Homework 5

Question 1

Consider the above closed loop system in Figure 1.

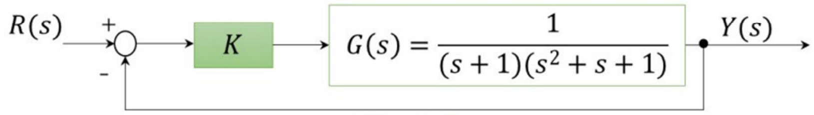


Figure 1

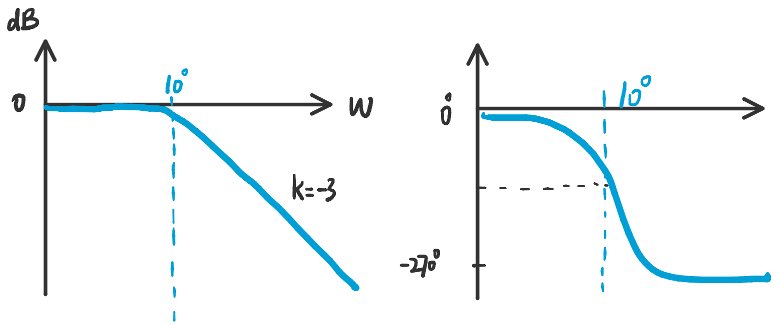
a) Sketch the Bode plot of the transfer function $G(s)$. *(4 Points)*

b) Sketch the Nyquist plot based on (a). *(4 Points)*

c) Using Nyquist stability criterion to determine all values of the feedback gain K that stabilizes the closed-loop system. *(6 Points)*

(a). $G(s) = \frac{1}{s+1} \cdot \frac{1}{s^2+s+1}$

$\downarrow$ $w=1$ $\quad$ $\downarrow$ $w=1$

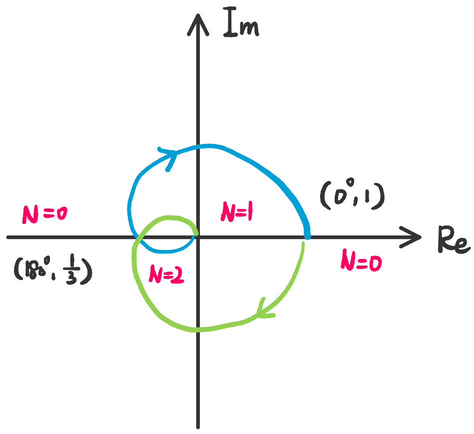


(b). $G(jw) = \dfrac{1}{(1-2w^2) + j(2w-w^3)}$

$\phi(w) = \tan^{-1}\left(\dfrac{2w-w^3}{1-2w^2}\right)$

$\Rightarrow \begin{cases} \text{when } \phi(w) = 180^\circ, & w = \sqrt{2} \\ \text{when } \phi(w) = 0^\circ, & w = 0 \end{cases}$

$\Rightarrow \begin{cases} M(w=\sqrt{2}) = \frac{1}{3} \\ M(w=0) = 1 \end{cases}$



(c). $G(s) = \frac{1}{s+1} \cdot \frac{1}{s^2+s+1} \Rightarrow$ No RHP poles $\Rightarrow P = 0$

Based on the equation

$Z = N - P$

N has to be zero to achieve stability

$\therefore -\frac{1}{k} < -\frac{1}{3}$ or $-\frac{1}{k} > 1$

$\therefore k \in (-1, 0) \cup (0, 3)$

Question 2

Consider the closed loop system in Figure 2.

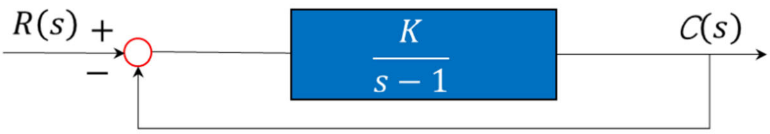
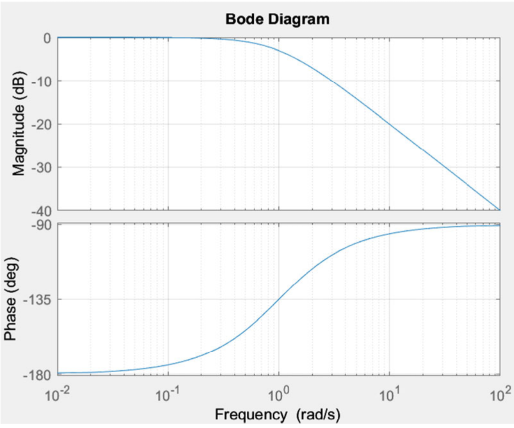


Figure 2.

Determine the critical value of K for stability using the Nyquist stability criterion.

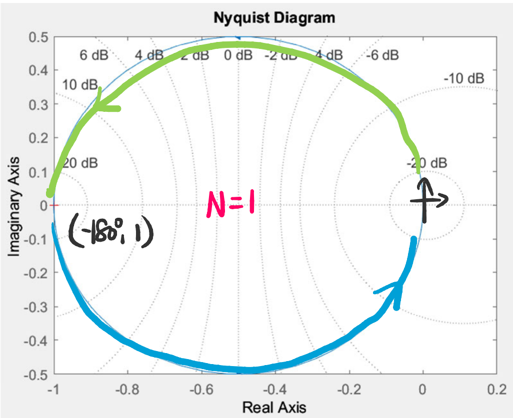
(6 Points)



$$G(s) = \frac{1}{s-1} \Rightarrow \# P_{RHP} = 1$$

$$\Downarrow$$

$$\begin{cases} M(w) = \dfrac{1}{\sqrt{1+w^2}} \\ \phi(w) = \tan^{-1}\left(\dfrac{w}{-1}\right) \end{cases} \Rightarrow \begin{cases} \phi(w=0) = -180^\circ, \; M(w=0) = 1 \\ \phi(w=\infty) = -90^\circ, \; M(w\to\infty) = 0 \end{cases}$$



Based on the equation

$$Z = N - P$$

$$Z = N - 1$$

$\therefore$ N should be +1

$\therefore -\frac{1}{K} \in (-1, 0) \Rightarrow K > 1$