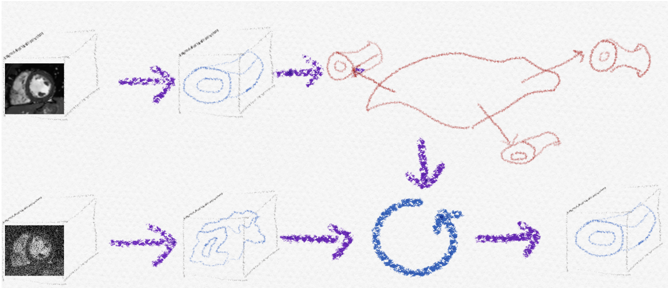
P2 Cardiac motion estimation

March 20, 2022



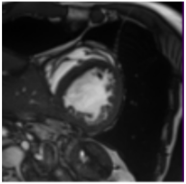
Setup

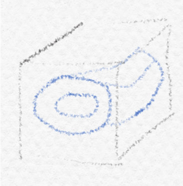
- Needed data: batch of images of the cardiac cycle sliced in z- and t-dimension. Typical batch will have a shape like

$$(30, 8, 144, 144),$$

with

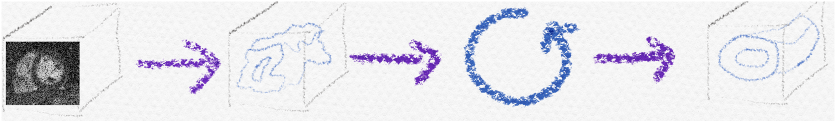
- 30 t slices.
- 8 z slices $\longrightarrow$ take middle one $\longrightarrow$ can be omitted.
- 144x144 pixel images.



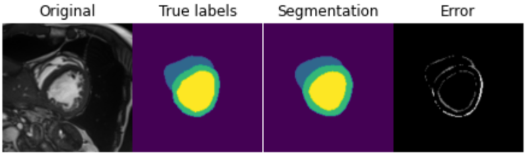


Task

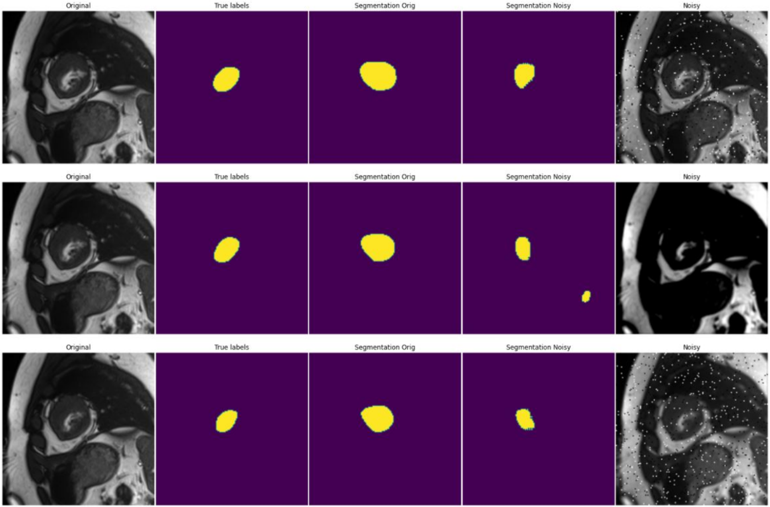
- Goal: Extract a model from noisy data



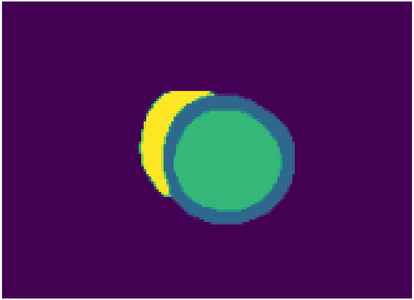
- Segmentation is performed by a NN (AE) for every time slice

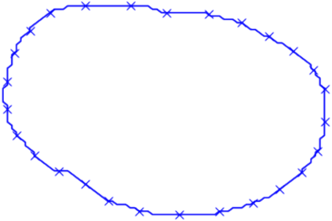


Segmentation of noisy data



Extract boundary shape from segmentation





(with `skimage.measure.find_contours`)

Learn statistical ensemble of curves in shape space

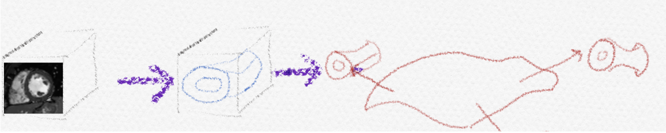


Figure: Manifold of Bezier curves representing cardiac cycles.

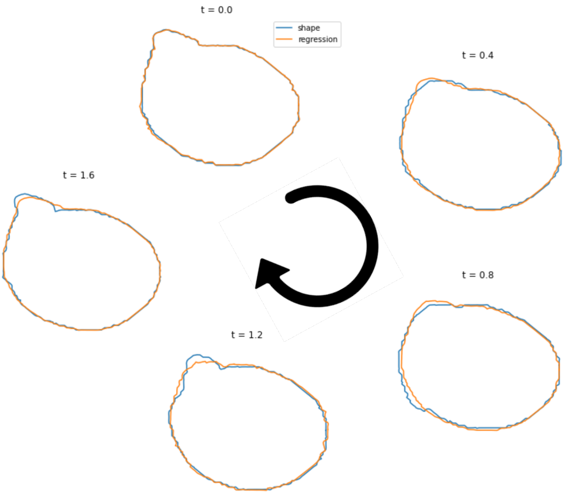
Extract statistics from ensemble $\{\beta_i\}_{i=1}^{N}$ of smooth trajectories on the *tangent space of the mean*:

$$\mathcal{N}_{\mathcal{T}(\mu)}(0, C), \qquad \mu = \mathrm{argmin}_\beta \sum_i d^2(\beta_i, \beta), \qquad C = \frac{1}{N} V V^T,$$

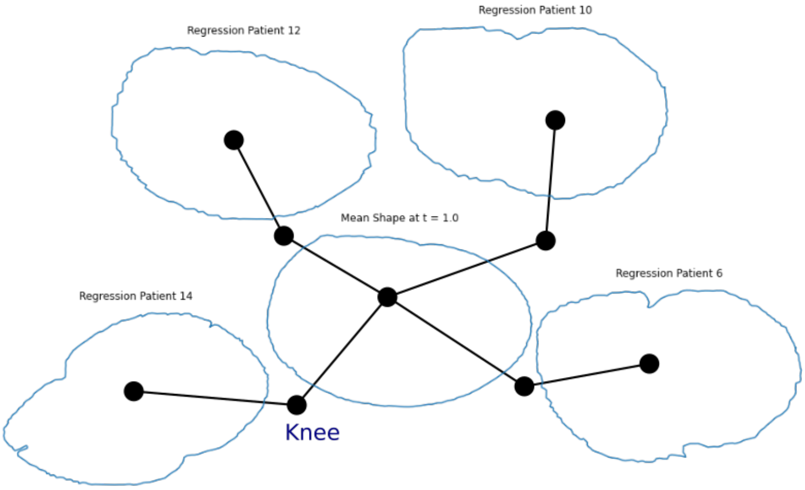
where

- $v_i = \log_\mu(\beta_i)$ is the logarithmic mapping of the curves onto the tangent space
- $V = [v_1, \ldots, v_N]$

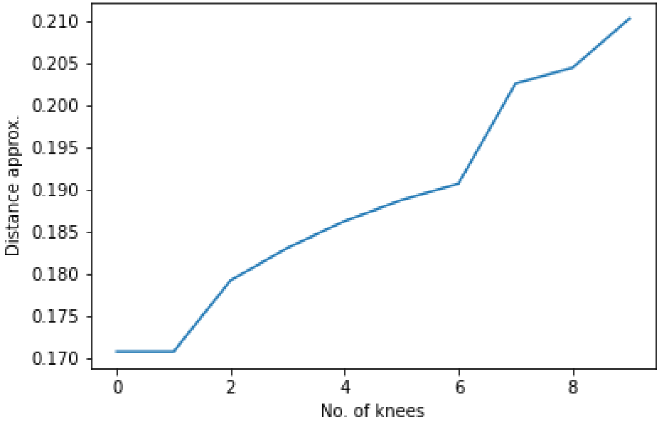
Blue: Cardiac contour evolution over time
Orange: Regressed shape interpolation



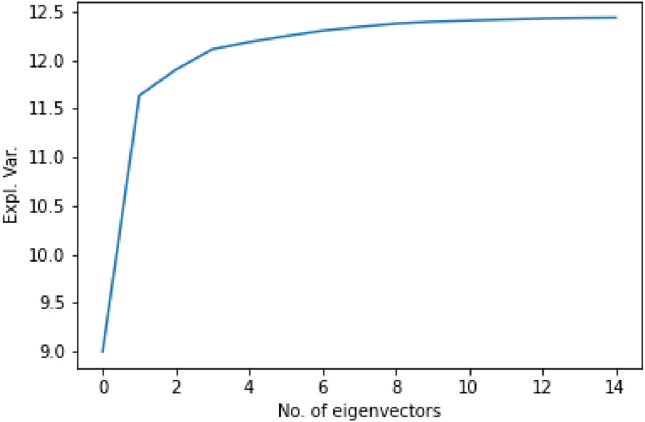
Approximating Mean Shape on a Manifold



Approx. Distance between two shapes increases with number of knees



Explained Variance of PCA computed with the Gram-Matrix



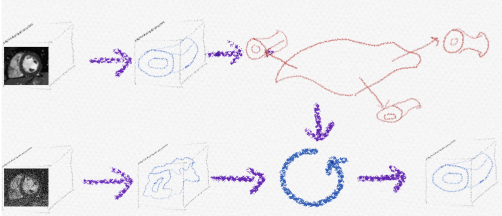
For noisy data: combine datafit with regularization based on statistics

- Represent one cardiac cycle as a curve β in Kendall shape space. Fit:

$$\min_{\beta} \sum_{i} d_{\Sigma}^{2}(x_i, \beta(t_i)) + \lambda d^{M}(\mu, \beta),$$

where

  - $d_{\Sigma}(x_i, \beta(t_i))$ is the datafit
  - $\lambda d^{M}(\mu, \beta)$ is regularization, i.e. penalization of deviation from the statistics of the smooth ensemble $\mathcal{N}_{\mathcal{T}(\mu)}(0, C)$.



Computing the regularization term

Denote $w = \log_\mu(\beta)$, then

$$(d^M(\mu,\beta))^2 = w^\intercal C^{-1} w = \tilde{w}^\intercal \tilde{w}, \qquad \text{(Mahalanobis distance)}$$

where $w = V\tilde{w}$.

Problem: We don't have $\log_\mu(\beta) \longrightarrow$ no v_i, no V, no C.

But: $\tilde{w} = \operatorname{argmin}_{w'} \|w - Vw'\|_2^2 = (\underbrace{V^\intercal V}_{G})^{-1} V^\intercal w = G^{-1} V^\intercal w$,

where $G = (g(v_i, v_j))_{i,j}$, $V^\intercal w = \sum_{i=1}^N g(v_i, w) e_i$ and

$$\begin{aligned} g(v,w) &= \frac{1}{2}(g(v,v) + g(w,w) - g(v-w, v-w)) \\ &= \frac{1}{2}(d^2(\mu,\beta_v) + d^2(\mu,\beta_w)) - g(v-w, v-w)) \\ &\approx \frac{1}{2}(d^2(\mu,\beta_v)) + d^2(\mu,\beta_w)) - d^2(\beta_v,\beta_w)) \qquad \text{(computable!)} \end{aligned}$$