黑体辐射

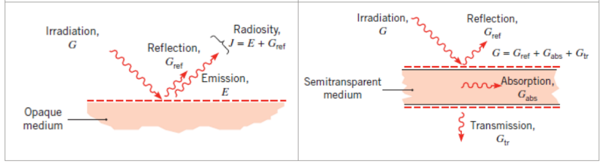
TABLE 12.2 Blackbody Radiation Functions

λT (μm·K)	$F_{(0\to\lambda)}$	$I_{\lambda,b}(\lambda,T)/\sigma T^5$ (μm·K·sr)$^{-1}$	$I_{\lambda,b}(\lambda,T)/I_{\lambda,b}(\lambda_{max},T)$
200	0.000000	0.375034×10^{-27}	0.000000
400	0.000000	0.490335×10^{-13}	0.000000
600	0.000000	0.104046×10^{-8}	0.000014
800	0.000016	0.991126×10^{-7}	0.001372
1,000	0.000321	0.118505×10^{-5}	0.016406
1,200	0.002134	0.523927×10^{-5}	0.072534
1,400	0.007790	0.134411×10^{-4}	0.186082
1,600	0.019718	0.249130	0.344904
1,800	0.039341	0.375568	0.519949
2,000	0.066728	0.493432	0.683123
2,200	0.100888	0.589649×10^{-4}	0.816329
2,400	0.140256	0.658866	0.912155
2,600	0.183120	0.701292	0.970891
2,800	0.227897	0.720239	0.997123
2,898	0.250108	0.722318×10^{-4}	1.000000
3,000	0.273232	0.720254×10^{-4}	0.997143
3,200	0.318102	0.705974	0.977373
3,400	0.361735	0.681544	0.943551
3,600	0.403607	0.650396	0.900429
3,800	0.443382	0.615225×10^{-4}	0.851737
4,000	0.480877	0.578064	0.800291
4,200	0.516014	0.540394	0.748139
4,400	0.548796	0.503253	0.696720
4,600	0.579280	0.467343	0.647004
4,800	0.607559	0.433109	0.599610
5,000	0.633747	0.400813	0.554898
5,200	0.658970	0.370580×10^{-4}	0.513043
5,400	0.680360	0.342445	0.474092
5,600	0.701046	0.316376	0.438002
5,800	0.720158	0.292301	0.404671
6,000	0.737818	0.270121	0.373965
6,200	0.754140	0.249723×10^{-4}	0.345724
6,400	0.769234	0.230985	0.319783
6,600	0.783199	0.213786	0.295973
6,800	0.796129	0.198008	0.274128
7,000	0.808109	0.183534	0.254090
7,200	0.819217	0.170256×10^{-4}	0.235708
7,400	0.829527	0.158073	0.218842
7,600	0.839102	0.146891	0.203360
7,800	0.848005	0.136621	0.189143
8,000	0.856288	0.127185	0.176079
8,500	0.874608	0.106772×10^{-4}	0.147819
9,000	0.890029	0.901463×10^{-5}	0.124801
9,500	0.903085	0.765338	0.105956
10,000	0.914199	0.653279×10^{-5}	0.090442
10,500	0.923710	0.560522	0.077600
11,000	0.931890	0.483321	0.066913
11,500	0.939959	0.418725	0.057970
12,000	0.945098	0.364394×10^{-5}	0.050448
13,000	0.955139	0.279457	0.038689
14,000	0.962898	0.217641	0.030131
15,000	0.969981	0.171866×10^{-5}	0.023794
16,000	0.973814	0.137429	0.019026
18,000	0.980860	0.908240×10^{-6}	0.012574
20,000	0.985602	0.623310	0.008629
25,000	0.992215	0.276474	0.003828
30,000	0.995340	0.140469×10^{-6}	0.001945
40,000	0.997967	0.473891×10^{-7}	0.000656
50,000	0.998953	0.201605	0.000279
75,000	0.999713	0.418597×10^{-8}	0.000058
100,000	0.999905	0.135752	0.000019

$$F_{(0\to\lambda)} \equiv \frac{\int_0^\lambda E_{\lambda,b}\,d\lambda}{\int_0^\infty E_{\lambda,b}\,d\lambda} = \frac{\int_0^\lambda E_{\lambda,b}\,d\lambda}{\sigma T^4} = \int_0^{\lambda T} \frac{E_{\lambda,b}}{\sigma T^5}\,d(\lambda T) = f(\lambda T) \qquad (12.34)$$

辐射基本概念

不透明材料 (Opaque)	半透明材料



ρ	反射率 Reflectivity	$\rho = \frac{G_{ref}}{G}$
α	吸收率 Absorptivity	$\alpha = \frac{G_{abs}}{G}$
ε	发射率 Emissivity	
τ	透射率 Transmissivity	$\tau = \frac{G_{tr}}{G}$

I	强度	Intensity
E	放射	Emission
G	照射	Irradiation
$J = E + G_{反射} = E + \rho G$	辐射度	Radiosity
q	净热辐射率	Net radiation heat rate

能量守恒

任意介质	$\rho + \alpha + \tau = 1$
不透明介质	$\rho + \alpha = 1$

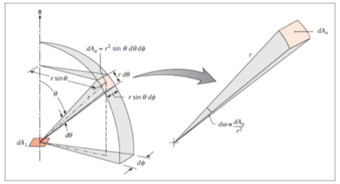
热力平衡 (吸收=发射)

$\varepsilon = \alpha$

推导结论

$\varepsilon = 1 - \rho$

立体角 计算公式



$$d\omega = \frac{dA_n}{r^2}$$

$$= \sin\theta\, d\theta\, d\phi$$

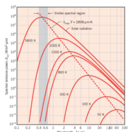
一个完整半球的立体角计算如下

$$\omega_{半球} = \int_0^{2\pi}\int_0^{\frac{\pi}{2}} \sin\theta\, d\theta\, d\phi = 2\pi \text{ sr}$$

黑体辐射 (理想物体)
Blackbody Radiation

普朗克分布 Planck distribution

$$E_{\lambda,b}(\lambda_{波长}, T_{温度}) = \pi I_{\lambda,b}(\lambda, T) = \frac{C_1}{\lambda^5\left[\exp\left(\frac{C_2}{\lambda T}\right) - 1\right]}$$



$C_1 = 3.742 \times 10^8\ W \cdot \mu m^4/m^2$

$C_2 = 1.439 \times 10^4\ \mu m \cdot K$

韦恩定律 Wien's displacement law

$\lambda_{Max} T = C_3 = 2898\ \mu m$

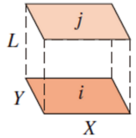
斯蒂芬-玻尔兹曼定律 The Stefan-Boltzmann Law

$E_b = \sigma T^4 \qquad E_b = \pi I_b = \int_0^\infty E_{\lambda,b}\, d\lambda = \sigma T^4$

$\sigma = 5.67 \times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4$

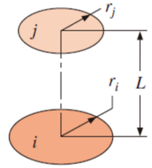
伯茨曼常数

View Factor 角系数



$$F_{ij} = \frac{2}{\pi \bar{X}\bar{Y}}\left\{\ln\left[\frac{(1+\bar{X}^2)(1+\bar{Y}^2)}{1+\bar{X}^2+\bar{Y}^2}\right]^{1/2} + \bar{X}(1+\bar{Y}^2)^{1/2}\tan^{-1}\frac{\bar{X}}{(1+\bar{Y}^2)^{1/2}} + \bar{Y}(1+\bar{X}^2)^{1/2}\tan^{-1}\frac{\bar{Y}}{(1+\bar{X}^2)^{1/2}} - \bar{X}\tan^{-1}\bar{X} - \bar{Y}\tan^{-1}\bar{Y}\right\}$$

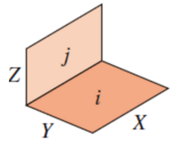
$\bar{X} = \frac{X}{L}$, $\bar{Y} = \frac{Y}{L}$



$R_i = r_i/L,\ R_j = r_j/L$

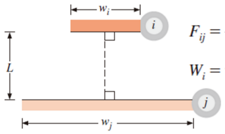
$$S = 1 + \frac{1 + R_j^2}{R_i^2}$$

$$F_{ij} = \frac{1}{2}\{S - [S^2 - 4(r_j/r_i)^2]^{1/2}\}$$



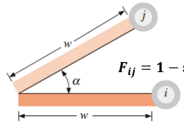
$H = Z/X,\ W = Y/X$

$$F_{ij} = \frac{1}{\pi W}\left(W\tan^{-1}\frac{1}{W} + H\tan^{-1}\frac{1}{H} - (H^2+W^2)^{1/2}\tan^{-1}\frac{1}{(H^2+W^2)^{1/2}} + \frac{1}{4}\ln\left\{\frac{(1+W^2)(1+H^2)}{1+W^2+H^2}\left[\frac{W^2(1+W^2+H^2)}{(1+W^2)(W^2+H^2)}\right]^{W^2} \times \left[\frac{H^2(1+H^2+W^2)}{(1+H^2)(H^2+W^2)}\right]^{H^2}\right\}\right)$$

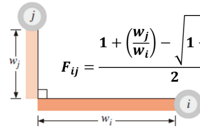


$$F_{ij} = \frac{[(W_i+W_j)^2+4]^{1/2} - [(W_j-W_i)^2+4]^{1/2}}{2W_i}$$

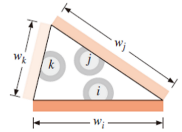
$W_i = w_i/L,\ W_j = w_j/L$



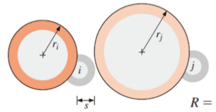
$$F_{ij} = 1 - \sin\left(\frac{\alpha}{2}\right)$$



$$F_{ij} = \frac{1 + \left(\frac{w_j}{w_i}\right) - \left[1 + \left(\frac{w_j}{w_i}\right)^2\right]^{1/2}}{2}$$



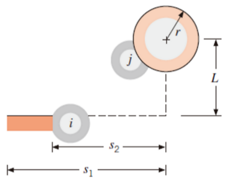
$$F_{ij} = \frac{w_i + w_j - w_k}{2w_i}$$



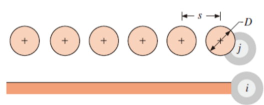
$$F_{ij} = \frac{1}{2\pi}\left\{\pi + [C^2 - (R+1)^2]^{1/2} - [C^2 - (R-1)^2]^{1/2} + (R-1)\cos^{-1}\left[\left(\frac{R}{C}\right) - \left(\frac{1}{C}\right)\right] - (R+1)\cos^{-1}\left[\left(\frac{R}{C}\right) + \left(\frac{1}{C}\right)\right]\right\}$$

$R = r_j/r_i,\ S = s/r_i$

$C = 1 + R + S$



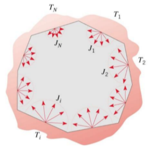
$$F_{ij} = \frac{r}{s_1 - s_2}\left[\tan^{-1}\frac{s_1}{L} - \tan^{-1}\frac{s_2}{L}\right]$$



$$F_{ij} = 1 - \left[1 - \left(\frac{D}{s}\right)^2\right]^{1/2} + \left(\frac{D}{s}\right)\tan^{-1}\left[\left(\frac{s^2 - D^2}{D^2}\right)^{1/2}\right]$$

辐射角系数 法则

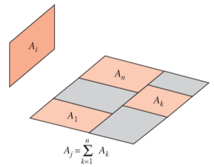
- **Reciprocity Relation**
  $A_i F_{ij} = A_j F_{ji}$
- **Summation Rule**
  $\sum_{j=1}^{N} F_{ij} = 1$



细分面 Subdivided Surface

$$F_{ij} = \sum_{k=1}^{n} F_{ik} \qquad A_j F_{ji} = \sum_{k=1}^{n} A_k F_{ki}$$

$$F_{ji} = \frac{\sum_{k=1}^{n} A_k F_{ki}}{\sum_{k=1}^{n} A_k}$$

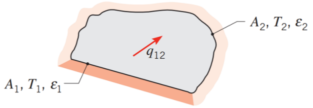


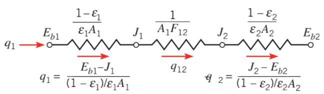
辐射阻

$$R_{表面辐射阻} = \frac{1-\varepsilon}{\varepsilon A} \qquad R_{空间辐射阻} = \frac{1}{AF}$$

Case 1 两表面 Two-surface Enclosure

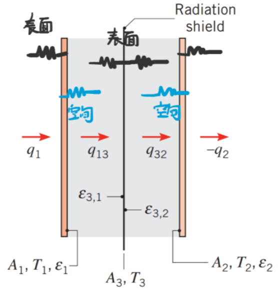
最简单类型，辐射热交换仅发生在两个表面

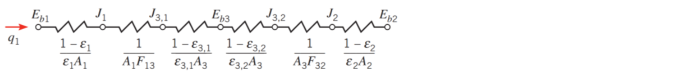




$$q_1 = \frac{\sigma(T_1^4 - T_2^4)}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1} + \frac{1}{A_1 F_{12}} + \frac{1-\varepsilon_2}{\varepsilon_2 A_2}}$$

Case 2 辐射屏障 Radiation Shield





Case 3 重辐射 Reradiating Surface

$J = G$

$J = E_b \Rightarrow$ 没有电流 q