Power of a Statistical Test

(8.5)

Type I Error: Review

Type I Error: Reject H_0 when H_0 is true

Hypothesis testing:

Set maximum acceptable rate of Type I error:

α <- significance level

Choose a test with the most power to detect H_A.

Power

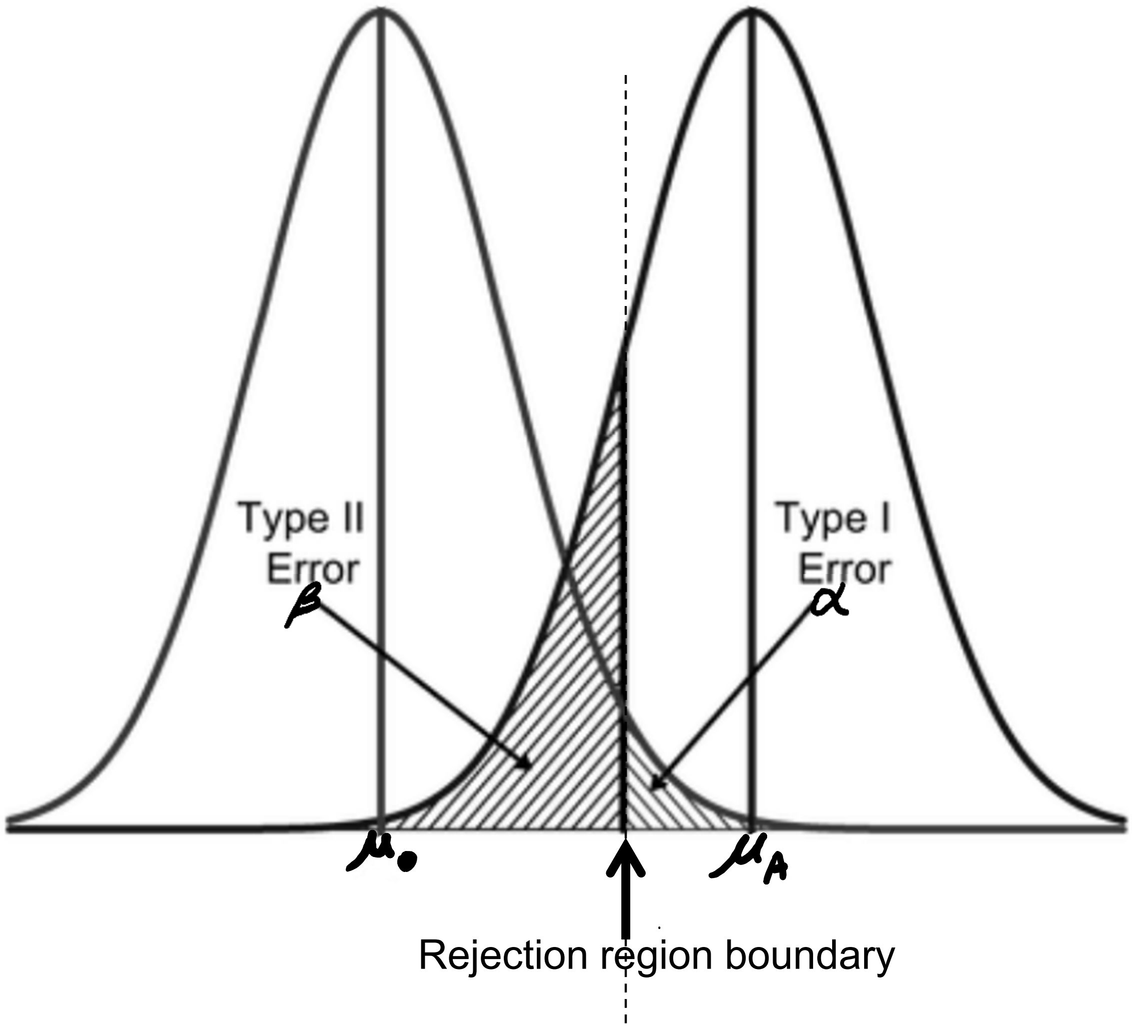
The power of a statistical test is the probability that the test correctly rejects H_0 (*when H_A is true*).

The **power** of a statistical test is related to Type II error.

- β is the probability of Type II error

$$\begin{aligned} \text{Power} &= 1 - P[\text{Type II Error}] \\ &= 1 - \beta \end{aligned}$$

Type I and Type II Errors

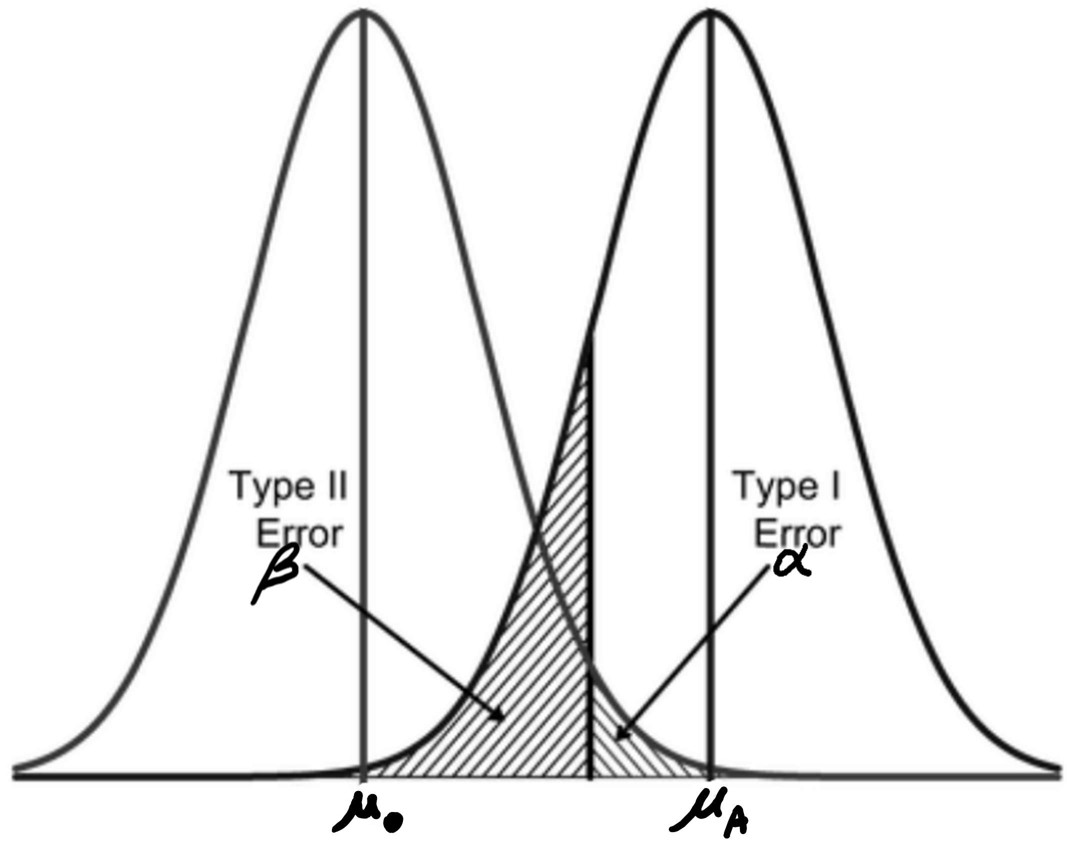


Type I and Type II Errors

Example:
H_0 : $\mu = 100$
H_A : $\mu > 100$

$\sigma = 10$, n = 100

Define a rejection region at $\alpha = 0.01$.



Power Function

The power function shows the probability of rejecting H_0 at different values of θ.

Note: The power function is sometimes denoted differently by different people. E.g.: $\beta(\theta)$, $B(\theta)$, or $\boldsymbol{K}(\boldsymbol{\theta})$.

- The β in this power function above is **not** the same as the β for type II error.
    - β is the name (letter) of the power function.
    - Need to look at context
- Textbook sometimes calls $\beta(\theta)$ as "K(μ)" when dealing with the mean.

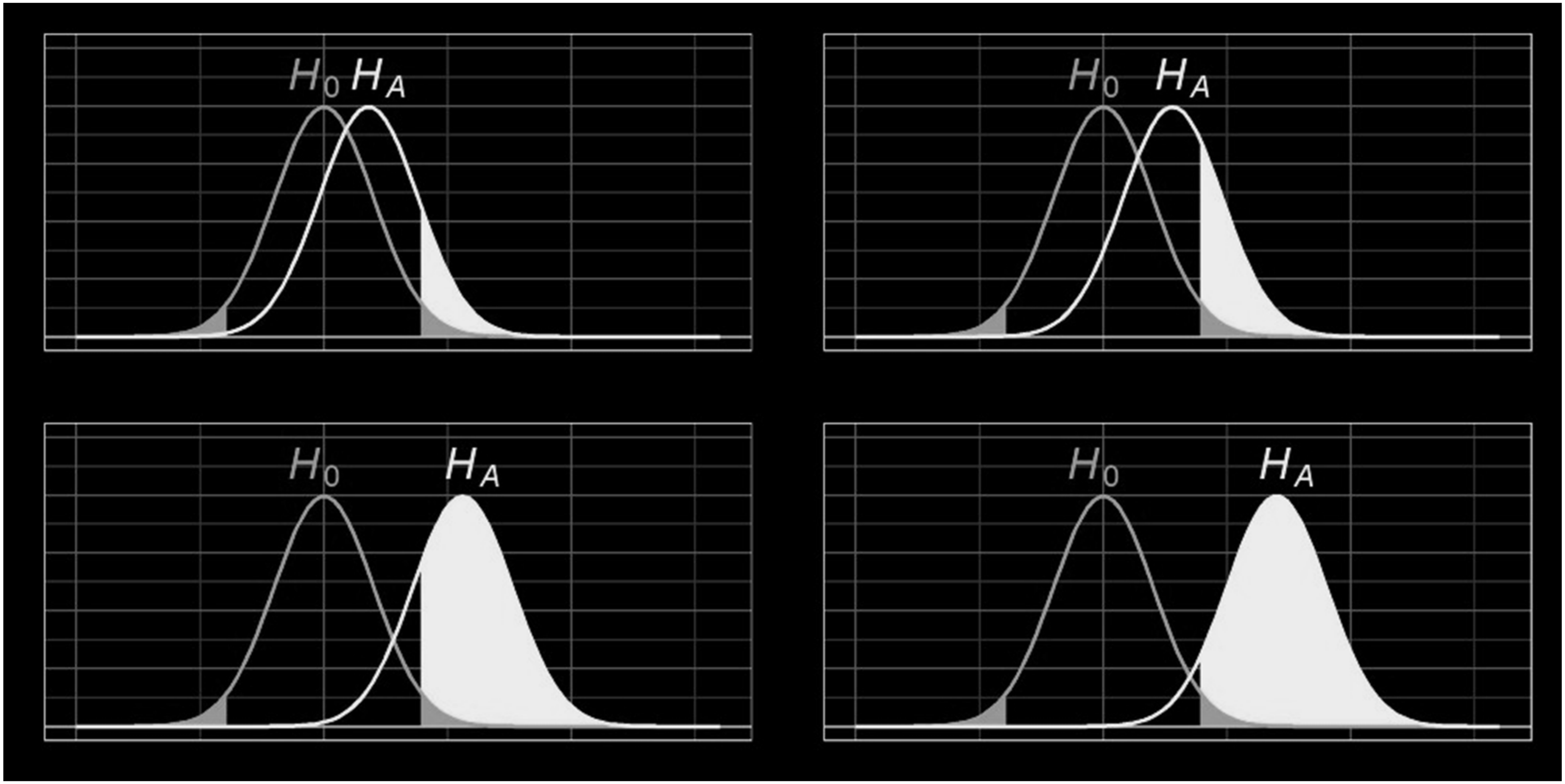


Image Source: https://towardsdatascience.com/power-analysis-made-easy-dfee1eb813a

Example 8.5-2

Let $X_1, X_2, \ldots, X_n$ be a random sample $\sim N(\mu, 100)$. Let $n = 25$.

Suppose we want to test whether the true mean is 60 (H_0) versus if it is greater than 60 (H_A).

H_0: $\mu = 60$

H_A: $\mu > 60$

Test statistic:

Suppose we choose a test that reject H_0 if and only if $\bar{x} \geq 62$.

What are the consequences of this test *(what does the power curve look like)?*

$\beta(\theta)$

here, $K(\mu)$

Power Function

If the true mean under H_A is μ, and X $\sim$ N(μ,100),
then $\bar{X} \sim$ N(μ, 100/n) = N(μ, 4)

power function

The probability of rejecting H_0 is given by

$K(\mu)$ = P$[\bar{X} \geq 62\ ;\ \mu]$

$$= P\left[\frac{\bar{X}-\mu}{2} \geq \frac{62-\mu}{2}\ ;\ \mu\right] \qquad = P\left[Z \geq \frac{62-\mu}{2}\ ;\ \mu\right]$$

Table 8.5-1 Values of the power function	
μ	$K(\mu)$
60	0.1587
61	0.3085
62	0.5000
63	0.6915
64	0.8413
65	0.9332
66	0.9772

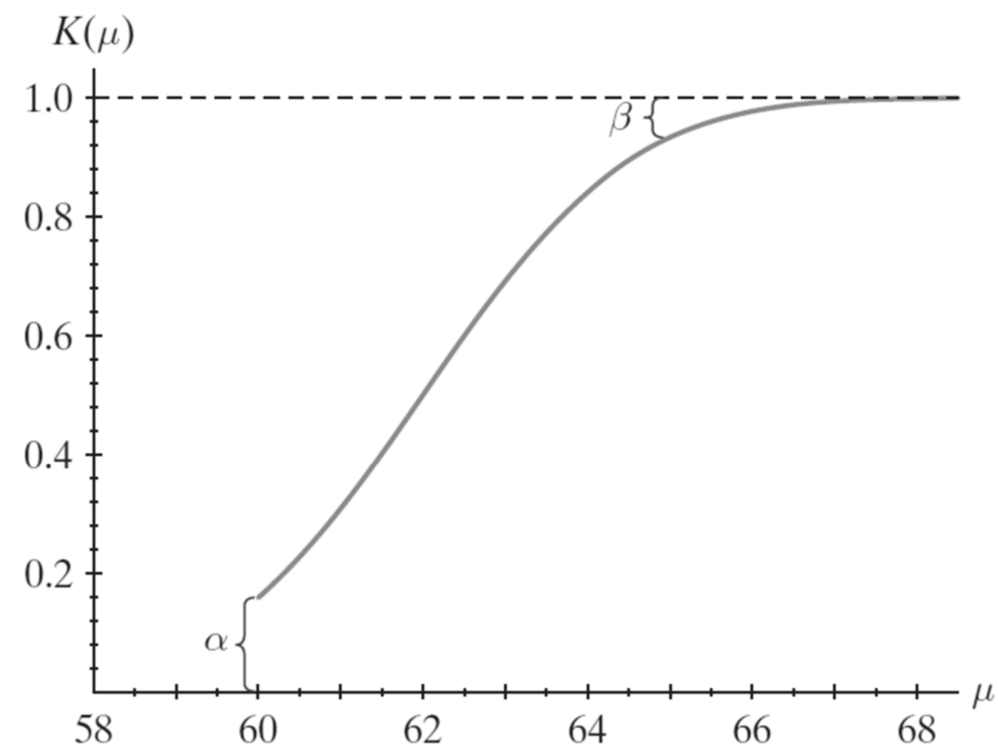


Figure 8.5-2 Power function
$K(\mu) = 1 - \Phi([62 - \mu]/2)$

Ideal power function?

- In the previous example, what would an ideal power function look like?

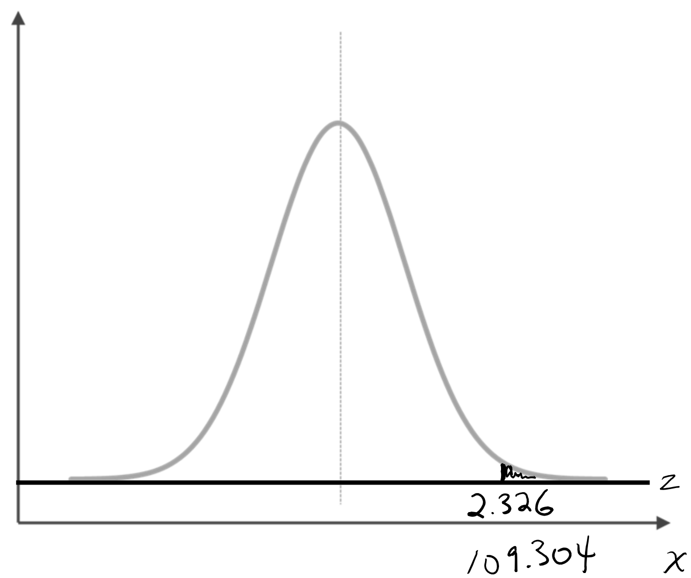
Example 2

Assume that the number of grams of caffeine that Albert ingests every day follows an approximately normal distribution with unknown mean and standard deviation 16.

Let $n = 16$, $\alpha = 0.01$

Test H_0: $\mu = 100$ vs H_A: $\mu > 100$.

Define a rejection region for H_0.



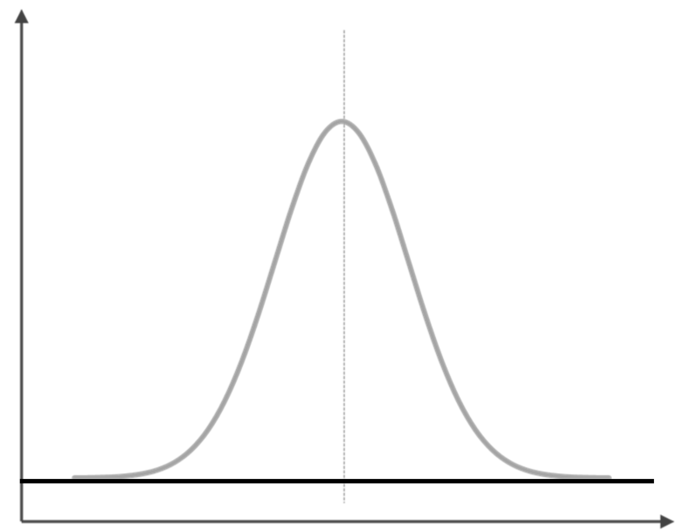
Example 2

What is the power at μ = 108?

Power = P[$\bar{X} \geq 109.304$ | μ = 108]

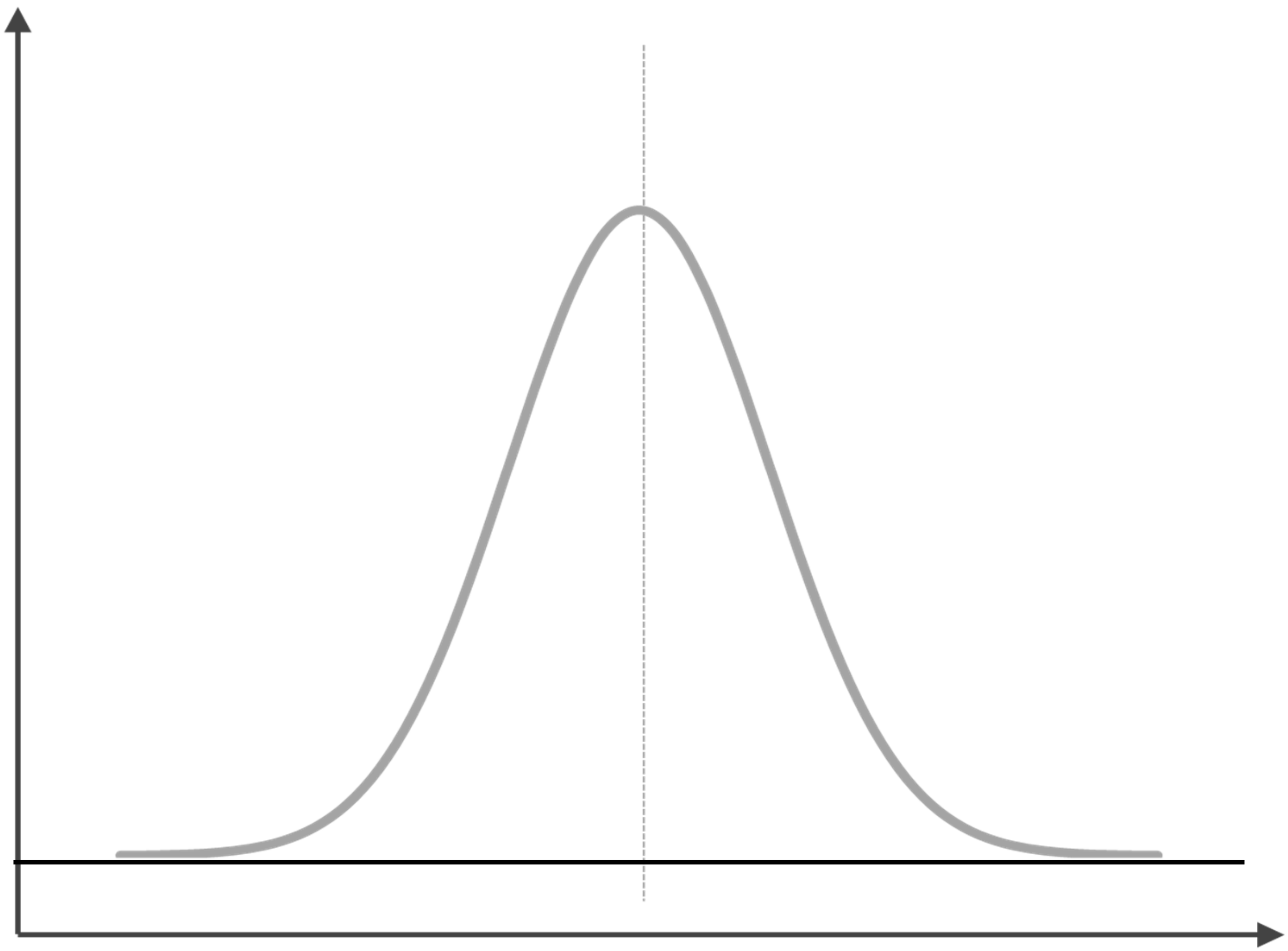
= P[Z $\geq \frac{109.304 - 108}{16/\sqrt{16}}$] = P[Z $\geq$ 0.326]

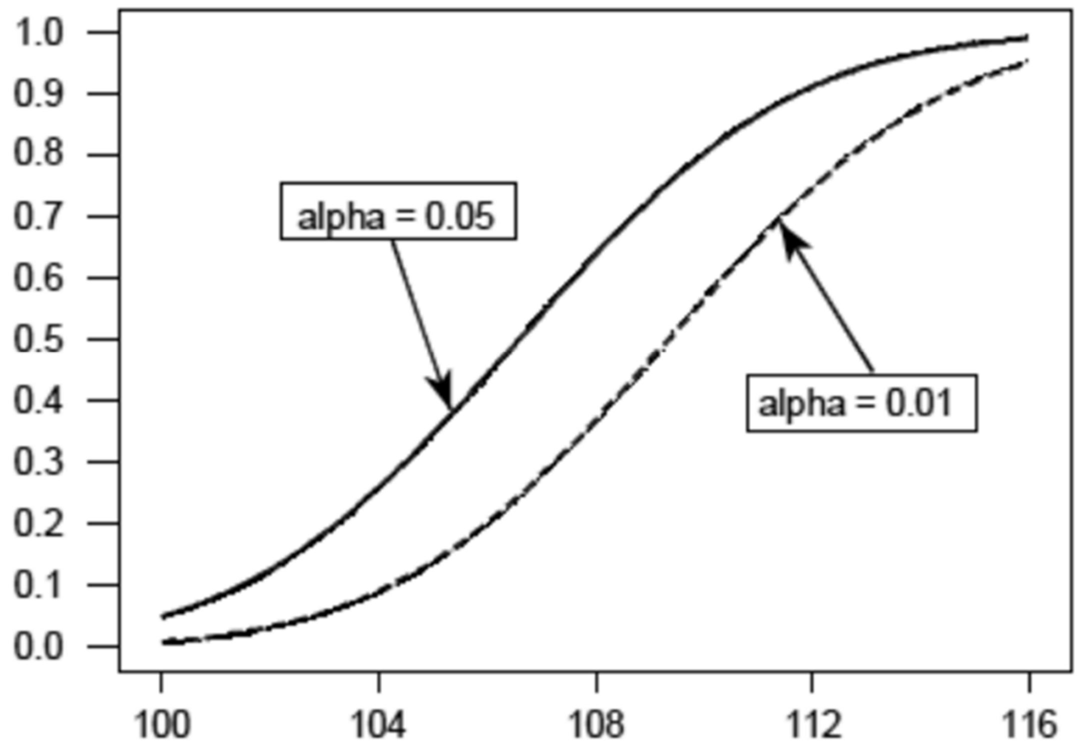
= 0.3722



What if we used α = 0.05?

Cutoff = 106.58, Power = P[$\bar{X} \geq 106.58] = 0.6404$



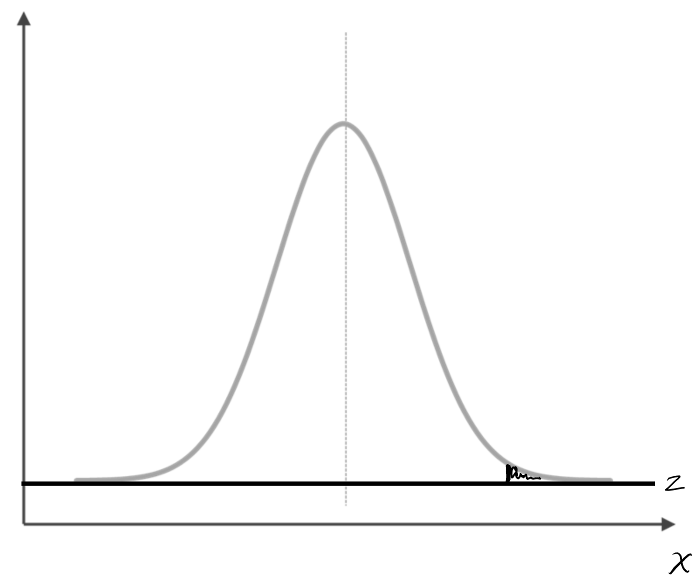

1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
alpha = 0.05
alpha = 0.01
100
104
108
112
116

Example 3

Suppose we have a normally distributed random sample with n = 16, s = 8.

We wish to test H_0: μ = 50 vs H_A: $\mu \neq 50$.

What is the power of a level α = 0.05 test?



Example 3

Suppose we have a normally distributed random sample with n = 36.

We wish to test H_0: μ = 50 vs H_A: $\mu \neq 50$.

What is the power of a level α = 0.05 test?