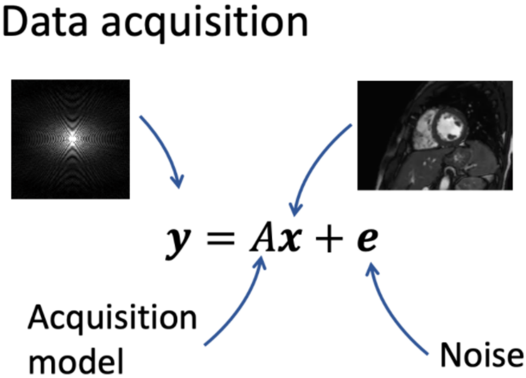
Temporally dependent TV/TGV regularisation

Aim: solve the inverse problem

$$y = Ax + e,$$

where A is the acquisition model, e is the noise, x is the unknown ground truth and y is the noisy observation.



Reconstruct the ground truth by minimizing

$$\min \|Ax - y\|^2 + \lambda TV(x)$$

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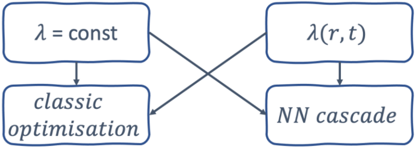
Problem: how to choose the regularizing parameter λ?
So far: choose the lambda constant
Now: develop reconstruction approach where the regularisation parameter is a function of space and time
We have two different approaches: a model based approach and a neural network based approach



NN-based approach

Idea: Learn a neural network with parameters θ by minimizing

$$x_{pred} = \arg\min \|Ax - y\|^2 + \lambda_\theta TV(x)$$

in order to get an x_{pred} and then optimize the network via the loss function

$$\min_\theta \|x - x_{pred}\|^2$$

NN-based approach

First step: Solve

$$\min \frac{1}{2}\|Ax - y\|_2^2 + \|\Lambda G x\|_1$$

for a fixed $\Lambda = \mathrm{diag}(\lambda_1, ..., \lambda_M)$ and a finite differences operator $G = [G_x, G_y, G_t]$ via a splitting strategy.

NN-based approach

First step: Solve

$$\min \frac{1}{2}\|Ax - y\|_2^2 + \|\Lambda G x\|_1$$

for a fixed $\Lambda = \mathrm{diag}(\lambda_1, ..., \lambda_M)$ and a finite differences operator $G = [G_x, G_y, G_t]$ via a splitting strategy. Introduce a auxiliary variable $z = Gx$ and solve

$$\min_{x,z} \frac{1}{2}\|Ax - y\|_2^2 + \|\Lambda z\|_1 + \frac{\beta}{2}\|Gx - z\|_2^2$$

with $\beta > 0$ and solve in an alternating manner. Input is the initial $x_0 = A^H y$

NN-based approach

Update z: For fixed x, minimize

$$\min_z \frac{1}{\beta}\|\Lambda z\|_1 + \frac{1}{2}\|Gx - z\|_2^2$$

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Solution: $z^* = (z_1^*, ..., z_M^*)$

$$z_i^* = S_{\lambda_i/\beta}((Gx)_i)$$

for the soft-thresholding operator S.

NN-based approach

Update z: For fixed x, minimize

$$\min_{z} \frac{1}{\beta}\|\Lambda z\|_1 + \frac{1}{2}\|Gx - z\|_2^2$$

Update x: For fixed z, minimize

$$\min_{x} \frac{1}{2}\|Ax - y\|_2^2 + \frac{\beta}{2}\|Gx - z\|_2^2$$

and thus solve the linear system $Hx = b$ via CG-method, where

$$H = A^H A + \beta G^H G$$
$$b = A^H y + \beta G^H z$$

NN-based approach

Results:

model-based approach

Sharing the screen