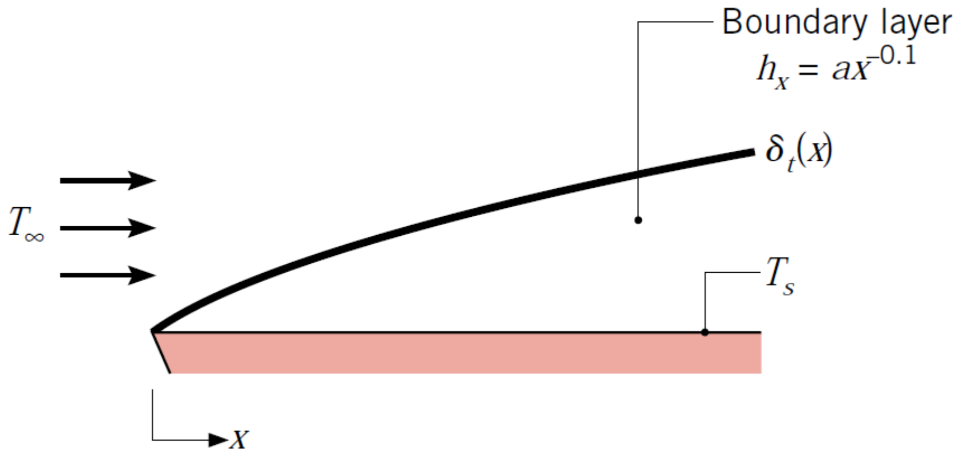
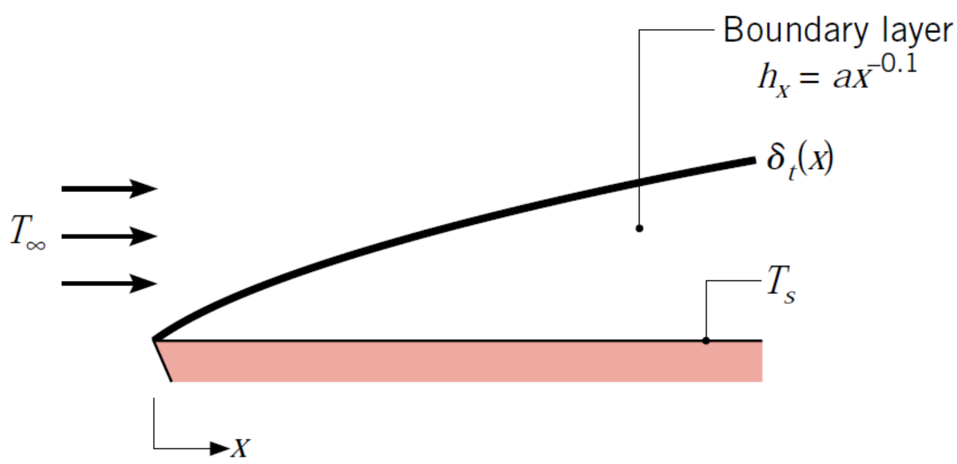


Example: 6.1 from Text



- Do we have local *h* or average *h*?

$$\bar{h} = \frac{1}{L}\int_o^L h dx$$

$$= \frac{1}{x}\int_o^x ax^{-0.1} dx$$

$$= 1.11ax^{-0.1}$$

$$= 1.11h_x$$

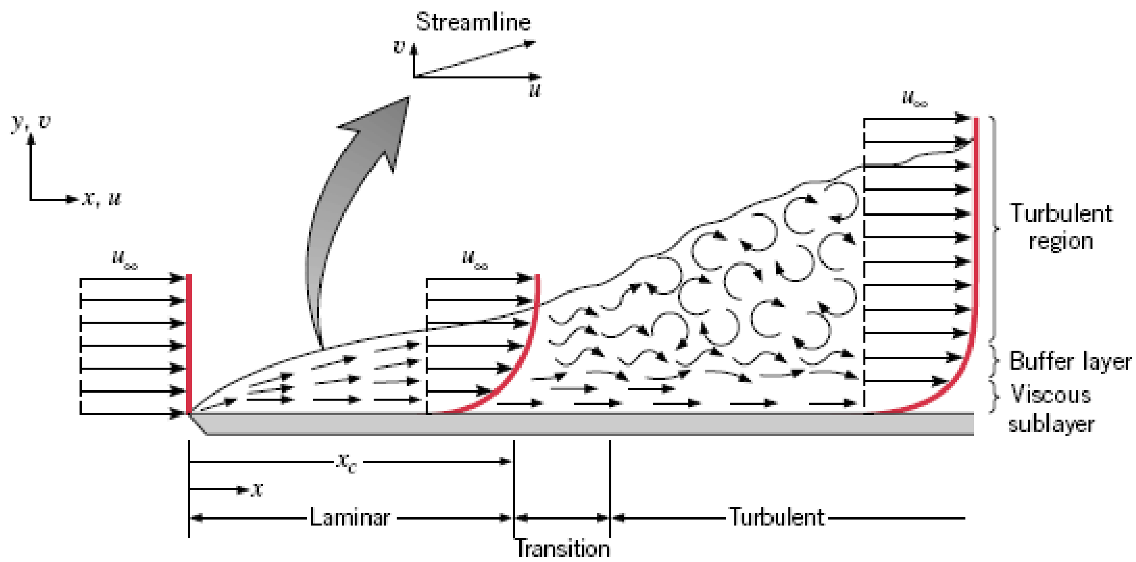
What is missing in our Flow?
Laminar and Turbulent

Section 6.3

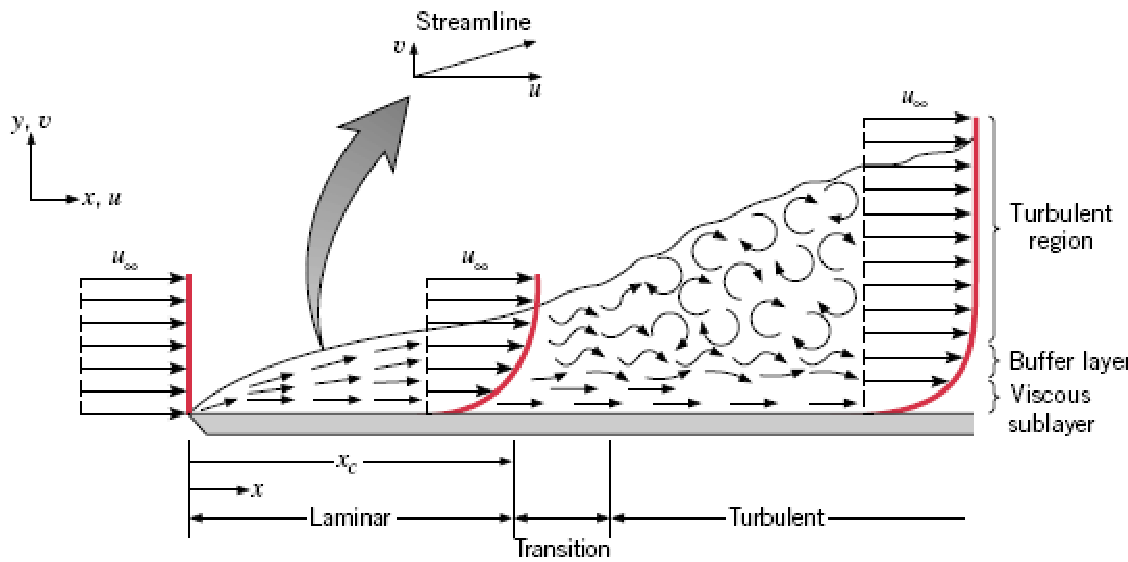
https://www.youtube.com/watch?v=9A-uUG0WR0w

Laminar vs Turbulent Velocity Boundary Layer (BL)

ZJUI



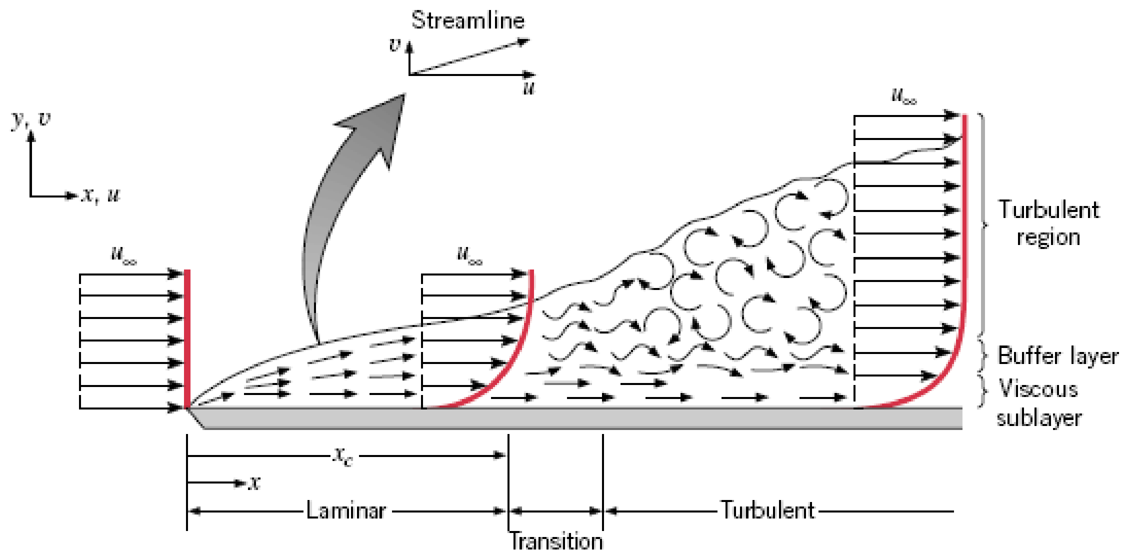
- **First Step to solve HT convection problems**
  - Determine the boundary layer flow type! Laminar or Turbulent!
  - Convection rates depend on these conditions.
  - How do you judge the flow condition?



- **Laminar BL**
  - Highly ordered streamlines
  - Laminar BL grows with x
    - ➢ du/dy at wall ($y = 0$) becomes smaller
    - ➢ smaller surface shear stress, τ_s
  - BL grows until Transition happens

Laminar vs Turbulent Velocity Boundary Layer (BL)

ZJUI



- **Turbulent BL**
  - Highly irregular with random 3D mixing! Complex!
  - High speed fluid from top replace slow speed ones at surface
  - Vortices appear and disappear rapidly near to plate
    - ➢ Velocity and pressure fluctuations in BL
  - Separated into 3 parts:
    - ➢ **Viscous sublayer** – diffusion with nearly linear y-velocity profile
    - ➢ **Buffer sublayer** – diffusion and turbulent mixing comparable
    - ➢ **Turbulent region** – turbulent mixing with flat y-velocity profile

ZJUI

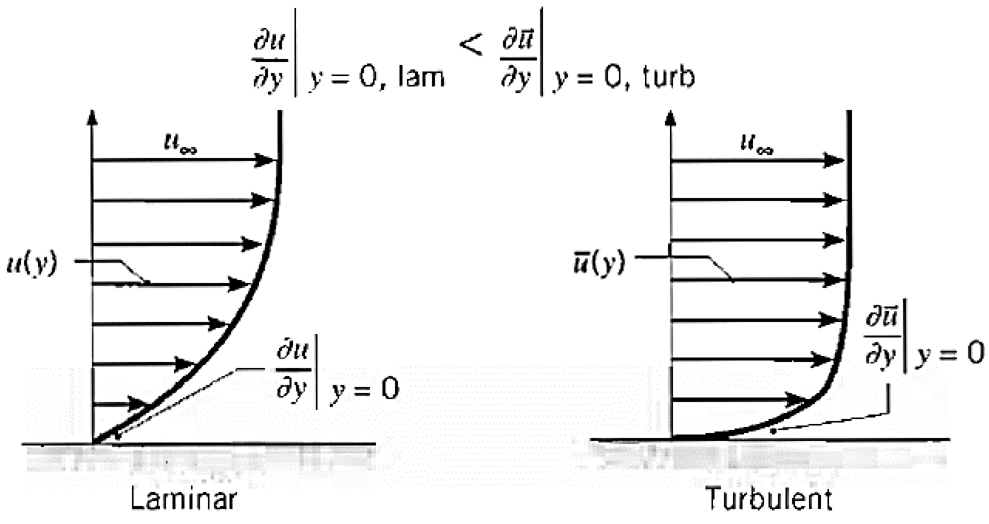


FIGURE 6.7 Comparison of laminar and turbulent velocity boundary layer profiles for the same free stream velocity.[3]

- **Velocity Profile in Laminar vs Turbulent BL**
  - Which is steeper at the surface?

    Turbulent

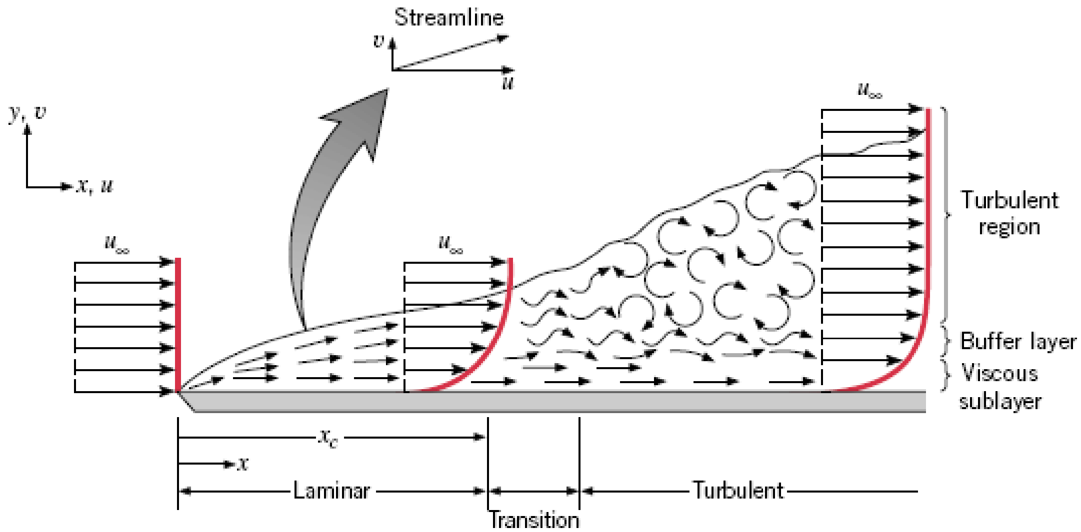
  - Which will give a larger surface shear stress?

    Turbulent

  - Why the profile in Turbulent BL looks flat away from surface?

    Mixing of faster fluid layer from the top

Velocity Boundary Layer Transition

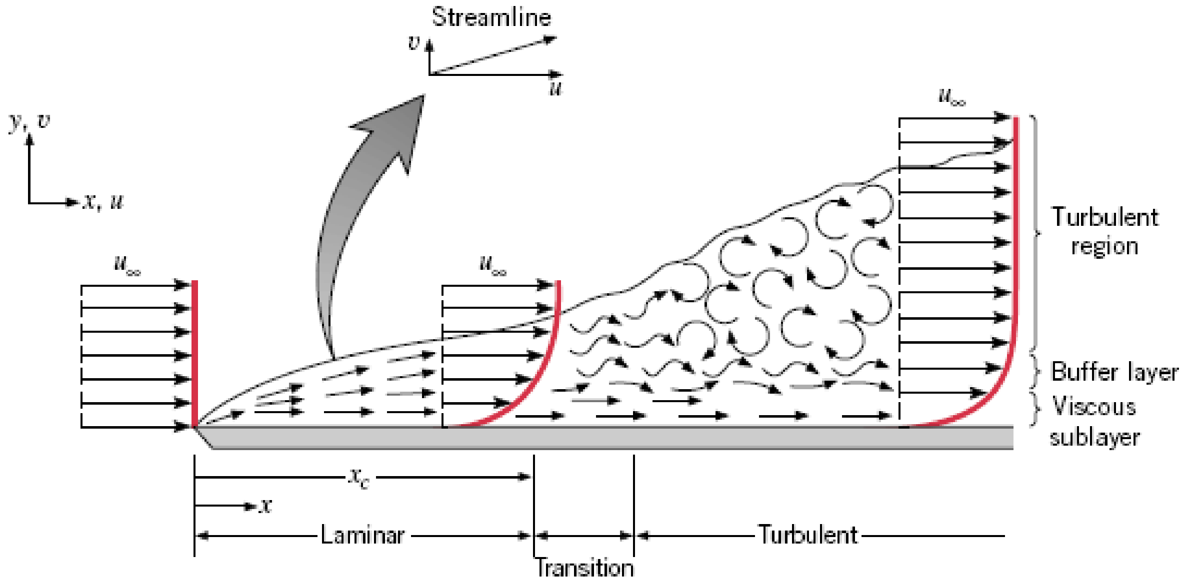


- What causes transition from laminar to turbulent flow?
  - Disturbance from ….

- Reynolds number quantifies the transition from laminar to turbulent flow

$$\boxed{Re_x \equiv \frac{\rho u_\infty x}{\mu} = \frac{inertia}{viscous} = \frac{amplifying}{dissipating}}$$

Velocity BL Transition



- **Transition criterion for a flat plate in parallel flow:**

$$Re_{x,c} \equiv \frac{\rho u_\infty x_c}{\mu} \rightarrow \text{ critical Reynolds number}$$

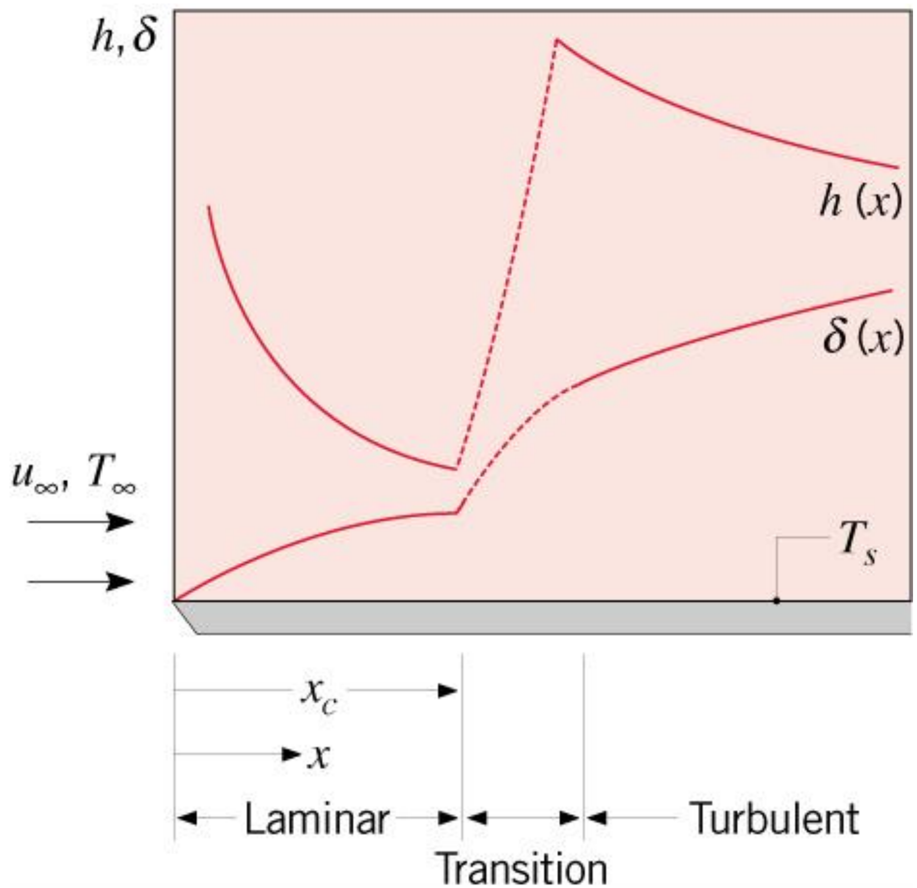
$$x_c \rightarrow \text{ location at which transition to turbulence begins}$$

$$10^5 \lesssim Re_{x,c} \lesssim 3 \times 10^6$$

$\boldsymbol{Re_{x,c}} = \mathbf{5 \times 10^5}$ is typically used.

Has transition happened if $Re_L < Re_{x,c}$? If $Re_L > Re_{x,c}$?

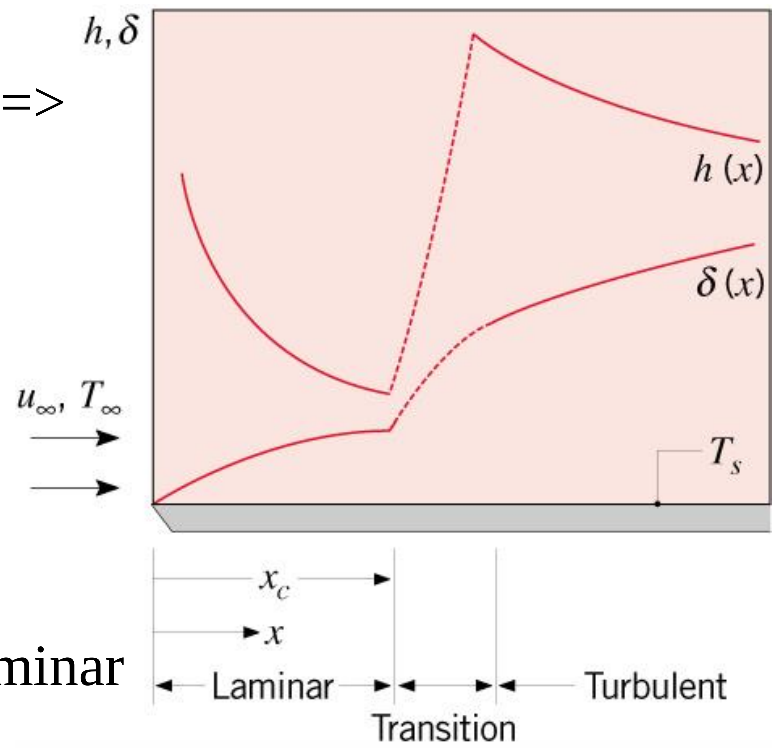
- Effect of transition on boundary layer thickness and local convection coefficient:



Velocity and Thermal BL

- Velocity => particle motions carry energy => heat transfer happens
- Velocity profile important for temperature profile

$$h \equiv \frac{-k_f \partial T / \partial y\big|_{y=0}}{T_s - T_\infty}$$



- Why does the local *h*, decay in the laminar region with increasing *x*?
  - Increase Thermal BL => smaller ∂*T*/∂*y* at suface
- Why does *h* increase significantly with transition, despite BL increases
  - Turbulent mixing => more layers having higher velocity
- Why does *h* decay in the turbulent region with increasing *x*?

Example: 6.4 from Text

Water flows at a velocity $u_\infty = 1$ m/s over a flat plate of length $L = 0.6$ m. Consider two cases, one for which the water temperature is approximately 300 K and the other for an approximate water temperature of 350 K. In the laminar and turbulent regions, experimental measurements show that the local convection coefficients are well described by

$$h_{\text{lam}}(x) = C_{\text{lam}}x^{-0.5} \qquad h_{\text{turb}}(x) = C_{\text{turb}}x^{-0.2}$$

where x has units of m. At 300 K,

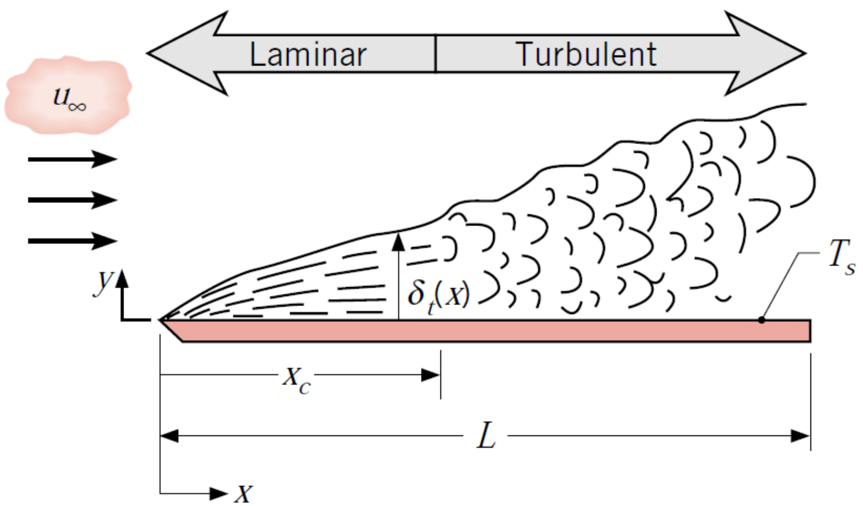
$$C_{\text{lam},300} = 395\ \text{W/m}^{1.5}\cdot\text{K} \qquad C_{\text{turb},300} = 2330\ \text{W/m}^{1.8}\cdot\text{K}$$

while at 350 K,

$$C_{\text{lam},350} = 477\ \text{W/m}^{1.5}\cdot\text{K} \qquad C_{\text{turb},350} = 3600\ \text{W/m}^{1.8}\cdot\text{K}$$

As is evident, the constant, C, depends on the nature of the flow as well as the water temperature because of the thermal dependence of various properties of the fluid.

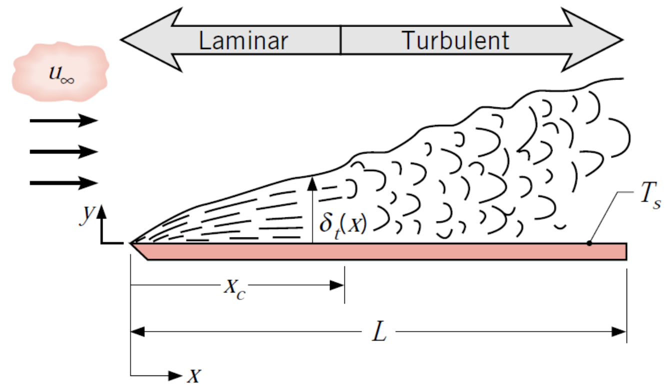
Determine the average convection coefficient, $\bar{h}$, over the entire plate for the two water temperatures.

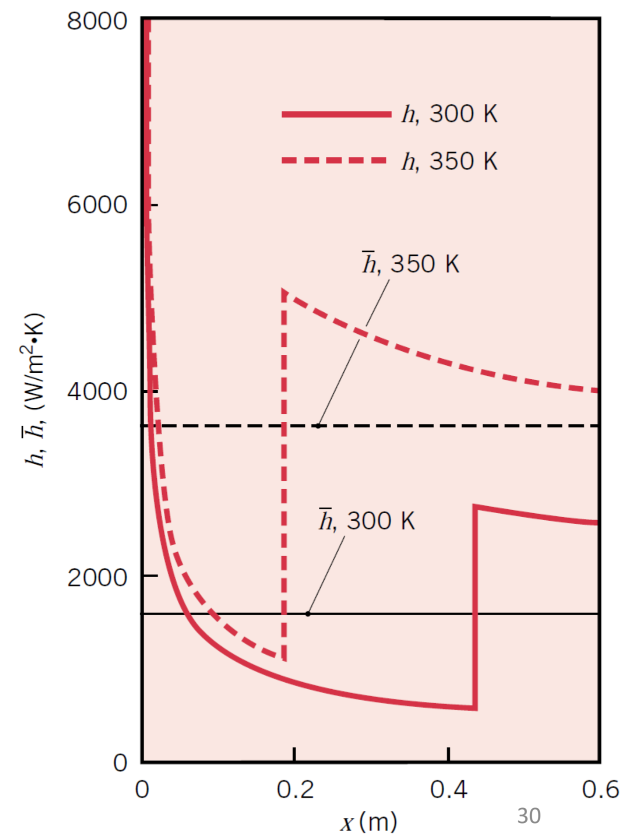


Example: 6.4 from Text

- Where does the laminar BL end and Turbulent BL begin?
  - $Re_{x_c} = \frac{\rho u_\infty x_c}{\mu} = 5x10^5$
- Will this location be the same for 300°C and 350°C?
  - No
- So there is mixed flow on the plate, how to find the average h?

$$\bar{h} = \frac{1}{L}\int_{o}^{L} h dx = \frac{1}{L}\left[\int_{o}^{x_c} h_{lam} dx + \int_{x_c}^{L} h_{turb} dx\right]$$





Summary

- Boundary Layers
  - Velocity
  - Thermal
- Velocity profile and temperature profile affect surface shear stress and heat transfer
- Heat transfer at the surface related to local *h*
- Laminar and Turbulent flow
  - Flow characteristics
  - *h* characteristics

Introduction to Convection: BL Equations and Similarity

Chapter Six

Sections 6.4 – 6.5

1. What are the common boundary layer approximations?

2. What are the *Re*, *Pr* and *Nu* number?

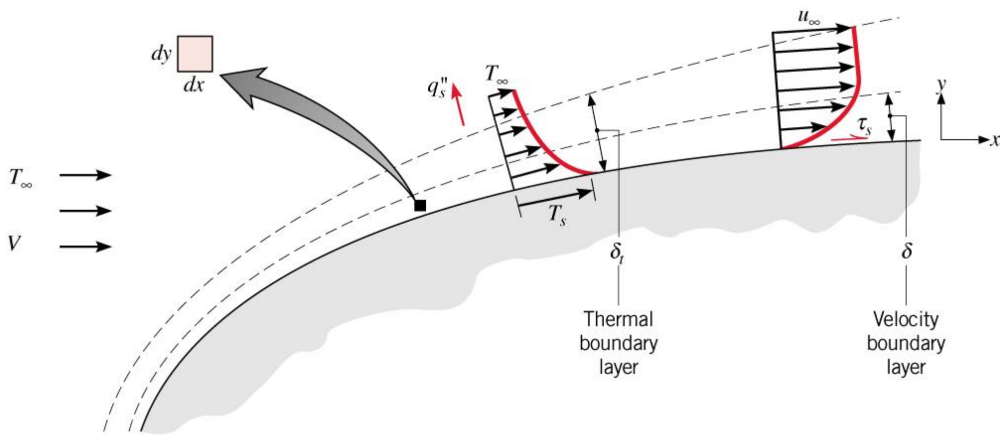
3. What variables are used for non-dimensionalizing BL equations?

4. What do T^* and *Nu* depend on?

Boundary Layer Equations

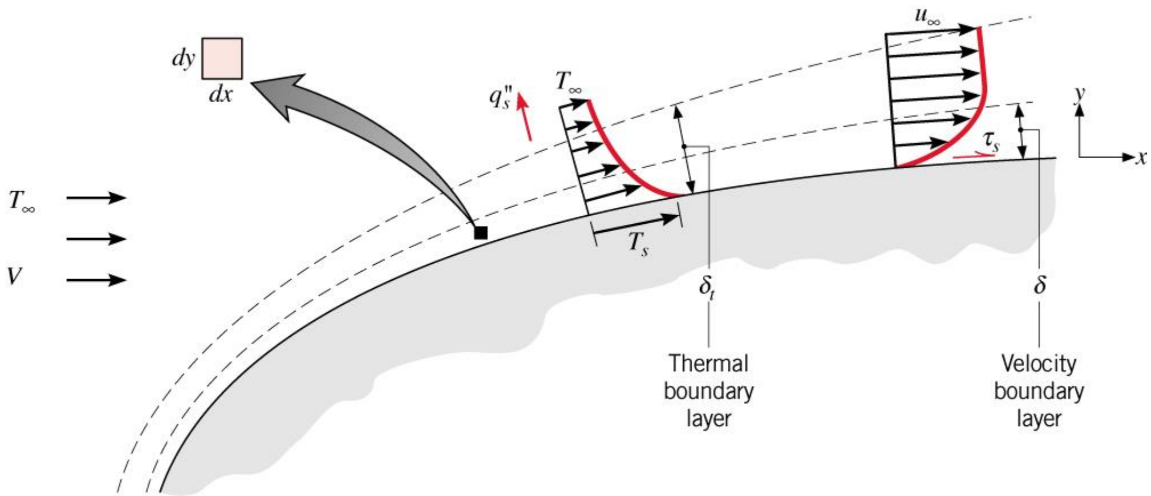
Section 6.4

Boundary Layer Equations



Consider concurrent velocity and thermal boundary layer development for steady, two-dimensional, incompressible flow with constant fluid properties (μ, c_p, k) and negligible body forces.

- Why BLs are formed?
  - Velocity BL due to … $u_\infty \neq u_s$
  - Thermal BL due to … $T_\infty \neq T_s$



How to solve for the velocity and temperature in the BL so that we can find the HT?

- Apply

  Step 1: Conservation of mass

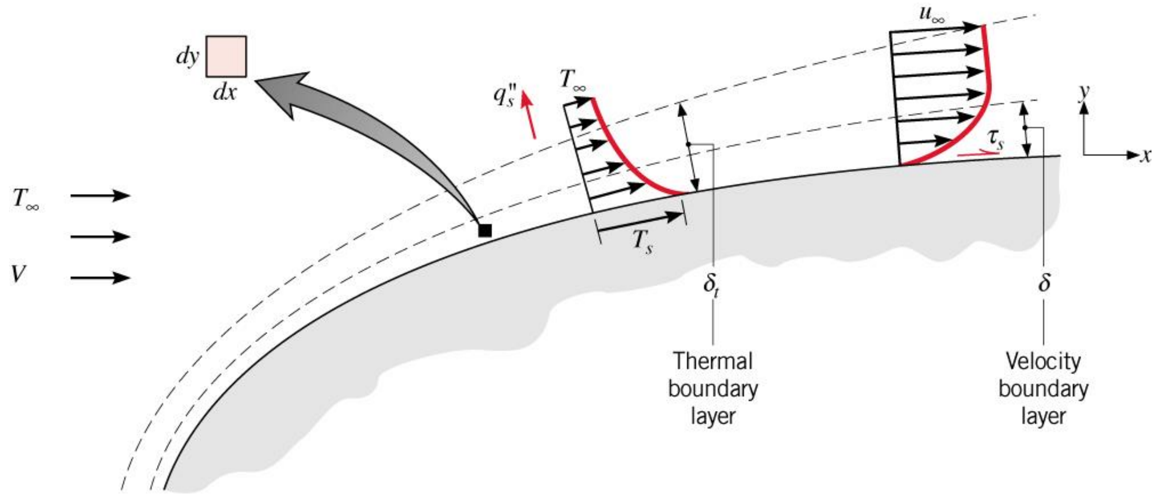
  Step 2: Conservation of energy

  Step 3: Newton's 2nd Law of Motion

  to a differential control volume (*dx*, *dy*, *dz*) and invoke **the boundary layer approximations.**

The resulting equations for steady-state, 2D incompressible flow with constant properties are found in Appendix F and G of textbook.

Boundary Layer Equations – General



- Steady 2D incompressible flow (u, v) with constant fluid properties and negligible body forces to a differential control volume (dx, dy)

- Conservation of mass (steady, 2D, incompressible, $\dot{m}_{in} = \dot{m}_{out}$) $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

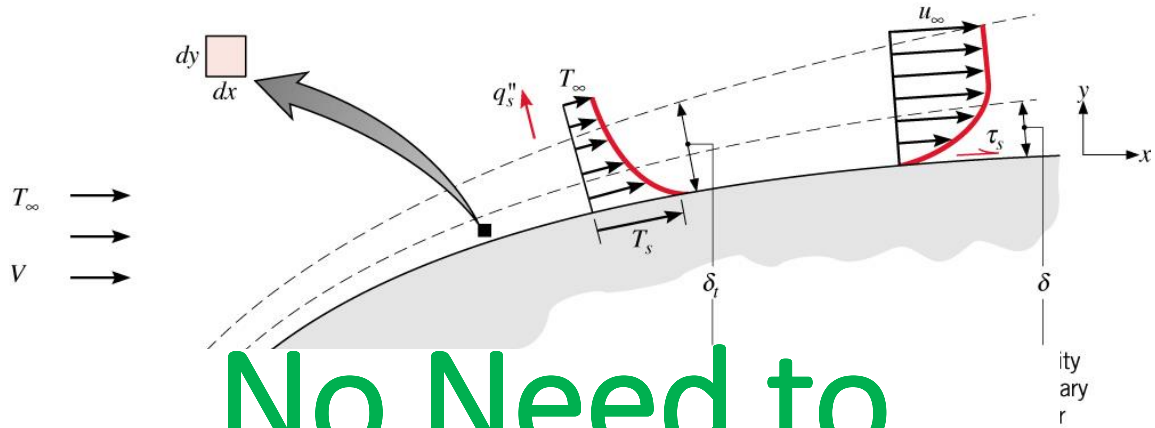
- Conservation of energy $$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\Phi + \dot{q} \qquad \text{(D.4)}$$

where $$\mu\Phi \equiv \mu\left\{\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 + 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2\right]\right\} \qquad \text{(D.5)}$$

- Newton's 2nd Law of Motion $$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + X \qquad \text{(D.2)}$$

$$\rho\left(u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right) = -\frac{\partial p}{\partial y} + \mu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + Y \qquad \text{(D.3)}$$

Boundary Layer Equations – General



No Need to Remember But to Know Meaning of each term

- Steady 2D inco… …nd negligible body forces to a d…
- Conservation of … $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$
- Conservation of … (D.4)

(D.5)

- Newton's 2nd La… (D.2)

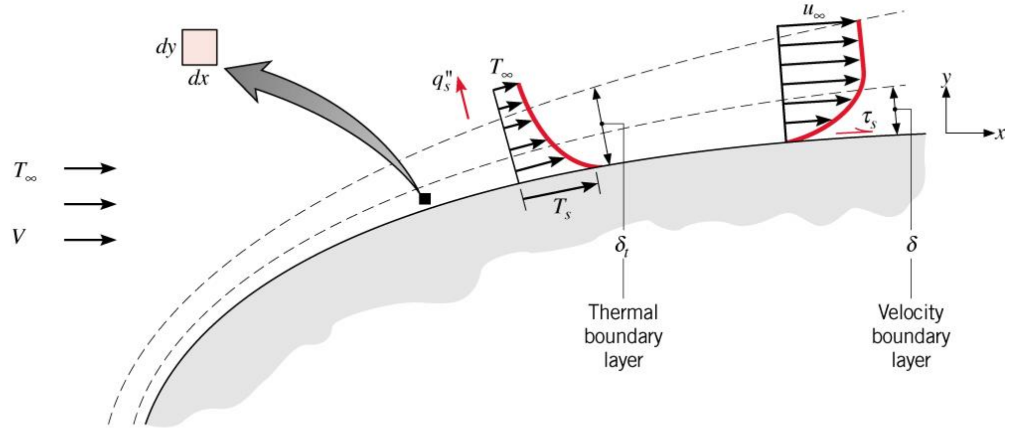
$\rho\left(u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}\right) = -\frac{\partial}{\partial y} + \mu\left(\frac{\partial}{\partial x^2} + \frac{\partial}{\partial y^2}\right) + \gamma$ (D.3)

Boundary Layer Approximations

Simplify the BL Equations

Section 6.4

Boundary Layer Approximations



Steady 2D incompressible (=> constant density) flow with velocity (u, v), constant fluid properties, and negligible body forces and heat generation to a differential control volume (dx, dy)

- The boundary layer approximations:
  - Very thin compared to the object length => dy << dx

Velocity Boundary Layer:

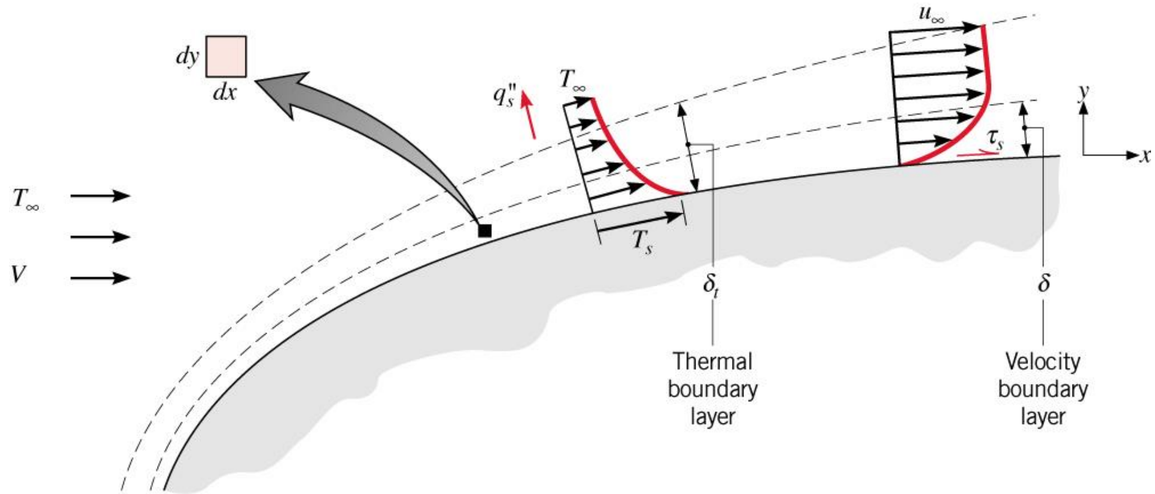
$$\frac{\partial^2 u}{\partial x^2} \ll \frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial p}{\partial x} \approx \frac{dp_\infty}{dx}$$

Thermal Boundary Layer:

$$\frac{\partial^2 T}{\partial x^2} << \frac{\partial^2 T}{\partial y^2}$$

Boundary Layer Equations – Approximated



- Steady 2D incompressible flow (u, v) with constant fluid properties and negligible body forces and heat generation to a differential control volume (dx, dy)

- Conservation of mass (steady, 2D, incompressible, $\dot{m}_{\text{in}} = \dot{m}_{\text{out}}$)

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

- Conservation of energy

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\boxed{\frac{\partial^2 T}{\partial x^2}}+\frac{\partial^2 T}{\partial y^2}\right)+\mu\Phi\boxed{+\dot{q}} \qquad \text{(D.4)}$$

where

$$\mu\Phi=\mu\left\{\left(\frac{\partial u}{\partial y}\boxed{+\frac{\partial v}{\partial x}}\right)^2\boxed{+2\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial y}\right)^2\right]}\right\} \qquad \text{(D.5)}$$

- Newton's 2nd Law of Motion

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}\right)=-\frac{\partial p_\infty}{\partial x}+\mu\left(\boxed{\frac{\partial^2 u}{\partial x^2}}+\frac{\partial^2 u}{\partial y^2}\right)\boxed{+X} \qquad \text{(D.2)}$$

$$\rho\left(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}\right)=-\frac{\partial p_\infty}{\partial y}+\mu\left(\boxed{\frac{\partial^2 v}{\partial x^2}}+\frac{\partial^2 v}{\partial y^2}\right)\boxed{+Y} \qquad \text{(D.3)}$$

ZJUI

- Conservation of Mass: $\dot{m}_{\text{in}} = \dot{m}_{\text{out}}$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (6.27)$$

  - Where is the mass in this conservation of mass?

- Newton's Second Law of Motion:

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial p}{\partial x} + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) + X$$

x–direction:

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp_\infty}{dx} + \nu\frac{\partial^2 u}{\partial y^2} \qquad (6.28)$$

  - What is the physical significance of each term in (6.28)? Which is from fluid momentum, pressure, and viscous forces

- Conservation of Energy:

$$\rho c_p\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}\right)=k\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right)+\mu\Phi+\dot{q} \qquad \text{(D.4)}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad \text{(6.29)}$$

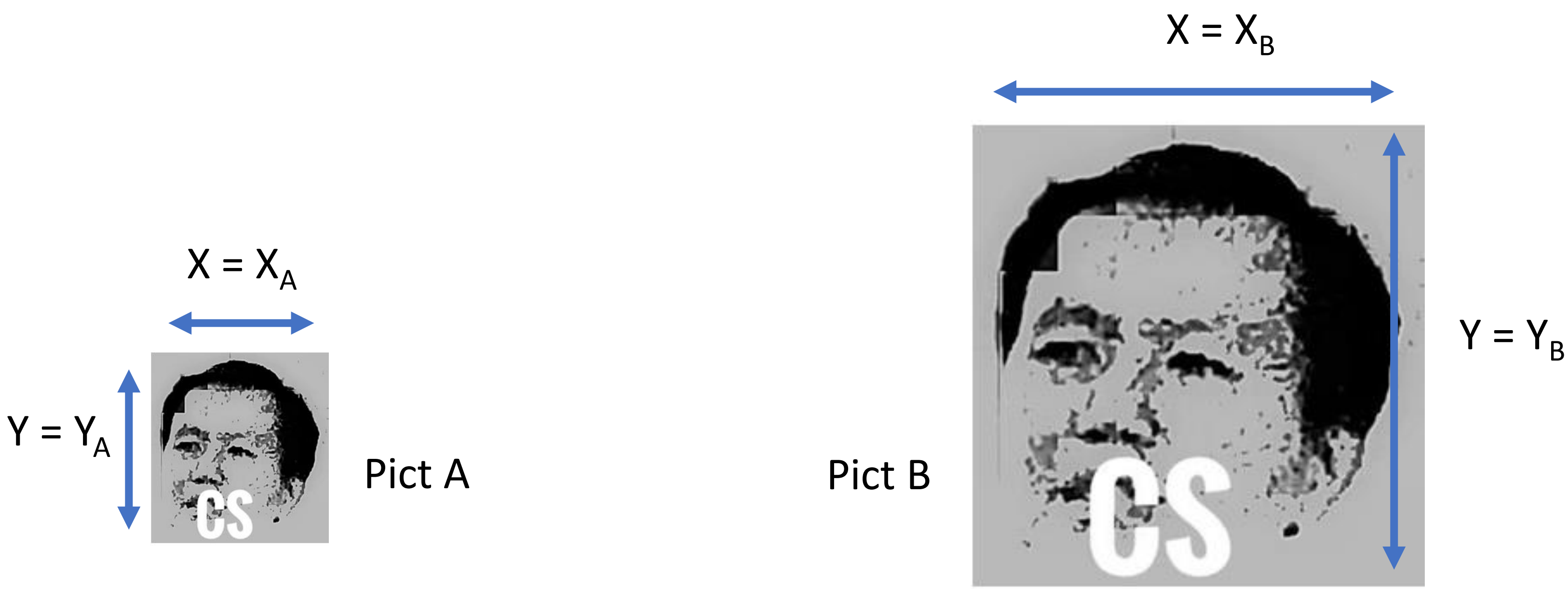
- What is the physical significance of each term in the (6.29)? Which is due to fluid motion carrying heat, viscous, and heat conduction?

Boundary Layer Similarity
Use similarity to solve unknowns

Section 6.5

Similarity Considerations :

- Principle of similarity:
  - Similarity parameters **facilitate results for geometrically similar surfaces experiencing different conditions.**



- Are these two pictures similar?
- If so, we can extract the value of a certain quality from one picture if we know the value of this quality from another picture.
- Example: **Pretty = *f*(X, Y)**

Similarity Considerations :

- Principle of similarity:
  - Similarity parameters **facilitate results for geometrically similar surfaces experiencing different conditions.**

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp_\infty}{dx}+\nu\frac{\partial^2 u}{\partial y^2}$$ (6.28) Conservation of Energy (Approx.)

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2$$ (6.29) Newton's 2nd Law (Approx.)

  - Can you see anything similar between these two equations?

  - What are you interested to get from a velocity boundary layer?
  - What are you interested to get from a thermal boundary layer?

u $or\ \tau_s$

$T\ or\ q''$ (or h)

Similarity Considerations : (optional)

- Independent variables are:

  Geometrical: Size (L), Location (x,y)

  Hydrodynamic: Velocity (V)

  Fluid Properties:

  Hydrodynamic: ρ, μ

  Thermal : c_p, k

- Dependent boundary layer variables of interest are:

$$\tau_s \text{ and } q'' \text{ (or } h\text{)}$$

Fluid BL,

$$u = f(x, y, L, V, \rho, \mu)$$
$$\tau_s = f(x, L, V, \rho, \mu)$$

Thermal BL,

$$T = f\left(x,\ y,\ L,\ V,\ \rho,\ \mu,\ c_p,\ k, T_s, T_\infty\right)$$
$$h = f\left(x,\ L,\ V,\ \rho,\ \mu,\ c_p,\ k, T_s, T_\infty\right)$$

ZJUI

Get key similarity parameters from non-dimensionalizing the momentum and energy equations.

- Recast BL equations using dimensionless independent and dependent variables.

$$x^* \equiv \frac{x}{L} \qquad y^* \equiv \frac{y}{L}$$

$$u^* \equiv \frac{u}{V} \qquad \upsilon^* \equiv \frac{\upsilon}{V}$$

$$T^* \equiv \frac{T - T_s}{T_\infty - T_s}$$

What is the expression for $\frac{du^*}{dx^*}$?

Similarity Considerations : (Optional)

ZJUI

- Get the following normalized forms of the x-momentum and energy equations are obtained:

The Momentum equation

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{dp_{\infty}}{dx}+\nu\frac{\partial^2 u}{\partial y^2} \qquad (6.28)$$

$$\Rightarrow \quad u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial y^*}=-\frac{dp^*}{dx^*}+\frac{1}{Re_L}\frac{\partial^2 u^*}{\partial y^{*2}} \qquad \mathbf{(6.35)}$$

The Energy equation

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\underbrace{\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2}_{\text{(neglected)}} \qquad (6.29)$$

$$\Rightarrow \quad u^*\frac{\partial T^*}{\partial x^*}+v^*\frac{\partial T^*}{\partial y^*}=\frac{1}{Re_L Pr}\frac{\partial^2 T^*}{\partial y^{*2}} \qquad \mathbf{(6.36)}$$

In these equations,
Only Re_L and Pr are not normalized.
So, if Re_L and Pr are the same for different situations, solutions will be similar.

Independent Dimensionless Numbers

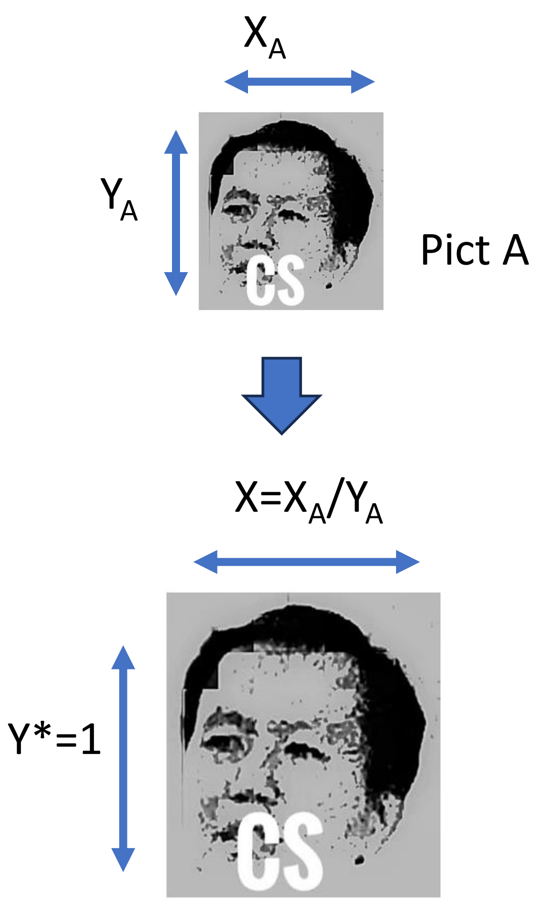
$$Re_L \equiv \frac{\rho V L}{\mu} = \frac{VL}{\nu} \rightarrow \text{ the Reynolds Number} \qquad (6.41)$$

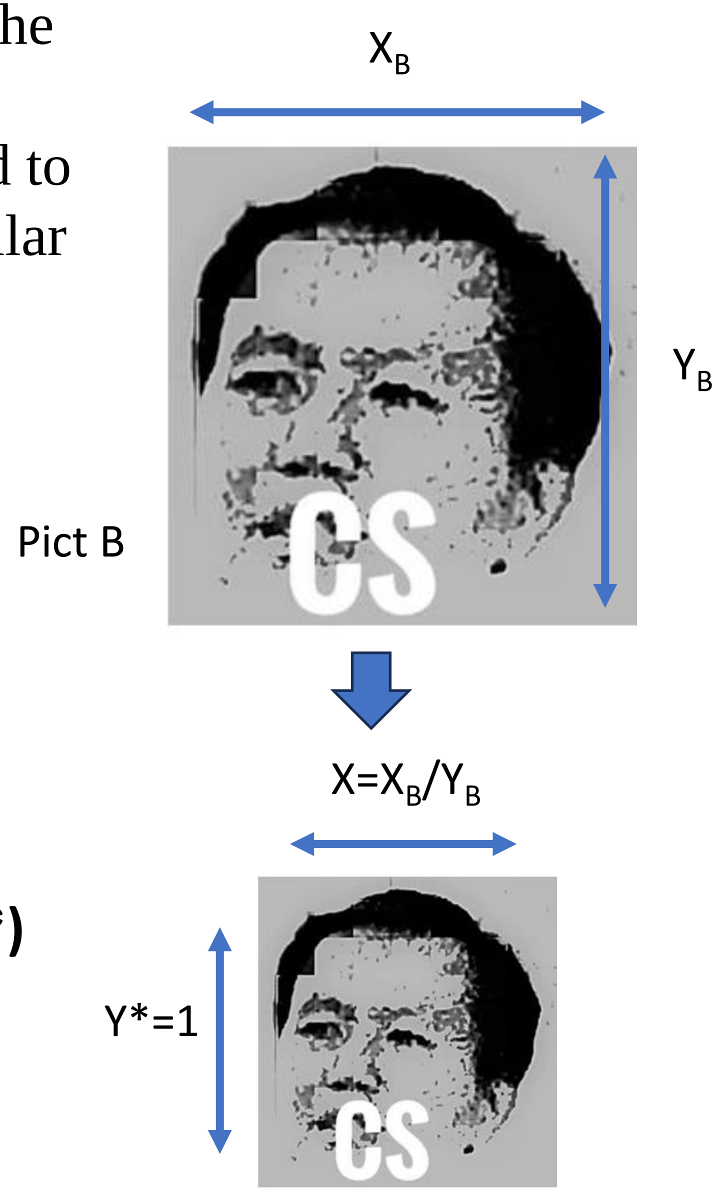
$$Pr \equiv \frac{c_p \mu}{k} = \frac{\nu}{\alpha} \rightarrow \text{ the Prandtl Number} \qquad (6.42)$$

What is the meaning of the Reynolds and Prandtl number?
(more next lecture)

ZJUI

- Normalize by Y (divide by Y), make the pictures to be as similar as possible.
- See what else is independent that need to be matched for these pictures to be similar



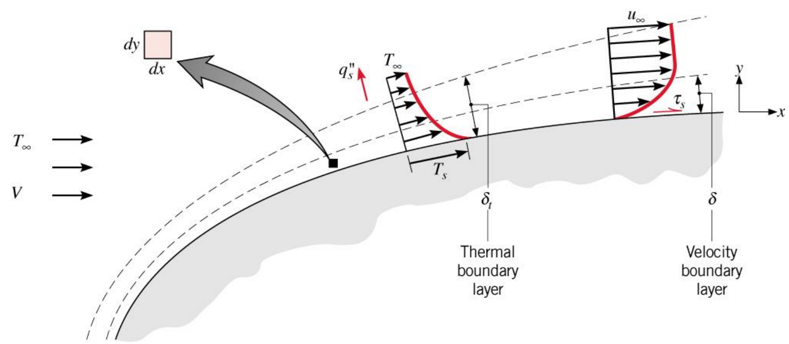


Pretty = *f*(X/Y, Y*)

Dependent Dimensionless Numbers: Velocity

For a prescribed geometry from **(6.35)**,

$$u^* = f\left(x^*, y^*, Re_L\right)$$



Shear stress: $$\tau_s = \mu \frac{\partial u}{\partial y}\bigg|_{y=0} = \left(\frac{\mu V}{L}\right)\frac{\partial u^*}{\partial y^*}\bigg|_{y^*=0}$$

The dimensionless shear stress, or local friction coefficient, is then

$$C_f \equiv \frac{\tau_s}{\rho V^2/2} = \frac{2}{Re_L}\frac{\partial u^*}{\partial y^*}\bigg|_{y^*=0} \qquad (6.45)$$

So,

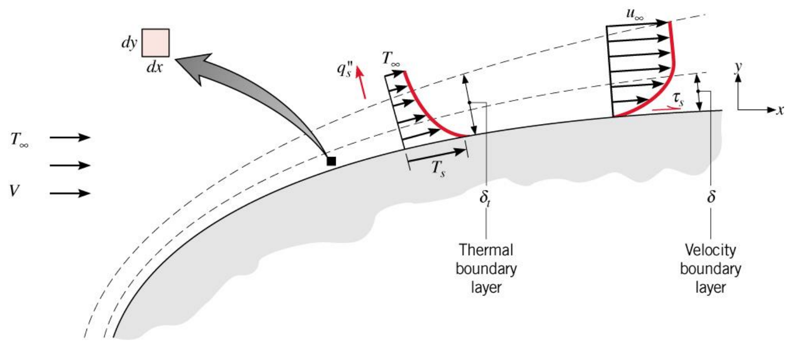
$$\boxed{C_f = \frac{2}{Re_L} f\left(x^*, Re_L\right) \qquad (6.46)}$$

Eq (6.46) means for this geometry, C_f can be found for a wide range of *x*, *V*, *L* (from x and Re). No need to re-analyze!

Dependent Dimensionless Numbers: Temperature

For a prescribed geometry from **(6.36)**,

$$T^* = f\left(x^*, y^*, Re_L, Pr\right)$$



Heat transfer coefficient:

$$h = \frac{-k_f \partial T / \partial y\big|_{y=0}}{T_s - T_\infty} = -\frac{k_f}{L}\frac{(T_\infty - T_s)}{(T_s - T_\infty)}\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = +\frac{k_f}{L}\frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0}$$

We define a **NEW** dimensionless number called the Nusselt number. The local convection coefficient is then

$$\boxed{Nu \equiv \frac{hL}{k_f} = \frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = f(x^*, Re_L, Pr) \qquad (6.48;\ 6.49)}$$

$Nu \rightarrow$ local Nusselt number = dimensionless temperature gradient dependent on *x**, *Re,* and *Pr.*

Average Nusselt number:

- $\overline{Nu}$ Obtained by integrating over the *x*-distance => does not change with *x* or *x**

Similarity Considerations :

ZJUI

$X=X_A/Y_A$

$Y^*=1$

Pict A

$X=X_B/Y_B$

$Y^*=1$

Pict B

Pretty = f(X/Y, Y*)

If $(X/Y)_A == (X/Y)_B$,

- **$Pretty_A = Pretty_B$**

- **But Pretty = Color/Roundness**

So we can find Roundness of B if we know Roundness of A

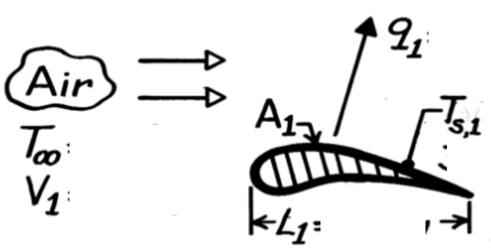
Consider the average *Nu* number:

$$\boxed{\overline{Nu} \equiv \frac{hL}{k_f}} = \frac{\partial T^*}{\partial y^*}\bigg|_{y^*=0} = \boxed{f(x^*, Re_L, Pr)}$$

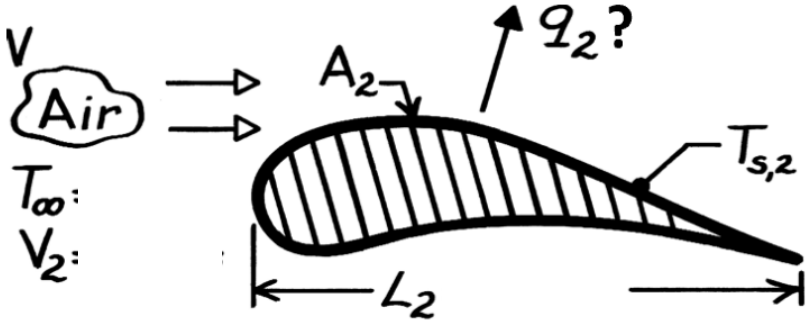
For two different geometries Case A and Case B :

- if the Independent Dimensionless Numbers are the same in the Green box => Dependent Dimensionless Numbers in Red box are the same.

Case A



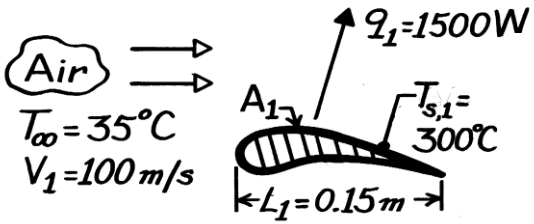
Case B

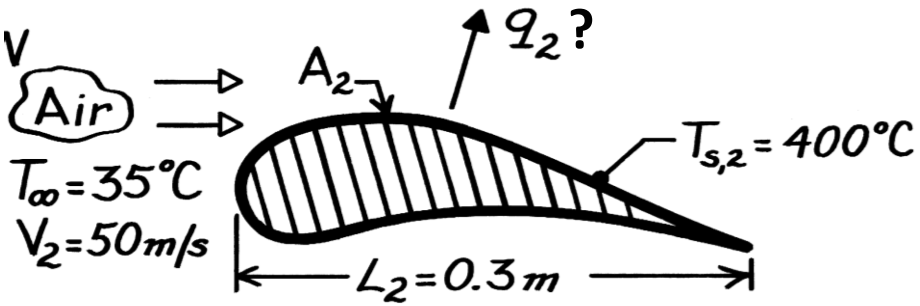


Summary

- BL equations
- BL approximations
- Non-dimensionalizing variables
- *Re* and *Pr* number and their meaning
- What is *Nu* number

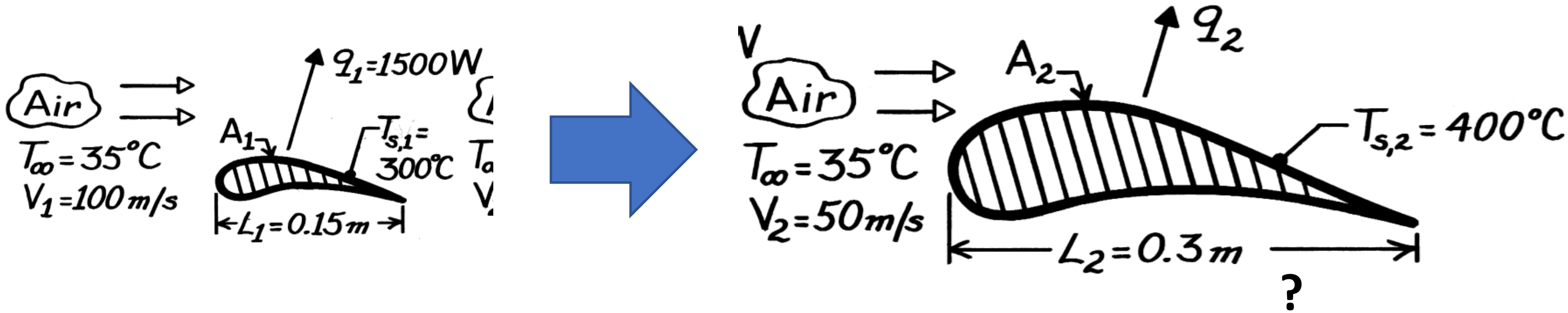
Problem 6.24: Determination of heat transfer rate for prescribed turbine blade operating conditions from wind tunnel data obtained for a geometrically similar but smaller blade. The blade surface area may be assumed to be $(A_s \propto L)$ directly proportional to its characteristic length.





- What is the use of a wind tunnel?
- Why make a model?
- What needs to be the same in the model and real object for experiment to be true?

Example 2



ASSUMPTIONS: (1) Steady-state conditions, (2) Constant properties, (3) Surface area A is directly proportional to characteristic length L, (4) Negligible radiation, (5) Blade shapes are geometrically similar.

ANALYSIS: For a prescribed geometry, for both objects:

$$\overline{Nu} = \frac{\bar{h}L}{k} = f\left(Re_L, Pr\right).$$

Example 2

The Reynolds numbers for the blades are

$$Re_{L,1} = (V_1 L_1 / \nu_1) = 15 / \nu_1 \qquad Re_{L,2} = (V_2 L_2 / \nu_2) = 15 / \nu_2$$

Same Re for both: $Re_{L,1} = Re_{L,2}$.

With constant properties, $Pr_1 = Pr_2$

Therefore,

$$\overline{Nu}_2 = \overline{Nu}_1$$

$$(\bar{h}_2 L_2 / k_2) = (\bar{h}_1 L_1 / k_1)$$

$$\bar{h}_2 = \frac{L_1}{L_2} \bar{h}_1 = \frac{L_1}{L_2} \frac{q_1}{A_1 (T_{s,1} - T_\infty)}$$

ZJUI

The heat rate for the *second blade* is then

$$q_2 = \bar{h}_2 A_2 \left(T_{s,2} - T_\infty\right) = \frac{L_1}{L_2} \frac{A_2}{A_1} \frac{\left(T_{s,2} - T_\infty\right)}{\left(T_{s,1} - T_\infty\right)} q_1$$

$$q_2 = \frac{T_{s,2} - T_\infty}{T_{s,1} - T_\infty} q_1 = \frac{(400 - 35)}{(300 - 35)}(1500 \text{ W})$$

$$q_2 = 2066 \text{ W.}$$

COMMENTS: (i) The variation in ν from Case 1 to Case 2 (due to different temperatures) would cause $Re_{L,2}$ to differ from $Re_{L,1}$. However, for air and the prescribed temperatures, this non-constant property effect is small.

(ii) If the Reynolds numbers were not equal $\left(Re_{L,1} \neq Re_{L2}\right)$, knowledge of the specific form of $f\left(Re_L, Pr\right)$ would be needed to determine h_2.