PHYSICS
(Properties of Matter)
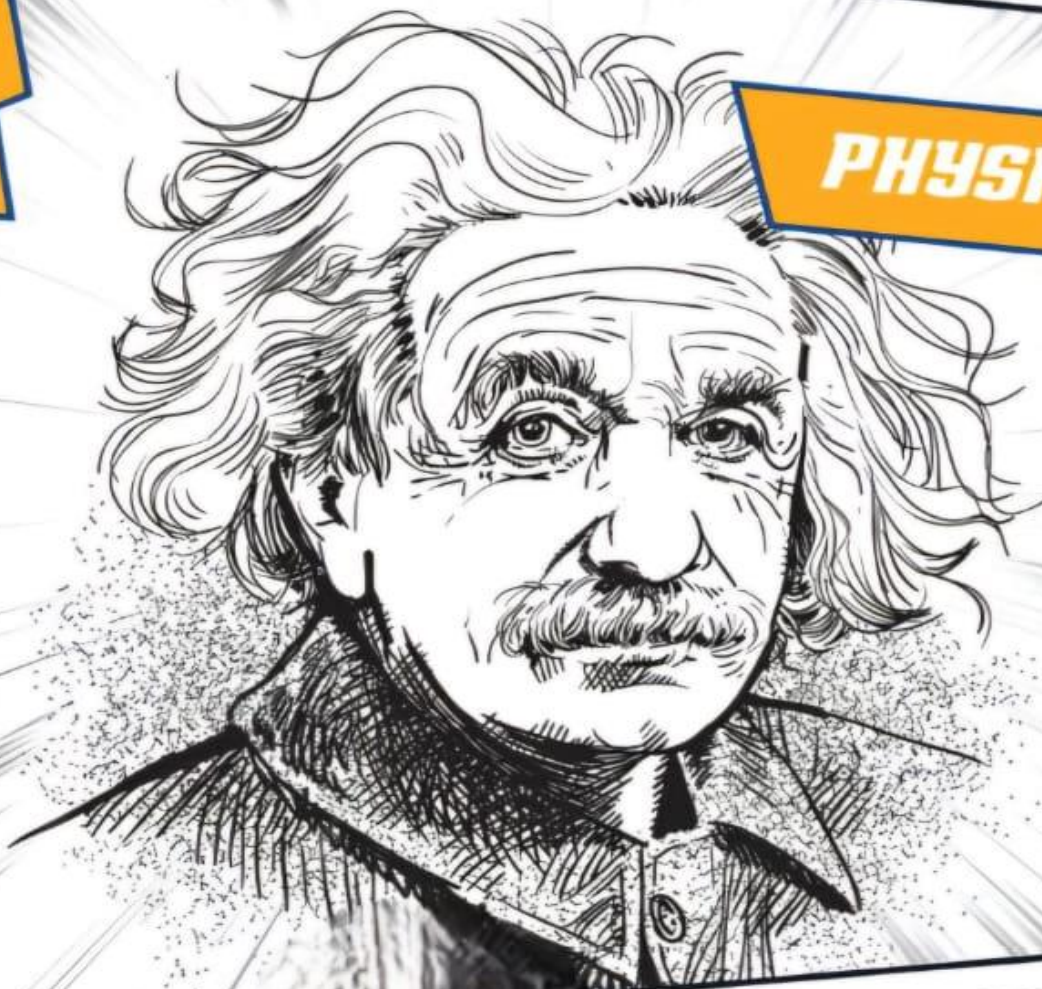### Moment of inertia

2024

NO. **6**

1ST YEAR

PHYSICS DEP,

A.ELSHABASY

DR / A.ELSHABASY

Whatsapp & Group

HI MARVEL, WE HAVE SUPER HEROS TO

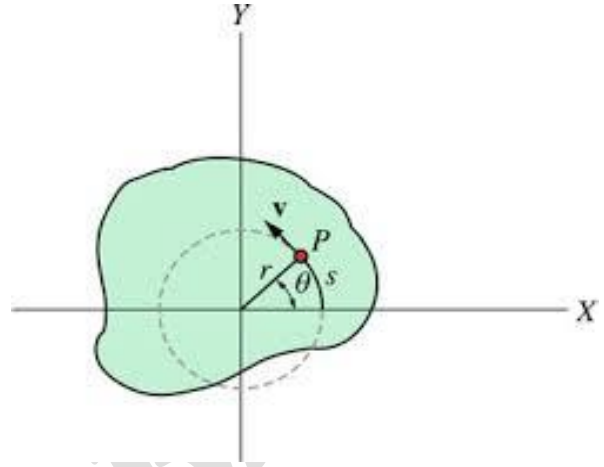## Rotation of rigid bodies

$$\theta = \frac{s}{r} \qquad s = \theta r$$

arc length   angular position

$$\theta(rad) = \frac{\pi}{180}\,\theta(deg)$$

angular velocity (ω):

▶ The ratio of the angular position of rigid obj to the time interval

$$\omega = \frac{d\theta}{dt}$$

angular acceleration (α):

▶ The ratio of change in angular speed to time interval:

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

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Quantities	translational motion	Rotation motion
Position: x	s	θ
Speed: v	$v = \dfrac{ds}{dt}$	$\omega = \dfrac{d\theta}{dt}$
Acceleration: a	$a = \dfrac{dv}{dt}$	$\alpha = \dfrac{d\omega}{dt}$

Kinetic energy:

$$k.E_t = \frac{1}{2}mv^2$$

Let us consider an object rotates about a fixed z-axis with angular speed (ω) where the object located at distance (r) if the mass (m) and tangential speed (v) then. Now → Every particle in rigid obj has same (w) but No the same (v).

▶ **The tangential speed depends on the distance (r):**

$$v = \omega r$$

$$k_R.E = \Sigma_{i=1}\frac{1}{2}\,m_i v_i^2 = \Sigma_i \frac{1}{2}m_i r_i^2 \omega_i^2$$

$$k.E_R = \Sigma \frac{1}{2}\,I\,\omega^2$$

Rotational kinetic energy.

• Where I → moment of inertia

$$I = \Sigma_i\,m_i r_i^2$$

Moment of inertia (I)

▶ is a measure of the resistance of object to change in its rotational motion or its translational motion

مقياس لمقاومة الجسم للتغير في حالته الدورانية حول محور معين

$$I = \int r^2 \, dm$$

Note that

Every particle move around different axes we use I_G which passes through center of gravity

$$I_G = Mk^2$$

القصور الذاتي عند مركز الجسم   **Raduis of gyration**

Parallel-axis Theorem:

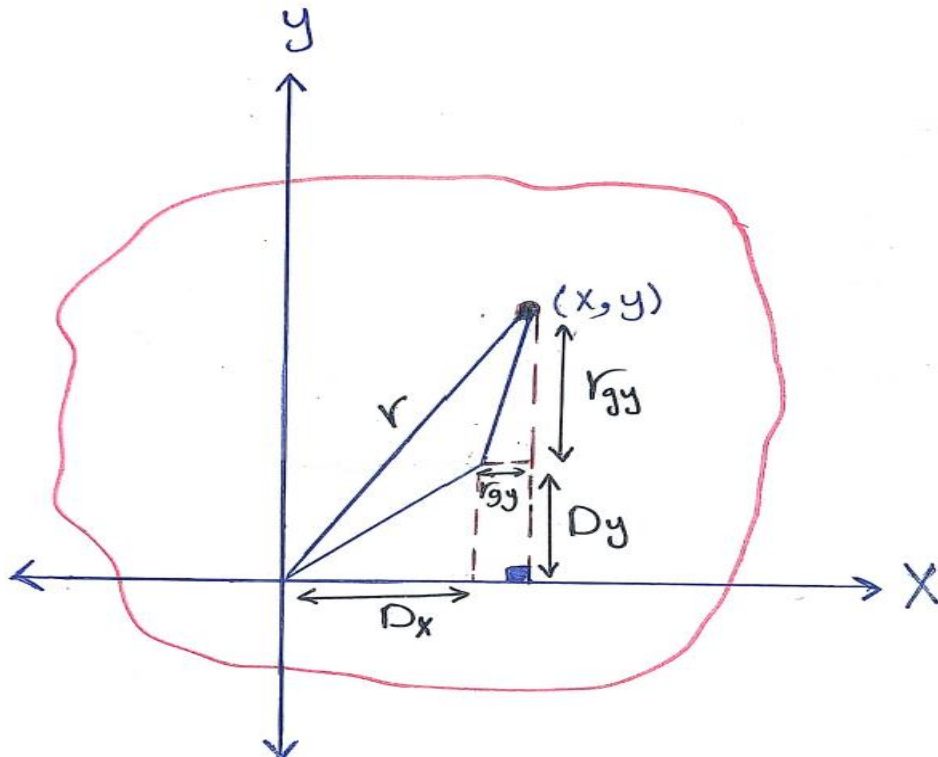▶ The calculation of moment of inertia about axis

$$I = I_G + MD^2$$

العزم عند المركز   **(D)** العزم عند نقطة

D → distance between center and point we need.

Prove of parallel-axis Theorem:

$$r^2 = r_x^2 + r_y^2$$

$$r_x = r_{gx} + D_x$$

$$r_y = r_{gy} + D_y$$

$$I_\circ = r^2 dm = \left[(D_x + r_{gx})^2 + (D_y + r_{gy})^2 \right] dm$$

$$I_\circ = \left([D_x^2 + 2D_x r_{gx} + r_{gx}^2] + [D_y^2 + 2D_y r_{gy} + r_{gy}^2] \right) dm$$

$$I_\circ = (D_x^2 + D_y^2) dm + 2D_x r_{gx} dm + 2D_y r_{gy} dm + (r_{gx}^2 + r_{gy}^2) dm$$

$$I_\circ = (D_x^2 + D_y^2) dm + (r_{gx}^2 + r_{gy}^2) dm$$

$$I_\circ = MD^2 + I_G$$

Mass density

1) Linear mass density $\lambda = \frac{m}{l}$ (mass / length) λ = lambda

2) Surface mass density $\sigma = \frac{m}{A}$ (mass / area) σ = sigma

3) Volume mass density $\rho = \frac{m}{V}$ (mass / volume) ρ = rho

Methods to solve problems

1 − *Find* linear [λ] *Or Area* [σ] *Or Volume* [ρ] mass density

2 − Find dm

3 − find I$_G$

$$I_G = \int r^2 \, dm$$

ال r هى المسافة بين المركز والشريحة الانته واخدها

4 −find D

ال D هى المسافة بين المركز والنقطة العايز احسب عندها عزم القصور الزاتى
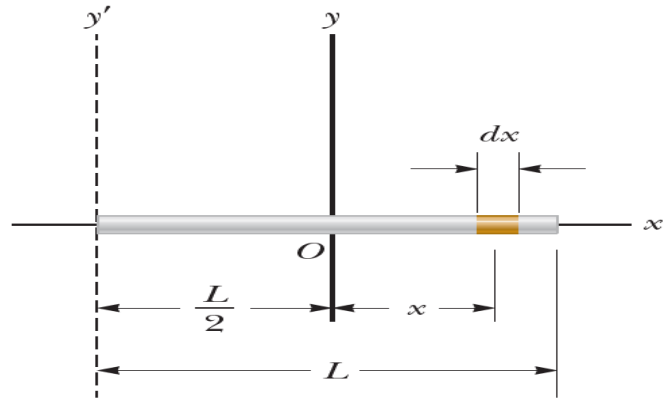
5 − applied parallel-axis theorem

$$I = I_G + MD^2$$

ملحوظة ال $I = I_G$ لو بتحسب عزم القصور الذاتى عند المركز علشان ال D حتكون بصفر

Example 1

Calculate the moment of inertia of **uniform rigid rod** of length L and mass M about an axis perpendicular to the rod through one end.

Solution

1 – linear mass density (λ)

$$\lambda = \frac{M}{L}$$

2 – Find dm

$$dm = \lambda dx$$

3 – find I_G
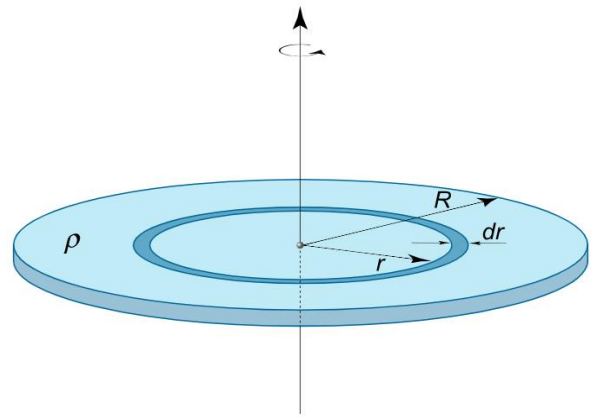
$$I_G = \int r^2 \, dM$$

$$I_G = \int x^2 \lambda dx$$

$$= \lambda \left[\frac{x^3}{3}\right]_{\frac{-L}{2}}^{\frac{L}{2}} = \frac{\lambda}{3}\left[\frac{L^3}{8} - \frac{-L^3}{8}\right] = \frac{\lambda}{3}\frac{L^3}{4} = \frac{\frac{M}{L}}{3}\frac{L^3}{4} = \frac{1}{12}ML^2$$

4 – find D

$$\therefore D = \frac{L}{2}$$

5 – applied parallel-axis theorem

$$I = I_G + MD^2 \qquad \therefore I = \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2 = \frac{ML^2}{3}$$

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Example 2

Find Moment of inertia for Solid disc

| Solution |

1 — Area mass density

$$\sigma = \frac{M}{A} = \frac{M}{\pi r^2}$$

2 — Find dm

$$M = \sigma A$$

$$dM = \sigma dA$$

$$\because A = \pi r^2$$

$$dA = 2\pi r dr$$

$$dM = 2\pi \sigma r dr$$

3 — find I$_G$

$$I_G = \int r^2 dM$$

$$= \int_{O}^{R} 2\pi \sigma r^3 dr$$

$$= 2\pi \sigma \left[\frac{r^4}{4}\right]_{O}^{R}$$

$$\mathbf{I = \frac{2\pi M R^4}{4\pi R^2} = \frac{1}{2}MR^2}$$
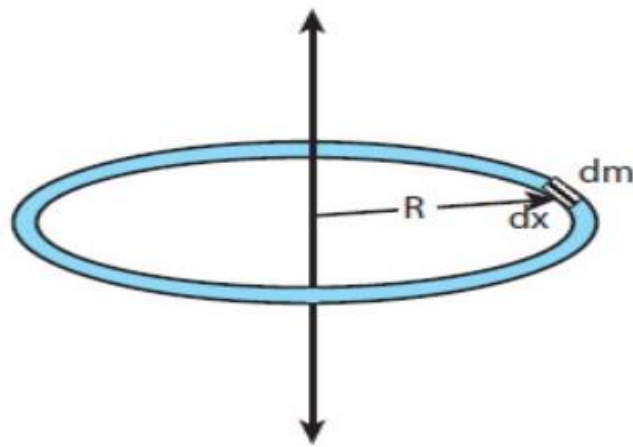
4 — find D = 0

5 — applied parallel-axis theorem $(I = I_G + MD^2)$

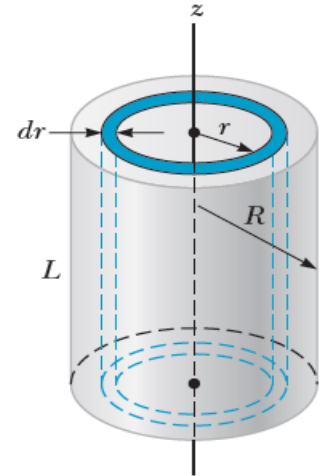$$I = I_G = \frac{2\pi MR^4}{4\pi R^2} = \frac{1}{2}MR^2$$

Example 3

Find Moment of inertia for Ring

Solution

$$I = \int r^2 dm = r^2 \int_O^M dm = r^2 m$$

Example 4

Find Moment of inertia for Solid Cylinder

Solution

1 –Volume mass density (ρ)

$$\rho = \frac{M}{V} = \frac{M}{\pi R^2 h}$$

2 –Find dm

$$\boldsymbol{dm = \rho dV}$$

$$dV = 2\pi h r dr$$

$$dm = 2\pi \rho h r dr$$

3 – find I_G

$$I_G = \int r^2 dm = \int_{O}^{R} 2\pi h \rho r^3 dr = \frac{1}{2}\pi h \rho R^4$$

4 –find $D = 0$

5 – applied parallel-axis theorem ($I = I_G + MD^2$)

$$\because \rho = \frac{M}{\pi R^2 h} \rightarrow I = \frac{1}{2}\pi h \frac{M}{\pi R^2 h} R^4$$

$$\boldsymbol{I = \frac{1}{2} M R^2}$$