线性化

线性 $\dot{\theta}_1 = f_1(\theta_1, \theta_2, T_e) = \theta_2$

非线性 $\dot{\theta}_2 = f_2(\theta_1, \theta_2, T_e) = -\frac{g}{l}\sin\theta_1 + \frac{1}{ml^2}T_e$

线性化 →

$$\dot{\theta}_1 = \theta_2$$

$$\dot{\theta}_2 = -\frac{g}{l}\theta_1 + \frac{1}{ml^2}T_e$$

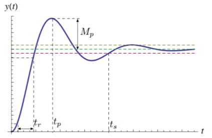
求偏导 ↓

$$\frac{\partial f_2}{\partial \theta_1} = -\frac{g}{l}\cos\theta_1 \qquad \frac{\partial f_2}{\partial \theta_2} = 0 \qquad \frac{\partial f_2}{\partial T_e} = \frac{1}{ml^2}$$

代0 ↓

$$\left.\frac{\partial f_2}{\partial \theta_1}\right|_{\theta_1=0} = -\frac{g}{l} \qquad \left.\frac{\partial f_2}{\partial \theta_2}\right|_{\theta_2=0} = 0 \qquad \left.\frac{\partial f_2}{\partial T_e}\right|_{T_e=0} = \frac{1}{ml^2}$$

Spec

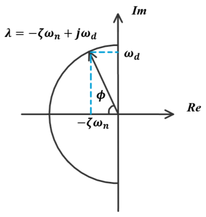


$$t_{r-rise} = \frac{1.8}{\omega_n}$$

$$t_{p-peak} = \frac{\pi}{\omega_d}$$

$$t_{s-settle} \approx \frac{3}{\sigma} = \frac{3}{\zeta\omega_n}$$

$$M_p = \exp\left(-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}\right)$$



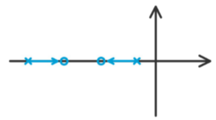
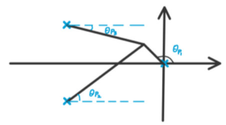
$$\lambda = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$$

$$\omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$R_{半径} = |\lambda| = \omega_n$$

$$\phi_{角度} = \cos^{-1}\zeta$$

根轨迹 $1 + K \cdot G(s) = 0$

Rule 1	$G(s) = \frac{b(s)}{a(s)}$ m 阶 n 阶 m,n谁大就有几条轨迹，从极点出发，向零点靠近
Rule 2	实轴的根轨迹存在于从右往左数的奇数个 zero/pole 的左边 
Rule 3	(n-m) 条根轨迹到无穷
Rule 4	交点 $\sigma = \frac{\Sigma_{极点} - \Sigma_{零点}}{n_{极点} - m_{零点}}$ 夹角 $\theta = \frac{2k+1}{n-m}\pi$ 轨迹点 $\Sigma_{零点角度} - \Sigma_{极点角度} = k\pi$ 出发角 $180° -$ 其他角 
Rule 5	虚轴截距，代入 $s = j\omega$
Rule 6	分离点计算 $b'a = a'b$ 检查 break 点是否在 根轨迹上，如果不在就没有 break 点

Nyquist 稳定性

$$F(s) = 1 + KG(s) = 1 + K\frac{b(s)}{a(s)} = \frac{a(s) + Kb(s)}{a(s)}$$

Z = N + P

Z = F(s) 右极点数

P = F(s) 极点数

N = 顺时针绕 -1/K 次数

Routh 稳定性

s^n	1	a_2	a_4	a_6
s^{n-1}	a_1	a_3	a_5	补0
s^{n-2}	$b_1 = -\frac{1}{a_1}\det\begin{pmatrix}1 & a_2\\ a_1 & a_3\end{pmatrix} = a_2 - \frac{a_3}{a_1}$	$b_2 = -\frac{1}{a_1}\det\begin{pmatrix}1 & a_4\\ a_1 & a_5\end{pmatrix} = a_4 - \frac{a_5}{a_1}$	$b_3 = -\frac{1}{a_1}\det\begin{pmatrix}1 & a_6\\ a_1 & a_7\end{pmatrix} = -\frac{1}{a_1}(a_7 - a_1a_6)$	…
s^{n-3}	$c_1 = -\frac{1}{b_1}\det\begin{pmatrix}a_1 & a_3\\ b_1 & b_2\end{pmatrix} = a_3 - \frac{a_1b_2}{b_1}$	$c_2 = -\frac{1}{b_1}\det\begin{pmatrix}a_1 & a_5\\ b_1 & b_3\end{pmatrix} = a_5 - \frac{a_1b_3}{b_1}$	补0	
…				
s^1	*	*		
s^0	*			

若**所有系数** & **第一列**所有数字为正，则 $p\ stable$

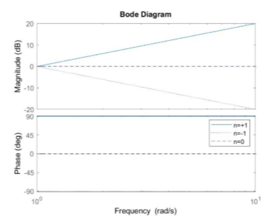
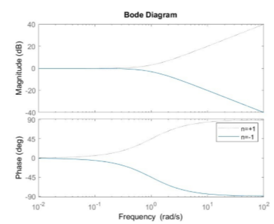
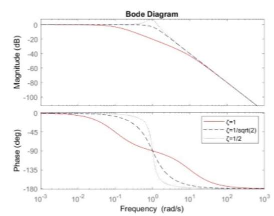
Routh 低阶推论

$$P(s) = s^2 + a_1s + a_2$$
$$a_1, a_2 > 0$$

$$P(s) = s^3 + a_1s^2 + a_2s + a_3$$
$$a_1, a_2, a_3 > 0$$
$$a_1 \cdot a_2 > a_3$$

$$P(s) = s^4 + a_1s^3 + a_2s^2 + a_3s + a_4$$
$$a_1, a_2, a_3, a_4 > 0$$
$$a_1 \cdot a_2 > a_3$$
$$a_1a_2a_3 - a_3^2 - a_1^2a_4 > 0$$

伯德图

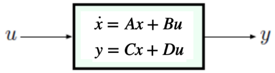
Type 1 积分 + 微分 + 常量	Type 2 一阶	Type 3 二阶
$G(s) = K_0(s)^n$	$G(s) = (Ts+1)^{\pm 1}$	$G(s) = (T^2s^2 + 2\zeta Ts + 1)^{\pm 1}$
$G(j\omega) = K_0(j\omega)^n$	$G(j\omega) = (j\omega\tau + 1)^{\pm 1}$	$G(j\omega) = \left[\left(\frac{j\omega}{\omega_n}\right)^2 + 2\zeta\frac{j\omega}{\omega_n} + 1\right]^{\pm 1}$
		

Gain Margin	$\phi(\omega) = 180°$ 时 $M(\omega)$ 到 1 的系数
Phase Margin	$M(\omega) = 1$ 时 $\phi(\omega) + 180°$ 的值

$$M(\omega) = |G(j\omega)| \qquad L(\omega) = 20 \cdot \log_{10} M(\omega)$$

$$\phi(\omega) = -\tan^{-1}\left(\frac{虚部}{实部}\right)$$

状态空间 (开环)



$$G(s) = \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D}$$
$$\det(s\boldsymbol{I} - \boldsymbol{A}) = 0$$

- A矩阵的行列式 就是 特征矩阵
- A矩阵的行列式的根 就是 特征值
- A矩阵的特征值就是 $G(s)$ 的极点
- **当A矩阵特征值在LHP时 传递函数$G(s)$稳定**

能控性矩阵 Controllability Matrix

$$\boldsymbol{\mathcal{C}} = [\boldsymbol{B} \mid \boldsymbol{A}^1\boldsymbol{B} \mid \boldsymbol{A}^2\boldsymbol{B} \ldots]$$
$$= [A^0B \mid A^1B \mid A^2B \ldots]$$

$\boldsymbol{\mathcal{C}}$ 矩阵可逆⟺能控

能观性矩阵 Observability Matrix

$$\boldsymbol{\mathcal{O}} = \begin{bmatrix} \boldsymbol{C} \\ \boldsymbol{CA}^1 \\ \vdots \\ \boldsymbol{CA}^{n-1} \end{bmatrix}$$

$\boldsymbol{\mathcal{O}}$ 矩阵可逆⟺能观

能控标准型 (单输入输出)
Controllable Canonical Form (CCF)

$$\dot{\vec{x}} = \boldsymbol{A}\vec{x} + \boldsymbol{B}u$$
$$y = \boldsymbol{C}\vec{x}$$

$$\boldsymbol{A} = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ * & * & * & \cdots & * \end{pmatrix} \quad \boldsymbol{B} = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

CCF 永远可控

能观标准型 (单输入输出)
Observable Canonical Form (OCF)

$$\dot{\vec{x}} = \boldsymbol{A}\vec{x} + \boldsymbol{B}u$$
$$y = \boldsymbol{C}\vec{x}$$

$$\boldsymbol{A} = \begin{pmatrix} 0 & 0 & \cdots & 0 & * \\ 1 & 0 & \cdots & 0 & * \\ 0 & 1 & \cdots & 0 & * \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & * \end{pmatrix} \quad \boldsymbol{C} = \begin{pmatrix} 0 & 0 & \cdots & 0 & 1 \end{pmatrix}$$

OCF 永远可观
不一定可控

将能控系统 转化为 CCF格式

$$\boldsymbol{A} = \begin{pmatrix} -15 & 8 \\ -15 & 7 \end{pmatrix} \quad \boldsymbol{B} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

第一步 (判断能控性)

$$\boldsymbol{\mathcal{C}} = \begin{pmatrix} 1 & -7 \\ 1 & -8 \end{pmatrix} \Rightarrow \det\boldsymbol{\mathcal{C}} = \text{-1} \Rightarrow \text{可控}$$

第二步 (确定 $\boldsymbol{\mathcal{C}}(\bar{\boldsymbol{A}}, \bar{\boldsymbol{B}})$)

$$\bar{\boldsymbol{A}} = \begin{pmatrix} 0 & 1 \\ * & * \end{pmatrix} \quad \bar{\boldsymbol{B}} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\boldsymbol{\mathcal{C}}(\bar{\boldsymbol{A}}, \bar{\boldsymbol{B}}) = [\bar{\boldsymbol{B}} \mid \bar{\boldsymbol{A}}\bar{\boldsymbol{B}}] = \begin{pmatrix} 0 & 1 \\ 1 & -8 \end{pmatrix}$$

第三步 (计算 T)

$$\boldsymbol{T} = \boldsymbol{\mathcal{C}}(\bar{\boldsymbol{A}}, \bar{\boldsymbol{B}}) \cdot [\boldsymbol{\mathcal{C}}(\boldsymbol{A}, \boldsymbol{B})]^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & -8 \end{pmatrix}\begin{pmatrix} 8 & -7 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$
$$\boldsymbol{T} = [\boldsymbol{\mathcal{O}}(\bar{\boldsymbol{A}}, \bar{\boldsymbol{C}})]^{-1}\boldsymbol{\mathcal{O}}(\boldsymbol{A}, \boldsymbol{C})$$

第四步 (应用坐标变换)

$$\left.\begin{aligned} \dot{x} &= \boldsymbol{A}x + \boldsymbol{B}u \\ y &= \boldsymbol{C}x \end{aligned}\right\} \xRightarrow{T\text{ 矩阵}} \left\{\begin{aligned} \dot{\bar{x}} &= \bar{\boldsymbol{A}}\bar{x} + \bar{\boldsymbol{B}}u \\ y &= \bar{\boldsymbol{C}}\bar{x} \end{aligned}\right. \quad \begin{aligned} \bar{A} &= TAT^{-1} \\ \bar{B} &= TB \\ \bar{C} &= CT^{-1} \end{aligned}$$

开环 CCF格式 特征多项式

$$A = \begin{pmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -a_n & -a_{n-1} & -a_{n-2} & \cdots & -a_1 \end{pmatrix} \quad B = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$$

特征多项式

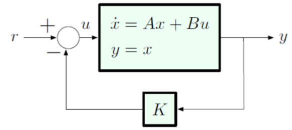
$$\det(Is - A) = s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n$$

2×2逆矩阵快捷计算方式

$$M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \det M \neq 0 \implies M^{-1} = \frac{1}{\det M}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

状态空间 (闭环)

控制极点配置 (闭环) $\mathcal{K}$



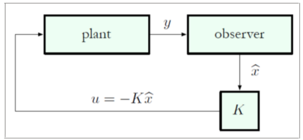
$$G(s) = \frac{Y(s)}{R(s)} = \boldsymbol{C}(\boldsymbol{Is} - \boldsymbol{A} + \boldsymbol{BK})^{-1}\boldsymbol{B} + \boldsymbol{D}$$
$$\det[Is - (A - BK)] = 0$$

闭环 CCF格式 特征多项式

$$\det(Is - A + BK) = s^n + (a_1 + k_n)s^{n-1} + \cdots + (a_{n-1} + k_2)s + (a_n + k_1)$$

观测极点配置 $\mathcal{L}$

对于自治系统 (Autonomous) 即 $u(t) = 0$



System	$\begin{cases} \dot{x} = Ax \\ y = Cx \end{cases}$
Observer	$\dot{\hat{x}} = (A - LC)\hat{x} + Ly$ $L = \boldsymbol{Output\ injection\ matrix}$
Error	$e = x - \hat{x}$ $\dot{e} = (A - LC)e$

结论:

A-LC 矩阵 特征值 为观测极点 (Observer Pole)
A-LC 矩阵 所有特征值实部小于0则收敛稳定
且特征值实部越负，收敛越快

观测极点配置 (闭环)

$$\begin{cases} \dot{x} = \boldsymbol{A}x \\ y = \boldsymbol{C}x \end{cases} \quad \boldsymbol{A} = \begin{pmatrix} 0 & 0 & \cdots & 0 & -a_n \\ 1 & 0 & \cdots & 0 & -a_{n-1} \\ 0 & 1 & \cdots & 0 & -a_{n-2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -a_1 \end{pmatrix} \quad \boldsymbol{C} = \begin{pmatrix} 0 & 0 & \cdots & 0 & 1 \end{pmatrix}$$

开环 OCF 特征方程

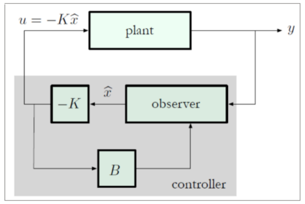
$$\begin{aligned} \det(Is - A) &= \det[(Is - A)^T] = \det(Is - A^T) \\ &= s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + a_n \end{aligned}$$

闭环 OCF 特征方程

$$\det(Is - A + LC) = s^n + (a_1 + l_n)s^{n-1} + \cdots + (a_{n-1} + l_2)s + (a_n + l_1)$$

该特征方程中，每一项的系数都单独被 l_i 影响，可单独修改

对于非自治系统 (Autonomous) 即 $u(t) \neq 0$



System	$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \end{cases}$
Observer	$\dot{\hat{x}} = (A - LC)\hat{x} + Ly + \boldsymbol{Bu}$ $L = \boldsymbol{Output\ injection\ matrix}$
Controller	$u = -K\hat{x}$
Error	$e = x - \hat{x}$ $\dot{e} = (A - LC)e$

分开看法则 Separation Principle

系统 状态空间 (原)

$$\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix} = \begin{pmatrix} A & -BK \\ LC & A - LC - BK \end{pmatrix}\begin{pmatrix} x \\ \hat{x} \end{pmatrix}$$

系统 状态空间 (坐标变换)

$$\begin{pmatrix} \dot{x} \\ \dot{e} \end{pmatrix} = \begin{pmatrix} I & 0 \\ I & -I \end{pmatrix}\begin{pmatrix} \dot{x} \\ \dot{\hat{x}} \end{pmatrix}$$

$$\begin{pmatrix} \dot{x} \\ \dot{e} \end{pmatrix} = \begin{pmatrix} A - BK & BK \\ 0 & A - LC \end{pmatrix}\begin{pmatrix} x \\ e \end{pmatrix}$$

特征方程

$$\det\begin{pmatrix} Is - A + BK & BK \\ 0 & Is - A + LC \end{pmatrix}$$
$$= \det(Is - A + BK) \cdot \det(Is - A + LC) = 0$$

分开看法则 (条件: 线性系统)

闭环特征值为

$$\det(Is - A + BK) = 0$$
$$\det(Is - A + LC) = 0$$

两者共同的特征值

Concept Question 概念题

What are the properties of Causal Linear Time invariant Systems?

- State only depend on past states but not future: consider only time. t>0

States some of the advantages in using state-space design

- Reveal more internal architecture than representation using transfer function
- Matrix representation facilitate computer analysis
- More convenient for modeling MIMO system problems.

State the key reason for using an estimator in feedback control

When the system is not readily available, too costly or impractical to measure state-variable

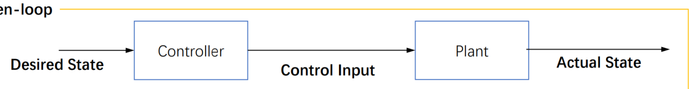
PID Parameter

P: Simplest to implement, but not always sufficient for stabilization

D: Helps achieve stability, improves time response (Arbitrary pole placement only valid for 2nd-order response. We still have control over 2 dominant poles)

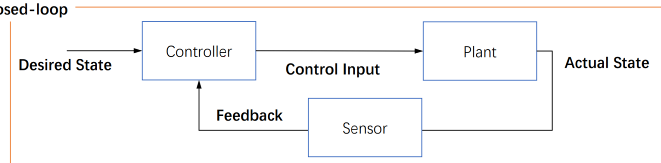
I: Essential for perfect steady-state tracking of constant reference and rejection of constant disturbance (It can destabilize the system if feedback loop is broken)

Open-loop



Through preestablished model, generate the command that will achieve the desired state.

Closed-loop



Through sensors, we are able to **feedback** the measurement to produce the command that will minimize the error between desired and actual targeted profile.

Plant	system being controlled
Sensor	Measure the quantity that is subject to control
Actuator	act on the plant
Controller	processes thee sensor signals and drives the actuators
Control Law	the rule for mapping sensor signals to actuator signals

P: Simplest to implement, but not always sufficient for stabilization

D: Helps achieve stability, improves time response (Arbitrary pole placement only valid for 2nd-order response. We still have control over 2 dominant poles)

I: Essential for perfect steady-state tracking of constant reference and rejection of constant disturbance (It can destabilize the system if feedback loop is broken)

拉普拉斯变换

① $\mathcal{L}\{1\} = \frac{1}{s}$ $\qquad \int_0^\infty e^{-st} \cdot 1\, dt = \frac{1}{s}$

② $\mathcal{L}\{e^t\} = \frac{1}{s-1}$ $\qquad \int_0^\infty e^{-st} \cdot e^t dt$

$\mathcal{L}\{e^{ct}\} = \frac{1}{s-c}$

③ $\mathcal{L}\{\cos t\} = \frac{s}{s^2+1}$ $\qquad \cos t = \frac{e^{it} - e^{-it}}{2}$

$\mathcal{L}\{\cos ct\} = \frac{s}{s^2+c^2}$

④ $\mathcal{L}\{\sin t\} = \frac{1}{s^2+1}$

$\mathcal{L}\{\sin ct\} = \frac{c}{s^2+c^2}$

⑤ $\mathcal{L}\{t^n\} = \frac{n!}{s^{n+1}}$ $\qquad \int_0^\infty e^{-st} \cdot t^n dt$ 　令$k = st$

$dk = sdt$

$= \int_0^\infty \left(\frac{k}{s}\right)^n e^{-k} \frac{dk}{s} = \frac{1}{s^{n+1}} \int_0^\infty k^n e^{-k} dk = \frac{n!}{s^{n+1}}$

⑥ $\mathcal{L}\{|t|\} = \frac{1}{s(e^s - 1)}$

⑦ $\mathcal{L}\{t^n e^{ct}\} = \frac{n!}{(s-c)^{n+1}}$

Free + Forced Response

$$x(t) = x_h(t) + x_p(t)$$

Transient + Steady-state response

$$x(t) = x_{tr}(t) + x_{ss}(x)$$

超前补偿器 (加上零点的同时，加上一个极点)

$$H(s) = \frac{s-z}{s-p} \qquad 其中|z| < |p|$$

零点在极点的左边

滞后补偿器

$$H(s) = \frac{s-z}{s-p} \qquad 其中|z| > |p|$$

零点在极点的右边

PD 控制

改善系统瞬态响应

PI 控制

减少系统稳态误差

$$H(s) = \frac{s+z}{s+0} = \boxed{\underbrace{1}_{P} + \underbrace{\frac{z}{s}}_{I}}$$

比例积分