Exercises

1- A particle moves in a straight line with S.H.M. of periodic time 4 secs. If the particle started to move from rest at a distance 4 ft. from the centre of the motion find the time taken in moving a distance of 1 ft. and it velocity then.

2- A particle moves in a straight line with S.H.M, of amplitude a and periodic time $\frac{2\pi}{w}$. At the centre of motion, the particle receives a blow in the direction of motion which increases the magnitude of its velocity by the amount 3wa. Find the new amplitude.

3- A particle moves in a straight line with an acceleration which is always directed towards a fixed point 0 and is equal to $w^2 x$ where x is the distance from 0.

If the particle starts to move with velocity V towards O at a distance a from it, prove that its motion is S.H. with amplitude $\sqrt{a^2+\frac{V^2}{w^2}}$ and

$x = a \cos wt - \frac{V}{w} \sin wt$ where t is measured from the starting point.

4- A particle moves in a straight line with S.H.M. such that its velocity is equal to 3 ft./sec. and 4 ft./sec. at the distances 4 ft. and 3 ft. respectively from the centre of motion. Find the periodic time and the amplitude.

5- A particle moves about a fixed point O, in a straight line AOA', a simple harmonic motion of periodic time $\frac{2\pi}{w}$ and amplitude OA = a.

After time $\frac{\pi}{6w}$ from reaching the end A, the particle receives a blow in the direction of motion which increases the magnitude of its velocity by the amount w a . Show that the new amplitude is $a\sqrt{3}$.

6-A rod AB moves in a plane such that the end B describes a circle of centre C and radius a with uniform angular velocity w and the end A moves in a straight line passing through C. Prove that if AB = a, the motion of A is simple harmonic. Find the amplitude and the periodic time of this motion.

7- The velocity v for a particle moving in the axis Ox in terms of its distance x from O is given by the relation

$$v^2 = n^2 (8ax - x^2 - 12a^2)$$

where n , a are constants. Prove that the motion is simple harmonic of amplitude 2a, and that the time of the motion from x = 4a to x = 6a is equal to $\frac{\pi}{2n}$. Also find the time of motion from x = 3a to x = 5a.

8- Assuming that the earth attracts points inside it with a force which varies as the distance from its centre , show that, if a straight frictionless airless tunnel be made from one point of the earth's surface to any other point, a train would traverse the tunnel in slightly less than three – quarters of an hour.

9- 7- A particle moves in a straight line with an acceleration which is always directed towards a fixed point O and is equal to $w^2 x$ where x is the distance from O. If the particle starts to move with velocity V away from O and at a distance b from it, prove that

$$x = b \cos wt + \frac{V}{w} \sin wt .$$

Find the amplitude and the initial phase angle.

10-A , B are the ends of the path of a particle of mass 4.9 gms. moving with S.H.M. in a straight line. If the velocity of the particle at a point P is equal to 44 cms./sec. and the time from A to P is 1.5 secs. and from P to B is 4.5 secs. find the two distances PA and PB . Find also the maximum momentum of the particle and the maximum force acting on it.

11-A particle moves in a straight line with S,H.M. It passes by two points A , B on its path at a distance 22 inches appart with the same velocity. If the time taken for the motion from A to B is 2 secs. and the time taken to return to B again is also 2 secs., find the amplitude and the frequency of the motion.

12-A particle of mass 3 lbs. moves with S.H.M. in a straight line whose ends are A , B. P is a point on the path distant 16 ft. from A and 36 ft. from B.

If the velocity of the particle at P in equal to 144 ft./sec. find the two times of motion from A to P and from P to B .

Find the maximum force acting on the particle and the maximum kinetic energy.

13- The length of a spring increases by $\frac{1}{2}$ **inch** if the suspended weight increases by 1 lb. The spring is suspended vertically carrying a mass 4 lbs. in equilibrium. The mass is pulled down a distance of 3 inches and then released. Find the periodic time. Also find the velocity and acceleration of the particle when its distance from the equilibrium position is 1 inch.

14- One end of an elastic string of natural length 2b and modulus of elasticity mg is fixed to a point O on a smooth horizontal table and the other end is tied to a particle of mass m which rests on the table at O. If the particle is projected horizontally with velocity $2\sqrt{bg}$ find the time of a complete oscillation.

15- A particle of mass m is tied at the middle point of an elastic string PQ of natural length 2b and modulus of elasticity mg. The two ends P , Q are fixed at two points on a smooth horizontal table at a distance 2b apart. The particle is initially on the table at a distance $\frac{3}{2}b$ from P and $\frac{1}{2}b$ from Q and then released. Prove that its motion is S.H. Find the amplitude and the periodic time.

16- A particle of mass 10 lbs. is suspended by a spring. At the position of equilibrium the extension of the spring is 10 inches. The particle is pulled down a distance of $\frac{1}{2}$ **inch** and then released. Find the time of a complete oscillation, and the velocity of the particle after rising 1 inch, and the force in the spring at the maximum height. Is it tension or compression ?

17- A bead is tied at the middle point of an elastic string joining two points P,Q on a smooth horizontal table. If the bead is given a small displacement in a direction perpendicular to PQ and then let go. Prove that the resulting motion is approximately simple harmonic. (consider the tension constant throughout the motion).

If PQ = 9 ft. and the tension in the string is twice the weight of the bead and the given displacement is $\frac{1}{2}$ **inch** find the periodic time and the maximum velocity of the bead.

18- The lower end of a spring of natural length 4 ft. and modulus of elasticity 4 mg pdls is fixed such that the axis of the spring is vertical, A particle of mass m rests on the other end. Another particle of the same mass falls from a height of 1 ft. on the first particle. Find the distance described by the common mass till it first comes to rest, and the time taken to reach this position.

19- A particle of mass m is tied to one endof an elastic string of natural length a and modulus of elasticity $\frac{1}{2}mg\tan^2\theta$ where θ is fixed at the point C. If the particle is let to fall from rest at C, prove that the distance described downwards is equal to

$a\cot^2\frac{\theta}{2}$, in time given by $\sqrt{\frac{2a}{g}}\,[1+(\pi-\theta)\cot\theta]$.

20- A small ring slides on a smooth straight horizontal wire. The ring is tied to one end of an elastic string of natural length a and modulus of elasticity λ. The other end is fixed at a point in the same horizontal plane passing through the wire and at a distance $b > a$ from it. If the ring is given a small displacement on the wire from the equilibrium position and then released, prove that the motion is simple harmonic and find its periodic time.

21. A light spring is kept compressed by the action of a given force; the force is suddenly reversed; prove that the greatest subsequent extension of the string is three times its initial contraction.

22. A particle is suspended by an elastic string of natural length a from a fixed point C. If the particle is let to fall from rest when the string is vertical of length a and the particle is below C, and when the particle reaches its lowest point half its mass falls. Prove that the other half will rise a distance 2d above the initial position where d is the extension of the string in the original equilibrium position.

If the particle when at its lowest position receives a downward blow to move with velocity u, show that it will rise a distance $\frac{u^2}{2g}$ above the initial position. Find τ in each case.

23- Two particles, of masses m and m', are connected by an elastic string whose coefficient of elasticity is λ ; they are placed on a smooth table, the distance between them being a, the natural length of the string.

The particle m is projected with velocity V along the direction of the string produced; find the motion of each particle, and show that in the subsequent motion the greatest length of the string is $a+Vp$, and that the string is next at its natural length after time πp, where $p^2 = \frac{mm'}{m+m'}\frac{a}{\lambda}$.