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Gaussian Noise Filtering from ECG by Wiener Filter and Ensemble Empirical Mode Decomposition

Kang-Ming Chang · Shing-Hong Liu

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Abstract Empirical mode decomposition (EMD) is a powerful algorithm that decomposes signals as a set of intrinsic mode function (IMF) based on the signal complexity. In this study, partial reconstruction of IMF acting as a filter was used for noise reduction in ECG. An improved algorithm, ensemble EMD (EEMD), was used for the first time to improve the noise-filtering performance, based on the mode-mixing reduction between near IMF scales. Both standard ECG templates derived from simulator and Arrhythmia ECG database were used as ECG signal, while Gaussian white noise was used as noise source. Mean square error (MSE) between the reconstructed ECG and original ECG was used as the filter performance indicator. FIR Wiener filter was also used to compare the filtering performance with EEMD. Experimental result showed that EEMD had better noise-filtering performance than EMD and FIR Wiener filter. The average MSE ratios of EEMD to EMD and FIR Wiener filter were 0.71 and 0.61, respectively. Thus, this study investigated an ECG noise-filtering procedure based on EEMD. Also, the optimal added noise power and trial number for EEMD was also examined.

Keywords ECG · Gaussian noise · Wiener filter · Ensemble empirical mode decomposition

K.-M. Chang (✉)
Asia University,
Taichung, Taiwan, Republic of China
e-mail: changkm@asia.edu.tw

S.-H. Liu
Department of Computer Science and Information Engineering,
Chaoyang University of Technology,
Taichung, Taiwan, Republic of China

1 Introduction

ECG is a vital sign monitoring measurement of heart activity. During ECG measurement, there may be various noises, such as muscle contraction, baselines wander, and power-line interferences, which interfered with the ECG information identification. Therefore, ECG noise reduction is an important issue and widely studied for many years [1–3]. Traditional noise reduction method is based on standard filter processing, either by low-pass filter or high-pass filter. A low-pass filter was designed to remove high-frequency noise, while a high-pass filter was designed to remove low-frequency vibration, such as baseline wander and respiration interference. Although there were numerous advanced signal processing methods applied on the studies of ECG noise reduction, such as wavelet [4, 5], adaptive filter [6], and independent component analysis [7], it is still an interesting and attractive approach to investigate the ECG filtering characteristics based on partial reconstruction of intrinsic mode function (IMF). IMF is an intermediate product of empirical mode decomposition (EMD), a pre-processing algorithm of Hilbert–Huang transforms (HHTs). HHT was introduced by Huang [8], which is a general signal analysis technique and has been widely used in many fields in recent years. There are two steps involved in HHT. The first step involves the EMD to extract IMF. The second step is the Hilbert transform of the decomposed IMF to obtain time-frequency distribution. EMD is based on the iterative computation of maximum extreme and minimum extreme function. The residual signal, called IMF, is extracted after EMD.

The EMD is adaptive and signal-dependent. This property is valuable for biomedical signal investigation and can be widely used. For example, dynamic behavior of atrial fibrillation from surface ECG was studied by

Huang [9]. Blanco-Velasco developed an EMD-based algorithm to remove the baseline wander and high-frequency noise of ECG [10]. Nimunkar and Tompkin added a pseudo-high-frequency noise to IMF, as an aid to remove power-line noise on ECG. They also developed complete ECG signal-processing procedures, such as R-peak detection and feature extraction, based on HHT approaches [11].

Ensemble EMD (EEMD) is an improved algorithm of EMD, to reduce the mode-mixing effect between the next to IMF scales. The principle of EEMD is to add additional white noise into the signal with many trials. Noise in each trial is different, and the added noise can be cancelled out on an average, if the trial number is high enough. Thus, the existing part is the signal, as more and more trials are added in the ensemble [12].

In this study, an interesting ECG-filtering approach was developed based on the low IMF scales contain high frequency parts and conversely, and partial reconstruction without low-scale IMF removes high frequency noise. Thus, the decomposed IMF behaves as the filter bank. Besides, the low-pass filter performance for ECG with EMD and EEMD was the major concern in this study.

Wiener filter is another noise-filtering approach used in this study. Wiener filter is a well-developed and popular class of optimal filters, which uses the signal and noise characteristics that are available [13]. Wiener filter theory is based on the minimization of difference between the filtered output and desired output. Least mean square method was used to adjust the filter coefficients to reduce the square of the difference between the desired and the actual signal after filtering. There are many studies on Wiener filter application on biomedical signal analysis, such as on stress ECG [14], time-frequency ECG representation, and filtering [15]. In this study, the FIR Wiener filter was adopted to compare EEMD filter performances. Gaussian white noise was used as a general frequency noise source and was added to the clean ECG signal.

The structure of this study is organized as follows: Section 1 presents the introduction, Section 2 provides the method description, Section 3 describes the result, while Sections 4 and 5 present the discussion and conclusion, respectively.

2 Method

2.1 ECG and Gaussian Noises Preparation

There were two ECG groups used in this study, one was the standard ECG template derived from ECG simulator; the other group was from the Arrhythmia ECG database in MIT/BIH database. The standard ECG template used in this study was prepared from ECG simulator, type number PS-2210 Patient Simulator, with sampling frequency of 360 Hz, down sampled from 1,000 Hz, duration of 180 s, preset heart rate of 80 BPM, and online low-pass filter of 0–35 Hz. The standard ECG template was noise free, without baseline drift and high frequency noise, and was marked as $x(t)$ in this study. Figure 1 (top) illustrates the typical standard ECG template segments.

The second ECG group was the real ECG signal derived from the Arrhythmia ECG database in MIT-BIH [16]. Sixteen subjects in this database were randomly chosen, each data containing 30 min durations, with sampling frequency of 360 Hz.

Gaussian white noise was used as the noise source and embedded in the ECG signal. In this study, the Gaussian noise signal was generated by Matlab code awgn.m, denoted as $n(t)$, with the determined signal-to-noise ratio (SNR) ranging from 2 to 18 dB, with the step of 2 dB. The SNR value greater than 18 dB was not included in this study, because the signal was rather "clean" when the SNR was greater than 18 dB. Higher SNR showed less noise part embedded and a "cleaner" ECG signal. The contaminated ECG segment with SNR values of 2 dB and 10 dB are illustrated in Fig. 1 (middle and bottom), respectively.

The contaminated ECG was denoted as $x_1(t)$, and $x_1(t)= x(t)+n(t)$. The noise assessed by SNR was defined as follows:

$$\mathrm{SNR} = \frac{\sum_{t=0}^{L-1} x^2(t)}{\sum_{t=0}^{L-1} n^2(t)} \tag{1}$$

where L is the length of the signal.

2.2 FIR Wiener Filter

The Wiener filter was derived from assuming minimum mean square error (MMSE) when both the statistical properties of the signal and noise were known. It comprised the ratio of cross correlation of the desired signal and noise to the auto correlation of noise signal. In this study, the FIR type was adopted, and the formula of the Wiener filter is given as [13]:

$$w = R_{X_1X_1}^{-1} R_{X_1X} \tag{2}$$

Where w is the FIR Wiener filter coefficients, and the cross correlation of $x_1(t)$ and $x(t)$,R_{X_1X}, autocorrelation of $x_1(t)$, $R_{X_1X_1}$ were estimated. The $x_1(t)$ and $x(t)$, represent the input signal and desired signal corresponding to $x_1(t)$ and $x(t)$ introduced in the earlier section, respectively. Wiener filter is especially useful when the power

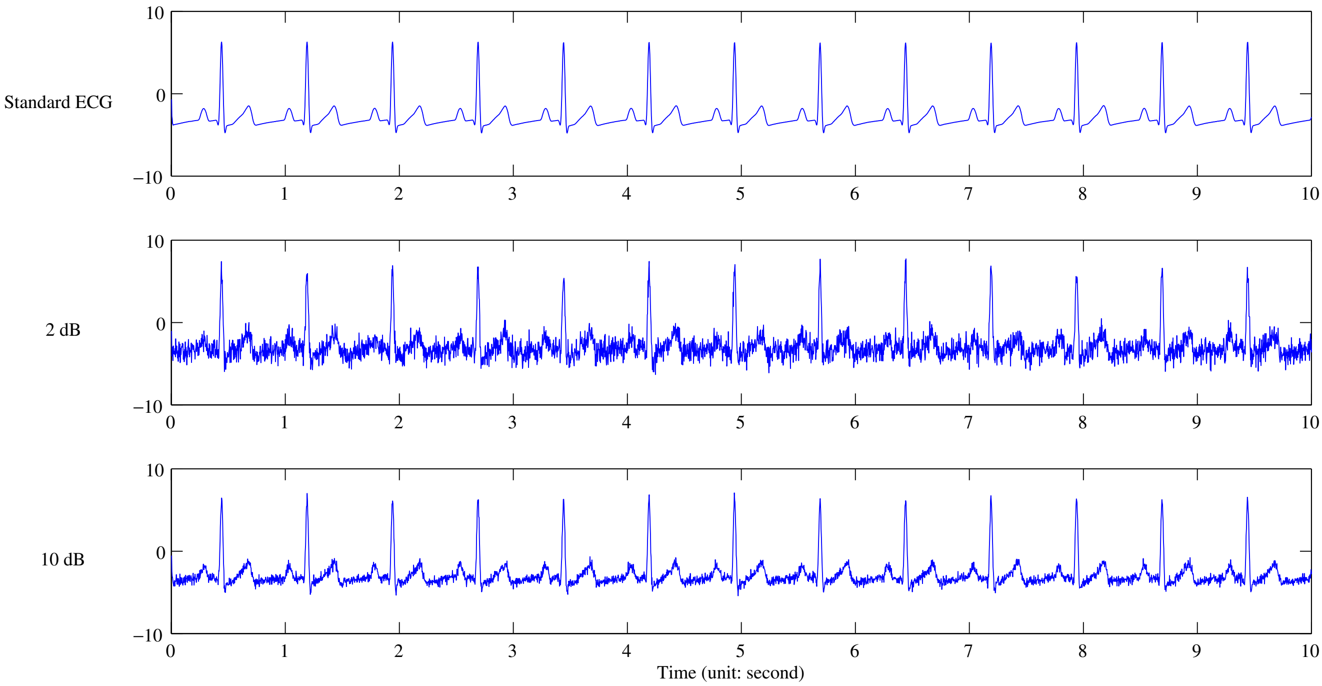

Figure 1 (*Top*) Standard ECG template for 10 s duration. (*Middle*) Contaminated ECG template with SNR 2 dB noise for 10 s duration. (*Bottom*) Contaminated ECG template with SNR 10 dB noise for 10 s duration.

spectrums of input signal and noise overlap and are not separable by tradition low-pass filter [13]. In this study, the FIR Wiener filter was derived from Matlab function firwiener.m, with filter order ranging from 100 to 300.

2.3 EMD Algorithm

The standard EMD algorithm was derived using following steps [8]:

(1) Identify all the extreme (maxima and minima) peaks of the signal (DC component of signal was removed before preprocessing), $s(t)$.
(2) Generate the upper and lower envelope by the cubic spline interpolation of the extreme peaks developed in step (1).
(3) Calculate the mean function of the upper and lower envelope, $m(t)$.
(4) Calculate the difference signal, $d(t)=s(t)-m(t)$.
(5) If $d(t)$ becomes a zero-mean process, then the iteration is stopped and $d(t)$ is considered as the first IMF, named $c_1(t)$; otherwise, go to step (1) and replace $s(t)$ with $d(t)$.
(6) Calculate the residue signal, $r(t)=s(t)-c_1(t)$.
(7) Repeat the procedure from steps (1) to (6) to obtain the second IMF, named $c_2(t)$. To obtain $c_n(t)$, continue the steps (1)–(6) after n iterations. The process is stopped when the final residual signal, $r(t)$, is obtained as a monotonic function.

At the end of the procedure, a residue $r(t)$ and a collection of n IMF were derived and named from $c_1(t)$ to $c_n(t)$. Hence, the original signal can be represented as:

$$s(t) = \sum_{i=1}^{n} c_i(t) + r(t), \tag{3}$$

where $r(t)$ is often regarded as $c_{n+1}(t)$.

The low IMF scales were mainly the high-frequency components of signal, while the high IMF scales were the low-frequency components of signal. Thus, an EMD-based low-pass filter was developed using the partial reconstruction of the selected IMF scale, which is given as:

$$\mathrm{REMD_K} = \sum_{i=k}^{n+1} c_i(t), \tag{4}$$

When k=1, the REMD_1 was equivalent to the original noise-contaminated ECG.

2.4 EEMD Algorithm

The EEMD algorithm is as follows [12]:

(1) Add a white-noise series, $n(t)$, to the targeted signal, $x(t)$, in the following description, $x_1(t)=x(t)+n(t)$. The added noise power from 5 to 25 dB was used to investigate the EEMD performance.

(2) Decompose the data $x_1(t)$ using the EMD algorithm, as described in Section 2.3.
(3) Repeat Steps (1) and (2) until the pre-set trial numbers, each time with different added white-noise series of the same power. The new IMF combination, $c_{ij}(t)$ is achieved, where i is the iteration number and j is the IMF scale.
(4) Estimate the mean (ensemble) of the final IMF of the decompositions as the desired output.

$$EEMD\_c_j(t) = \frac{\sum_{i=1}^{nt} c_{ij}(t)}{nt}, \tag{5}$$

where nt denotes the trial numbers.

Similar to EMD, an EEMD-based partial reconstruction of ensemble IMF can be defined as:

$$\mathrm{REEMD_K} = \sum_{j=k}^{n+1} EEMD\_c_j(t) \tag{6}$$

2.5 ECG Reconstruction Performance Measurement

A mean square error (MSE) between filter ECG output and clean ECG was used to measure the filter performance and can be defined as:

$$\mathrm{MSE} = \frac{\sum_{t=0}^{L-1} \left[x(t) - \widehat{x}(t)\right]^2}{L}, \tag{7}$$

where $x(t)$ is one of the clean ECG groups, either standard ECG template or Arrhythmia ECG database. $\widehat{x}(t)$ is either the filter output by the Wiener filter or the partial IMF-reconstructed ECG by EMD or EEMD. The lower MSE value represents better filtering performance. The MSE for Gaussian noise is an average of 10 times repetitions.

3 Results

3.1 Wiener Filter Characteristics

Figure 2 demonstrated both the FIR Wiener filter spectrum (order=300) and standard ECG template with 10 dB Gaussian noise. As expected, the Wiener filter was performed as a low-pass filter. Besides, the phase of the FIR Wiener filter was found to obey the linear phase speculation.

The MSE was used to examine the filtering characteristics of the Wiener filter. Figure 3 shows the MSE with filter order from 10 to 400 under SNR from 2 to 18 dB with standard ECG template. The MSE value decreased both when the signal SNR value and the FIR Wiener filter order increased. As the Wiener filter order increased, the MSE decreased and finally achieved a "steady-state". The "steady-state" Wiener filter order for standard ECG template was 300, while the filter order was around 100 for Arrhythmia ECG database.

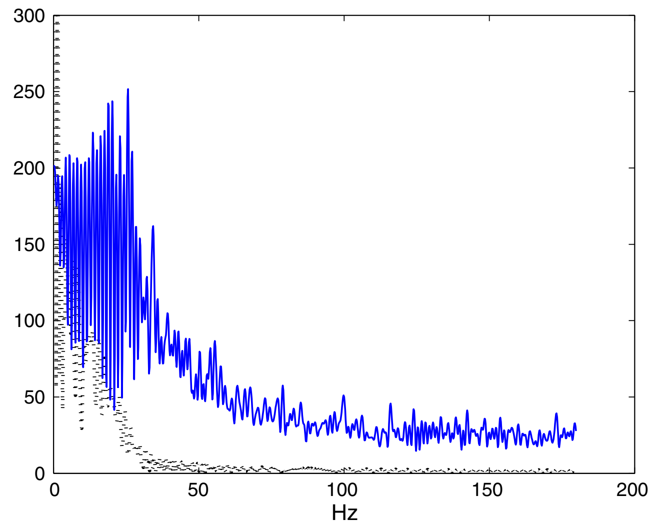

Figure 2 Power spectrum of Wiener filter (filter order=300, *solid line*), SNR 10 dB Gaussian noise contaminated standard ECG template (*dash line*).

Figure 4 presents Wiener filtered ECG for standard ECG template. It is obvious that the Gaussian noise is significantly reduced after Wiener filter processing. There is also no phase delay for filtered ECG with careful examination on R-wave location, due to the linear phase property of FIR Wiener filter.

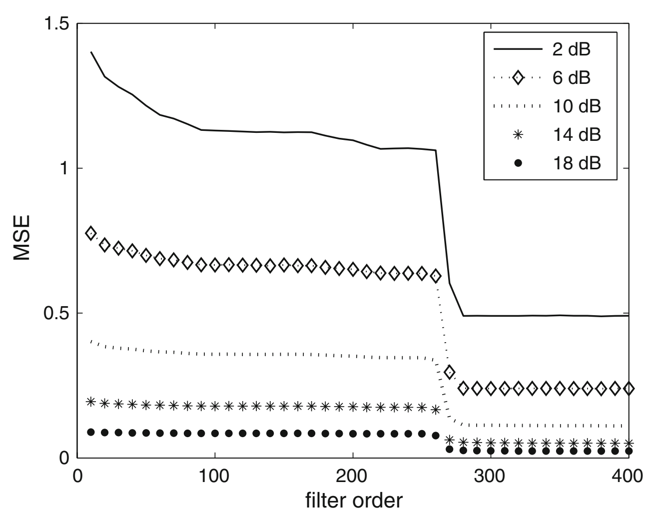

Figure 3 MSE of standard ECG template with FIR Wiener filter. The x-axis is filter order, and y-axis is MSE value. SNR value of 2 dB (*solid line*), 6 dB (*diamond*), 10 dB (*dot line*), 14 dB (*star*) and 18 dB (*circle*) are also illustrated.

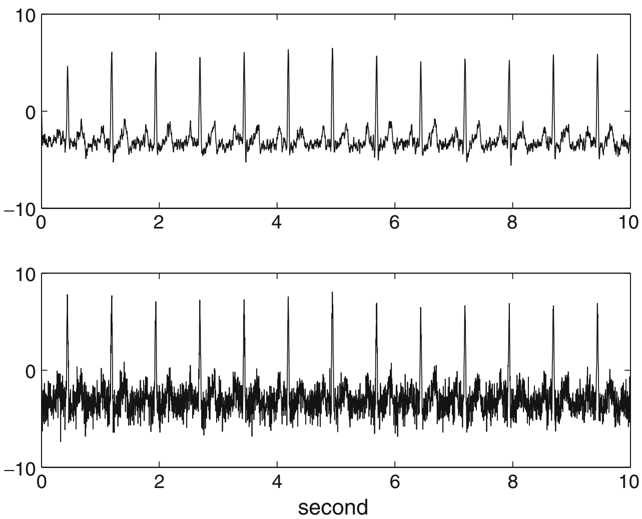

Figure 4 (*Top*) FIR Wiener filtered (filter order=300) ECG of noisy standard ECG template (SNR=10 dB) (*Bottom*).

3.2 EMD Decomposition of Standard ECG Template

The EMD-derived IMF distributions of standard ECG template without/with Gaussian noise are shown in Figs. 5 and 6, respectively. As expected from EMD algorithm, the IMF components were extracted according to the signal complexity, such as a filter bank that filters out simple waveform from high-frequency component. Thus, the noise components were filtered in the lower IMF scale. The QRS complex of standard ECG template was distributed widely from scale 1 to 2, as shown in Fig. 5. The first three and the first two IMF scales of ECG with 2- and 10-dB Gaussian noise, were mainly noise, as shown in Fig. 6a and b, respectively. The QRS component was distributed in the middle scales of the IMF. Furthermore, the fourth IMF scale in Fig. 6a and the third IMF scale in Fig. 6b both contained QRS component and contaminated Gaussian noise. This was also called as the “mode-mixing effect” between near IMF scales.

3.3 EEMD Decomposition of Standard ECG Template

The corresponding EEMD-derived IMF distribution of Fig. 6 is illustrated in Fig. 7. The improvement of EEMD with regard to EMD is the mode-mixing reduction between the near IMF scales. However, the fourth IMF scale in Fig. 7a and the third IMF scale in Fig. 7b can be observed to contain more QRS components and less Gaussian noise than the same IMF scale given in Fig. 6. The corresponding IMF spectrum of Figs. 6 and 7 is shown in Fig. 8, which further demonstrates that EEMD reduces IMF mode-mixing between the near IMF scales. The first two IMF spectrum of EEMD had more concentrated and localized high frequency spectrum. The third IMF contained the ECG-spectrum component that ranged below 40 Hz. On the other hand, there was a significant spectrum overlapping between the IMF scales at first, second, and third scale on EMD with spectrum below 40 Hz. Thus, evidently EEMD improved the EMD mode-mixing performance and acted as a better filter-like noise reduction method. With the partial reconstruction of IMF by removing the low scale with noise-contained IMF, high frequency noise was removed and a “filtered” clean ECG was obtained as the output. In the following section, this concept will be further examined by MSE parameter to check the reconstructed ECG performance by EMD, EEMD, and the Wiener filter.

Trial number and added noise power were two influential EEMD factors on the partial IMF-reconstruction performance. Figure 9 shows the MSE of various EEMD reconstruction scale versus trial numbers. As trial number increased, the MSE value was significantly decreased, and reached a saturated state with a slow MSE decrease after sufficient trial-number calculations on EEMD. Each partial reconstruction, $REEMD_k$, obeyed the same trend for k=1 to 13. The larger trial number achieved low MSE value, but it also required larger computation load. Trial number of 100 was sufficient for the standard ECG template data to achieve an acceptable MSE performance, compromised with computation load consideration.

The added noise was the other important EEMD factor, which represented an extra noise source added to the noise-contaminated signal, and followed with EMD computation. The ensemble (average) EMD of each IMF was observed to reduce the mode mixing, and thus, enhanced the separation of signal and noise. Table 1 lists the MSE of added noise from 5 to 25 dB. As shown in Table 1, the added noise of 5 dB had the best MSE performance than the other added noise power. High added noise enhanced the noise component, and further separated the noise and signal into distinct IMF scales.

In standard ECG template case, similar MSE performance was observed for 5 dB and 10 dB added Gaussian noise with a trial number of 100 (MSE=0.1891 and MSE=0.1892, respectively). To investigate the optimal EEMD parameters of trial number and added noise, the detail information on the last row of Table 1 was examined, which indicated that the MSE performance ratio of trial number=500 to trial number=100 were around 1.0405 to 1.0025. This result demonstrated there was only 0.25–4.05% reduction performance with a trial number=500 than trial number=100. On the other hand, added noise of 10 dB had less MSE performance variation than 5 dB added noise (1.33% vs. 4.05%). From results, 10 dB added noise and a trial number=00 were chosen as the optimal EEMD parameters with respect to the absolute MSE and MSE variation performance.

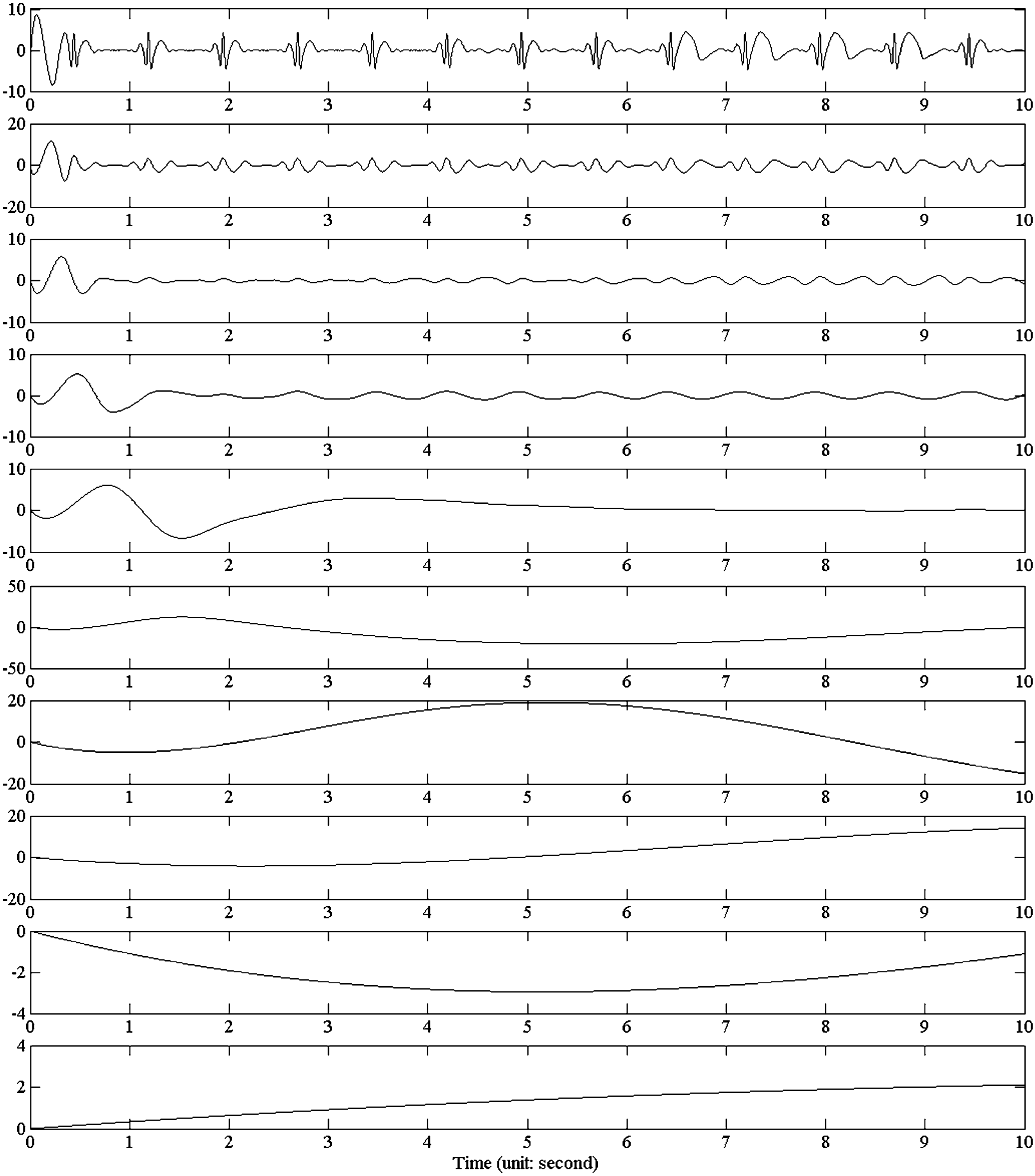

Figure 5 IMF distribution of EMD for standard ECG template (100 times repeats average). From top to bottom is the first IMF scale to 10th IMF scale. Unit of x-axis is 10 s, and it's the same unit for Figs. 6, 7, 8 and 9.

3.4 ECG Filtering Performance of Wiener Filter, EMD, and EEMD

The overall MSE performance of standard ECG template with various SNR is listed in Table 2. The minimum MSE scales of 2 dB for EMD and EEMD were at $REMD_4$ and $REEMD_4$, respectively, while for the SNR from 4 to 18 dB, the minimum MSE scale were at $REMD_3$ and $REEMD_3$, respectively. The EEMD had lower MSE than EMD of the same reconstruction IMF scale. When compared with the

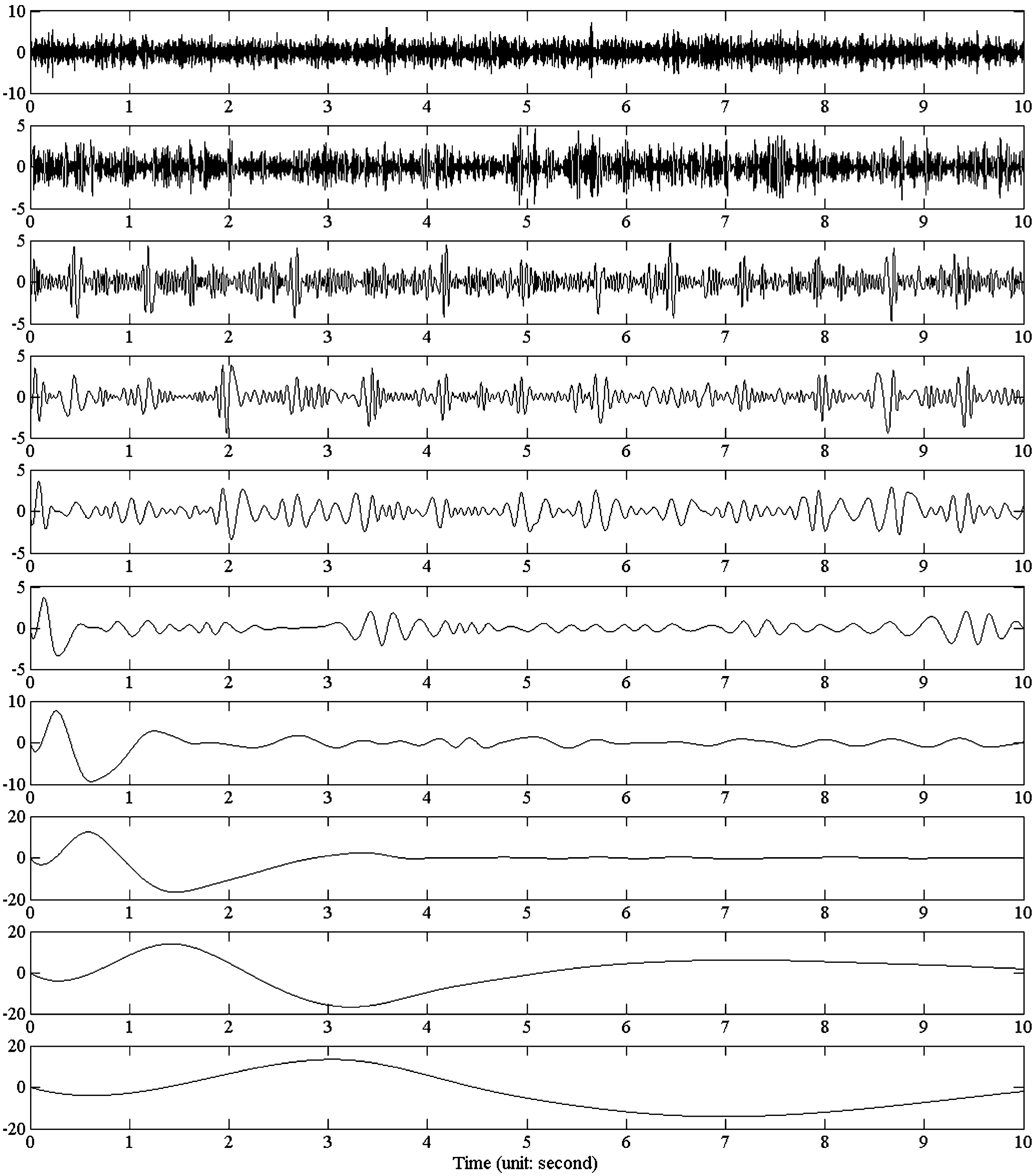

Figure 6 IMF distribution of EMD for standard ECG template with **a** SNR 2 dB noise; **b** 10 dB noise.

FIR Wiener filter, the minimum MSE of EEMD was lower than that of the FIR Wiener filter within the order 200, but greater than that of the FIR Wiener filter with order 300, for SNR from 2 to 18 dB. Thus, the FIR Wiener filter demonstrated superior noise reduction performance than EEMD with enough filter order in standard ECG template case.

The MSE performance of Arrhythmia ECG listed in Table 3 showed two impressive results. The first result was the MSE performance of EEMD was superior to EMD and

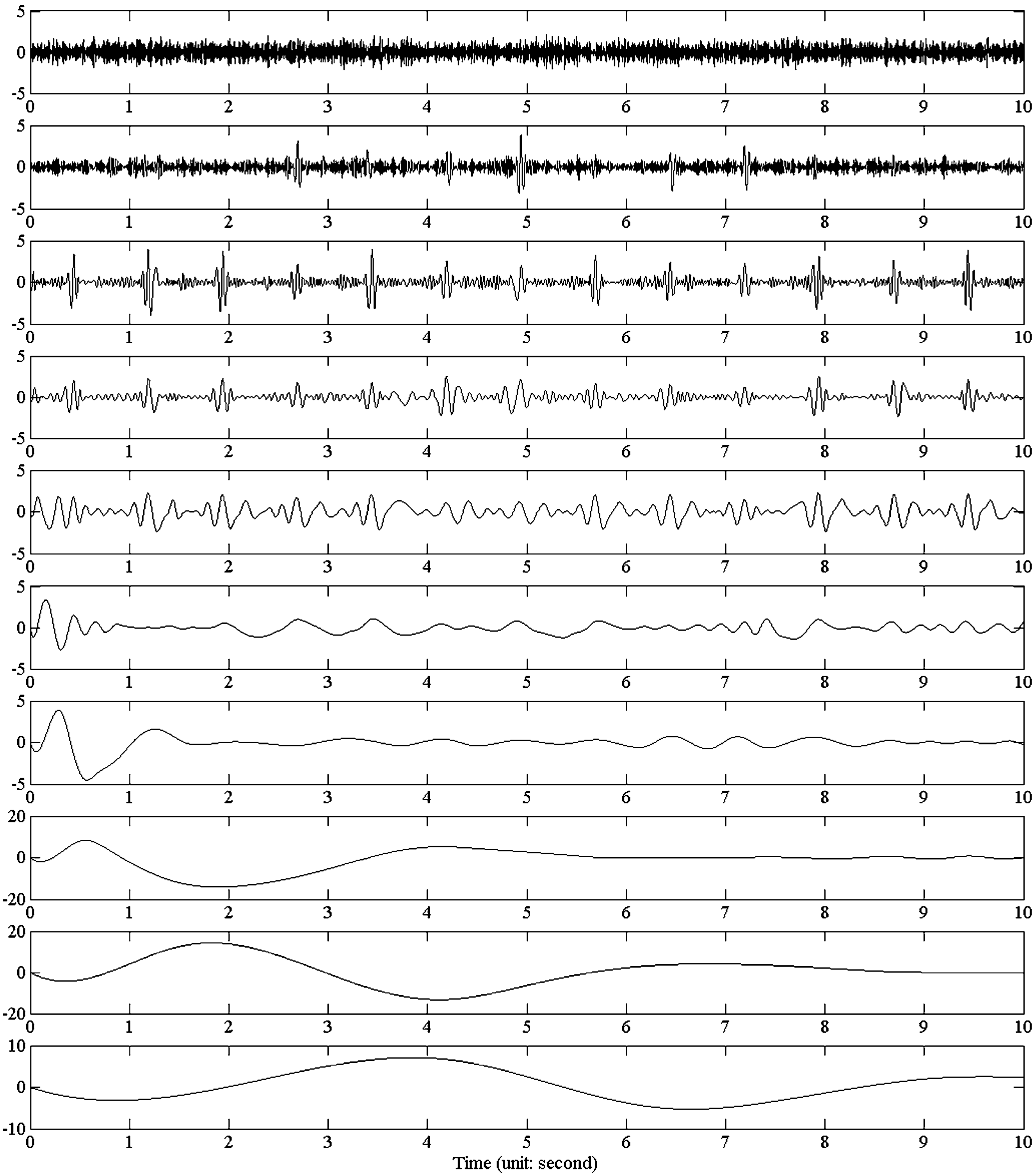

Figure 6 (continued).

to Wiener filter. For all the subjects in Arrhythmia ECG database, the minimum MSE of EEMD was lower than that of EMD and FIR Weiner filter. The minimum MSE ratio of EEMD to EMD was around 0.71±0.06, while the minimum MSE ratio of EEMD to the FIR Wiener filter (order=10) was around 0.61±0.09. The MSE of EMD was slightly lower than that of the FIR Wiener filter (order=10). Also, the MSE performances of the FIR Wiener filter and EEMD for Arrhythmia ECG database were different from that for standard ECG template.

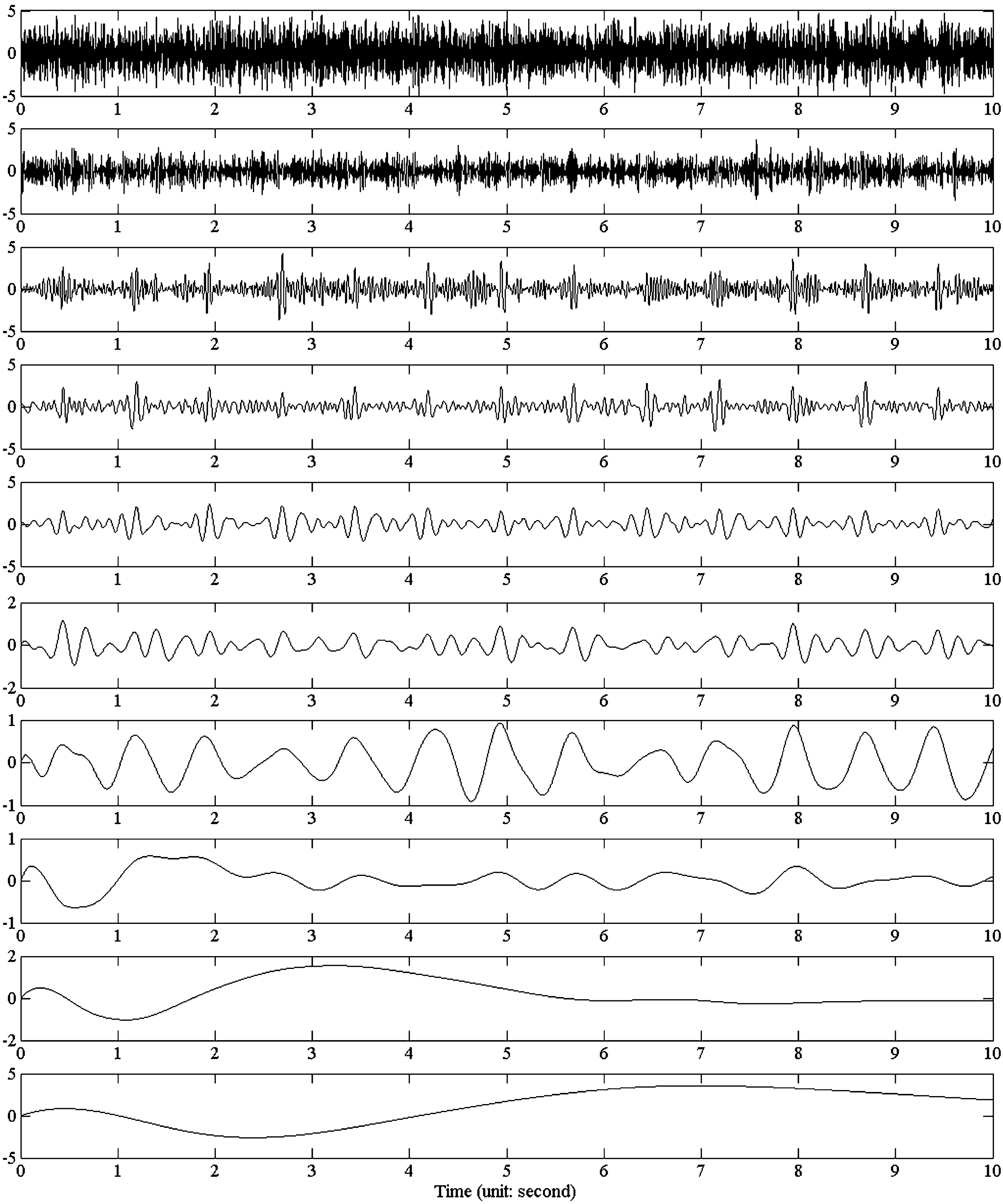

Figure 7 IMF distribution of EEMD for standard ECG template with **a** SNR 2 dB noise; **b** 10 dB noise.

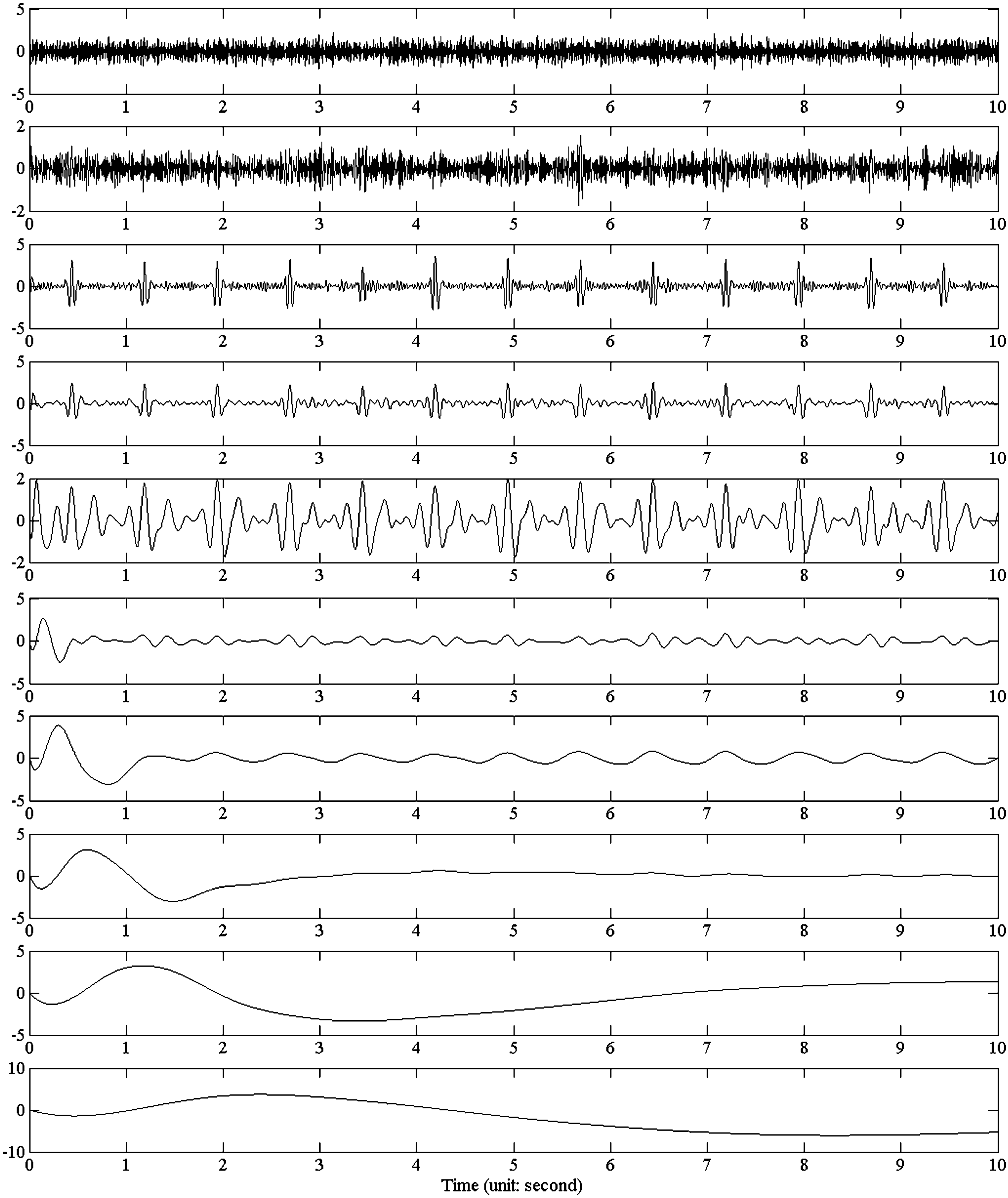

Figure 7 (continued).

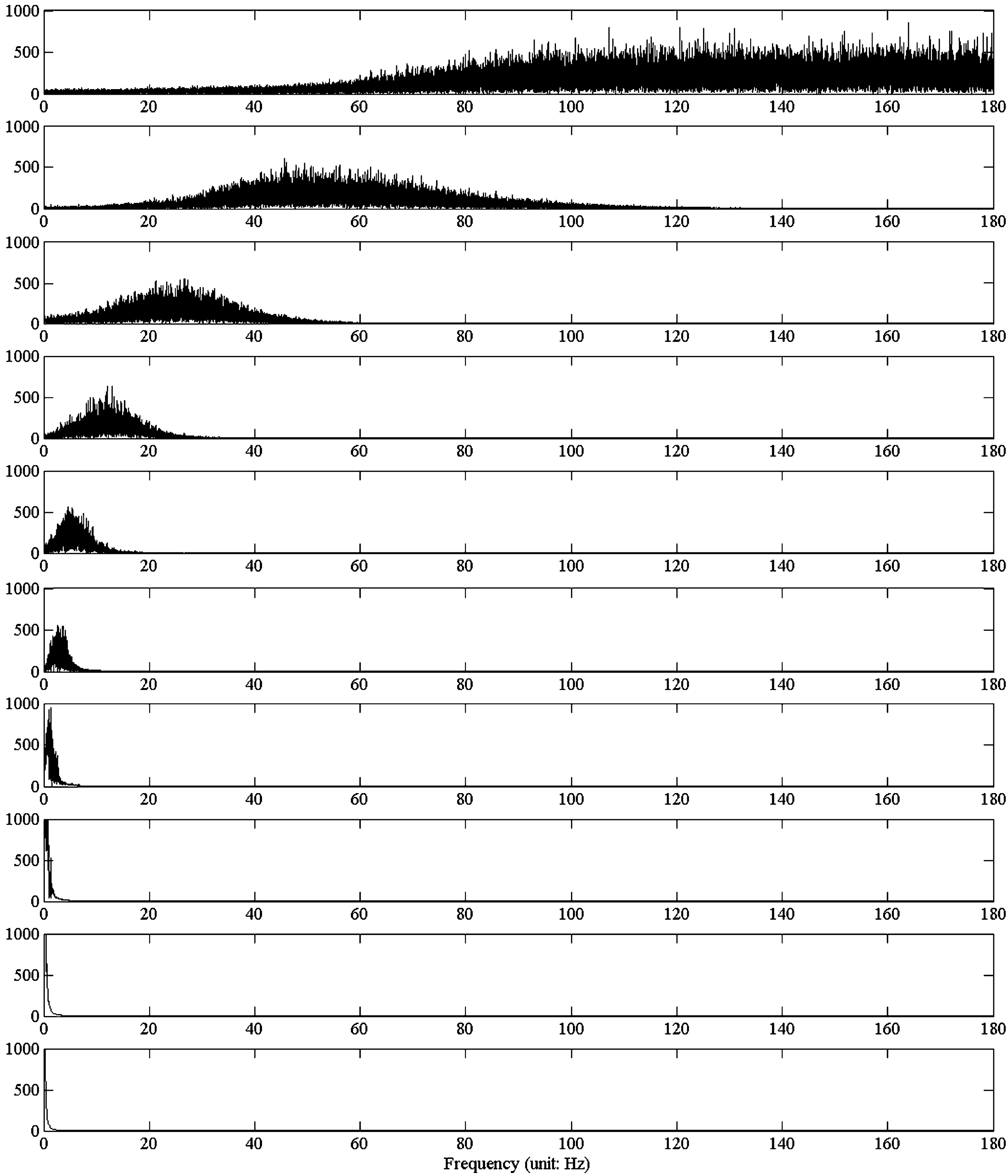

Figure 8 IMF power spectrum distribution for standard ECG with 10 dB Gaussian noise by **a** EEMD; **b** EMD.

The other impressive result was the minimum MSE was located mainly at $REEMD_2$ and $REEMD_3$. There was no prior knowledge to estimate which IMF scale contained the noise component; thus, the partial reduction of IMF without these IMF scales might achieve a minimum MSE. Therefore, the only way to determine the minimum MSE of IMF scale was through trial-and-error approach.

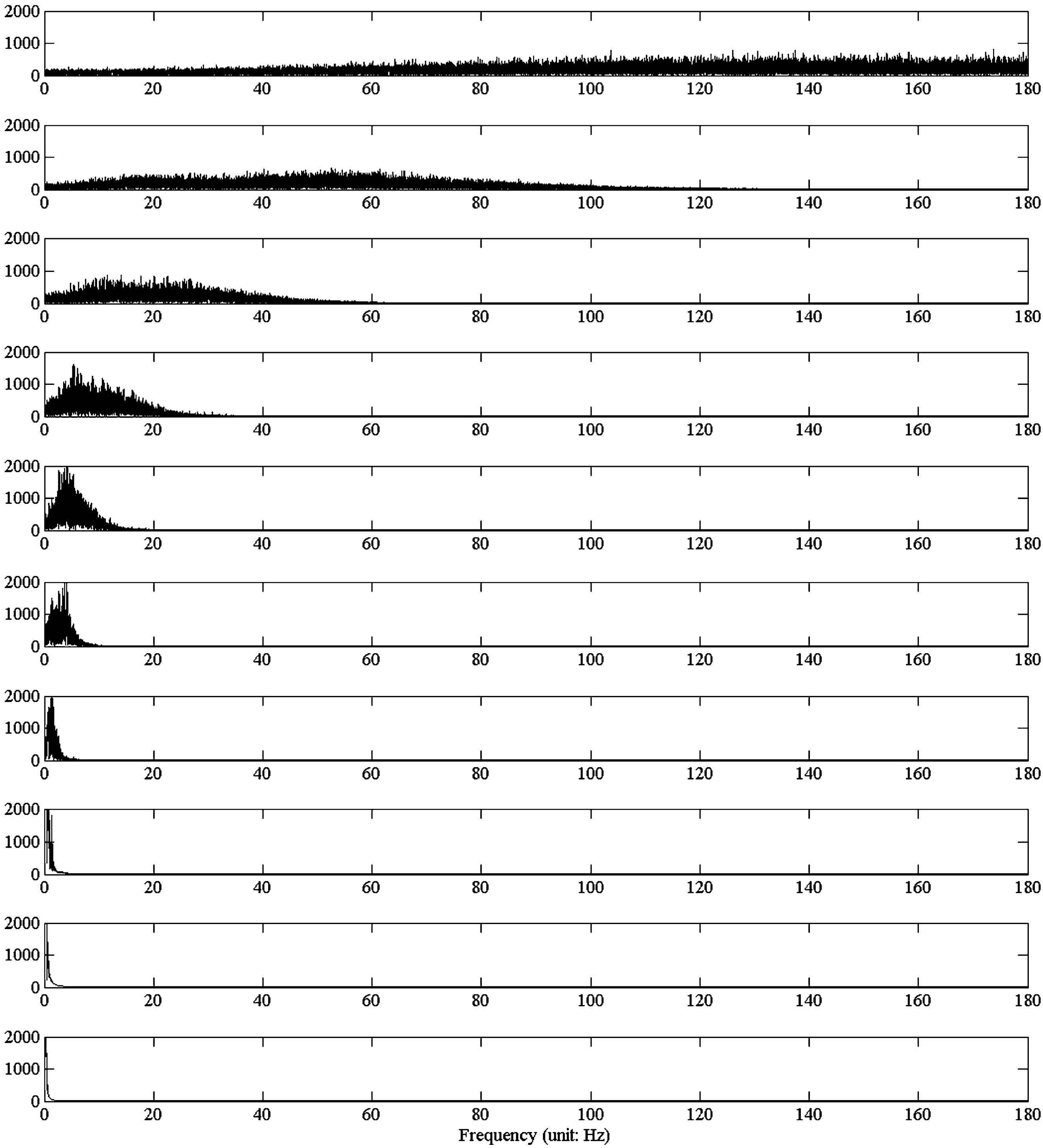

Figure 8 (continued).

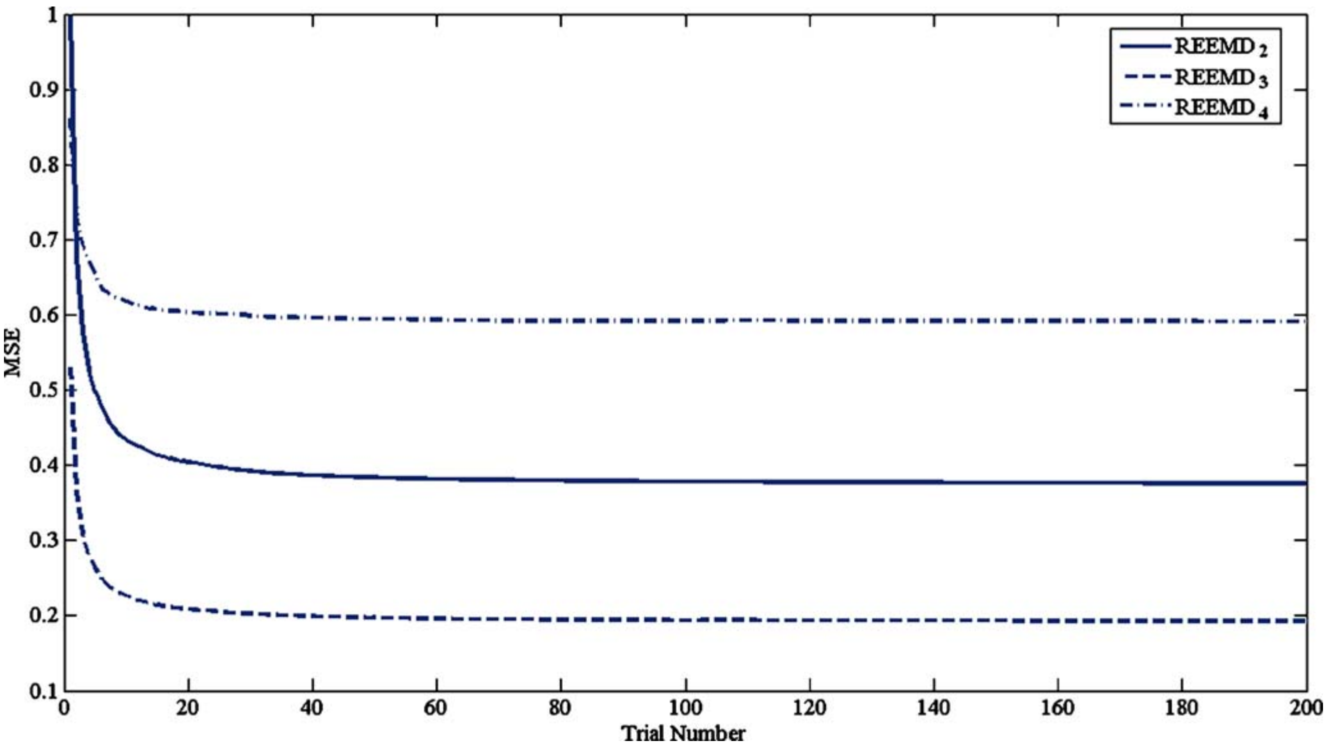

Figure 9 Trial number effect of EEMD with partial reconstruction for standard ECG template with 10 dB noise, and 10 dB added noise for $REEMD_1$ (*dash line*), $REEMD_2$ (*solid line*) and $REEMD_3$ (*dash line with triangle mark*).

Table 1 MSE value of $REEMD_3$ for standard ECG template (SNR=10 dB) with various added noise under two trial number, 100 and 500, respectively. "Ratio" is the MSE ratio between trial numbers 100 to trial number 500.

added noise (dB)	5	10	15	20	25
trial no.=100	0.1891	0.1892	0.2010	0.2213	0.2430
trial no.=500	0.1817	0.1867	0.2004	0.2207	0.2424
Ratio	1.0405	1.0133	1.0029	1.0029	1.0025

Table 2 MSE value of standard ECG template with various SNR noise. Where EMD was with 100 times computation and EEMD was with trial number=100 and 10 dB added noise. W_100 means the FIR Wiener order with order 100. W_200 and W_300 are with filter order 200 and 300, respectively.

SNR	2	4	6	8	10	12	14	16	18
$REMD_1$	6.741	4.249	2.680	1.688	1.069	0.673	0.425	0.268	0.169
$REMD_2$	2.962	1.870	1.177	0.741	0.469	0.296	0.187	0.118	0.075
$REMD_3$	1.520	0.977	0.627	0.422	0.305	0.249	0.238	0.250	0.270
$REMD_4$	1.284	1.038	0.833	0.806	0.780	0.773	0.795	0.821	0.851
$REEMD_1$	6.765	4.337	2.691	1.712	1.068	0.682	0.427	0.273	0.170
$REEMD_2$	2.488	1.578	0.974	0.598	0.371	0.233	0.145	0.092	0.057
$REEMD_3$	1.201	0.758	0.473	0.296	0.187	0.124	0.087	0.062	0.047
$REEMD_4$	0.972	0.783	0.675	0.615	0.584	0.569	0.567	0.561	0.558
W_100	1.129	0.878	0.666	0.494	0.359	0.255	0.179	0.124	0.085
W_200	1.095	0.856	0.652	0.483	0.352	0.252	0.177	0.123	0.084
W_300	0.490	0.346	0.241	0.165	0.113	0.077	0.053	0.036	0.025

Table 3 MSE value of Arrhythmia ECG by EMD (10 times repeat average), EEMD (trial number=100, 10 dB added noise) and Wiener filter (order=10 and order=100, denoted as W_10 and W_100, respectively) under SNR=10 dB noise.

Subjects NO.	$REMD_2$	$REMD_3$	$REMD_4$	$REEMD_2$	$REEMD_3$	$REEMD_4$	W_10	W_100
101	*126.9*	229.7	594.4	*97.4*	148.4	458.0	141.6	140.4
102	*83.3*	117.7	204.6	*60.0*	90.1	167.3	79.4	77.7
103	*189.4*	596.3	1,391.7	*147.0*	374.2	1,199.7	218.3	215.8
104	*151.6*	197.5	359.7	*109.5*	152.3	288.3	136.6	136.4
105	291.0	*180.6*	550.3	232.4	*128.1*	328.0	259.6	257.6
106	*245.8*	375.7	1,036.4	*192.5*	226.9	808.9	266.4	263.6
107	1,303.1	*771.7*	962.0	1,039.1	*574.9*	679.8	1,061.7	1,025.4
108	161.9	*103.2*	175.8	128.4	*76.9*	113.8	131.6	126.5
109	439.5	*237.2*	848.7	351.2	*179.7*	329.8	347.7	334.1
201	*67.1*	69.7	215.2	53.4	*38.6*	159.5	67.5	66.6
202	157.1	*131.3*	484.0	125.4	*76.3*	336.5	152.1	150.2
203	439.3	*279.7*	599.9	348.7	*206.5*	378.8	378.3	375.6
205	*72.5*	235.2	505.5	*55.0*	155.9	439.8	83.6	83.2
207	223.6	*129.7*	155.1	178.4	99.9	*97.5*	165.8	163.9
208	413.7	*361.2*	789.0	329.8	*232.0*	606.0	401.0	398.6
209	*140.3*	551.7	1,168.8	*103.3*	364.3	1,037.7	161.1	158.8

IMF scale with minimum MSE for EMD and EEMD was in italics

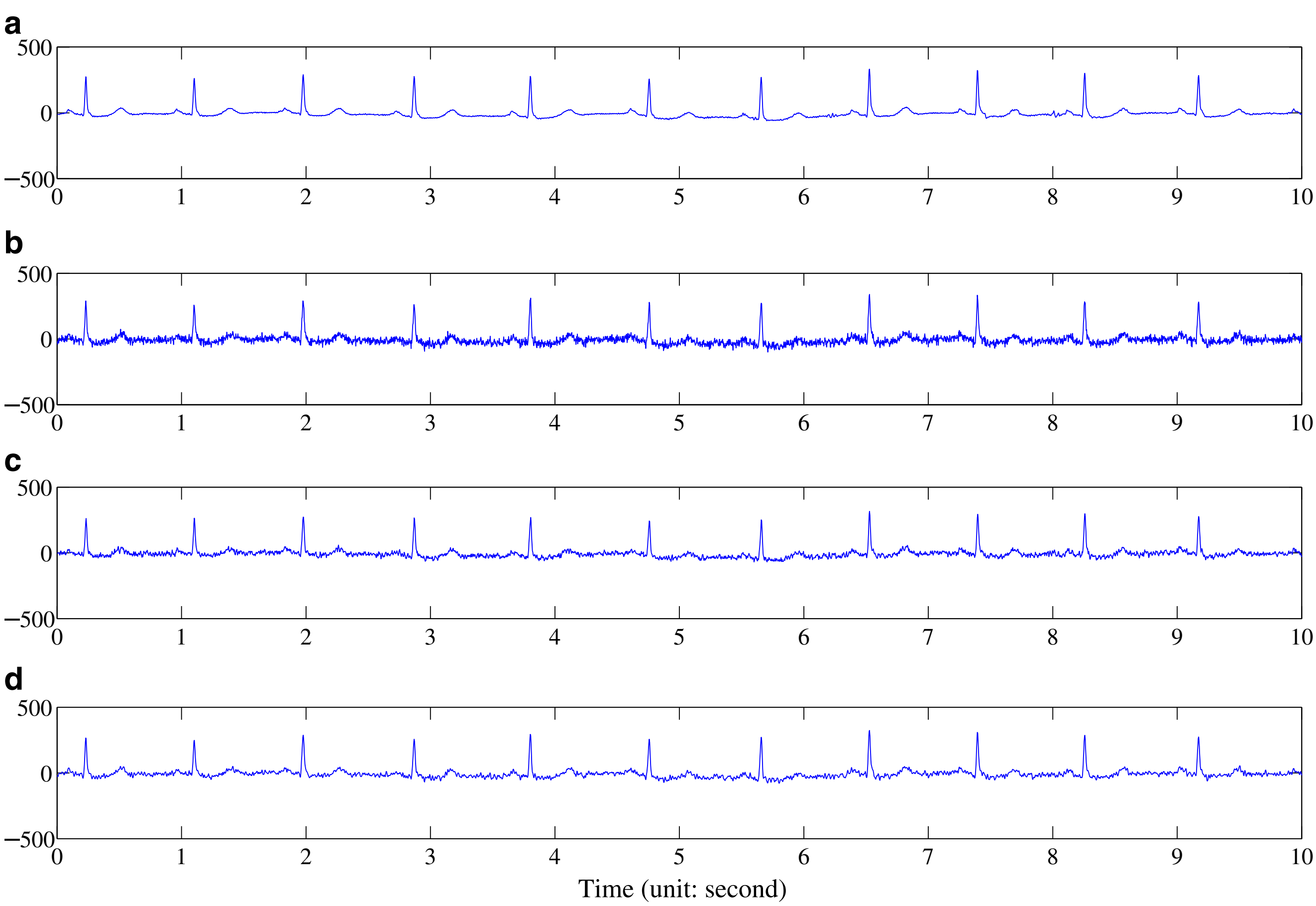

Figure 10 Arrhythmia ECG processing (data 101, 2nd channel) **a** Raw 101 ECG, **b** ECG with 10 dB noise, **c** Wiener filter output of (**b**) with filter order=10, and output MSE=141.6, **d** $REEMD_2$ output of (**b**) with trial number =10 and 10 dB added noise, the output MSE=97.4).

The last two columns of Table 3 showed that the MSE performance of FIR Wiener filter of order=10 and order=100 were similar. The FIR Wiener-filtered ECG and ECG from partial reconstruction of EEMD for Arrhythmia ECG were shown in Fig. 10. The "filtered" ECGs by both methods were visibly "clean" when compared with the original uncontaminated ECG.

4 Discussion

This study was mainly based on the concept the partial reconstruction of EMD-derived IMF to remove noise similar as a "filter". EMD is a newly developed algorithm [17], and can separate signal parts into separate IMF scales by signal complexity. EMD is also suitable for nonlinear and nonstationary signals, with adaptive IMF scales. There is no distinct spectrum range for the fixed IMF scale; it is a signal-dependent method that is neither similar to traditional filter nor wavelet. This property offers both advantage as well as disadvantage for EMD-based signal analysis.

When noise was added, EMD collected the high frequency component in the low-order IMF scale. When SNR = 2 dB, the noise power was high and the first three IMF scales were distributed for noise; thus, the QRS component was found from the fourth IMF. Therefore, the minimum MSE with 2 dB noise was found at $REMD_4$, which indicates the deletion of first three IMF scales will reduce noise. On the other hand, for the other noise power with SNR from 4 to 18 dB, the noise was distributed in the first two IMF scales, and therefore, the minimum MSE was at $REMD_3$.

The main reason for the improved noise reduction performance by EEMD than EMD was the reduction of IMF mode mixing. The minimum MSE was found at the same IMF scale both for EMD and EEMD. Both added noise and ensemble IMF after computation by trial-number time improved separation of signal and noise. With 10 dB added noise and sufficient trial number, there was an improved noise-filtering performance by EEMD than EMD, with the minimum MSE ratio of EEMD to EMD was around 0.71.

However, unlike the previous study by Blanco-Velasco, we only used the partial reconstruction method, which is easier and straightforward. Blanco-Velasco used a QRS-reserved window filter to extract the QRS component in the first several IMF [10] that was just owing to the mode mixing by EMD. However, the EEMD just improved it and reduced the mode-mixing effect; therefore, a QRS-reserved window filter was not necessary.

Compared with the minimum MSE value of EEMD and FIR Wiener filter for Arrhythmia database, the MSE ratio between EEMD and the Wiener filter was around 0.61, indicating that EEMD had higher noise reduction performance than FIR Wiener filter. The Arrhythmia data was the real ECG data, and this result showed that EEMD had better performance in the real ECG environment. The FIR Wiener filter attenuated noise and demonstrated an "optimal" filtering solution, in terms of minimum MSE sense. On the other hand, EEMD concentrated the noise component in the first several IMF scales.

Although EEMD was superior to the FIR Wiener filter on filtering performance in real ECG signal, the Wiener filter had lesser computation requirement than EEMD. For the tradeoff between MSE performance and computation efficiency, EEMD with trial number 100 and 10 dB added noise was considered as a good choice. In addition, the Wiener filter order of 300 was also considered as an acceptable solution for ECG template.

In the future, the idea of partial IMF reconstruction by EEMD to remove noise can be widely used and applied to various biomedical signals, such as respiration, power line interference, and muscle contraction noise on ECG, especially when signal and noise is not band restricted. In the future, with the development of a rule to decide the selected IMF scale, an ECG noise filtering procedure can be expected.

5 Conclusion

EEMD-based partial reconstruction with selected added noise and sufficient trial number is a simple and effective approach to remove Gaussian noise in ECG. EEMD improved the previous EMD algorithm, and had a better performance than FIR Wiener filters, under sufficient trial-number iteration and added noise power.

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Kang-Ming Chang received the Ph.D. degree in the Department of Electrical and Control Engineering at the National Chiao-Tung University, Hsinchu, Taiwan, R. O.C., in 2006. Since 2006, he has been an Assistant Professor in the Department of Photonics and Communication Engineering, Asia University, Taichung, Taiwan, R.O.C., and he is also the Director of Distance Learning and Digital Content Center of Asia University. His current research interest includes home-care monitoring systems, biomedical signal processing, especially on sleep, meditation and pulse wave analysis.

Shing-Hong Liu received the B.S. degree in electronic engineering from the Feng-Jia University, Taizhon, Taiwan, R.O.C., in 1990, the M.S. degree in biomedical engineering from the National Cheng-Kung University, Tainan, Taiwan, R.O.C., in 1992 and the Ph.D. degree in the Department of Electrical and Control Engineering at the National Chiao-Tung University, Hsinchu, Taiwan, R.O.C., in 2002.

Now, he is an Associate Professor in the Department of Computer Science and Information Engineering, Chaoyang University of Technology, Taichung, Taiwan, R.O.C. His current research interests are biomedical signal processing, fuzzy control and designing biomedical instrument.