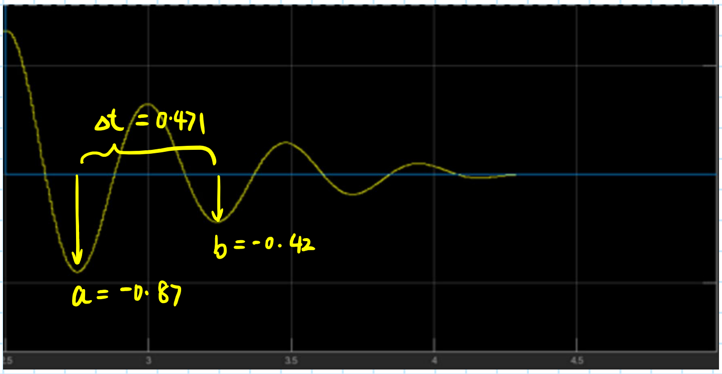
# Lab 4 Post-Lab

1. In the case of the free response, label each plot with the corresponding case and order the plots accordingly.

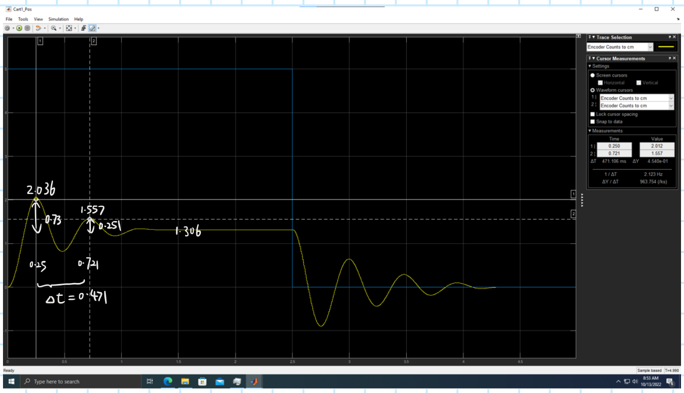



2. Copy the table below into your report and enter the appropriate data. Show your work used to estimate $\omega_n$ and $\zeta$ on the plot for the corresponding case, in the case that the system is underdamped. If the system is overdamped, identify this on the plot and in the table.

| Experiment Data Reduction Table | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Approx. System Parameters | | | Theoretical Values | | Experimental Values | |
| | $c$ | $m$ | $k$ | $\zeta$ | $\omega_n$ | $\zeta$ | $\omega_n$ |
| Case # | Ns/m | kg | N/m | – | rad/s | – | rad/s |
| 1 | 8 | 0.55 | 780 | | | | |
| 2 | 8 | 2.55 | 780 | 0.08969 | 17.4895 | 0.0747 | 10.19 |
| 3 | 75 | 2.55 | 780 | | | | |

$$\begin{cases} \dfrac{a}{b} = e^{\frac{2\pi\xi}{\sqrt{1-\xi^2}}} = \dfrac{0.87}{0.42} \\ \Delta t = \dfrac{2\pi}{\omega_n\sqrt{1-\xi^2}} = 0.471 \end{cases} \Rightarrow \begin{cases} \xi = 0.0747522 \\ \omega_n = 10.1942 \end{cases}$$

3. In the case of the step response, label each plot accordingly.




4. Determine the maximum relative overshoot, the rise time, the settling time, and the peak time. Use these measurements to estimate $\omega_n$ and $\zeta$. Identify the equations used in your calculation on the corresponding plot, and make sure to check the units.

maximum value $h_{tp} = 2.036$

rise time $t_r = 0.141$

settling time $t_s = 1.269$

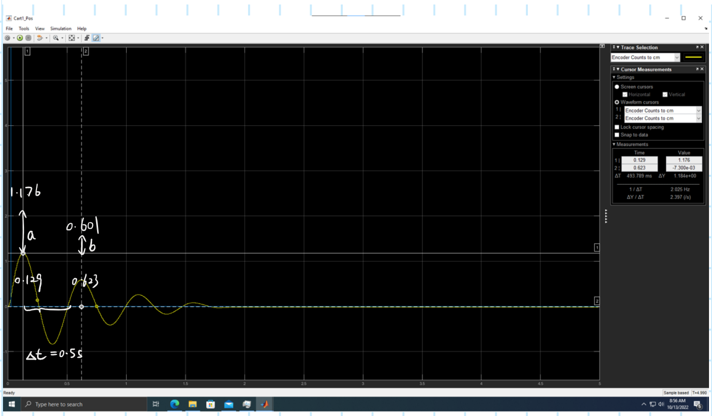
peak time $t_p = 0.25$

$$\begin{cases} \dfrac{x(t_{p_1}) - x_{ss}}{x(t_{p_2}) - x_{ss}} = \dfrac{a}{b} = e^{\frac{2\pi\xi}{\sqrt{1-\xi^2}}} \\ t_{p_2} - t_{p_1} = \Delta t = \dfrac{2\pi}{\omega_n\sqrt{1-\xi^2}} \end{cases} \Longrightarrow \begin{cases} \xi = 0.1675 \\ \omega_n = 13.53 \end{cases}$$

5. Estimate the stiffness $k$ from the steady-state displacement given the pulse amplitude 5 N. Proceed to estimate $m$ and $c$ from the step response. Show your work on the corresponding plot.

$$\begin{cases} \xi = \dfrac{c}{2\sqrt{km}} = 0.1675 \\ \omega_n = \sqrt{\dfrac{k}{m}} = 13.53 \\ k = \dfrac{5}{f(\infty)} = \dfrac{5}{0.01036} \end{cases} \Rightarrow \begin{cases} c=9.47924 \text{ Ns/m} \\ k=382.848 \text{ N/m} \\ m=2.09137 \text{ kg} \end{cases}$$

6. In the case of the impulse response, label the plot accordingly.




7. Use the value of $k$ obtained from the steady-state response to a step input, and estimate $m$ and $c$ from the impulse response. Show your work on the corresponding plot.

$$\begin{cases} \dfrac{a}{b} = e^{\frac{2\pi\xi}{\sqrt{1-\xi^2}}} = 1.9567 \\ \Delta t = \dfrac{2\pi}{\omega_n\sqrt{1-\xi^2}} = \dfrac{1}{2} \end{cases} \Rightarrow \begin{cases} \xi = 0.10623 \\ \omega_n = 12.6379 \end{cases}$$

$$\left.\begin{array}{l} \xi = \dfrac{c}{2\sqrt{km}} \\ \omega_n = \sqrt{\dfrac{k}{m}} \\ k = 382.848 \end{array}\right\} \Rightarrow \begin{cases} c = 6.436 \text{ Ns/m} \\ m = 2.397 \text{ kg} \end{cases}$$