Exercises

1- A particle moves on a straight line such that its distance x ft. measured from a fixed point 0 on the line is given in terms of the time t secs by the equation $x = t^3 - 4t^2 + 5t - 2.$

a- Where is the particle when its velocity vanishes.

b- Find the position of the particle when its acceleration vanishes.

c- Find the acceleration of the particle when its velocity is equal to 8 ft./ sec.

2- If $x^2 = t^2 + 6t + 5$

prove that the acceleration is inversely proportional to the cube of the distance .

3- If the relation between distance x and time t for a particle which moves in a straight line is given by

$$x = 3 \cos 2t + 4 \sin 2t$$

prove that

$$v^2 = 4(25 - x^2) \;;\; f = -4x.$$

4- A particle moves in the positive direction of the x-axis with acceleration $(4t+3) cm/sec^2$. The particle starts motion from a point distant 9 cms. From the origin 0 and after 3 secs its velocity becomes 22 cms/sec. Find its velocity and distances from O after time t secs. When and where the velocity of the particle vanishes.

5- A particle starts motion from rest at the origin in the positive direction of the x-axis. If the acceleration of the particle after time t is given by

$$f = k\, e^{-nt}$$

where k , n are constants. Prove that the acceleration f and the velocity v after the particle has moved a distance x satisfy the relation

$$n x = (nt - 1)v + f t.$$

6- The distance x of a moving particle on a straight line from a fixed point O on that line is related to the time t by the equation

$$x = A e^{nt} + B e^{-nt}$$

where A , B , n are constants. Prove that

$$v^2 = n^2 (x^2 - 4AB) \quad , \qquad f = n^2 x.$$

7- For the motion of a particle in a straight line, if v^2 is quadratic in x, prove that the acceleration of the particle at any position varies as the distance between this position and a fixed point on the line.

8- A particle starts motion from rest in a straight line such that its acceleration after time t secs is equal to $\left[3\sin t + (t+1)^{-2}\right]$ ft./sec^2. Find the distance described after time t secs from the beginning.

9- A particle moves in a straight line such that its distance x ft. from a fixed point O on the line is given by the equation

$$x = 2t^3 - 3t^2 - 12t + 18$$

where t is the time in secs. Find the position of the particle and its acceleration when its velocity vanishes.

10- A ladder of length 30 ft. slides in a vertical plane such that its upper end touches a vertical wall and its lower end touches a horizontal ground. Find the velocity of the upper end when the lower one is at a distance of 18 ft. from the wall and is moving with velocity 8 ft./sec.

11- A particle starts to move from rest in a straight line with an acceleration which increases with constant time rate from 1 ft./sec^2. to 4 ft./sec^2. in one second. Prove that the particle will move a distance of 1 ft. in this second.

12- A particle moves in a straight line Ox with an acceleration of magnitude $w^2 x$ directed always far from the origin O. If the particle is initially projected towards O with velocity w a from a point distant a from O, prove that its distance from O at time t is given by $x = a e^{-wt}$.

13- A particle moves in a straight line with a retardation of magnitude μv^2 where v is the velocity, μ is a constant. If the particle starts motion from the origin with velocity V prove that the velocity v and the time t depend on x by the following relations:

$$v = V e^{-\mu x} \quad , \quad t = \frac{1}{\mu V}(e^{\mu x} - 1).$$

14- The relation between the velocity and the distance for a particle moving in a straight line is given by $v = V_0 e^{-kx}$ where V_0, k are constants. Find x, v, f as functions of time t.

15- The frictional forces acting upon a train which is moving in a straight line with velocity v ft./sec. causes a retardation of magnitude $a + bv^2$ where a,b are constants. If the engine is stopped when the train was moving with velocity v_0 ft./sec., prove that it moves a distance

$\frac{1}{2b}\ln(1 + \frac{bv_0^2}{a})$ before it comes to rest.

16- A particle moves in a straight line under the action of a central repulsive force from a fixed point O on that line and of magnitude proportional to the distance from it. The particle is projected towards O from the point p distant 10 ft. from O with velocity v_0 just enough for the particle to reach O. Prove that if the particle is projected from P towards O with velocity $\frac{1}{2}v_0$, it will come to rest instantaneously at a point distant $5\sqrt{3}$ ft. from O.

17- A particle is attracted by a force to a fixed point varying inversely as the n^{th} power of the distance; if the velocity acquired by it in falling from an infinite distance to a distance a from the centre is equal to the velocity that would be acquired by it in falling from rest at a distance a to a distance $\frac{a}{4}$, show that $n = 3/2$.

18- A particle is moving in a straight line under the action of an acceleration of magnitude $k^2(x+\frac{a^4}{x^3})$ towards the origin. If it starts from rest at a distance a, show that it will arrive at the origin in time $\frac{\pi}{4k}$. Find also the time taken by the particle from the initial position to the point $x=\frac{a}{\sqrt{2}}$.

19.A particle is projected from the origin in the positive direction of the x-axis with initial velocity v_0 and moves with a retardation of magnitude kv^2 at any moment, where v is the velocity of the particle at this moment and k is a constant. Prove that

$$v=\frac{v_0}{1+kv_0t}=v_0e^{-kx}\ ,\ x=\frac{1}{k}\ln(1+kv_0t).$$

20. A particle moves in a straight line with acceleration equal to $\mu \div$ the n^{th} power of the distance from a fixed point O in the straight line. If it be projected towards 0, from a point at a distance a, with the velocity it would have acquired in falling from infinity, show that it will reach 0 in time

$$\frac{2}{n+1}\sqrt{\frac{n-1}{2\mu}}\,a^{\frac{n+1}{2}}$$