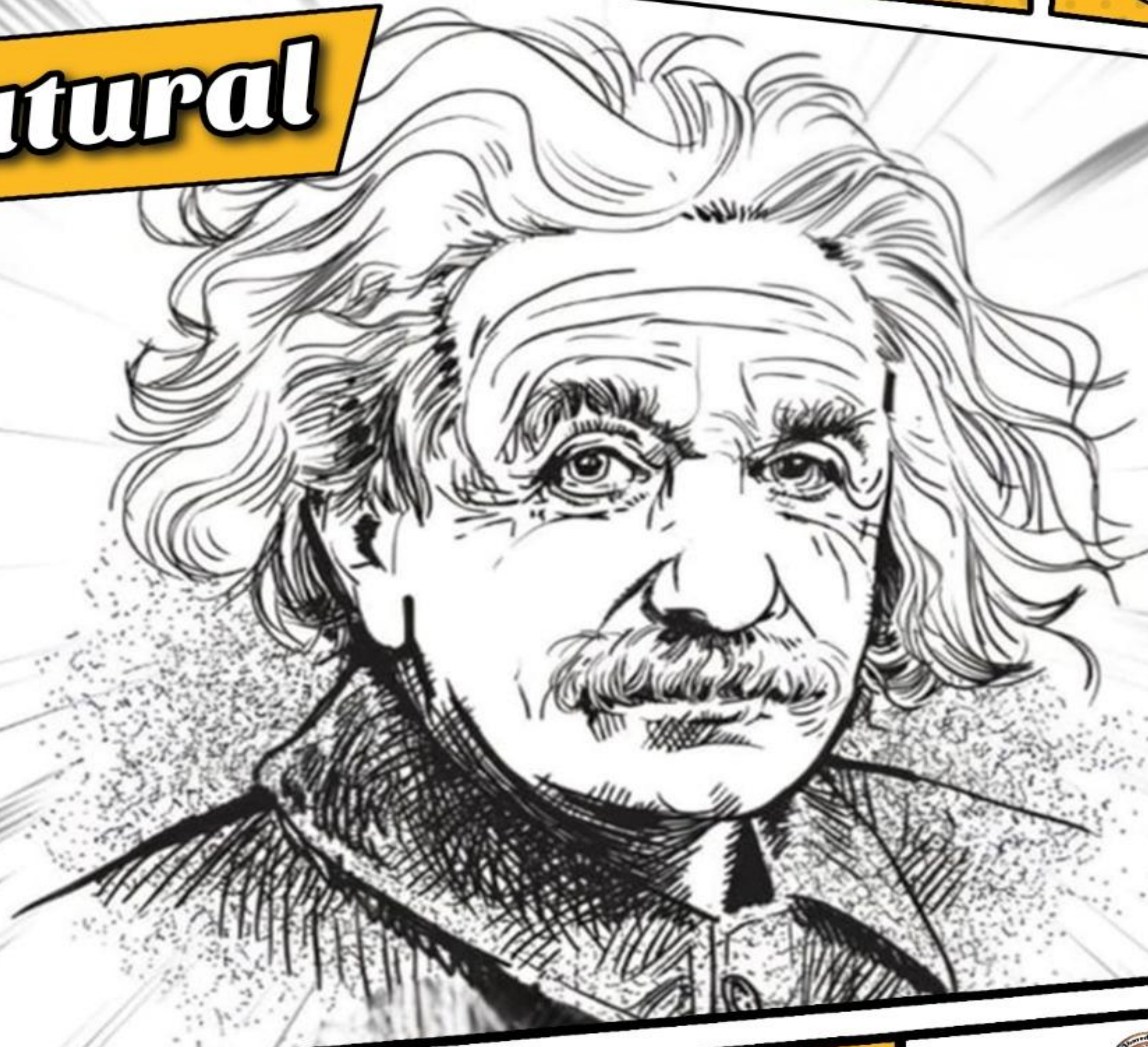
Dr/A.ElShabasy
physics

2025

NO 2

Natural



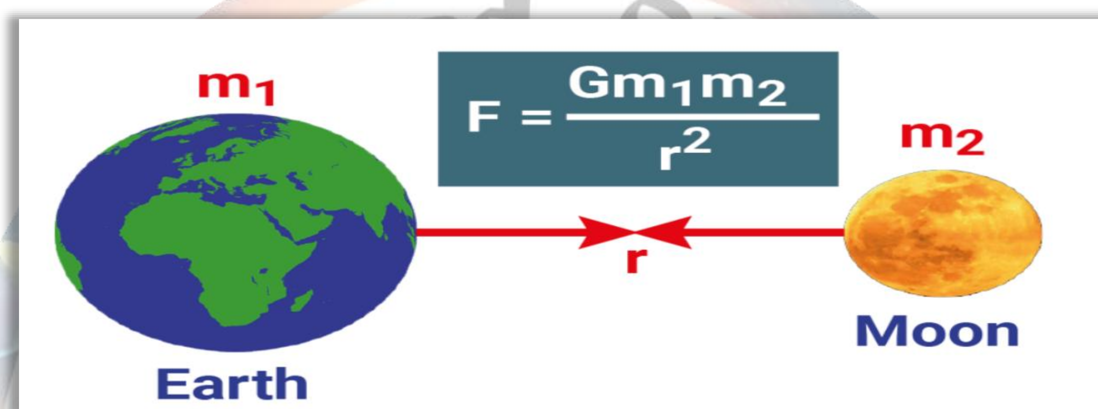
Physics 1 Dr.Elsahbasy

WhatsApp group

Properties of Matter
(Gravitation)

Gravitation

Newton's Law of Universal Gravitation



- Every two particles in the Universe attract each other's with a force

كل جسيمين لهما كتله في الكون يتجاذبان بقوة تُعرف بقوة الجاذبية.

$$F_g = G\frac{m_1 m_2}{r^2}$$

$$F_{12} = -G\frac{m_1 m_2}{r^2}\,\widehat{r_{12}}$$

Where:

☞ G: called the universal gravitational constant. ($G = 6.67 \times 10^{-11}\ N.m^2/kg^2$)

☞ M_1: mass of the first body

☞ M_2: mass of the second body

☞ r: the distance between the two bodies

- The magnitude of the force exerted by the Earth on a particle of mass m near the Earth's surface is

$$F_g = G\frac{M_E m}{R_E^2}$$

Where:

☞ **m:** mass of the body

☞ M_E**:** mass of the earth

☞ R_E**:** radius of the earth

Example 1

Two objects with masses of 500 kg and 800 kg are placed 3 meters apart. Calculate the gravitational force between them. Use the gravitational constant

$$G = 6.674 \times 10^{-11} N \cdot m^2 \cdot kg^{-2}$$

Solution

Example 2

Three 0.3 kg billiard balls are placed on a table at the corners of a right triangle, Calculate the gravitational force on the ball at the right angle if the triangle sides lengths are 0.3, 0.4 and 0.5 m.

Solution

Example 3

Three objects of masses $m_1 = 2m\ kg$, $m_2 = 3m\ kg$, $m_3 = 4m\ kg$ are positioned on the x- axis at $0m, 1m,$ and $3m$ respectively. Calculate the net gravitational force acting on the object m_2.

Solution

1. Calculate the gravitational force between m_1 and m_2:

The distance $d_{12} = x_2 - x_1 = 1 - 0 = 1m.$

$$F_{12} = \frac{G \cdot m_1 \cdot m_2}{d_{12}^2} = \frac{G \cdot (2m) \cdot (3m)}{(1)^2} = \frac{6Gm^2}{1} = 6Gm^2$$

2. Calculate the gravitational force between m_3 and m_2:

The distance $d_{23} = x_3 - x_2 = 3 - 1 = 2m$.

$$F_{23} = \frac{G \cdot m_2 \cdot m_3}{d_{23}^2} = \frac{G \cdot (3m) \cdot (4m)}{(2)^2} = \frac{12Gm^2}{4} = 3Gm^2$$

3. Calculate the net gravitational force on m_2:

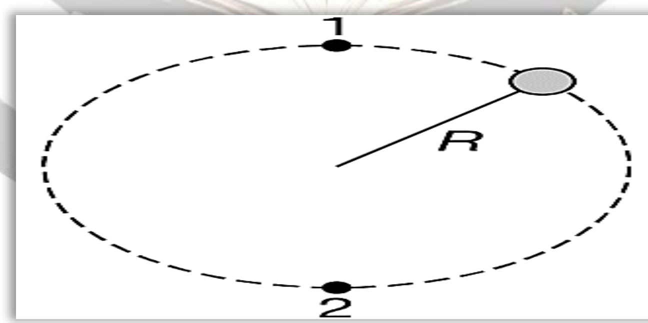
$$F_{net} = F_{12} - F_{23} = 6Gm^2 - 3Gm^2 = 3Gm^2$$

Free-Fall Acceleration and the Gravitational

The weight (w) of an object with mass (m) is calculated using the formula:

$$\boldsymbol{w = mg}$$

The gravitational attraction between the Earth (with mass M_E and radius R_E) and an object of mass m located at the surface of the Earth is given by:

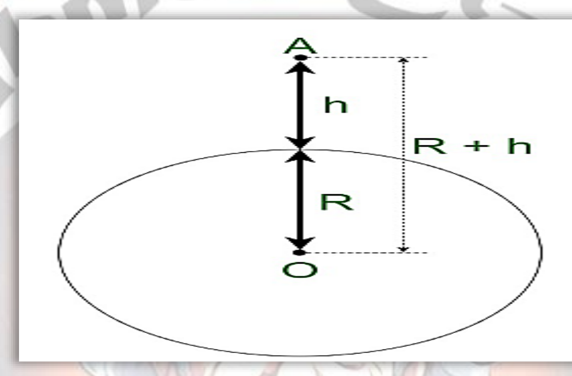


$$\boldsymbol{F_g = G\frac{M_E m}{R_E^2}}$$

$$mg = G\frac{M_E m}{R_E^2}$$

$$g = G\frac{M_E}{R_E^2}$$

- If the object of mass m located **a distance h above the Earth's**



surface

or a distance r from the Earth's center, then

$$g = \frac{GM_E}{r^2} = \frac{GM_E}{(R_E + h)^2}$$

Example

If you want the gravitational acceleration to be *8.7 m/s²*, at what height *h* above the Earth's surface must you be?

Solution

Example

At what height h from the Earth's surface is the gravitational force 16% of the gravitational force at the surface of the Earth? (Use R_E as the Earth's radius)

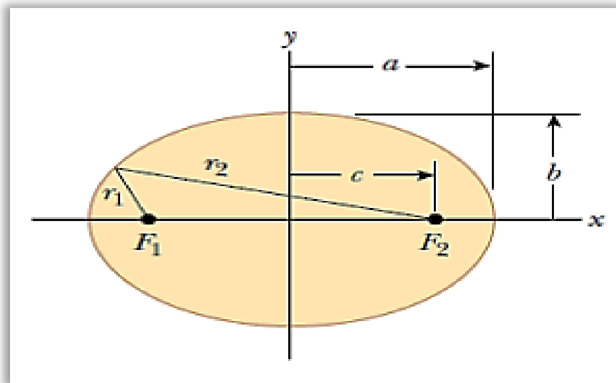
Solution

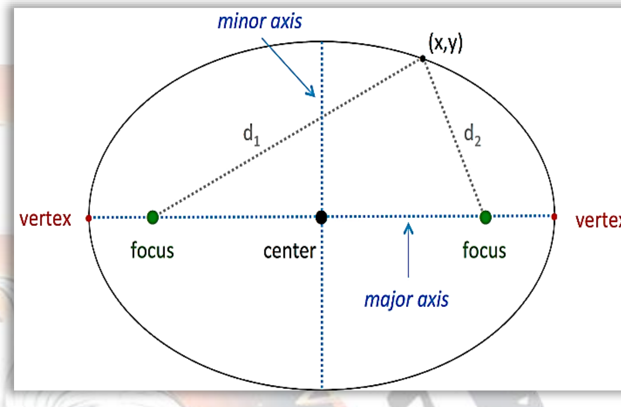
Example

At what distance r from the center of the Earth is the gravitational acceleration 36% of that at the Earth's surface?

Solution

Kepler's laws





An ellipse is defined by two points called foci (F_1 *and* F_2). The curve consists of points where the sum of the distances from each focus (r_1 *and* r_2) is **constant**.

- ☞ The longest distance on the ellipse is called the major axis $(2a)$
- ☞ the shortest distance is the minor axis $(2b)$.
- ☞ The foci are positioned at a distance c from the center, with the relation

$$\boldsymbol{a^2 = b^2 + c^2}$$

Kepler's First Law

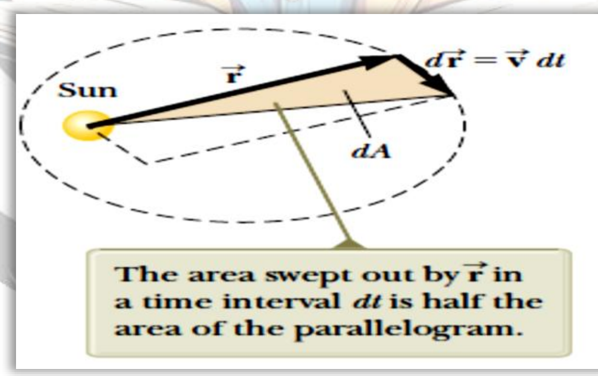
- **All planets move in elliptical orbits with the Sun at one focus.**

Vert important notes

- ☞ The **maximum distance** between the planet and the Sun is $a + c$ (**aphelion**).
- ☞ The **minimum distance** is $a - c$ (**perihelion**).
- ☞ The paths for these objects can include parabolas ($e = 1$)
- ☞ hyperbolas ($e > 1$),
- ☞ e represents the eccentricity الانحراف($e = c/a$).

Kepler's second law

- The radius vector drawn from the Sun to a planet sweeps out equal areas in equal time intervals



الخط الذي يربط بين الشمس والكوكب (أي نصف القطر) يقطع مساحات متساوية خلال ازمنه متساوية

- Consider a planet of mass M_P moving about the Sun in an elliptical orbit. the angular momentum L of the planet is a constant

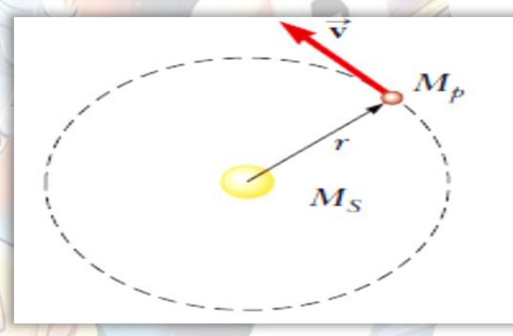
$$L = r \times P = M_P\, r\, v = constant$$

In a time, interval the radius vector r sweeps out an area dA

$$dA = \frac{1}{2}\, r\, dr = \frac{1}{2}\, r\, vdt = \frac{L}{2\, M_P}\, dt$$

Kepler's Third Law

- The square of the orbital period of any planet is proportional to the cube of the semi major axis of the elliptical orbit.



$$T^2 \propto a^3$$

- Consider a planet of mass M_P that is assumed to be moving about the Sun (mass M_S) in a circular orbit.

$$F_g = M_P a \rightarrow \frac{GM_s M_P}{r^2} = M_P\left(\frac{v^2}{r}\right)$$

- The orbital speed of the planet is $v = \frac{2\pi r}{T}$

$$\frac{GM_s}{r^2} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r}$$

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right) r^3 = K_s r^3$$

$$K_s = \left(\frac{4\pi^2}{GM_s}\right) = 2.97 \times 10^{-19}\ s^2/m^3$$

$\underline{when\ r = a}$

$$T^2 = \left(\frac{4\pi^2}{GM_s}\right) a^3 = K_s a^3$$

- The constant of proportionality K_s is independent of the mass of the **planet**.
- $T_E = 1$ **solar year,** $a_E = 1Au$
- If we were to consider the orbit of a satellite such as the Moon about the Earth, the constant would have a different value, with the Sun's mass replaced by the Earth's mass; that is
- ***Depend on Mass of Earth but not depended on Mass of sun***

$$K_E = \frac{4\pi^2}{GM_E}$$

Example 1

A planet orbits the Sun in 2.5 Earth years. Determine its distance from the Sun in astronomical units (AU).

Solution

- Use Kepler's third law:

$$\left(\frac{T}{T_E}\right)^2 = \left(\frac{a}{a_E}\right)^3, where\ T_E = 1\ Year\ and\ a_E = 1\ AU$$

$$\left(\frac{2.5}{1}\right)^2 = \left(\frac{r}{1}\right)^3$$

$$(2.5)^2 = r^3$$

$$6.25 = r^3$$

Example 2

A moon orbits its planet in 0.5 Earth years. What is the distance between the moon and the planet in terms of the distance between the Earth and the Sun (AU)?

Solution

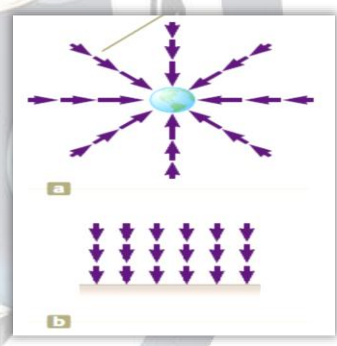
Example 3

A moon orbits its planet in 0.5 Earth years. What is the distance between the moon and the planet in terms of the distance between the Earth and the Sun (AU)?

Solution

The gravitational field

$$\vec{g} = \frac{\vec{F_g}}{m} = -\frac{GM_E}{r^2}\hat{r}$$



where the negative sign indicates that **the field points toward the center of the Earth** as shown in figure.

☞ ***The potential energy of a system at distance (r)***

$$U(r) = -\frac{GM_E m}{r}$$

At certain height above Earth's surface.

$$U(r) = -\frac{GM_E m}{R_E + h}$$

Example

1. Calculate the height h where the energy is 50% of the energy at the Earth's surface. (Let R_E be the Earth's radius).

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2. Calculate the height h where the energy is 10% of the energy at the Earth's surface. (Let R_E be the Earth's radius).

Solution

☞ ***The kinetic energy of a system***

$$K = \frac{1}{2}mv^2$$

☞ ***Total mechanical energy E in Planetary and Satellite Motion***

$$E = K + U$$

$$E = \frac{1}{2}mv^2 - \frac{GMm}{r}$$

$$F_g = ma \rightarrow \frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\frac{1}{2}mv^2 = \frac{GMm}{2r}$$

$$E = \frac{GMm}{2r} - \frac{GMm}{r}$$

$$E = -\frac{GMm}{2r} \; (\,circular\ orbits)$$

- We find also that

$$E = -\frac{GMm}{2a} \; (\,elliptical\ orbits)$$

✶ **From conservation energy**

$$E_i = E_f$$

$$\frac{1}{2}mv_i^2 - \frac{GMm}{r_i} = \frac{1}{2}mv_f^2 - \frac{GMm}{r_f}$$

Escape speed

✶ is the minimum speed the object must have at the Earth's surface to approach an infinite separation distance from the Earth.

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$

We know

$$\rho_E = \frac{M_E}{V}$$

So we can conclude

$$M_E = \rho_E \times \frac{4}{3}\pi R_E^3$$

$$v_{esc} = \sqrt{\frac{2G}{R_E}\rho_E \times \frac{4}{3}\pi R_E^3}$$

$$v_{esc} = \sqrt{2G\,\rho_E \times \frac{4}{3}\pi R_E^2}$$

$$v_{esc} = \sqrt{2G\,\rho_E \times \frac{4}{3}\pi}\quad R$$

$$v_{esc} \propto R$$

Example 1

A spacecraft is launched from the surface of Earth. Given the following values:

- Gravitational constant $G = 6.674 \times 10^{-11} Nm^2/kg^2$
- Mass of Earth $M_E = 5.972 \times 10^{24} kg$
- Radius of Earth $R_E = 6.371 \times 10^6 m$

Calculate the escape velocity required for the spacecraft to leave the Earth's gravitational influence.

Solution

$$v_{esc} = \sqrt{\frac{2GM_E}{R_E}}$$

$$v_{esc} = \sqrt{1.251 \times 10^8} \approx 11.180\ m/s$$

Example 2

A planet has a density of 5500 kg/m³ and a radius of 800 km. Calculate the escape velocity. ($G = 6.67 \times 10^{-11} N.m^2/kg^2$).

Solution

$$\boldsymbol{v_{esc} = \sqrt{\frac{8\pi G\rho R^2}{3}}}$$

Given:

- Density $\rho = 5500\ kg/m^3$
- Radius $R = 800 \times 10^3\ m$
- Gravitational constant $G = 6.67 \times 10^{-11} N.m^2/kg^2$.

Substituting these values:

$$v_{esc} = \sqrt{\frac{8\pi \times 6.67 \times 10^{-11} \times 5500 \times (800 \times 10^3)^2}{3}} = v_{esc} \approx 3.27\ km/s$$