

Gauss elimination vs. LU decomposition

In this tutorial, you will compare Gauss elimination and LU decomposition. Consider the system of equations

$$\begin{aligned}8x_1 + 2x_2 + 7x_3 &= 2 \\4x_1 - x_2 + 4x_3 &= -3 \\-4x_1 - \frac{1}{2}x_2 + x_3 &= 1\end{aligned}$$

- (a) Form the augmented matrix and find the solution (by hand) following the algorithm for Gauss elimination you learned in Linear Algebra. Verify your answer with the function programmed in class.
- (b) Take the (3 by 3) coefficient matrix and compute its decomposition into factors L and U according to the algorithm in lecture 5, slides 31 – 33.
- (c) Find the solution using the factors L and U and forward/backward substitution. Verify your (intermediate) solution with the function programmed in class.

Here is a question to consider:

- Write the system of equations as $Wax = r$ (where $r = (2, -3, 1)^t$). Now suppose you have to solve this system with many different right-hand sides r_i ($i = 1, \dots, N$). Which of the two methods would be preferable and why?

All code (Gauss elimination, LU decomposition, forward and backward substitution) is available from the Course Code repository. Report your conclusions, mistakes and ideas on the discord channel dedicated to this tutorial!