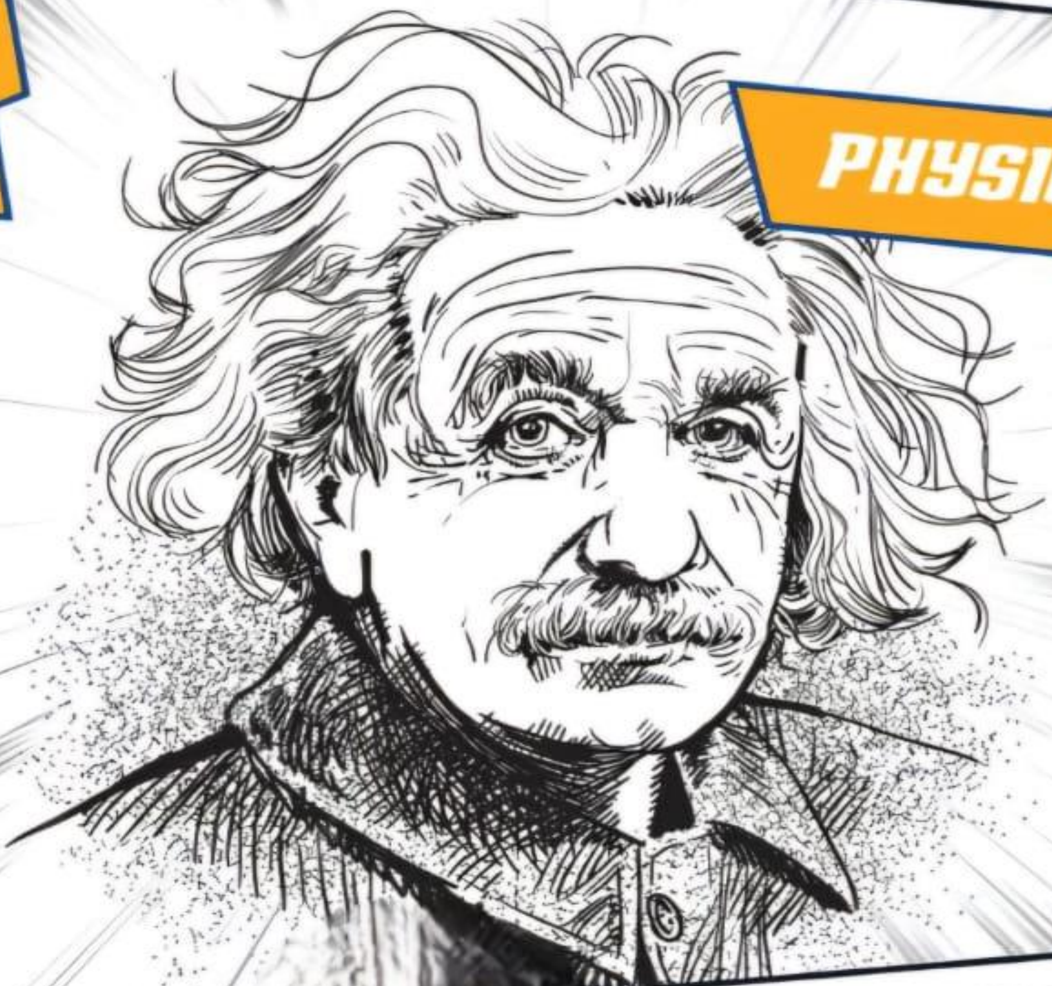
PHYSICS
(Properties of Matter)
Moment of inertia

2024

NO. 7

1ST YEAR

PHYSICS DEP.



A.ELSHABASY

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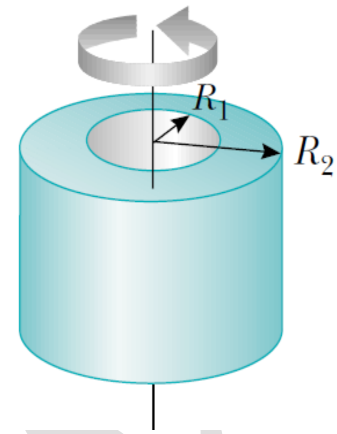
Whatsapp & Group

HI MARVEL,WE HAVE SUPER HEROS TO

Example 5

Hollow cylinder



Solution

1 – Volume mass density (ρ)

$$\rho = \frac{M}{V}, but\ now\ \rightarrow V = \pi h(R_2^2 - R_1^2)$$

2 – Find dm

$$dV = 2\pi rhdr$$

$$dM = \rho dV = 2\pi\rho hrdr$$

3 – find I_G

$$I_G = \int r^2 dm = \int_{R_1}^{R_2} 2\pi\rho hr^3 dr$$

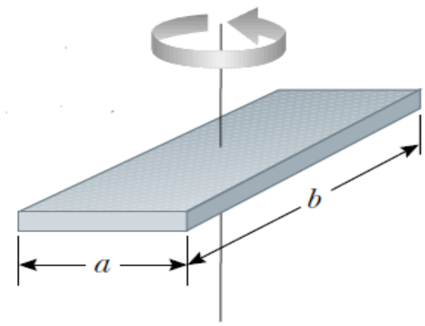
$$I_G = 2\pi\rho h\left[\frac{r^4}{4}\right]_{R_1}^{R_2} = 2\pi\rho h\frac{R_2^4 - R_1^4}{4}$$

$$\because \rho = \frac{M}{\pi h(R_2^2 - R_1^2)}$$

$$I_G = 2\pi h\frac{M}{\pi h(R_2^2 - R_1^2)}\frac{(R_2^4 - R_1^4)}{4} \rightarrow (R_1^2 + R_2^2)(R_2^2 - R_1^2)$$

$$\boldsymbol{I_G = \frac{1}{2}M(R_2^2 + R_1^2)}$$

Example 6



Rectangular Plate

Solution

1 −Area mass density

$$\sigma = \frac{M}{a.b}$$

2 −Find dm

$$dm = da\, db$$

3 − find I_G

$$I = \int r^2 dm$$

$$= \int\int (x^2 + y^2)\sigma dx dy$$

$$= \sigma \int_{-b/2}^{b/2} \left[\int_{-a/2}^{a/2} ((x^2 + y^2)\, dx \right] dy$$

$$= 4\,\sigma \int_{0}^{b/2} \left[\int_{0}^{a/2} ((x^2 + y^2)\, dx \right] dy$$

$$= 4\,\sigma \int_{0}^{b/2} \frac{x^3}{3} + y^2 x]_{\mathbf{0}}^{a/2}\, dy$$

$$= 4\,\sigma \int_0^{b/2} \left(\frac{a^3}{24} + y^2\,\frac{a}{2}\right) dy$$

$$= 4\,\sigma \left(\frac{a^3}{24}y + \frac{y^3}{3}\,\frac{a}{2}\right|_0^{b/2}$$

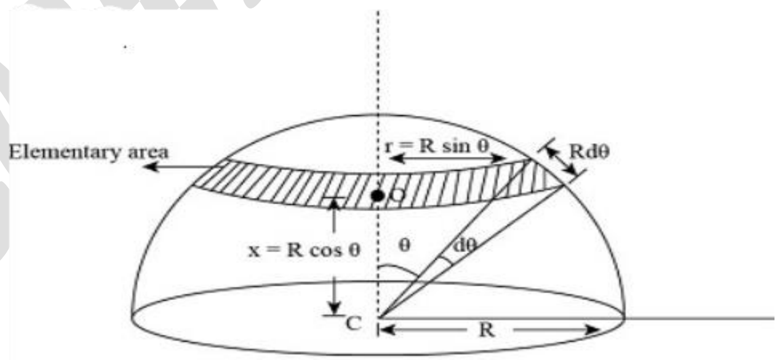
$$= 4\,\sigma \left(\frac{a^3}{24}\,\frac{b}{2} + \frac{b^3}{24}\,\frac{a}{2}\right)$$

$$= \sigma \left(\frac{a^3 b}{12} + \frac{a b^3}{12}\right) \rightarrow \sigma = \frac{M}{a.b}$$

$$\boldsymbol{I = \frac{M(a^2 + b^2)}{12}}$$

Example 7

Spherical shell



Solution

1 –Area mass density

$$\sigma = \frac{M}{A}$$

2 –Find dm

$$dA = 2\pi R sin\theta (R d\theta)$$

$$\boldsymbol{dA \;=\; 2\pi R^2 sin\theta \; d\theta}$$

احفظ قيمتها وتعالى اسئلنى جت منين

$$dm = \sigma\, dA$$

$$dm = \sigma 2\pi R^2 \sin\theta\, d\theta$$

3 – find I_G

$$dI = r^2 dm = (R\sin\theta)^2 \sigma 2\pi R^2 \sin\theta\, d\theta$$

$$dI = \sigma\, 2\pi R^4 \sin^3\theta\, d\theta$$

$$I = 2\pi R^4 \sigma \int_0^{\pi} \sin^2\theta \sin\theta\, d\theta$$

$$I = (2\pi R^4 \sigma) \int_0^{\pi} (1 - \cos^2\theta)\sin\theta\, d\theta$$

$$I = (2\pi R^4 \sigma) \int_0^{\pi} (\sin\theta - \sin\theta\cos^2\theta)\, d\theta$$

$$I = (2\pi R^4 \sigma)\left[-\cos\theta + \frac{\cos^3\theta}{3}\right]_0^{\pi}$$

$$= (2\pi R^4 \sigma)\left[-(-1) + \frac{(-1)^3}{3} - (1 + \frac{1}{3})\right]$$

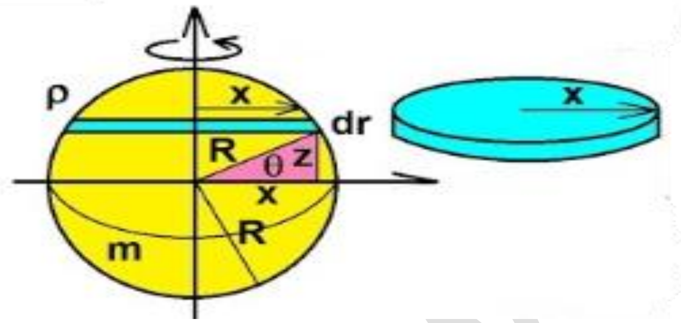
$$\therefore I = \frac{2\pi}{3}[4\sigma\pi R^4]R^2$$

$$\because \sigma = \frac{M}{4\pi R^2}$$

$$\boldsymbol{I = \frac{2}{3} M R^2}$$

Example 8

Solid sphere



Solution

1 −Volume mass density (ρ)

$$\rho = \frac{M}{\frac{4}{3}\pi R^3}$$

2 −Find dm

$$dV = 4\pi r^2 dr$$

$$dm = \rho dV = 4\pi\rho r^2 dr$$

3 − find I_G

$$dI = \frac{2}{3} r^2 dm$$

$$dI = \frac{2}{3} r^2 (4\pi\rho r^2) dr = \frac{2}{3}(4\pi)\rho r^4 dr$$

$$I = \int_0^R \frac{8\pi\rho}{3} r^4 dr = \frac{8\pi\rho}{3}\left[\frac{r^5}{5}\right]_0^R$$

$$\sigma = \frac{M}{\frac{4}{3}\pi R^2}$$

$$\therefore I = \frac{8\pi}{3} = \frac{M}{\frac{4}{3}\pi R^2} = \frac{R^5}{5} = \frac{2}{5} MR^2$$

Summary

Shape	Moment of inertia
① Rod	$\rightarrow \frac{1}{12}ML^2$ $\rightarrow \frac{1}{3}ML^2$
② Solid disc	$\frac{1}{2}MR^2$
③ Ring	MR^2
④ Solid Cylinder	$\frac{1}{2}MR^2$
⑤ Hallow cylinder	$\frac{1}{2}M(R_1^2+R_2^2)$
⑥ Rectangular Plate	$I=\frac{M(a^2+b^2)}{12}$
⑦ Spherical shell	$I=\frac{2}{3}MR^2$
⑧ Solid sphere	$\frac{2}{5}MR^2$