Exercises

1. A train of mass 225 tons was moving with velocity 48m.p.h. when it starts to ascend an inclined plane of slope $\frac{1}{75}$. If the engine force is 2.5 tons wt. and the resistance due to friction is constant and is equal to 3360 lbs. wt. find the distance described by the train before it comes to rest. (consider the plane makes an angle α to the horizontal, i.e. $\tan\alpha = \frac{1}{75}$, and since α is small $\therefore \sin\alpha \cong \frac{1}{75}$).
2. An engine, of mass 105 tons, pulls after it a train of mass 30 tons by a chain between them. If the resistance acting on the engine is $\frac{1}{100}$ of its weight, while that acting on the train is $\frac{1}{150}$ of its weight, find the tension in the chain if the engine force is equal to 6000 lbs, wt.
3. A train of mass 210 tons whose engine force is 4 tons wt. and the resistance to its motion is 20 lbs. wt. per ton while the force of its brakes is 400 lbs. wt. per ton. If the train starts motion from rest up an inclined plane of slope $\frac{1}{224}$ and when its velocity becomes 45m.p.h. the steam is shut off and the brakes are used. Find the time that elapses before the train comes to rest and the distance described in this time.
4. A train, whose mass is 112 tons, is traveling at the uniform rate of 25m.p.h. on a horizontal track, and the resistance due to air, friction, etc. is 16 lbs. wt. per ton. Part of the train, of mass 12 tons, becomes detached. Assuming that the force exerted by the engine is the same throughout, find the velocity of the train and the distance between it and the detached part when this part comes to rest.

5. A particle of mass 200 lbs. is attached to one end of a light string which passes over a fixed smooth pulley. The other end of the string is pulled and the particle starts to move upwards. If the tension is initially equal to 250 lbs. wt. and decreases uniformly with the rate of 1 lb. wt. per ft. through which the particle rises, find the velocity of the particle after rising a distance of 30 ft.
6. To determine the coefficient of friction between two materials experimentally, a particle of mass 10 lbs. made from one material is projected with velocity 30 ft./sec. over a horizontal surface made from the second material. Find the coefficient of friction if the particle comes to rest after moving a distance of 45 ft.
7. A driver uses the brakes of its car suddenly. The car slides a distance of 32 ft. in 2 secs. before stopping. If the retardation was uniform in this interval prove that the coefficient of friction between the car and the ground is $\frac{1}{2}$.
8. A particle of mass 2 Kgms. is projected with velocity 10m./sec. up an inclined plane whose slope is $\frac{3}{4}$. If the particle comes to rest after moving 8.3m. find the time taken and the coefficient of friction.
9. A car ascends an inclined plane of slope $\frac{1}{20}$. The air resistance is proportional to the square of the velocity. If the motor is shut off when the velocity of the car was 90 km.p.h. and the air resistance was 5% the weight of the car at this velocity; prove that the car has moved a distance of 435 metres before it comes to rest.
(assume g = 10 m. /sec^2)

10-A particle is projected vertically upwards with velocity u from a point A, and when it reaches a point B, another particle is projected upwards with the same velocity u from A. If the two particles meet at B prove that the distance AB is equal to $\frac{8}{9}$ the maximum height attained by the first particle.

11-A heavy particle is projected vertically upwards with velocity u in a medium, the resistance of which is $\frac{mg}{u^2}\tan^2\alpha$ times the square of the velocity, where m is the mass of the particle, α is a constant.

Show that the particle will return to the point of projection with velocity $u\cos\alpha$, after a time

$$\frac{u}{g}\cot\alpha\left(\alpha+\ln\frac{\cos\alpha}{1-\sin\alpha}\right).$$

12-In starting a train the pull of the engine on the rails is at first constant and equal to P; and after the speed attains a certain value V the pull of the engine becomes equal to $\frac{PV}{v}$, where v is the velocity of the train.

Prove that the time t and distance x from the start are given by

$$t=\frac{M}{2pv}(v^2+V^2),$$

$$x=\frac{M}{3pv}\left(v^3+\frac{1}{2}V^3\right),$$

Where M is the combined mass of the engine and train.

13- A particle is projected with velocity 20 ft./sec. at an angle $\sin^{-1}\frac{3}{5}$ with the horizontal. Find the greatest height, the time of flight, the horizontal range and the equation of the path. Find also when and

where the direction of motion makes an angle. $\tan^{-1}\frac{3}{4}$ with the horizontal and find the velocity there.

14- Find the velocity and direction of projection of a shot which passes in a horizontal direction just over the top of a wall of height 75 ft. at a distance of 150 ft. from the point of projection.

15- A shot is fired from a tower whose height is 3z with a velocity of $\sqrt{2gz}$. Find the greatest horizontal distance from the foot of the tower and the angle of projection in this case. If the shot is fired at the end of that distance to hit the top of the tower, find the least velocity of projection.

16- A shot is fired horizontally from the top of a tower. If it hits a target at a horizontal distance a and a vertical distance b downwards, find the velocity of projection and prove that there is another direction with which this velocity could hit the same target. Find the ratio between the times of flight in the two cases.

17- If the maximum range for a particle which is projected up an inelined plane at an angle of 30° with the horizontal is $\frac{64}{3}$ ft., find the velocity of projection. If the particle is projected with the same velocity down the plane, find the maximum range and the time of flight. Find the angle of projection in each case.

18- A ball is projected from a point at a height z above the ground and at a distance a from a smooth vertical wall.
If the ball is projected with a horizontal velocity V_0 towards the wall and $a < \sqrt{\frac{2z}{g}}\, V_0$, prove that it will hit the ground at a distance

$e\left(\sqrt{\frac{2z}{g}}\,V_0 - a\right)$ from the wall where e is the coefficient of restitution between the ball and the wall.

19-A particle is projected with a velocity of 26 ft./sec. at an angle $\tan^{-1}\frac{5}{12}$ with the horizontal. Find the greatest height attained the time of flight, the horizontal range and determine the direction of motion of the particle when it is at a height of 1 ft.

20-A projectile is fired from the top of a tower whose height is 64 ft. with a velocity of 80 ft./sec. in a direction making an angle $\tan^{-1}\frac{3}{4}$ with the horizontal. Prove that it hits a target at a distance of 256 ft. from the foot of the tower. Find the maximum height above the ground and the direction of motion when hitting the target.

21-A particle moves in a straight line Ox under a central attractive force $m\mu/x^2$ towards the origin O where m is the mass of the particle and μ is a constant. If it starts from rest at a distance a, show that it will arrive at the origin in time $\pi a^{3/2}/\sqrt{8\mu}$.

22-A particle is projected vertically upwards with initial velocity V_1 in a medium whose resistance is mkv^2 where k is a constant.

If the particle returns to the point of projection with velocity V_2 prove that $\frac{1}{V_2^2} = \frac{1}{V_1^2} + \frac{k}{g}$

where g is the acceleration of gravity.

23-A particle of mass m is projected vertically upwards with initial velocity V_0 in a medium whose resistance is mkv^2 where k is a constant and v is the velocity of the particle at any moment. Prove that the velocity v is given, in terms of the time t, measured from the initial position, by the equation

$$v = u \tan(\alpha - k u t)$$

where u is the limiting velocity of the particle and $\alpha = \tan^{-1}(v_0 / u)$. Find the relation between distance and time and prove that the maximum height is equal to $\frac{1}{k} \ln(\sec \alpha)$.