Chapter 4
Photon Transfer Theory

4.1 Photon Transfer Relation

A functional block diagram for a typical CCD/CMOS camera system is illustrated in Fig. 4.1. The system shown is described by the six transfer functions related to the semiconductor, pixel detector, and electronics that process the video signal. The input to the camera is expressed in units of the average number of incident photons per pixel (P), and the final output signal is given as the average DN encoded for each pixel. The output is related to the input gain relation

$$\frac{S(\mathrm{DN})}{P} = \mathrm{QE_I}\eta_\mathrm{i} A_\mathrm{SN} A_\mathrm{SF} A_\mathrm{CDS} A_\mathrm{ADC}, \tag{4.1}$$

where the individual gain functions are given in Table 4.1.

The signal and shot noise parameters at each point in the block diagram are listed in Table 4.2.

The gain functions contained in Eq. (4.1) are difficult to measure individually with good precision ($<$1% rms), especially those parameters related to the internal workings of the sensor. The PT method provides us with a solution to find the overall camera transfer function [Eq. (4.1)] accurately without knowing individual transfer functions. However, the PT technique is only fully applicable if a detector's response is shot noise-limited as explained below. Fortunately, this is the case for solid state sensors such as CCD and CMOS imagers.

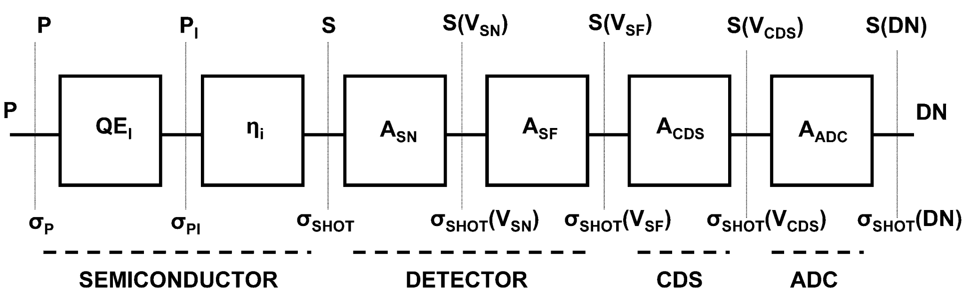

Figure 4.1 Typical solid state camera system showing internal gain functions and signal and noise parameters.

Table 4.1 Camera gain functions.

Parameter	Gain Function	Symbol
Quantum efficiency	P_I/P	QE_I
Quantum yield gain	S/P_I	η_i
Sense node gain	$S(V_{SN})/S$	A_{SN}
Source follower gain	$S(V_{SF})/S(V_{SN})$	A_{SF}
CDS gain	$S(V_{CDS})/S(V_{SF})$	A_{CDS}
ADC gain	$S(DN)/S(V_{CDS})$	A_{ADC}

Table 4.2 Camera signal and noise parameters.

Parameter	Average Signal	Noise (rms)
Incident photons	P	$\sigma_{SHOT}(P)$
Interacting photons	P_I	$\sigma_{SHOT}(P_I)$
Sense node electrons	S	σ_{SHOT}
Sense node voltage	$S(V_{SN})$	$\sigma_{SHOT}(V_{SN})$
Source follower voltage	$S(V_{SF})$	$\sigma_{SHOT}(V_{SF})$
CDS voltage	$S(V_{CDS})$	$\sigma_{SHOT}(V_{CDS})$
ADC signal	$S(DN)$	$\sigma_{SHOT}(DN)$

The general PT formula will now be derived. Figure 4.2 shows a general camera block diagram where the input signal exhibits shot noise characteristics. That is, from Eq. (3.2),

$$\sigma_A = A^{1/2}, \tag{4.2}$$

where A is the mean input signal level, and σ_A is the input noise standard deviation (or rms).

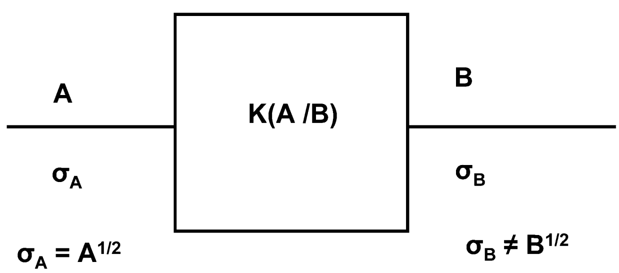

Figure 4.2 Black box camera system with a constant $K(A/B)$ used to transfer output signal (B) and noise (σ_B) measurements to the input.

A sensitivity constant defined as $K(A/B)$ relates and transfers output signal and noise measurements to the input. In other words,

$$A = BK(A/B) \tag{4.3}$$

and

$$\sigma_A = \sigma_B K(A/B), \tag{4.4}$$

where B and σ_B are the measured output mean signal level and noise standard deviation, respectively. Units for output B and input A will be different; generally, A is specified in absolute physical units that describe shot noise characteristics (photons and electrons), whereas B is specified in relative nonphysical units generated by an amplifier or analog-to-digital converter (volt and DN).

At this point in the analysis $K(A/B)$ is unknown. However, substituting Eqs. (4.3) and (4.4) into Eq. (4.2) and solving for $K(A/B)$ yields the desired result:

$$K(A/B) = \frac{B}{\sigma_B^2}, \tag{4.5}$$

where $K(A/B)$ is referred to the sensitivity constant of the system. Equation (4.5) is called the *PT relation*, an important equation that is the basis of the PT technique. Note that $K(A/B)$ is simply found by measuring output statistics (i.e., mean and noise variance) without knowledge of the individual camera transfer functions.

For a given set of measured B and σ_B output quantities, there is only one unique value for $K(A/B)$ that will satisfy the special input condition $\sigma_A = A^{1/2}$. It is also important to note from Eq. (4.5) that $\sigma_B \neq B^{1/2}$, because if that were the case, $K(A/B)$ would be forced to unity. As will be shown later, this would be an extremely rare setting for a camera system (mere coincidence). Also, it will be shown that $K(A/B)$ is adjusted to achieve optimum camera performance. $K(A/B)$ is usually tuned optimally by changing the voltage gain of an amplifier stage (typically block 5 in Fig. 4.1). Note that increasing the gain between A and B causes the sensitivity constant $K(A/B)$ to decrease.

It should be emphasized that gain functions are employed when referring the input of a measuring device to its output (i.e., from left to right in Fig. 4.1). Sensitivity functions are used to transfer output measurements to the input (i.e., from

Table 4.3 Sense node sensitivities ($\eta_i = 1$).

Sensitivity Parameter	Symbol	Sensitivity	Gain
Sense node	$K_{SN}(e^-/V_{SN})$	$S(V_{SN})/\sigma_{SHOT}(V_{SN})^2$	A_{SN}
Source follower	$K_{SF}(e^-/V_{SF})$	$S(V_{SF})/\sigma_{SHOT}(V_{SF})^2$	$A_{SN}A_{SF}$
CDS	$K_{CDS}(e^-/V_{CDS})$	$S(V_{CDS})/\sigma_{SHOT}(V_{CDS})^2$	$A_{SN}A_{SF}A_{CDS}$
ADC	$K_{ADC}(e^-/DN)$	$S(DN)/\sigma_{SHOT}(DN)^2$	$A_{SN}A_{SF}A_{CDS}A_{ADC}$

Table 4.4 Interacting photon sensitivities ($\eta_i \geq 1$).

Sensitivity Parameter	Symbol	Sensitivity	Gain
Interacting photon	$K_{SN}(P_I/e^-)$	$S(V_{SN})/\sigma_{SHOT}(V_{SN})^2$	η_i
ADC	$K_{ADC}(P_I/\mathrm{DN})$	$S(\mathrm{DN})/\sigma_{SHOT}(\mathrm{DN})^2$	$\eta_i A_{SN} A_{SF} A_{CDS} A_{ADC}$

Table 4.5 Incident photon sensitivities ($\eta_i \geq 1$).

Sensitivity Parameter	Symbol	Sensitivity	Gain
Incident photon	$K_{PI}(P/P_I)$	$(\mathrm{QE_I})^{-1}$	$\mathrm{QE_I}$
ADC	$K_{ADC}(P/\mathrm{DN})$	$(\mathrm{QE_I})^{-1}S(\mathrm{DN})/\sigma_{SHOT}(\mathrm{DN})^2$	$\mathrm{QE_I}\eta_i A_{SN} A_{SF} A_{CDS} A_{ADC}$

right to left in Fig. 4.1). For example, to convert output units to input units, multiply the output by the sensitivity factor $K(A/B)$. Equation (4.5) can now be applied to derive the most commonly applied sensitivity camera constants listed in Tables 4.3–4.5.

4.2 Sense Node Sensitivities

4.2.1 Sense node sensitivity

Figure 4.3 shows the sense node region, common to all CCD and CMOS detectors, where signal charge is converted to a working voltage and buffered by a source follower amplifier (i.e., the third block shown in Fig. 4.1). Relationships linking sense node capacitance, V/e$^-$ sense node gain, e$^-$/V sensitivity, and charge and voltage on this node are related through the differential equation

$$C_{SN} = q\frac{dS}{dV_{SN}}, \tag{4.6}$$

where C_{SN} is the sense node capacitance (F), V_{SN} is the sense node voltage, S is the signal (e$^-$), and q is the charge of an electron (1.6×10^{-19}C).

Integrating Eq. (4.6) with respect to V_{SN} yields

$$S = \frac{1}{q}\int_{V_{SN}}^{V_{REF}} C_{SN} dV_{SN}, \tag{4.7}$$

where V_{REF} is the reference voltage on the sense node after the sense node is reset. The limits of integration are set for negative (i.e., electron) changing signals on the sense node.

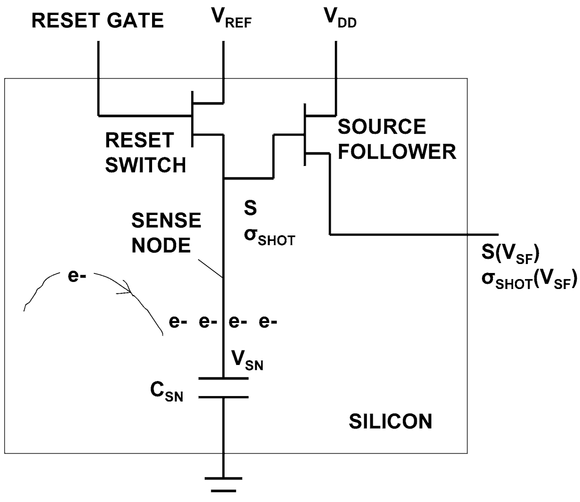

Figure 4.3 Typical CCD/CMOS sense node region where signal charge and related noise are converted to a working output voltage.

For linear detectors, C_{SN} is a constant and invariant to signal. Chapter 7 analyzes the situation when the sense node capacitance varies with signal. Integrating Eq. (4.7) with a fixed C_{SN} yields

$$S = \frac{C_{\mathrm{SN}}(V_{\mathrm{REF}} - V_{\mathrm{SN}})}{q}. \tag{4.8}$$

The sense node voltage is

$$V_{\mathrm{SN}} = V_{\mathrm{REF}} - S(V_{\mathrm{SN}}), \tag{4.9}$$

where $S(V_{\mathrm{SN}})$ is the sense node signal voltage.

Equation (4.9) reduces Eq. (4.8) to

$$S = \frac{C_{\mathrm{SN}} S(V_{\mathrm{SN}})}{q}. \tag{4.10}$$

From Eq. (4.10), the sense node sensitivity is

$$K_{\mathrm{SN}}(\mathrm{e}^-/V_{\mathrm{SN}}) = \frac{S}{S(V_{\mathrm{SN}})} = \frac{C_{\mathrm{SN}}}{q} = \frac{1}{A_{\mathrm{SN}}}. \tag{4.11}$$

From the PT Eq. (4.5), the shot noise voltage on the sense node is

$$\sigma_{\mathrm{SHOT}}(V_{\mathrm{SN}}) = \left[\frac{S(V_{\mathrm{SN}})}{K_{\mathrm{SN}}(\mathrm{e}^-/V_{\mathrm{SN}})}\right]^{1/2}; \quad \eta_{\mathrm{i}} = 1. \tag{4.12}$$

4.2.2 Sense node to source follower sensitivity

The source follower output voltage is given by

$$V_{\mathrm{SF}} = V_{\mathrm{SF\_OFF}} - S(V_{\mathrm{SF}}), \tag{4.13}$$

where V_{SF} is the source follower output voltage, $S(V_{\mathrm{SF}})$ is the source follower signal voltage, and $V_{\mathrm{SF\_OFF}}$ is the DC source follower offset voltage.

The sense node to source follower sensitivity node is found through the PT Eq. (4.5), which yields

$$K_{\mathrm{SF}}(\mathrm{e}^-/V_{\mathrm{SF}}) = \frac{S(V_{\mathrm{SF}})}{\sigma_{\mathrm{SHOT}}(V_{\mathrm{SF}})^2}; \quad \eta_{\mathrm{i}} = 1. \tag{4.14}$$

In terms of sense node sensitivity,

$$K_{\mathrm{SF}}(\mathrm{e}^-/V_{\mathrm{SF}}) = \frac{K_{\mathrm{SN}}(\mathrm{e}^-/V_{\mathrm{SN}})}{A_{\mathrm{SF}}}, \tag{4.15}$$

where A_{SF} is the source follower voltage gain.

Example 4.1

Determine $K_{\mathrm{SF}}(\mathrm{e}^-/V_{\mathrm{SF}})$, S, σ_{SHOT}, $K_{\mathrm{SN}}(\mathrm{e}^-/V_{\mathrm{SN}})$, A_{SN}, and C_{SN} from the following data measurements made at the output of the source follower:

$$S(V_{\mathrm{SF}}) = 0.016 \text{ V}$$

$$\sigma_{\mathrm{SHOT}}(V_{\mathrm{SF}}) = 0.000315 \text{ V}$$

Assume $A_{\mathrm{SF}} = 0.9$ V/V, and $\eta_{\mathrm{i}} = 1$ (i.e., visible or near IR light).

Solution:
Applying Eq. (4.14) to the data yields

$$K_{\mathrm{SF}}(\mathrm{e}^-/V_{\mathrm{SF}}) = \frac{0.016}{0.000315^2} = 1.61 \times 10^5 \text{ e}^-/\text{V}.$$

Converting the source follower signal voltage to electrons,

$$S = K_{\mathrm{SF}}(\mathrm{e}^-/V_{\mathrm{SF}}) \times S(V_{\mathrm{SF}}) = 2560 \text{ e}^-.$$

Converting the source follower noise voltage to electrons,

$$\sigma_{\mathrm{SHOT}} = K_{\mathrm{SF}}(\mathrm{e}^-/V_{\mathrm{SF}}) \times \sigma_{\mathrm{SHOT}}(V_{\mathrm{SF}}) = 51 \text{ e}^- \text{ rms}.$$

From Eq. (4.15), the sense node sensitivity is

$$K_{\mathrm{SN}}(\mathrm{e}^-/V_{\mathrm{SN}}) = (1.61 \times 10^5) \times 0.9 = 1.45 \times 10^5 \text{ e}^-/\text{V}.$$

The reciprocal of the sense node sensitivity is the sense node gain,

$$A_{\mathrm{SN}} = \frac{1}{K_{\mathrm{SN}}(\mathrm{e}^-/V_{\mathrm{SN}})} = 6.9 \times 10^{-6}\ \mathrm{V/e}^-.$$

From Eqs. (4.10) and (4.15), the sense node capacitance is

$$C_{\mathrm{SN}} = (1.6 \times 10^{-19}) \times 0.9 \times (1.45 \times 10^5) = 2.01 \times 10^{-14}\ \mathrm{F}.$$

It is amazing that only two relative measurements (signal and noise voltages) at the output of the source follower produce such a wealth of absolute information through the PT technique.

4.2.3 Sense node to CDS sensitivity

For positive going signals, the CDS output voltage is given by

$$V_{\mathrm{CDS}} = V_{\mathrm{CDS\_OFF}} + S(V_{\mathrm{CDS}}), \tag{4.16}$$

where V_{CDS} is the CDS output voltage, $S(V_{\mathrm{CDS}})$ is the CDS signal voltage, and $V_{\mathrm{CDS\_OFF}}$ is the CDS offset voltage.

The sense node to CDS sensitivity is found through the PT Eq. (4.5), which yields

$$K_{\mathrm{CDS}}(\mathrm{e}^-/V_{\mathrm{CDS}}) = \frac{S(V_{\mathrm{CDS}})}{\sigma_{\mathrm{SHOT}}(V_{\mathrm{CDS}})^2}; \quad \eta_{\mathrm{i}} = 1. \tag{4.17}$$

In terms of sense node sensitivity,

$$K_{\mathrm{CDS}}(\mathrm{e}^-/V_{\mathrm{CDS}}) = \frac{K_{\mathrm{SN}}(\mathrm{e}^-/V_{\mathrm{SN}})}{A_{\mathrm{SF}}A_{\mathrm{CDS}}}, \tag{4.18}$$

where A_{CDS} is the CDS voltage gain.

4.2.4 Sense node to ADC sensitivity

For positive going signals, the ADC output signal is

$$\mathrm{DN}_{\mathrm{ADC}} = S_{\mathrm{ADC\_OFF}}(\mathrm{DN}) + S(\mathrm{DN}), \tag{4.19}$$

where $\mathrm{DN}_{\mathrm{ADC}}$ is the raw ADC output signal, $S(\mathrm{DN})$ is the true signal, and $S_{\mathrm{ADC\_OFF}}(\mathrm{DN})$ is the ADC offset level.

From PT, the sense node to ADC sensitivity is

$$K_{\mathrm{ADC}}(\mathrm{e}^-/\mathrm{DN}) = \frac{S(\mathrm{DN})}{\sigma_{\mathrm{SHOT}}(\mathrm{DN})^2}; \quad \eta_{\mathrm{i}} = 1. \tag{4.20}$$

In terms of sense node sensitivity,

$$K_{\mathrm{ADC}}(\mathrm{e}^-/\mathrm{DN}) = \frac{K_{\mathrm{SN}}(\mathrm{e}^-/V_{\mathrm{SN}})}{A_{\mathrm{SF}}A_{\mathrm{CDS}}A_{\mathrm{ADC}}}. \tag{4.21}$$

Example 4.2

Find $K_{\mathrm{ADC}}(\mathrm{e}^-/\mathrm{DN})$, S, σ_{SHOT}, $K_{\mathrm{SN}}(\mathrm{e}^-/V_{\mathrm{SN}})$, C_{SN}, and A_{SN} from the following data taken at the output of the ADC:

$$\mathrm{DN}_{\mathrm{ADC}} = 10{,}800$$

$$S_{\mathrm{ADC\_OFF}}(\mathrm{DN}) = 800$$

$$\sigma_{\mathrm{SHOT}}(\mathrm{DN}) = 50$$

Assume $A_{\mathrm{SF}} = 0.9$ V/V, $A_{\mathrm{CDS}} = 10$ V/V, $A_{\mathrm{ADC}} = 3250$ DN/V, and $\eta_{\mathrm{i}} = 1$. Convert DN signal and noise levels to electrons.

Solution:
From Eq. (4.19), the ADC signal is

$$S(\mathrm{DN}) = 10{,}800 - 800 = 10{,}000.$$

From Eq. (4.20), the ADC sensitivity is

$$K_{\mathrm{ADC}}(\mathrm{e}^-/\mathrm{DN}) = \frac{10{,}000}{50^2} = 4.$$

Signal and noise in electrons are

$$S = 4 \times 10{,}000 = 40{,}000\ \mathrm{e}^-$$

$$\sigma_{\mathrm{SHOT}} = 40{,}000^{1/2} = 200\ \mathrm{e}^-$$

From Eq. (4.21), the sense node sensitivity is

$$K_{\mathrm{SN}}(\mathrm{e}^-/V_{\mathrm{SN}}) = 0.9 \times 10 \times 3250 \times 4 = 1.17 \times 10^5\ \mathrm{e}^-/\mathrm{V}.$$

From Eq. (4.11), the sense node capacitance is

$$C_{\mathrm{SN}} = (1.6 \times 10^{-19}) \times 1.17 \times 10^5 = 1.87 \times 10^{-14}\ \mathrm{F}.$$

The sense node gain is

$$A_{\mathrm{SN}} = \frac{1}{K_{\mathrm{SN}}(\mathrm{e}^-/V_{\mathrm{SN}})} = \frac{1}{1.17 \times 10^5\ \mathrm{e}^-/\mathrm{V}} = 8.54 \times 10^{-6}\ \mathrm{V}/\mathrm{e}^-.$$

4.3 Interacting Photon Sensitivities

4.3.1 Interacting photon sensitivity

The interacting photon sensitivity is defined by the quantum yield,

$$K_{\mathrm{SN}}(P_{\mathrm{I}}/\mathrm{e}^-) = (\eta_{\mathrm{i}})^{-1}. \tag{4.22}$$

4.3.2 Interacting photon to ADC sensitivity

The interacting photon to ADC sensitivity, from PT Eq. (4.5), is

$$K_{\mathrm{ADC}}(P_{\mathrm{I}}/\mathrm{DN}) = \frac{S(\mathrm{DN})}{\sigma_{\mathrm{SHOT}}(\mathrm{DN})^2} \quad \text{for all } \eta_{\mathrm{i}}. \tag{4.23}$$

Noting that

$$K_{\mathrm{ADC}}(P_{\mathrm{I}}/\mathrm{DN}) = \frac{K_{\mathrm{ADC}}(\mathrm{e}^-/\mathrm{DN})}{\eta_{\mathrm{i}}}, \tag{4.24}$$

and solving for η_{i} yields

$$\eta_{\mathrm{i}} = \frac{K_{\mathrm{ADC}}(\mathrm{e}^-/\mathrm{DN})}{K_{\mathrm{ADC}}(P_{\mathrm{I}}/\mathrm{DN})} = \frac{K_{\mathrm{ADC}}(\mathrm{e}^-/\mathrm{DN})\sigma_{\mathrm{SHOT}}(\mathrm{DN})^2}{S(\mathrm{DN})}. \tag{4.25}$$

Example 4.3

Find $K_{\mathrm{ADC}}(P_{\mathrm{I}}/\mathrm{DN})$, $K_{\mathrm{ADC}}(\mathrm{e}^-/\mathrm{DN})$, η_{i}, and the photon energy for the following data taken at the output of the ADC:

$\eta_{\mathrm{i}} = 1$ (i.e., visible or near IR stimulus)
$S(\mathrm{DN}) = 10{,}000$
$\sigma_{\mathrm{SHOT}}(\mathrm{DN}) = 70$

$\eta_{\mathrm{i}} > 1$ (working wavelength)
$S(\mathrm{DN}) = 25{,}000$
$\sigma_{\mathrm{SHOT}}(\mathrm{DN}) = 700$

Solution:

$$\eta_{\mathrm{i}} = 1$$

From Eq. (4.20),

$$K_{\mathrm{ADC}}(\mathrm{e}^-/\mathrm{DN}) = \frac{10{,}000}{70^2} = 2.04;$$

$$\eta_{\mathrm{i}} > 1.$$

From Eq. (4.23),

$$K_{\mathrm{ADC}}(P_{\mathrm{I}}/\mathrm{DN}) = \frac{25{,}000}{700^2} = 0.051.$$

From Eq. (4.25), the quantum yield is

$$\eta_{\mathrm{i}} = \frac{2.04}{0.051} = 40 \ \mathrm{e}^-/\text{interacting photon}.$$

The quantum yield corresponds to a photon energy of approximately $40 \times 3.65 = 146$ eV [Eq. (2.9)].

4.4 Incident Photon Sensitivities

4.4.1 Incident photon sensitivity

The incident photon sensitivity is equal to the interacting QE found experimentally [i.e., Eq. (2.4)]:

$$K_{\mathrm{PI}}(P/P_{\mathrm{I}}) = \mathrm{QE}_{\mathrm{I}}^{-1}. \tag{4.26}$$

4.4.2 Incident photon to ADC sensitivity

Incident photon to ADC sensitivity is given by

$$K_{\mathrm{ADC}}(P/\mathrm{DN}) = \frac{K_{\mathrm{ADC}}(P_{\mathrm{I}}/\mathrm{DN})}{\mathrm{QE}_{\mathrm{I}}} \quad \text{for all } \eta_{\mathrm{i}}. \tag{4.27}$$

Example 4.4

From Example 4.3, find $K_{\mathrm{ADC}}(P/\mathrm{DN})$ assuming $\mathrm{QE}_{\mathrm{I}} = 0.5$.

Solution:
From Eq. (4.27),

$$K_{\mathrm{ADC}}(P/\mathrm{DN}) = \frac{0.051}{0.5} = 0.102.$$

4.5 Photon Transfer General Derivation

$K_{\mathrm{ADC}}(\mathrm{e}^-/\mathrm{DN})$ in terms of output DN statistics can be derived on first principles starting with the equation

$$S(\mathrm{DN}) = \frac{P_{\mathrm{I}}}{K_{\mathrm{ADC}}(\mathrm{e}^-/\mathrm{DN})}, \tag{4.28}$$

where unity quantum yield is assumed. The variance of Eq. (4.28) is found by the propagation of errors formula, i.e.,

$$\sigma^2_{\mathrm{SHOT}}(\mathrm{DN}) = \left[\frac{\partial S(\mathrm{DN})}{\partial P_{\mathrm{I}}}\right]^2 \sigma^2_{P_{\mathrm{I}}} + \left[\frac{\partial S(\mathrm{DN})}{\partial K_{\mathrm{ADC}}(\mathrm{e}^-/\mathrm{DN})}\right]^2 \sigma^2_{K_{\mathrm{ADC}}(\mathrm{e}^-/\mathrm{DN})}. \tag{4.29}$$

Performing the differentiation yields

$$\sigma_{\mathrm{SHOT}}(\mathrm{DN})^2 = \left[\frac{1}{K_{\mathrm{ADC}}(\mathrm{e}^-/\mathrm{DN})}\right]^2 \sigma^2_{P_{\mathrm{I}}} + \left[\frac{-P_{\mathrm{I}}}{K_{\mathrm{ADC}}(\mathrm{e}^-/\mathrm{DN})^2}\right]^2 \sigma^2_{K_{\mathrm{ADC}}(\mathrm{e}^-/\mathrm{DN})}. \tag{4.30}$$

Assuming $\sigma^2_{K(\mathrm{e^-/DN})} = 0$ (i.e., if a sufficient number of pixels are sampled) and $\sigma^2_{P_\mathrm{I}} = P_\mathrm{I}$, Eq. (4.30) reduces to

$$\sigma_\mathrm{SHOT}(\mathrm{DN})^2 = \left[\frac{1}{K_\mathrm{ADC}(\mathrm{e^-/DN})}\right]^2 P_\mathrm{I}. \tag{4.31}$$

Substituting Eq. (4.28) into Eq. (4.31) and solving for $K_\mathrm{ADC}(\mathrm{e^-/DN})$ yields

$$K_\mathrm{ADC}(\mathrm{e^-/DN}) = \frac{S(\mathrm{DN})}{\sigma^2_\mathrm{SHOT}(\mathrm{DN})}. \tag{4.32}$$

4.6 Effective Quantum Yield

4.6.1 Photon event charge sharing

As long as all of the charge generated by a photon is collected by the target pixel and not by neighboring pixels, Eq. (4.25) can be applied to determine the quantum yield. However, charge may be shared if the diameter of the initial electron cloud immediately after a photon interacts is comparable to the pixel size. This effect may take place for high-energy soft x-rays where cloud diameter increases with photon energy.

Charge sharing also occurs when a pixel's active volume is only partially depleted—that is, regions of the pixel where electric fields do not exist, allowing electrons to wander into neighboring pixels.[1] The charge diffusion problem is presented in Figs. 4.4 through 4.5. Figure 4.4 shows 5.9-keV soft x-ray photon events ($1620\ \mathrm{e^-}$) generated by a CCD that exhibits a significant charge diffusion problem. Photons that penetrate and interact below the pixel's depletion region diffuse into many pixels around the target pixel. Figure 4.5 shows the charge diffusion problem for three 9-MeV grazing incident protons that interact with the same CCD. The protons ionize the silicon atoms, generating a signal charge that initially collects in single pixels along the track. However, as the proton goes deeper into the silicon, beyond the depletion region, the charge cloud grows in size and occupies more than a dozen pixels before the proton finally stops.

The quantum yield measured will be lower than expected if electrons from a photon event are shared among the pixels. The measurement effect is a reduction of the true, measured shot noise. That is,

$$\sigma_\mathrm{SHOT} = (\eta_\mathrm{E} S)^{1/2}, \tag{4.33}$$

where η_E is the effective quantum yield that is measured. If charge sharing takes place, the effective quantum yield will always be less than the ideal quantum yield (i.e., $\eta_\mathrm{E} < \eta_\mathrm{i}$).

Quantum yield is measured more accurately with larger pixel sensors because the pixels represent a larger target for the charge cloud to collect. Also, pixels can be summed to improve the measurement (e.g., 2×2 sum). This can be achieved either on-chip for a CCD or off-chip for a CMOS sensor.

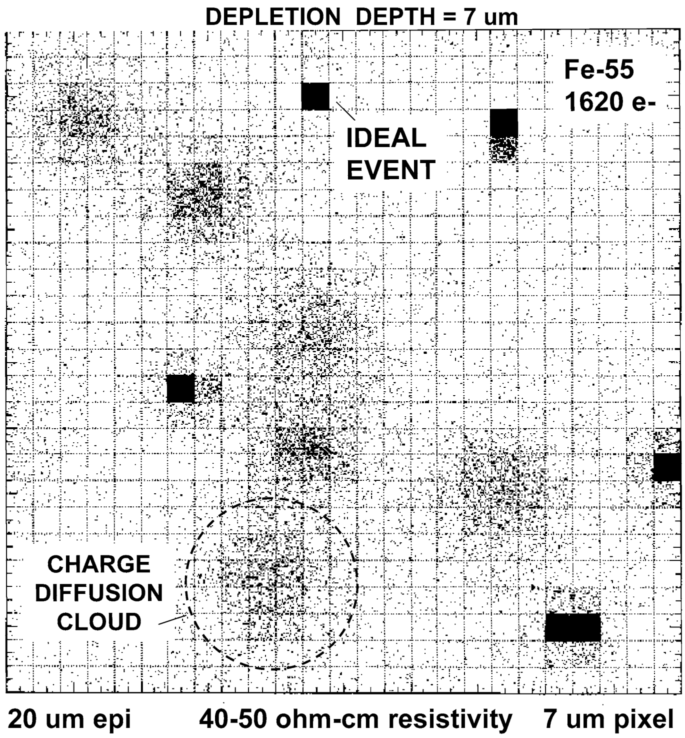

Figure 4.4 Fe-55 1620 e^- x-ray photons taken from a CCD that exhibits charge diffusion and collection problems.

4.6.2 Charge collection efficiency

The effective quantum yield measured represents a figure of merit for how well pixels collect signal charge. Charge collection efficiency (CCE) is defined as[1]

$$\mathrm{CCE} = \frac{\eta_{\mathrm{E}}}{\eta_{\mathrm{i}}}. \tag{4.34}$$

Figure 4.6 shows a 5.9-keV x-ray quantum yield histogram taken from a CCD with 15-μm pixels. The array was uniformly illuminated with x-rays (approximately five x-rays per pixel). The quantum yield was calculated using Eq. (4.25) for many different 40×40 pixel subarrays across the sensor. Data were then displayed in the histogram shown. The average effective quantum yield measured was 900 e^- significantly less than the ideal quantum yield of 1620 e^-. The less-than-optimum response was caused by field-free silicon that resulted in charge diffusion and sharing as well as some recombination loss within the substrate of the device.

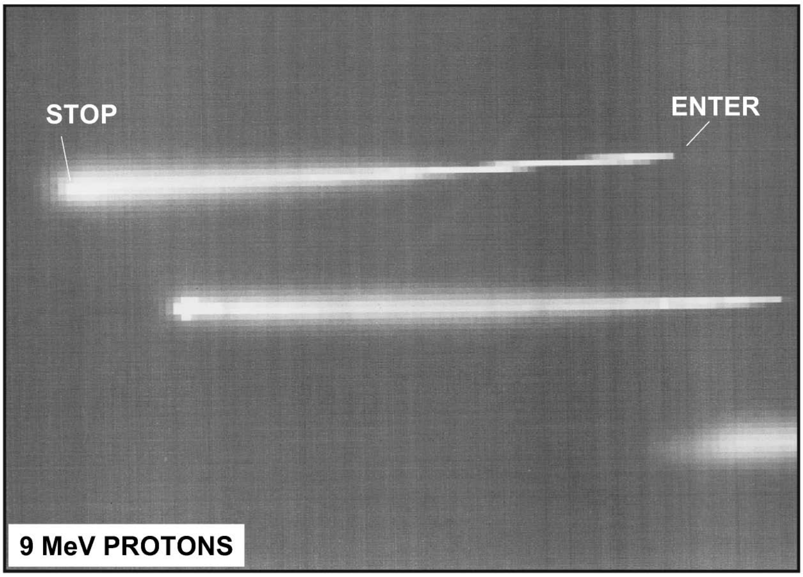

Figure 4.5 9 MeV proton events showing charge diffusion characteristics by the same CCD in Fig. 4.4.

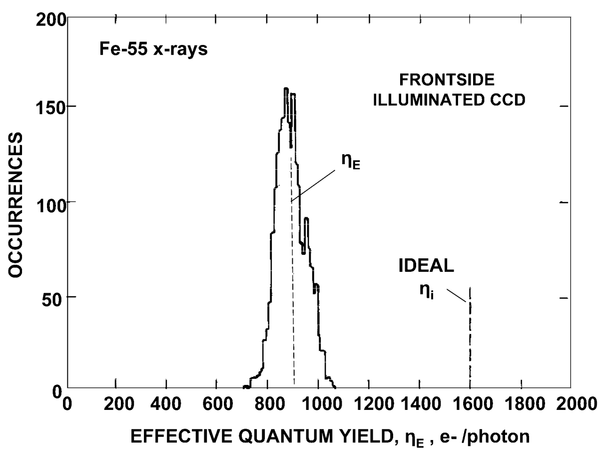

Figure 4.6 CCD Fe-55 histogram showing that the effective quantum yield is less than the ideal quantum yield because of charge collection problems.

Example 4.5

Calculate the CCE for the PT results in Fig. 4.6.

Solution:
From Eq. (4.34),

$$\text{CCE} = \frac{900}{1620} = 0.556.$$

Important Points

1. PT relations [e.g., $K_{\text{ADC}}(\text{e}^{-}/\text{DN})$] are found by measuring the output signal statistics. When finding the relations, the camera can be treated as a black box as long as the input exhibits shot noise behavior.
2. Given the output mean and variance quantities, only one unique PT relation satisfies the input shot noise statistics (i.e., $\text{noise} = \text{signal}^{1/2}$).
3. The PT relation allows relative output measurement units (DN, volt) to be converted to absolute input physical units (electrons and photons).
4. The PT relation is used to determine internal camera transfer functions [e.g., sense node gain (V/e^{-}) and sense node capacitance].
5. Quantum efficiency and quantum yield performance parameters are found through the PT relation.
6. Charge sharing between pixels lowers shot noise and quantum yield measurements.
7. The PT relation is used to measure pixel CCE performance of a detector.