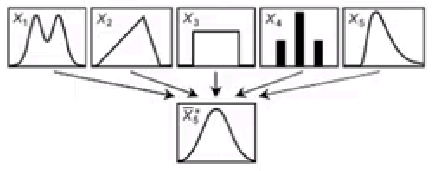
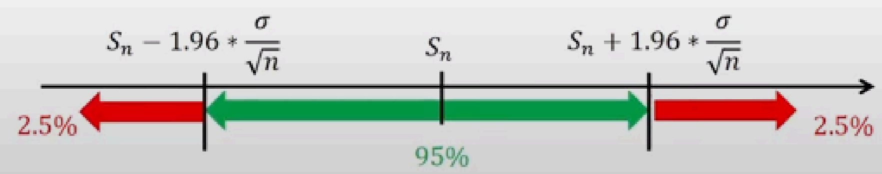
Teoremas

2 important theorems

- Law of large numbers
  - Let $X_1, \dots, X_n, \dots$ be i.i.d. random variables with mean μ and variance σ^2
  - Define the sample mean as $S_n = \frac{1}{n}[X_1 + \dots + X_n]$
  - Then the sample mean converges to the real mean: $S_n \to \mu$ a.s.
- Central Limit Theorem
  - The sample mean S_n
    - Is itself a random variable
    - It is normally distributed $N\left(\mu, \frac{\sigma^2}{n}\right)$



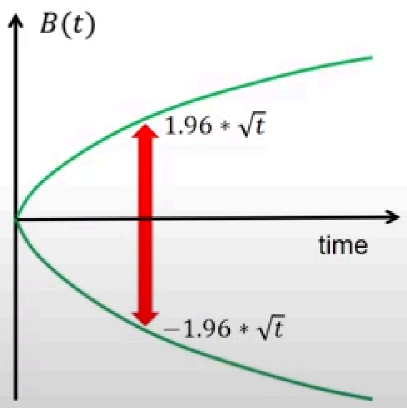
- We can now compute the confidence interval of μ as a function of S_n



Movimiento Browniano

Brownian motion

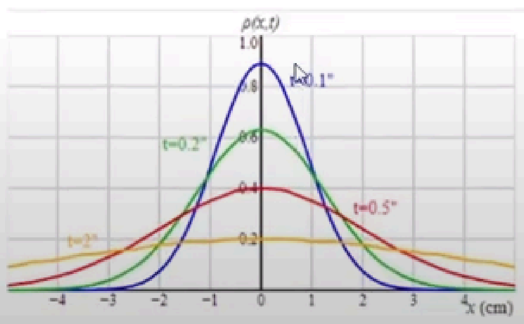
- We can describe the Brownian motion based on
  - Normal distribution
  - Confidence intervals
- Start with a particle at the origin
  - $B(0) = 0$ a.s.
- At time t the particle moves randomly
  - We do not know the exact position
  - But we do know its distribution
    - $B(t) \sim N(0, t)$
- At time t the confidence interval 95% is
  - $[-1.96 * \sqrt{t},\ +1.96 * \sqrt{t}]$



Propiedades

Properties of the Brownian motion

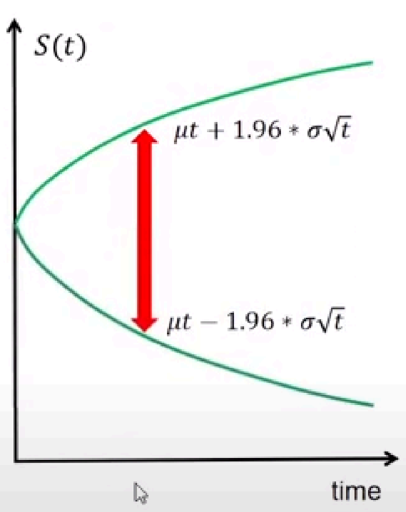
- Independence (Markov property)
  - $B(t+h) - B(t) \sim N(0, h)$
- Paths $t \to B(t)$
  - Are continuous
  - But not differentiable
- Scaling
  - If $B(t) \sim N(0, t)$ then
    - $S(t) = \mu + \sigma B(t) \sim N(\mu, \sigma^2 t)$
  - Confidence interval for $S(t)$ is
    - $[\mu - 1.96 * \sigma\sqrt{t}, \mu + 1.96 * \sigma\sqrt{t}]$



Proceso estacástcio

General Stochastic Processes

- Assume a standard Brownian motion
  - $B(t) \sim N(0, t)$
- We add drift and volatility
  - (1) $S(t) = \mu t + \sigma B(t)$
  - This is the classical model for financial returns
- Itô Process
  - Generalisation of the (1)
  - It is customary to use the differential notation
    - (2) $dS(t) = \mu(t, S)dt + \sigma(t, S)dB(t)$
- The discrete version of the Itô process (2) is
  - (3) $S(t+h) - S(t) = \mu(t, S)h + \sigma(t, S)B(h)$
  - Monte Carlo simulations assume (3) with $B(h) \sim \sqrt{h}\, N(0,1)$
- Examples
  - Geometric Brownian motion: $dS(t) = \mu S(t)dt + \sigma S(t)dB(t)$
  - Ornstein-Uhlenbeck: $dS(t) = \theta(\mu - S(t))dt + \sigma dB(t)$



Montecarlo Risk

Referencia Mauricio Labadie. 02-05-2019 Geometría de portafolio de inversión

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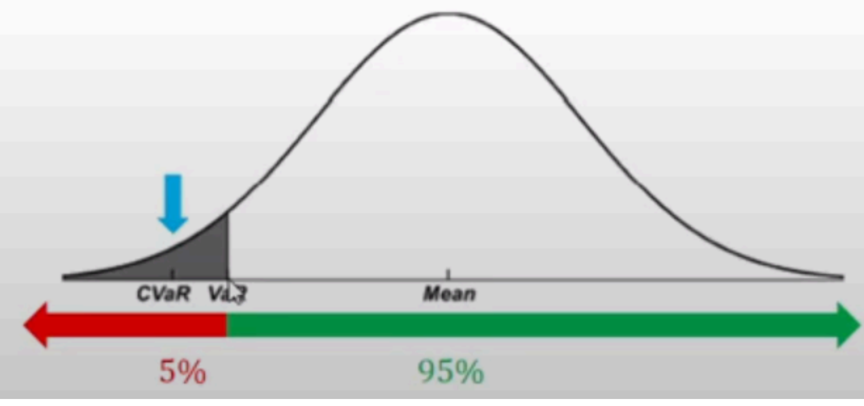
Valor del riesgo

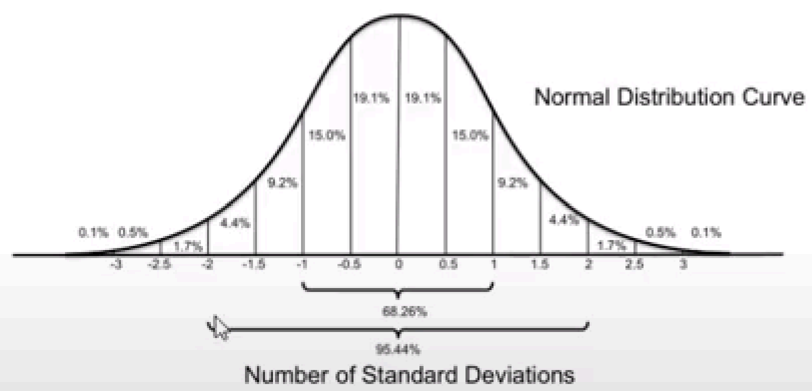
Value at Risk

- The Value at Risk of level $q \in (0,1)$ is a number $VaR(q)$ such that
  - $P[x \geq VaR(q)] = q$
- Equivalently, $P[x \leq VaR(q)] = 1 - q$
- For normal random variables $N(\mu, \sigma^2)$ we have
  - $VaR(95\%) = \mu - 1.64 * \sigma$
- Conditional Value at Risk
  - $CVaR(95\%)$ is the average of all losses at the left tail of the $VaR(95\%)$



Intervalos de confianza

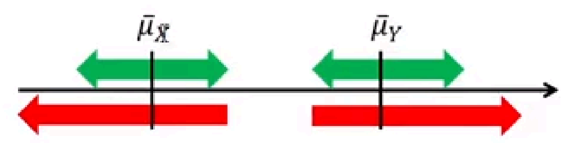
- We start with a random variable X with mean μ
- The confidence interval with level $q \in (0,1)$ around μ is
  - $CI(q) = [\mu - K(q),\ \mu + K(q)]$
  - Where $K(q) > 0$ is such that $P[x \in CI(q)] = q$
- For normal random variables $N(\mu, \sigma^2)$ we have
  - $CI(95\%) = [\mu - 1.96 * \sigma,\ \mu + 1.96 * \sigma]$



Actuación

Difference of performance

- We have a finite number of data points $t > 0$
  - X_t and Y_t where $t = 1, 2, \dots, N$
- Compute their sample means
  - $\bar{\mu}_X$ and $\bar{\mu}_Y$
- Suppose that we found that $\bar{\mu}_X < \bar{\mu}_Y$
  - Is it statistically significant?
- If we have disjoint confidence intervals
  - Problem solved
- If the confidence intervals intersect
  - Reduce the confidence level
  - Add more data points
  - Use a Student T-test for 2 means



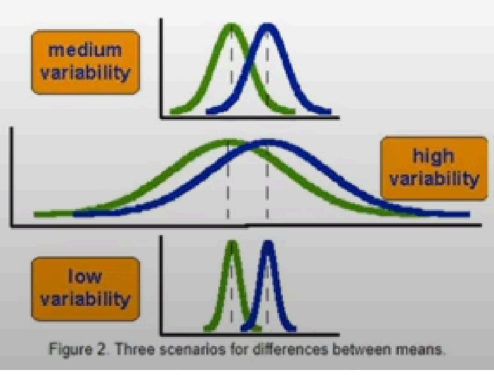


Figure 2. Three scenarios for differences between means.