Lecture 1 - Heat Transfer: Physical Origins and Rate Equations

Chapter One

Course Structure

- Syllabus
- Lecture: Mon/Wed 9:00 AM @ Lecture LTN-A 325
- Six different Labs spread across 12 weeks of 1 hour each
  - Every Thurs at ZJUI Experimental Building D338, beginning from 8 am!

Course Structure

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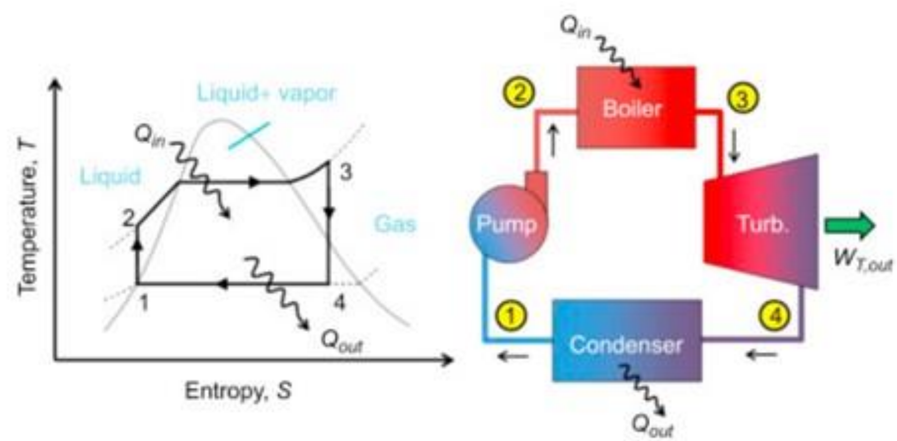
- Blackboard (BB) for Syllabus/Announcement/Lab/HWs/PPTs.
- Exams on Paper
- Office hours
  - every Monday evening
  - Library café 7.30pm
- Wechat group with TAs

Content

1. What are the common modes of heat transfer?
2. What do these symbols mean in heat transfer and what are their units:
   - Q
   - q
   - q''
3. What are the rate laws used for the different modes of heat transfer? Define the variables used.
4. Write down the general equation for thermal resistance and the thermal resistance for 1D heat conduction.

ME200 Thermodynamics

- Deals with initial and final states of an interaction process
- No much on how and how fast energy is transferred



ME320 Heat Transfer

- How and how fast the energy is transferred

Global and China Outlook – Efficient heat exchange

Trichlorofluoromethane
(CFC-11)

- Can we find a replacement for HFC with GMP < 1 that is affordable, non-toxic, not dangerous?
- **Can we design a new efficient aircon that uses a different cooling mechanism?**

Adopted from Prof Arun Majumdar, 1st May 2020

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Exhaust Pipe thermoelectric generator (TEG)

Industrial exhaust thermoelectric generator (TEG)

+5% Fuel efficiency == -7.5 million metric

- 5 quadrillion Btu wasted/year == 14 trillion

.

’s TEG

Study thermal transport properties can make you into the next Jack Ma

https://www.alphabetenergy.com/
https://energy.gov/sites/prod/files/2014/03/f13/fleurial.pdf
http://news.nationalgeographic.com/energy/2015/08/150817-power-plant-pollution-depends-on-the-weather/

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习近平总书记在十九大报告中明确提到“**推进能源生产和消费革命,构建清洁低碳、安全高效的能源体系**”。在二十大报告继续加强推进绿色能源。

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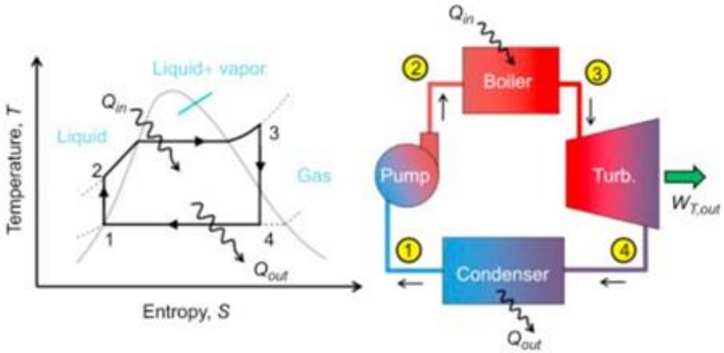
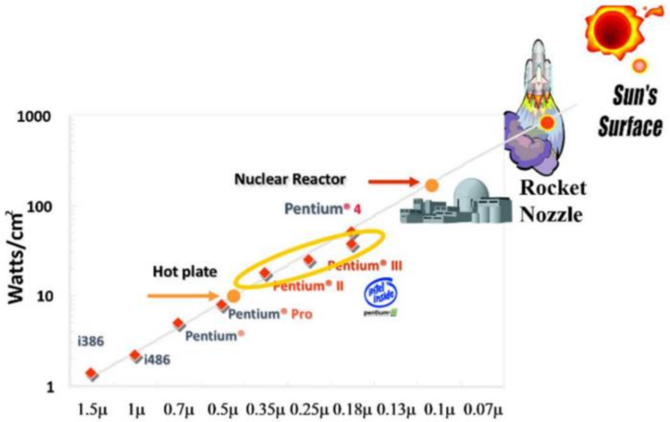
Steam Power Plant

- Any heat transfer in this cycle?



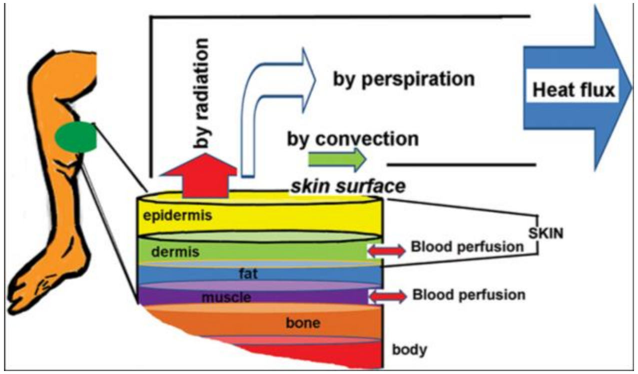
CPU

- Failure of Moore's law
- Limited clock speed as too much heat generated



BioHeat

- We are 37°C partly due to HT
- Laser surgery
- Preserving organs



Many more….

More applications

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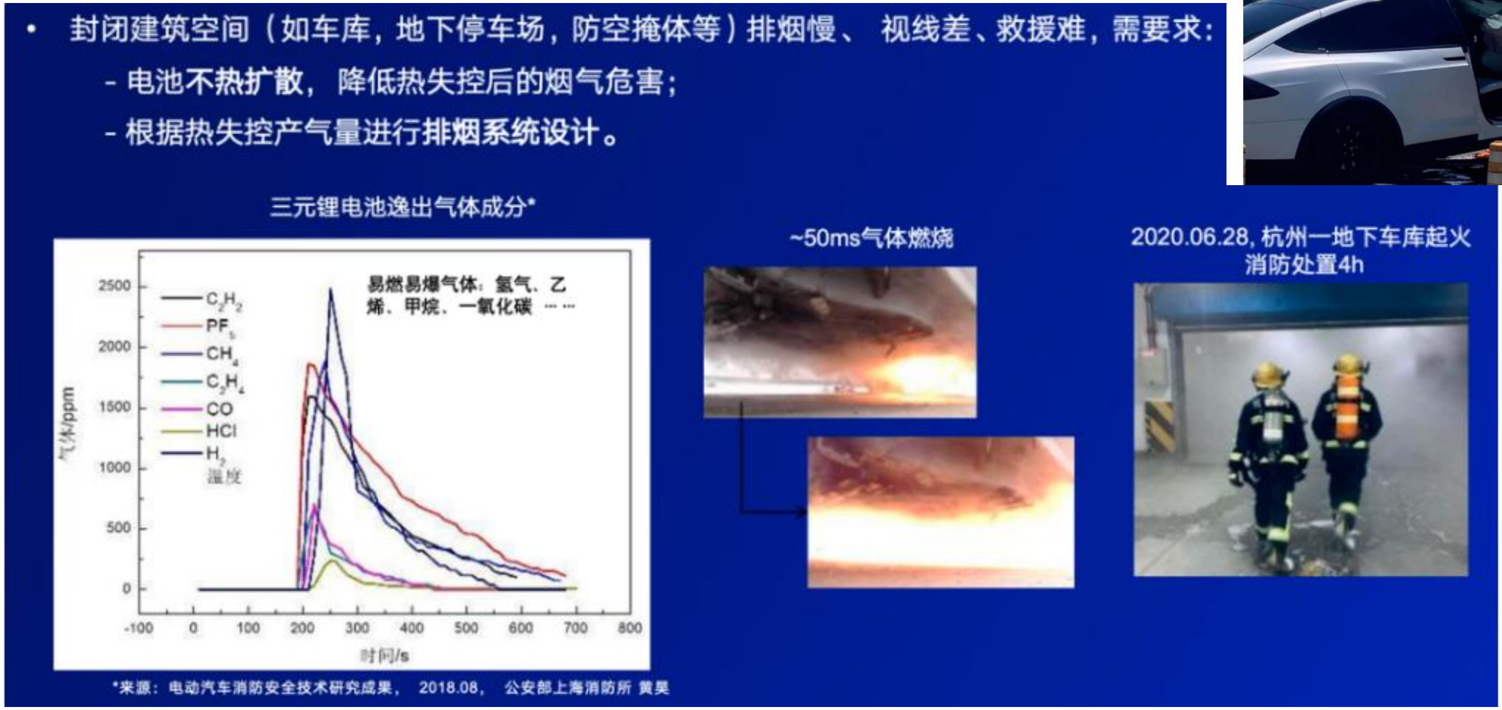
身为一个80后，还是很关心林志颖的车祸事件——他所驾驶的车辆由于行驶途中向右偏移，导致车头撞向路边桥墩，在车上还有小儿子Jenson。目前父子俩均没有生命危险，但林志颖受伤的情况就相对比较严重一些，右肩膀等多处骨折，颜面也因撞击造成脸骨骨折，额头部位也有破相，需要一定的时间恢复。由于撞击的部位在头部，所以接下来的72小时仍是关键期。

这件事点燃的不只是一辆特斯拉，更是点燃了大众对电动汽车的安全性的再度怀疑，所以我想花一些时间来聊聊这个案例，以及电动汽车的安全性。



What is heat transfer?

Heat transfer is *thermal energy in transit* due to a temperature difference.
=> Temperature difference leads to heat transfer

What is thermal energy?

Thermal energy comes from:

- translation, rotation, vibration and electronic states of atoms and molecules
- microscopic activities give rise to temperature

Thermal Energy, Temperature and Heat Transfer

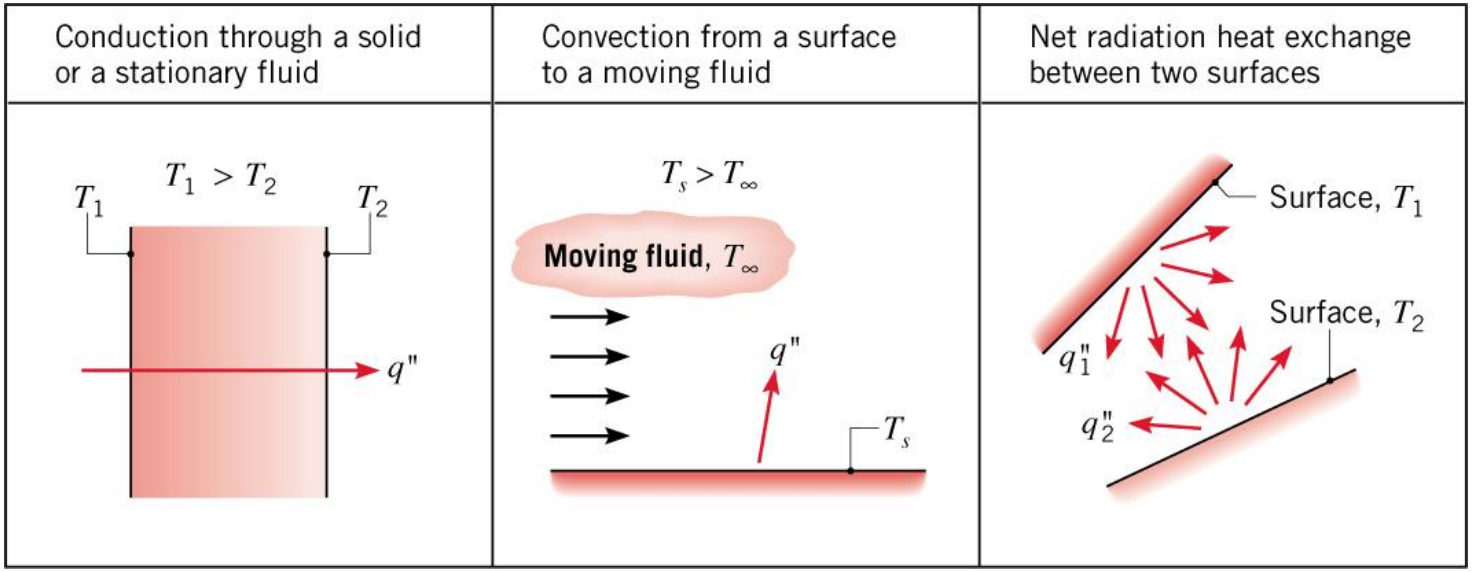
Quantity	Meaning	Symbol (Units)
Thermal Energy+	Energy associated with microscopic behavior of matter	U or u (J or J/kg)
Temperature	A means of indirectly assessing the amount of thermal energy stored in matter	T (K or ^{o}C)
Heat Transfer	Thermal energy transport due to temperature gradients	
Heat	Amount of thermal energy transferred over a time interval	Q (J)
Heat Rate	Thermal energy transfer per unit time	q (W)
Heat Flux	Thermal energy transfer per unit time and surface area	q'' (W/m^2)

+

$U \rightarrow$ Thermal energy of system

$u \rightarrow$ Thermal energy per unit mass of system

Modes of Heat Transfer



Conduction: In a solid or/and a stationary fluid (gas or liquid) due to random motion of its constituent atoms, molecules and /or electrons. (Diffusion-like!)

Convection: Combined influence of random diffusion and bulk **motion of atoms/molecules etc** for fluid flow over a surface.

Radiation: Energy that is emitted by matter due to changes in the electron configurations of its atoms or molecules and is transported as electromagnetic waves (or photons).

Note:

- Conduction and convection require **a temperature gradient across a material medium**.
- Radiation **does not require a material medium** and occurs most efficiently in a vacuum

Questions: Modes of Heat Transfer

Classify the modes of heat transfer in the following cases:

- Touch something hot/cold
- Heat flow from a microprocessor
- Removing heat from microprocessor using a heat sink
- Cooking on your stove (Electric/Gas)
- Baking in your oven
- Heat from the sun
- Heat from the moon
- Boiling your water

Conduction

Lec 3 -13

Heat Conduction: **diffusive** (normally) heat transfer process in solids across a **temperature gradient**

General (vector) form of Fourier's law

$$\vec{q''} = -k\underbrace{\nabla T}$$

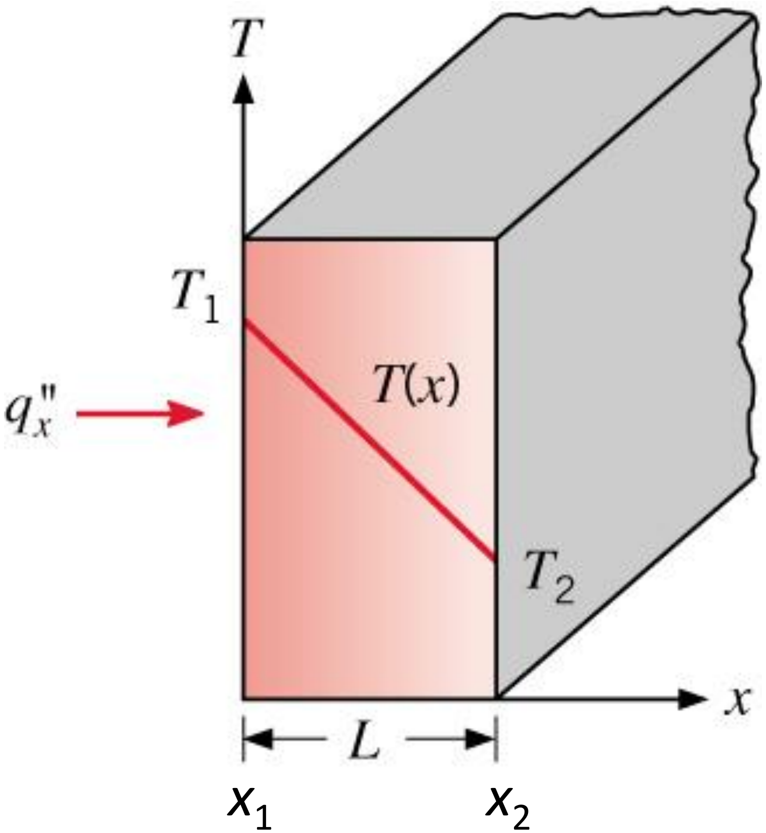
Heat flux | Thermal conductivity | Temperature gradient

$\mathrm{W/m^2}$ | $\mathrm{W/m \cdot K}$ | °C/m or K/m

Why is there a – ve sign in Fourier's law?
Look at next slide!

Consider a simple example:

One-dimensional, steady conduction across a plane wall of constant thermal conductivity:



$$q''_x = -k\frac{dT}{dx} = -k\frac{T_1 - T_2}{x_1 - x_2}$$

$$q''_x = k\frac{T_1 - T_2}{L} \qquad (1.2)$$

Heat rate (W): $q_x = q''_x \cdot A$

As dT/dx is -ve but q” is +ve, there has to be a –ve sign!

Example 1: Conduction

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EXAMPLE 1.1

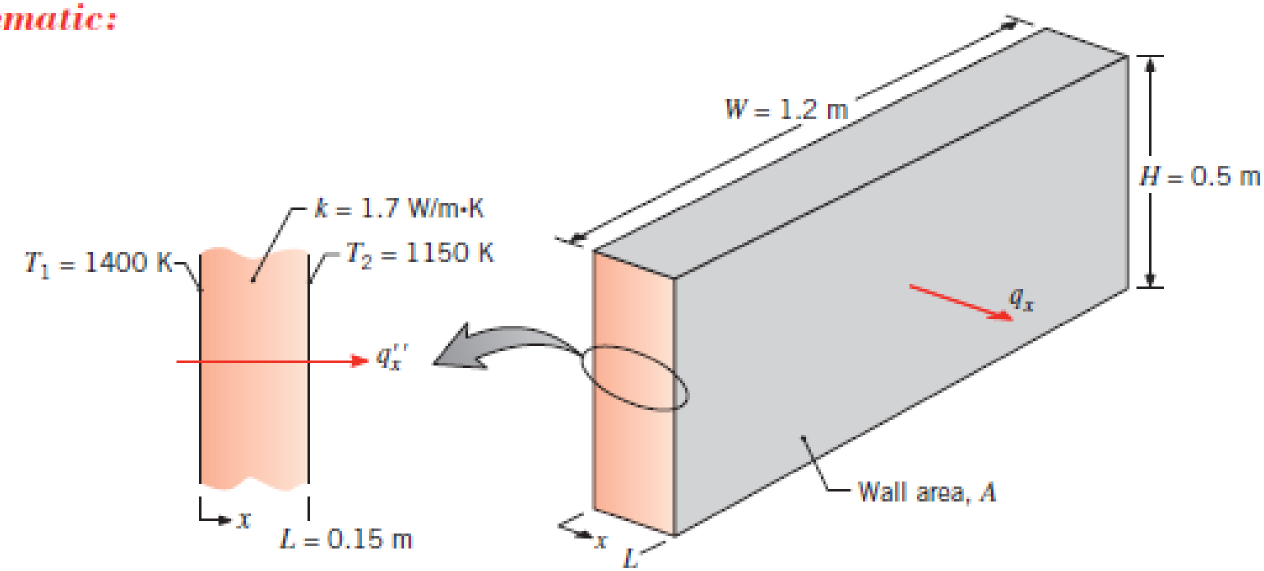
The wall of an industrial furnace is constructed from 0.15-m-thick fireclay brick having a thermal conductivity of 1.7 W/m · K. Measurements made during steady-state operation reveal temperatures of 1400 and 1150 K at the inner and outer surfaces, respectively. What is the rate of heat loss through a wall that is 0.5 m $\times$ 1.2 m on a side?

SOLUTION

Known: Steady-state conditions with prescribed wall thickness, area, thermal conductivity, and surface temperatures.

Find: Wall heat loss.

Schematic:



Assumptions:

1. Steady-state conditions.
2. One-dimensional conduction through the wall.
3. Constant thermal conductivity.

Analysis: Since heat transfer through the wall is by conduction, the heat flux may be determined from Fourier's law. Using Equation 1.2, we have

$$q''_x = k\frac{\Delta T}{L} = 1.7\ \mathrm{W/m\cdot K} \times \frac{250\ \mathrm{K}}{0.15\ \mathrm{m}} = 2833\ \mathrm{W/m^2}$$

The heat flux represents the rate of heat transfer through a section of unit area, and it is uniform (invariant) across the surface of the wall. The heat loss through the wall of area $A = H \times W$ is then

$$q_x = (HW)\,q''_x = (0.5\ \mathrm{m} \times 1.2\ \mathrm{m})\,2833\ \mathrm{W/m^2} = 1700\ \mathrm{W} \qquad \triangleleft$$

Convection

Lec 14 -25

Convection: Heat transfer with **fluid motion**

- Especially across fluid-solid interface with fluid and solid at two different temperatures

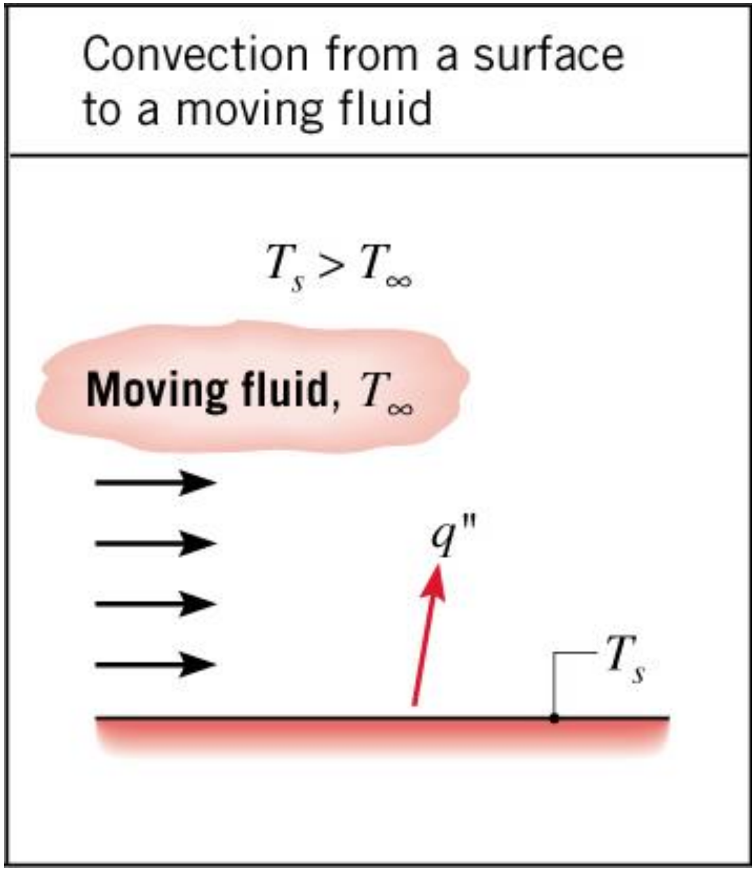
Note:

Convection

- Random motion of fluidic constituents (diffusive) and
- Macroscopic motion of fluid

Advection

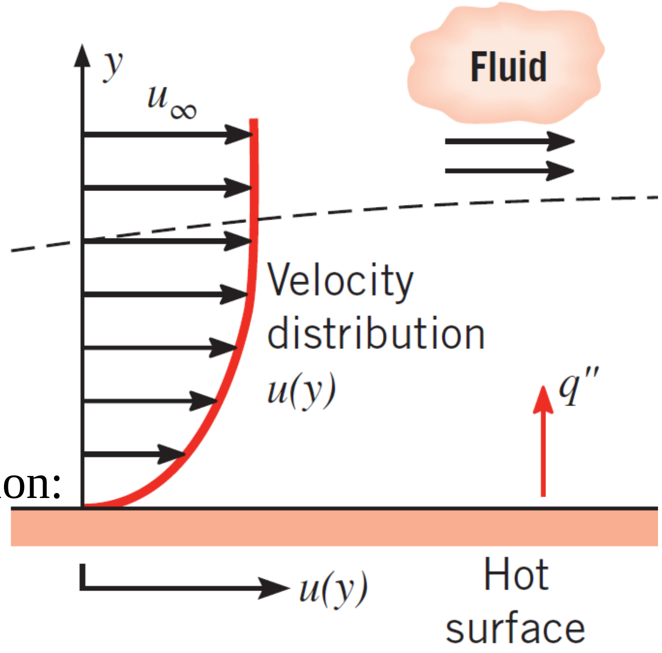
- Part of Convection
- Macroscopic motion of fluid



Convection over Fluid-Solid interface develops:

- Velocity/Hydrodynamic boundary layers
- Thermal boundary layers:

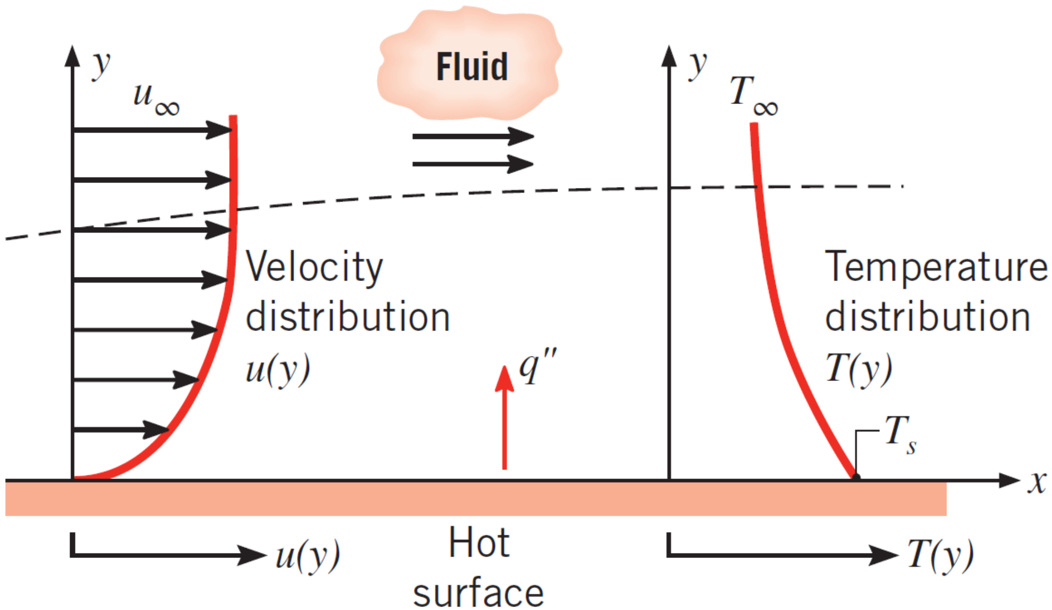
Non-slip condition:
$u(y=0) = 0$



Newton's law of cooling: $$q'' = h\left(T_s - T_\infty\right) \quad (1.3a)$$

h: Convection heat transfer coefficient $(\mathrm{W/m^2 \cdot K})$

Note: T_∞ used to describe fluid temperature



Convection

- Random motion of fluidic constituents (diffusive)
    - Near surface
- Macroscopic motion of fluid
    - Throughout the boundary layer
- Forced Convection (Lec 14 -24)
- Free (Natural) Convection – density difference (Lec 25)

Radiation

Lec 26 -37

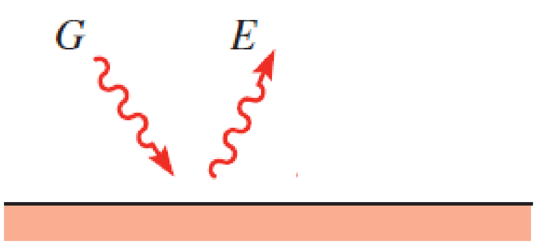
Heat Transfer Rates: Radiation

- Any surface above $\boxed{\text{absolute zero}}$ temperature

- Changes in electronic configuration of atoms/molecules
- Energy carried by $\boxed{\text{electromagnetic waves}}$

- Can occur across $\boxed{\text{vacuum}}$

- Involves
  - Emission, *E*
  - Irradiation, *G* (Absorption of radiation from the surroundings)



Surface of emissivity ε, absorptivity α, and temperature T_s

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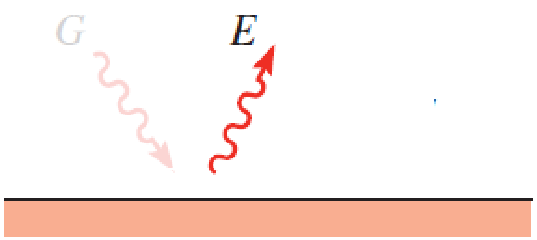


Surface of emissivity ε, absorptivity α, and temperature T_s

Energy outflow due to Emission:

$$E = \varepsilon E_b = \varepsilon \sigma T_s^4 \qquad (1.5)$$

E : Emissive power $\left(\mathrm{W/m^2}\right)$

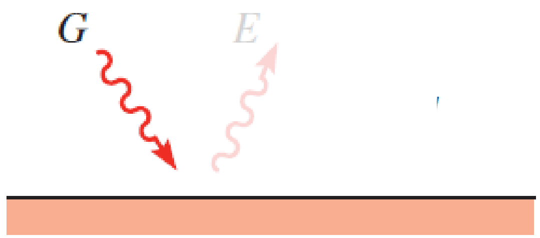
ε : Surface emissivity $\left(0 \le \varepsilon \le 1\right)$

E_b : Emissive power of a blackbody (the perfect emitter)

σ : Stefan-Boltzmann constant $\left(5.67\times10^{-8}\ \mathrm{W/m^2 \cdot K^4}\right)$

Emissivity (ε) describes how efficient a surface emits energy relative to a **blackbody**.

Heat Transfer Rates: Radiation inflow



Energy absorption due to irradiation:

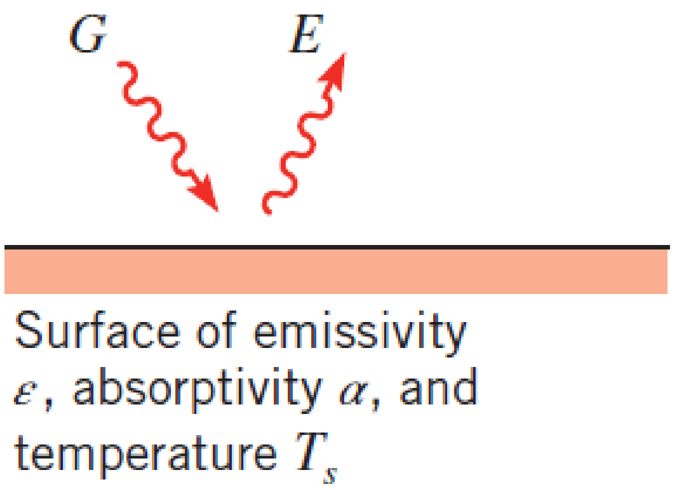
$$G_{\text{abs}} = \alpha G \qquad (1.6)$$

G_{abs} : Absorbed incident radiation (W/m^2)
α : Surface absorptivity $(0 \le \alpha \le 1)$
G : Irradiation (W/m^2)

Absorbed energy will increase/decrease the thermal energy of a material?

What object will have $\alpha = 1$?

Other than being absorbed, what else can happen to the incident irradiation, *G*?



Net radiation flux experienced by surface:

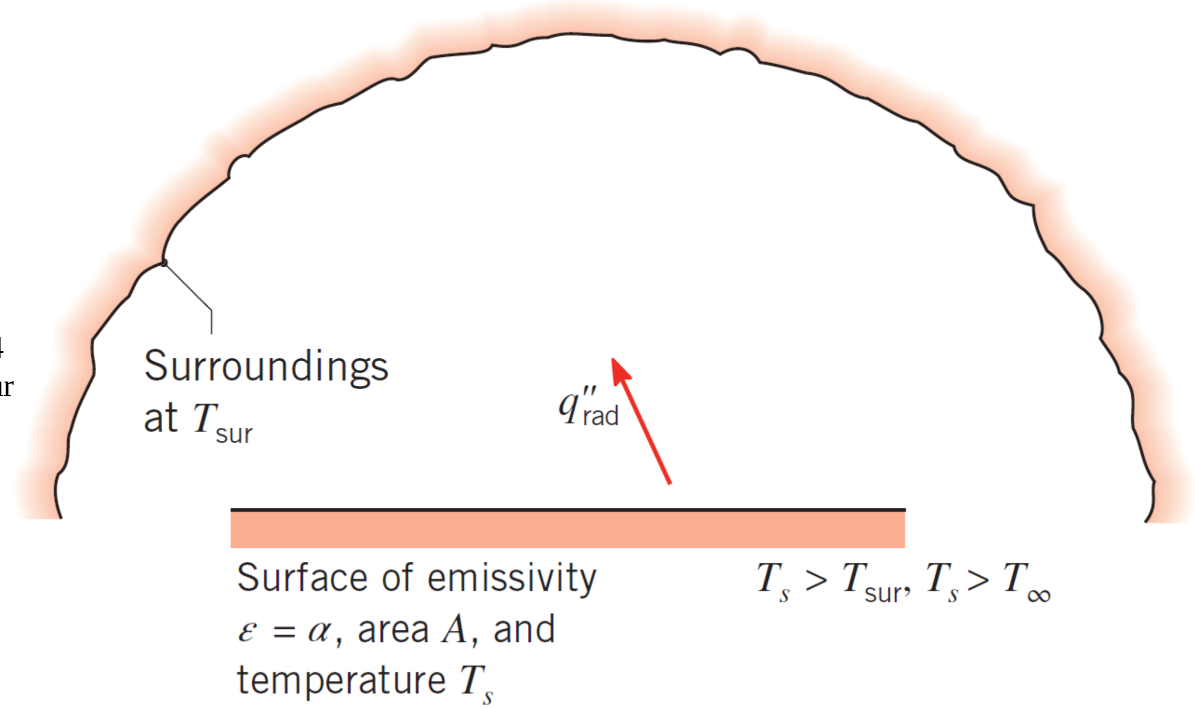
q''_{rad} **= emission - absorption**

Special but common case: small surface exposed to large surroundings of uniform temperature

Examples:

- Inside a large room
- Inside a furnace

$$G = G_{sur} = \sigma T_{sur}^4$$



If $\alpha = \varepsilon$, the net radiation heat flux from the surface due to exchange with the surroundings is:

$$q''_{rad} = \text{emission - absorption}$$

$$= \varepsilon E_b(T_s) - \alpha G = \varepsilon\sigma\left(T_s^4 - T_{sur}^4\right) \qquad (1.7)$$

Heat Transfer Rates: Summary

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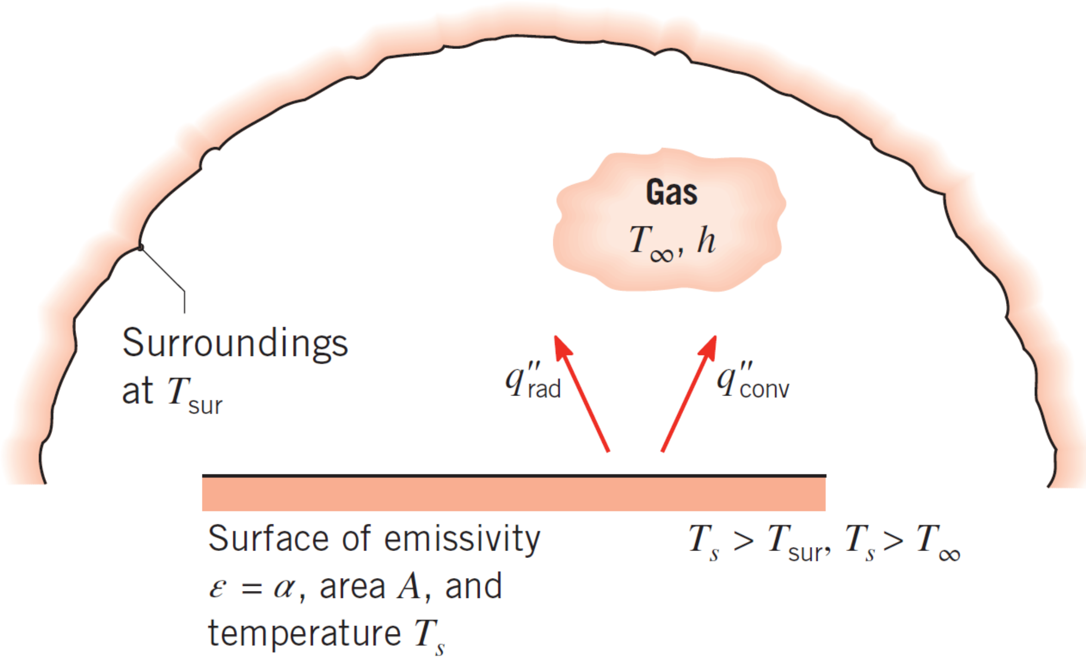
Table 1.5 Summary of heat transfer processes

Mode	Mechanism(s)	Rate Equation	Equation Number	Transport Property or Coefficient
Conduction	Diffusion of energy due to random molecular motion	$q''_x(\mathrm{W/m^2}) = -k\dfrac{dT}{dx}$	(1.1)	k (W/m · K)
Convection	Diffusion of energy due to random molecular motion plus energy transfer due to bulk motion (advection)	$q''(\mathrm{W/m^2}) = h(T_s - T_\infty)$	(1.3a)	h (W/m^2 · K)
Radiation	Energy transfer by electromagnetic waves	$q''(\mathrm{W/m^2}) = \varepsilon\sigma(T_s^4 - T_{sur}^4)$ or $q(\mathrm{W}) = h_r A(T_s - T_{sur})$	(1.7) (1.8)	ε h_r (W/m^2 · K)

Note: T_∞ used to describe fluid temperature

Note: T_{sur} used to describe surrounding temperature



For multi-mode heat transfer, for example with convection and radiation,

$$q'' = \boxed{q''_{conv} + q''_{rad}} = \boxed{h(T_s - T_\infty) + h_r(T_s - T_{sur})} \qquad (1.10)$$

Heat Transfer Rates: Thermal Resistance

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Rate Equation

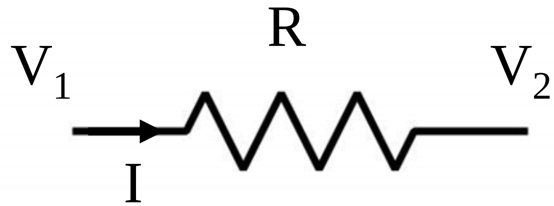
$$q''_x\,(\mathrm{W/m^2}) = -k\frac{dT}{dx}$$

$$q''(\mathrm{W/m^2}) = h(T_s - T_\infty)$$

$$q''(\mathrm{W/m^2}) = \varepsilon\sigma(T_s^4 - T_{sur}^4)$$
$$\text{or } q\,(\mathrm{W}) = h_r A(T_s - T_{sur})$$

In electrical circuit,

Eqn: $V = IR$

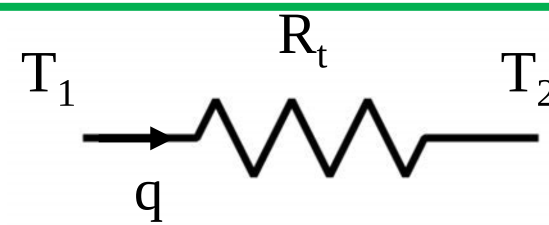


V – voltage drives the electron around
I – flow of electrons
R – resistance to the electron flow

Similarly, in thermal transport,

dT – temperature difference drives the heat flow
q – flow of thermal energy, (q”A)
R_t – thermal resistance

Eqn:
$$\boxed{q = \frac{dT}{R_t} \qquad (1.11)}$$



Which area to use?

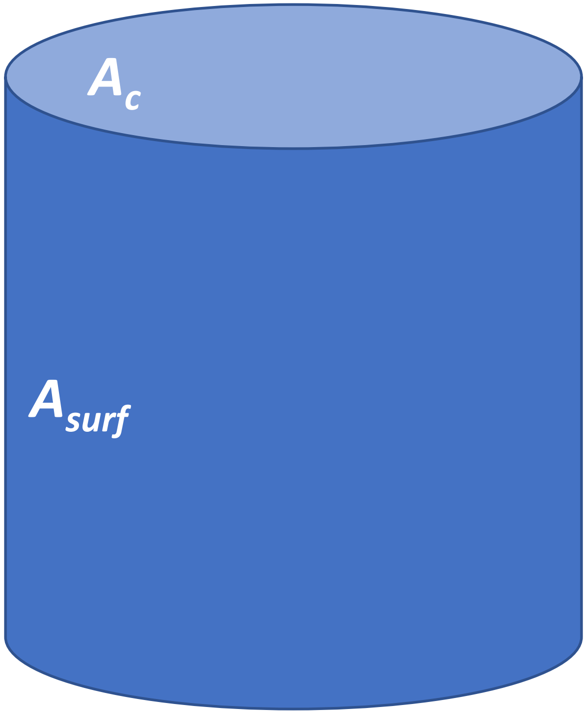
- Conduction, q = -$kA\frac{dT}{dx}$
- Convection and Radiation, q = -$hA(T - T_{\infty})$

and q = -$hA(T^4 - T_s^4)$

Depends on the heat transfer process!

Conduction, A will be perpendicular to heat flow.

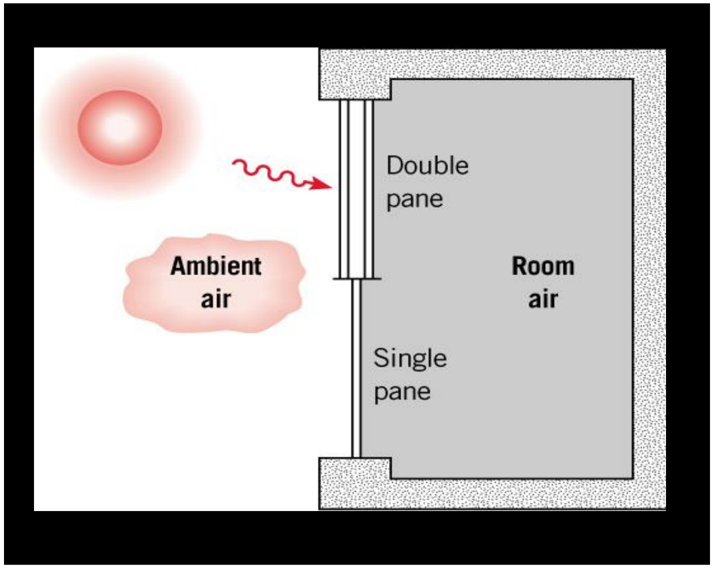
Convection and Radiation, A could be any or both.

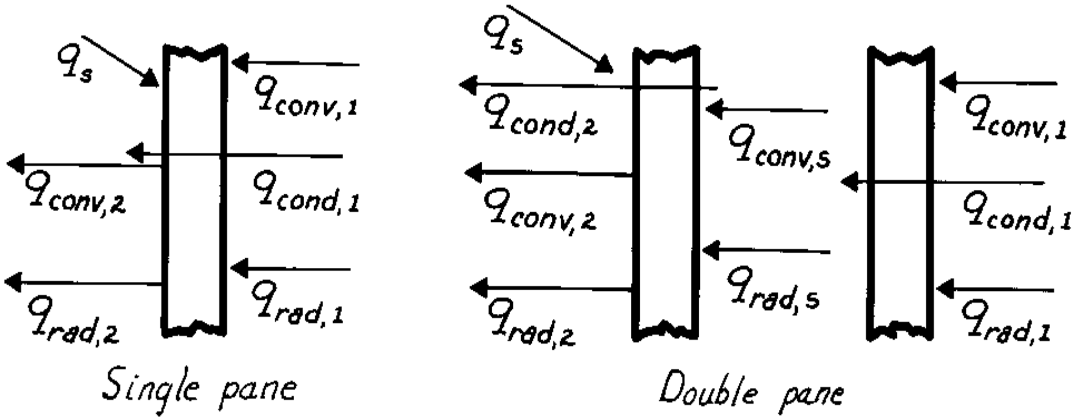


Summary

- Different types of heat transfer mechanisms
- Equations describing these mechanisms
- Variables and their meanings

Problem 1.63(a): Process identification for single-and double-pane windows





$q_{\text{conv},1}$ Convection from room air to inner surface of first pane

$q_{\text{rad},1}$ Net radiation exchange between room walls and inner surface of first pane

$q_{\text{cond},1}$ Conduction through first pane

$q_{\text{conv},s}$ Convection across airspace between panes

$q_{\text{rad},s}$ Net radiation exchange between outer surface of first pane and inner surface of second pane (across airspace)

$q_{\text{cond},2}$ Conduction through a second pane

$q_{\text{conv},2}$ Convection from outer surface of single (or second) pane to ambient air

$q_{\text{rad},2}$ Net radiation exchange between outer surface of single (or second) pane and surroundings such as the ground

q_s Incident solar radiation during day; fraction transmitted to room is smaller for double pane

Content: So what are the answers?

1. What are the common modes of heat transfer?
2. What do these symbols mean in heat transfer and what are their units:
   - *Q*
   - *q*
   - *q“*
3. What are the rate laws used for the different modes of heat transfer? Define the variables used.
4. Write down the general equation for thermal resistance and the thermal resistance for 1D heat conduction.

Relationship to Thermodynamics

Chapter One

Section 1.3

1. What is the main conservation law used in solving heat transfer problems?
2. Write down the modified law in both the rate form and "over a time interval" form. Define the variables.
3. In a control volume, when will certain variables in the modified 1^{st} law go to zero?
4. In a control surface, which two variables in the 1^{st} modified law become zero? Why?

Thermodynamics

- The ***amount*** of heat passed from one state to another

Heat Transfer

- ***How fast and through what mechanisms*** to pass heat from one state to another.
- Conservation laws, First and Second laws of thermodynamics to find out!
    - Conservation of mass
    - Conservation of momentum
    - First Law : Conservation of Energy (most important)
    - Second Law : Entropy => Heat flows from high to low temperatures

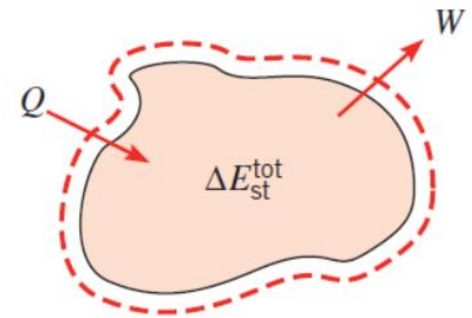
aka CONSERVATION OF ENERGY

$$\Delta E_{st}^{total} = Q - W \qquad (1.12a)$$

where ΔE_{st}^{tot} is the change in the total energy stored in the system, Q is the *net* heat transferred to the system, and W is the *net* work done by the system.

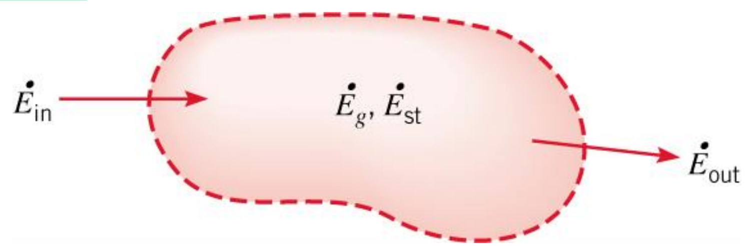
In **a closed system** (no mass crosses boundary), Energy crosses boundary through

- Heat transfer
- Work done on or by the system



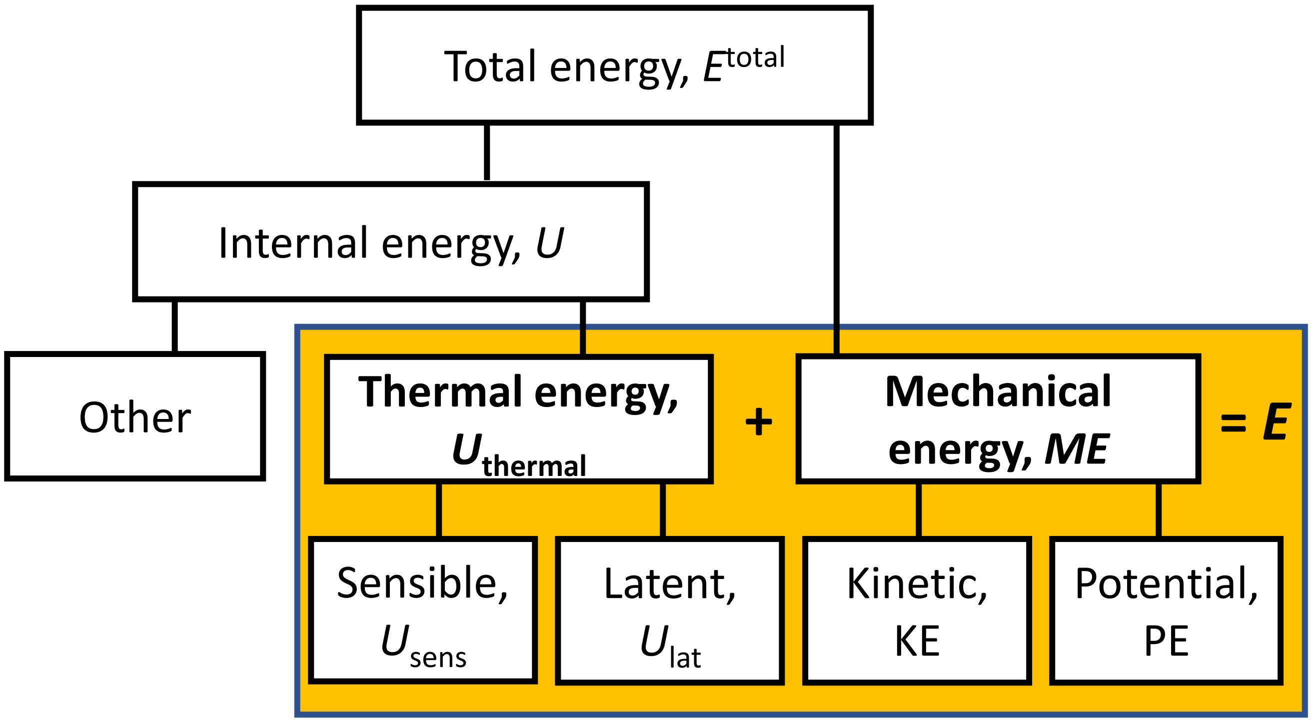
In an **open system (control volume)**, Energy crosses boundary through

- As in closed system, plus…
- Mass flows into or out of system and carries energy with it.



Total energy is conserved.

- What does the total energy consist of?



HT is often interested in the yellow portion which is **Thermal and Mechanical energy, $E = U_{sens} + U_{lat} + KE + PE$**

Thermal and Mechanical energy,

$$E = U_{\text{thermal}} + \text{KE} + \text{PE}$$

- KE = $0.5mV^2$ (normally)
- PE = mgh (normally)

- $U_{\text{thermal}} = U_{\text{sens}} + U_{\text{lat}}$
  - U_{sens} : translational, rotational, vibrational motions of atoms/molecules
  - U_{lat} : inter-atomic/inter-molecular forces influencing phase changes

NOTE:

- **Strictly speaking, *E* is not total energy but only thermal and mechanical energy**

Modified 1^{st} Law

$$\Delta E_{st}^{total} = Q - W$$

$$\Delta E_{st} = E_{in} - E_{out} + E_g \quad (1.12b)$$

Increase in thermal and mechanical energy

thermal and mechanical energy that enters the system

thermal and mechanical energy that leaves the system

thermal and mechanical energy generated iside the system over a time interval.

- Unit: **Joules (J)**

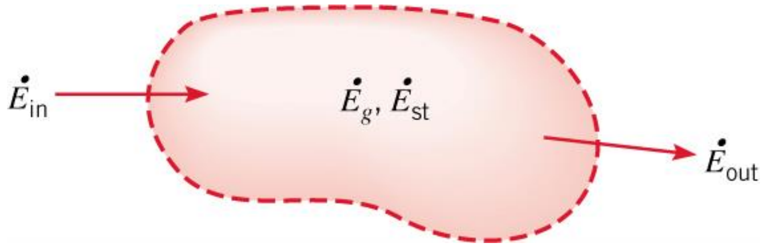
Alternatively,

$$\dot{E}_{st} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_g \quad (1.12c)$$

- **Rate** is considered where $\dot{E} = \frac{dE}{dt}$
- Unit: **J/s or W**

Control volume and rate of change

$$\dot{E}_{st} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_{g}$$



Note representation of system by a control surface (dashed line) at the boundaries.

Surface Phenomena

$\dot{E}_{in}, \dot{E}_{out}$: rate of thermal and/or mechanical energy transfer across the control surface due to heat transfer, fluid flow and/or work interactions.

Volumetric Phenomena

$\dot{E}_{g}$: rate of thermal and mechanical energy generation in a system from another energy form

$\dot{E}_{st}$: rate of change of thermal and mechanical energy stored in a system

Factors in Surface Phenomena

$\dot{E}_{in}, \dot{E}_{out}$: rate of thermal and/or mechanical energy transfer across the control surface due to heat transfer, fluid flow and/or work interactions.

Includes:

- Heat transfer, $\boldsymbol{q_{cond}, q_{conv}, q_{rad}}$
- Work, $\boldsymbol{\dot{W}}$
- Mass flow, $\boldsymbol{\dot{m}(u_{thermal} + pv + 0.5V^2 + gh)}$
    - ***pv*** is called the flow work due to work done by pressure moving the fluid. ***p*** is pressure and ***v*** specific volume.
    - **Enthalpy,** $\boldsymbol{i = u_{thermal} + pv}$

Factors in Volumetric Phenomena

$\dot{E}_g$: rate of thermal and mechanical energy generation in a system from another energy form

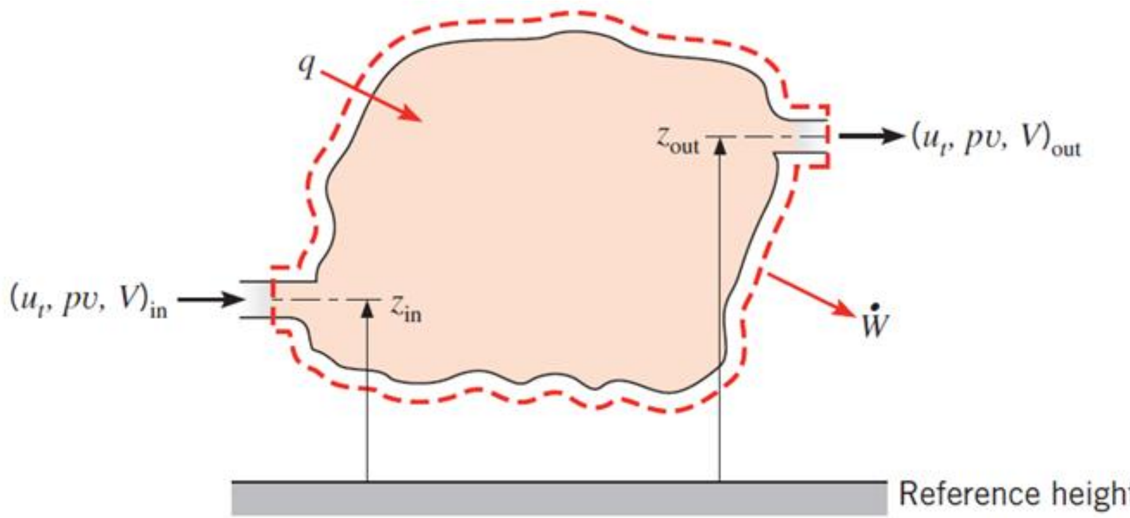
- Thermal is only one type of energy
- Others: Nuclear, Chemical, Electrical, etc
- Conversion from Others into Thermal or Mechanical energy is $\dot{E}_g$

$\dot{E}_{st}$: rate of change of thermal and mechanical energy stored in a system

- At **steady-state**, $\dot{E}_{st} = 0$

Example 1: Surface Phenomena

Consider this control volume, write down $\dot{E}_{in}$ and $\dot{E}_{out}$



$$\dot{E}_{in} = \dot{m}(u_{thermal} + pv + 0.5V^2 + gz)_{in} + q$$

$$\dot{E}_{out} = \dot{m}(u_{thermal} + pv + 0.5V^2 + gz)_{out} + \dot{W}$$

Control Surface

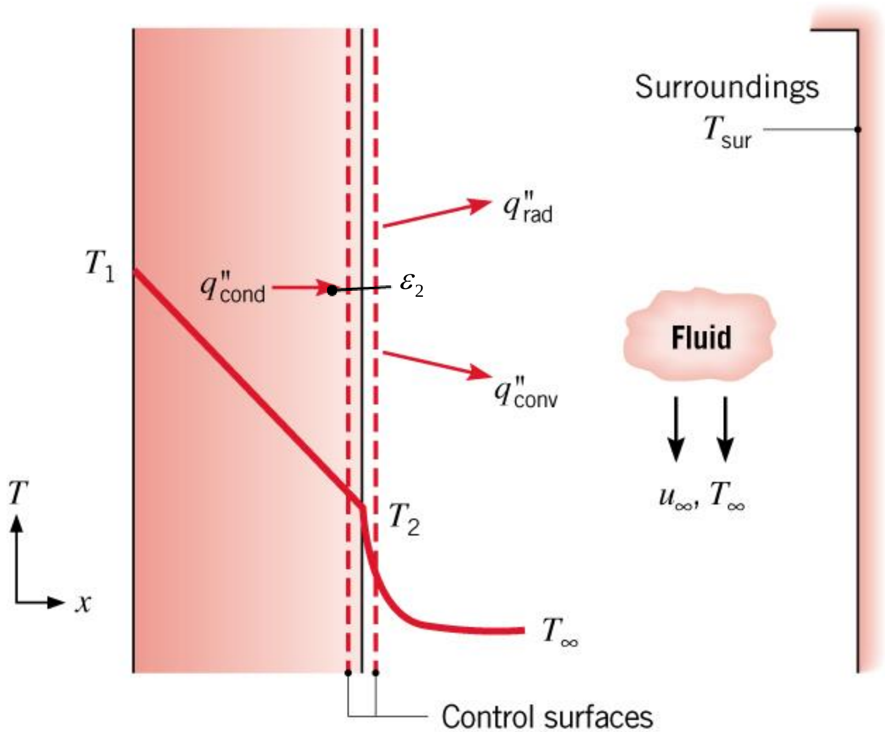
Control Surface vs Control Volume

Special case of control volume with **No volume or mass inside**

- Energy storage and generation not considered even if they occur in the material bounded by the surface.
- Valid for both steady-state and transient conditions.

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = 0 \qquad \textbf{(1.13)}$$

Control Surface ENERGY BALANCE example



$$\dot{E}_{in} - \dot{E}_{out} = 0$$

$$q''_{cond} - q''_{conv} - q''_{rad} = 0$$

$$k\frac{T_1 - T_2}{L} - h(T_2 - T_\infty) - \varepsilon_2\sigma\left(T_2^4 - T_{sur}^4\right) = 0$$

Using 1st Law

1. Draw dashed lines to denote the control volume/surface on a schematic of the system.

2. Choose appropriate time basis,

   i.e., over a time interval or rate

3. What are the E_{in}, E_{out}, E_{g}, E_{st}?

4. Apply $\Delta E_{\mathrm{st}} = E_{\mathrm{in}} - E_{out} + E_{\mathrm{g}}$ or $\dot{E}_{st} = \dot{E}_{in} - \dot{E}_{out} + \dot{E}_{g}$

5. Substitute appropriate expressions for terms in the energy equation

6. Solve for the unknown quantity

Example 2

IHT | **EXAMPLE 1.4**

A long conducting rod of diameter D and electrical resistance per unit length R'_e is initially in thermal equilibrium with the ambient air and its surroundings. This equilibrium is disturbed when an electrical current I is passed through the rod. Develop an equation that could be used to compute the variation of the rod temperature with time during the passage of the current.

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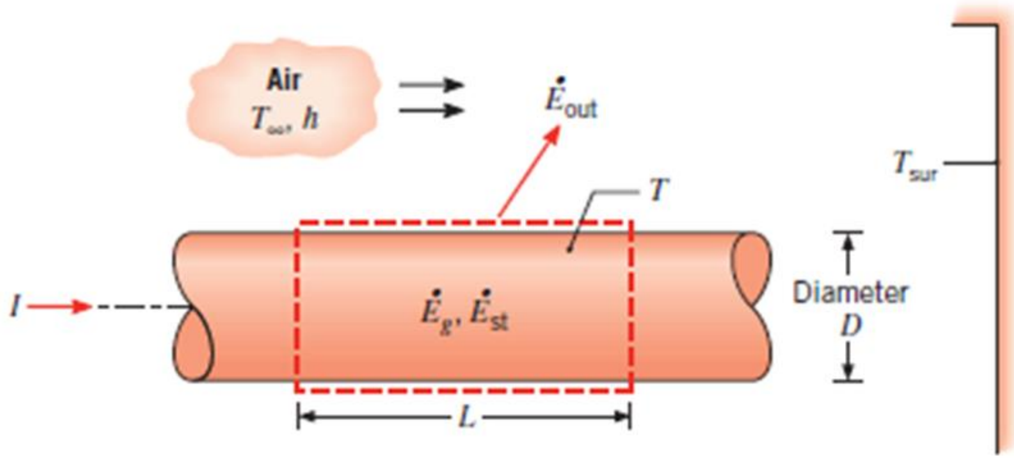
Known: Temperature of a rod of prescribed diameter and electrical resistance changes with time due to passage of an electrical current.

Find: Equation that governs temperature change with time for the rod.

Schematic:



1. Draw dashed lines to denote the control volume/surface on a schematic of the system.

Assumptions:

1. At any time t, the temperature of the rod is uniform.
2. Constant properties (ρ, c, $\varepsilon = \alpha$).
3. Radiation exchange between the outer surface of the rod and the surroundings is between a small surface and a large enclosure.

Analysis: The first law of thermodynamics may often be used to determine an unknown temperature. In this case, there is no mechanical energy component. So relevant terms include heat transfer by convection and radiation from the surface, thermal energy generation due to ohmic heating within the conductor, and a change in thermal energy storage. Since we wish to determine the rate of change of the temperature, the first law should be applied at an instant of time. Hence, applying Equation 1.12c to a control volume of length L about the rod, it follows that

4. Apply first law

$$\boxed{\dot{E}_g - \dot{E}_{\text{out}} = \dot{E}_{\text{st}}}$$

2. Choose appropriate time basis, i.e., over time or rate

where thermal energy generation is due to the electric resistance heating,

$$\boxed{\dot{E}_g = I^2 R'_e L}$$

3. What are the E_{in}, E_{out}, E_g, E_{st}?

Heating occurs uniformly within the control volume and could also be expressed in terms of a volumetric heat generation rate $\dot{q}(\text{W/m}^3)$. The generation rate for the entire control volume is then $\dot{E}_g = \dot{q}V$, where $\dot{q} = I^2 R'_e/(\pi D^2/4)$. Energy outflow is due to convection and net radiation from the surface, Equations 1.3a and 1.7, respectively,

$$\boxed{\dot{E}_{\text{out}} = h(\pi D L)(T - T_\infty) + \varepsilon\sigma(\pi D L)(T^4 - T_{\text{sur}}^4)}$$

and the change in energy storage is due to the temperature change,

$$\boxed{\dot{E}_{\text{st}} = \frac{dU_t}{dt} = \frac{d}{dt}(\rho V c T)}$$

The term $\dot{E}_{\text{st}}$ is associated with the rate of change in the internal thermal energy of the rod, where ρ and c are the mass density and the specific heat, respectively, of the rod material,

Example 2

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and V is the volume of the rod, $V = (\pi D^2/4)L$. Substituting the rate equations into the energy balance, it follows that

$$\dot{E}_g - \dot{E}_{out} = \dot{E}_{st}$$

$$\boxed{I^2R'_eL - h(\pi DL)(T - T_\infty) - \varepsilon\sigma(\pi DL)(T^4 - T^4_{sur}) = \rho c\left(\frac{\pi D^2}{4}\right)L\,\frac{dT}{dt}}$$

5. Substitute appropriate expressions for terms in the energy equation

Hence

$$\boxed{\frac{dT}{dt} = \frac{I^2R'_e - \pi Dh(T - T_\infty) - \pi D\varepsilon\sigma(T^4 - T^4_{sur})}{\rho c(\pi D^2/4)}} \quad \triangleleft$$

6. Solve for the unknown quantity

Example 3

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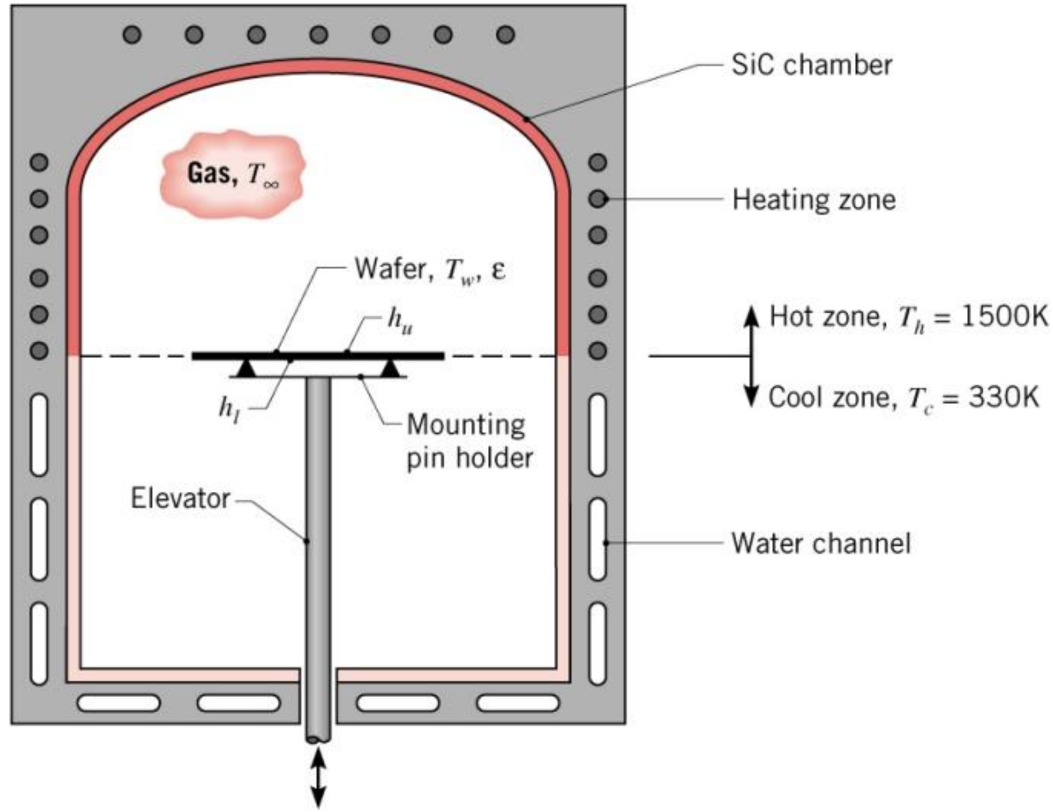
A furnace for processing semiconductor materials is formed by a silicon carbide chamber that is zone-heated on the top section and cooled on the lower section. With the elevator in the lowest position, a robot arm inserts the silicon wafer on the mounting pins. In a production operation, the wafer is rapidly moved toward the hot zone to achieve the temperature-time history required for the process recipe.



In this position, the top and bottom surfaces of the wafer exchange radiation with the hot and cool zones, respectively, of the chamber. The zone temperatures are T_h = 1500 K and T_C = 330 K, and the emissivity and thickness of the wafer are 0.65 and d 0.78 mm, respectively. With the ambient gas at T_∞=700 K, convection coefficients at the upper and lower surfaces of the wafer are 8 and 4 W/m^2 K, respectively. The silicon wafer has a density of 2700 kg/m^3 and a specific heat of 875 J/kg K. Determine

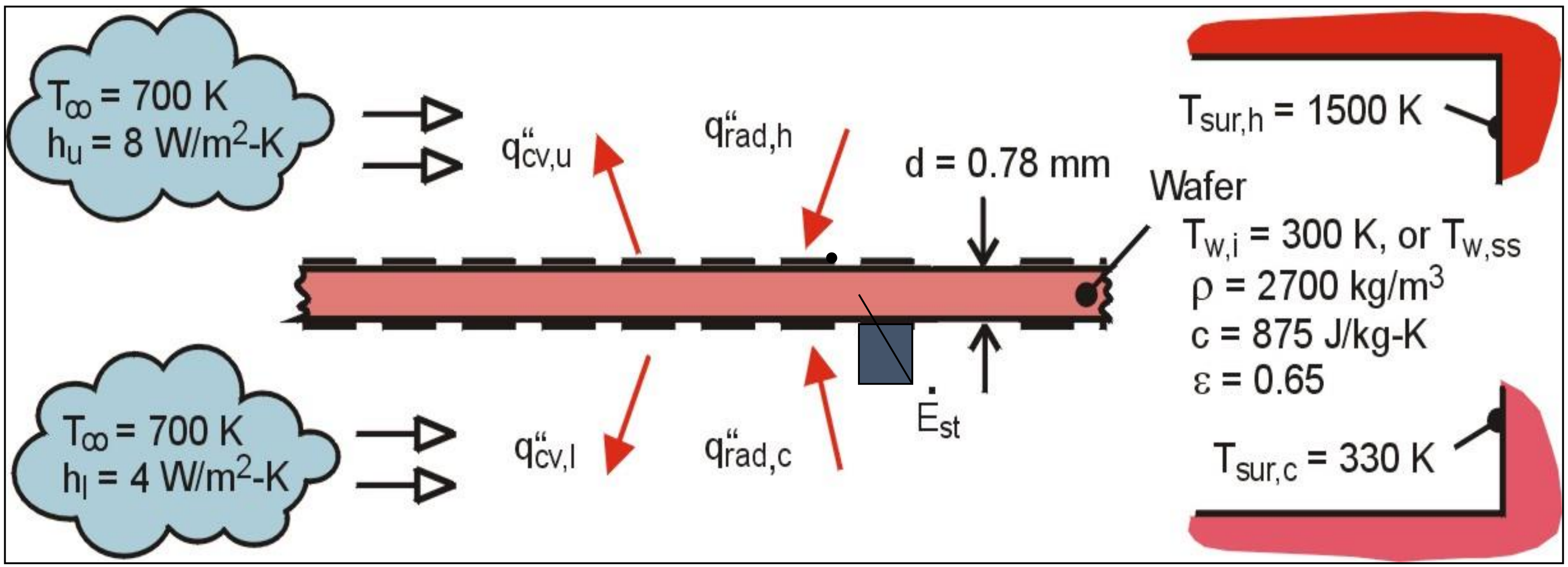
(a) the initial rate of change of the wafer temperature and

(b) the steady-state temperature

KNOWN: Silicon wafer positioned in furnace with top and bottom surfaces exposed to hot and cool zones, respectively.

FIND: (a) Initial rate of change of the wafer temperature from a value of $T_{w,i}$ = 300K, (b) **steady-state** temperature.

SCHEMATIC:



ASSUMPTIONS: (1) Wafer temperature is uniform, (2) Hot and cool zones have uniform temperatures, (3) Radiation exchange is between small surface (wafer) and large enclosure (chamber, hot or cold zone), (4) $\alpha = \varepsilon$, and (5) Negligible heat losses from wafer to pin holder.

ANALYSIS: The energy balance on the wafer includes convection to the upper (u) and lower (l) surfaces from the ambient gas, radiation exchange with the hot- and cool-zones and an energy storage term for the transient condition. Hence, from conservation of energy:

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} + \dot{E}_g = \dot{E}_{\text{st}}$$

$$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} + 0 = \dot{E}_{\text{st}}$$

$$q''_{\text{rad},h} + q''_{\text{rad},c} - q''_{\text{cv},u} - q''_{\text{cv},l} = \rho c d \frac{d\,T_w}{dt}$$

in per unit surface area

$$\varepsilon\sigma\left(T_{\text{sur},h}^4 - T_w^4\right) + \varepsilon\sigma\left(T_{\text{sur},c}^4 - T_w^4\right) - h_u\left(T_w - T_\infty\right) - h_l\left(T_w - T_\infty\right) = \rho c d \frac{d\,T_w}{dt}$$

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(a) For the initial condition, the time rate of change of the wafer temperature is determined using the foregoing energy balance with $T_{i,w}$ = 300K

LHS:

$$0.65 \times 5.67 \times 10^{-8} \mathrm{W/m^2 \cdot K^4} \left(1500^4 - 300^4\right) \mathrm{K^4} + 0.65 \times 5.67 \times 10^{-8} \mathrm{W/m^2 \cdot K^4} \left(330^4 - 300^4\right) \mathrm{K^4}$$
$$-8\,\mathrm{W/m^2 \cdot K} \left(300 - 700\right) \mathrm{K} - 4\,\mathrm{W/m^2 \cdot K} \left(300 - 700\right) \mathrm{K}$$

RHS:

$$2700\,\mathrm{kg/m^3} \times 875\,\mathrm{J/kg \cdot K} \times 0.00078\,\mathrm{m} \left(dT_w / dt\right)_i$$

Therefore:

$$\left(dT_w/dt\right)_i = 104\ \mathrm{K/s}$$

(b) For the steady-state condition, the energy storage term is zero, and the energy balance can be solved for the steady-state wafer temperature, $T_w = T_{w,ss}$.

$$0.65\sigma\left(1500^4 - T_{w,ss}^4\right)\mathrm{K}^4 + 0.65\sigma\left(330^4 - T_{w,ss}^4\right)\mathrm{K}^4$$
$$-8\,\mathrm{W/m^2\cdot K}\left(T_{w,ss} - 700\right)\mathrm{K} - 4\,\mathrm{W/m^2\cdot K}\left(T_{w,ss} - 700\right)\mathrm{K} = 0$$

$$T_{w,ss} = 1251\ \mathrm{K}$$

Summary

- 1st law of thermodynamics
- Modified to focus on conservation of thermal and mechanical energy
- Applying 1st law to solve problems