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1. Find $\bar{a} \wedge \bar{b}$ for the following problems:
   i. $\bar{a} = 2\hat{\imath} - \hat{\jmath} + 2\hat{k}$, $\bar{b} = -\hat{\imath} + 3\hat{\jmath} - \hat{k}$
   ii. $\bar{a} = 4\hat{\imath} + \hat{\jmath} + 5\hat{k}$, $\bar{b} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$
   iii. $\bar{a} = \langle \frac{1}{2}, 0, \frac{1}{2} \rangle$, $\bar{b} = \langle 4,6,0 \rangle$
   iv. $\bar{a} = \langle 8,1,6 \rangle$, $\bar{b} = \langle 1,-2,10 \rangle$
2. Prove the law of sines for a triangle ABC using vector notation.
3. Show that $A(4,-2,-16)$, $B(0,-10,-4)$ and $C(-6,-22,14)$are collinear.
4. Given that the three points $A(2,3,1), B(5,7,4)$ and $C(\mathrm{c}_1, \mathrm{c}_2, \mathrm{c}_3)$are collinear. If the length from A to C is exactly twice as long as from A to B, find the coordinates of point C**.**
5. Find a unit vector that is perpendicular to both $\bar{a}$, $\bar{b}$
   i. $\bar{a} = 2\hat{\imath} + 7\hat{\jmath} - 4\hat{k}$, $\bar{b} = \hat{\imath} + \hat{\jmath} - \hat{k}$
   ii. $\bar{a} = \langle -1,-2,4 \rangle$, $\bar{b} = \langle 4,-1,0 \rangle$
   iii. $\bar{a} = \langle 1,-2,4 \rangle$, $\bar{b} = \langle 2,1,-2 \rangle$
6. Find three vectors perpendicular to the vector $2\hat{\imath} + 7\hat{\jmath} - 4\hat{k}$.
7. Verify that $\bar{a} \cdot (\bar{a} \wedge \bar{b}) = 0$ and $\bar{b} \cdot (\bar{a} \wedge \bar{b}) = 0$ for:

i. $\bar{a} = 5\hat{\imath} - 2\hat{\jmath} + \hat{k}$, $\bar{b} = 2\hat{\imath} - 7\hat{k}$

ii. $\bar{a} = \langle \frac{1}{2}, \frac{-1}{4}, 0 \rangle$, $\bar{b} = \langle 2, -2, 6 \rangle$

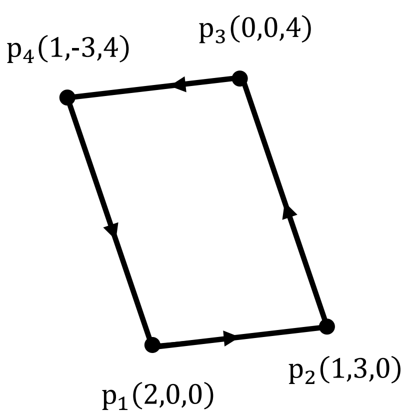
8. Let $\bar{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$, $\bar{b} = -\hat{\imath} + 2\hat{\jmath} + 5\hat{k}$ *and* $\bar{c} = 3\hat{\imath} + 6\hat{\jmath} - \hat{k}$, find the indicated vector or scalar in the following problems:

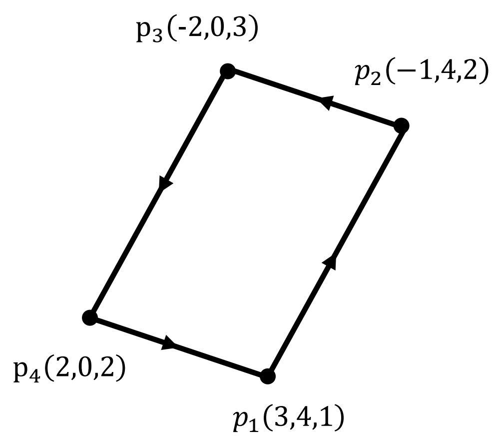
   i. $\bar{a} \wedge \bar{b}$ ii. $\bar{b} \wedge \bar{c}$

   iii. $\bar{a} \wedge (\bar{b} + \bar{c})$ iv. $\bar{a} \cdot (\bar{b} \wedge \bar{c})$

   v. $(2\bar{a}) \cdot (\bar{a} \wedge 2\bar{b})$

9. Verify that the given quadrilateral is parallelogram, and find its area





10. Calculate the area of the parallelogram $ABCD$, where $A(1,2), B(2,3), C(5,4)$ and $D(4,2)$.

11. Find the area of the triangle determined By the given vertices:

i. $A(1,1,1)$, $B(1,2,1)$, $C(1,1,2)$
ii. $A(1,2,4)$, $B(1,-1,3)$, $C(-1,-1,2)$
iii. $A(3,-1,2)$, $B(1,-1,3)$, $C(4,-3,1)$
iv. $A(2,4,-7)$, $B(3,7,18)$, $C(-5,12,8)$

12. Find the volume of the parallelepiped for which the given vectors are three edges:
   i. $\bar{a} = \hat{\imath} + \hat{\jmath}$, $\bar{b} = -\hat{\imath} + 4\hat{\jmath}$
   $and \quad \bar{c} = 2\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$
   ii. $\bar{a} = 3\hat{\imath} + \hat{\jmath} + \hat{k}$, $\bar{b} = \hat{\imath} + 4\hat{\jmath} + \hat{k},$
   $and \quad \bar{c} = \hat{\imath} + \hat{\jmath} + 5\hat{k}$
13. Determine whether of the following vectors are coplanar.

$$\bar{a} = 4\hat{\imath} + 6\hat{\jmath}\,, \bar{b} = -2\hat{\imath} + 6\hat{\jmath} - 6\hat{k}, \bar{c} = \frac{5}{2}\hat{\imath} + 3\hat{\jmath} + \frac{1}{2}\hat{k}$$

14. If $\bar{a}, \bar{b}, \bar{c}$ are coplanar vectors, then we can write

$$\bar{a} = m\bar{b} + n\bar{c}$$

where m, n are constants.
15. Determine whether the four points

$$A(1,1,-2), B(4,0,-3),$$
$$C(1,-5,10), D(-7,2,4)$$

lie in the same plane.

16. Let a force $\bar{F} = 4\hat{\imath} - 2\hat{\jmath} + \hat{k}$ acts at the point $P(3,1,0)$. Find the moment of this force about $(1,0,2)$.
17. If a force $\bar{F} = \langle c, 2c, 0 \rangle$ acts at the point $P(1,1,0)$, and the magnitude of the moment of this force about the origin is 3. Determine c.
18. Prove or disprove $\bar{a} \wedge (\bar{b} \wedge \bar{c}) = (\bar{a} \wedge \bar{b}) \wedge \bar{c}$
19. Prove $|\bar{a} \wedge \bar{b}|^2 + (\bar{a} \cdot \bar{b})^2 = |\bar{a}|^2 |\bar{b}|^2$
20. Does $\bar{a} \wedge \bar{c} = \bar{a} \wedge \bar{b}$ imply that $\bar{c} = \bar{b}$
21. Show that $(\bar{a} + \bar{b}) \wedge (\bar{a} - \bar{b}) = 2\bar{b} \wedge \bar{a}$
22. Calculate $\bar{a} \cdot (\bar{b} \wedge \bar{c})$, $\bar{a} \wedge (\bar{b} \wedge \bar{c})$, where

$$\bar{a} = \hat{\imath} + \hat{\jmath} + \hat{k}$$
$$\bar{b} = 3\hat{\imath} + 2\hat{k}$$
$$\bar{c} = 2\hat{\imath} + 2\hat{\jmath} + 2\hat{k}$$

23. Consider the vector equation $\bar{a} \wedge \bar{x} = \bar{b}$ in the space, where $\bar{a} \neq 0$. Show that
    (a) $\bar{a} \cdot \bar{b} = 0$
    (b) $\bar{x} = \frac{\bar{b} \wedge \bar{a}}{|\bar{a}|^2} + k\bar{a}$ is a solution to the equation, for any scalar k.
24. For all vectors $\bar{a}, \bar{b}, \bar{c}, \bar{d}$ in space, show that:

i. $\bar{a} \wedge \left(\bar{b} \wedge \bar{c}\right) + \bar{b} \wedge (\bar{c} \wedge \bar{a}) + \bar{c} \wedge \left(\bar{a} \wedge \bar{b}\right) = 0$

ii. $\left(\bar{a} \wedge \bar{b}\right) \cdot \left(\bar{c} \wedge \bar{d}\right) = (\bar{a} \cdot \bar{c})\left(\bar{b} \cdot \bar{d}\right) - \left(\bar{a} \cdot \bar{d}\right)\left(\bar{b} \cdot \bar{c}\right)$

iii. $\left(\bar{a} \wedge \bar{b}\right) \wedge \left(\bar{c} \wedge \bar{d}\right) = \left[\bar{d} \cdot \left(\bar{a} \wedge \bar{b}\right)\right]\bar{c} - \left[\bar{c} \cdot \left(\bar{a} \wedge \bar{b}\right)\right]\bar{d}$

iv. $\left(\bar{a} \wedge \bar{b}\right) \wedge \left(\bar{c} \wedge \bar{d}\right) = \left[\bar{a} \cdot \left(\bar{c} \wedge \bar{d}\right)\right]\bar{b} - \left[\bar{b} \cdot \left(\bar{c} \wedge \bar{d}\right)\right]\bar{a}$

v. $\left(\bar{a} \wedge \bar{b}\right) \wedge \left(\bar{c} \wedge \bar{d}\right) = \left[\bar{a} \cdot \left(\bar{b} \wedge \bar{d}\right)\right]\bar{c} - \left[\bar{a} \cdot \left(\bar{b} \wedge \bar{c}\right)\right]\bar{d}$

vi. $\left(\bar{a} \wedge \bar{b}\right) \cdot \left(\left(\bar{b} \wedge \bar{c}\right) \wedge (\bar{c} \wedge \bar{a})\right) = \left(\bar{a} \cdot \left(\bar{b} \wedge \bar{c}\right)\right)^2$

vii. $\bar{a} \cdot \left(\bar{b} \wedge \left(\bar{c} \wedge \left(\bar{a} \wedge \bar{b}\right)\right)\right) = 0$