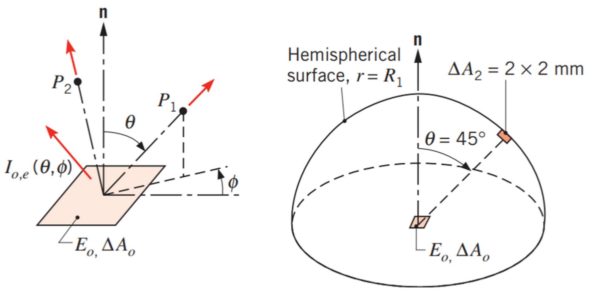
ME 320 Homework 8

Consider a 5-mm-square, diffuse surface ΔA_o having a total emissive power of $E_o = 4000\ \mathrm{W/m^2}$. The radiation field due to emission into the hemispherical space above the surface is diffuse, thereby providing a uniform intensity $I(\theta, \phi)$. ~~Moreover, if the space is a nonparticipating medium (nonabsorbing, nonscattering, and nonemitting),~~ the intensity is independent of radius for any (θ, ϕ) direction. Hence intensities at any points P_1 and P_2 would be equal.



(a) What is the rate at which radiant energy is emitted by ΔA_o, q_{emit}?

(b) What is the intensity $I_{o,e}$ of the radiation field emitted from the surface ΔA_o?

(c) Beginning with Equation 12.13 and presuming knowledge of the intensity $I_{o,e}$, obtain an expression for q_{emit}.

$$q_{emit} = \int_h I_{o,e} \Delta A_o \cos\theta \sin\theta \, d\theta \, d\phi$$

(d) Consider the hemispherical surface located at $r = R_1 = 0.5$ m. Using the conservation of energy requirement, determine the rate at which radiant energy is incident on this surface due to emission from ΔA_o.

(a). $q_{emit} = E_o \cdot \Delta A_o = 4000\ \mathrm{W/m^2} \times (5\mathrm{mm})^2 = 0.1\ \mathrm{W}$

这是什么

(b). $I_{o,e} = \dfrac{E_o}{\pi} = \dfrac{4000\ \mathrm{W/m^2}}{\pi} = 1273\ \mathrm{W/m^2 \cdot sr}$

(c). 公式推导

$$q_e = \int_h I_{o,e} \cdot \Delta A_o \cdot \cos\theta \cdot \sin\theta \, d\theta \, d\phi$$

$$= I_{o,e} \Delta A_o \int_0^{2\pi} \int_0^{\frac{\pi}{2}} d\phi \cdot \frac{1}{2} \sin 2\theta \, d\theta$$

$$= I_{o,e} \cdot \Delta A_o \int_0^{\frac{\pi}{2}} 2\pi \cdot \frac{1}{2} \sin 2\theta \cdot d\theta$$

$$= I_{o,e} \cdot \Delta A_o \left[-\frac{\pi}{2} \cos(2\theta) \right]_0^{\frac{\pi}{2}}$$

$$= \pi \cdot I_{o,e} \Delta A_o$$

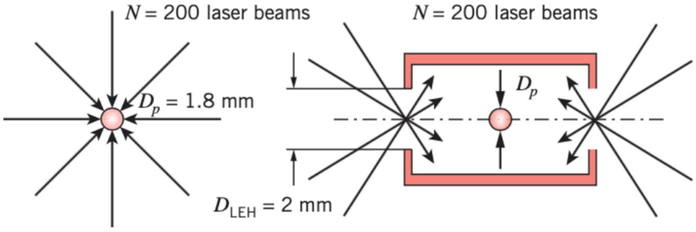
(d). $A_1 = \frac{1}{2} \cdot 4\pi R_1^2 = 1.57\ \mathrm{m^2}$

$$G_1 = \frac{q_{emit}}{A_1} = \frac{0.1\mathrm{W}}{1.57\mathrm{m^2}} = 0.0637\ \mathrm{W/m^2}$$

The extremely high temperatures needed to trigger nuclear fusion are proposed to be generated by laser-irradiating a spherical pellet of deuterium and tritium fuel of diameter $D_p = 1.8$ mm. 核反应堆

(a) Determine the maximum fuel temperature that can be achieved by irradiating the pellet with 200 lasers, each producing a power of $P = 500$ W. The pellet has an absorptivity $\alpha = 0.3$ and emissivity $\varepsilon = 0.8$.

(b) The pellet is placed inside a cylindrical enclosure. Two laser entrance holes are located at either end of the enclosure and have a diameter of $D_{LEH} = 2$ mm. Determine the maximum temperature that can be generated within the enclosure.



(a) (b)

- **Pellet is very small compared to the environment, so what is the radiation area?**
- **Assume that all the laser that enters the enclosure is totally absorbed.**

(a). 吸收能量 = 辐射能量

$$\alpha \cdot N \cdot P = A \cdot \varepsilon \sigma T_p^4$$

$$\alpha \cdot N \cdot P = 4\pi \left(\frac{D_p}{2}\right)^2 \cdot \varepsilon \cdot \sigma \cdot T_p^4$$

$$\Downarrow$$

$$T_p = \sqrt[4]{\frac{\alpha \cdot N \cdot P}{\pi D_p^2 \cdot \varepsilon \cdot \sigma}} = 15967 \text{ K}$$

(b). 吸收能量 = 辐射能量

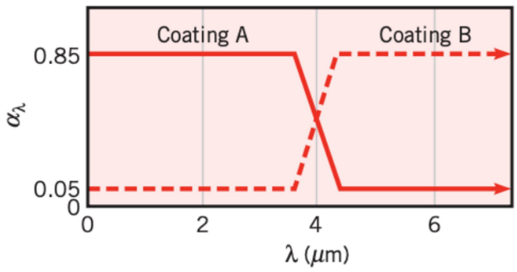
$$N \cdot P = A \cdot \sigma T_p^4$$

$$N \cdot P = 2 \cdot \left(\pi \frac{D_{LEH}^2}{4}\right) \sigma \cdot T_p^4$$

$$\Downarrow$$

$$T_p = \sqrt[4]{\frac{2 \cdot N \cdot P}{\pi \cdot D_{LEH}^2 \cdot \sigma}} = 23017 \text{ K}$$

Two special coatings are available for application to an absorber plate installed below the cover glass described in Example 12.9. Each coating is diffuse and is characterized by the spectral distributions shown.



Which coating would you select for the absorber plate? Explain briefly. For the selected coating, what is the rate at which radiation is absorbed per unit area of the absorber plate if the total solar irradiation at the cover glass is $G_S = 1000\ \mathrm{W/m^2}$?

$$\alpha = \frac{G_{abs}}{G_i}$$

?

Assume the change in the absorptivity occurs sharply at $\lambda = 4$ um.

Example 12.9 is about a cover glass on a flat-plate solar collector with a spectral transmissivity that has a value of 0.9 between 0.3um and 2.5um but 0 everywhere else.

根据公式 $E = \frac{h}{\lambda_{波长}}$，λ 波长越小，携带能量越高

所以应该选择 Coating A，因为其吸收的波长更短

$$\alpha_A = 0.85 \times F_{0\text{-}4\mu m} + 0.05 \times F_{4\text{-}\infty\, \mu m}$$

根据 Figure 12.12，太阳温度 = 5800K

$$\lambda_{4\mu m} \times T_{太阳} = 4\mu m \times 5800K = 23200\ \mu m \cdot K$$

根据 Table 12.12

$$\lambda_{\mu m} T = 23200\ \mu m K \Rightarrow F_{0\text{-}4\mu m} = 0.99$$

$$\Rightarrow F_{4\mu m\text{-}\infty} = 0.01$$

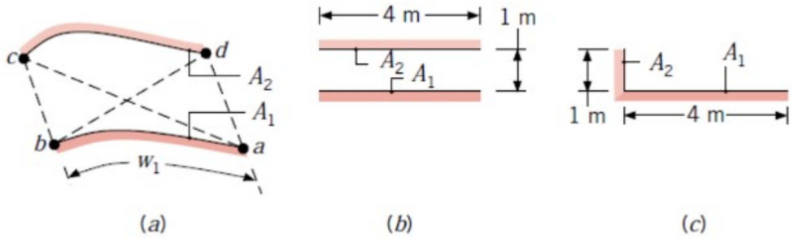
$$\Rightarrow \alpha_A = 0.85 \times 0.99 + 0.05 \times 0.01 \approx 0.85$$

$$\Rightarrow G_{abs} = \alpha_A \cdot \tau \cdot G_s = 0.85 \times 0.84 \times 1000\ \mathrm{W/m^2} = 714\ \mathrm{W/m^2}$$

?

The "crossed-strings" method of Hottel [13] provides a simple means to calculate view factors between surfaces that are of infinite extent in one direction. For two such surfaces (*a*) with unobstructed views of one another, the view factor is of the form

$$F_{12} = \frac{1}{2w_1}[(ac + bd) - (ad + bc)]$$



Use this method to evaluate the view factors F_{12} for sketches (*b*) and (*c*). Compare your results with those from the appropriate graphs and analytical expressions.

View Factor

Calculation Based on Equation

View Factor For (b).

$$F_{12} = \frac{1}{2W}\left[(ac+bd) - (ad+bc)\right]$$

$$= \frac{1}{2\times4}\left[(\sqrt{17}+\sqrt{17}) - (1+1)\right]$$

$$= \frac{\sqrt{17}-1}{4} = \boxed{0.7807}$$

View Factor for (c).

$$F_{12} = \frac{1}{2W}\left[(ac+bd) - (ad+bc)\right]$$

$$= \frac{1}{2\times4}\left[(4+1) - (\sqrt{17}+0)\right]$$

$$= \frac{5-\sqrt{17}}{8} = \boxed{0.1096}$$

Calculation Based on Table

View Factor for (b) from table 13.1

$$F_{12} = \frac{\sqrt{(W_1+W_2)^2+4} - \sqrt{(W_2-W_1)^2+4}}{2W_1}$$

$$W_1 = \frac{w_1}{L}, \quad W_2 = \frac{w_2}{L}$$

$$F_{12} = \frac{\sqrt{64+4} - \sqrt{4}}{2\times4}$$

$$= \frac{\sqrt{68}-2}{8}$$

$$= \boxed{0.7807}$$

View Factor for (c) from table 13.1

$$F_{12} = \frac{1+\left(\frac{w_2}{w_1}\right) - \sqrt{1+\left(\frac{w_2}{w_1}\right)^2}}{2}$$

$$= \frac{1+\frac{1}{4} - \sqrt{1+\left(\frac{1}{4}\right)^2}}{2}$$

$$= \boxed{0.1096}$$