Sheet

[1] Differentiate the following functions:

1) $y = \dfrac{1 + x - 4\sqrt{x}}{x}$

2) $y = (x^2 + 1)^2 \left(x + 5 + \dfrac{1}{x}\right)^3$

3) $y = \sqrt[7]{x^2} - x^3 + 8$

4) $y = e^{\sin^{-1} x}$

5) $y = \tan^2 x + x \cos^3 x$

6) $y = e^{\sqrt{x+1}} \ln(\tan x)$

7) $y = 10^x \log x$

8) $y = (\sec^{-1} x^4)^3$

9) $y = \cos^{-1}\left(\dfrac{a + b\cos x}{b + a\cos x}\right)$

10) $y = \tan^{-1}\sqrt{\left(\dfrac{1 - \cos x}{1 + \cos x}\right)}$

11) $y = e^{\ln(\sin^{-1} x^2)}$

12) $y = \frac{x\, e^{\sin^{-1} x}}{\sqrt{1-x^2}}$

13) $y = \ln\left(x + \sqrt{x^2 + 1}\right)$

14) $y = x^3 \cosh(x^2) - \coth(x^{-1})$

15) $\mathrm{y} = \tanh(x^2 + \sin x)$

16) $y = sech^2\left(\sqrt{cos^2 x + 1}\right)$

[2] Find $\frac{dy}{dx}$ for the following functions

1) $y = (x)^{\sin x}$

2) $y = \frac{x^x \sin^3 x}{e^{\cot(3x)}\,(x^2+4)^7}$

3) $y = \frac{x^2\,\sqrt[3]{7x-14}}{e^{\sin x}\,(25-x^2)^4}$

4) $y = \frac{(x-1)^3(1-\tan x)^4}{x^{\ln x}\,(2-\cos x)^2}$

5) $y = \sqrt[3]{\frac{x(x+5)e^x}{sin^3 x}}$

6) $\sqrt{y} = \ln\left(\frac{\tan^{-1} 3x}{(x+1)^2 \cosh x}\right)$

[3] Find the first derivative for the following functions

1) $x^{\sin x}\,.\,y = (\sin x)^x$

2) $x^y = \sin(x^3 + e^{7x^2})$

3) $y = \log_7\left(\frac{\sin x \cos x}{e^{2x}\,2^x}\right)^5$

4) $y = x[\log_5(x^2 - 2x)]^3$

5) $y = x^{\sin x}\,.\cot^4(\sqrt{1 - x^2})$

6) $y = \frac{x^x\left(2-\cot^{-1} x\right)^{x^2}}{x^{\cos x}\,(1-2\ln x)^5}$.

[4] Find $\frac{dy}{dx}$ by implicit differentiation

1) $\log_3\left(\sqrt{x^3+y^3}\right)+\sec^4(x^2y^3)=2$

2) $y^2+1+xy=e^{xy}$

3) $e^{x^2y}=2x+2y$

4) $x\ \tan^{-1}y=y\ \tan^{-1}x$

[5] Find $\frac{d^2y}{dx^2}$ for the parametric functions

1) $x=\sqrt{1-t^6}$, $y=\sin^{-1}(t^3)$

2) $x=\ln(1+t^2)$, $y=\tan^{-1}(t)$

3) $x=sin\,(ln\,t)$, $y=ln(sin\,t)$

4) $x=cos\,\theta+\theta\,sin\,\theta$, $y=sin\,\theta-\theta\,cos\,\theta$

[6] If $y=x\sin\frac{1}{x}$. Prove that $x^4y''+y=0$

[7] If $x=\sec^2(t^2+1)$, $y=\cos(t^2+1)$. Prove that $y''=\frac{6}{x^4}$.

[8] If $x=\tan\left(\frac{\sqrt{t}}{t^2+1}\right)$, $y=\sec\left(\frac{\sqrt{t}}{t^2+1}\right)$. Show that $y''=y^{-3}$.

[9] If $x=2\,(1-\cos\theta)$, $y=2\sin^2\theta$. Show that $y''=1$.

Good Luck