Consider $G(s) = \dfrac{10}{0.5s+1}$

$$G(j\omega) = \frac{10}{0.5j\omega+1} \quad , \quad M(\omega) = \frac{10}{\sqrt{0.25\omega^2+1}}$$

$$20\log_{10} M(\omega) = 20\log_{10} 10 - 20\log_{10}\sqrt{0.25\omega^2+1}$$

$$= 20 - 20\log\sqrt{0.25\omega^2+1}$$

$$\phi(\omega) = 0 - \tan^{-1}(0.5\omega)$$

- when $\omega \to 0$ $(\omega \ll 2)$

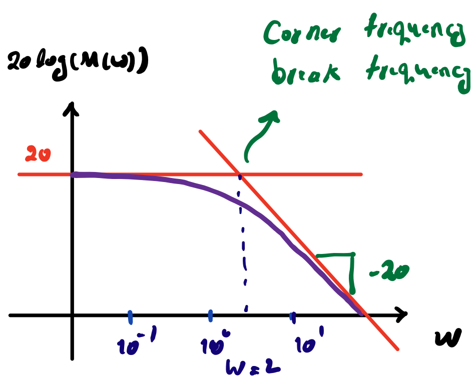
$20\log M(\omega) = 20\text{ dB}$ $\rightsquigarrow$ DC gain (low frequencies magnitude)

- when $\omega \to \infty$ $(\omega \gg 2)$

$$20\log M(\omega) \cong 20 - 20\log\sqrt{0.25\omega^2}$$

$$= 20 - 10\log 0.25\omega^2$$

$$= 20 - 10\log 0.25 - 20\log\omega$$



We would like to find where these two lines cross ?

$$20 = 20 - 10\log 0.25 - 20\log\omega$$

$$20\log\omega = -10\log 0.25$$

$$\log\omega = -0.5\log 0.25$$

$$\rightsquigarrow \omega = 10^{(-0.5\log 0.25)} = \left(10^{\log 0.25}\right)^{-0.5}$$

$= (0.25)^{-0.5}$

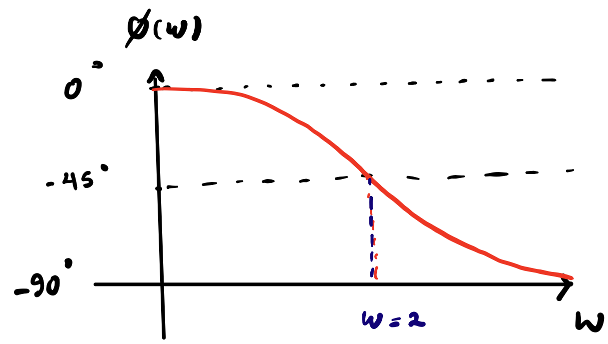
$= \sqrt{(0.25)^{-1}}$

$= \sqrt{4} = 2$

$\phi(w \to 0) = 0$

$\phi(w \to \infty) = -90^\circ$

$\phi(w = 2) = -45^\circ$



For a second order system: $(0 < \zeta < 1)$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$G(j\omega) = \frac{\omega_n^2}{-\omega^2 + j\,2\zeta\omega_n\omega + \omega_n^2} = \frac{1}{-\omega^2/\omega_n^2 + j\,2\zeta\,\omega/\omega_n + 1}$$

$$M(\omega) = |G(j\omega)| = \frac{1}{\sqrt{\left(1-\left(\omega/\omega_n\right)^2\right)^2 + \left(2\zeta\left(\omega/\omega_n\right)\right)^2}}$$

$$20 \log M(\omega) = -20 \log \sqrt{\left(1-\left(\omega/\omega_n\right)^2\right)^2 + \left(2\zeta\left(\omega/\omega_n\right)\right)^2}$$

$$= -10 \log \left(\left(1-\left(\omega/\omega_n\right)^2\right)^2 + \left(2\zeta\left(\omega/\omega_n\right)\right)^2 \right)$$

$$\phi(\omega) = 0 - \tan^{-1}\left(\frac{2\zeta\left(\omega/\omega_n\right)}{1-\left(\omega/\omega_n\right)^2} \right)$$

- For low ω $(\omega \ll \omega_n)$: $20 \log M(\omega) \approx 0$, $\phi(\omega) \approx 0$

- For high ω $(\omega \gg \omega_n)$: $20 \log M(\omega) = -10 \log \left(\left(\omega/\omega_n\right)^4 + \left(2\zeta\left(\omega/\omega_n\right)\right)^2 \right)$

$$\approx -10 \log \left(\left(\omega/\omega_n\right)^4 \right)$$

$$\approx -40 \log\left(\omega/\omega_n\right) , \quad \phi(\omega) \approx -180^\circ$$