Example: Compute x_{ss} for $\frac{1}{4}\dot{x} + 3x = \underbrace{\frac{9}{5}\sin(\frac{1}{2}t)}_{u(t)}$

$X(s) = \mathcal{L}\{x(t)\}$, $U(s) = \mathcal{L}\{u(t)\}$ $\qquad \omega = \frac{1}{2}$, $U_0 = \frac{9}{5}$

$$G(s) = \frac{X(s)}{U(s)} = \frac{1}{\frac{1}{4}s + 3} \qquad M(\omega) = |G(j\omega)| = \frac{1}{\sqrt{9 + \omega^2/16}} \rightsquigarrow M(\omega = \frac{1}{2}) = 0.33$$

$$G(j\omega) = \frac{1}{3 + j\,\omega/4} \qquad \phi = \angle G(j\omega) = 0 - \tan^{-1}\left(\frac{\omega/4}{3}\right) \rightsquigarrow \phi(\omega = \frac{1}{2}) = -0.04 \text{ rad/s}$$

$$x_{ss}(t) = 0.33 \times \frac{9}{5} \times \sin(\frac{1}{2}t - 0.04)$$

$$= 0.6 \sin(\frac{1}{2}t - 0.04)$$

Example: $4\ddot{x} + 4\dot{x} + 13x = 2.4 \sin(2t)$, x_{ss}?

$u(t) = 2.4 \sin(2t)$, $\omega = 2$ rad/s, $U_0 = 2.4$

$$G(s) = \frac{1}{4s^2 + 4s + 13} \qquad M(\omega) = |G(j\omega)| = \frac{1}{\sqrt{(13 - 4\omega^2)^2 + (4\omega)^2}}$$

$$G(j\omega) = \frac{1}{(13 - 4\omega^2) + j\,4\omega} \qquad M(\omega = 2) = 0.117$$

$$\phi(\omega) = 0 - \tan^{-1}\left(\frac{4\omega}{13 - 4\omega^2}\right)$$

$$\phi(\omega = 2) = -1.93 \text{ rad}$$

$$x_{ss}(t) = 0.117 \times 2.4 \times \sin(2t - 1.93)$$

Big picture: ODE, TF } represent dynamical systems

- ODE $\longrightarrow$ TF(s)
- Usually we care about the $x_{ss}(t)$
- Any input u(t) can be written as a sum of sinusoidal terms
- If we know response to sin $\longrightarrow$ we know response to any u(t)
- Frequency response $G(j\omega)$ is determined by the $M(\omega)$ and $\phi(\omega)$

Bode plots: Are graphical representation of $G(j\omega)$

- Given $G(j\omega) = |G(j\omega)| e^{\angle G(j\omega)} = M(\omega) e^{\phi(\omega)}$
- The plots consist of: $\begin{cases} 20 \log_{10}(M(\omega)) \text{ vs } \omega \\ \phi(\omega) \text{ vs } \omega \end{cases}$
- semilog scale $\rightsquigarrow$ log in frequency
- $M(\omega)$ is plotted in decibel (dB)
- TF given by poly/poly and any poly can be broken down with $(Ts+1)$ and $(s^2+2\zeta\omega_n s+\omega_n^2)$

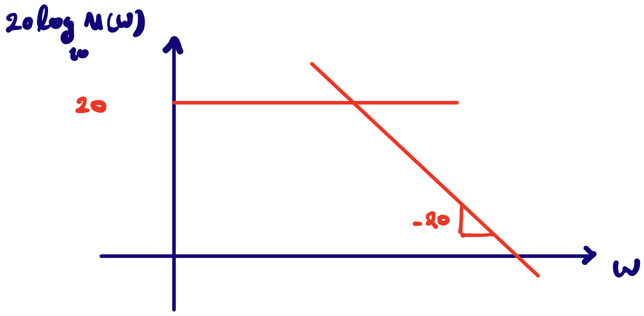
Bode plots of 1st order system:

Consider $G(s) = \dfrac{10}{0.5s+1}$ $\rightsquigarrow$ $\phi = 0 - \tan^{-1}\left(\dfrac{0.5\omega}{1}\right)$

$G(j\omega) = \dfrac{10}{0.5\omega j + 1}$ $\rightsquigarrow$ $M(\omega) = \dfrac{10}{\sqrt{1+0.25\omega^2}}$

$$20\log_{10} M(\omega) = 20\log_{10}\frac{10}{\sqrt{1+0.25\omega^2}} = 20\log_{10} 10 - 20\log\left(\sqrt{0.25\omega^2+1}\right)$$

$$= 20 - 10\log\left(0.25\omega^2+1\right)$$



- when $\omega \to 0$ $(\omega \ll 2)$

$20\log_{10} M(\omega) = 20\log 10 - \underbrace{10\log(0.25\omega^2+1)}_{\approx 0}$

$= 20\ dB$

↳ DC gain

(low frequencies magnitude)

- when $\omega \to \infty$ $(\omega \gg 2)$

$20\log_{10} M(\omega) = 20 - 10\log(0.25\omega^2)$

$= 20 - 10\log 0.25 - 20\log\omega$