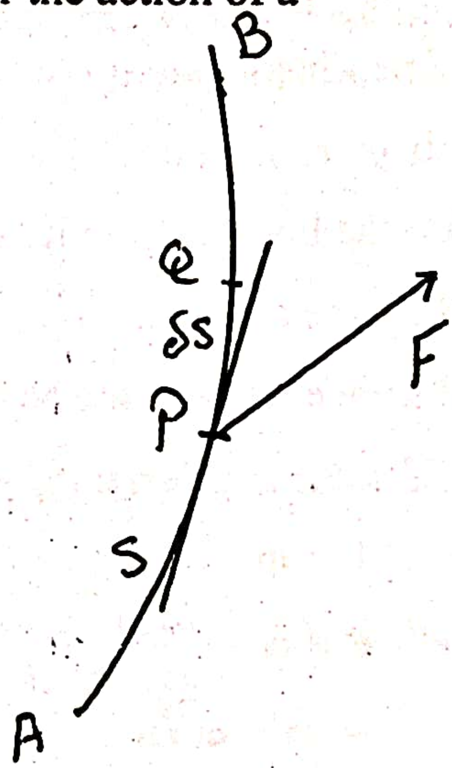
Work, Energy, Power, Impulse and Collision of Elastic Bodies

1- Work :

A force is said to do work when it moves its point of application in the direction of the force. The work is measured by the product of the force and the distance through which the point of application is moved in the direction of the force.

Consider a particle moving along the curve AB under the action of a variable force F.

The motion of the particle along the curve could be considered as composed of successive infinitesimal displacements as $PQ = \delta s$ and so we can consider F as constant in magnitude and direction through the displacement δs .



Let θ be the angle between F and δs, then the element of work done by the force F through the displacement δs is given by

$$\delta W = F(\delta s \cos\theta) = (F\cos\theta)\delta s = \underline{\vec{F}} \cdot \overrightarrow{\delta s}.$$

and the work done in moving the particle from A to B is

$$W = \int F \cos\theta \; ds = \int \vec{F} \cdot \overrightarrow{ds}$$

where the integral is evaluated along the curve from A to B.

If the particle moves along a straight line under the action of a force F along the line from the point x_1 to the point x_2, the work done in the displacement $(x_2 - x_1)$ is

$$\int_{x_1}^{x_2} F\,dx .$$

unit of work = unit of force x unit of distance

i.e. dyne. cm. or poundal. ft.

(1 dyne. cm. = 1 erg. & 1 Joule = 10^7 ergs.)

2- <u>Energy</u>:

The energy of a body is its capacity for doing work and is of two kinds, Kinetic and Potential.

The kinetic energy of a body is the energy which it possesses by virtue of its motion, and is measured by the amount of work done by the acting force on the body to gain its velocity.

Assume a particle of mass m gains a velocity v starting from rest in a distance x under the action of a constant force F.

If f is the acceleration,

$\therefore \quad F = mf$

i.e. $\quad F = m\,v\,\dfrac{dv}{dx}$

$\therefore \quad F\,dx = m\,v\,dv$

and the work done through the distance x is

$$\int_0^x F\,dx = \int_0^v m\,v\,dv = \frac{1}{2} m\,v^2$$

i.e. the kinetic energy of a particle of mass m when its velocity is v is given by $T = \frac{1}{2} m\,v^2$.

The potential energy of a body is the work it can do by means of its position in passing from its present configuration to some standard configuration (usually called its zero position).

For a particle of mass m at a height h above the ground, the potential energy is $V = m g h$. (the earth's surface is taken as the zero of potential energy).

In general, if the body is acted upon by a force $F(x)$ in the direction of the x-axis and its distance from the origin is x, then, by taking the origin as the zero position, the potential energy is given by

$$V = \int_x^0 F dx = -\int_0^x F dx.$$

3- Principle of work and the law of Conservation of Energy:

If a body moves along a straight line under the action of a force F along that line such that its velocity was v_1 at the position x_1 and becomes v_2 at the position x_2, therefore from the relation

$$m f = F$$

$$m v \frac{dv}{dx} = F$$

$$\therefore \int_{v_1}^{v_2} m v \, dv = \int_{x_1}^{x_2} F \, dx .$$

$$\therefore \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = \int_{x_1}^{x_2} F \, dx \qquad (1)$$

i.e. the work done through the displacement $(x_2 - x_1)$ equals the change in kinetic energy.

This principle is called the principle of work and energy and also holds if the body moves on any curve.

Writing equation (1) in the form

$$\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = \int_{x_1}^{0} F dx + \int_0^{x_2} F dx$$

$$= -\int_0^{x_1} F dx + \int_0^{x_2} F dx$$

$$\therefore \quad T_2 - T_1 = V_1 - V_2$$

where T is the kinetic energy and V is the potential energy.

$$\therefore \quad T_2 + V_2 = T_1 + V_1$$

$$\therefore \quad T + V = \text{constant} \qquad (2)$$

i.e. the sum of the kinetic and potential energies at any position is constant.

This is called the law of conservation of energy and also holds if the body moves on any curve.

4- Power:

Power is the time rate of doing work.

i.e. $\text{Power} = P = \frac{dw}{dt}$.

If P is constant, that $P = \frac{w}{t}$. i.e. the work done in unit time.

Unit of power = unit of work per unit time i.e. erg./ sec. or pdl. ft./ sec.

(1 watt = 10^7 erg./sec. = 1 Joule/ sec.

& 1 H.P. = 1 Hourse Power = 550 lb. wt. ft./ sec.

= 550 x 32 pdl. ft./sec.)

5- Impulse:

When a force is constant in magnitude and direction, the impulse is the product of the force and the time during which it acts.

If a constant force $\vec{F}$ acts upon a body, the impulse $\vec{I}$ of this force through the interval of time between the times t_1, t_2 is defined by the equation

$$\vec{I} = \vec{F}(t_2 - t_1).$$

$\vec{I}$ is a vector in the direction of $\vec{F}$.

When the force $\vec{F}$ is variable, $\vec{F}(t)$ say, its impulse through the interval $(t_2 - t_1)$ is defined by :

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}(t)\,dt. \qquad (1)$$

If m is the mass of the moving body, $\vec{v}$ its velocity at time t, its momentum at time t is $m\vec{v}$ and so, using Newton's second law, we get

$$\vec{F}(t) = \frac{d}{dt}(m\vec{v})$$

$$\therefore \quad \vec{I} = \int_{v_1}^{v_2} d(m\vec{v}),$$

$$\text{i.e.} \quad \vec{I} = m\vec{v}_2 - m\vec{v}_1 \qquad (2)$$

where $\vec{v}_1$ is the velocity of the body at time t_1, $\vec{v}_2$ its velocity at time t_2.

The law of impulse (2) states that: The impulse in any direction is equal to the change of momentum in that direction.

Unit of impulse = unit of force x unit of time

= unit of mass x unit of acc. X unit of time

= unit of mass x unit of velocity

= unit of momentum

∴ unit of impulse is

gm. Cm./sec. Or lb. ft. / sec.

6- <u>Impulsive Forces</u>:

An impulsive force is a very great force acting for a very short time, so that the change in the position of the particle during the time the force acts on it may be neglected.

Its whole effect is measured by its impulse, or the change of momentum produced.

7- <u>Conservation of Linear Momentum</u>:

Assume collision happens between two bodies A, B of masses m_1, m_2 whose velocities just before impact are $\vec{v}_1$, $\vec{v}_2$ respectively.

Let their velocities after impact be $\vec{v}'_1$, $\vec{v}'_2$.

By the third law of motion, the action of A on B is equal and apposite to that of B on A.

Let the impulsive reaction between A,B be $\vec{R}$ and since the impulse of B on A is equal to the change in momentum of A and the impulse of A on B is equal to the change in momentum of B,

$$\therefore \qquad \vec{R} = m_1 \vec{v}'_1 - m_1 \vec{v}_1 ,$$

$$\vec{R} = m_2 \vec{v}'_2 - m_2 \vec{v}_2 .$$

adding we get
$$m_1 \vec{v}'_1 - m \underline{v}_1 + m_2 \vec{v}'_2 - m_2 \vec{v}_2 = 0$$

i.e.
$$m_1 \vec{v}'_1 + m_2 \vec{v}'_2 = m_1 \vec{v}_1 + m_2 \vec{v}_2 . \qquad (1)$$

Equation (1) represents the law of conservation of linear momentum for two impinging bodies.

It states that the sum of the momenta of the two masses, measured in the same direction, is unaltered by their impact.

It could be applied only when no external impulses act on either or both bodies.

Also if the sum of the external impulses resolved in any direction is zero, equation (1) is true in that direction.

8- Collision of elastic bodies:

If two balls one made of lead and the other made of glass, say are dropped from the same height upon the ground, the distances through which they rebound will be different. Now the velocities of these bodies are the same on first touching the ground; but since they rebound through different heights, their velocities on leaving the ground must be different.

The property of the bodies which causes these differences in their velocities after leaving the ground is called their Elasticity.

When a ball is dropped from a certain height above the ground and after collision it rebounds to the same height, collision is called perfectly elastic and in this case the velocity of the ball after impact is equal to the velocity before and no loss of kinetic energy by impact.

When the ball comes to rest after reaching the ground, collision is called inelastic and the ball looses all its kinetic energy by impact we shall consider the impact of smooth elastic bodies. In this case the mutual reaction between the two bodies will be in the direction normal to their surfaces at the point of contact.

And if each body is a smooth sphere, the common normal is the line joining their centres.

Two bodies are said to impinge directly when the direction of motion of each is along the common normal at the point at which they touch.

They are said to impinge obliquely when the direction of motion of either, or both, is not along the common normal at the point of contact.

The direction of this common normal is called the line of impact.

9- Newton's experimental law :

Newton found, by experiment, that, if two bodies impinge and u_1 , u_2 are the components of their velocities in the direction of the line of impact before collision, u_1' , u'_2 are the components of their velocities in the same direction after collision then

$$u_1' - u'_2 = -e(u_1 - u_2)$$

i.e. $$\frac{u_1' - u'_2}{u_1 - u_2} = -e .$$

This law means that:

The relative velocity resolved along the line of impact after impact is in a constant ratio to the relative velocity before impact resolved in the same direction, and is of opposite sign.

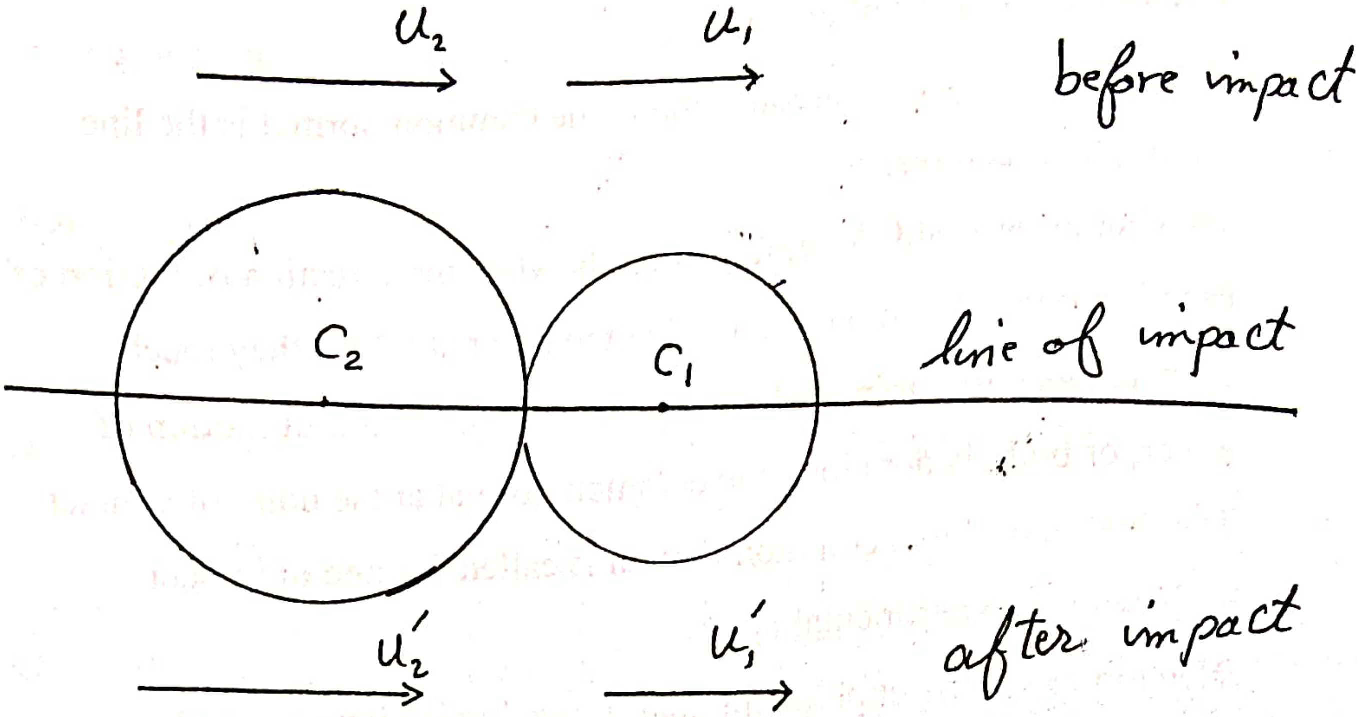
The constant e depends on the substances of which the bodies are made, and is independent of the masses of the bodies. It is called the coefficient of restitution.

In most cases $0 < e < 1$.

For perfectly elastic bodies $e = 1$.

For inelastic bodies $e = 0$.

10- Direct impact of two smooth spheres :



A smooth sphere of mass m_2 impinges directly with velocity u_2 on another smooth sphere of mass m_1 moving in the same direction with velocity u_1.

The law of conservation of linear momentum gives

$$m_1 u'_1 + m_2 u'_2 = m_1 u_1 + m_2 u_2 . \qquad (1)$$

Newton's experimental law gives:

$$u'_1 - u'_2 = -e(u_1 - u_2). \qquad (2)$$

Multiplying (2) by m_2, and adding to (1), we have

$$(m_1 + m_2)\, u'_1 = (m_1 - e m_2) u_1 + m_2 (1+e) u_2 .$$

Again multiplying (2) by m_1, and subtracting from (1), we have

$$(m_1 + m_2)\, u'_2 = m_1 (1+e) u_1 + (m_2 - e m_1) u_2 .$$

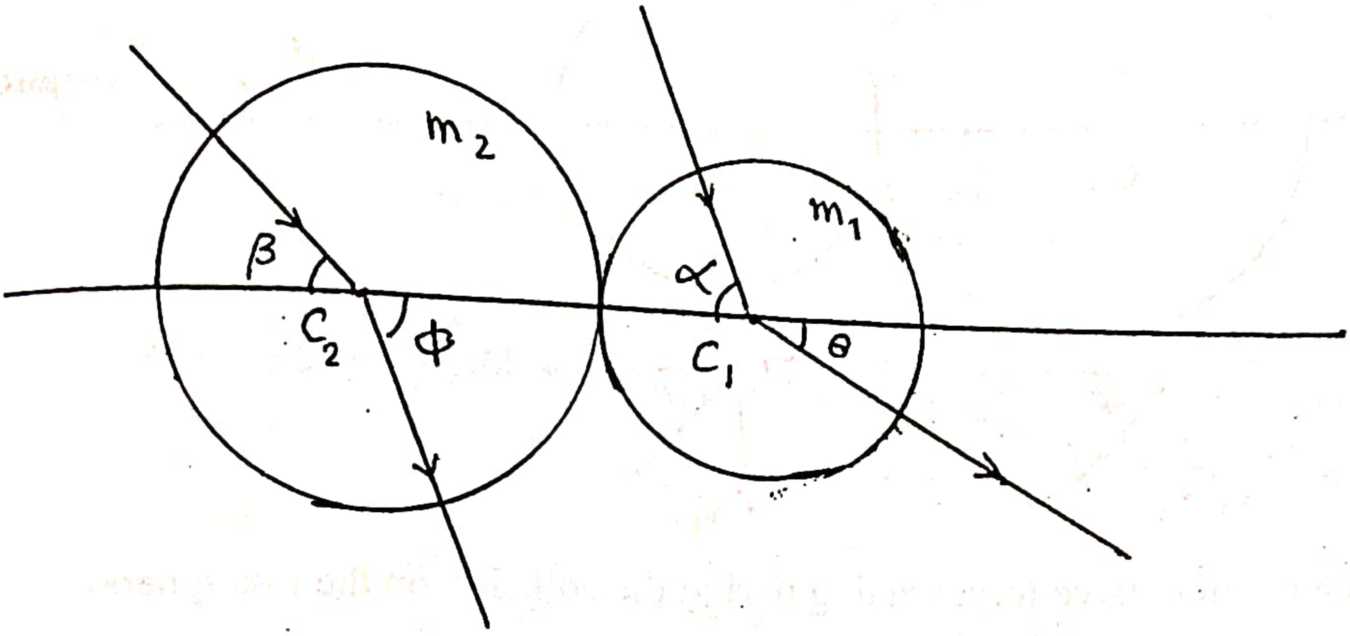
These two equations give the velocities after impact.

Corollary. If we put $m_1 = m_2$ and $e = 1$, we have

$$u'_1 = u_2 \quad , \quad u'_2 = u_1$$

hence if two equal perfectly elastic balls impinge directly they interchange their velocities.

11. <u>Oblique impact of two spheres:</u>

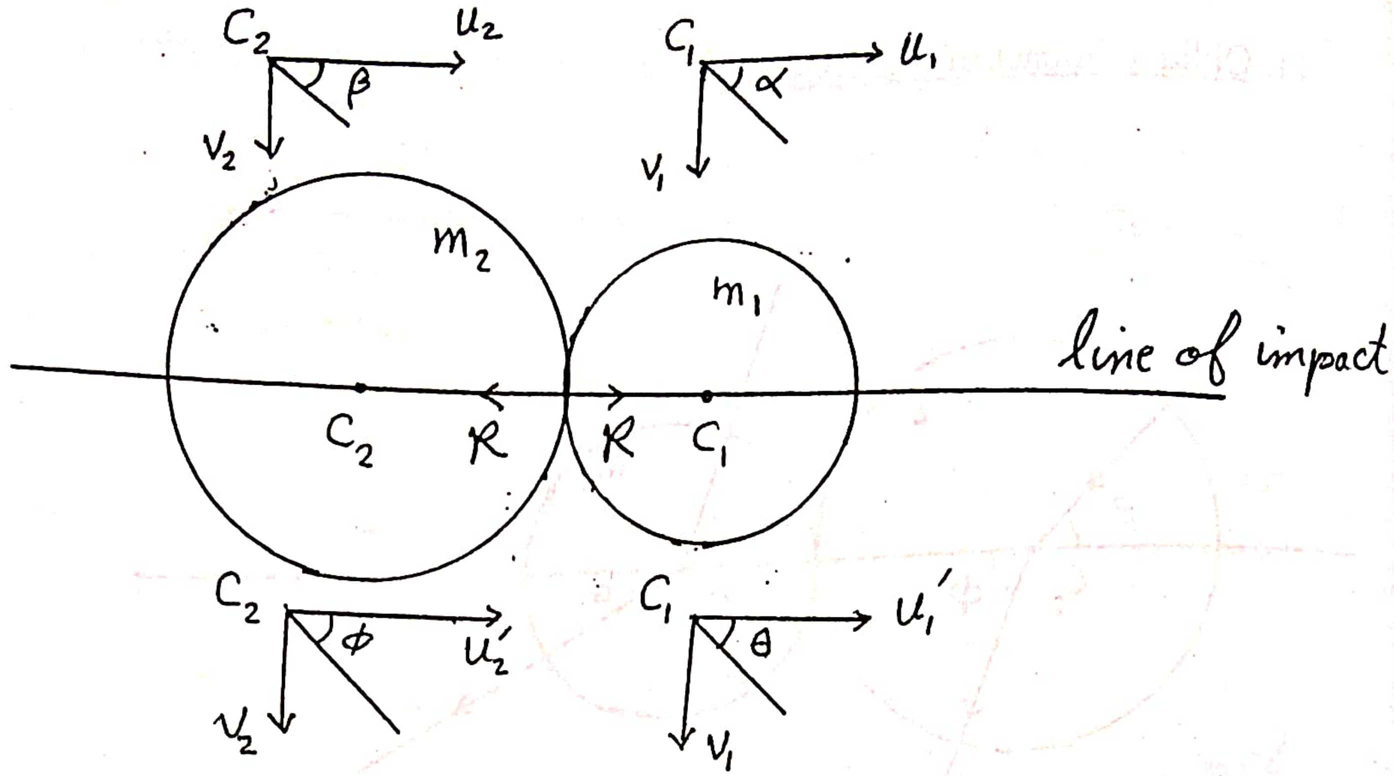


In this case we resolve the velocities of the two spheres before impact along and perpendicular to the line of centres. Let u_1, v_1 be the components of the velocity of the sphere of mass m_1 in these directions before impact and let u'_1, v'_1 be its components in the same directions after impact.

Let u_2, v_2, u'_2, v'_2 be the same quantities relative to the second sphere of mass m_2.

Since the spheres are smooth, there is no impulse perpendicular to the line of centres and hence the resolved parts of velocities of the two spheres in the direction perpendicular to C_1C_2 remain unaltered.

Hence $$v'_1 = v_1 \quad , \quad v'_2 = v_2 . \qquad (1)$$



Since the impulsive forces acting during the collision on the two spheres along their line of centres are equal and opposite, the total momentum along this line remains unaltered

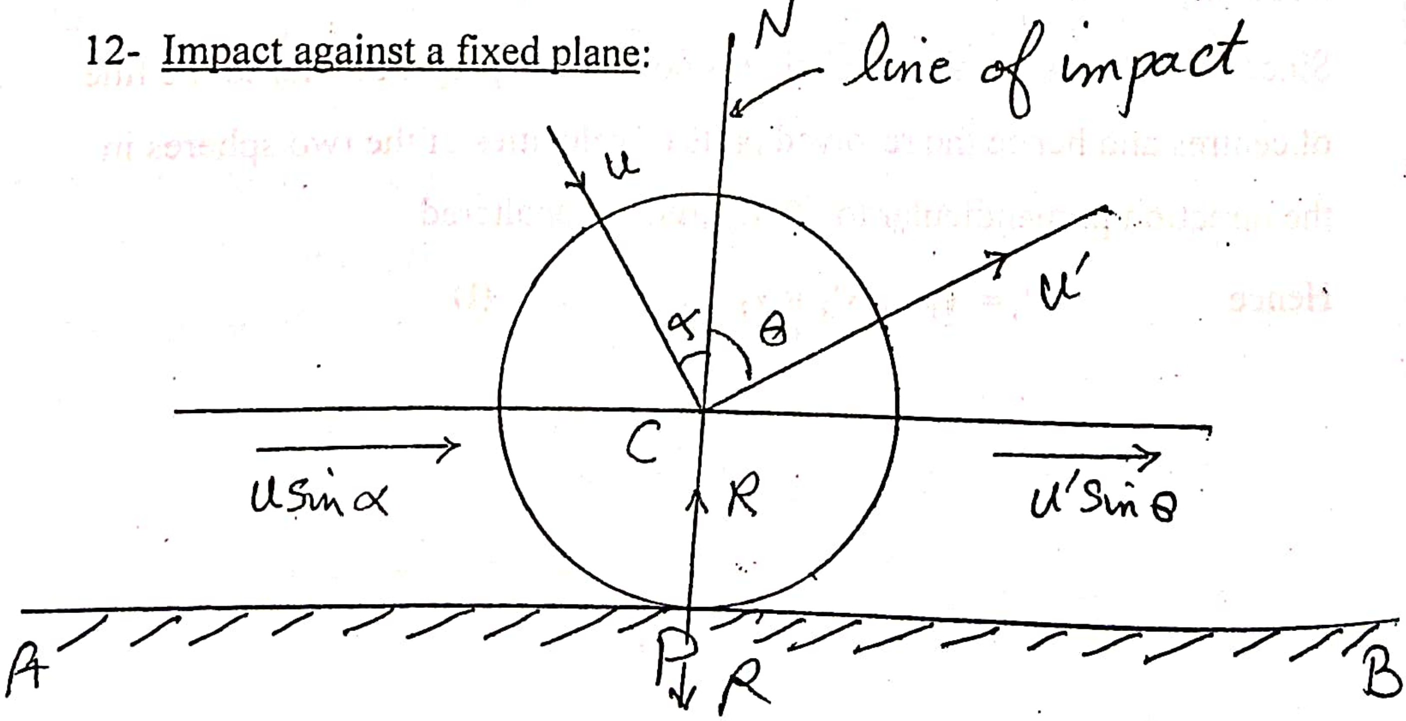
$$\therefore \quad m_1 u'_1 + m_2 u'_2 = m_1 u_1 + m_2 u_2 \qquad (2)$$

Newton's experimental law gives

$$u'_1 - u'_2 = -e\,(u_1 - u_2). \qquad (3)$$

Solving (2) , (3) we obtain u'_1 , u'_2 as in the case of direct impact. Using (1), the motion of the two spheres after impact is completely determined.

12- Impact against a fixed plane:



A smooth sphere whose mass is m and whose coefficient of restitution is e, impinges obliquely on a fixed plane.

Let AB be the fixed plane, P the point at which the sphere impinges, and PN the normal to the plane at P so that PN passes through the centre C of the sphere.

Let u, u' be the velocities of the sphere before and after impact as indicated in the figure.

Since the plane is smooth, there is no force parallel to the plane; hence the velocity of the sphere resolved in a direction parallel to the plane is unaltered.

$$\therefore \; u' \sin\theta = u \sin\alpha . \qquad (1)$$

By Newton's experimental law, we get

$$u'\cos\theta - 0 = -e(-u\cos\alpha - 0)$$

i.e. $$u'\cos\theta = e\,u\cos\alpha \qquad (2)$$

From (1) and (2), by squaring and adding, we have

$$u'^2 = u^2\,(\sin^2\alpha + e^2\cos^2\alpha),$$

and, by division,

$$\tan\theta = \frac{1}{e}\tan\alpha .$$

These two equations give the magnitude and direction of the velocity after impact.

The impulse R acting on the sphere

= the change in its momentum

$$\therefore \quad R = m\,u'\cos\theta - (-m\,u\cos\alpha)$$
$$= m\,(u'\cos\theta + u\cos\alpha)$$

and using (2),

$\therefore \quad R = m\,u(1+e)\cos\alpha .$

<u>Cor.</u>1 If the impact be direct, we have $\alpha = 0$.

$$\therefore \; \theta = 0 \quad , \text{and} \quad u' = eu .$$

<u>Cor.</u>2 If $e = 1$, we have

$$\theta = \alpha \quad , \text{and} \quad u' = u .$$

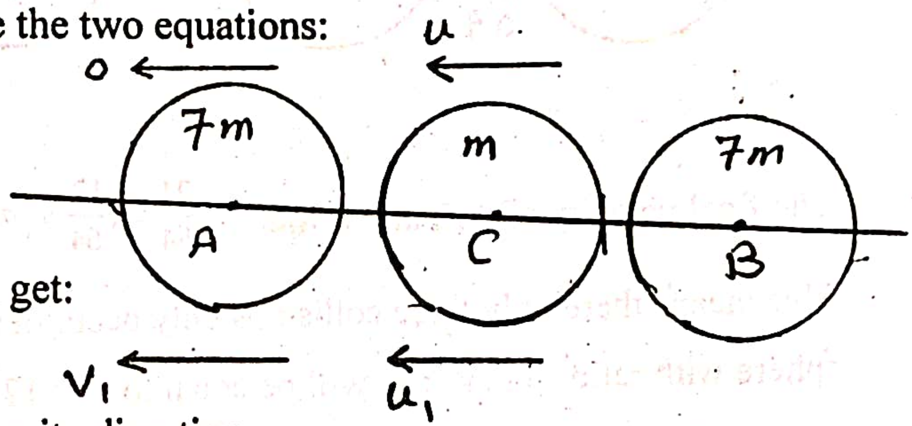
$$=\frac{1}{2}mu^2-\left[2x\frac{1}{2}m\frac{3u^2}{25}(1+e)^2+\frac{1}{2}m\frac{u^2}{25}(2-3e)^2\right]$$

$$=\frac{1}{2}m\frac{3u^2}{5}\left(1-e^2\right)=\frac{3}{5}T\left(1-e^2\right)$$

Where $T=\frac{1}{2}mu^2$ is the kinetic energy before impact.

2) Three perfectly elastic spheres A, B, C of masses 7m,7m,m respectively are at rest with their centers along a straight line and C lies between A and B. The sphere C is projected towards A with velocity u. Prove that three collisions only occur between the three spheres and the ratio between their final velocities is 21 : 12: 1 .

Assume v_1 , u_1 be the velocities of the spheres A,C after the direct impact between them. The law of conservation of momentum and Newton's experimental law give the two equations:



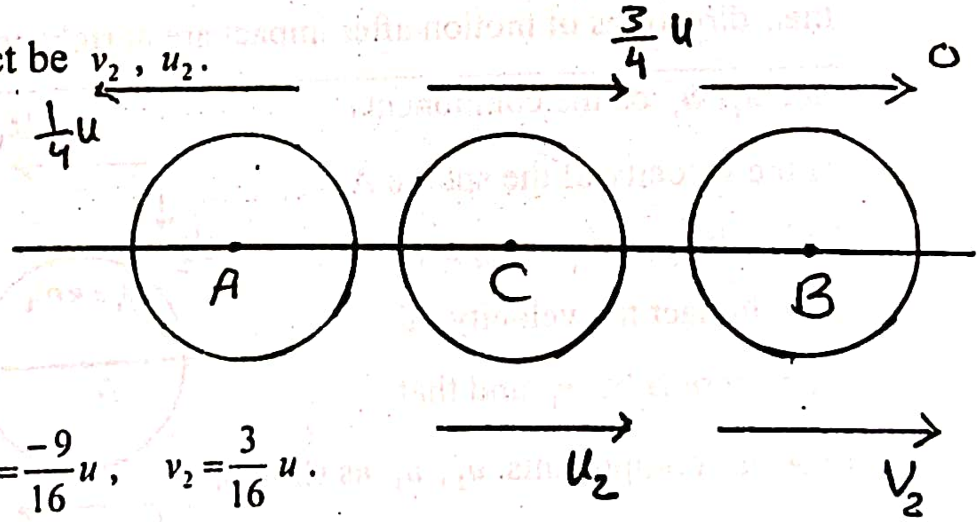
$$7mv_1+m\,u_1=mu+0$$
$$\Rightarrow 7v_1+u_1=u \quad ,$$
$$v_1-u_1=-e(0-u)=u \quad (e=1)$$

Solving these two equations we get:

$$v_1=\frac{1}{4}u \quad , \quad u_1=\frac{-3}{4}u \; .$$

The negative sign means in opposite direction.
After that, another direct impact occurs between the spheres B,C.

Let their velocities after impact be v_2 , u_2.



As before

$$\left.\begin{aligned} u_2+7v_2&=\frac{3}{4}u \, , \\ u_2-v_2&=\frac{-3}{4}u \end{aligned}\right\}$$

$$\therefore\; u_2=\frac{-9}{16}u \, , \quad v_2=\frac{3}{16}u \, .$$

Therefore there is another collision between A and C.

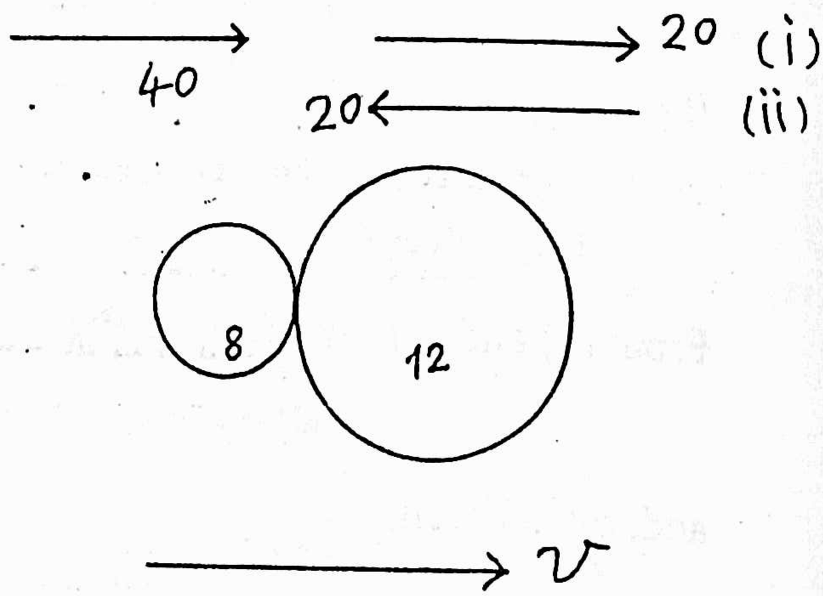
Let their velocities after impact be v_3 , u_3.

1) A sphere of mass 8 lbs. and moving with velocity 40 ft./sec., overtakes a sphere of mass 12 lbs. and moving with velocity 20 ft./sec.

The two spheres form one body. Find its common velocity if the two Spheres were moving before impact:

(i) In the same direction.

(ii) In apposite directions.



Assume the common velocity after impact be v.

By the law of conservation of linear momentum, we have in case (i):

$$(8+12)v = 8 \times 40 + 12 \times 20 = 560$$

i.e. $v = 28$ ft./sec.

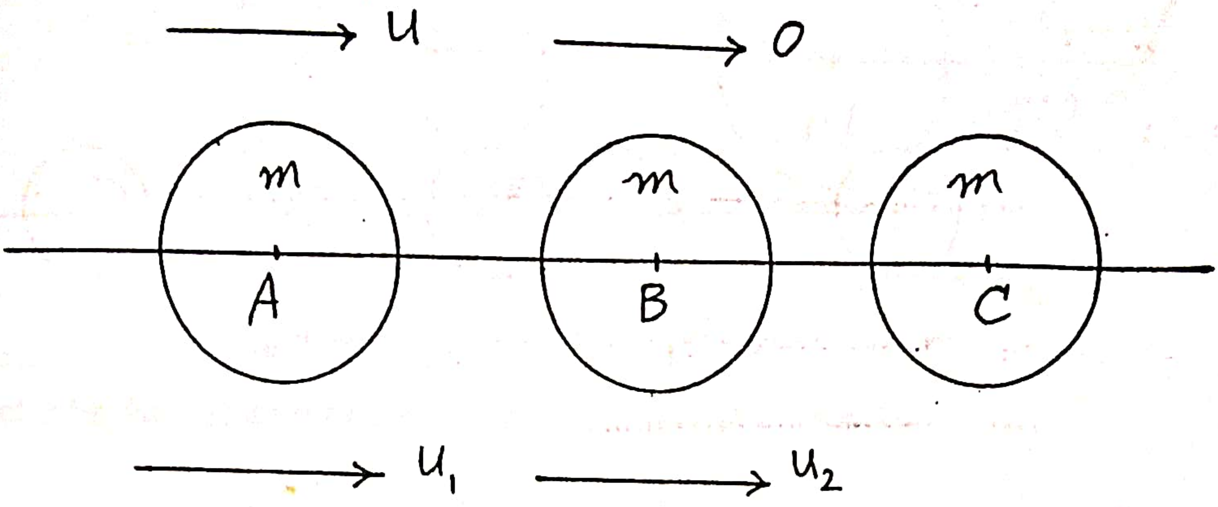
and in case (ii):

$$(8+12)v = 8 \times 40 - 12 \times 20 = 80$$

i.e. $v = 4$ ft./sec.

2) Three equal elastic spheres A, B, C are at rest along a straight line on a smooth horizontal table. A is projected towards B with velocity u. If the coefficient of restitution be $\frac{1}{2}$ for all collisions prove that two collisions

only occur between A and B and the ratio between the final velocities of the three spheres is 13 : 15 : 36 .



Assume u_1 , u_2 be the velocities of the spheres A,B after the direct impact between them. The law of conservation of momentum and Newton's experimental law give the two equations:

$$m u_1 + m u_2 = m u + 0,$$
$$u_1 - u_2 = -e(u - 0)$$

i.e.

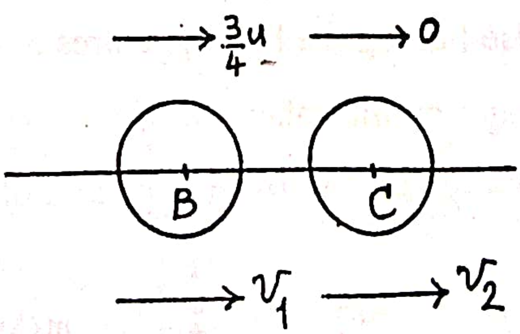
$$\left.\begin{aligned} u_1 + u_2 &= u , \\ u_1 - u_2 &= -\frac{1}{2} u \end{aligned}\right\}$$

Solving these two equations we get:

$$u_1 = \frac{1}{4} u \ , \quad u_2 = \frac{3}{4} u \ .$$

After that, another direct impact occurs between the spheres B,C. Let their velocities after impact be v_1 , v_2 .

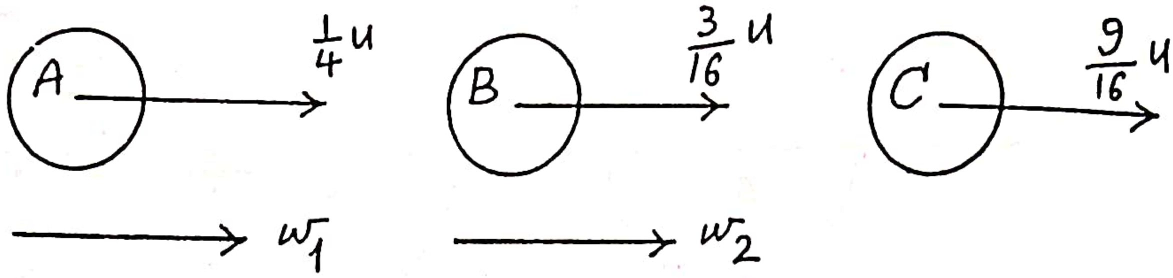
As before



$$\left.\begin{aligned} v_1 + v_2 &= \frac{3}{4} u , \\ v_1 - v_2 &= -\frac{3}{8} u \end{aligned}\right\}$$

$$\therefore\ v_1 = \frac{3}{16}u\ ,\quad v_2 = \frac{9}{16}u\ .$$

Now the three spheres are moving as shown:



Since $u_1 > v_1$, a second impact between A,B will occur. Let w_1, w_2 be their velocities after this impact, therefore we have the two equations:

$$m w_1 + m w_2 = \frac{1}{4}mu + \frac{3}{16}mu\ ,$$

$$w_1 - w_2 = -\frac{1}{2}\left(\frac{1}{4}u - \frac{3}{16}u\right)$$

$$\text{i.e.}\quad \left.\begin{aligned} w_1 + w_1 &= \frac{7}{16}u\ , \\ w_1 - w_2 &= -\frac{1}{32}u \end{aligned}\right\}$$

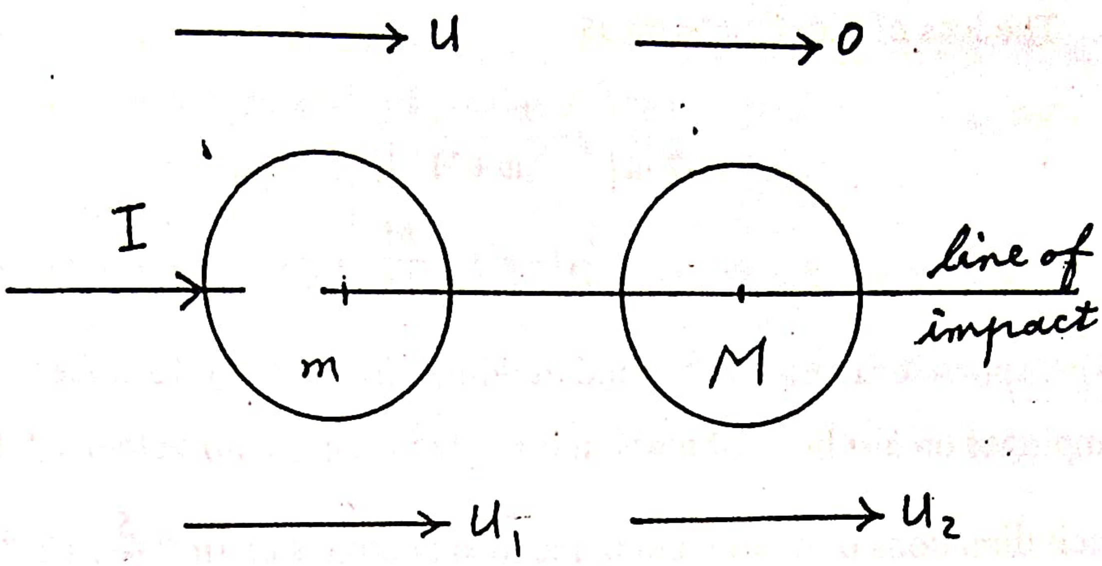
$$\therefore\ w_1 = \frac{13}{64}u\ ,\ w_2 = \frac{15}{64}u\ .$$

$$\because\ w_1 < w_2 < v_2$$

no further collisions will occur and the ratio between the final velocities of the three spheres, $\because\ w_1 : w_2 : v_2$ will be equal to $13 : 15 : 36$.

3) Two smooth spheres of the same volume and of masses M , m (M > m) are at rest on a smooth horizontal table. The sphere m is given an impulse I along the line of centres of the two spheres and then impinges on the sphere M. If e is the coefficient of restitution between the two spheres prove that the loss of kinetic energy by impact is

$$\frac{1}{2}(1-e^2)\frac{MI^2}{m(M+m)}.$$



Let u be the velocity of the sphere m directly after impulse,

$$\therefore \; mu - 0 = I \;, \qquad \text{i.e.} \qquad u = \frac{I}{m}.$$

If u_1, u_2 are the velocities of the two spheres m , M after the direct collision between them, the law of conservation of momentum and Newton's experimental law give :

$$m u_1 + M u_2 = m u \,,$$

$$u_1 - u_2 = -e\,u \,.$$

Solving these two equations we obtain:

$$u_1 = \frac{m - eM}{m+M} u \;, \qquad u_2 = \frac{m\,(1+e)}{m+M} u \,.$$

Now, the sum of kinetic energies of the two spheres before impact =

$$= \frac{1}{2} m u^2 = \frac{1}{2} \frac{I^2}{m} \,.$$

The sum of kinetic energies after impact

$$= \frac{1}{2} m u_1^2 + \frac{1}{2} M u_2^2$$

$$= \frac{1}{2} \frac{m u^2}{(m+M)^2} \left[(m - eM)^2 + m M (1+e)^2 \right]$$

$$= \frac{1}{2} \frac{m u^2}{(m+M)^2} \left[m\,(m+M) + e^2 M (m+M) \right]$$

$$= \frac{1}{2} \frac{m u^2 (m + e^2 M)}{(m+M)} = \frac{1}{2} \frac{I^2 (m + e^2 M)}{m\,(m+M)} \,.$$

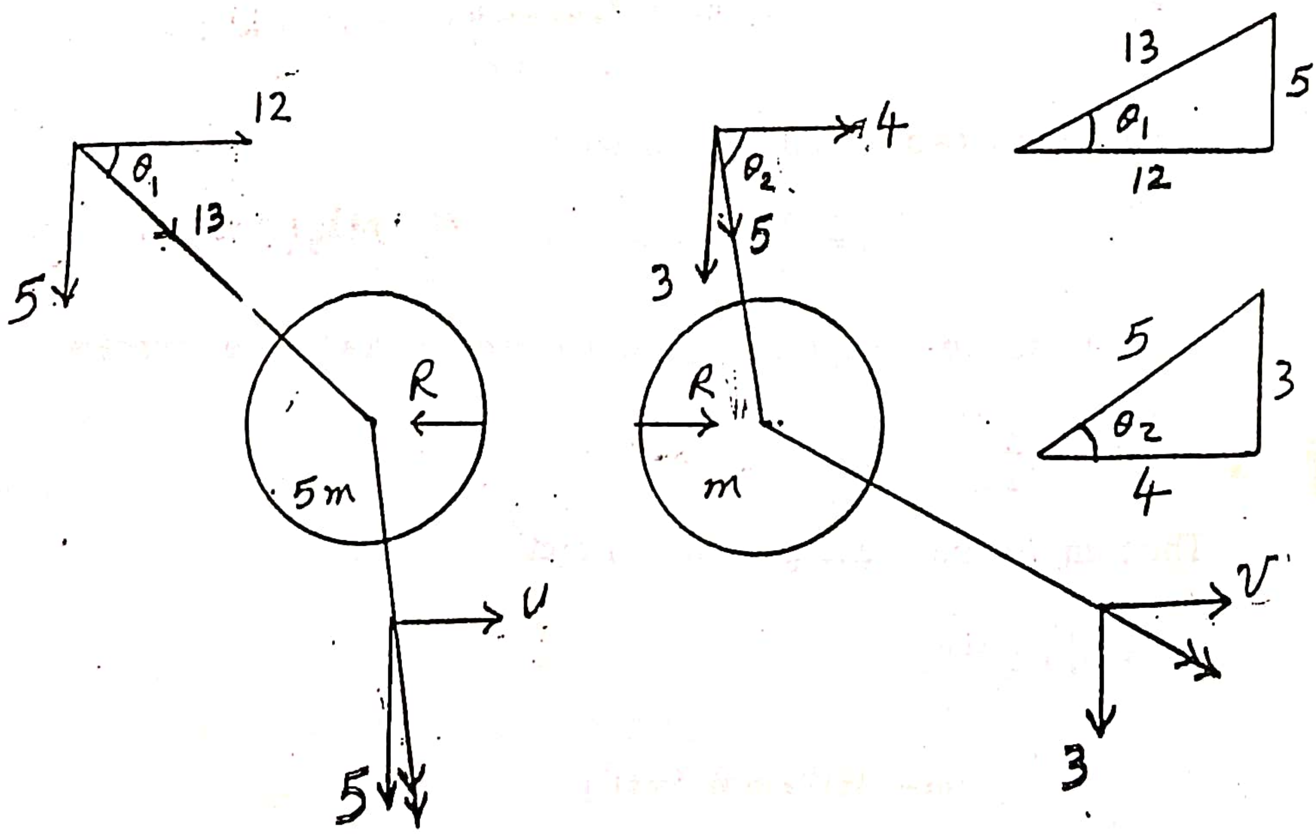
$\therefore$ The loss of kinetic energy is

$$\frac{1}{2}\frac{I^2}{m}\left[1-\frac{m+e^2M}{m+M}\right]$$

$$=\frac{1}{2}(1-e^2)\,\frac{MI^2}{m(M+m)}.$$

4) A sphere of mass 5m lbs. and moving with velocity 13 ft./sec., impinges on a sphere, of mass m lbs. and moving with velocity 5 ft./sec., their directions of motion being inclined at angles of $\sin^{-1}\frac{5}{13}$, $\sin^{-1}\frac{3}{5}$ respectively to the line of centres; if the coefficient of restitution be $\frac{1}{2}$, find the magnitudes and directions of their velocities after the impact.

Find the impulsive reaction between the two spheres and the loss of kinetic energy by impact.



Let u , v be the components of the velocities of the two spheres 5m, m in the direction of the line of centres after impact.

Since the two spheres are smooth, the components of their velocities in the direction perpendicular to the line of centres remain unaltered as shown in figure.

Applying the law of conservation of momentum and Newton's experimental law in the direction of the line of centres we get:

$$5m\,u + m\,v = 5m \times 12 + m \times 4 = 64m\ ,$$

$$u - v = -\frac{1}{2}(12-4)\ ,$$

i.e. $$\left.\begin{aligned} 5u + v &= 64\ , \\ u - v &= -4 \end{aligned}\right\}$$

$\therefore\ u = 10$ ft./sec. , $v = 14$ ft./sec.

Hence the velocity of the sphere 5m after impact has magnitude equal to $\sqrt{u^2+5^2} = \sqrt{125} = 5\sqrt{5}$ ft./sec. In a direction making an angle = $\tan^{-1}\frac{1}{2}$ with the line of centres. Also the velocity of the sphere m after impact has magnitude $\sqrt{v^2+3^2} = \sqrt{205}$ ft./sec. in a direction making an angle = $\tan^{-1}\frac{3}{14}$ with the line of centres.

The impulsive reaction R acting on the sphere m is equal to the change in its linear momentum

i.e. $R = m(v-4) = 10m$ lb.ft./sec.

The loss of kinetic energy by impact

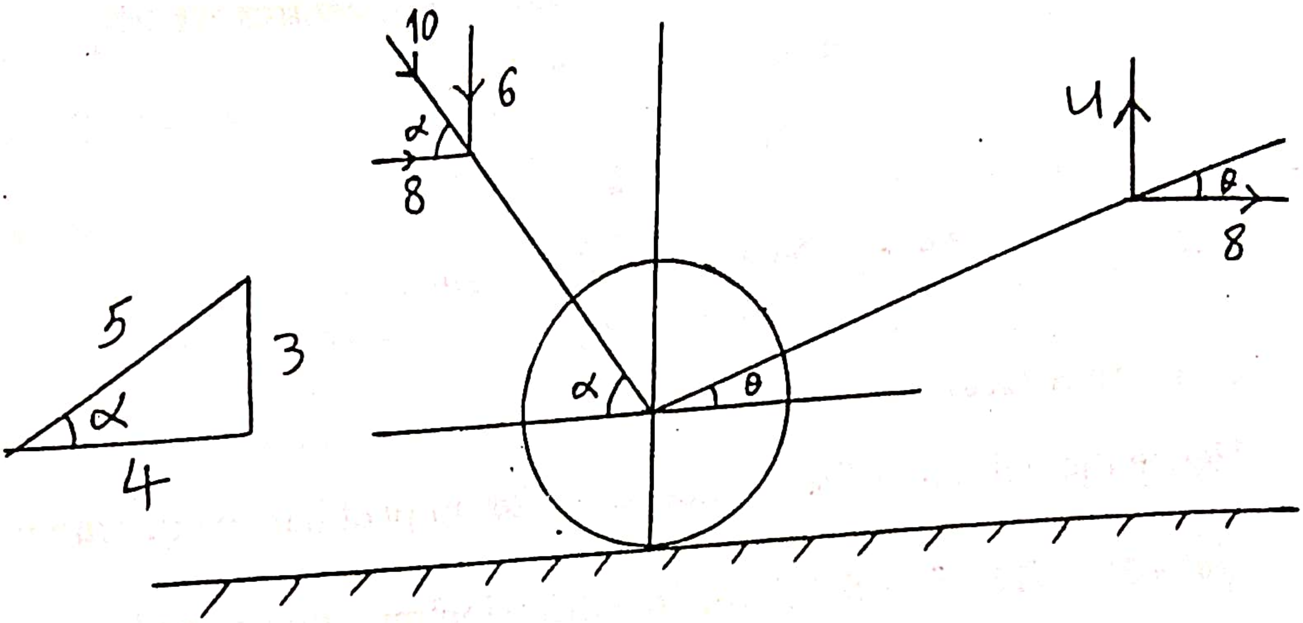
= sum of kinetic energies before impact

- sum of kinetic energies after impact

$$= \frac{1}{2}(5m \times (13)^2 + m \times 5^2) - \frac{1}{2}(5m \times 125 + m \times 205)$$

= 20 m poundal, ft.

5) A ball, moving with a velocity of 10 ft./sec., impinges on a smooth fixed plane in a direction making an angle $\tan^{-1}\frac{3}{4}$ with the plane; if the coefficient of restitution be $\frac{2}{3}$, find the velocity of the ball after the impact.



The component of the velocity of the sphere in the direction parallel to the plane will remain unaltered, i.e. will be 8 ft./sec.

Let the component in the direction perpendicular to the plane, i.e. in the direction of the line of impact, after impact be equal to u.

Applying Newton's experimental law, we get

$$u - 0 = -e(-6-0)$$

$$u = 4\,\text{ft./sec.}$$

Hence the velocity of the bal er impact has magnitude equal to

$$\sqrt{8^2+u^2} = \sqrt{80} = 4\sqrt{5}\,\text{ft./sec.}$$

in a direction making an angle θ with the plane where

$$\tan\theta = \frac{4}{8} = \frac{1}{2}, \qquad \text{i.e. } \theta = \tan^{-1}\frac{1}{2}.$$

Exercises

1- A sphere of mass 10 lbs. and moving with velocity 8 ft./sec. overtakes a sphere of mass 8 lbs. and moving in the same direction with velocity 6 ft./sec. If the coefficient of restitution be $\frac{1}{2}$, find the velocities of the spheres after impact and the impulsive reaction between them.

2- Two spheres of masses m , m' impinge. If the collision is direct and V is their relative velocity before impact e is the coefficient of restitution prove that the loss of kinetic energy by impact is

$$\frac{1}{2}\,\frac{m m' V^2 (1-e^2)}{m+m'}.$$

3- Two smooth spheres of the same volume and of masses M , m (M > m) are at rest on a smooth horizontal table. The sphere m is given an impulse I along the line of centers of the two spheres and then impinges on the sphere M. If e is the coefficient of restitution between the two spheres prove that the loss of kinetic energy by impact is

$$\frac{1}{2}(1-e^2)\frac{M I^2}{m(M+m)}.$$

4-A sphere of mass 8 lbs. and moving with velocity 4ft./sec. impinges on a sphere of mass 4 lbs. and moving with velocity 2 ft./sec. , their directions of motion before impact making angles of 30° and 60° with the line of centers. If $e=\frac{1}{2}$ find their velocities and directions of motion after impact.

5- A sphere of mass m and moving with velocity 10 ft./sec. impinges obliquely on a second sphere at rest, whose mass is 2m, in a direction making an angle of 30° with the line of centres. If $e=\frac{1}{2}$ find their velocities and directions of motion after impact.

6- A sphere of mass 5m lbs. and moving with velocity 13 ft./sec., impinges on a sphere, of mass m lbs. and moving with velocity 5 ft./sec., their directions of motion being inclined at angles of $\sin^{-1}\frac{5}{13}$, $\sin^{-1}\frac{3}{5}$ respectively to the line of centers; if the coefficient of restitution be $\frac{1}{2}$, find the magnitudes and directions of their velocities after the impact. Find the impulsive reaction between the two spheres and the loss of kinetic energy by impact.

7- A sphere moving with a velocity of 10 ft. /sec. impinges at an angle of 45° on a smooth plane; find its velocity and direction of motion after the impact; the coefficient of restitution being $\frac{4}{5}$.

8- A sphere of mass m lies on a smooth horizontal table between another sphere of mass m' and a fixed vertical plane. If the sphere m is projected on the table towards m' and the coefficient of restitution between the two spheres and between the sphere m and the plane equals $\frac{3}{5}$. prove that the sphere m comes to rest after collision with the sphere m' to the second time if m' = 15 m.

9-A billiard table in the form of a rectangle of dimensions 8 ft., 6 ft. Find the position of the point on the shorter side from which the ball could be projected and the direction of its initial velocity such that the ball traces a rectangle and returns after collision with the other three sides to its initial position exactly given that e = 4/9.

10-If a ball overtake a ball of twice its own mass moving with one-seventh of its velocity, and if the coefficient of restitution between them be $\frac{3}{4}$, show that the first ball will, after striking the second ball, remain at rest.

11-Two equal perfectly elastic balls impinge; if their directions of motion before impact be at right angles, show that their directions of motion after impact are at right angles also.

12-A sphere of mass 8 lbs. and moving with velocity 40 ft./sec., overtakes a sphere of mass 12 lbs. and moving with velocity 20 ft./sec. The two spheres form one body. Find its common velocity if the two Spheres were moving before impact:

(i)In the same direction.

(ii)In apposite directions.

14- A ball, moving with a velocity of 10 ft./sec., impinges on a smooth fixed plane in a direction making an angle $\tan^{-1}\frac{3}{4}$ with the plane; if the coefficient of restitution be $\frac{2}{3}$, find the velocity of the ball after the impact.

13- Three equal elastic spheres A, B, C are at rest along a straight line on a smooth horizontal table. A is projected towards B with velocity u. If the coefficient of restitution be $\frac{1}{2}$ for all collisions prove that two collisions only occur between A and B and the ratio between the final velocities of the three spheres is 13 : 15 : 36 .
