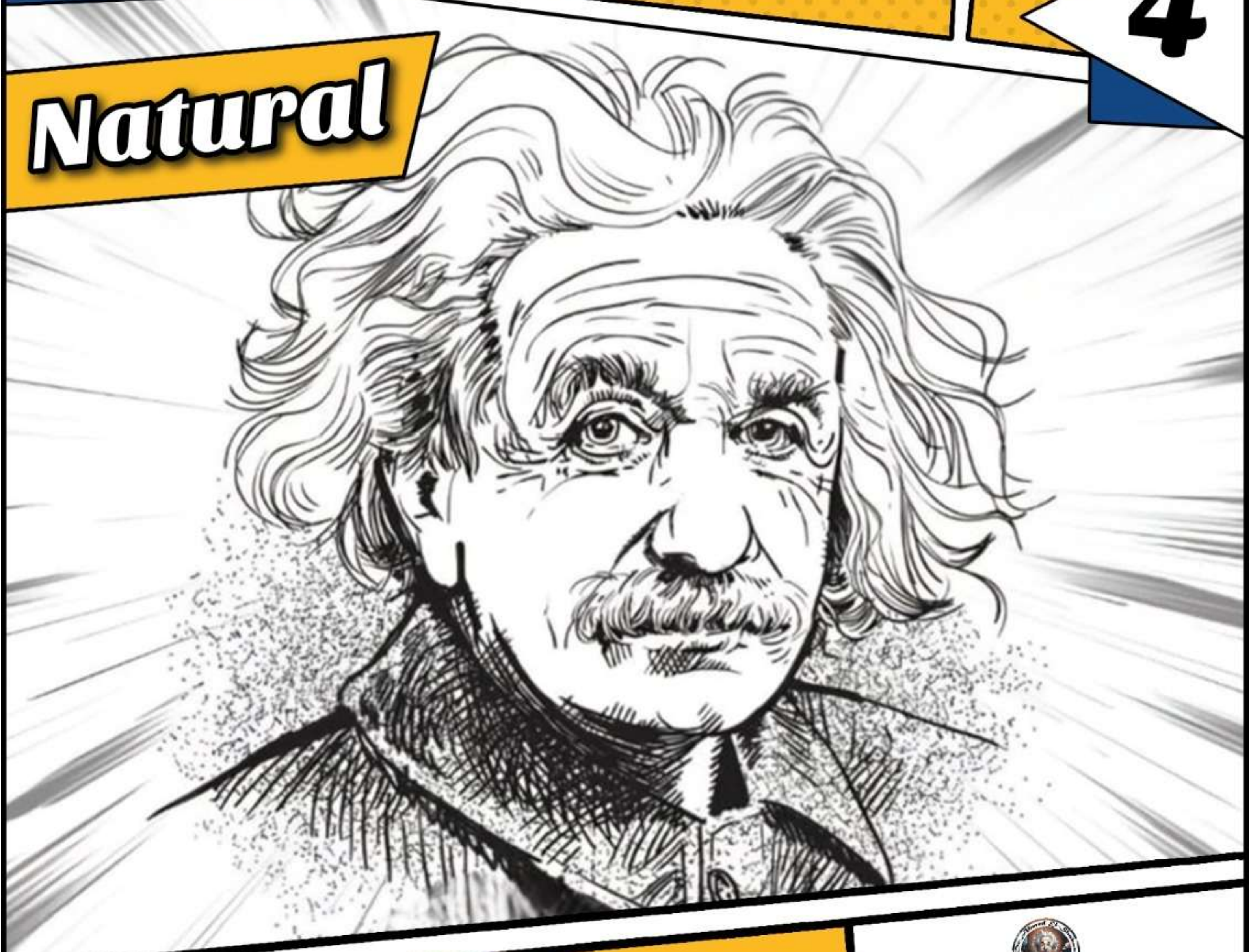
Dr/A.ElShabasy physics

2025

NO 4

Natural



Properties of Matter (Static Dynamic and Viscosity)

Physics 1 Dr.Elsahbasy

WhatsApp group

Fluids

Fluids

- materials that can flow, and they include both **gases** and **liquids**.

Fluids we study

1. Statics

☞Steady flow

- The flow is said to be **steady**, or **laminar**, if each particle of the fluid follows a smooth path such that the paths of different particles never cross each other
- The path taken by a fluid particle under steady flow is called a **streamline**
- The velocity of the particle is always **tangent to the streamline**

☞Dynamic

- dynamics when the fluid is in motion
- Continuity - Bernoulli – viscosity - Poiseuille

Unsteady flow (Turbulent)

- An extreme kind of flow in which the velocity of the fluid particles at a point change in both magnitude and direction

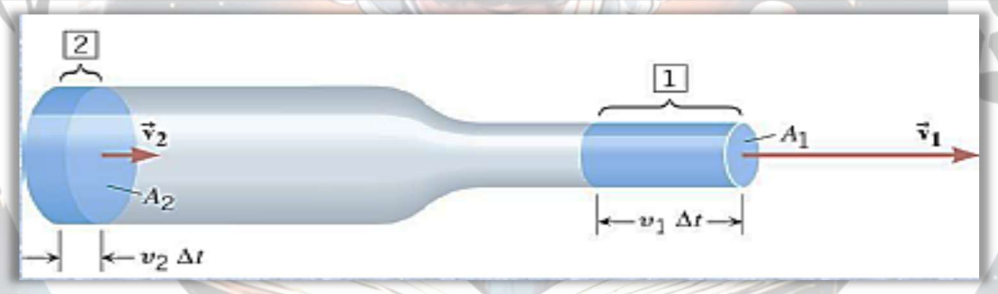
- The streamlines intersect

Steady flow fluids:

In the model of ideal fluid flow

1) **The fluid is non-viscous:** the internal friction is neglected
2) **The flow is steady** in steady (laminar) flow, all particles passing through a point have the same velocity
3) **The fluid is incompressible**, the density of an incompressible fluid is constant

The Equation of Continuity (mass conservation):



☞ If a fluid enters one end of a pipe at a certain rate, then fluid must also leave at the same rate

☞ The equation of continuity mass conservation law

$$\because m_1 = m_2$$

$$\because m = \rho . V$$

$$\therefore \rho_1 V_1 = \rho_2 V_2$$

$$\because V = A\ dx$$

$$\therefore \rho_1 . A_1 . dx_1 = \rho_2 . A_2 . dx_2$$

$$\because v = \frac{dx}{\Delta t}$$

$$\because dx = v . \Delta t$$

$$\therefore \rho_1 . A_1 . v_1 . \Delta t_1 = \rho_2 . A_2 . v_2 . \Delta t_2$$

$$\because \rho_1 = \rho_2 = \rho$$

$$\because \Delta t_1 = \Delta t_2 = \Delta t$$

$$\boldsymbol{A_1 . v_1 = A_2 v_2 = Q_v}$$

volume flow rate (Q_v)

The volume of fluid per unit time flows through a tube is called the

$$Q_V = \frac{V}{\Delta t} = \frac{A . \Delta X}{\Delta t} = Av$$

$$\boldsymbol{Q_v = \frac{V}{\Delta t} = A_1\, v_1 = A_2\, v_2 = constant}$$

Example

Water flows uniformly through a pipe with a diameter of 100 cm at one end at a speed of 10 m\s. If the diameter of the other end is 200 cm

Find: a) **Fluid flow rate**

b) **The speed of the fluid flow**

Answer

$$Q = A_1 v_1$$

- $A_1 = \frac{\pi\,(1\text{m})^2}{4} \approx 0.785\, m^2$
- $v_1 = 10\, m/s$

$$Q \approx 0.785 \times 10 \approx 7.85\, m^3/s$$

$$A_1 v_1 = A_2 v_2 \Rightarrow v_2 = \frac{A_1 v_1}{A_2}$$

- $A_2 = \frac{\pi\,(2\text{m})^2}{4} \approx 3.142\, m^2$

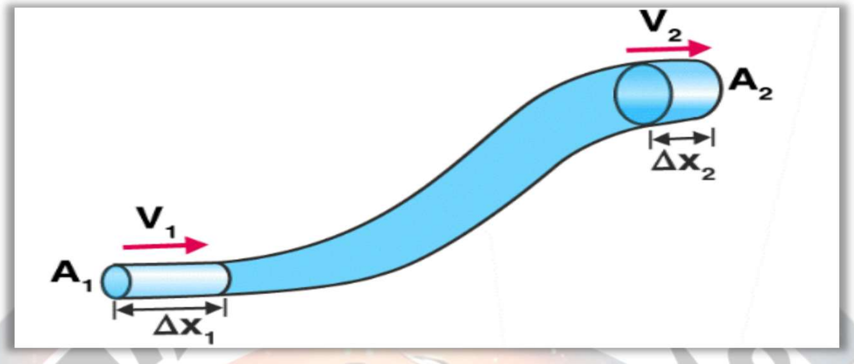
$$v_2 = \frac{0.785 \times 10}{3.142} \approx 2.5\, m/s$$

Example

A water pipe has a diameter of $10cm$ at point A and $5cm$ at point B. The speed of water at point A is $2\, m/s$. Calculate the speed of water at point B.

Answer

Bernoulli's Equation (Energy conservation)



- If we have a non-viscous liquid flowing through a tube of irregular cross-section, so its velocity changes from section 1 to section 2.

The amount of work done to move the fluid a distance ΔX_1 :

$$\boldsymbol{W = F\,\Delta X}$$

$$W_1 = P_1 A_1 \Delta X_1$$

$$\because A_1 \Delta X_1 = V$$

$$W_1 = P_1\,V$$

The amount of work done to move the fluid a distance ΔX_2 :

$$W_2 = P_2 A_2 \Delta X_2$$

$$W_2 = P_2\,V$$

$$\because \Delta W = W_1 - W_2$$

$$\boldsymbol{\Delta W = (P_1 - P_2)V} \rightarrow 1$$

$$\because \Delta W = \Delta K.E + \Delta P.E \rightarrow 2$$

Where:

Change in kinetic energy ΔK.E → $\because \boldsymbol{K.E = \frac{1}{2}mv^2}$

$$\because \boldsymbol{\Delta K.E = K.E_2 - K.E_1 = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2} \rightarrow 3$$

Change in potential energy of the fluid ΔP.E → $\because \boldsymbol{P.E = mgh}$

$$\because \boldsymbol{\Delta P.E = mgh_2 - mgh_1} \rightarrow 4$$

من المعادلات 1،2،3،4

$$\because \boldsymbol{\Delta W = \Delta K.E + \Delta P.E}$$

$$\therefore (P_1 - P_2)V = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgh_2 - mgh_1$$

بقسمة طرفي المعادلة على V

$$\therefore \boldsymbol{(P_1 - P_2) = \frac{1}{2}\frac{m}{V}v_2^2 - \frac{1}{2}\frac{m}{V}v_1^2 + \frac{m}{V}gh_2 - \frac{m}{V}gh_1}$$

$$\because \rho = \frac{m}{V}$$

$$\therefore \boldsymbol{(P_1 - P_2) = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho gh_2 - \rho gh_1}$$

$$\therefore \boldsymbol{P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2}$$

- ***In horizontal tube*** (الانبوبة الافقية) $h_1 = h_2$

Example

Water flows through a horizontal pipe at a speed of $3m/s$ and pressure of $200.000\ Pa$. If the diameter of the pipe is reduced to half, what is the pressure at the narrower section? (Assume incompressible flow.)

Answer

1. **Calculate the cross-sectional areas:**

$$let = d_1 = d, then\ d_2 = \frac{d}{2}$$

$$A_1 = \frac{\pi d^2}{4}, A_2 = \frac{\pi\left(\frac{d}{2}\right)^2}{4} = \frac{\pi d^2}{16}$$

2. **Apply the continuity equation:**

$$A_1 v_1 = A_2 v_2 \Rightarrow v_2 = \frac{A_1 v_1}{A_2} = \frac{4A_2 v_1}{A_2} = 4v_1 = 4 \times 3 = 12\ m/s$$

3. **Apply Bernoulli's equation:**

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

$$200.000 + \frac{1}{2}(1000)(3^2) = p_2 + \frac{1}{2}(1000)(12^2)$$

$$200.000 + 4.500 = p_2 + 72.000$$

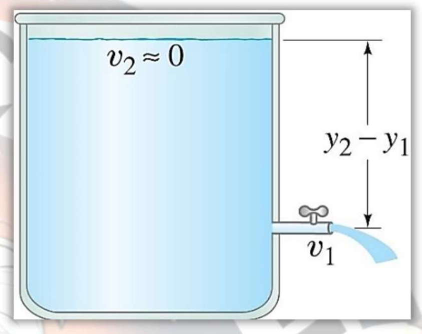
$$p_2 = 200.000 + 4.500 - 72.000 = 132.500\ Pa$$

Applications of Bernoulli's equation

1. Liquid flows from a tank that has a side opening at the bottom

(Torricelli's Theorem)

✶	$V_2 > V_1$
✶	$P_1 = P_2$
✶	$A_1 > A_2$



From Bernoulli's equation

$$\therefore P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

بقسم المعادلة على ρ

$$\therefore \frac{P_1}{\rho} + \frac{1}{2}v_1^2 + g h_1 = \frac{P_2}{\rho} + \frac{1}{2}v_2^2 + g h_2$$

$$\because P_1 = P_2$$

$$h_1 = h \ , \ \ h_2 = 0$$

$$v_1 = 0$$

$$\therefore gh = \frac{1}{2}v_2^2$$

$$v_2 = \sqrt{2gh}$$

Example 1

A tank with water has a hole at a height of *2* meters from the bottom. The height of the water surface above the hole is *8* meters. Find the speed of water as it exits the hole.

Answer

- **Using Torricelli's Theorem:**

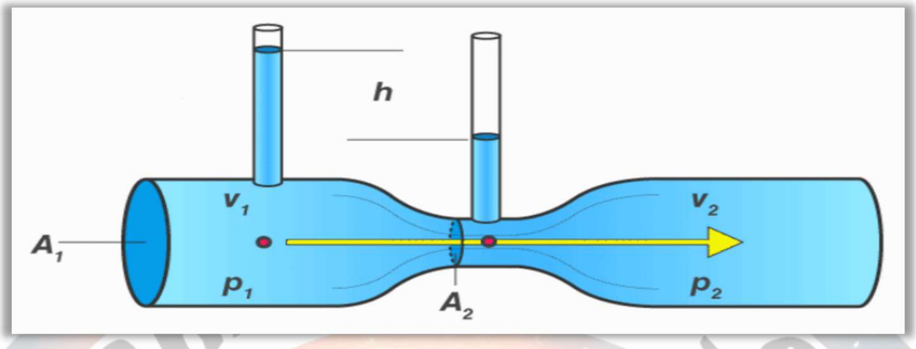
$$v = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 8} \approx \sqrt{165.96} \approx 12.53\, m/s$$

Example 2

A tank is filled with water to a height of *10* meters. If a hole is made *3* meters above the bottom of the tank, calculate the speed of water as it exits the hole.

Answer

The Venturi Meter



$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

$$h_1 = h_2$$

$$P_1 - P_2 = \frac{1}{2}\rho (v_2^2 - v_1^2)$$

$$\rho g h = \frac{1}{2}\rho (v_2^2 - v_1^2)$$

$$v_2^2 - v_1^2 = 2 g h$$

$$v_1^2\left(\frac{v_2^2}{v_1^2} - 1\right) = 2gh$$

$$A_1 v_1 = A_2 v_2$$

$$\frac{v_2}{v_1} = \frac{A_1}{A_2}$$

$$\boxed{v_1 = \sqrt{\frac{2 g h}{\left(\frac{A_1}{A_2}\right)^2 - 1}}}$$

Example 3

In a Venturi meter, the diameter of the wide section is $0.15m$, and the diameter of the narrow section is $0.07m$. The height difference in the manometer is $0.3m$. Find the velocity v_1 in the wide section. Assume $g = 9.81m/s^2$.

Answer

1. Calculate the area ratio:

$$A_1 = \frac{\pi(0.15)^2}{4} \approx 0.0177m^2,\ A_2 = \frac{\pi(0.07)^2}{4} \approx 0.00385m^2$$

$$\frac{A_1}{A_2} = \frac{0.0177}{0.00385} \approx 4.6$$

2. Calculate the velocity v_1:

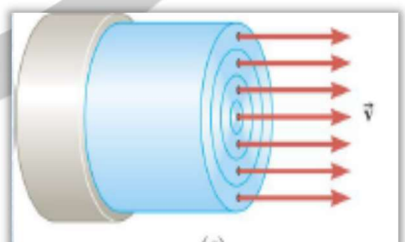
$$v_1 = \sqrt{\frac{2\ .\ 9.81.\ 0.3}{4.6^2 - 1}} \approx \sqrt{\frac{5.886}{21.16 - 1}} \approx \sqrt{\frac{5.886}{20.16}} \approx \sqrt{0.291} \approx 0.54m/s$$

Viscosity

- resistance of a fluid (liquid or gas) to a change in shape, or movement of neighboring portions relative to one another.

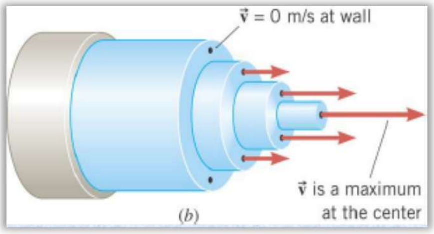
☞Flow of an ideal fluid

- *No viscosity to hinder the fluid layers*
- *Every layer of an ideal fluid moves with the same velocity, even the layer next to the wall*

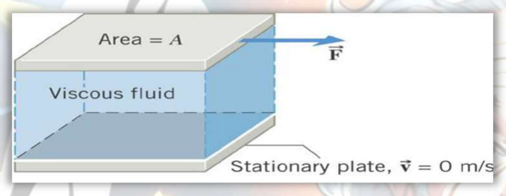


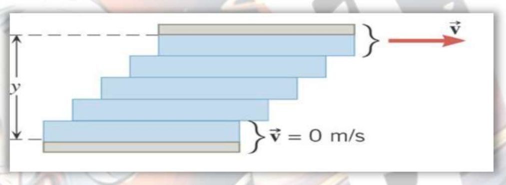
(a)

☞Flow of a viscous fluid



- *Fluid layers have different velocities*
- *The fluid at the center of the pipe has the greatest velocity*
- *the fluid layer next to the wall surface does not move at all, because it is held tightly by intermolecular forces.*
- *Consider a viscous fluid between two parallel plates The top plate is free to move and the bottom one is stationary*





- *When the top plate moves the intermediate fluid layers can slide over each other The velocity of each layer is different, changing uniformly from at the top plate to zero at the bottom plate.*

the tangential force

The magnitude of the tangential force required to move a fluid layer at a constant speed is given by:

$$F = \eta \frac{A.v}{\mathbf{d}}$$

$$\eta = \frac{Fd}{Av}$$

Where :

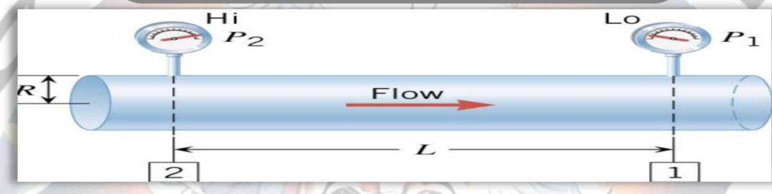
η → *coefficient of viscosity* $(kg \setminus m.s)$

F → *tangential force*

d → *distance d between the two plates*

$$1\ Poise = 10^{-1}\ N.\frac{s}{m^2}$$

Poiseuille's law



$$Q_v = \frac{\pi r^4\,(P_2 - P_1)}{8\eta L} = \frac{\pi r^4(\Delta P)}{8\eta L}$$

Prove that

$$F = \frac{\eta\, A\, dv(r)}{dr}$$

$$F = \frac{2\,\eta\,\pi\,r\,L\,dv(r)}{dr}$$

$$F = (P_1 - P_2)A = (P_1 - P_2)\pi r^2$$

$$(P_1 - P_2)\pi r^2 = \frac{2\,\eta\,L\,\pi\,r\,dv}{dr}$$

$$dv = \frac{(P_1 - P_2)\,r\,dr}{2\eta L}$$

$$\int dv = \int_0^r \frac{\Delta P r}{2\eta L}\,dr$$

$$v(r) = \frac{\Delta P}{2\eta L} \times \frac{r^2}{2} = \frac{\Delta P}{4\eta L} r^2$$

$$dQ_V = v(r)dA$$

$$\int dQ_V = \int_0^r \frac{\Delta P}{4\eta L}\ 2\pi r^3\ dr$$

$$Q_V = \frac{\Delta P \pi}{2\eta L} \int_0^R r^3\ dr$$

$$\boxed{Q_V = \frac{\Delta P\ \pi R^4}{8\eta L}}$$

Stock's law

$$F = K\ r^a v^b \eta^c$$

$$\left(MLT^{-2}\right) = L^a\left(LT^{-1}\right)^b\left(ML^{-1}M^{-2}\right)^c$$

$$\left(MLT^{-2}\right) = L^{a+b-c}\ M^c\ T^{-b-c}$$

$$C = 1$$

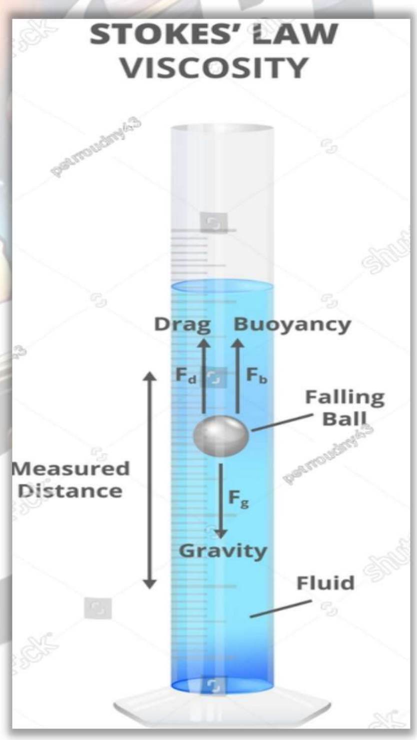
$$-2 =- b - C$$

$$b = 1$$

$$1 = a + b - C$$

$$\mathrm{a} = 1$$

$$F = Krv\eta$$



$$K = 6\pi$$

$$\boxed{\therefore\ F = 6\pi\ \eta\ vr}$$

where:

- ***F*:** Magnitude of the force (in Newtons, N).
- ***η*:** Coefficient of viscosity of the fluid (in Pascal-seconds, Pa.s or N.s/m²).
- ***r*:** Radius of the sphere (in meters, m).
- ***v*:** Speed of the sphere (in meters per second, m/s).

$$\eta = \frac{2}{9}\frac{r^2}{v}(\rho_s - \rho_L)g$$

Where (r) is the radius of iron bal.

- **(v)** is the iron ball velocity.
- **(g)** is the acceleration due to gravity.
- **(ρ_s)** is the density of iron
- **(ρ_L)** is the density of liquid.

Example

A sphere falls inside a viscous fluid for a time of 12 seconds. The viscosity of the fluid is 1.5 poise, the density of the sphere is $2.5 g/cm^3$, and the density of the fluid is $1.0 g/cm^3$. Calculate the distance traveled by the sphere during this time.

Answer

Example

A bubble rises in a liquid with a viscosity of 0.9 poise and a density of $0.8 g/cm^3$. If the bubble travels 150 cm in 15 seconds, calculate the radius of the bubble.

Answer

Reynolds number N_R

It is the number that determines if the flow is turbulent or laminar and it can be given as:

$$\boxed{N_R = \frac{\rho v d}{\eta}}$$

- ρ: Density of the fluid
- v: Average speed of the fluid along the direction of flow
- d: Diameter of the tube
- η: Viscosity of the fluid

Reynolds Number (RN)	Flow Type	Description
$R_N < 2000$	Streamline Flow (Laminar)	Smooth and orderly flow.
$2000 < R_N < 3000$	Unstable Flow	Can be streamline, but sensitive to disturbances.
$R_N > 3000$	Turbulent Flow	irregular flow.