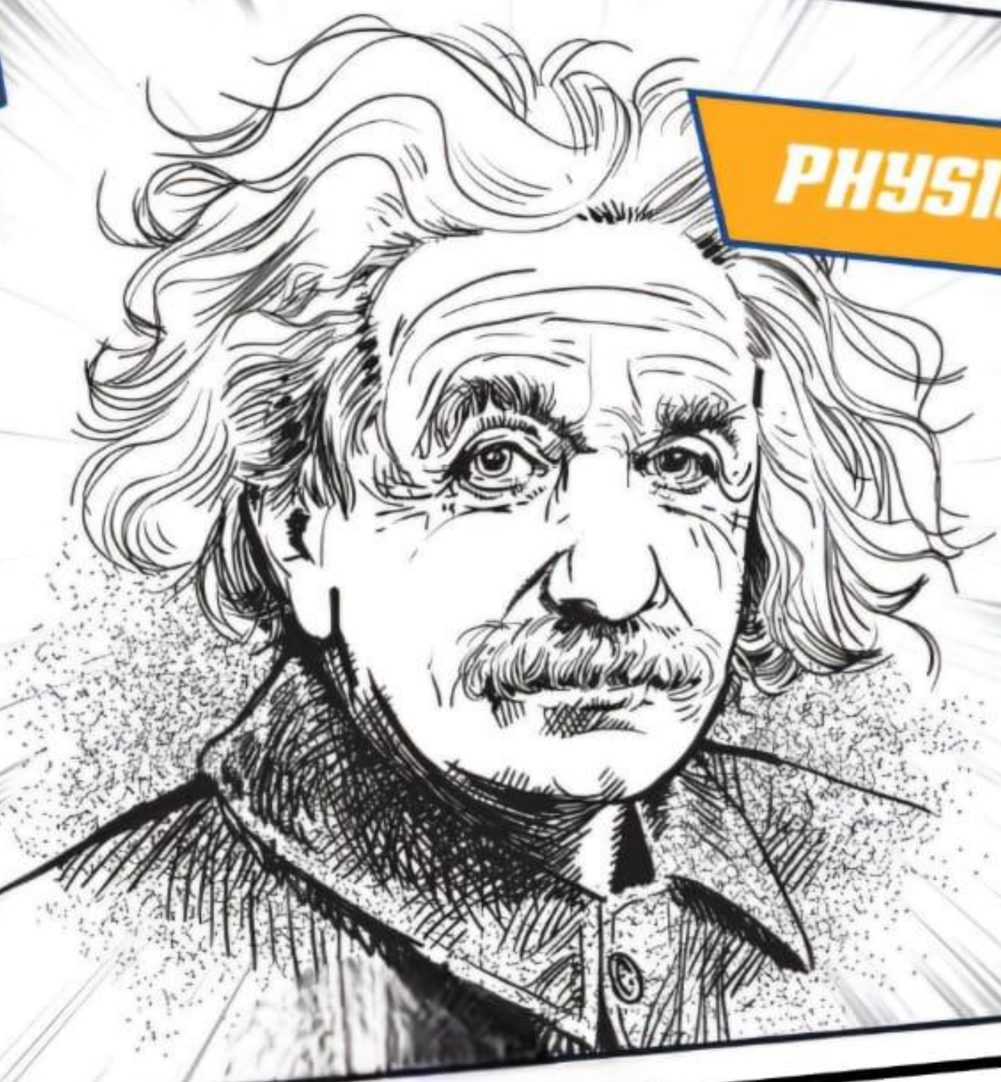
PHYSICS
(Properties of Matter)
Simple harmonic Motion (Last part)

2024

NO. 8

1ST YEAR

PHYSICS DEP.



تم بحمد الله انهاء المنهج بالتوفيق

A.ELSHABASY

DR/ A.ELSHABASY

Whatsapp & Group

HI MARVEL, WE HAVE SUPER HEROS TO

Simple harmonic Motion

Periodic Motion:

- It's Motion of object repeated regularly after Fixed time interval

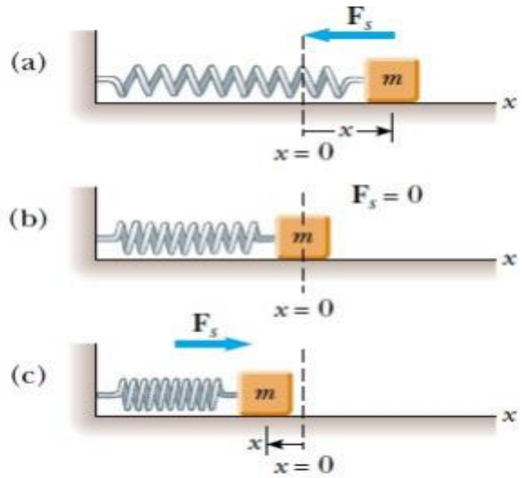
Examples:

1. **Motion of earth around sun.**
2. **Motion of Moon around earth.**

Simple harmonic Motion

- is a type of oscillatory motion where an object moves back and forth around a stable equilibrium position.

Motion of object attach to spring



❶ Restoring Force (F_S)

- if displace from equilibrium Position $X\ cm$
- The Spring exert Force on block, That Proportional to Position and This Force given by **Hook's Law**

$$\because F_s \propto x$$

$$\therefore F_s = -Kx \rightarrow \textit{Restoring}$$

where:

x → Displacement From equilibrium Position.

k → (Spring Const) $(\frac{N}{m})$

- IF $x = 0$, $F_s = 0$
- if x Positive (right), F_s is to left.
- IF x Negative (left), F_s is to right

$\because$ By Applying Newton Second law

$$\because F_x = ma_x$$

$$\therefore -Kx = ma_x$$

$$\therefore a_x = \frac{-k}{m} x$$

Where:

- a_x → acceleration of block which Proportion with Position
- If $x =$ Amplitude (A)

$$\therefore a = \frac{-k}{m} A$$

Mathematical Representation of S.H.M:

$$\because a_x = \frac{-k}{m} x$$

$$\because a = \frac{d^2x}{dt^2}$$

$$\therefore \frac{d^2x}{dt^2} = -\frac{K}{m} x$$

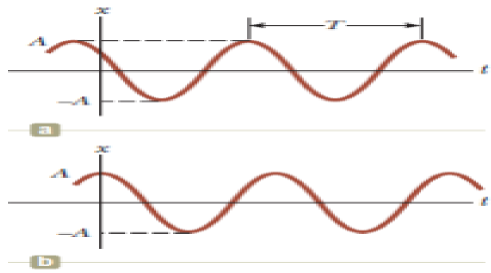
→ ***Equation of S.H.M***

$$\therefore \omega^2 = \frac{k}{m}$$

$$\therefore \frac{d^2x}{dt^2} = -\omega^2 x$$

$$\boxed{\therefore \boldsymbol{x(t) = A\cos(\omega t + \emptyset)}}$$

Where:



- A → Amplitude
- ω → Angular Velocity (rad)
- $\emptyset$ → intial Phase angle

look:

- The quantity $(\omega t - \emptyset)$ Called Phase of Motion
- The function $x(t)$ is Periodic after $t = T$ it increases by $2\pi(rad)$

$$\therefore [\omega(t + T) + \emptyset] - [(\omega t + \emptyset)] = 2\pi$$

$$\therefore \omega t + \omega T + \emptyset - \omega t - \emptyset = 2\pi$$

$$\therefore \omega T = 2\pi$$

$$\boxed{\therefore \boldsymbol{T = \frac{2\pi}{\omega}}}$$

$$\because \omega = \sqrt{\frac{k}{m}}$$

$$\boxed{\therefore \boldsymbol{T = 2\pi\sqrt{\frac{m}{k}}}}$$

Frequency of S.H.M

- Number of oscillations Per unit time

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

Linear Velocity of S.H.M

$$\because x(t) = A\cos(\omega t + \emptyset)$$

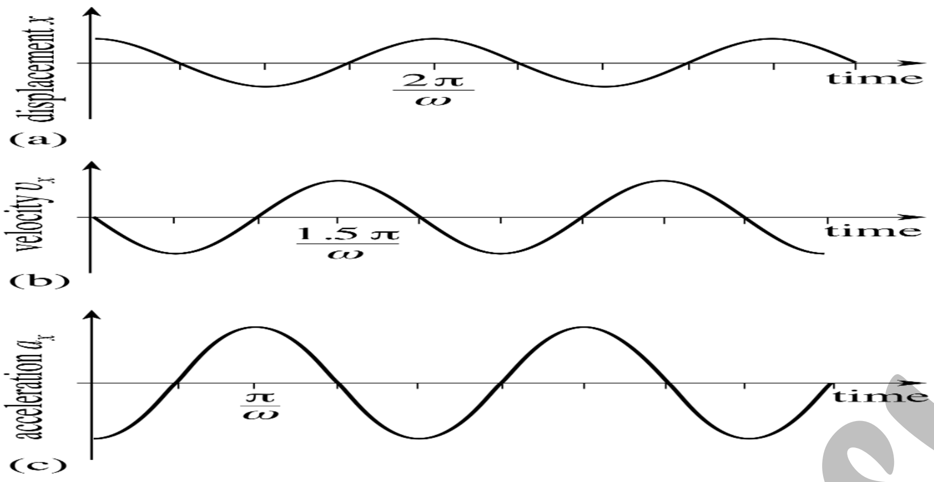
$$\because v = \frac{dx}{dt} = -A\omega \sin(\omega t + \emptyset)$$

$$\because v_{max} = A\omega = 2\pi f A = \sqrt{\frac{k}{m}}\, A$$

Linear acceleration of S.H.M

$$\because a = \frac{d^2x}{dt^2} = -Aw^2\cos(wt + \emptyset)$$

$$\because a_{max} = A\omega^2 = \frac{k}{m}A$$

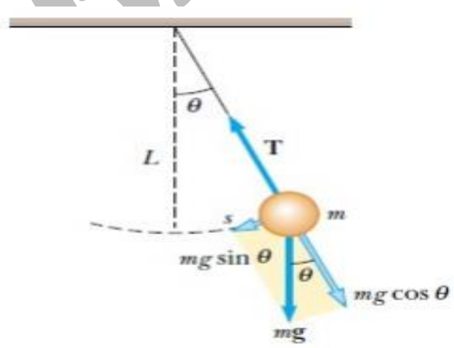


Pendulum

❶ Simple Pendulum:

- The Periodic time of simple Pendulum

$$\therefore T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$

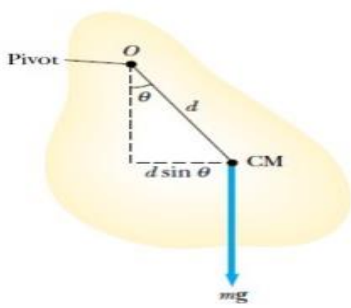


Where:

- L → length of string
- g → acceleration of gravity

❷ Compound Pendulum:

$$\because T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgd}}$$



Where:

- I → moment of inertia (kg.m^2)
- m → mass of Pendulum (kg)

Energy of simple harmonic Motion

① Kinetic energy (K.E):

$$K.E = \frac{1}{2}mv^2$$

$$K.E = \frac{1}{2}m\omega^2A^2sin^2(\omega t + \emptyset)$$

$$\because \omega^2 = \frac{k}{m}$$

$$\boldsymbol{K.E = \frac{1}{2}KA^2sin^2(\omega t + \emptyset)}$$

② Potential energy (u):

$$\boldsymbol{\therefore U = \frac{1}{2}kx^2 = \frac{1}{2}kA^2cos^2(\omega t + \emptyset)}$$

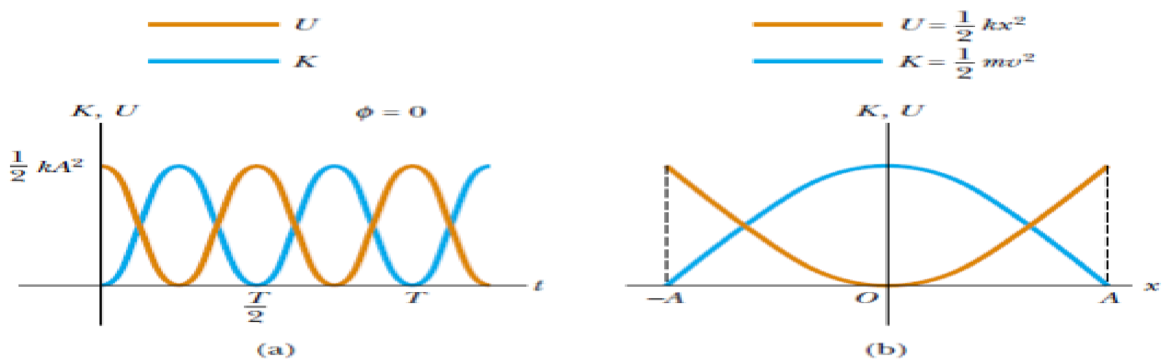
③ Total energy (E):

$$E = k.E + u$$

$$= \frac{1}{2}kA^2\{sin^2(\omega t + \emptyset) + cos^2(\omega t + \emptyset)\}$$

$$= \frac{1}{2}kA^2 = const$$

$$\boldsymbol{E = kx^2 + \frac{1}{2}mv^2 = \frac{1}{2}kA^2}$$



Velocity of S.H.M

$$\because E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}KA^2$$

$$\therefore mv^2 = k(A^2 - x^2)$$

$$\therefore v^2 = \frac{k}{m}(A^2 - x^2)$$

$$\boxed{\therefore \boldsymbol{v = \pm\sqrt{\frac{k}{m}(A^2 - x^2)}}}$$

$$\therefore v = \pm w\sqrt{A^2 - x^2)}$$

Damped oscillation

- The oscillatory Motion is not ideal System For real Cases The oscillatory Motion affected by some Forces Such as Friction, which retard Motion

$$\because \sum F_X = F_s - F_{retard}$$

$$= -kx - bv_x = ma_x$$

$$\because R_x = -bv_x$$

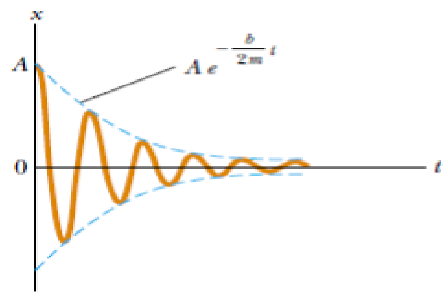
retarding Force

$$\therefore -Kx - b\frac{dx}{dt} = m\frac{d^2x}{dt^2}$$

$$\therefore x = Ae^{\frac{-b}{2m}t}\cos(\omega t + \emptyset)$$

$$\boxed{\therefore \boldsymbol{\omega} = \sqrt{\frac{\boldsymbol{k}}{\boldsymbol{m}} - \left(\frac{\boldsymbol{b}}{\boldsymbol{2m}}\right)^{\boldsymbol{2}}}}$$

Where:



- K → Spring Constant
- m → mass of block
- b → damping coefficient
- The Amplitude of oscillation decreases with time and system is called ***damped oscillator***
- if $\boldsymbol{R_m = b\,v_{max} < KA} \rightarrow$ The system Said ***under damping*** as b increase A is decreased.
- If $\boldsymbol{R_m = bV_m > KA}$ and $\boldsymbol{\frac{b}{2m} > \omega_0}$. The system Said ***over damped.***
- if $\boldsymbol{\frac{b_c}{2m} = \omega_0}$ The system does not oscillate and called ***critically damping***
- If damping increases The time taken to reach to equilibrium Position increase.
- For critically and over damping There is no Angular Frequency.
- The loses in Mechanical energy is Transformed to internal energy in object and retarding Medium