Lab 3 Pre-Lab

1. Determine the decay rate for the exponentially decaying signal $x(t) = 4e^{-t/3}$.

decay rate $\gamma = \frac{1}{3}$

2. After what time does the value of the exponentially decaying signal $x(t) = 2e^{-t/6}$ equal the fraction $2/e$ of its initial value?

$x(0) = 2$

$x(t) = x(0) \cdot \frac{2}{e} = \frac{4}{e} \Rightarrow 2e^{-\frac{t}{6}} = \frac{4}{e}$

$e^{-\frac{t}{6}} = 2 \cdot e^{-1}$

$-\frac{t}{6} = \ln 2 - 1$

$t = 6(1 - \ln 2)$

3. Determine the slowest decay rate for the sum of exponentially decaying signals

$$x(t) = \frac{1}{2}e^{-t/3} - \frac{1}{3}e^{-2t} + 18e^{-t/2}.$$

slowest decay rate: $\gamma_{min} = \frac{1}{3}$

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4. Suppose that the signal

$$x(t) = 2e^{-t/3} + e^{-t} + 3e^{-t/2}$$

is measured in the presence of noise distributed uniformly in the interval $[-0.25, 0.25]$. Are the decay rates well separated? Are measurements dominated by the component with the slowest decay rate on any time interval?

1. Yes, it's well separated
2. No, because their coeffcients are different

5. Find the free response of a linear, first order system with time constant $3/2$ and initial value 4.

$$x(t) = 4e^{-\frac{2}{3}t}$$

6. Find the two-units step response of a linear, first order system with time constant 3.

$$x(t) = 2\int_0^t e^{\frac{-(t-\tau)}{3}} d\tau = 6(1-e^{-\frac{t}{3}})$$

7. Suppose that the response $x(t)$ of a linear, first order system with $f(t) = 0$ for $t \geq 0$ is approximately equal to the fraction $1/e$ of its initial value $x(0)$ after time $5/3$. Estimate the corresponding time constant.

Free response

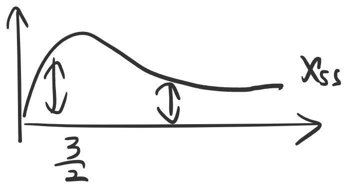
$\because x(t) = x(0)e^{-\frac{t}{T}}$

$\therefore x(\frac{5}{3}) = \frac{1}{e} \cdot x(0)$

$$\frac{-\frac{5}{3}}{T} = -1$$

$$T = \frac{5}{3}$$

8. Suppose that the response $x(t)$ of a linear, first order system with $f(t)$ $=c$ constant for $t \geq 0$ and $x(0) = 0$ differs from its steady-state value by approximately a fraction $2/e$ of the initial difference after time $3/2$. Estimate the corresponding time constant.



$$\dot{x}(t) + \frac{1}{T}x(t) = f(t) = c$$

$$\frac{dx}{dt} + \frac{x}{T} = c$$

$$\frac{1}{c-\frac{x}{T}}\,dx = dt$$

$$-T\cdot\ln\left(c-\frac{x}{T}\right) = t + b$$

$$\ln\left(c-\frac{x}{T}\right) = \frac{t+b}{-T}$$

$$c - \frac{x}{T} = e^{-\frac{t+b}{T}}$$

$$x = CT - T\cdot e^{-\frac{t+b}{T}}$$

$\because x(0) = 0$

$\therefore b = T\cdot\ln c \qquad \therefore x(t) = CT\left(1-e^{-\frac{t}{T}}\right)$

$$\because \begin{cases} \dfrac{x(\frac{3}{2}) - x_{ss}}{x(\frac{3}{2}) - 0} = \dfrac{2}{e}, \\ \lim\limits_{t\to\infty} x(t) = CT \\ x(\frac{3}{2}) = CT\left(1-e^{-\frac{3}{2T}}\right) \end{cases}$$

$$\therefore\ T = \frac{-3}{2\ln\left(\frac{2}{2-e}\right)}$$

9. A linear, first order system whose response $x(t)$ satisfies the differential equation

$$T\frac{\mathrm{d}x}{\mathrm{d}t}(t) + x(t) = Kf(t)$$

has time constant T and gain K. How would you estimate T and K from measurements of the response $x(t)$ when $f(t) = 1$ for $t \geq 0$?

$\because T\frac{dx}{dt} + x = k$

$T\frac{dx}{dt} = k - x$

$T \cdot \frac{1}{k-x} dx = dt$

$\int T \frac{1}{k-x} dx = \int dt$

$-T\ln(k-x) = t + C$

$K - x = e^{\frac{t+C}{-T}}$

$x = K - e^{-\frac{t+C}{T}}$

$x = K - C \cdot e^{-\frac{t}{T}}$

① measuring x_{ss} at $t \to \infty$ to get K

② measuring $x(0)$

$\therefore x(0) = K - C$

$\therefore$ we can determine C

③ measure how long it takes $x(t) = K - Ce^{-1}$

$\therefore$ we can get $T = t$