

**sheet of limits and continuity**  
**Exercises:**

1.  $\lim_{x \rightarrow 3} \frac{3x(2x-1)}{x^2+2}$

2.  $\lim_{x \rightarrow 5} \frac{x-5}{x^2-25}$

3.  $\lim_{x \rightarrow -3} \frac{x^2+4x+3}{x^2-9}$

4.  $\lim_{x \rightarrow 2} \frac{x^2+7x+10}{x-2}$

5.  $\lim_{x \rightarrow -1} \frac{x^2-1}{x+1}$

6.  $\lim_{x \rightarrow -2} \frac{x^2-x-6}{x^2+2x}$

7.  $\lim_{y \rightarrow 3} \frac{5y^2+8y^2-3}{y^2-9}$

8.  $\lim_{x \rightarrow 1} \frac{\frac{1}{x}-1}{x-1}$

9.  $\lim_{x \rightarrow 0} \frac{\sqrt{5x+4}-2}{x}$

10.  $\lim_{x \rightarrow 1} \frac{x^4-1}{x+1}$

11.  $\lim_{x \rightarrow 2} \frac{x-2}{x^2+x-6}$

12.  $\lim_{x \rightarrow 9} \frac{\sqrt{x}-3}{x-9}$

13.  $\lim_{x \rightarrow 2} \frac{\sqrt{x+2}-2}{x-2}$

14.  $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x^2+3}-2}$

15.  $\lim_{x \rightarrow 1} \frac{\sqrt{x^2+8}-3}{x+1}$

16.  $\lim_{x \rightarrow -1} \frac{\sqrt{x^2+8}-3}{x+1}$

17.  $\lim_{x \rightarrow -1} \frac{\sqrt{x+5}-2}{x+1}$

18.  $\lim_{x \rightarrow 2} \frac{4-x^2}{3-\sqrt{x^2+5}}$

19.  $\lim_{x \rightarrow 1} \frac{x-1}{\sqrt{x+3}-2}$

20. Suppose  $\lim_{x \rightarrow c} f(x) = 5$  and  $\lim_{x \rightarrow c} g(x) = -2$ . Find

1.  $\lim_{x \rightarrow c} f(x)g(x)$

2.  $\lim_{x \rightarrow c} 2f(x)g(x)$

$$3. \lim_{x \rightarrow c} (f(x) + 3g(x))$$

$$4. \lim_{x \rightarrow c} \frac{f(x)}{f(x)-g(x)}$$

**21.** Suppose  $\lim_{x \rightarrow 4} f(x) = 0$  and  $\lim_{x \rightarrow 4} g(x) = -3$ . Find

$$1. \lim_{x \rightarrow 4} (g(x) + 3)$$

$$2. \lim_{x \rightarrow 4} xf(x)$$

$$3. \lim_{x \rightarrow 4} (g(x))^2$$

$$4. \lim_{x \rightarrow 4} \frac{g(x)}{f(x)-1}$$

**22.** Suppose  $\lim_{x \rightarrow b} f(x) = 7$  and  $\lim_{x \rightarrow b} g(x) = -3$ . Find

$$1. \lim_{x \rightarrow b} (f(x) + g(x))$$

$$2. \lim_{x \rightarrow b} f(x) \cdot g(x)$$

$$3. \lim_{x \rightarrow b} 4g(x)$$

$$4. \lim_{x \rightarrow b} f(x)/g(x)$$

**23.** Suppose that  $\lim_{x \rightarrow -2} p(x) = 4$ ,  $\lim_{x \rightarrow -2} r(x) = 0$ , and  $\lim_{x \rightarrow -2} s(x) = -3$ . Find

$$1. \lim_{x \rightarrow -2} (p(x) + r(x) + s(x))$$

$$2. \lim_{x \rightarrow -2} p(x) \cdot r(x) \cdot s(x)$$

$$3. \lim_{x \rightarrow -2} (-4p(x) + 5r(x))/s(x)$$

$$\mathbf{24.} \lim_{x \rightarrow -1} \frac{\sin(x^2-x-2)}{x+1}$$

$$\mathbf{25.} \lim_{\theta \rightarrow 0} \frac{\sin \sqrt{2}\theta}{\sqrt{2}\theta}$$

$$\mathbf{26.} \lim_{t \rightarrow 0} \frac{\sin kt}{t} \text{ (k constant)}$$

$$\mathbf{27.} \lim_{y \rightarrow 0} \frac{\sin 3y}{4y}$$

$$\mathbf{28.} \lim_{h \rightarrow 0} \frac{h}{\sin 3h}$$

$$\mathbf{29.} \lim_{x \rightarrow 0} \frac{\tan 2x}{x}$$

$$\mathbf{30.} \lim_{t \rightarrow 0} \frac{2t}{\tan t}$$

$$31. \lim_{x \rightarrow 0} \frac{x \csc 2x}{\cos 5x}$$

$$32. \lim_{x \rightarrow 0} 6x^2(\cot x)(\csc 2x)$$

$$33. \lim_{x \rightarrow 0} \frac{x+x \cos x}{\sin x \cos x}$$

$$34. \lim_{x \rightarrow 0} \frac{x^2 - x + \sin x}{2x}$$

$$35. \lim_{\theta \rightarrow 0} \frac{1 - \cos \theta}{\sin 2\theta}$$

$$36. \lim_{x \rightarrow 0} \frac{x - x \cos x}{\sin^2 3x}$$

$$37. \lim_{t \rightarrow 0} \frac{\sin(1 - \cos t)}{1 - \cos t}$$

$$38. \lim_{h \rightarrow 0} \frac{\sin(\sin h)}{\sin h}$$

$$39. \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta + \sin 2\theta}$$

$$40. \lim_{x \rightarrow 0} \frac{\sin 5x}{\sin 4x}$$

$$41. \lim_{x \rightarrow 1} \frac{\sin(1 - \sqrt{x})}{x - 1}$$

$$42. \lim_{\theta \rightarrow 0} \frac{\theta}{\cos \theta}$$

$$43. \lim_{\theta \rightarrow 0} \frac{\sin \theta}{\cot 2\theta}$$

$$44. \lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 8x}$$

$$45. \lim_{y \rightarrow 0} \frac{\sin 3y \cot 5y}{y \cot 4y}$$

### Using the Sandwich Theorem

46. If  $\sqrt{5 - 2x^2} \leq f(x) \leq \sqrt{5 - x^2}$  for  $-1 \leq x \leq 1$ , find  $\lim_{x \rightarrow 0} f(x)$ .

47. If  $2 - x^2 \leq g(x) \leq 2 \cos x$  for all  $x$ , find  $\lim_{x \rightarrow 0} g(x)$ .

48. IF the inequalities

$$1 - \frac{x^2}{6} < \frac{x \sin x}{2 - 2 \cos x} < 1$$

hold for all values of  $x$  close to zero. Evaluate

$$\lim_{x \rightarrow 0} \frac{x \sin x}{2 - 2 \cos x}?$$

49. a. Suppose that the inequalities

$$\frac{1}{2} - \frac{x^2}{24} < \frac{1 - \cos x}{x^2} < \frac{1}{2}$$

hold for values of  $x$  close to zero. Evaluate

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2}?$$

**50.**  $\lim_{x \rightarrow \infty} \sqrt{\frac{8x^2 - 3}{2x^2 + x}}$

**51.**  $\lim_{x \rightarrow \infty} \left( \frac{x^2 + x - 1}{8x^2 - 3} \right)^{1/3}$

**52.**  $\lim_{x \rightarrow \infty} \left( \frac{1 - x^3}{x^2 + 7x} \right)^5$

**53.**  $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 - 5x}}{\sqrt[3]{x^3 + x - 2}}$

**54.**  $\lim_{x \rightarrow \infty} \frac{2\sqrt{x} + x^{-1}}{3x - 7}$

**55.**  $\lim_{x \rightarrow \infty} \frac{2 + \sqrt{x}}{2 - \sqrt{x}}$

**56.**  $\lim_{x \rightarrow \infty} \frac{\sqrt[3]{x} - \sqrt{x}}{\sqrt[3]{x} + \sqrt{x}}$

**57.**  $\lim_{x \rightarrow \infty} \frac{x^{-1} + x^{-4}}{x^{-2} - x^{-3}}$

**58.**  $\lim_{x \rightarrow \infty} \frac{2x^{5/3} - x^{1/3} + 7}{x^{8/5} + 3x + \sqrt{x}}$

**59.**  $\lim_{x \rightarrow \infty} \frac{\sqrt[3]{x} - 5x + 3}{2x + x^{2/3} - 4}$

**60.** For what value of  $a$  is

$$f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$$

continuous at every  $x$ ?

**61.** For what value of  $b$  is

$$g(x) = \begin{cases} x, & x < -2 \\ bx^2, & x \geq -2 \end{cases}$$

continuous at every  $x$ ?

**62.** For what values of  $a$  is

$$f(x) = \begin{cases} a^2x - 2a, & x \geq 2 \\ 12, & x < 2 \end{cases}$$

continuous at every  $x$ ?

**62.** For what value of  $b$  is

$$g(x) = \begin{cases} \frac{x-b}{b+1}, & x < 0 \\ x^2 + b, & x > 0 \end{cases}$$

continuous at every  $x$ ?

**63.** For what values of  $a$  and  $b$  is

$$f(x) = \begin{cases} -2, & x \leq -1 \\ ax - b, & -1 < x < 1 \\ 3, & x \geq 1 \end{cases}$$

continuous at every  $x$ ?

**64.** For what values of  $a$  and  $b$  is

$$g(x) = \begin{cases} ax + 2b, & x \leq 0 \\ x^2 + 3a - b, & 0 < x \leq 2 \\ 3x - 5, & x > 2 \end{cases}$$

continuous at every  $x$ ?