Pairs selection and trading

Outline

- What is pairs trading?
- Pair selection.
- Strategy development
- Parameter tuning, potential improvements and future challenges.

Pairs trading: Key idea

- Capitalize in market imbalances between two or more securities, in anticipation of making money when the inequality is corrected in the future [2004, Whistler]

- Find two securities that have *moved together* in the recent past.

- Look at the spread between the prices. Buy the spread if it is below a lower threshold and sell it once it crosses the threshold. Conversely, if the spread is above a (upper) threshold, short the spread and close the short once it drops below the (upper) threshold.

Features of pairs trading

- The idea is to profit from temporary mis-pricings of assets.
- Long positions are hedged with short positions in same/ related sectors.
- An example of a *market-neutral strategy* - generates profits under uptrend or downtrend.

Intuitive explanation: Adapted from Murray, 1994

- A drunk customer sets out from a bar and starts walking in the streets.
- Accompanying dog thinks "I can't let him get to far off. After all, my role is to protect him!"
- So, the dog assesses how far the drunk is and moves in to close the gap .

- Two observers, Sasha and Peng, bet on the drunk's and the dog's position.
- They observe the dog and the drunk individually but their path looks like a random walk (growing variance, lack of predictability.)
- They hit upon an idea of just finding the drunk since the dog won't be far away. ☺
- They are correct since the gap between the two fellows should occasionally open and close but never being out of control, a notion formalized in the co-integration test.

- There are a couple of ways in which we build on this idea. One is to model the spread explicitly as a mean-reverting process (Ornstein-Uhlenbeck process) and predict the parameters of the process.
- The other is to find a linear combination of the asset price series which would be stationary (and hence mean reverting.)
- We use the latter approach. One way to do this is set a rolling linear regression with a look-back window.
- This introduces an extra parameter (look-back window length) over which we need to optimize.

Kalman Filter approach

- For this project, we use a Kalman filter to dynamically track the hedging ratio (the ratio of number of shares of one asset to the number of shares of the other asset) in order to keep the spread stationary (and hence mean reverting.)
- We treat the true hedge ratio as an unobserved hidden variable and attempt to estimate it with "noisy" observations (price data of each asset.)
- Before we go on to the mathematics of Kalman filter and trading rules, it is very important to select the *right* pair for the strategy.

Kalman Filter Model

- State space equation:

$$\theta_{t+1} = \theta_t + w_t; \; w_t \sim N(0, W_t)$$

- To form the observation equation, choose one of the price series as y_t and the other as x_t.

$$y_t = (x_t, 1)\theta_t + v_t; \;\; v_t \sim N(0, V_t).$$

- To wit, let $\theta_t | D_{t-1} \sim N(a_t, R_t)$ i.e the prior view of θ at time t, given our knowledge at time $t-1$ is a multivariate normal distribution with mean a_t and covariance matrix R_t. Then

$$\theta_t | D_t \sim N(m_t, C_t).$$

Kalman filter algorithm

$$
\begin{aligned}
&a_t = m_{t-1} \text{ (prior mean)}\\
&R_t = C_{t-1} + w_t \text{ (prior covariance matrix)}\\
&f_t = (x_t, 1)m_{t-1} \text{ (predicted value of observation at time } t\text{)}\\
&e_t = y_t - f_t \text{ (forecast error)}\\
&Q_t = (x_t, 1)R_t(x_t, 1)^T + V_t\\
&A_t = R_t(x_t, 1)^T Q_t^{-1}\\
&m_t = m_{t-1} + A_t e_t \text{ (posterior mean)}\\
&C_t = R_t - A_t(x_t, 1)R_t^T \text{ (posterior covariance matrix)}
\end{aligned}
$$

Trading rules

The role of the Kalman filter is to calculate θ_t, e_t and Q_t.

Trading Rules

1. $e_t < -\text{multiple} \times \sqrt{Q_t}$. Long the spread.
2. $e_t \geq -\text{multiple} \times \sqrt{Q_t}$. Exit long. Close all positions.
3. $e_t > \text{multiple} \times \sqrt{Q_t}$. Short the spread.
4. $e_t \leq -\text{multiple} \times \sqrt{Q_t}$. Exit short. Close all positions.

Longing the spread means longing N units of y_t and selling short $\lfloor \theta_t^0 N \rfloor$ of x_t, where N controls the size of our positions

Hyper parameter tuning and other stuff

- We hard-coded certain values in to our code. Ideally, we should do a cross validation to get the best parameters depending on our preferred choice of performance metric(s).
- The choice of clustering algorithm in pair selection also needs to be investigated.
- See what time of the day is best for trading a *particular* pair of assets.

Future work

- Note that this strategy works well if a pair of asset prices are co-integrated. In that case, the trading rules are clear since the *spread* is expected to be a mean-reverting process.
- What if a pair of asset prices are not co-integrated but some function of them is? For instance, log or square prices can be co-integrated.
- In this case, how do we about generating the signals? How can we take advantage of temporary mis-pricing of log (or square) of asset prices?
- Non-linear Kalman filter? Groups trading?