Lab 5 Pre-Lab

1. Find the natural frequencies and mode shapes of a linear, time-invariant mechanical system with

$$M = \begin{pmatrix} 3 & 0 \\ 0 & 3/4 \end{pmatrix}, K = \begin{pmatrix} 4 & -3 \\ -3 & 5 \end{pmatrix}.$$

$$\because\ 0 = \det(K - M\cdot w^2) = \frac{9}{4}w^4 - 18w^2 + 11 = \left(\frac{3}{2}w^2 - 11\right)\left(\frac{3}{2}w^2 - 1\right)$$

$$\therefore\ w_1 = \sqrt{\frac{22}{3}},\ w_2 = \sqrt{\frac{2}{3}}$$

$$(K - M w_1^2)\begin{pmatrix} A_1 \\ A_2 \end{pmatrix} = 0 \Rightarrow 6A_1 = -A_2$$

$$(K - M w_2^2)\begin{pmatrix} B_1 \\ B_2 \end{pmatrix} = 0 \Rightarrow B_1 = \frac{3}{2}B_2$$

2. Suppose that an undamped, linear, time-invariant, two-degree-of-freedom mechanical system has a mode shape

$\zeta = 0$ $\quad \ddot{x} + 0 + \omega_n^2 x = 0$

$$\begin{pmatrix} 4 \\ -3 \end{pmatrix}$$

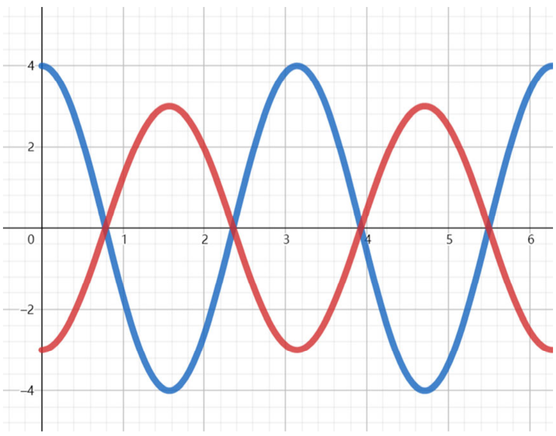
$\omega_n = 2$

with corresponding natural frequency 2. Sketch the steady-state response for the two degrees of freedom if the inputs $f_1(t)$ and $f_2(t)$ are both harmonic with frequency 1.98 and there is small damping present.

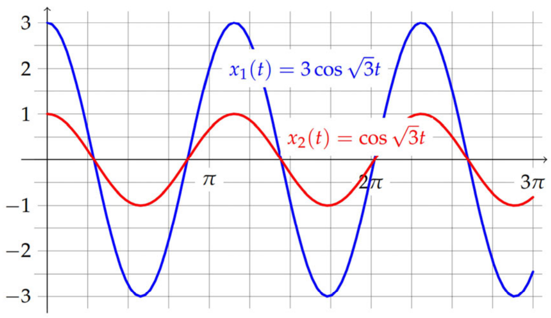
$f = 1.98$

$x_1(t) = 4\cos(2t)$

$x_2(t) = -3\cos(2t)$



3. Consider a linear, time-invariant, two-degree-of-freedom mechanical system that has two natural frequencies whose ratio is not a rational number. Suppose that a combination of impulses $f_1(t)$ and $f_2(t)$ results in the response shown in the graph below. Use this to determine one of the natural frequencies and the corresponding mode shape.



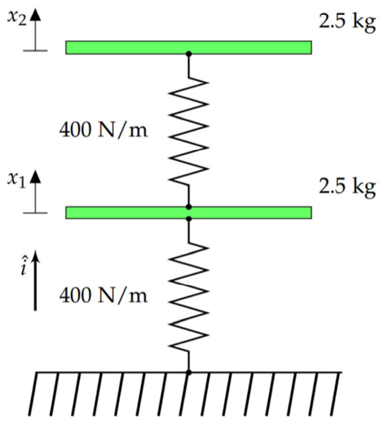
$\because$ $x_1(t)$ and $x_2(t)$ are harmonic

$\therefore$ They must be free response

From the graph we know $w = \sqrt{3}$ column matrix is $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$

phase difference is π

4. Consider the two-degree-of-freedom mechanical suspension, and upper plate, respectively, relative to the undeformed configuration of the two springs.



Compute the natural frequencies and the corresponding mode shapes.

$$M \begin{pmatrix} \ddot{x}_1(t) \\ \ddot{x}_2(t) \end{pmatrix} + K \cdot \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$M = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix}, \quad K = \begin{pmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{pmatrix}$$

$$w^2 = \frac{m_1 k_2 + m_2 k_1 + m_2 k_2 \pm \sqrt{(m_1 k_2 + m_2 k_1 + m_2 k_2)^2 - 4 m_1 m_2 k_1 k_2}}{2 m_1 \cdot m_2}$$

$\therefore W_1 = 20.467, \quad W_2 = 7.8175$