Lec06 and 07: Extended Surfaces

Chapter Three

Section 3.2 and 3.6

Announcement

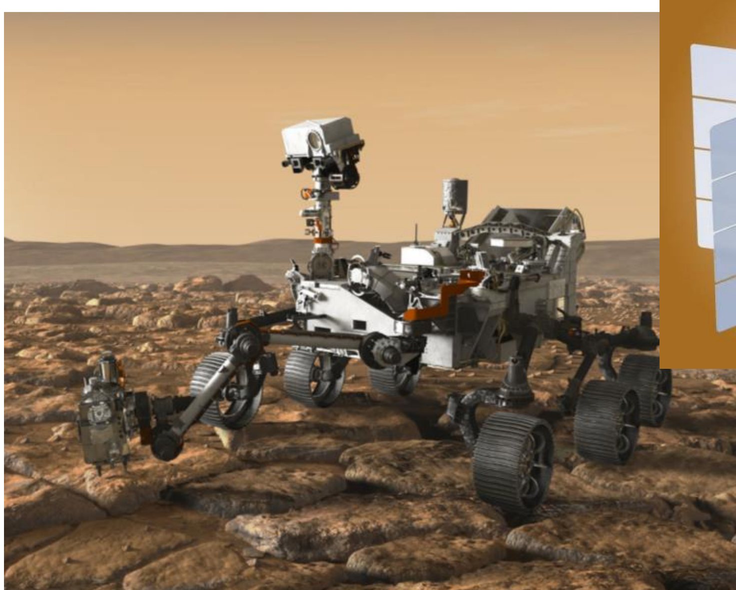
- HW03 will be up by Wed
- See you in Lab for Lab02 tomorrow

1. Under what situations is a fin useful?
2. What is the governing differential equation for the fin heat transfer?
3. How many boundary conditions are needed to solve the fin equation? What are they?
4. Define fin effectiveness. Will a fin be always more effective than the base surface for heat transfer?
5. Of the four BCs at the end of a fin, which will make a more effective fin?
6. Define efficiency of a fin.

Extended Surface

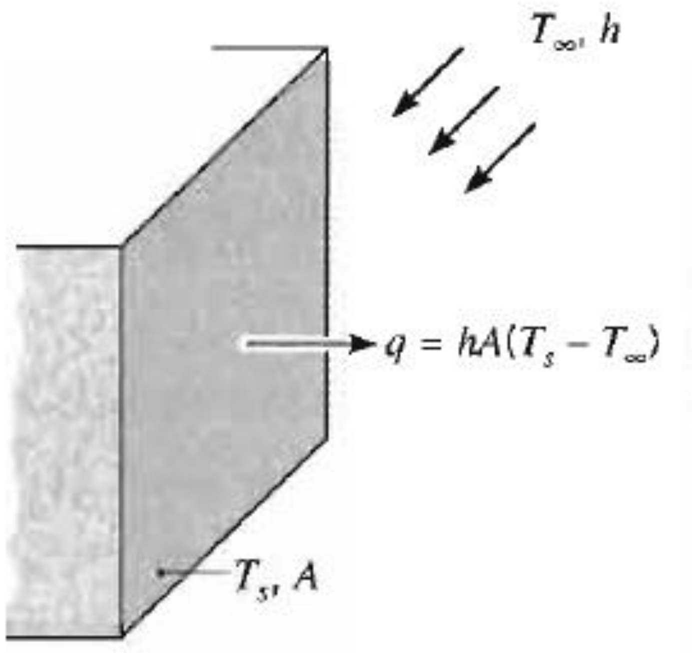
1D Conduction + Convection

Extended Surfaces



Why use Extended Surfaces

Consider a flat surface from which you want to increase the heat transfer to the surrounding,



How can you do that?

A. Increase *h*

- Increase fluid speed

B. Reduce *T*$_\infty$

- Cool the incoming fluid

C. Increase *A*

- Increase surface area

Often **C option** is easier to achieve as it does not depend too much on the operation conditions

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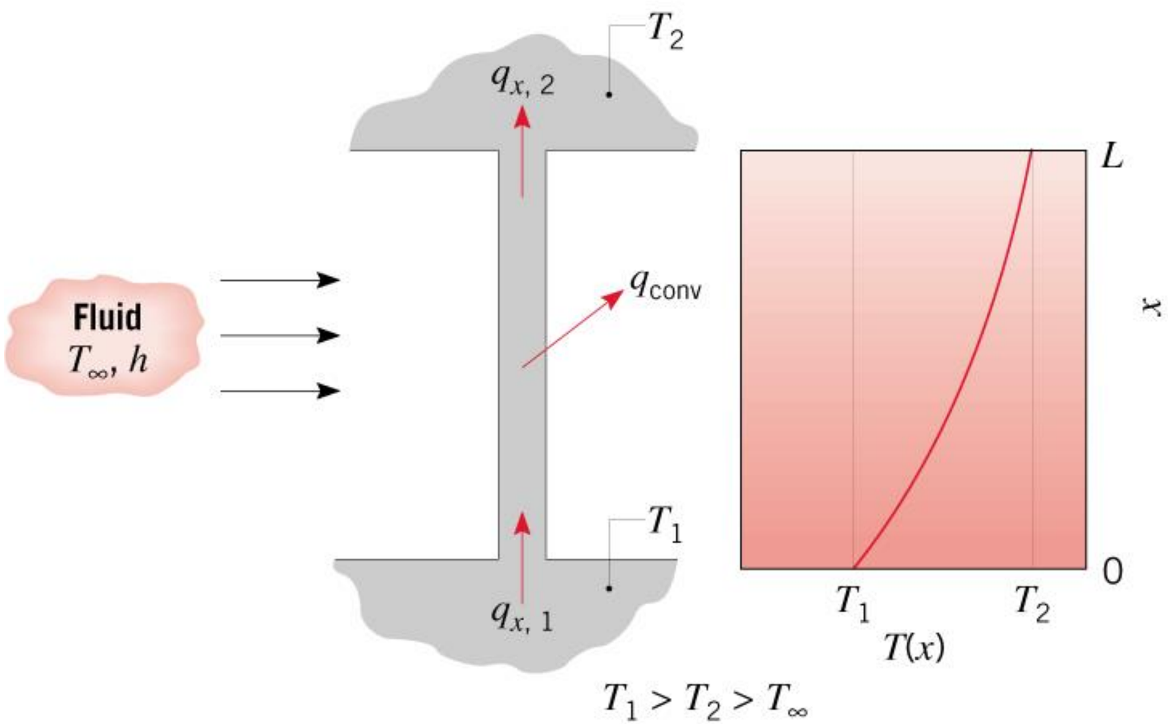
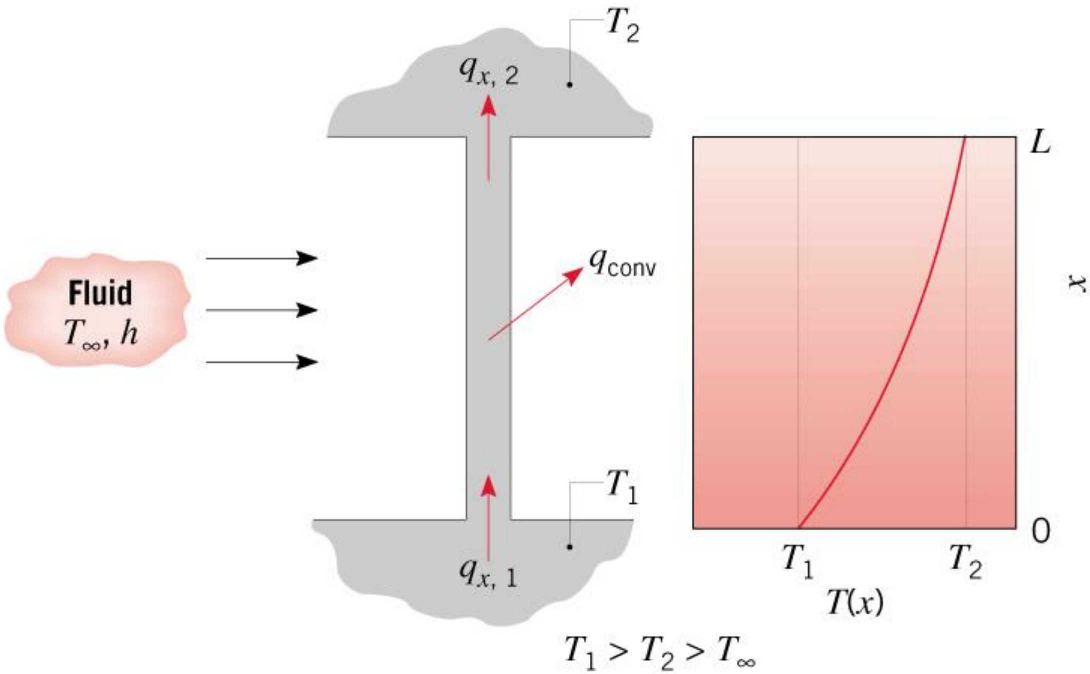
An extended surface is a solid

- With *assumed 1D* conduction through the solid
- Heat is also transferred by convection (and/or radiation) perpendicular to conduction direction
- Combined conduction-convection/radiation system or a fin





- If heat is transferred from the surface to the fluid by convection, what equation quantifies this heat loss?
  => $-kdT/dx = h(T_s - T_\infty)$

- Is heat transfer by conduction in *x*-direction REALLY one-dimensional?
  => No, due to convection along the x-direction

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So when may the assumption of one-dimensional conduction be viewed as an excellent approximation?

$\Rightarrow$ high k of the fin

$\Rightarrow$ thin fin approximation => thickness, t ~ 0

In HT, we are as usual interested in the **temperature distribution** and **heat flux**,

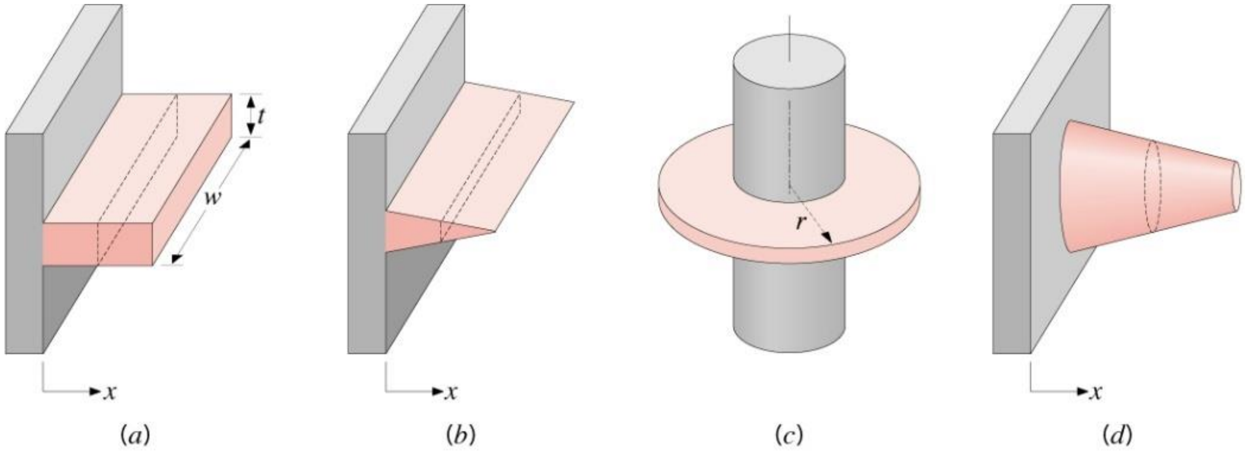
- How does $q_{\text{cond},x}$ vary with ***x*** ? (See next few slides)

- What is the actual functional dependence of the temperature distribution in the solid? (See next few slides)

Extended surfaces may exist in many situations

- Commonly used as fins to enhance heat transfer by increasing the surface area available for convection and radiation.

- Some typical fin configurations:



Straight fins of (a) uniform and (b) non-uniform cross sections; (c) annular fin, and (d) pin fin of non-uniform cross section.

Fin Equation

1D Conduction + Convection

Assumptions:

- one-dimensional, steady-state conduction
- constant conductivity (k) and uniform cross-sectional area (A_c), with negligible generation $(\dot{q}=0)$ and radiation $(q''_{\text{rad}}=0)$, the *fin equation* is of the form:

$$\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$$

$$q_{in} - q_{out} = 0$$

$$q_x - q_{x+dx} - dq_{conv} = 0 \qquad (3.61)$$

$$q_x - \left(q_x + \frac{dq_x}{dx}dx\right) - dq_{conv} = 0$$

$$-\frac{dq_x}{dx}dx - dq_{conv} = 0$$

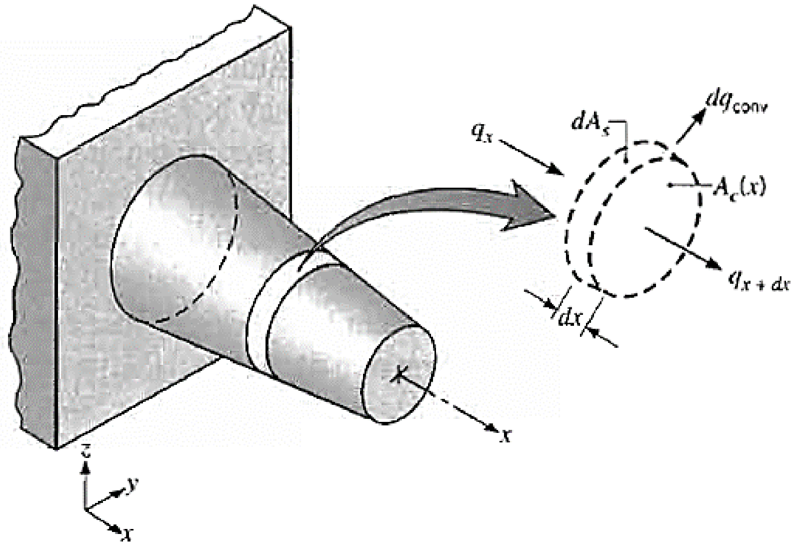


FIGURE 3.15 Energy balance for an extended surface.

$$-\frac{dq_x}{dx}dx - dq_{conv} = 0$$

With

- $q_x = -kA_c\frac{dT}{dx}$
- $dq_{conv} = hdA_s(T - T_\infty) \quad where \quad dA_s = Pdx$
- Beware A_c ! $=$ A_s

$$\boxed{\frac{d^2T}{dx^2} + (\frac{1}{A_c}\frac{dA_c}{dx})\frac{dT}{dx} - \frac{h}{kA_c}\frac{dA_s}{dx}(T - T_\infty) \quad = 0 \quad (3.66)}$$

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$$\frac{d^2T}{dx^2} + \left(\frac{1}{A_c}\frac{dA_c}{dx}\right)\frac{dT}{dx} - \frac{h}{kA_c}\frac{dA_s}{dx}(T - T_\infty) = 0$$

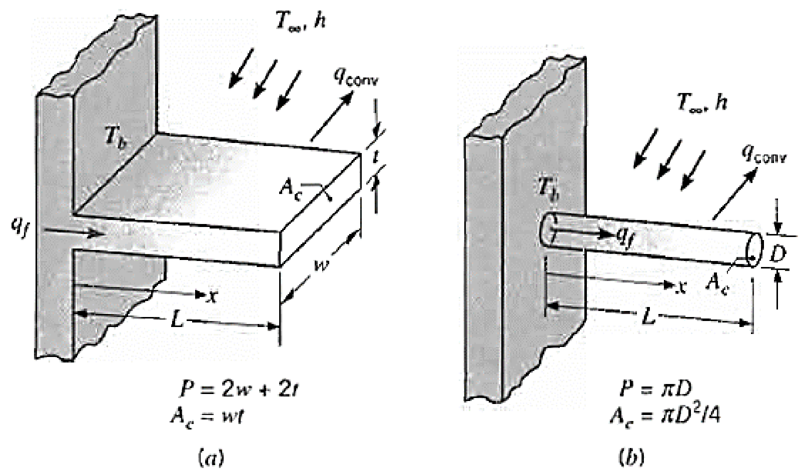

FIGURE 3.16 Straight fins of uniform cross section. (*a*) Rectangular fin. (*b*) Pin fin.

1. With **uniform X-section area,**

$$\boxed{\frac{d^2T}{dx^2} - \frac{hP}{kA_c}(T - T_\infty) = 0 \qquad (3.67)}$$

2. Define $m^2 \equiv (hP/kA_c)$ and the reduced temperature $\theta \equiv T - T_\infty$, (why do we define a θ?)

$$\boxed{\frac{d^2\theta}{dx^2} - m^2\theta = 0 \qquad (3.69)}$$

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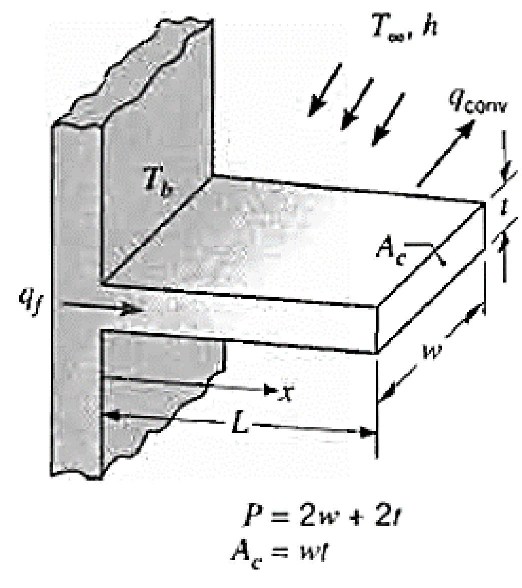
$\frac{d^2\theta}{dx^2} - m^2\theta = 0$ is second order in space, so it requires 2 boundary conditions



1. Base ($x = 0$) condition

$$\theta(0) = T_b - T_\infty \equiv \theta_b$$

2. Tip ($x = L$) conditions

A. Convection: $-k d\theta/dx|_{x=L} = h\theta(L)$

B. Adiabatic: $d\theta/dx|_{x=L} = 0$

C. Fixed temperature: $\theta(L) = \theta_L$

D. Infinite fin: $\theta(L = \infty) = 0$ valid for ($mL > 2.65$)

Using Condition D (infinite tip) as an example

ZJUI

Start with:

$$\frac{d^2\theta}{dx^2} - m^2\theta = 0$$

Can get a temperature distribution:

$$\boxed{\theta(x) = C_1 e^{mx} + C_2 e^{-mx} \qquad (3.71)}$$

BCs:

Base: $\theta(0) = T_b - T_\infty \equiv \theta_b$

$$\theta(0) = C_1 e^0 + C_2 e^{-0}$$

$$\theta_b = C_1 + C_2 \qquad (3.74)$$

Infinite fin: $\theta(L = \infty) = 0$ (valid if $mL > 2.65, know\ why?\ Clue:\ adiabatic$)

$$\theta(L) = C_1 e^{m\infty} + C_2 e^{-m\infty}$$

$$0 = C_1 e^{m\infty}$$

$$0 = C_1$$

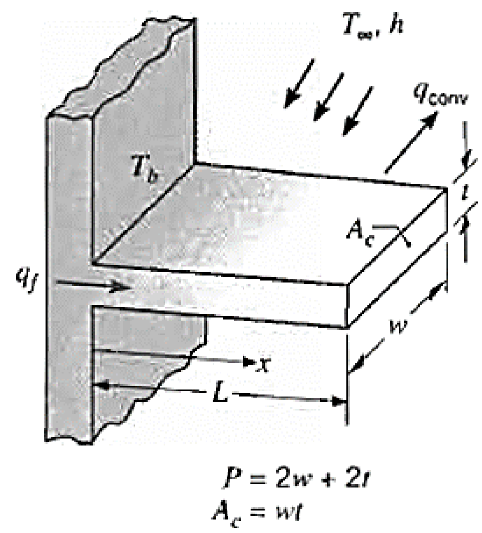
Therefore, $\theta(x) = \theta_b e^{-mx}$ which is the temperature distribution along the fin.

To find Fin Heat Rate (heat loss/gain by fin):

M1. Calculate $\dot{E}_{out}$

$$q_f = \int_{A_f} h\theta(x)dA_s \qquad (3.78)$$

A_f is the fin surface area that also includes the tip area



(a)

M2. Calculate $\dot{E}_{in}$ using Fourier law at x =0,

$$q_f = q_b = -kA_c \frac{d\theta}{dx}\Big|_{x=0} \qquad (3.76)$$

Of course, we can also use M1 = M2 to find other unknowns.

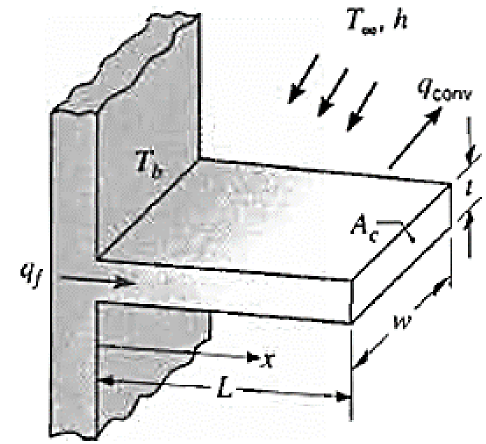
Using Condition D (infinite tip) as an example

So, $q_f = q_b = -kA_c \frac{d\theta}{dx}|_{x=0}$

$$= -kA_c \frac{d(\theta_b e^{-mx})}{dx}|_{x=0}$$

$$= mkA_c\theta_b e^{-m0}$$

$$\boxed{= \sqrt{\frac{hP}{kA_c}} kA_c\theta_b = \sqrt{hPkA_c}\theta_b = M \quad (3.85)}$$



$P = 2w + 2t$
$A_c = wt$
(a)

 ZJUI

TABLE 3.4 Temperature distribution and heat loss for fins of uniform cross section

Case	Tip Condition $(x = L)$	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx\vert_{x=L}$	$\dfrac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$M\dfrac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$
B	Adiabatic $d\theta/dx\vert_{x=L} = 0$	$\dfrac{\cosh m(L-x)}{\cosh mL}$	$M \tanh mL$
C	Prescribed temperature: $\theta(L) = \theta_L$	$\dfrac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$	$M\dfrac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$
D	Infinite fin $(L \rightarrow \infty)$: $\theta(L) = 0$	e^{-mx}	M

$\theta \equiv T - T_\infty$ $\quad$ $m^2 \equiv hP/kA_c$

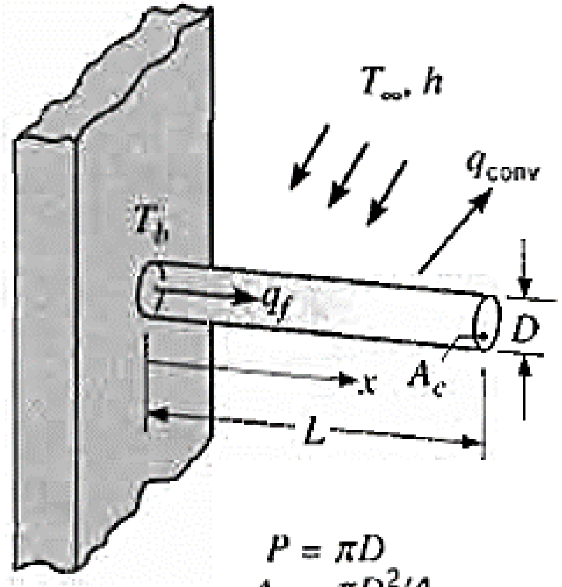
$\theta_b = \theta(0) = T_b - T_\infty$ $\quad$ $M \equiv \sqrt{hPkA_c}\,\theta_b$

Performance Parameters for a Fin

Will this Fin get an A+?

Is fin always good for HT?

- Fin with finite k is also a conduction resistance.
- Longer L, smaller A, smaller k will increase $R_{\mathrm{t,fin}}$
- $$R_{t,fin} = \frac{\theta_b}{q_f} \qquad (3.88)$$



How do we know a fin is good? Can we characterize the fin performance?

1. Compare HT with and without fin. Hopefully with fin works better!

   - Fin Effectiveness, ε_f

2. Compare HT of a fin in ideal case with actual case.

Ideal case means the whole fin is at the base temperature, θ_b. Why?

- Fin Efficiency, η_f

1. **Fin Effectiveness**:

$$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = \frac{R_{t,b}}{R_{t,fin}} \qquad (3.86, 3.90)$$

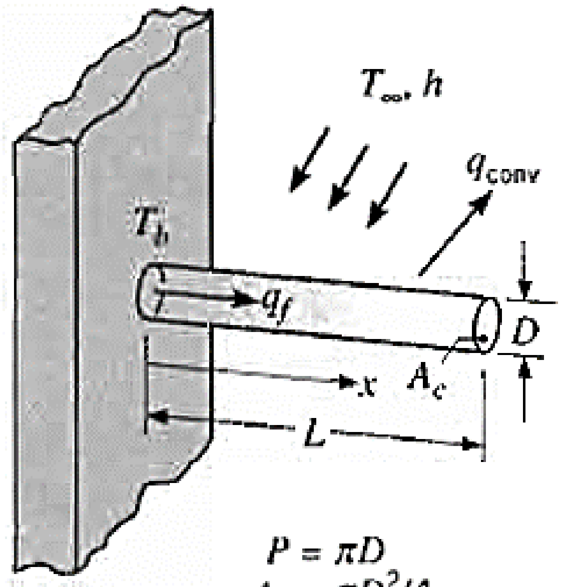
- Ratio of heat flow or equivalently of resistance between base and fin
- Rule of thumb, normally $\varepsilon_f > 2$ means fin usage is justified.

(Why > 1 but < 2 not justified?)

Taking the example of an infinite fin,

$$q_f = \sqrt{hPkA_{c,b}}\theta_b$$

$$\varepsilon_f = \left(\frac{kP}{hA_{c,b}}\right)^{0.5} \qquad (3.87)$$



So, what is the effect of

- k, P, h, $A_{c,b}$ on the effectiveness?
- $P/A_{c,b}$
- Should fin be needed more in gas or liquid? Given that $h_{gas} < h_{liquid}$

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2. Fin Efficiency (quite useful as you will see later):

$$\eta_f \equiv \frac{q_f}{q_{f,\ \max}} = \frac{q_f}{hA_f\theta_b} \qquad \text{where } 0 \le \eta_f \le 1 \tag{3.91}$$

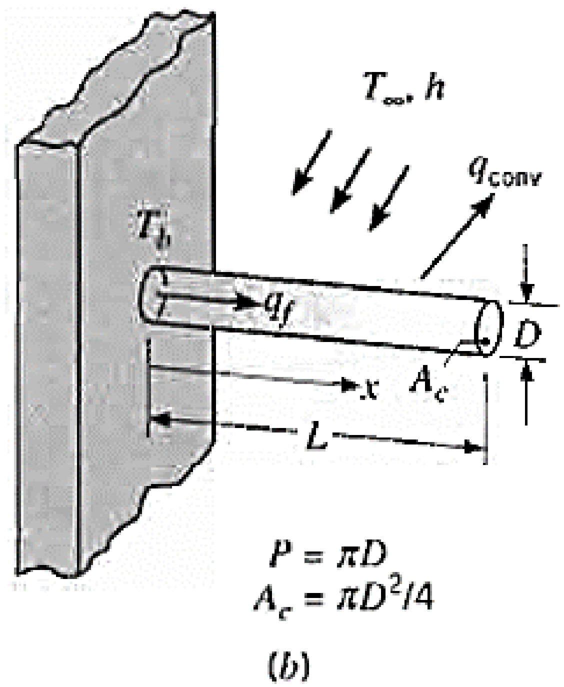
How is the efficiency affected by the thermal conductivity of the fin?

We can rewrite the Fin Resistance as :

$$R_{t,f} = \frac{\theta_b}{q_f} = \frac{1}{hA_f\eta_f} \tag{3.93}$$

Be very very **very careful of A_f and $A_{c,b}$**

A_f includes tip area



Uniform or Non-uniform Fin

Non-uniform fin

The simple thing about this is (as seen in slide 13):

$$-\frac{dq_x}{dx}dx - dq_{conv} = 0$$

With

- $q_x = -kA_c\frac{dT}{dx}$
- $dq_{conv} = hdA_s(T - T_\infty)$

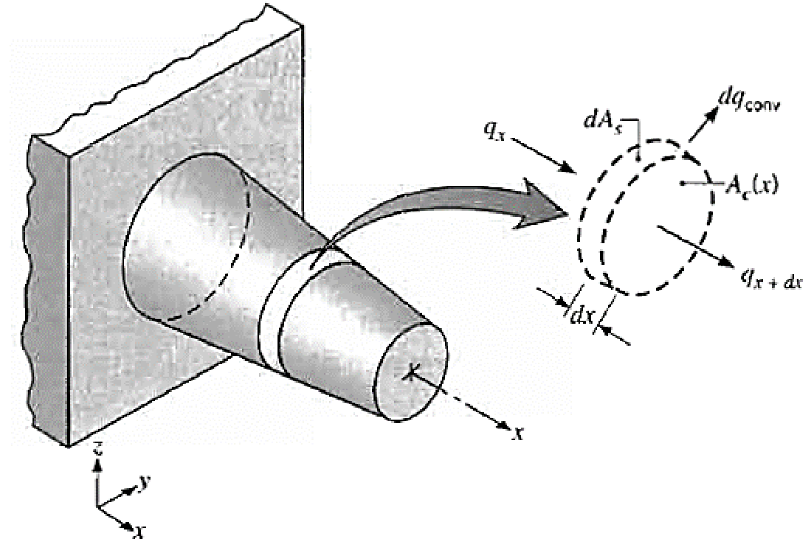


FIGURE 3.15 Energy balance for an extended surface.

$$\frac{d(kA_c\frac{dT}{dx})}{dx}dx - h(T - T_\infty)dA_s = 0$$

$$kA_c\frac{d^2T}{dx^2}dx + k\frac{d(A_c)}{dx}\frac{dT}{dx}dx - h(T - T_\infty)dA_s = 0$$

$$\boxed{\frac{d^2T}{dx^2} + (\frac{1}{A_c}\frac{dA_c}{dx})\frac{dT}{dx} - \frac{h}{kA_c}\frac{dA_s}{dx}(T - T_\infty) = 0 \quad (3.66)}$$

But….. You will get a set of complex solutions.

But….. we have precalculated tables (Table 3.5 in Text) and graphs (Figs. 3.18 and 3.19) for some common cases!!! Example:

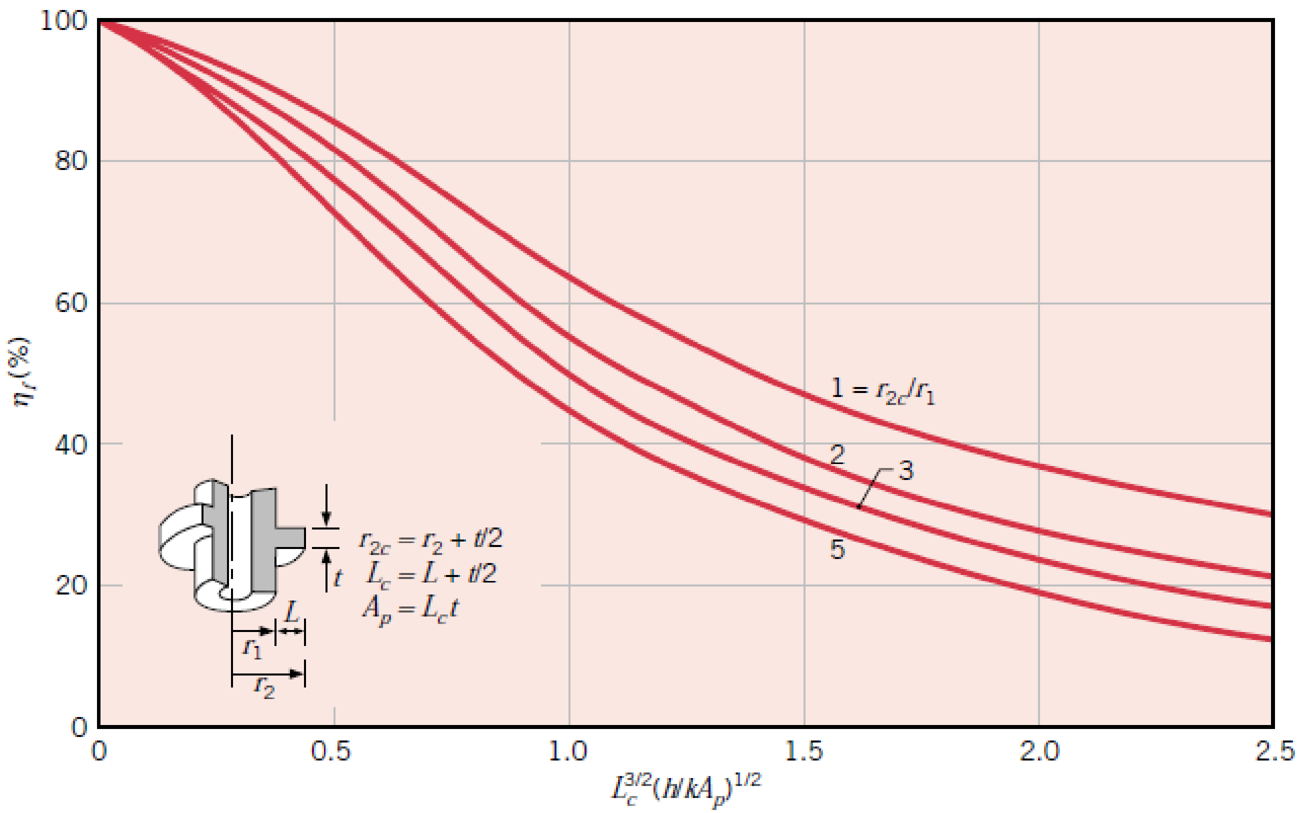


FIGURE 3.19 Efficiency of annular fins of rectangular profile.

From here, one possibility is to use: $$\eta_f \equiv \frac{q_f}{q_{f,\,\max}} = \frac{q_f}{hA_f\theta_b} \qquad (3.91)$$

Choice is motivated by:

- Less cost to achieve what you want
- Cost => material and manufacturing
  - Smaller volume => less material cost
  - Simple shape => less manufacturing cost

- Often, complex shapes give you a better heat transfer but with a higher cost! No free lunch!

Summary

- Integrate Fourier Law directly to get results for **1D, steady-state, no heat generation** problem.
- Fin and equation
  - Heat Equation
  - Simplifications – uniform x-section are and constant ***k***
  - 4 BCs
  - Effectiveness
  - Efficiency
- Uniform vs non-uniform fin

Example 1

A very long rod 5 mm in diameter has one end maintained at 100°C. The surface of the rod is exposed to ambient air at 25°C with a convection heat transfer coefficient of 100 $W/m^2 \cdot K$.

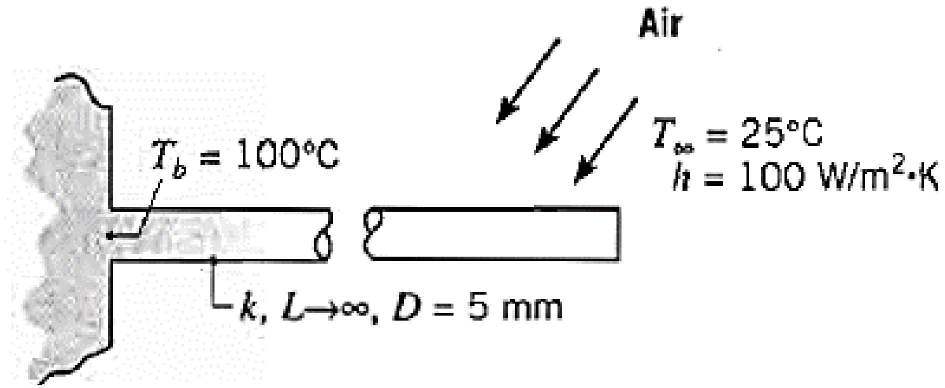
1. Determine the temperature distributions along rods constructed from pure copper, 2024 aluminum alloy, and type AISI 316 stainless steel. What are the corresponding heat losses from the rods?
2. Estimate how long the rods must be for the assumption of *infinite length* to yield an accurate estimate of the heat loss.

Known: A long circular rod exposed to ambient air.

Find:

1. Temperature distribution and heat loss when rod is fabricated from copper, an aluminum alloy, or stainless steel.
2. How long rods must be to assume infinite length.

Schematic:



Assumptions:

1. Steady-state conditions.
2. One-dimensional conduction along the rod.
3. Constant properties.
4. Negligible radiation exchange with surroundings.
5. Uniform heat transfer coefficient.
6. Infinitely long rod.

Properties: Table A.1, copper $[T = (T_b + T_\infty)/2 = 62.5°\text{C} \approx 335 \text{ K}]$: $k = 398$ W/m · K. Table A.1, 2024 aluminum (335 K): $k = 180$ W/m · K. Table A.1, stainless steel, AISI 316 (335 K): $k = 14$ W/m · K.

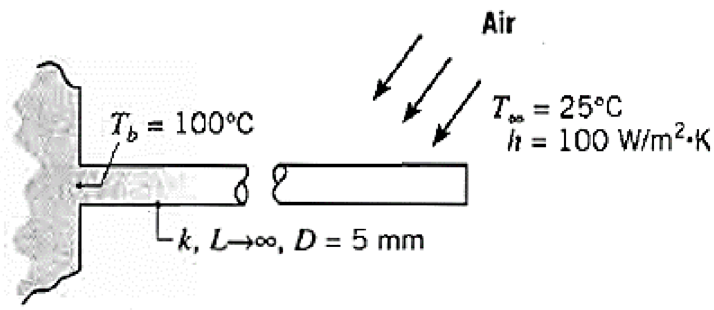
1. Determine the temperature distributions along rods constructed from pure copper, 2024 aluminum alloy, and type AISI 316 stainless steel. What are the corresponding heat losses from the rods?

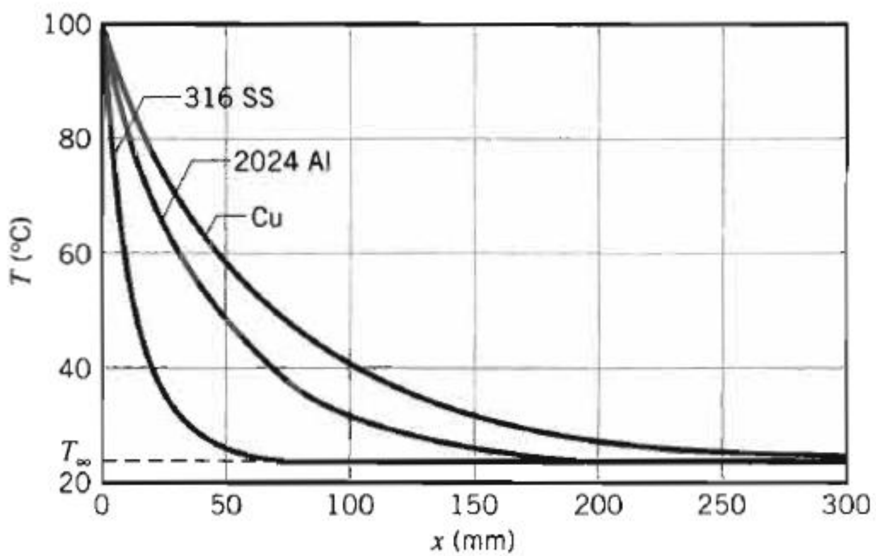
Very long rod => L approaches ∞ (from Slide 16, use which equation?),

Use infinite rod condition:

$$\theta(x) = \theta_b e^{-mx} \text{ where } m = (hP/kA_c)^{0.5}$$

$$q_f = \theta_b\ (hPkA_c)^{0.5}$$





So, no need very long to be "infinite" **for temperature distribution**

Example 1

2. Estimate how long the rods must be for the assumption of *infinite length* to yield an accurate estimate of the heat loss.

Very long rod => L approaches ∞ and $T_{\text{end}} = T_\infty$

No temperature difference with the environment => no heat loss => adiabatic

Compare q_f for "infinite long" and "adiabatic" fin,

$$\frac{q_{adiabatic}}{q_{infinite}} = \tanh mL$$

$$1 = \tanh mL$$

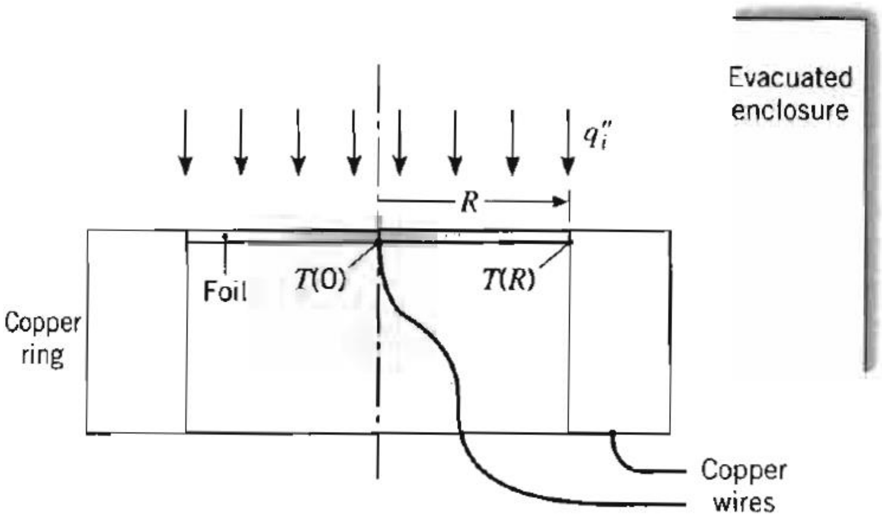
But the above is mathematically undefined!

$$0.99 = \tanh mL$$ (here we assume when 99% of the heat loss is reached, it is good enough)

$$mL = 2.65$$

$$L = 2.65/m$$

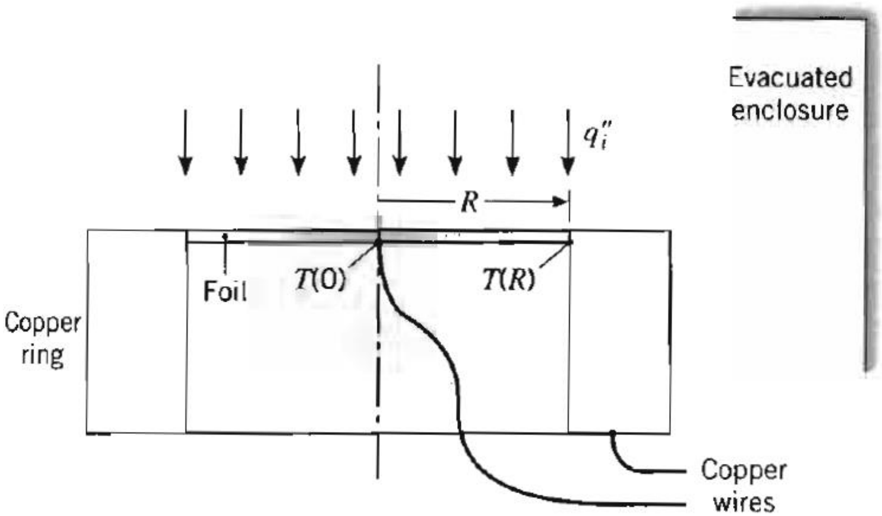
Example 2



The radiation heat gage shown in the diagram is made from constantan metal foil, which is coated black and is in the form of a circular disk of radius R and thickness t. The gage is located in an evacuated enclosure. The incident radiation flux absorbed by the foil, q_i'', diffuses toward the outer circumference and into the larger copper ring, which acts as a heat sink at the constant temperature $T(R)$. Two copper lead wires are attached to the center of the foil and to the ring to complete a thermocouple circuit that allows for measurement of the temperature difference between the foil center and the foil edge, $\Delta T = T(0) - T(R)$.

1. Where is the fin?
2. What are the BCs?
3. Which Fin equation to use?

Example 2



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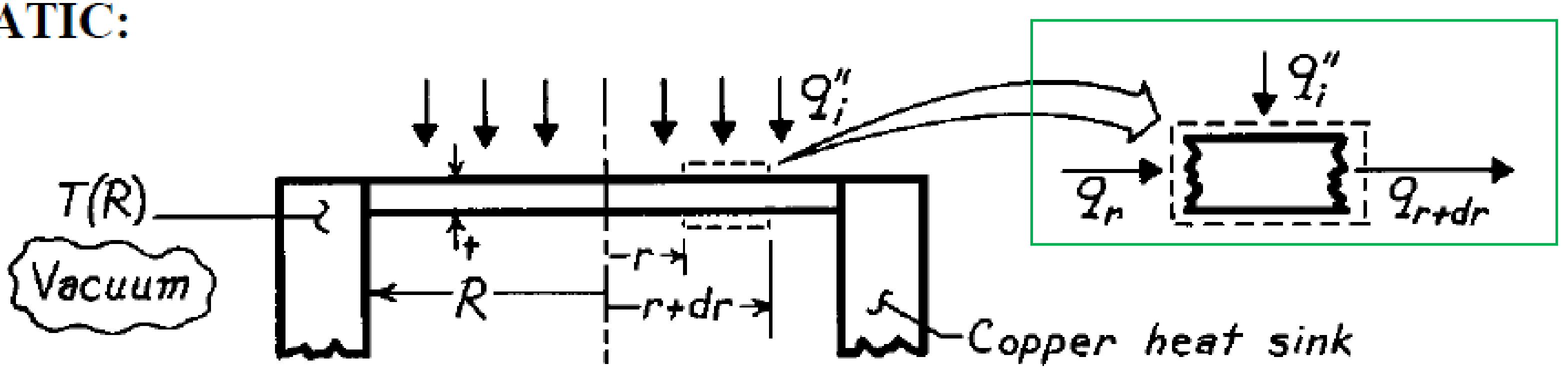
SCHEMATIC:



More Extended Surfaces: Fin Array

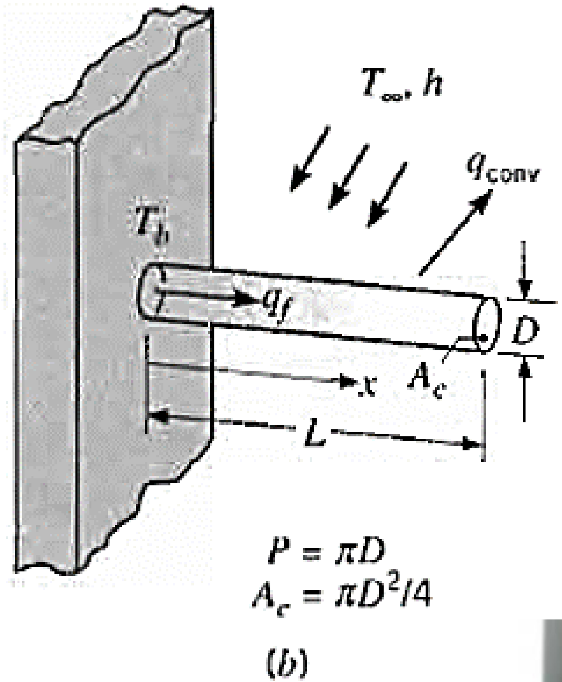
Chapter Three

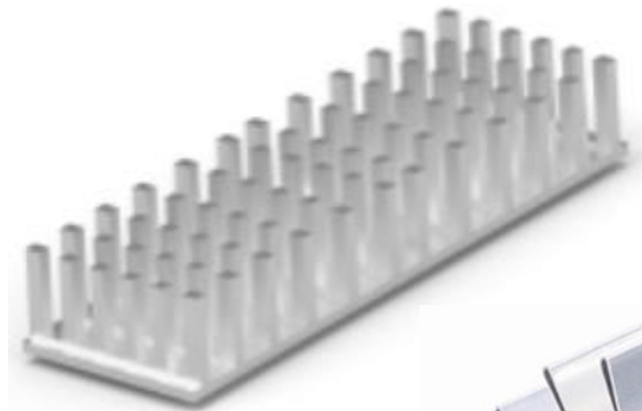
Section 3.6

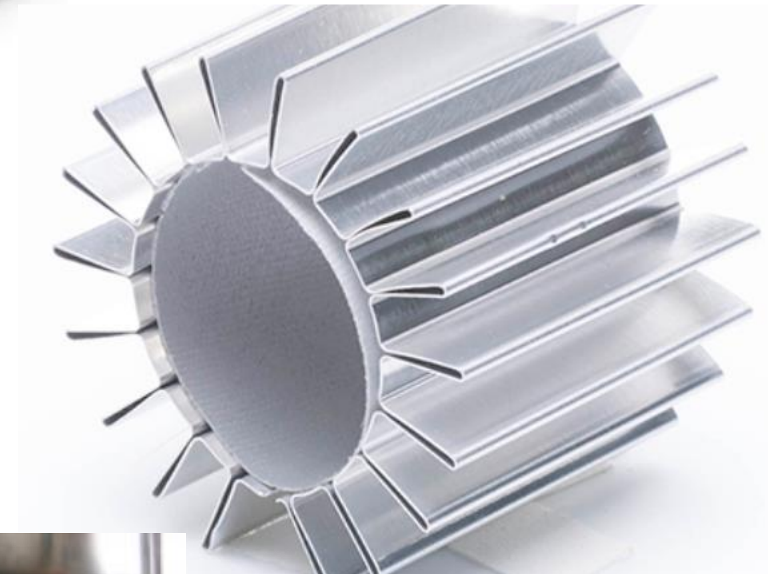
1. What is the total heat rate q_t for a fin array?
2. Where in a thermal circuit is the thermal resistance for a fin array located with respect to the thermal resistance from the base area?
3. What caused the contact resistance, $R''_{t,c}$ at the base of the fin?
4. How is $R''_{t,c}$ included in the thermal circuit?
5. What is the expression overall surface efficiency of a surface with a fin array? With and without contact resistance.
6. What is the easiest way to analyze a fin array?

Fin Array

Fin Array

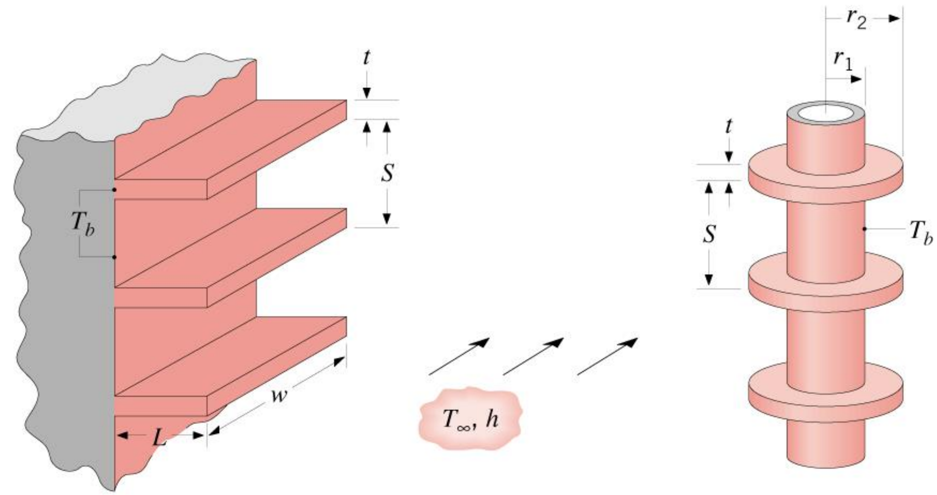






Fin Array

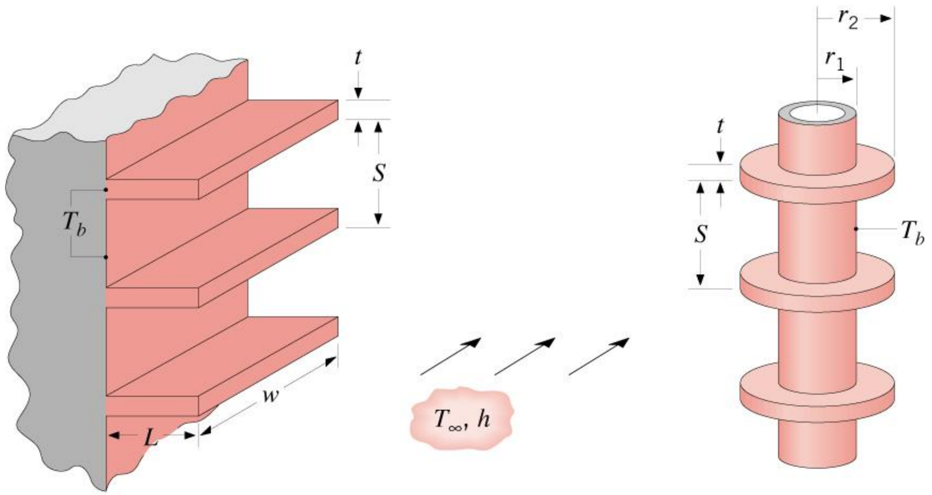
Many similar fins on a surface.
Common fins are
(a) rectangular
(b) annular



How to analyze fin array?
(a) One fin of the same type first
(b) Consider the rest (normally periodically arranged)

The efficiency of a fin is given by,

$$\eta_f = \frac{q_f}{hA_f\theta_b}$$



- Surface area of a fin be A_f
- Total exposed base area be A_b

- Total fin surface area:

$$\boxed{A_t = NA_f + A_b} \qquad (3.104)$$

Number of fins

Total area of exposed base (*prime* surface)

- Total heat rate, q_t:

$$q_t = Nq_f + q_b \qquad (3.105)$$

$$= \boxed{N(\eta_f hA_f\theta_b) + hA_b\theta_b}$$

Fin Array

$$q_t = N(\eta_f h A_f \theta_b) + h A_b \theta_b$$

$$= h\left(N\eta_f A_f + A_b\right)\theta_b$$

$$= h\left(N\eta_f A_f + A_t - N A_f\right)\theta_b$$

$$= h A_t \left(N\eta_f \frac{A_f}{A_t} + 1 - N\frac{A_f}{A_t}\right)\theta_b$$

$$= h A_t(\eta_f \frac{N A_f}{A_t} + 1 - \frac{N A_f}{A_t})\theta_b \quad \text{(A)}$$

- But we can also define an overall efficiency and resistance:

$$q_t = \frac{\theta_b}{R_{t,o}} = \eta_o h A_t \theta_b \qquad \text{(B)}$$

Therefore, Overall surface efficiency (η_o) and resistance ($R_{t,o}$) using (A) and (B):

$$\eta_o = 1 - \frac{NA_f}{A_t}\left(1 - \eta_f\right) \qquad (3.107)$$

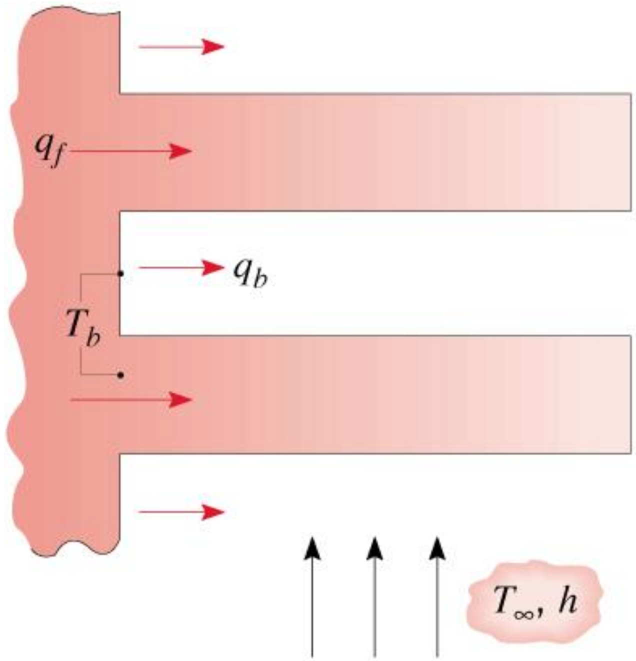
$$R_{t,o} = \frac{\theta_b}{q_t} = \frac{1}{\eta_o h A_t} \qquad (3.108)$$

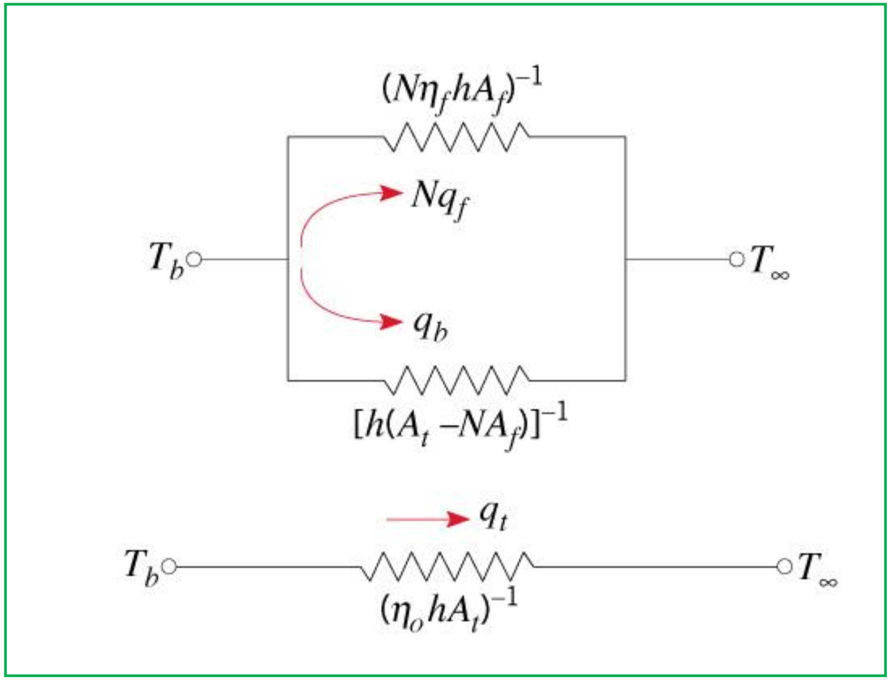
Thermal Resistance Circuit for Fin Array

Array without Contact resistance

Equivalent Thermal Circuit:

$$q_t = \eta_o h A_t \theta_b = \frac{\theta_b}{R_{t,o}} \qquad \text{where } R_{t,o} = \frac{\theta_b}{q_t} = \frac{1}{\eta_o h A_t}$$





Array with Contact resistance

Effect of Surface Contact Resistance given as $R''_{t,c}$ [unit: $(K.m^2)/W$]:

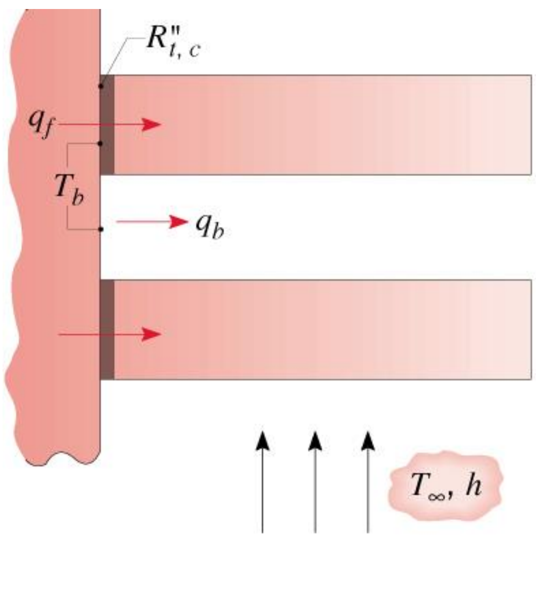
How to include $R''_{t,c}$ in the thermal circuit?

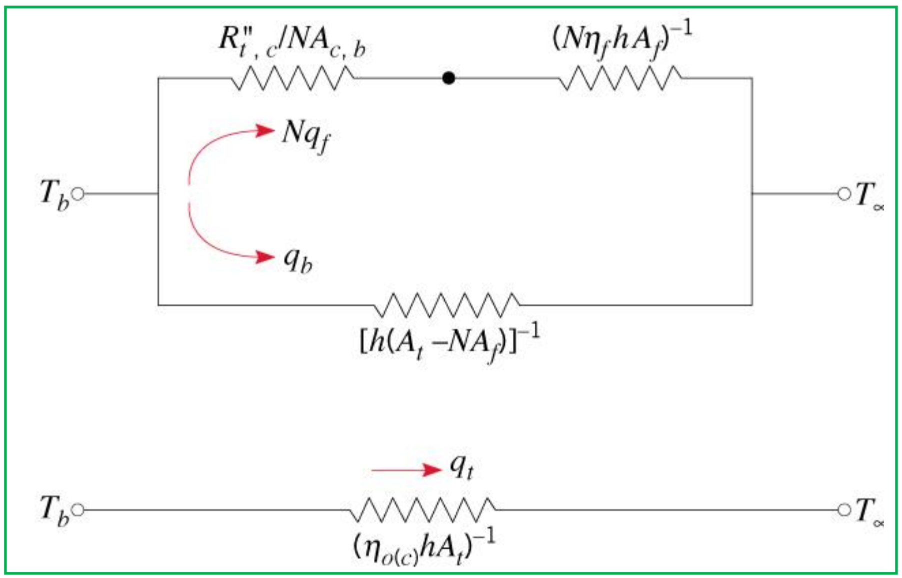
- Consider resistance of a fin, R_f

$$q_f = \eta_f h A_f (T - T_b) \;\Rightarrow\; R_f = \frac{1}{\eta_f h A_f} \qquad \text{[unit: K/W]}$$

But $R''_f = \dfrac{1}{\eta_f h}$ [unit: $(K.m^2)/W$]

So, in the same form, $R_{t,c} = \dfrac{R''_{t,c}}{A_{c,b}}$





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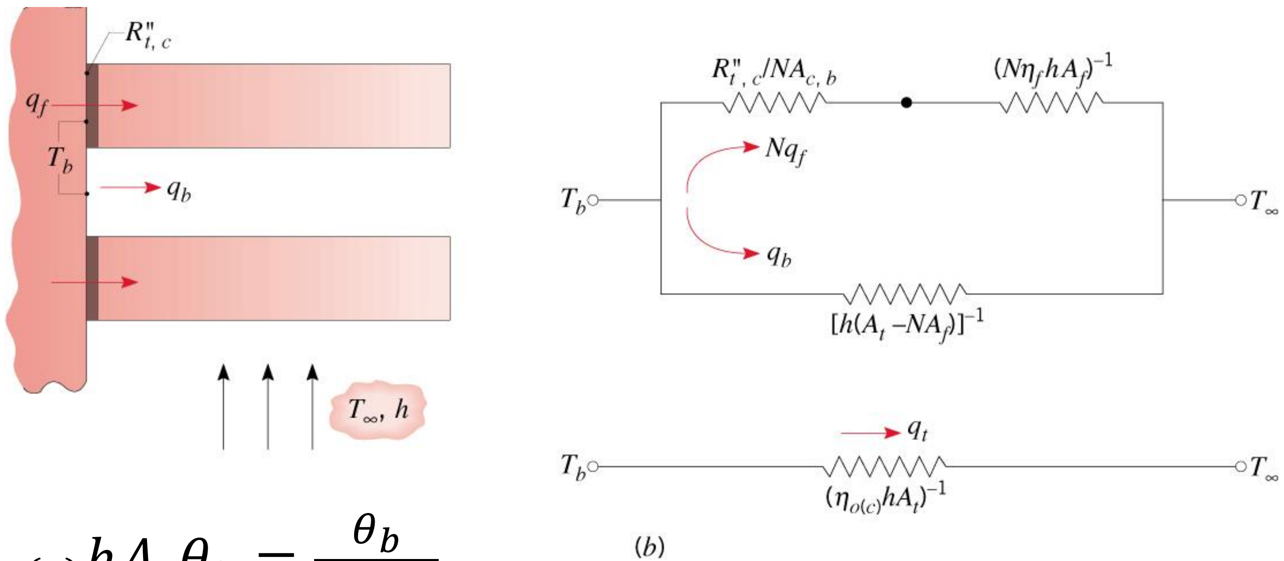
Effect of Surface Contact Resistance given as $R''_{t,c}$:



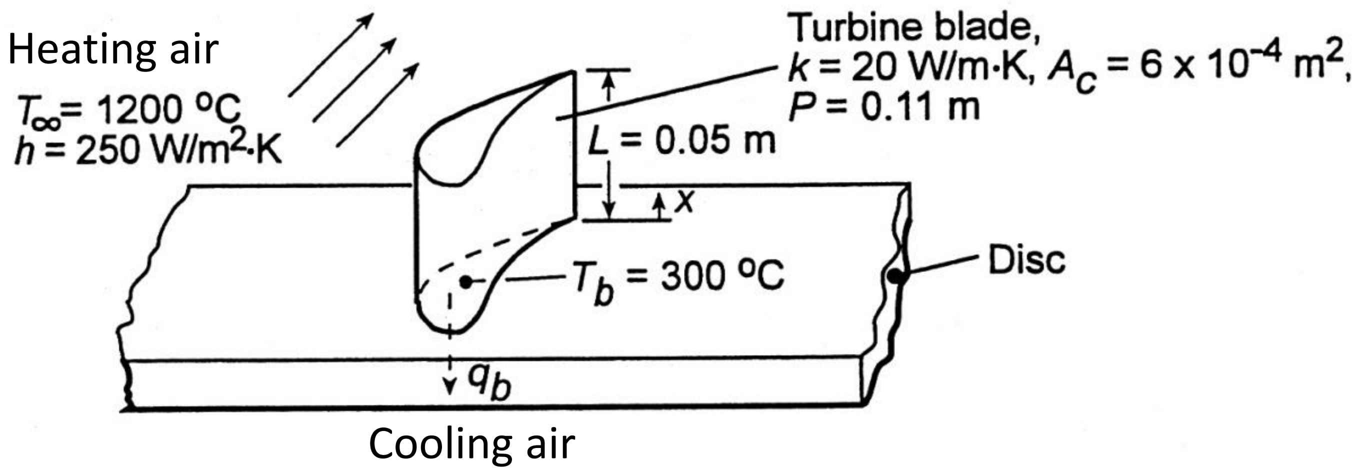
- $q_t = \eta_{o(c)} h A_t \theta_b = \dfrac{\theta_b}{R_{t,o(c)}}$
- $\eta_{o(c)} = 1 - \dfrac{N A_f}{A_t}\left(1 - \dfrac{\eta_f}{C_1}\right)$ (3.110a) => Efficiency with Contact resistance

  where $C_1 = 1 + \eta_f h A_f \left(R''_{t,c}/A_{c,b}\right)$ (3.110b)
- $R_{t,o(c)} = \dfrac{1}{\eta_{o(c)} h A_t}$ (3.109)

Summary

- Fin array
  - Overall Efficiency
  - With and without Contact Resistance

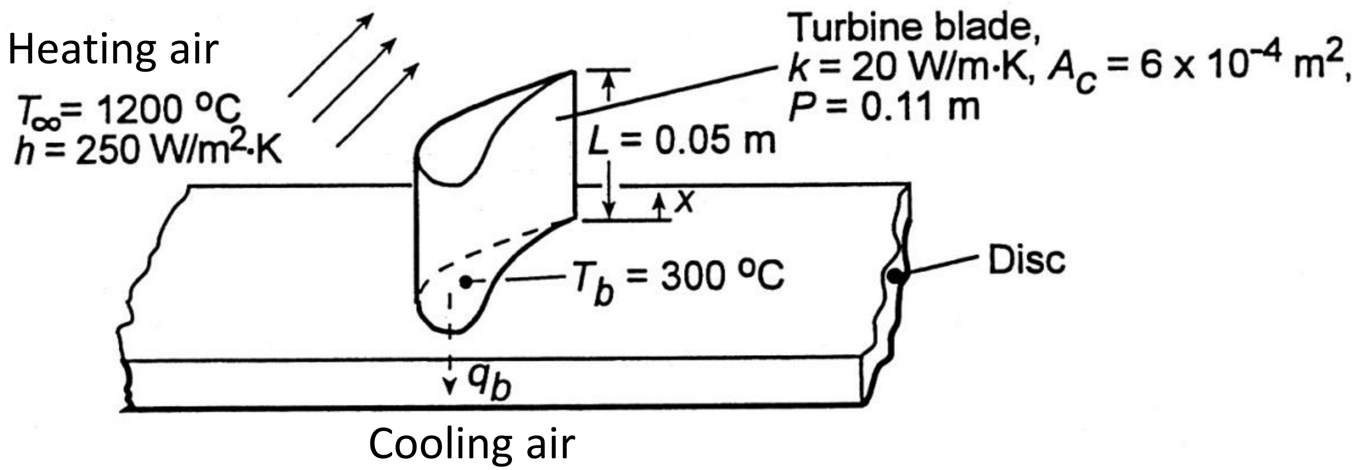
Assessment of cooling scheme for gas turbine blade. Cooling air in the Disc to maintain the base at 300 °C. Determination of whether blade temperatures are less than the maximum allowable value (1050°C) for prescribed operating conditions and evaluation of blade cooling rate. Assume adiabatic condition at the tip of the blade.



ANALYSIS: Conditions in the blade are determined by Case B (adiabatic tip) of Table 3.4.

- Where is the fin?
- Is the cross-sectional area uniform?
- Where will the maximum temperature be?

Assessment of cooling scheme for gas turbine blade. Cooling air in the Disc to maintain the base at 300 ºC. Determination of whether blade temperatures are less than the maximum allowable value (1050°C) for prescribed operating conditions and evaluation of blade cooling rate. Assume adiabatic condition at the tip of the blade.



ASSUMPTIONS: (1) One-dimensional, steady-state conduction in blade, (2) Constant *k*, (3) Adiabatic blade tip, (4) Negligible radiation.

Example 1

(a) The maximum temperature existing at $x = L$, Eq. 3.80 yields

$$\frac{T(L) - T_\infty}{T_b - T_\infty} = \frac{1}{\cosh mL}$$

where $m = (hP/kA_c)^{1/2}$

$$= \left(250\ \mathrm{W/m^2 \cdot K} \times 0.11\mathrm{m}/20\mathrm{W/m \cdot K} \times 6 \times 10^{-4}\mathrm{m^2}\right)^{1/2}$$
$$= 47.87\ \mathrm{m^{-1}}$$

$$mL = 47.87\ \mathrm{m^{-1}} \times 0.05\ \mathrm{m} = 2.39$$
$$\Rightarrow \cosh mL = \cosh(2.39) = 5.51$$

Hence,

$$T(L) = 1200^\circ\mathrm{C} + (300 - 1200)^\circ\mathrm{C}/5.51 = 1037^\circ\mathrm{C}$$

and, *subject to the assumption of an adiabatic tip*, the operating conditions are acceptable.

(b) With $M = (hPkA_c)^{1/2}\theta_b$

$$= \left(250W/m^2 \cdot K \times 0.11m \times 20W/m \cdot K \times 6 \times 10^{-4}m^2\right)^{1/2}(-900^oC)$$

$$= -517W,$$

Heat transfer to the fin,

$$q_f = M \tanh m L$$
$$= -517\text{W}(0.983)$$
$$= -508\text{W}$$

Hence,

$$q_b = -q_f$$
$$= 508\text{W}$$

COMMENTS:

- Heat transfer is to the base of the blade.
- Radiation losses from the blade surface contribute to reducing the blade temperatures.

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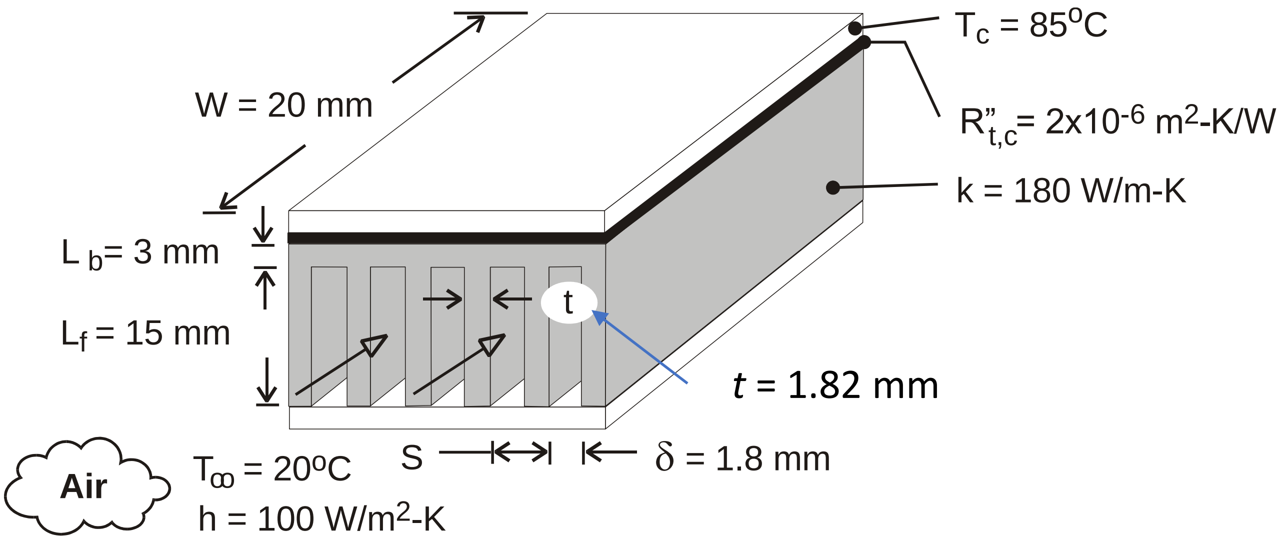
Determination of maximum allowable power q_c for a 20 mm x 20 mm electronic chip whose temperature is not to exceed $T_c = 85^\circ\text{C}$, when the chip is attached to an air-cooled heat sink with N = 11 fins of prescribed dimensions and adiabatic tips.

SCHEMATIC:

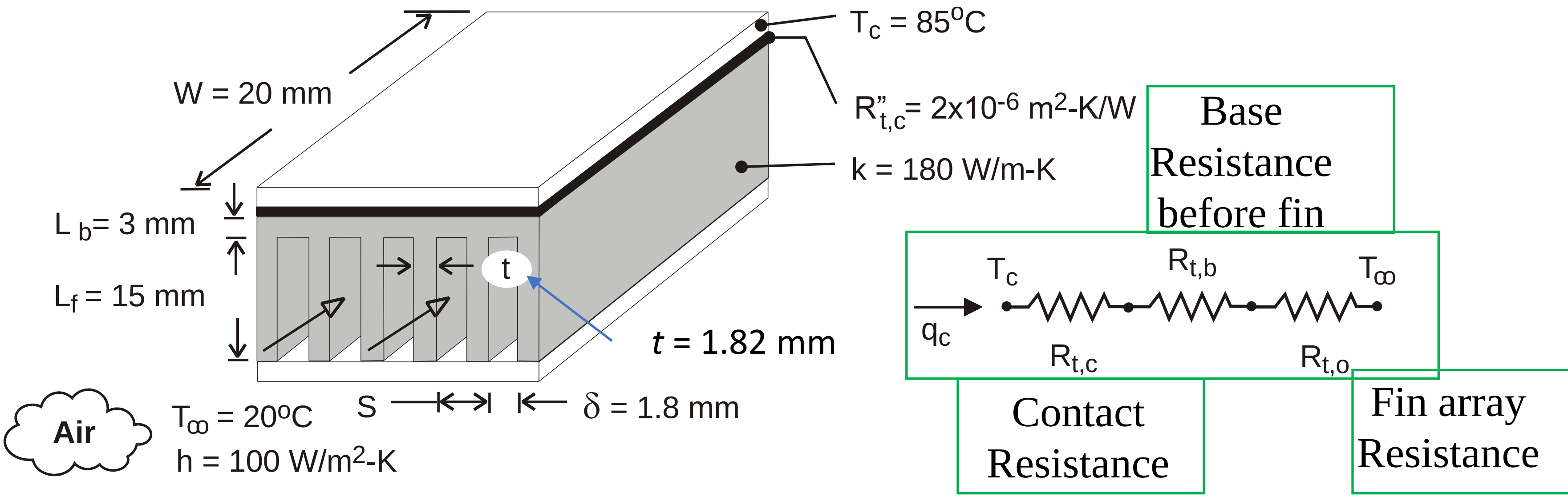


ASSUMPTIONS: (1) Steady-state, (2) One-dimensional heat transfer, (3) Isothermal chip, (4) Negligible heat transfer from top surface of chip, (5) Negligible temperature rise for air flow, (6) Uniform convection coefficient associated with air flow through channels and over outer surface of heat sink, (7) Negligible radiation

Example 2: Fin Array

How does the thermal resistance circuit look like?

SCHEMATIC:



ANALYSIS: (a) From the thermal circuit,

$$q_c = \frac{T_c - T_\infty}{R_{\text{tot}}} = \frac{T_c - T_\infty}{R_{t,c} + R_{t,b} + R_{t,o}}$$

$$R_{t,c} = \boxed{\frac{R''_{t,c}}{\mathrm{W}^2}} = 2 \times 10^{-6}\mathrm{m}^2 \cdot \mathrm{K/W}/(0.02\mathrm{m})^2 = 0.005\mathrm{K/W}$$

$$R_{t,b} = \boxed{L_b/k\left(\mathrm{W}^2\right)} = 0.003\mathrm{m}/180\mathrm{W/m}\cdot\mathrm{K}(0.02\mathrm{m})^2$$
$$= 0.042\mathrm{K/W}$$

From Eqs. (3.108), (3.107), and (3.104)

$$R_{t,o} = \frac{1}{\eta_o h A_t}, \quad \eta_o = 1 - \frac{N A_f}{A_t}\left(1 - \eta_f\right)$$

Find A_t,

$$A_f = \boxed{2WL_f} = 2 \times 0.02\mathrm{m} \times 0.015\mathrm{m} = 6 \times 10^{-4}\ \mathrm{m}^2$$

$$A_b = W^2 - N(tW) = (0.02\mathrm{m})^2 - 11(0.182 \times 10^{-3}\ \mathrm{m} \times 0.02\mathrm{m}) = 3.6 \times 10^{-4}\ \mathrm{m}^2$$

$$A_t = NA_f + A_b = 6.96 \times 10^{-3}\ \mathrm{m}^2$$

To find η_f of a fin,

As adiabatic tip,

$$mL_f = (2h/kt)^{1/2} = (2*100\ \text{W/m}^2\cdot\text{K}/180\ \text{W/m}\cdot\text{K} / 0.182 \times 10^{-3}\text{m})^{1/2}\ (0.015\text{m})$$
$$= 1.17$$

=> $\tanh mL_f = 0.824$

$$\eta_f = \frac{q_f}{hA_f\theta_b} = \frac{M \tanh\ mL_f}{hPL_f\theta_b} = \frac{\tanh\ mL_f}{mL_f} = \frac{0.824}{1.17} = 0.704$$

$$So,\ \eta_o = 1 - \frac{NA_f}{A_t}\left(1 - \eta_f\right) = 0.719$$

$$R_{t,o} = \frac{1}{\eta_o h A_t} = 2\ K/W$$

$$q_c = \frac{(85 - 20)°\text{C}}{(0.005 + 0.042 + 2.00)\text{K/W}}$$
$$= 31.8\text{W}$$

COMMENTS: The heat sink significantly increases the allowable heat dissipation. If it were not used and heat was simply transferred by convection from the surface of the chip with $h = 100\ \text{W/m}^2\cdot\text{K}, R_{\text{tot}} = 2.05\ \text{K/W}$ from Part (a) would be replaced by $R_{\text{conv}} = 1/hW^2 =$ 25 K/W, yielding $q_c =$ 2.60 W.

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1. **Classical method (no or minimum assumptions)**
   - Use Energy Balance, $\dot{E}_{in} - \dot{E}_{out} + \dot{E}_g = \dot{E}_{st}$
   - Get heat equation
   - Solve and apply BCs
   - Get temperature distribution
   - Apply Fourier's law to get heat flux (q") /heat rate ($\dot{q}$)

2. **Alternative method (only when 1D, steady-state, no heat generation)**
   - Apply Fourier Law

$$q_x = -kA\frac{dT}{dx} \qquad (3.26)$$

$$q_x \int_{x_o}^{x} \frac{dx}{A(x)} = -\int_{T_o}^{T} k(T)dT$$

where area (A) is usually dependent on x

thermal conductivity (k) is usually dependent on T