Energy Method:

保守系统 (无阻尼 + 摩擦)

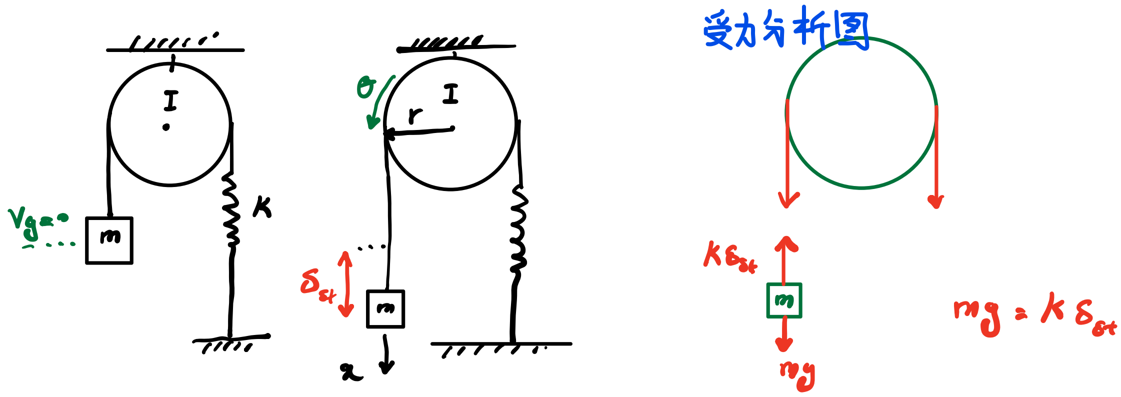
For a conservative system (no damping or friction), we can use the principle of conservation energy:

Total energy : $T+V$ ⇝ $\boxed{\frac{d}{dt}(T+V)=0}$ 总能量变化为0

potential energy 旋转动能

kinetic energy 平移动能

ex :



Soln:

动能:

平移 $T_{block} = \frac{1}{2}m\dot{x}^2$

旋转 $T_{pulley} = \frac{1}{2}I\dot{\theta}^2$

⇝ $T = T_{block} + T_p = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\dot{\theta}^2$

势能:

弹性 $V_s = \frac{1}{2}k(x+\delta_{st})^2$

重力 $V_g = -mg(x+\delta_{st})$

⇝ $V = V_s + V_g = \frac{1}{2}k(x+\delta_{st})^2 - mg(x+\delta_{st})$ 消去②

Total energy : $T+V = \frac{1}{2}m\dot{x}^2 + \frac{1}{2}I\dot{\theta}^2 + \frac{1}{2}k(x+\delta_{st})^2 - mg(x+\delta_{st})$

Geometric Constraint: $r\theta = x$ 几何关联

$\Big\downarrow$ 求导

Energy Method: $\frac{d}{dt}(T+V) = m\dot{x}\ddot{x} + I\dot{\theta}\ddot{\theta} + k(x+\delta_{st})\dot{x} - mg\dot{x}$

$\frac{d(r\theta)}{dt} = \dot{x} \sim r\dot{\theta} = \dot{x}$

$\frac{d(r\dot{\theta})}{dt} = \ddot{x} \sim r\ddot{\theta} = \ddot{x}$

$$\frac{d}{dt}(T+V) = mr^2\dot{\theta}\ddot{\theta} + I\dot{\theta}\ddot{\theta} + kr^2\theta\dot{\theta} + \overset{0}{\cancel{\overset{\overset{mg}{\|}}{k\delta_{st}}\, r\dot{\theta} - mgr\dot{\theta}}}$$

$$= mr^2\dot{\theta}\ddot{\theta} + I\dot{\theta}\ddot{\theta} + kr^2\theta\dot{\theta} = 0$$

$$= \boxed{(mr^2+I)\ddot{\theta} + kr^2\theta = 0}$$