

1.3 Conditional Probability

Conditional Probability

	Early (E)	Late (L)	Totals
Red (R)	5	8	13
Yellow (Y)	3	4	7
Totals	8	12	20

What is the probability of selecting a red bulb? (marginal)

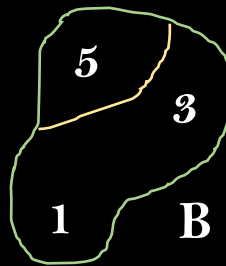
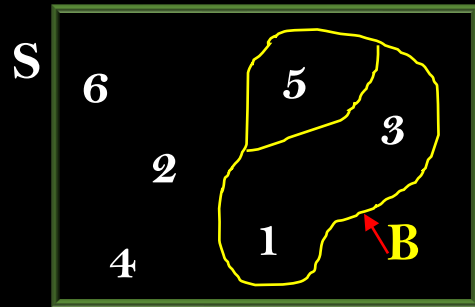
What is the probability of selecting a red bulb if you know the flower will bloom early? (conditional)

Conditional Probability

A **conditional probability** is a probability that is updated to take into account the (known) occurrence of another event.

Conditional Probability Example

- With a fair die being rolled once, define $A = \{5\}$
- Then, $P[A] = 1/6$
- What if someone rolls the die and doesn't tell us the number showing. Tells us only that event $B = \{\text{odd number}\}$ occurs?



Conditional Probability

Definition 1.3-1

The **conditional probability** of an event A , given that event B has occurred, is defined by

$$P(A | B) = \frac{P(A \cap B)}{P(B)},$$

provided that $P(B) > 0$.

Multiplication Rule

Definition 1.3-2

The probability that two events, A and B , both occur is given by the **multiplication rule**,

$$P(A \cap B) = P(A)P(B | A),$$

provided $P(A) > 0$ or by

$$P(A \cap B) = P(B)P(A | B)$$

provided $P(B) > 0$.

Examples

For a randomly selected off-campus student at UIUC on any given day, assume:

$$\begin{aligned}P[\text{Bikes to campus}] &= 0.4, \\P[\text{Rides bus to campus}] &= 0.3, \\P[\text{Does both}] &= 0.04.\end{aligned}$$

1. What is the probability that a student bikes to campus, given that they ride the bus?
2. What is the probability that a student bikes to campus, given that they don't ride the bus?

Examples

For a randomly selected off-campus student at UIUC on any given day, assume:

$$\begin{aligned}P[\text{Bikes to campus}] &= 0.4, \\P[\text{Rides bus to campus}] &= 0.3, \\P[\text{Does both}] &= 0.04.\end{aligned}$$

1. What is the probability that a student bikes to campus, given that they ride the bus?
2. What is the probability that a student bikes to campus, given that they don't ride the bus?

- 3. Suppose you know that a student does not bike to campus. Find the probability that this student does not take the bus.

$P[\text{Bikes to campus}] = 0.4,$
 $P[\text{Rides bus to campus}] = 0.3,$
 $P[\text{Does both}] = 0.04.$

Suppose you draw two cards out of a standard deck (52 cards) without replacement.

Let A = first card is an Ace

B = second card is an Ace

4. What is the probability that you will draw two aces (out of two draws)?

5. Find the probability at that least one of the two cards is an Ace.

6. If you draw 4 cards, what is the probability that you get exactly 2 Aces?

6a. If you draw 10 cards, what is the probability that you get exactly 3 diamonds?

7. Two fair 6-sided dice are rolled. What is the probability that the number on the first die was at least as large as 4 given that the sum of the two dice was 8?

7. Two fair 6-sided dice are rolled. What is the probability that the number on the first die was at least as large as 4 given that the sum of the two dice was 8? (Using R)

To sample rolling two die:

```
dieRoll <- sample(1:6, 2, replace = TRUE)
```

Using that roll, check to see if the sum is equal to 8 AND the first die is at least as large as 4:

```
sum(dieRoll) == 8 && dieRoll[1]>=4
```

```
probAB <- mean(replicate(10000,  
  {dieRoll <- sample(1:6, 2, replace = TRUE)  
    sum(dieRoll) == 8 && dieRoll[1]>=4}))
```

```
probB <- mean(replicate(10000,  
  {dieRoll <- sample(1:6, 2, replace = TRUE)  
    sum(dieRoll) == 8  }))
```