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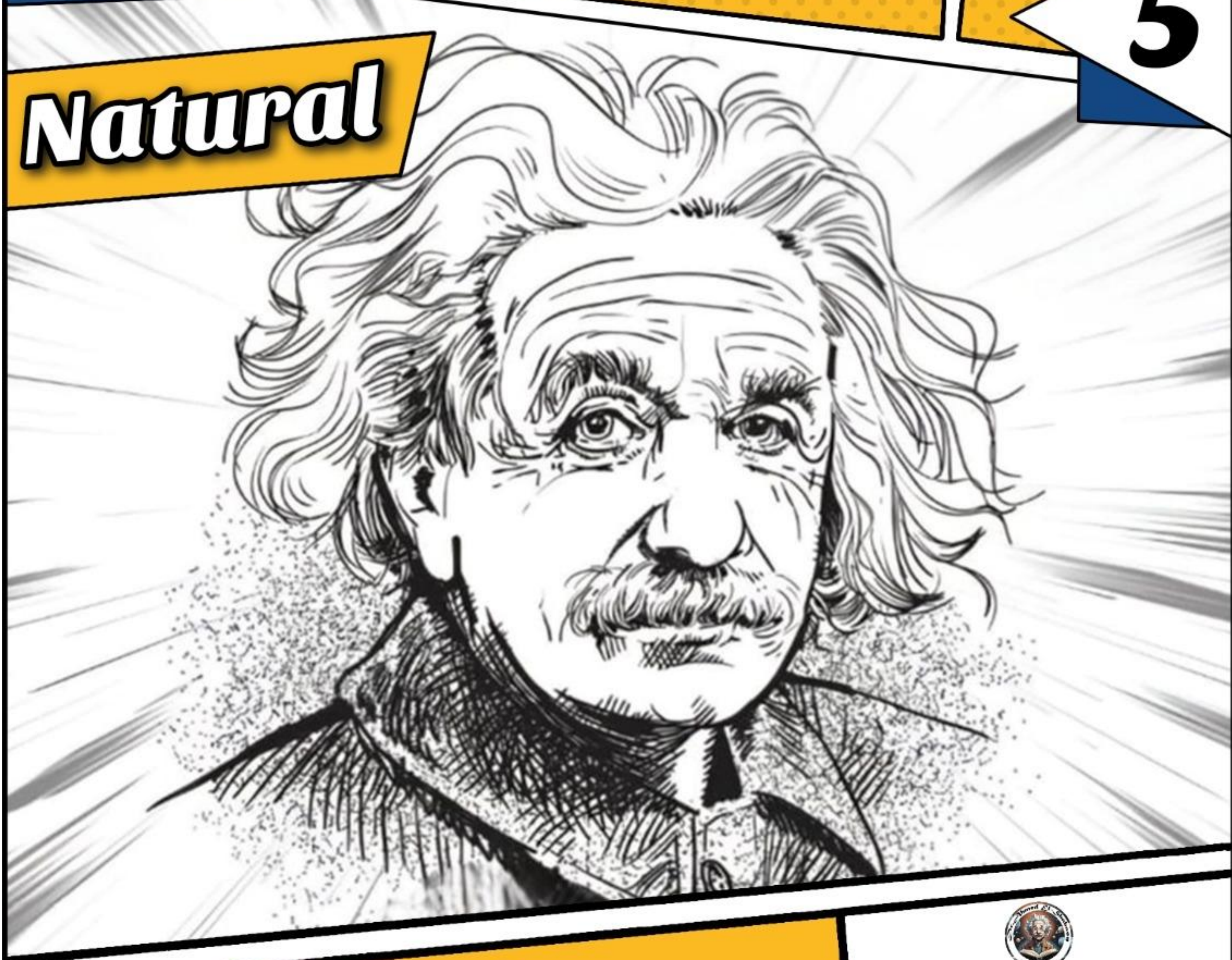
Natural



Heat (Thermal Expansion)

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Thermal expansion

☞ ***In general:*** materials expand with rise in temperature and contract on cooling.

Three types for thermal expansion

① Linear thermal expansion coefficient (α)

Depends on:

a) Initial length

b) Change in temperature.

c) Actual material

Definition

☞ It is defined as the fractional change in length per change in temperature.

$$\alpha = \frac{\frac{\Delta L}{L_o}}{\Delta T} = \frac{\Delta L}{\Delta T\ L_o} = K^{-1}$$

$$\therefore \Delta L = \alpha\ \Delta T\ L_o$$

Example

The railway from Alexandria to Cairo is 100 km long in two days. Calculate the difference in the length between (-5℃)and 25℃ , linear expansion coefficient of iron = $12 \times 10^{-6} K^{-1}$

Answer

$$\Delta T = 25 - (-5) = 30\ K$$

$$\Delta L = \alpha\ \Delta T\ L_o$$

$$= 12 \times 10^{-6} \times 30 \times 100 \times 10^{3} = 36m$$

Example

An iron ring of inner diameter $3.453\ cm$ at $20oC$ is to fit on a piston cylinder of diameter $3.458\ cm$. To what temperature must the ring be heated so that it just fits over the piston. Linear expansion of iron (α) = $12\ x\ 10^{-6}\ K^{-1}$

Answer

$$\Delta L = 3.458 - 3.453 = 0.005\ cm = 5\ x\ 10^{-3}\ cm = 5\ x\ 10^{-5}\ m$$

$$Lo = 3.453\ cm = 3.453\ x\ 10^{-2}\ m$$

$$\Delta L = Lo\ \alpha \Delta T$$

$$5\ x\ 10^{-5} = 3.453\ x\ 10^{-2}\ x\ 12\ x\ 10^{-6}\ \Delta T$$

$$5\ x\ 10^{-5} = 4.252\ x\ 10^{-7}\ \Delta T$$

$$\frac{5\ x\ 10^{-5}}{4.252\ x\ 10^{-7}} = \Delta T$$

$$\Delta T = 117.6\,K$$

$$T_f = 20 + 117.6 = 137.6$$

Example

A copper ring with an inner diameter of 4.500 cm at 25°C needs to fit over a rod with a diameter of 4.505 cm. To what temperature must the copper ring be heated so that it just fits over the rod? The coefficient of linear expansion for copper is $\alpha = 16.5 \times 10^{-6} K^{-1}$

Answer

② Area thermal expansion coefficient (β)

$$A_o = {L_o}^2$$

$$L = L_o + \Delta L$$

$$A = (L_o + \Delta L)^2$$

$$A = (L_o + \alpha\, \Delta T\, L_o)^2$$

$$A = L_o^2 + 2L_o\, \alpha\, \Delta T\, L_o + \alpha^2\, \Delta T^2\, L_o^2$$

α^2 is neglected

$$A = L_o^2 + 2L_o\, \alpha\, \Delta T\, L_o$$

$$A = A_o + 2A_o\alpha\, \Delta T$$

$$A - A_o = 2A_o\alpha\, \Delta T$$

$$\beta = 2\alpha$$

$$\Delta A = A_o \beta \Delta T$$

Example

A sheet of steel measure $10m \times 12m$ at a temperature of 20°C Determine increasing area of the sheet when its temperature rises to 100°C given $\alpha = 12 \times 10^{-6} K^{-1}$

Answer

First solution

$$\beta = 2\alpha = 2 \times 12 \times 10^{-6} = 24 \times 10^{-6} K^{-1}$$

$$\Delta T = 100 - 20 = 80°\text{C}$$

$$\Delta A = A_o \beta \Delta T = 10 \times 12 \times 24 \times 10^{-6} \times 80 = 0.2304\ m^2$$

Second solution

$$10\text{ m: } \Delta L = \alpha\ \Delta T\ L_o = 10 \times 12 \times 10^{-6} \times 80 = 9.6 \times 10^{-3} m$$

$$L = L_o + \Delta L = 10 + 9.6 \times 10^{-3} = 10.0096\ m$$

$$12\text{ m: } \Delta L = \alpha\ \Delta T\ L_o = 12 \times 12 \times 10^{-6} \times 80 = 0.01152\ m$$

$$L = L_o + \Delta L = 12 + 0.01152 = 12.01152\ m$$

$$\text{New area (A)} = 10.0096 \times 12.01152 = 120.23051\ m^2$$

$$change\ in\ area = 120.23051 - 120 = 0.23051\ m^2$$

Example

A copper sheet measures $5\,m \times 8\,m$ at a temperature of 30°C. Determine the increase in the area of the sheet when its temperature rises to 150°C. The coefficient of linear expansion for copper is

$\alpha = 16.5 \times 10^{-6} K^{-1}$

Answer

③ Volume thermal expansion coefficient (γ)

Prove that: Volume thermal expansion coefficient = 3 linear thermal expansion coefficient

$$V = (L_o + \alpha\,\Delta T\,L_o)^3$$

$$V = {L_o}^3 + \cancel{\alpha^3\,\Delta T^3\,L_o^3} + 3L_o^3\,\Delta T \alpha\,L_o + 3\,\cancel{\alpha^2\,\Delta T^2\,L_o^2}\,L_o$$

$$V = L_o^3 + 3L_o^3 \Delta T \alpha$$

$$V = V_o + 3V_o \Delta T \alpha$$

$$\boxed{\Delta V = V_o \gamma \Delta T}$$

$$V - V_o = 3V_o \Delta T \alpha$$

$$\boxed{\gamma = 3\alpha}$$

Example

A steel tank can hold a volume $of\ 4\ m^3\ at\ 20C^0$. Calculate its likely capacity were its temperature to rise to $80oC$. Linear expansion coefficient of steel = $12\ x\ 10^{-6}\ K^{-1}$

Answer

$$V_o = 4\ m^3$$

$$\gamma = 3\ \alpha$$

$$3\ \ 12\ x\ 10^{-6} = 36\ \ 10^{-6}\ K^{-1}$$

$$\Delta T = 80 - 20 = 60\ K$$

$$\Delta V = Vo\ \gamma\ \Delta T$$

$$= 4 \times 36 \times 10^{-6} \times 60$$

$$= 8.64\ x\ 10^{-3}\ m^3$$

$$V = V_0 + \Delta V$$

$$V = 4 + 8.64\ x\ 10^{-3} = 4.008\ 64\ m^3$$

Example

You have a water tank made of aluminum with a volume of 3 m^3 at a temperature of 25°C. Calculate its volume when the temperature rises to 75°C. The volumetric expansion coefficient of aluminum is approximately $7 \times 10^{-5} K^{-1}$

Answer