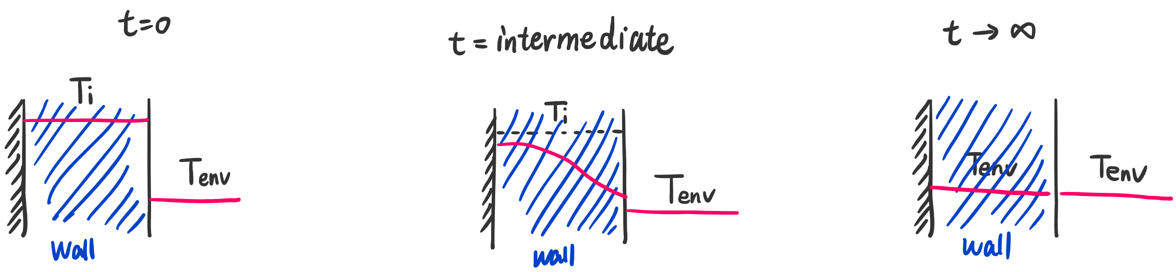
ME 320 Homework 2

I.

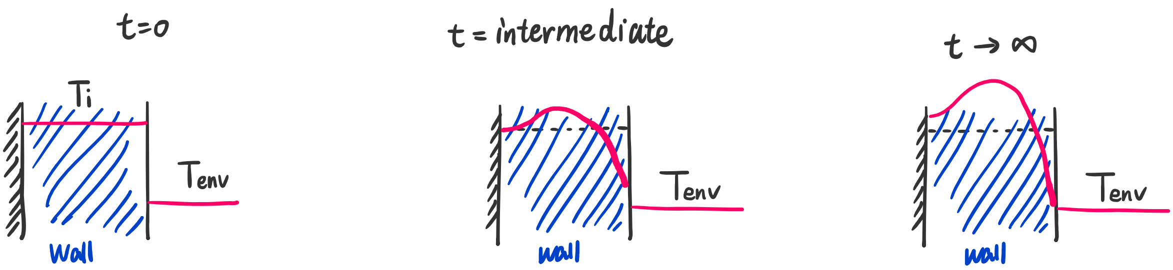
Assume a 1D plane wall, draw its temperature profile at t = 0, t -> ∞, and an intermediate t value when

1) **The wall is at an initial temperature T_i and insulated on one side with the other side exposed to an environment at constant temperature. No heat generation inside the wall.**
2) **The wall is at an initial temperature T_i and insulated on one side with the other side exposed to an environment at constant temperature. There is volumetric heat generation (g) inside the wall.**
3) **The wall is at an initial temperature T_i with exposed sides. Each of the sides is in contact with a different surrounding of a different temperature but has the same convective heat transfer to the wall. Here, $T_1 > T_i > T_2$.**

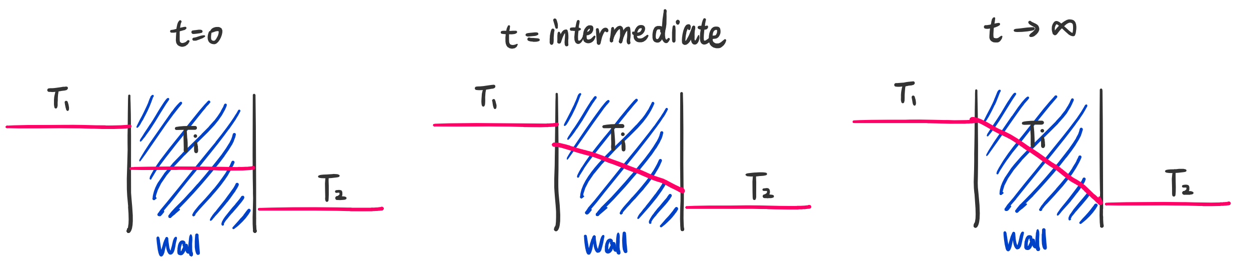
(1)



(2)

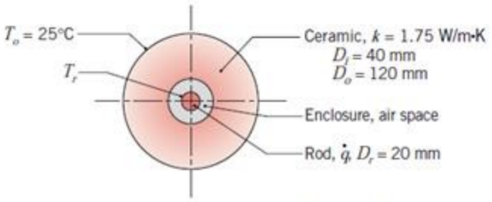


(3)



II. 3.48

Electric current flows through a long rod generating thermal energy at a uniform volumetric rate of $\dot{q} = 2 \times 10^6\ \mathrm{W/m^3}$. The rod is concentric with a hollow ceramic cylinder, creating an enclosure that is filled with air.



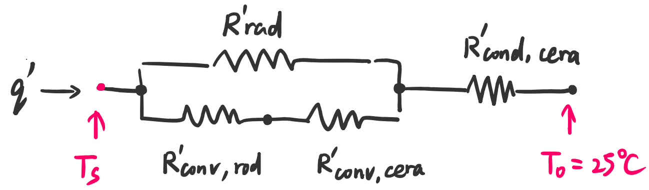
The thermal resistance per unit length due to radiation between the enclosure surfaces is $R'_{rad} = 0.30\ \mathrm{m \cdot K/W}$, and the coefficient associated with free convection in the enclosure is $h = 20\ \mathrm{W/m^2 \cdot K}$.

(a) Construct a thermal circuit that can be used to calculate the surface temperature of the rod, T_r. Label all temperatures, heat rates, and thermal resistances, and evaluate each thermal resistance.

(b) Calculate the surface temperature of the rod for the prescribed conditions.

为什么没有 $R_{cond,rod}$?

(a) 画出等效热阻电路，计算等效总热阻



$$R'_{rad} = 0.3\ \mathrm{mK/W}$$

$$R'_{conv,\,rod} = \frac{1}{h \cdot L_{rod}} = \frac{1}{h \cdot \pi \cdot D_r} = 0.8\ \mathrm{mK/W}$$

$$R'_{conv,\,cera} = \frac{1}{h \cdot L_{cera}} = \frac{1}{h \cdot \pi \cdot D_i} = 0.4\ \mathrm{mK/W}$$

$$R'_{cond,\,cera} = \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi k} = 0.1\ \mathrm{mK/W}$$

$$R'_{total} = R'_{cond,\,cera} + \left[\frac{1}{R'_{rad}} + \frac{1}{R'_{conv,\,rod} + R'_{conv,\,cera}}\right]^{-1} = 0.34\ \mathrm{mK/W}$$

(b) 计算rod表面温度 T_s

$$q' = \frac{T_s - T_0}{R'_{total}}$$

$$\dot{q} \cdot \pi \cdot R_r^2 = \frac{T_s - T_0}{R'_{total}}$$

$$T_s = 239°C$$

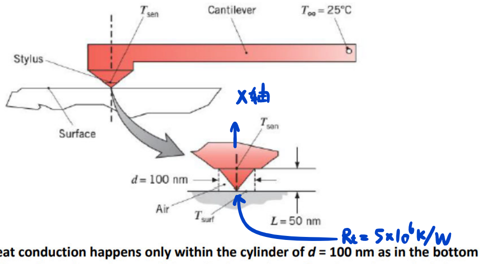
III. 3.40

A device used to measure the surface temperature of an object to within a spatial resolution of approximately 50 nm is shown in the schematic. It consists of an extremely sharp-tipped stylus and an extremely small cantilever that is scanned across the surface. The probe tip is of circular cross section and is fabricated of polycrystalline silicon dioxide. The ambient temperature is measured at the pivoted end of the cantilever as $T_\infty = 25°C$, and the device is equipped with a sensor to measure the temperature at the upper end of the sharp tip, T_{sen}. The thermal resistance between the sensing probe and the pivoted end is $R_t = 5 \times 10^6$ K/W.

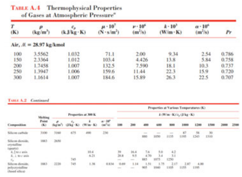
(a) Determine the thermal resistance between the surface temperature and the sensing temperature.

(b) If the sensing temperature is $T_{sen} = 28.5°C$, determine the surface temperature.

Hint: Although nanoscale heat transfer effects may be important, assume that the conduction occurring in the air adjacent to the probe tip can be described by Fourier's law and the thermal conductivity found in Table A.4.



HINT 2: Assume heat conduction happens only within the cylinder of d = 100 nm as in the bottom figure. Consider if heat flows from the top of this cylinder to the surface, how do you find the x-sectional area of the probe and the air that the heat passes through at any height (h) above this surface.



根据 Table A.2 + A.4

$k_{Si} = 1.38\ W/mK$	polycrystalline silicon dioxide (300K)
$k_{air} = 0.0263\ W/mK$	air (300K)

(a) 计算热阻 (T_{sur}与T_{sen}之间)

① 热阻公式

$$R_{sen} = \frac{T_{sur} - T_{sen}}{q}$$

② 傅立叶定律

$$q = -k_{air}\cdot A_{air}\cdot\frac{dT}{dx} - k_{Si}\cdot A_{Si}\cdot\frac{dT}{dx}$$

$$r = \frac{\frac{D}{2}}{L}\cdot x$$

$$A_{air} = \pi r^2$$

$$A_{Si} = \pi\left(\frac{D}{2}\right)^2 - A_{air}$$

$$\therefore\ q = -\frac{\pi D^2}{4L^2}\left(k_{air}(L^2-x^2) + k_{Si}\,x^2\right)\frac{dT}{dx}$$

$$\frac{q}{k_{air}(L^2-x^2)+k_{Si}\cdot x^2} = -\frac{\pi D^2}{4L^2}\cdot\frac{dT}{dx}$$

← T是一个关于x的函数

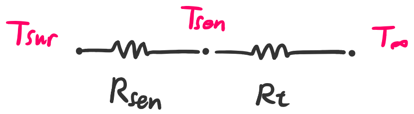
$$q\cdot\int_{x=0}^{x=L}\frac{1}{k_{air}(L^2-x^2)+k_{Si}\cdot x^2}\cdot dx = \int_{x=0}^{x=L} -\frac{\pi D^2}{4L^2}\cdot\frac{dT(x)}{dx}\cdot dx$$

$$q\cdot\frac{1}{\sqrt{k_{air}(k_{Si}-k_{air})}}\cdot\tan^{-1}\sqrt{\frac{k_{Si}-k_{air}}{k_{air}}} = -\frac{\pi D^2}{4L^2}\cdot(T_{sen}-T_{sur})$$

$$R_{sen} = \frac{4L^2}{\pi D^2}\cdot\frac{\tan^{-1}\sqrt{\frac{k_{Si}-k_{air}}{k_{air}}}}{\sqrt{k_{air}(k_{Si}-k_{air})}} = \frac{T_{sur}-T_{sen}}{q}$$

$$= 48\times10^6\ K/W$$

(b) 若$T_{sen} = 28.5°C$，计算 T_{sur}



$$\because\ \frac{T_{sur}-T_{sen}}{R_{sen}} = \frac{T_{sen}-T_\infty}{R_t}$$

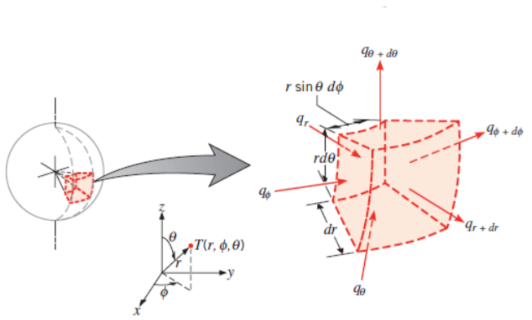
$$\therefore\ T_{sur} = 62\ °C$$

IV.

Derive the Heat Equation for a spherical coordinate system. Simplify the equation when there is no heat generation in the conduction material but is also undergoing a transient process.

Spherical Coordinates:

$$\boxed{\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(k\sin\theta\frac{\partial T}{\partial \theta}\right)+\dot{q}=\rho c_p\frac{\partial T}{\partial t} \qquad (2.29)}$$



能量守恒

$$\dot{E}_{in}-\dot{E}_{out}+\dot{E}_g=\dot{E}_{st}$$

$$\because Q=c_p\cdot m\cdot \Delta T$$

$$\therefore \dot{E}_{st}=\rho\cdot C_p\cdot\frac{\partial T}{\partial t}\cdot\underbrace{(r\cdot\sin\theta\, d\phi\cdot r\cdot d\theta\cdot dr)}_{\text{体积}}$$

$$\dot{E}_g=\dot{q}\,(r\cdot\sin\theta\, d\phi\cdot r\cdot d\theta\cdot dr)$$

$$\dot{E}_{in}-\dot{E}_{out}=-\frac{\partial q_r}{\partial r}\cdot dr-\frac{\partial q_\phi}{\partial \phi}\cdot d\phi-\frac{\partial q_\theta}{\partial \theta}\cdot d\theta$$

其中
$$\begin{cases} q_r=-k\cdot r\,d\theta\cdot r\sin\theta\cdot d\phi\cdot\dfrac{\partial T}{\partial r}\\ q_\theta=-k\cdot r\sin\theta\cdot d\phi\cdot dr\cdot\dfrac{1}{r}\cdot\dfrac{\partial T}{\partial \theta}\\ q_\phi=-k\cdot r\,d\theta\, dr\,\dfrac{1}{r\sin\theta}\cdot\dfrac{\partial T}{\partial \phi}\end{cases}$$

$$\therefore \frac{1}{r^2}\cdot\frac{\partial}{\partial r}\left(kr^2\cdot\frac{\partial T}{\partial r}\right)+\frac{1}{r^2\sin^2\theta}\cdot\frac{\partial}{\partial \phi}\left(k\frac{\partial T}{\partial \phi}\right)+\frac{1}{r^2\sin\theta}\cdot\frac{\partial}{\partial \theta}\left(k\sin\theta\cdot\frac{\partial T}{\partial \theta}\right)+\dot{q}=\rho\cdot C_p\cdot\frac{\partial T}{\partial t}$$