

**sheet of limits and continuity**

**Exercises:**

1.  $\lim_{x \rightarrow 3} \frac{3x(2x-1)}{x^2+2}$

2.  $\lim_{x \rightarrow 5} \frac{x-5}{x^2-25}$

3.  $\lim_{x \rightarrow -3} \frac{x^2+4x+3}{x^2-9}$

4.  $\lim_{x \rightarrow 2} \frac{x^2+7x+10}{x-2}$

5.  $\lim_{x \rightarrow -1} \frac{x^2-1}{x+1}$

6.  $\lim_{x \rightarrow -2} \frac{x^2-x-6}{x^2+2x}$

7.  $\lim_{y \rightarrow 3} \frac{5y^2+8y^2-3}{y^2-9}$

8.  $\lim_{x \rightarrow 1} \frac{\frac{1}{x}-1}{x-1}$

9.  $\lim_{x \rightarrow 0} \frac{\sqrt{5x+4}-2}{x}$

10.  $\lim_{x \rightarrow 1} \frac{x^4-1}{x+1}$

11.  $\lim_{x \rightarrow 2} \frac{x-2}{x^2+x-6}$

12.  $\lim_{x \rightarrow 9} \frac{\sqrt{x}-3}{x-9}$

13.  $\lim_{x \rightarrow 2} \frac{\sqrt{x+2}-2}{x-2}$

14.  $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x^2+3}-2}$

15.  $\lim_{x \rightarrow 1} \frac{\sqrt{x^2+8}-3}{x+1}$

14.  $\lim_{x \rightarrow -1} \frac{\sqrt{x^2+8}-3}{x+1}$

17.  $\lim_{x \rightarrow -1} \frac{\sqrt{x+5}-2}{x+1}$

18.  $\lim_{x \rightarrow 2} \frac{4-x^2}{3-\sqrt{x^2+5}}$

19.  $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x+3}-2}$

20. Suppose  $\lim_{x \rightarrow c} f(x) = 5$  and  $\lim_{x \rightarrow c} g(x) = -2$ . Find

1.  $\lim_{x \rightarrow c} f(x)g(x)$

2.  $\lim_{x \rightarrow c} 2f(x)g(x)$

3.  $\lim_{x \rightarrow c} (f(x) + 3g(x))$

4.  $\lim_{x \rightarrow c} \frac{f(x)}{f(x) - g(x)}$

**21.** Suppose  $\lim_{x \rightarrow 4} f(x) = 0$  and  $\lim_{x \rightarrow 4} g(x) = -3$ . Find

1.  $\lim_{x \rightarrow 4} (g(x) + 3)$

2.  $\lim_{x \rightarrow 4} xf(x)$

3.  $\lim_{x \rightarrow 4} (g(x))^2$

4.  $\lim_{x \rightarrow 4} \frac{g(x)}{f(x) - 1}$

**22.** Suppose  $\lim_{x \rightarrow b} f(x) = 7$  and  $\lim_{x \rightarrow b} g(x) = -3$ . Find

1.  $\lim_{x \rightarrow b} (f(x) + g(x))$

2.  $\lim_{x \rightarrow b} f(x) \cdot g(x)$

3.  $\lim_{x \rightarrow b} 4g(x)$

4.  $\lim_{x \rightarrow b} f(x)/g(x)$

**23.** Suppose that  $\lim_{x \rightarrow -2} p(x) = 4$ ,  $\lim_{x \rightarrow -2} r(x) = 0$ , and  $\lim_{x \rightarrow -2} s(x) = -3$ . Find

1.  $\lim_{x \rightarrow -2} (p(x) + r(x) + s(x))$

2.  $\lim_{x \rightarrow -2} p(x) \cdot r(x) \cdot s(x)$

3.  $\lim_{x \rightarrow -2} (-4p(x) + 5r(x))/s(x)$

**24.**  $\lim_{x \rightarrow -1} \frac{\sin(x^2 - x - 2)}{x + 1}$

**25.**  $\lim_{\theta \rightarrow 0} \frac{\sin \sqrt{2}\theta}{\sqrt{2}\theta}$

**26.**  $\lim_{t \rightarrow 0} \frac{\sin kt}{t}$  (k constant)

**27.**  $\lim_{y \rightarrow 0} \frac{\sin 3y}{4y}$

**28.**  $\lim_{h \rightarrow 0} \frac{h}{\sin 3h}$

**29.**  $\lim_{x \rightarrow 0} \frac{\tan 2x}{x}$

**30.**  $\lim_{t \rightarrow 0} \frac{2t}{\tan t}$

31.  $\lim_{x \rightarrow 0} \frac{x \csc 2x}{\cos 5x}$
32.  $\lim_{x \rightarrow 0} 6x^2(\cot x)(\csc 2x)$
33.  $\lim_{x \rightarrow 0} \frac{x + x \cos x}{\sin x \cos x}$
34.  $\lim_{x \rightarrow 0} \frac{x^2 - x + \sin x}{2x}$
35.  $\lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin 2\theta}$
36.  $\lim_{x \rightarrow 0} \frac{x - x \cos x}{\sin^2 3x}$
37.  $\lim_{t \rightarrow 0} \frac{\sin(1 - \cos t)}{1 - \cos t}$
38.  $\lim_{h \rightarrow 0} \frac{\sin(\sin h)}{\sin h}$
39.  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta + \sin 2\theta}$
40.  $\lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 4x}$
41.  $\lim_{x \rightarrow 1} \frac{\sin(1 - \sqrt{x})}{x - 1}$
42.  $\lim_{\theta \rightarrow 0} \frac{\theta}{\cos \theta}$
43.  $\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\cot 2\theta}$
44.  $\lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 8x}$
45.  $\lim_{y \rightarrow 0} \frac{\sin 3y \cot 5y}{y \cot 4y}$

**Using the Sandwich Theorem**

46. If  $\sqrt{5 - 2x^2} \leq f(x) \leq \sqrt{5 - x^2}$  for  $-1 \leq x \leq 1$ , find  $\lim_{x \rightarrow 0} f(x)$ .
47. If  $2 - x^2 \leq g(x) \leq 2 \cos x$  for all  $x$ , find  $\lim_{x \rightarrow 0} g(x)$ .
48. IF the inequalities

$$1 - \frac{x^2}{6} < \frac{x \sin x}{2 - 2 \cos x} < 1$$

hold for all values of  $x$  close to zero. Evaluate

$$\lim_{x \rightarrow 0} \frac{x \sin x}{2 - 2 \cos x}?$$

49. a. Suppose that the inequalities

$$\frac{1}{2} - \frac{x^2}{24} < \frac{1 - \cos x}{x^2} < \frac{1}{2}$$

hold for values of  $x$  close to zero. Evaluate

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}?$$

50.  $\lim_{x \rightarrow \infty} \sqrt{\frac{8x^2-3}{2x^2+x}}$

51.  $\lim_{x \rightarrow \infty} \left( \frac{x^2+x-1}{8x^2-3} \right)^{1/3}$

52.  $\lim_{x \rightarrow \infty} \left( \frac{1-x^3}{x^2+7x} \right)^5$

53.  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2-5x}}{\sqrt[3]{x^3+x-2}}$

54.  $\lim_{x \rightarrow \infty} \frac{2\sqrt{x}+x^{-1}}{3x-7}$

55.  $\lim_{x \rightarrow \infty} \frac{2+\sqrt{x}}{2-\sqrt{x}}$

56.  $\lim_{x \rightarrow \infty} \frac{\sqrt[3]{x}-\sqrt{x}}{\sqrt[3]{x}+\sqrt{x}}$

57.  $\lim_{x \rightarrow \infty} \frac{x^{-1}+x^{-4}}{x^{-2}-x^{-3}}$

58.  $\lim_{x \rightarrow \infty} \frac{2x^{5/3}-x^{1/3}+7}{x^{8/5}+3x+\sqrt{x}}$

59.  $\lim_{x \rightarrow \infty} \frac{\sqrt[3]{x}-5x+3}{2x+x^{2/3}-4}$

60. For what value of  $a$  is

$$f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$$

continuous at every  $x$ ?

61. For what value of  $b$  is

$$g(x) = \begin{cases} x, & x < -2 \\ bx^2, & x \geq -2 \end{cases}$$

continuous at every  $x$ ?

62. For what values of  $a$  is

$$f(x) = \begin{cases} a^2x - 2a, & x \geq 2 \\ 12, & x < 2 \end{cases}$$

continuous at every  $x$ ?

**62.** For what value of  $b$  is

$$g(x) = \begin{cases} \frac{x-b}{b+1}, & x < 0 \\ x^2 + b, & x > 0 \end{cases}$$

continuous at every  $x$ ?

**63.** For what values of  $a$  and  $b$  is

$$f(x) = \begin{cases} -2, & x \leq -1 \\ ax - b, & -1 < x < 1 \\ 3, & x \geq 1 \end{cases}$$

continuous at every  $x$ ?

**64.** For what values of  $a$  and  $b$  is

$$g(x) = \begin{cases} ax + 2b, & x \leq 0 \\ x^2 + 3a - b, & 0 < x \leq 2 \\ 3x - 5, & x > 2 \end{cases}$$

continuous at every  $x$ ?