Calculus I and Analytical Geometry

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Chapter 4

INTEGRATION

4. Integration of Powers of Trigonometric Functions

1) Integrating Powers of Sine and Cosine:

Integrals of the form:

$$\int \sin^n x \ \cos^m x \, dx$$

Case (1): if m or n is an odd positive integer, we use the identity

$$\sin^2 x + \cos^2 x = 1$$

to convert high powers of one trigonometric function into the other, leaving a single sine or cosine term in the integrand.

Example 1:

$$\int \sin^3 x \, dx$$

Solution

$$\int \sin^3 x \, dx = \int \sin^2 x \, . \sin x \;\; dx$$

$$= \int (1 - \cos^2 x) \, . \, sinx \, dx$$

$$= \int sinx \, dx - \int \cos^2 x \, . \, sinx \, dx$$

$$= \left[-\cos x + \frac{\cos^3 x}{3} \right] + C$$

Example 2:

$$\int \cos^5 x \, dx$$

Solution

$$\int \cos^5 x \, dx = \int \cos^4 x \ \cos x \, dx = \int (\cos^2 x)^2 . \cos x \, dx = \int (1 - \sin^2 x)^2 \ . \cos x \, dx$$

$$= \int (1 - 2\sin^2 x + \sin^4 x) \ \cos x \, dx$$

$$= \int \cos x \, dx + \int -2\sin^2 x . \cos x \, dx + \int \sin^4 x . \cos x \, dx$$

$$= \sin x - \frac{2}{3}\sin^3 x + \frac{\sin^5 x}{5} + C$$

Example 3:

$$\int \sin^3 x \cos^2 x \; dx$$

Solution

$$\int \sin^3 x \cos^2 x \; dx = \int \sin^2 x \; \cos^2 x \; \sin x \, dx$$

$$= \int (1 - \cos^2 x) \; \cos^2 x . \sin x \, dx = \int (\cos^2 x - \cos^4 x) \; \sin x \, dx$$

$$= -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + C$$

Example 4:

$$\int \sin^3 x \ \cos^{-5} x \ dx$$

Solution

$$\int \sin^3 x \ \cos^{-5} x \ dx = \int \sin^2 x \ \cos^{-5} x \ \sin x \ dx$$

$$= \int (1 - \cos^2 x) \ \cos^{-5} x \ . \sin x \ dx$$

$$= \int \ (\cos^{-5} x - \cos^{-3} x) . \sin x \ dx$$

$$= \left[- \frac{\cos^{-4} x}{-4} + \frac{\cos^{-2} x}{-2} \right] + C$$

$$= \frac{1}{4} \cos^{-4} x - \frac{1}{2} \cos^{-2} x + C$$

Case (2): if m and n are both even positive integers, we use the trigonometric identities

$$\sin^2 x = \frac{1}{2}(1 - \cos 2x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos 2x)$$

Example 5:

$$\int \sin^2 x \, dx$$

Solution

$$\int \sin^2 x \, dx = \int \frac{1}{2}(1 - \cos 2x) \, dx = \frac{1}{2}\left[x - \frac{\sin 2x}{2}\right] + C$$

Example 6:

$$\int \cos^4 x \, dx$$

Solution

$$\int \cos^4 x \, dx = \int (\cos^2 x)^2 \, dx = \int \left[\frac{1}{2}(1 + \cos 2x)\right]^2 \, dx$$

$$= \frac{1}{4} \int (1 + 2\cos 2x + \cos^2 2x) \, dx$$

$$= \frac{1}{4}\left[x + 2\frac{\sin 2x}{2} + \frac{1}{2}\int (1 + \cos 4x) \, dx\right]$$

$$= \frac{1}{4}\left[x + \sin 2x + \frac{1}{2}\left(x + \frac{\sin 4x}{4}\right)\right] + C$$

Example 7:

$$\int \sin^2 x \ \cos^2 x \, dx$$

Solution

$$\int \sin^2 x \ \cos^2 x \, dx = \int (\sin x \cos x)^2 \ dx$$

$$= \int \left(\frac{1}{2}\sin 2x\right)^2 dx = \frac{1}{4}\int \left(\frac{1}{2}(1 - \cos 4x)\right) dx$$

$$= \frac{1}{4}\left[\frac{1}{2}\left(x - \frac{\sin 4x}{4}\right)\right] + C = \frac{1}{4}\left[\frac{1}{2}x - \frac{1}{8}\sin 4x\right] + C$$

(home work) $\int \sin^4 x \ \cos^4 x \, dx$

Example 8:

$$\int \sin^2 x \ \cos^4 x \ dx$$

Solution

$$\int \sin^2 x \ \cos^4 x \ dx = \int \ (\sin x \cos x)^2 \cos^2 x \, dx$$

$$= \int \left(\frac{1}{2}\sin(2x)\right)^2 . \frac{1}{2}(1+\cos\,(2x))\;dx = \frac{1}{8}\int (\sin(2x))^2 + (\sin(2x))^2(\cos(2x))\;dx$$

$$= \frac{1}{8}\int \frac{1}{2}(1-\cos(4x)) + (\sin(2x))^2(\cos(2x))\;dx = \frac{1}{8}\int \frac{1}{2}(1-\cos(4x)) + (\sin(2x))^2(\cos(2x))\;dx$$

$$= \frac{1}{8}\left[\frac{1}{2}\left(x - \frac{1}{4}\sin(4x)\right) + \frac{1}{3*2}(\sin(2x))^3\right] + C = \left[\frac{1}{16}x - \frac{1}{64}\sin(4x) + \frac{1}{48}(\sin(2x))^3\right] + C$$

5) Integrals Involving Powers of Tangent and Secant

Integrals of the form

$$\int \tan^n x \; \sec^m x \, dx$$

Example 9:

$$\int \tan^6 x \; \sec^4 x \, dx$$

Solution

$$\int \tan^6 x \; \sec^4 x \, dx = \int tan^6 x \; sec^2 x \; . \boldsymbol{sec^2 x \, dx} = \int \tan^6 x \,(1+\tan^2 x) \, . \boldsymbol{sec^2 x \, dx}$$

$$= \int (\tan^6 x + \tan^8 x) \; . \boldsymbol{sec^2 x \, dx} = \frac{\tan^7 x}{7} + \frac{\tan^9 x}{9} + C$$

Example 10:

$$\int \tan^3 x \ \sec^6 x \, dx$$

Solution

$$\int \tan^3 x \ \sec^6 x \, dx = \int \tan^3 x \ \sec^4 x \ . \sec^2 x \ dx = \int \tan^3 x \,(\sec^2 x)^2 \ . \sec^2 x \ dx$$

$$= \int \tan^3 x \,(1 + \tan^2 x)^2 \ . \sec^2 x \ dx = \int \tan^3 x (1 + 2\tan^2 x + \tan^4 x) . \sec^2 x \ dx$$

$$= \int (\tan^3 x + 2\tan^5 x + \tan^7 x) . \sec^2 x \ dx = \frac{\tan^4 x}{4} + \frac{2\tan^6 x}{6} + \frac{\tan^8 x}{8} + C$$

Example 11:

$$\int \sec^4 2x \, dx$$

Solution

$$\int \sec^4 2x \, dx = \int \sec^2 2x \, . \sec^2 2x \, dx = \int (1 + \tan^2(2x)) \, \sec^2 2x \, dx$$

$$= \int \sec^2 2x \, dx \, + \int \tan^2(2x) \, . \sec^2 2x \, dx = \frac{1}{2}\left[\tan 2x + \frac{\tan^3 2x}{3}\right] + C$$

Homework : $\int csc^6 2x \;\; dx$

Example 12:

$$\int \tan^5 x \ \sec^2 x \, dx$$

Solution

$$\int \tan^5 x \ \sec^2 x \, dx = \int (\tan x)^5 \sec^2 x \ dx = \frac{(\tan x)^6}{6} + C$$

Example 13 :

$$\int \cot^2 x \ \csc^4 x \, dx$$

Solution

$$\int \cot^2 x \ \csc^4 x \, dx = \int \cot^2 x \,(1 + \cot^2 x) \ \boldsymbol{\csc^2 x \ dx} = \int (\cot^2 x + \cot^4 x) \ \boldsymbol{\csc^2 x \ dx}$$

$$= -\frac{cot^3 x}{3} - \frac{cot^5 x}{5} + C$$

Example 14:

$$\int \tan^3 x \ \sec^3 x \, dx$$

Solution

$$\int \tan^3 x \ \sec^3 x \, dx = \int \tan^2 x \ \sec^2 x \,.\, \boldsymbol{\sec x \ \tan x} \ dx$$

$$= \int (\sec^2 x - 1) \ \sec^2 x \ .\, \boldsymbol{\sec x \ \tan x} \ dx$$

$$= \int (\sec^4 x - \sec^2 x) \,.\, \boldsymbol{\sec x \ \tan x} \ dx = \frac{\sec^5 x}{5} - \frac{\sec^3 x}{3} + C$$

Example 15:

$$\int \tan^4 x \, dx$$

Solution

$$\int \tan^4 x \, dx = \int \tan^2 x \, \tan^2 x \, dx$$

$$= \int \tan^2 x \, . (\sec^2 x - 1) dx = \int (\sec^2 x \, \tan^2 x - \tan^2 x) \, dx$$

$$= \int (\tan x)^2 \sec^2 x \, dx - \int (\sec^2 x - 1) \, dx$$

$$= \frac{(\tan x)^3}{3} - (\tan x - x) + C$$

Example 16:

$$\int \tan^5 x \ dx$$

Solution

$$I = \int \tan^5 x \ dx \quad = \int \tan^3 x \tan^2 x \ dx$$

$$= \int \tan^3 x\,(\sec^2 x - 1)dx \quad = \int (\tan x)^3 \sec^2 x \ dx - \int \tan^3 x \ dx$$

$$= \frac{\tan^4 x}{4} - \int \tan x\,(\sec^2 x - 1)dx$$

$$= \frac{\tan^4 x}{4} - \int (\tan x)^1 \sec^2 x \ dx + \int \tan x \ dx$$

$$= \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \ln|\sec x| + \mathrm{C}$$

Example 17:

$$\int \cot^4 x$$

Solution

$$I = \int \cot^4 x \ dx \quad = \int \cot^2 x \cot^2 x \ dx$$

$$= \int \cot^2 x (\csc^2 x - 1) dx = -\int (\cot x)^2 (-\csc^2 x) \ dx - \int \cot^2 x \ dx$$

$$= -\frac{\cot^3 x}{3} - \int (\csc^2 x - 1) \ dx$$

$$= -\frac{\cot^3 x}{3} - [-\cot x - x] + \mathrm{c}$$

$$= -\frac{\cot^3 x}{3} + \cot x + x + \mathrm{c}$$

6) Integration by Trigonometric substitutions

1) Integrals that contain

$$\sqrt{a^2 - x^2} \;\; OR \;\; (a^2 - x^2)^{\frac{3}{2}} \;\; oR \;\; (a^2 - x^2)^{\frac{5}{2}}, \dots$$

$$\text{Let } x = a \sin\theta, \qquad dx = a\cos\theta \, d\theta$$

2) Integrals that contain

$$\sqrt{a^2 + x^2} \quad OR \quad \left(a^2 + x^2\right)^{\frac{3}{2}} \quad OR \dots$$

$$\text{Let } x = a\tan\theta, \qquad dx = a\sec^2\theta \, d\theta$$

3) Integrals that contain

$$\sqrt{x^2 - a^2} \; OR \; \left(x^2 - a^2\right)^{\frac{5}{2}} \; OR \dots$$

$$Let \; x = a\sec\theta, dx = a\sec\theta\tan\theta \, d\theta$$

- **Example 1:**

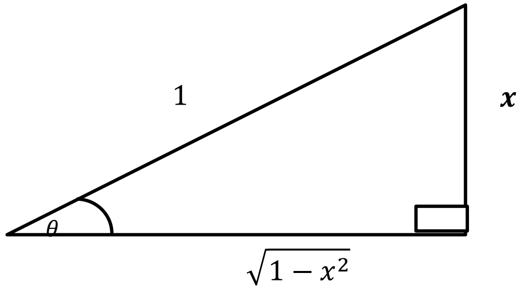
$$\int \frac{x^3}{\sqrt{1-x^2}}\,dx$$

Solution

Let $x = sin\,\theta\,, dx = cos\,\theta\,d\theta$

$$\sqrt{1-x^2} = \sqrt{1-sin^2\theta} = \sqrt{cos^2\theta} = cos\,\theta$$

$$\int \frac{x^3}{\sqrt{1-x^2}}\,dx = \int \frac{sin^3\theta}{cos\,\theta}\,.\,cos\,\theta\,d\theta$$



$$= \int sin^3\theta\,d\theta = \int sin^2\theta\,sin\,\theta\;d\theta$$

$$= \int (1-cos^2\theta)\,sin\,\theta\;d\theta = \int (sin\,\theta - cos^2\theta\,.\,sin\,\theta)d\theta = \left[-cos\,\theta + \frac{cos^3\theta}{3}\right] + C$$

$$= \left[-\sqrt{1-x^2} + \frac{1}{3}\left(\sqrt{1-x^2}\right)^3\right] + C$$

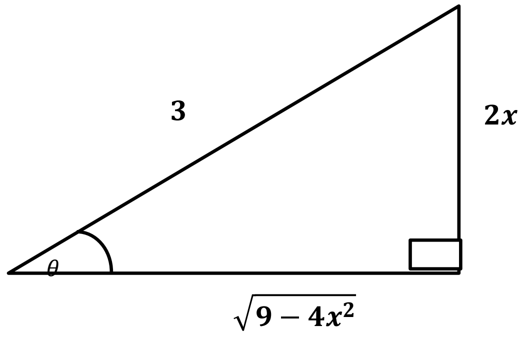
Example 2:

$$\int \frac{\sqrt{9-4x^2}}{x}\,dx$$

Solution

Let $2x = 3\sin\theta$, then $x = \frac{3}{2}\sin\theta \to dx = \frac{3}{2}\cos\theta\, d\theta$

$$\sqrt{9-4x^2} = \sqrt{9-9\sin^2\theta} = \sqrt{9\,(1-\sin^2\theta)} = 3\sqrt{\cos^2\theta} = 3\cos\theta$$



$$\therefore \int \frac{\sqrt{9-4x^2}}{x}\,dx = \int \frac{3\cos\theta}{\frac{3}{2}\sin\theta} \cdot \frac{3}{2}\cos\theta\, d\theta$$

$$= \int \frac{3\ \cos^2\theta}{\sin\theta}\,d\theta = 3\int \frac{1-\sin^2\theta}{\sin\theta}\,d\theta$$

$$= 3\int \left(\frac{1}{\sin\theta} - \frac{\sin^2\theta}{\sin\theta}\right) d\theta = 3\int (\csc\theta - \sin\theta)\,d\theta = 3[\,ln\,|\csc\theta - \cot\theta| + \cos\theta] + C$$

$$= 3\left[ln\left|\frac{3}{2x} - \frac{\sqrt{9-4x^2}}{2x}\right| + \frac{\sqrt{9-4x^2}}{3}\right] + C$$

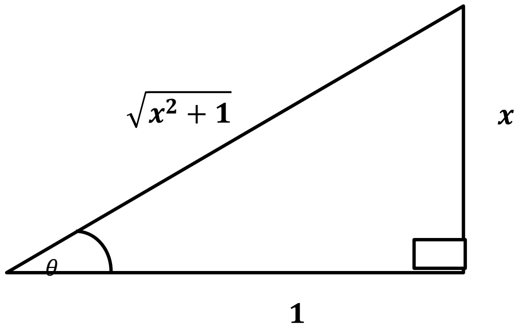
Example 3:

$$\int \frac{\boldsymbol{dx}}{\boldsymbol{x^2\sqrt{x^2+1}}}$$

Solution

Let $x = \tan\theta\,, dx = \sec^2\theta\ \ d\theta$

$$\sqrt{x^2+1} = \sqrt{\tan^2\theta + 1} = \sqrt{\sec^2\theta} = \sec\theta$$



$$I = \int \frac{\boldsymbol{dx}}{\boldsymbol{x^2\sqrt{x^2+1}}} = \int \frac{\sec^2\theta\ \ d\theta}{\tan^2\theta\ .\ \sec\theta} = \int \frac{\sec\theta}{\tan^2\theta} d\theta = \int \frac{1}{\cos\theta}\ .\ \frac{\cos^2\theta}{\sin^2\theta}\ d\theta$$

$$= \int \cos\theta\ .\ (\sin\theta)^{-2} d\theta = \frac{(\sin\theta)^{-1}}{-1} + C = -\frac{1}{\sin\theta} + C = -\text{cosec}\,\theta + C = -\frac{\sqrt{x^2+1}}{x} + C$$

Example 4:

$$\int \frac{\boldsymbol{dx}}{(\boldsymbol{9x^2+1})^{\boldsymbol{2}}}$$

Solution

Let $3x = \tan\theta$

$$\therefore x = \frac{1}{3}\tan\theta \rightarrow dx = \frac{1}{3}\sec^2\theta\; d\theta$$

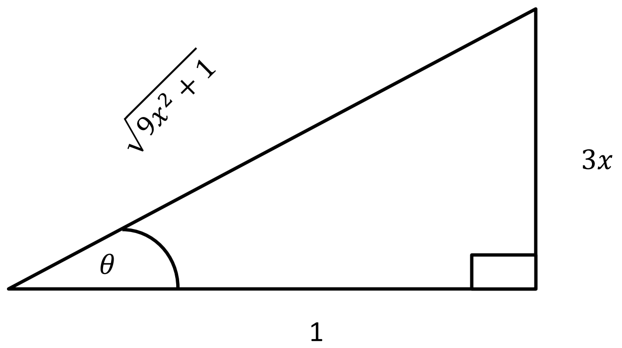
$$(9x^2+1)^2 = (\tan^2\theta + 1)^2 = (\sec^2\theta)^2 = \sec^4\theta$$

$$I = \int \frac{\frac{1}{3}\sec^2\theta\;\; d\theta}{\sec^4\theta} = \frac{1}{3}\int \frac{1}{\sec^2\theta}\; d\theta = \frac{1}{3}\int \cos^2\theta\; d\theta = \frac{1}{3}\cdot\frac{1}{2}\int (1+\cos 2\theta)d\theta$$

$$= \frac{1}{6}\left[\theta + \frac{sin\, 2\theta}{2}\right] + C = \frac{1}{6}\left[\theta + \frac{2\sin\theta\cos\theta}{2}\right] + C = \frac{1}{6}[\theta + \sin\theta\cos\theta] + C$$

$$I = \frac{1}{6}[\theta + \sin\theta\cos\theta] + C$$

$$= \frac{1}{6}\left[\tan^{-1}(3x) + \frac{3x}{\sqrt{9x^2+1}} . \frac{1}{\sqrt{9x^2+1}}\right]$$



Example 5:

$$\int \frac{\sqrt{x^2 - 64}}{x}\, dx$$

Solution

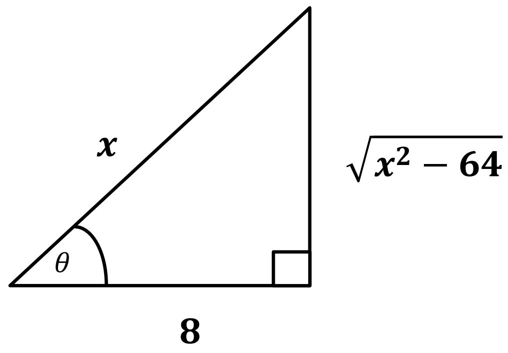
Let $x = 8 \sec\theta$, $dx = 8 \sec\theta \tan\theta\, d\theta$

$$\sqrt{x^2 - 64} = \sqrt{64 \sec^2\theta - 64} = 8\sqrt{\sec^2\theta - 1} = 8 \tan\theta$$

$$I = \int \frac{\sqrt{x^2 - 64}}{x}\, dx = \int \frac{8\tan\theta}{8\sec\theta} . 8 \sec\theta \tan\theta\, d\theta$$

$$= 8 \int \tan^2\theta\, d\theta = 8 \int (\sec^2\theta - 1)\, d\theta$$

$$= 8\,[\tan\theta - \theta] + C = 8\left[\frac{\sqrt{x^2 - 64}}{8} - \sec^{-1}\left(\frac{x}{8}\right)\right] + C$$



Example 6:

$$\int \frac{x^2}{(x^2-1)^{\frac{5}{2}}}\,dx$$

Solution

Let $x = sec\,\theta\,, dx = sec\,\theta\,tan\,\theta\,d\theta$

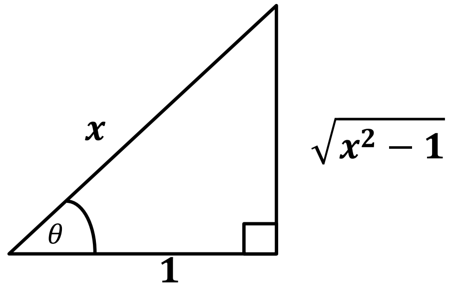
$$(x^2-1)^{\frac{5}{2}} = (sec^2\theta - 1)^{\frac{5}{2}} = (tan^2\theta)^{\frac{5}{2}} = tan^5\theta$$

$$I = \int \frac{x^2}{(x^2-1)^{\frac{5}{2}}}\,dx \;= \int \frac{sec^2\theta}{tan^5\theta}\,.\,sec\,\theta\,tan\,\theta\,d\theta$$

$$= \int \frac{sec^3\theta}{tan^4\theta}\,d\theta = \int \frac{1}{cos^3\theta}\,.\,\frac{cos^4\theta}{sin^4\theta}\,d\theta$$

$$= \int \frac{cos\,\theta}{sin^4\theta}\,d\theta = \int cos\,\theta\,(sin\,\theta)^{-4}d\theta$$

$$= \frac{(sin\,\theta)^{-3}}{-3} + C = \frac{-1}{3}\left[\frac{\sqrt{x^2-1}}{x}\right]^{-3} + C$$



$\because\, \boldsymbol{x = sec\,\theta}$

$\boldsymbol{cos\,\theta = \frac{1}{x}}$

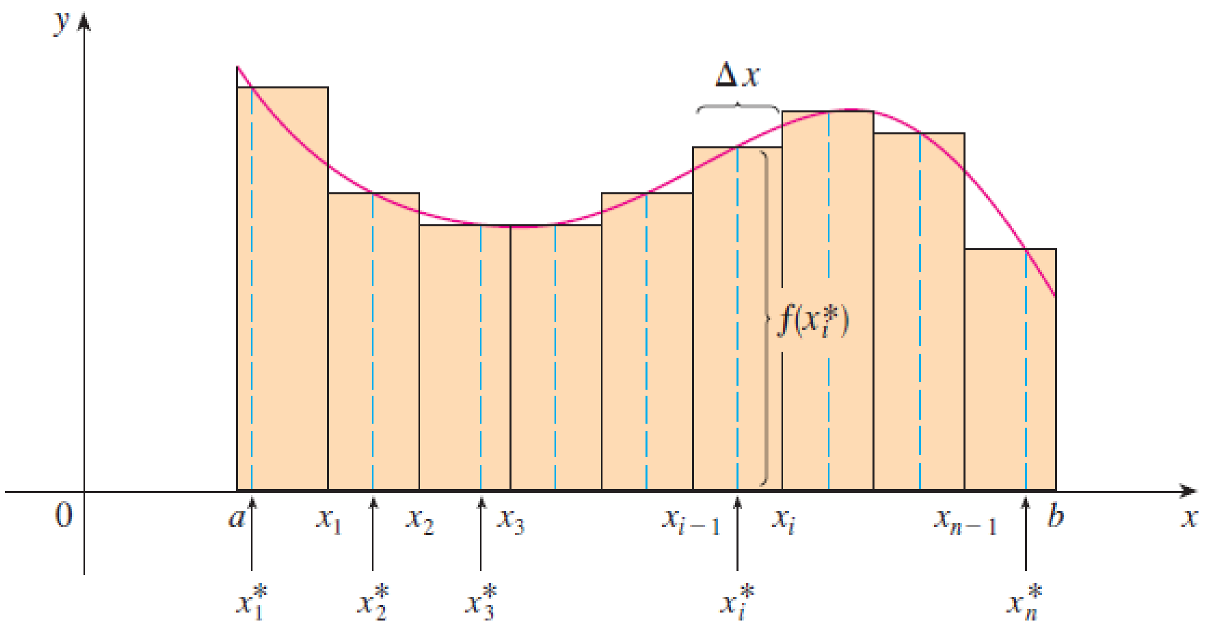
$\boldsymbol{sin\,\theta = \frac{\sqrt{x^2-1}}{x}}$

Definition of a Definite Integral

If f is a function defined for $a \leq x \leq b$, we divide the interval $[a, b]$ into n subintervals of equal width $\Delta x = (b - a)/n$. We let $x_0\ (= a), x_1, x_2, \ldots, x_n\ (= b)$ be the endpoints of these subintervals and we let $x_1^*, x_2^*, \ldots, x_n^*$ be any sample points in these subintervals, so x_i^* lies in the ith subinterval $[x_{i-1}, x_i]$. Then the definite integral of f from a to b is

$$\int_a^b f(x)\, dx \lim_{n \to \infty} \sum_{i=1}^{n} f(x_i^*)\, \Delta x$$

provided that this limit exists. If it does exist, we say that f is integrable on [a, b].



$$\boldsymbol{Area} = \lim_{\boldsymbol{n}\to\infty} [\boldsymbol{f}(x_1^*)\Delta\boldsymbol{x} + \boldsymbol{f}(x_2^*)\Delta\boldsymbol{x} + \cdots + \boldsymbol{f}(x_n^*)\Delta\boldsymbol{x}] = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\,\Delta x$$

Riemman sum

Upper limit of integration

The function is the integrand.

Integral sign

x is the variable of integration.

$$\underbrace{\int_a^b f(x)\,dx}_{\text{Integral of } f \text{ from } a \text{ to } b}$$

Lower limit of integration

When you find the value of the integral, you have evaluated the integral.

- **Fundamental theorem of calculus**

Suppose that $f(x)$ is continuous on $[a,b]$ and that $f(x)$ is a function such that $F'(x) = f(x)$, then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Example 1:

$$\int_1^3 (x^2 - 3)\,dx$$

Solution

$$\int_1^3 (x^2 - 3)\,dx = \left[\frac{x^3}{3} - 3x\right]_1^3 = \left(\frac{27}{3} - 9\right) - \left(\frac{1}{3} - 3\right) = (9 - 9) - \left(\frac{1-9}{3}\right) = -\left(\frac{-8}{3}\right) = \frac{8}{3}$$

Example 2:

$$\int_0^1 \frac{dx}{\sqrt{3+4x^2}}$$

Solution

$$\int_0^1 \frac{dx}{\sqrt{3+4x^2}} = \frac{1}{2}\int_0^1 \frac{2\,dx}{\sqrt{\left(\sqrt{3}\right)^2 + (2x)^2}}$$

$$= \frac{1}{2}\left[\sinh^{-1}\left(\frac{2x}{\sqrt{3}}\right)\right]_0^1 = \frac{1}{2}\left[\sinh^{-1}\left(\frac{2}{\sqrt{3}}\right) - \sinh^{-1}(0)\right] = \frac{1}{2}\sinh^{-1}\left(\frac{2}{\sqrt{3}}\right)$$

- **Properties of the definite integral:**

1) $$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx\,, \qquad where\ a < c < b$$

2) $$\int_a^b k\,f(x)dx = k\int_a^b f(x)dx$$

3) $$\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$$

4) $$\int_a^b f(x)\,dx = -\int_b^a f(x)dx$$

5) $$\int_a^b f(x)dx = \int_a^b f(y)dy = \int_a^b f(t)dt = \cdots$$

- The definite integral does not depend on the variable in the integration but depends on the lower & upper bounds.

- If $f(x)$ is even function $\left(f(-x) = f(x)\right)$

$$\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$$

- If $f(x)$ is add function $(f(-x) = -f(x))$

$$\int_{-a}^{a} f(x)dx = 0$$

Example 3 :

$$\int_{-1}^{1} x^3\, dx$$

Solution

$$\int_{-1}^{1} x^3\, dx = 0$$

Example 4:

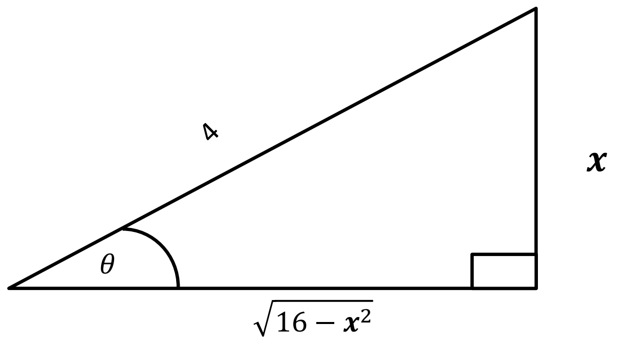
$$\int_1^2 \frac{dx}{x^2\sqrt{16 - x^2}}$$

Solution

Let $x = 4\sin\theta \rightarrow dx = 4\cos\theta\, d\theta$

$$I = \int_1^2 \frac{4\cos\theta\, d\theta}{16\sin^2\theta\sqrt{16 - 16\sin^2\theta}}$$

$$= \frac{1}{16}\int_1^2 \frac{\cos\theta\, d\theta}{\sin^2\theta\sqrt{1 - \sin^2\theta}} = \frac{1}{16}\int_1^2 \csc^2\theta\ \ d\theta$$



$$= \left[-\frac{1}{16}\cot\theta\right]_{x=1}^{x=2} = \left[-\frac{1}{16}\frac{\sqrt{16 - x^2}}{x}\right]_{x=1}^{x=2} = -\frac{1}{16}\left[\frac{\sqrt{16-4}}{2} - \frac{\sqrt{16-1}}{1}\right]$$

Example 5:

$$\int_0^1 \sin^{-1} x \, dx$$

Solution

$$u = \sin^{-1} x \qquad\qquad dv = dx$$

$$du = \frac{1}{\sqrt{1-x^2}} dx \qquad\qquad v = x$$

$$\int_0^1 \sin^{-1} x \, dx = [x \, sin^{-1} x]_{x=0}^{x=1} - \int_0^1 \frac{x}{\sqrt{1-x^2}} dx$$

$$= [x \, sin^{-1} x]_{x=0}^{x=1} + \left[\sqrt{1-x^2}\right]_{x=0}^{x=1} = \left[\frac{\pi}{2} - 0\right] + \left[\sqrt{1-1} - \sqrt{1-0}\right] = \frac{\pi}{2} - 1$$