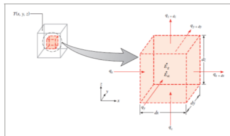
三种基本传热方式

$$q = q''A = \frac{\Delta T}{R} = \frac{\Delta T}{\frac{R''}{A}}$$

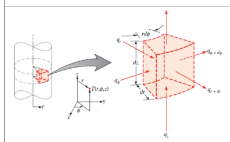
	热通量	热阻
Conduction	$q''_{cond} = -k\frac{dT}{dx}$	$R_{t,cond} = \frac{L}{kA}$
Convection	$q''_{conv} = h(T_s - T_\infty)$	$R_{t,conv} = \frac{1}{hA}$
Radiation	$q''_{rad} = \varepsilon\sigma(T_s^4 - T_{sur}^4)$ $q_{rad} = h_r A(T_s - T_{sur})$	$R_{t,rad} = \frac{1}{h_r A}$ (近似)

功率	P	W
导热速率 Heat rate	q	W
	$\dot{q}$	W/m^3
热通量 / 热流密度 Heat flux	q''	W/m^2
导热系数 Thermal conductivity	k	W/mK
热对流系数 convection heat transfer coefficient	h	W/m^2K
热辐射系数 radiation heat transfer coefficient	h_r	W/m^2K
热阻	R_t	K/W
	R'_t	mK/W
单位面积热阻	R''_t	m^2K/W

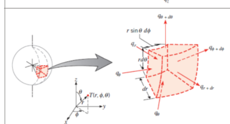
Heat Diffusion Equation (能量守恒)



$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\cancel{\frac{\partial T}{\partial y}}\right) + \frac{\partial}{\partial z}\left(k\cancel{\frac{\partial T}{\partial z}}\right) + \dot{q} = \rho c_p \cancel{\frac{\partial T}{\partial t}}$$



$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$



$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right) + \dot{q} = \rho c_p\frac{\partial T}{\partial t}$$

复合层导热 (空心+不产热)

	Plane Wall	Cylindrical Wall[a]	Spherical Wall[a]
Heat equation	$\frac{d^2T}{dx^2} = 0$	$\frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$	$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$
温度分布	$T_{s,1} - \Delta T\frac{x}{L}$	$T_{s,2} + \Delta T\frac{\ln(r/r_2)}{\ln(r_1/r_2)}$	$T_{s,1} - \Delta T\left[\frac{1-(r_1/r)}{1-(r_1/r_2)}\right]$
热通量 Heat flux (q'')	$k\frac{\Delta T}{L}$	$\frac{k\,\Delta T}{r\ln(r_2/r_1)}$	$\frac{k\,\Delta T}{r^2[(1/r_1)-(1/r_2)]}$
热流量 Heat rate (q)	$kA\frac{\Delta T}{L}$	$\frac{2\pi Lk\,\Delta T}{\ln(r_2/r_1)}$	$\frac{4\pi k\,\Delta T}{(1/r_1)-(1/r_2)}$
热阻 ($R_{t,cond}$)	$\frac{L}{kA}$	$\frac{\ln(r_2/r_1)}{2\pi Lk}$	$\frac{(1/r_1)-(1/r_2)}{4\pi k}$

瞬态传热

$$\frac{\theta}{\theta_i} = \frac{T - T_\infty}{T_i - T_\infty} = \exp\left[-\frac{h\cdot As}{\rho\cdot V\cdot c}\cdot t\right]$$

$$\tau = \frac{\rho\cdot V\cdot c}{h\cdot A_s} = \left(\frac{1}{hA_s}\right)\cdot(\rho Vc) = R_t\cdot C_t$$

Biot Number

$$Bi = \frac{h\cdot L_c}{k} = \frac{\frac{L_c}{kA_s}}{\frac{1}{hA_s}} \approx \frac{R_{cond}}{R_{conv}} \approx \frac{\Delta T_{solid}}{\Delta T_{solid-fluid}}$$

h = 对流 或 辐射传热系数

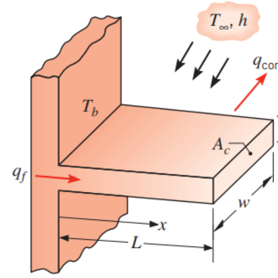
k = 固体传热系数

L_c = 特征长度

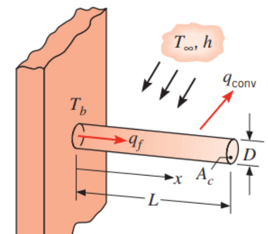
$Bi < 0.1$ 是好的

代表固体传热远快于对流或辐射

$L_c \rightarrow$ characteristic length of the solid
(V/A_s or distance associated with maximum spatial temperature difference)
Normally: $L/2$ for wall, r_o for sphere (sometimes $r_o/3$) and cylinder (sometimes $r_o/2$). But when is this "sometimes"?

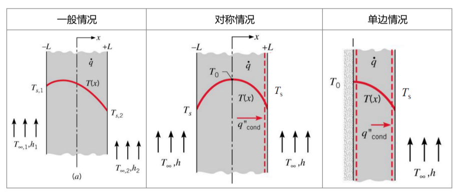


$P = 2w + 2t$
$A_c = wt$



$P = \pi D$
$A_c = \pi D^2/4$

墙产热模型 (均匀产热)



General Solution:

$$\boxed{T(x) = -(\dot{q}/2k)x^2 + C_1x + C_2} \quad (3.45)$$

边界条件

$x = -L, \quad T = T_{s,1}$
$x = +L, \quad T = T_{s,2}$

$$T(x) = \left(\frac{\dot{q}}{2k}L^2\right)\left(1 - \frac{x^2}{L^2}\right) + \left(\frac{T_{s,2} - T_{s,1}}{2}\right)\left(\frac{x}{L}\right) + \left(\frac{T_{s,1} + T_{s,2}}{2}\right)$$

鳍片均匀截面 散热公式

Case	Tip Condition ($x = L$)	Temperature Distribution θ/θ_b	Fin Heat Transfer Rate q_f
A	Convection heat transfer: $h\theta(L) = -kd\theta/dx\vert_{x=L}$	$\frac{\cosh m(L-x) + (h/mk)\sinh m(L-x)}{\cosh mL + (h/mk)\sinh mL}$	$M\frac{\sinh mL + (h/mk)\cosh mL}{\cosh mL + (h/mk)\sinh mL}$
B	Adiabatic: $d\theta/dx\vert_{x=L} = 0$	$\frac{\cosh m(L-x)}{\cosh mL}$	$M\tanh mL$
C	Prescribed temperature: $\theta(L) = \theta_L$	$\frac{(\theta_L/\theta_b)\sinh mx + \sinh m(L-x)}{\sinh mL}$	$M\frac{(\cosh mL - \theta_L/\theta_b)}{\sinh mL}$
D	Infinite fin ($L \rightarrow \infty$): $\theta(L) = 0$	e^{-mx}	M

$\theta \equiv T - T_\infty$ $\quad m^2 \equiv hP/kA_c$
$\theta_b = \theta(0) = T_b - T_\infty$ $\quad M \equiv \sqrt{hPkA_c}\theta_b$

$\sinh x = \frac{e^x - e^{-x}}{2}$ $\quad \cosh x = \frac{e^x + e^{-x}}{2}$ $\quad \tanh x = \frac{\sinh x}{\cosh x} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

鳍片效率

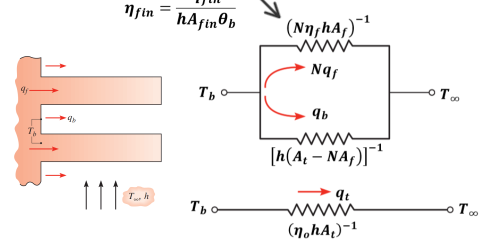
L↑ A↓ k↓ 会使得散热鳍片热阻增加

$$R_{t,f} = \frac{\theta_b}{q_f} = \frac{1}{hA_f\eta_f}$$

Fin Effectiveness 有无散热片差别	$\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = \frac{R_{t,b}}{R_{t,fin}}$ $\varepsilon_f = \frac{q_f}{hA_{c,b}\theta_b} = \frac{\sqrt{hPkA_{c,b}}\theta_b}{hA_{c,b}\theta_b} = \sqrt{\frac{kP}{hA_{c,b}}}$ 对于 infinite fin
Fin Efficiency 实际与理想散热片差别	$\eta_f = \frac{q_f}{q_{f,max}} = \frac{q_f}{hA_f\theta_b}$

鳍片阵列

$$R_{fin} = \frac{\theta_b}{q_{fin}}$$
$$\eta_{fin} = \frac{q_{fin}}{hA_{fin}\theta_b}$$



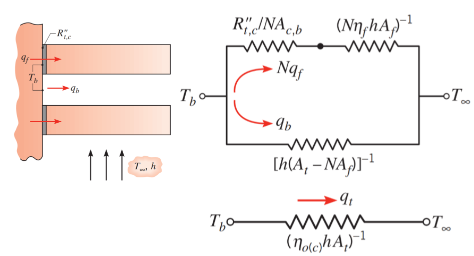
A_f = 单个鳍片表面积

A_b = 没被鳍片覆盖的面积

A_t = 总散热面积

$$A_t = NA_f + A_b$$

$$\begin{aligned} q_t &= Nq_f + q_b \\ &= \boldsymbol{N(\eta_f hA_f\theta_b) + hA_b\theta_b} \\ &= h(N\eta_f A_f + A_t - NA_f)\theta_b \\ &= hA_t\left(\eta_f\frac{NA_f}{A_t} + 1 - \frac{NA_f}{A_t}\right)\theta_b \\ &= \boldsymbol{\eta_{overall}hA_t\theta_b} \\ &= \boldsymbol{\frac{\theta_b}{R_{overall}}} \end{aligned}$$



$$\eta_{overall} = 1 - \frac{NA_f}{A_t}(1 - \eta_{fin})$$

$$R_{overall} = \frac{\theta_b}{q_{total}} = \frac{1}{\eta_{overall}hA_{total}}$$