1. Let $\bar{a} = 2\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$,
   $\bar{b} = -\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$,
   $\bar{c} = 3\hat{\imath} + 6\hat{\jmath} - \hat{k}$
   find the indicated vector or scalar in the following:
   i. $\bar{a} \cdot \bar{b}$
   ii. $\bar{b} \cdot \bar{c}$
   iii. $\bar{a} \cdot (\bar{b} + \bar{c})$
   iv. $(2\bar{b}) \cdot (3\bar{c})$
   v. $(2\bar{a}) \cdot (\bar{a} - 2\bar{b})$
   vi. $\left(\frac{\bar{a} \cdot \bar{b}}{\bar{b} \cdot \bar{b}}\right) \bar{b}$
2. Prove law of cosines using vector notation.
3. Determine which of the following vectors are orthogonal:
   i. $\bar{a} = 2\hat{\imath} + \hat{k}$, $\bar{b} = 3\hat{\imath} + 2\hat{\jmath} - \hat{k}$
   ii. $\bar{a} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$, $\bar{b} = \hat{\imath} - 4\hat{\jmath} + 6\hat{k}$
   iii. $\bar{a} = \hat{\imath} - \hat{\jmath} + \hat{k}$, $\bar{b} = -4\hat{\imath} + 3\hat{\jmath} + 8\hat{k}$
4. Determine a scalar $\boldsymbol{c}$ so that the following vectors are orthogonal:
   i. $\bar{a} = 2\hat{\imath} - c\hat{\jmath} + 3\hat{k}$, $\bar{b} = 3\hat{\imath} + 2\hat{\jmath} + 4\hat{k}$

ii. $\bar{a} = 2\hat{\imath} + c\hat{\jmath} + \hat{k}$, $\bar{b} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$

iii. $\bar{a} = c\hat{\imath} + \frac{1}{2}\hat{\jmath} + c\hat{k}$, $\bar{b} = -3\hat{\imath} + 4\hat{\jmath} + c\hat{k}$

5. Find a vector $\bar{v} = x\hat{\imath} + y\hat{\jmath} + \hat{k}$ that is orthogonal to both $\bar{a} = 3\hat{\imath} + \hat{\jmath} - \hat{k}$ and

$$\bar{b} = -3\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$$

6. Verify that the vector $\bar{c} = \bar{b} - \left(\frac{\bar{a}\cdot\bar{b}}{|\bar{a}|^2}\right)\bar{a}$ is orthogonal to the vector $\bar{a}$
7. Determine a scalar $\boldsymbol{c}$ so that the angle between $\bar{a} = \hat{\imath} + c\hat{\jmath}$, $\bar{b} = \hat{\imath} + \hat{\jmath}$ is $45°$
8. Find the angle θ between the following vectors:

i. $\bar{a} = 2\hat{\imath} + 4\hat{\jmath}$, $\bar{b} = -\hat{\imath} - \hat{\jmath} + 4\hat{k}$

ii. $\bar{a} = 2\hat{\imath} + 2\hat{\jmath} - \hat{k}$, $\bar{b} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$

iii. $\bar{a} = 3\hat{\imath} + 2\hat{\jmath} - 6\hat{k}$, $\bar{b} = 4\hat{\imath} - 3\hat{\jmath} + \hat{k}$

iv. $\bar{a} = \frac{1}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath} + \frac{3}{2}\hat{k}$, $\bar{b} = 2\hat{\imath} - 4\hat{\jmath} + 6\hat{k}$

9. Let $\bar{a} = \hat{\imath} - \hat{\jmath} + 3\hat{k}$, $\bar{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$ find

i. $comp_{\bar{b}}\bar{a}$ ii. $comp_{\bar{a}}\bar{b}$

iii. $comp_{\bar{a}}(\bar{b} - \bar{a})$ iv. $comp_{2\bar{b}}(\bar{a} + \bar{b})$

10. Find the $proj_{\bar{b}}\bar{a}$, $proj_{\bar{b}^{\perp}}\bar{a}$ for the following problems:
    i. $\bar{a} = -\hat{\imath} - 2\hat{\jmath} + 7\hat{k}$, $\bar{b} = 6\hat{\imath} - 3\hat{\jmath} - 2\hat{k}$
    ii. $\bar{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$, $\bar{b} = -2\hat{\imath} + 2\hat{\jmath} - \hat{k}$
11. Find the magnitude of the projection for the following:
    i. $\bar{a} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ on $\bar{b} = 4\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$
    ii. $\bar{a} = 4\hat{\imath} - 3\hat{\jmath} + \hat{k}$ on the line passing through the points $(2,3,-1)$ and $(-2,-4,3)$.
12. An object moves 2m due east from O to A and then $2\sqrt{2}\text{m}$ to the northeast from A to B. Evaluate the magnitude of $\overline{OB}$ and its angle with $\overline{OA}$. Also, determine the component of $\overline{AB}$ in the direction parallel to $\overline{OA}$.
13. Find the $proj_{(\bar{a}+\bar{b})}\bar{a}$, $proj_{(\bar{a}-\bar{b})^{\perp}}\bar{b}$ for the following two vectors:

$$\bar{a} = 4\hat{\imath} + 3\hat{\jmath} \quad , \quad \bar{b} = -\hat{\imath} + \hat{\jmath}$$

14. Find the work done if the point at which the constant force $\bar{F} = 4\hat{\imath} + 3\hat{\jmath} + 5\hat{k}$ is applied to an object moves from $P_1(3,1,-2) to\ P_2(2,4,6)$.
15. An object subjected to the action of three constant forces:

$$\bar{F}_1 = 2\hat{\imath} + \hat{\jmath} - 3\hat{k},$$
$$\bar{F}_2 = \hat{\imath} + c\hat{\jmath} + 2\hat{k},$$
$$\bar{F}_3 = -\hat{\imath} + 4\hat{\jmath} + \hat{k}.$$

The object undergoes a displacement equal to twice the length of $\hat{\imath} - 2\hat{\jmath} + \hat{k}$ in the direction of $2\hat{\imath} + \hat{\jmath} - \hat{k}$. If the total work done by the combined forces during this displacement is 20 J, determine c.