

Laboratory handout 7 – Nonlinear systems

Let f denote a function of n variables, such that $f(x_1, \dots, x_n)$ is the value of the function when the variables equal x_1 through x_n . The **partial derivative** of f with respect to its i th argument is then a new function $\partial f / \partial x_i$ of n arguments, and

$$\frac{\partial f}{\partial x_i}(x_1, \dots, x_n) = \lim_{h \rightarrow 0} \frac{f(\dots, x_i + h, \dots) - f(\dots, x_i, \dots)}{h} \quad (1)$$

Suppose that $x_1(t), \dots, x_n(t)$ are some known functions of time.

Then, if $g(t) = f(x_1(t), \dots, x_n(t))$, it follows that

$$\begin{aligned} \frac{dg}{dt}(t) &= \frac{\partial f}{\partial x_1}(x_1(t), \dots, x_n(t)) \dot{x}_1(t) + \dots \\ &\quad + \frac{\partial f}{\partial x_n}(x_1(t), \dots, x_n(t)) \dot{x}_n(t) \end{aligned} \quad (2)$$

For a function g of a single variable, the **Taylor expansion** about an operating point a is the power series

$$\sum_{i=0}^{\infty} \frac{g^{(i)}(a)}{i!} (x - a)^i, \quad g^{(i)}(x) = \frac{d^i g}{dx^i}(x). \quad (3)$$

For smooth functions, a **truncated Taylor expansion**

$$\sum_{i=0}^N \frac{g^{(i)}(a)}{i!} (x - a)^i, \quad (4)$$

for some integer N , is an approximation of the original function for $x \approx a$.

For a function f of n variables, the Taylor expansion about an operating point (a_1, \dots, a_n) is obtained by letting $\varepsilon = 1$ in the Taylor expansion of the function

$$g(\varepsilon) = f(a_1 + \varepsilon(x_1 - a_1), \dots, a_n + \varepsilon(x_n - a_n)) \quad (5)$$

about the operating point $\varepsilon = 0$. In this case, the first order truncation

$$g(\varepsilon) \approx g(0) + g'(0)\varepsilon \quad (6)$$

evaluated at $\varepsilon = 1$ implies that

$$f(x_1, \dots, x_n) \approx f(a_1, \dots, a_n) + \frac{\partial f}{\partial x_1}(a_1, \dots, a_n)(x_1 - a_1) + \dots + \frac{\partial f}{\partial x_n}(a_1, \dots, a_n)(x_n - a_n). \quad (7)$$

For a mechanical system with n degrees of freedom, **Lagrange's equations** are n equalities of the form

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}} \right) - \frac{\partial T}{\partial q} = F_q \quad (8)$$

where T is the **total kinetic energy** of the system, q is a **generalized coordinate**, i.e., one of n unknown time-dependent functions that are used to describe the configuration and motion of the mechanism, and F_q is a corresponding **generalized force**.

For **conservative mechanical systems**, the generalized force $F_q = -\partial V / \partial q$ in terms of the **total potential energy** V of the system that depends on q but not on \dot{q} . In this case, Lagrange's equations become

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) - \frac{\partial L}{\partial q} = 0 \quad (9)$$

where $L = T - V$ is the system **Lagrangian**.

Under physically reasonable assumptions about internal and external reaction forces acting on the mechanism, Lagrange's equations are equivalent to the equations of motion obtained from **Newton's 2nd law**.

A configuration of a mechanical system with generalized coordinates $(q_1, \dots, q_n) = (q_{1,0}, \dots, q_{n,0})$ is said to be an **equilibrium** if $(q_1(t), \dots, q_n(t)) = (q_{1,0}, \dots, q_{n,0})$ is a solution to Lagrange's equations for all t . If the system is placed in an equilibrium configuration, it will remain in that configuration for all time.

Slow motion of a mechanical system near an equilibrium may be understood in terms of a local approximation of the Lagrangian.

Specifically, given the Lagrangian $L(q_1, \dots, q_n, \dot{q}_1, \dots, \dot{q}_n)$, let the local approximation L_{loc} be given by the Taylor expansion of L about the operating point $(q_{1,0}, \dots, q_{n,0}, 0, \dots, 0)$ truncated to terms that are quadratic in $q_i - q_{i,0}$ and \dot{q}_i .

The equations

$$\frac{d}{dt} \left(\frac{\partial L_{\text{loc}}}{\partial \dot{q}_i} \right) - \frac{\partial L_{\text{loc}}}{\partial q_i} = 0, \quad i = 1, \dots, n \quad (10)$$

are the **linearized equations of motion** about the equilibrium. The behavior of solutions to the linearized equations of motion approximates the behavior of the mechanical system as long as motion remains slow and in the vicinity of the equilibrium.

As an alternative to first deriving a local approximation to the Lagrangian and then deriving the linearized equations of motion, it is possible to perform linearization directly on the original system of equations of motion. This is accomplished by substituting $q_i = q_{i,0} + \varepsilon(q_i - q_{i,0})$, $\dot{q}_i = \varepsilon \dot{q}_i$, and $\ddot{q}_i = \varepsilon \ddot{q}_i$ in the equations of motion, Taylor expanding each term about the operating point $\varepsilon = 0$, truncating at first order in ε , and letting $\varepsilon = 1$.

Exercises

1. Use Lagrange's equations to derive the equation of motion for a particle pendulum of mass m swinging in a vertical plane at a distance ℓ from a stationary suspension, under the influence of gravity.
2. Use Lagrange's equations to derive the equation of motion for a single-link pendulum consisting of a uniform slender rod of mass M and length ℓ swinging in a vertical plane about one of its end points, under the influence of gravity.

3. Find all equilibria of the mechanical system with Lagrangian

$$L = \frac{m\ell^2\dot{q}^2}{2} - mg\ell \sin q$$

in terms of the generalized coordinate q , and derive the corresponding linearized equations of motion.

4. Find the linearized equation of motion corresponding to an approximation of the nonlinear equation of motion

$$\ell\ddot{q} + g \cos q = 0$$

about the equilibrium $q = \pi/2$ by operating directly on the differential equation.

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Solutions

1. Introduce an inertial coordinate system with origin at the stationary suspension of the pendulum, with vertical y -axis, and with motion of the pendulum constrained to the xy -plane. Let q denote the counterclockwise angle made by the pendulum with the x -axis. It follows that the particle can be described in terms of the position vector

$$\vec{r} = \ell \cos q \hat{i} + \ell \sin q \hat{j},$$

where \hat{i} and \hat{j} are unit vectors along the x and y axis, respectively. Every configuration of the pendulum is described by some value of the generalized coordinate q .

If q varies with time, then the velocity of the particle is given by

$$\vec{v} = -\ell\dot{q} \sin q \hat{i} + \ell\dot{q} \cos q \hat{j}$$

The particle has kinetic energy

$$T = \frac{m}{2} (\vec{v} \bullet \vec{v}) = \frac{m\ell^2\dot{q}^2}{2}$$

and potential energy (due to gravity)

$$V = mg(\vec{r} \bullet \hat{j}) = mg\ell \sin q$$

It follows that the Lagrangian equals

$$L = \frac{m\ell^2\dot{q}^2}{2} - mg\ell \sin q$$

In particular,

$$\frac{\partial L}{\partial q} = -mg\ell \cos q, \quad \frac{\partial L}{\partial \dot{q}} = m\ell^2\dot{q}, \quad \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) = m\ell^2\ddot{q}$$

Lagrange's equations then imply that

$$m\ell^2\ddot{q} + mg\ell \cos q = 0$$

i.e.,

$$\ell\ddot{q} + g \cos q = 0.$$

2. Introduce an inertial coordinate system with origin at the stationary end point of the pendulum, with vertical y -axis, and with motion of the pendulum constrained to the xy -plane. Let q denote the counterclockwise angle made by the pendulum with the x -axis. It follows that the point along the pendulum at distance ρ from the origin can be described in terms of the position vector

$$\vec{r} = \rho \cos q \hat{i} + \rho \sin q \hat{j},$$

where \hat{i} and \hat{j} are unit vectors along the x and y axis, respectively. Every configuration of the pendulum is described by some value of the generalized coordinate q .

If q varies with time, then the velocity of the point along the pendulum at distance ρ from the origin is given by

$$\vec{v} = -\rho\dot{q} \sin q \hat{i} + \rho\dot{q} \cos q \hat{j}$$

Each such point may be associated with an infinitesimal mass

element dm . If the density is uniform throughout the pendulum, then $dm = \frac{M}{\ell} d\rho$ in terms of the total mass M and the length ℓ . Such a mass element has infinitesimal kinetic energy

$$dT = \frac{1}{2}(\vec{v} \bullet \vec{v}) dm = \frac{M\rho^2 \dot{q}^2}{2\ell} d\rho$$

and infinitesimal potential energy

$$dV = dm g(\vec{r} \bullet \hat{j}) = \frac{Mg\rho}{\ell} \sin q d\rho$$

The total kinetic energy then equals

$$\int dT = \frac{M\dot{q}^2}{2\ell} \int_0^\ell \rho^2 d\rho = \frac{M\ell^2 \dot{q}^2}{6}.$$

Similarly, the total potential energy equals

$$\int dV = \frac{Mg}{\ell} \sin q \int_0^\ell \rho d\rho = \frac{M\ell g}{2} \sin q$$

It follows that the Lagrangian equals

$$L = \frac{M\ell^2 \dot{q}^2}{6} - \frac{M\ell g}{2} \sin q$$

In particular,

$$\frac{\partial L}{\partial q} = -\frac{M\ell g}{2} \cos q, \quad \frac{\partial L}{\partial \dot{q}} = \frac{M\ell^2 \dot{q}}{3}, \quad \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}} \right) = \frac{M\ell^2 \ddot{q}}{3}$$

Lagrange's equations then imply that

$$\frac{M\ell^2 \ddot{q}}{3} + \frac{M\ell g}{2} \cos q = 0$$

i.e.,

$$\frac{2\ell}{3} \ddot{q} + g \cos q = 0.$$

3. In this case, Lagrange's equations are given by

$$\ell \ddot{q} + g \cos q = 0$$

If $q = q_0$ is an equilibrium, then $q(t) = q_0$ must be a solution to

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Lagrange's equations. In other words,

$$\cos q_0 = 0 \Rightarrow q_0 = \pm \frac{\pi}{2}.$$

The local approximation L_{loc} to the Lagrangian L near the equilibrium $\pi/2$ is obtained by substituting $q = \pi/2 + \varepsilon(q - \pi/2)$ and $\dot{q} = \varepsilon\dot{q}$ into L , Taylor expanding about $\varepsilon = 0$ up to second order in ε , and letting $\varepsilon = 1$. In this case,

$$L_{\text{loc}} = -mg\ell + \frac{m\ell}{2} \left(g \left(q - \frac{\pi}{2} \right)^2 + \ell \dot{q}^2 \right)$$

The linearized equation of motion then becomes

$$m\ell^2\ddot{q} - m\ell g \left(q - \frac{\pi}{2} \right) = 0.$$

Similarly, the local approximation L_{loc} to the Lagrangian L near the equilibrium $-\pi/2$ is obtained by substituting $q = -\pi/2 + \varepsilon(q - \pi/2)$ and $\dot{q} = \varepsilon\dot{q}$ into L , Taylor expanding about $\varepsilon = 0$ up to second order in ε , and letting $\varepsilon = 1$. In this case,

$$L_{\text{loc}} = mg\ell + \frac{m\ell}{2} \left(-g \left(q + \frac{\pi}{2} \right)^2 + \ell \dot{q}^2 \right)$$

The linearized equation of motion then becomes

$$m\ell^2\ddot{q} + m\ell g \left(q + \frac{\pi}{2} \right) = 0.$$

4. Here, substitution of $q = \pi/2 + \varepsilon(q - \pi/2)$, $\dot{q} = \varepsilon\dot{q}$ in the differential equation yields

$$\varepsilon\ell\ddot{q} - g \sin \left(\varepsilon \left(q - \frac{\pi}{2} \right) \right) = 0$$

Taylor expansion about $\varepsilon = 0$ to first order in ε then yields

$$\varepsilon\ell\ddot{q} - g\varepsilon \left(q - \frac{\pi}{2} \right) = 0$$

and the linearized equation of motion

$$\ell\ddot{q} - g \left(q - \frac{\pi}{2} \right) = 0$$

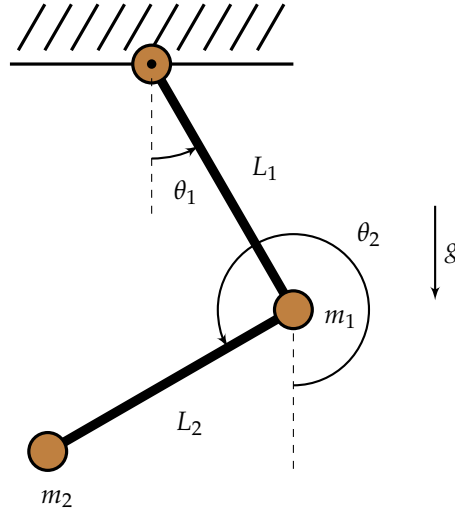
is obtained by letting $\varepsilon = 1$.

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Prelab Assignments

Complete these assignments before the lab. Show all work for credit.

1. Consider a double pendulum consisting of two particles swinging in a vertical plane under the influence of gravity, such that the first particle of mass m_1 is located at a distance L_1 from a stationary suspension, and the second particle of mass m_2 is located at a distance L_2 from the first particle, as shown in the figure.



- (a) Show that the kinetic energy of the mechanism equals

$$T = \frac{m_1 + m_2}{2} L_1^2 \dot{\theta}_1^2 + \frac{m_2}{2} L_2^2 \dot{\theta}_2^2 + m_2 L_1 L_2 \cos(\theta_1 - \theta_2) \dot{\theta}_1 \dot{\theta}_2$$

in terms of the angles θ_1 and θ_2 .

- (b) Show that the potential energy of the mechanism equals

$$V = -(m_1 + m_2) L_1 g \cos \theta_1 - m_2 L_2 g \cos \theta_2$$

- (c) Show that Lagrange's equations equal

$$\begin{aligned} (m_1 + m_2) L_1^2 \ddot{\theta}_1 + m_2 L_1 L_2 \cos(\theta_1 - \theta_2) \ddot{\theta}_2 \\ + m_2 L_1 L_2 \sin(\theta_1 - \theta_2) \dot{\theta}_2^2 + (m_1 + m_2) L_1 g \sin \theta_1 = 0 \end{aligned}$$

and

$$m_2 L_2^2 \ddot{\theta}_2 + m_2 L_1 L_2 \cos(\theta_1 - \theta_2) \ddot{\theta}_1 - m_2 L_1 L_2 \sin(\theta_1 - \theta_2) \dot{\theta}_1^2 + m_2 L_2 g \sin \theta_2 = 0$$

2. Determine the four equilibrium configurations of the equations of motion shown in Prob. 1 by letting $\dot{\theta}_1 = 0$, $\dot{\theta}_2 = 0$, $\ddot{\theta}_1 = 0$, and $\ddot{\theta}_2 = 0$ and solving for all possible values of θ_1 and θ_2 .
3. Show that the linearized equations of motion about the equilibrium configuration in which both particles are located below the stationary suspension are given by

$$(m_1 + m_2) L_1^2 \ddot{\theta}_1 + m_2 L_1 L_2 \ddot{\theta}_2 + (m_1 + m_2) g L_1 \theta_1 = 0$$

and

$$m_2 L_2^2 \ddot{\theta}_2 + m_2 L_1 L_2 \ddot{\theta}_1 + m_2 g L_2 \theta_2 = 0$$

Lab instructions³

³ These notes are an edited version of handouts authored by Andrew Alleyne.

Linear equations have a very specific form. This form makes linear systems relatively simple to analyze. However, many mechanical systems are inherently nonlinear. In order to make it possible to apply linear analysis methods to nonlinear systems, we must exercise an approximation process of linearizing nonlinear system equations.

This lab will demonstrate the significance of linearization techniques and investigate the accuracy and limit of such approximation techniques on a nonlinear mechanical system. The main purpose of this lab is to acquaint you with the main methods of analyzing nonlinear dynamical systems: nonlinear numerical simulation and analytical linear analysis via a linearized model.

The Pendubot

The Pendubot is a double pendulum found at the College of Engineering Control Systems Laboratory. It is used for educational purposes in several controls courses. The Pendubot consists of two rigid aluminum links. There are optical encoders at each of the joints of the Pendubot and the whole system is actuated at the first link's origin. The system is controlled via MATLAB commands. These will be introduced as needed.

A simplified mechanical mode of the Pendubot is a double pendulum consisting of two particles swinging in a vertical plane under the influence of gravity, as in Prob. 1 in the Prelab Assignments. A close match between experimental results and the predictions of such a model is achieved by assuming that $L_1 \approx 0.1508$ m, $L_2 \approx 0.1683$ m, $m_1 \approx 0.4373$ kg, and $m_2 \approx 0.1615$ kg.

The following nonlinear equations of motion were obtained in the Prelab Assignments.

$$\begin{aligned} p_1\ddot{\theta}_1 + p_3\cos(\theta_1 - \theta_2)\ddot{\theta}_2 + p_3\sin(\theta_1 - \theta_2)\dot{\theta}_2^2 + p_4g\sin\theta_1 &= 0 \\ p_2\ddot{\theta}_2 + p_3\cos(\theta_1 - \theta_2)\ddot{\theta}_1 - p_3\sin(\theta_1 - \theta_2)\dot{\theta}_1^2 + p_5g\sin\theta_2 &= 0 \end{aligned}$$

where $p_1 = (m_1 + m_2)L_1^2$, $p_2 = m_2L_2^2$, $p_3 = m_2L_1L_2$, $p_4 = (m_1 + m_2)L_1$, $p_5 = m_2L_2$. By solving for $\ddot{\theta}_1$ and $\ddot{\theta}_2$, we find

$$\begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} = \begin{pmatrix} M_{11}^{-1} & M_{12}^{-1} \\ M_{21}^{-1} & M_{22}^{-1} \end{pmatrix} \cdot \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$

where

$$M_{11}^{-1} = \frac{p_2}{p_1p_2 - p_3^2 \cos^2(\theta_2 - \theta_1)}, \quad M_{22}^{-1} = \frac{p_1}{p_1p_2 - p_3^2 \cos^2(\theta_2 - \theta_1)}$$

$$M_{12}^{-1} = M_{21}^{-1} = -\frac{p_3 \cos(\theta_2 - \theta_1)}{p_1p_2 - p_3^2 \cos^2(\theta_2 - \theta_1)}$$

$$q_1 = p_3\dot{\theta}_2^2 \sin(\theta_2 - \theta_1) - p_4g \sin \theta_1,$$

and

$$q_2 = -p_3\dot{\theta}_1^2 \sin(\theta_2 - \theta_1) - p_5g \sin \theta_2.$$

The linearized equations of motion

$$p_1\ddot{\theta}_1 + p_3\ddot{\theta}_2 + p_4g\theta_1 = 0$$

$$p_2\ddot{\theta}_2 + p_3\ddot{\theta}_1 + p_5g\theta_2 = 0$$

may also be written in the form

$$\begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} = \begin{pmatrix} M_{11}^{-1} & M_{12}^{-1} \\ M_{21}^{-1} & M_{22}^{-1} \end{pmatrix} \cdot \begin{pmatrix} q_1 \\ q_2 \end{pmatrix}$$

where

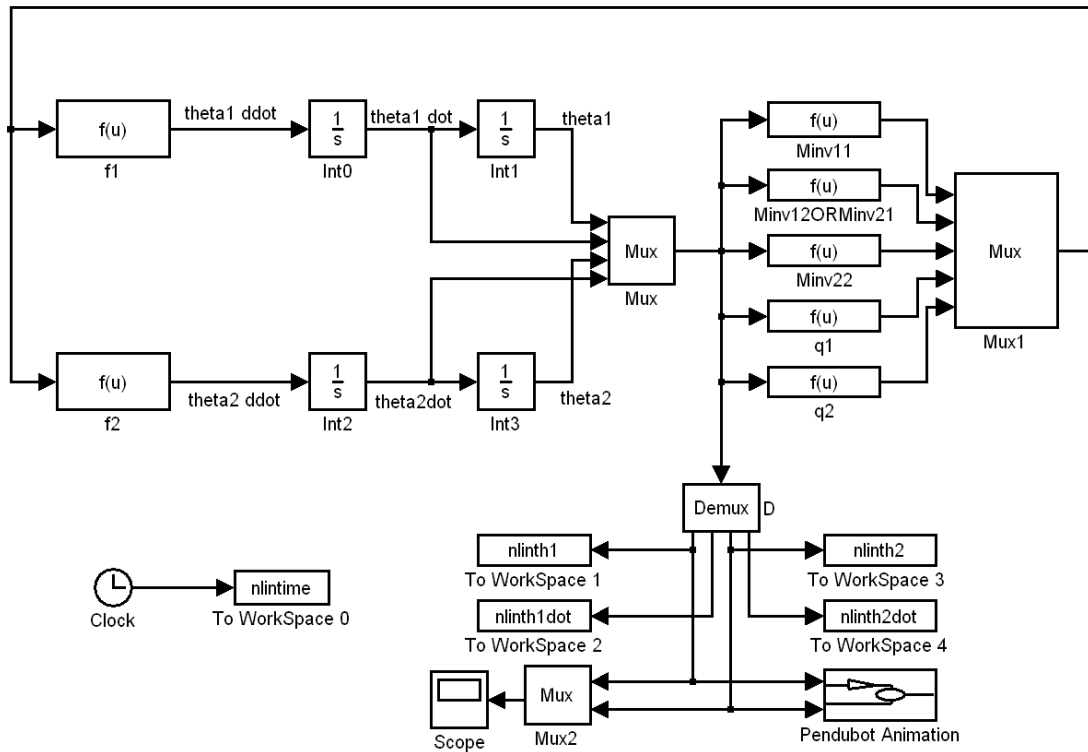
$$M_{11}^{-1} = \frac{p_2}{p_1p_2 - p_3^2}, \quad M_{22}^{-1} = \frac{p_1}{p_1p_2 - p_3^2}, \quad M_{12}^{-1} = M_{21}^{-1} = -\frac{p_3}{p_1p_2 - p_3^2}$$

and

$$q_1 = -p_4g\theta_1, \quad q_2 = -p_5g\theta_2.$$

Experiment I: Simulation

The equations of motion may be represented by the SIMULINK diagram shown below.



- The wires labeled $\theta_1 \ddot{\theta}$ and $\theta_2 \ddot{\theta}$ carry the signals $\ddot{\theta}_1(t)$ and $\ddot{\theta}_2(t)$.
- The integrators labeled Int0 , Int1 , Int2 , and Int3 output $\dot{\theta}_1(t)$, $\dot{\theta}_2(t)$, $\theta_1(t)$, and $\theta_2(t)$.
- The blocks labeled Mux and Mux1 output vectors with components given by the input signals.
- The five function blocks labeled Minv11 , Minv12ORMinv21 , Minv22 , q_1 , and q_2 output the signals M_{11}^{-1} , $M_{12}^{-1} = M_{21}^{-1}$, M_{22}^{-1} , q_1 , and q_2 .
- The two function blocks labeled f_1 and f_2 perform the matrix multiplications required to compute $\ddot{\theta}_1(t)$ and $\ddot{\theta}_2(t)$.
- The block labeled ToWorkspace0 outputs the time array to the MATLAB workspace for plotting.
- The blocks labeled ToWorkspace1 , ToWorkspace2 , ToWorkspace3 , and ToWorkspace4 output $\theta_1(t)$, $\dot{\theta}_1(t)$, $\theta_2(t)$, $\dot{\theta}_2(t)$ to the MATLAB

workspace for plotting.

- The block labeled Demux outputs the individual components from a vector input signal.
- The block labeled PendubotAnimation contains an animation function that shows the motion of the pendulum during simulation.

Experimental procedure

1. Start MATLAB using the icon found on your desktop.
2. Inside MATLAB, open the “read-only” file pendubot.mdl located in the directory N:\labs\me340\pendubot.
3. Save the file with the name nonlinear<yourNetID>.mdl in the directory C:\matlab\me340\.
4. Change MATLAB’s current directory to the location where your model file was saved by typing `cd c:\matlab\me340` at the MATLAB command prompt.
5. Click the SIMULINK model file to give it focus.
6. Change the initial value of the top-right integrator to 0.1 and verify that the other integrators have initial values equal to 0.
7. Use the given numerical values for L_1 , L_2 , m_1 , and m_2 to compute p_1 through p_5 , and enter these values at the MATLAB prompt, e.g.,

`>> p1=0.0136;`

Do the same for the g variable.

8. Run the model.
9. Next, save the file with the name linear<yourNetID>.mdl to the same directory.
10. Change the content of the function blocks to correspond to the linearized equations of motion. Rename the output variables to `linth1`, `linth2`, `linth1dot`, and `linth2dot` by double-clicking on the

corresponding components. Similarly, change the name of the time variable to `lintime`. Save the model.

11. Run the model.

12. On the MATLAB command prompt, type

```
>> plot(lintime, linth1, '-', nlintime, nlinth1, '--')
>> legend('Theta 1, Lin. Sys.', 'Theta 1, Nonlin. Sys.', 0)
```

Use this plot to verify that your linearization is correct, i.e., that the plots of the linear and nonlinear system response are very similar. If the responses are obviously different, go back and verify the content of the function blocks in the linearized model.

Experiment II: Validation

The Pendubot is an expensive piece of precision equipment. Please do not throw the arm wildly during experimentation.

In this section, the actual dynamics of the Pendubot will be recorded for two different initial conditions. The initial conditions that you give the Pendubot will be determined exactly, and they will be used as the initial conditions for the non-linear and linear SIMULINK models. After running the SIMULINK models, you will evaluate the ability of the mathematical model to reproduce the actual motion of the Pendubot.

Experimental procedure

1. Have your TA verify that the Pendubot is ready to record data.
2. The first set of data to be collected is for small initial displacements of $\theta_1 \approx 20^\circ$ and with $\theta_2 \approx 0^\circ$ (the second link will hang vertically at the start of data collection).

3. Start the data collection routine by entering

```
>> [acttime, actth1, a, b, c, d] = ...
    pend_collect(zeros(1000, 1));
```

on the MATLAB command line. Follow the on-screen instructions.

DO NOT hold the Pendubot by the encoder when displacing θ_1 !

4. Verify the measurement by entering

```
>> plot(acttime, actth1);
```

on the MATLAB command prompt. Here, the MATLAB variable actth1 contains the measured values of the angle θ_1 . You do not need to print this plot.

5. Zoom in on the plot to determine the exact initial conditions (positions) for your experimental data. Note that initial velocities are zero. Also record the time at which the Pendubot was dropped and assign its value to the MATLAB variable timeshift.
6. Run the linear and non-linear SIMULINK simulations using the initial condition for θ_1 that you just determined.
7. Enter the following set of commands on the MATLAB prompt in order to generate plots of the experimental and simulated data. Use the xlabel, ylabel, and title commands to title and label your each of the plots appropriately. Print this figure using the landscape orientation.

```
>> figure
>> subplot(2,2,1)
>> plot(acttime, acth1)
>> subplot(2,2,2)
>> plot(acttime, actth1, '-', nlintime+timeshift, nlinth1, '--')
>> legend('Actual \theta_1', 'Nonlinear Sim. \theta_1', 0)
>> subplot(2,2,3)
>> plot(nlintime, nlinth1, '-', lintime, linth1, '--')
>> legend('Nonlinear Simulation', 'Linear Simulation', 0)
>> subplot(2,2,4)
>> plot(acttime, actth1, '-', lintime+timeshift, linth1, '--')
>> legend('Actual \theta_1', 'Linear Sim. \theta_1', 0)
```


8. Repeat the steps above for a larger initial condition with $\theta_1 \approx 60^\circ - 70^\circ$ and with $\theta_2 \approx 0^\circ$.

Report Assignments

Complete these assignments during the lab. *Show all work for credit.*

1. Attach the print outs of experimental and simulated data generated in Experiment II.
2. Comment on the validity of the linear approximation as a function of the magnitude of the initial displacement θ_1 . Justify your answer by referring to the plots of experimental and simulated data.
3. Comment on the validity of the mathematical model as an approximation of the experimental system. Suggest two physical effects that are not included in the mathematical model that could be included to improve its applicability.
4. What could be improved in the ME340 labs? Do not remark on your TA. Try to focus your response on the lab manuals, pre-lab assignments, report assignments, experiments, and equipment.
5. What should not be changed in ME340 labs?
6. Identify the two labs that were the most difficult to understand.
 - Lab 1: Mathematical preliminaries, MATLAB
 - Lab 2: First-order systems, leaking tank, hydraulic motor
 - Lab 3: Block diagrams and simulation, SIMULINK
 - Lab 4: Second-order systems, single-degree-of-freedom mass-spring-damper mechanisms
 - Lab 5: Mode shapes and resonance, two-degree-of-freedom mass-spring-damper mechanisms
 - Lab 6: Continuous systems, vibrating beam
 - Lab 7: Nonlinear systems, Pendubot, Lagrange equations

7. Why were these two labs difficult to understand? (include all that apply)

- The lab manual's explanation was poor/confusing
 - The lab manual was missing important information.
 - The pre-lab assignments did not prepare me for the lab.
 - The lab procedure was too complicated.
 - I was unable to see the "big picture" after lab completion.
 - The equipment did not behave correctly.
 - The lectures did not prepare me adequately for the lab.
 - The mathematics was too complicated.
 - Other:
-