Maths Concepts & Qns

Video-5

→ codestorywithmik

→ CSwithMIK

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Motivation :-

Do you want to grow ???

↳ Come out of your comfort zone.

↳ Stop running from tough/new topics

MIK...

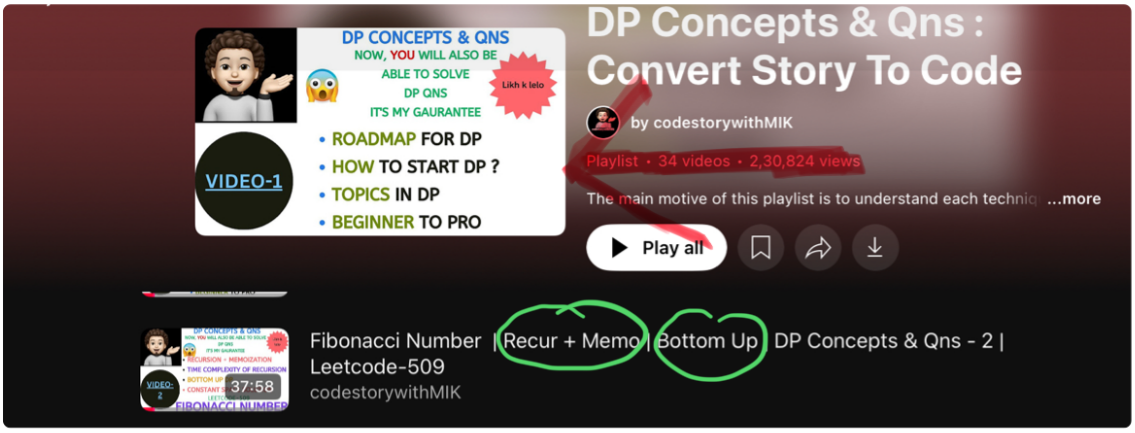
Remember, everything seems difficult in the beginning, slowly you start realising how easy it was...

Matrix Exponentiation

Powerful optimization technique, especially useful in problems involving linear recurrences.

For Example :-

$$F(n) = F(n-1) + F(n-2)$$



$$\boxed{F(n) = F(n-1) + F(n-2)}$$

Recursive soln → $T.C = O(2^n)$

Bottom up → $T.C = O(n)$

Linear Recurrence Relation

So, we can further improve our

solution from $\boxed{O(n) \text{ to } \log(n)}$

How??? :O

$$\boxed{F(n)} = \boxed{F(n-1)} + \boxed{F(n-2)}$$

$(n-2)^{th}$ $(n-1)^{th}$ $(n)^{th}$ $(n+1)^{th}$ $(n+2)^{th}$ . . .

$$\boxed{\begin{bmatrix} F(n) \\ F(n-1) \end{bmatrix}_{2*1} = T * \begin{bmatrix} F(n-1) \\ F(n-2) \end{bmatrix}_{2\times 1}}$$

$$\begin{bmatrix} F(n) \\ \end{bmatrix} = \begin{bmatrix} a*F(n-1) + b*F(n-2) \\ \end{bmatrix}$$

$$\begin{bmatrix} F(n-1) \end{bmatrix}_{2*1} \qquad \begin{bmatrix} c*F(n-1) + d*F(n-2) \end{bmatrix}_{2*1}$$

$$1(F(n-1))+F(n-2) \quad \cancel{F(n)} = a * F(n-1) + b * F(n-2) \rightarrow 1$$

$$1 * F(n-1) = c * F(n-1) + d * F(n-2) \rightarrow 2$$

$$a = 1 \quad , \quad b = 1$$

$$c = 1 \quad , \quad d = 0$$

$$T = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$\boxed{\begin{bmatrix} F(n) \\ F(n-1) \end{bmatrix}_{2*1} = T * \begin{bmatrix} F(n-1) \\ F(n-2) \end{bmatrix}_{2x1}}$$

$F(0) = 0$

$F(1) = 1$

$n = 2$

$F(n) = F(2)$??

$$\begin{bmatrix} F(2) \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} * \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

0, 1, 1, 2

n=0 n=1 n=2 n=3

$$= \begin{bmatrix} (1*1) + (1*0) \\ (1*1) + (0*0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} F(2) \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Result

Result[0][0] → 1

$$\begin{bmatrix} F(2) \\ F(1) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}}_{T} * \begin{bmatrix} F(1) \\ F(0) \end{bmatrix}$$

$$T = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$T^2 = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$$

$$T^2 * \begin{bmatrix} F(1) \\ F(0) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} * \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$n=3$

$$\begin{bmatrix} F(3) \\ F(2) \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

3rd Fibonacci nu.

$n = 3$

$$T^{2} * \begin{bmatrix} F(1) \\ F(0) \end{bmatrix}$$

$$n = 4 \qquad T^{3} * \begin{bmatrix} F(1) \\ F(0) \end{bmatrix}$$

$$n = 5 \qquad T^{4} * \begin{bmatrix} F(1) \\ F(0) \end{bmatrix}$$

Generalise:-

$$\boxed{\begin{bmatrix} F(n) \\ F(n-1) \end{bmatrix} = T^{n-1} * \begin{bmatrix} F(1) \\ F(0) \end{bmatrix}}$$

Degree Of 2

Recurrence

↳ It tells you how many previous terms it depends on.

Example :- $F(n) = \underbrace{F(n-1) + F(n-2)}_{\text{Two terms}}$

So, it's a "Degree 2" relation.

$$\begin{bmatrix} F(n) \\ F(n-1) \end{bmatrix} = T * \begin{bmatrix} F(n-1) \\ F(n-2) \end{bmatrix}$$

Example :-

$$F(n) = F(n-1) + F(n-2) + F(n-3)$$

degree = 3

$$\begin{bmatrix} F(n) \\ F(n-1) \\ F(n-2) \end{bmatrix}_{3*1} = T \begin{bmatrix} F(n-1) \\ F(n-2) \\ F(n-3) \end{bmatrix}_{3*1}$$

3*3

T

3*3

$$T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

Where is exponentiation ???

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$F(n) \longrightarrow \log(n)$?????

Remember, we had to find T^{n-1}

(Binary Exponent)

T^{n-1}

$n = 6$

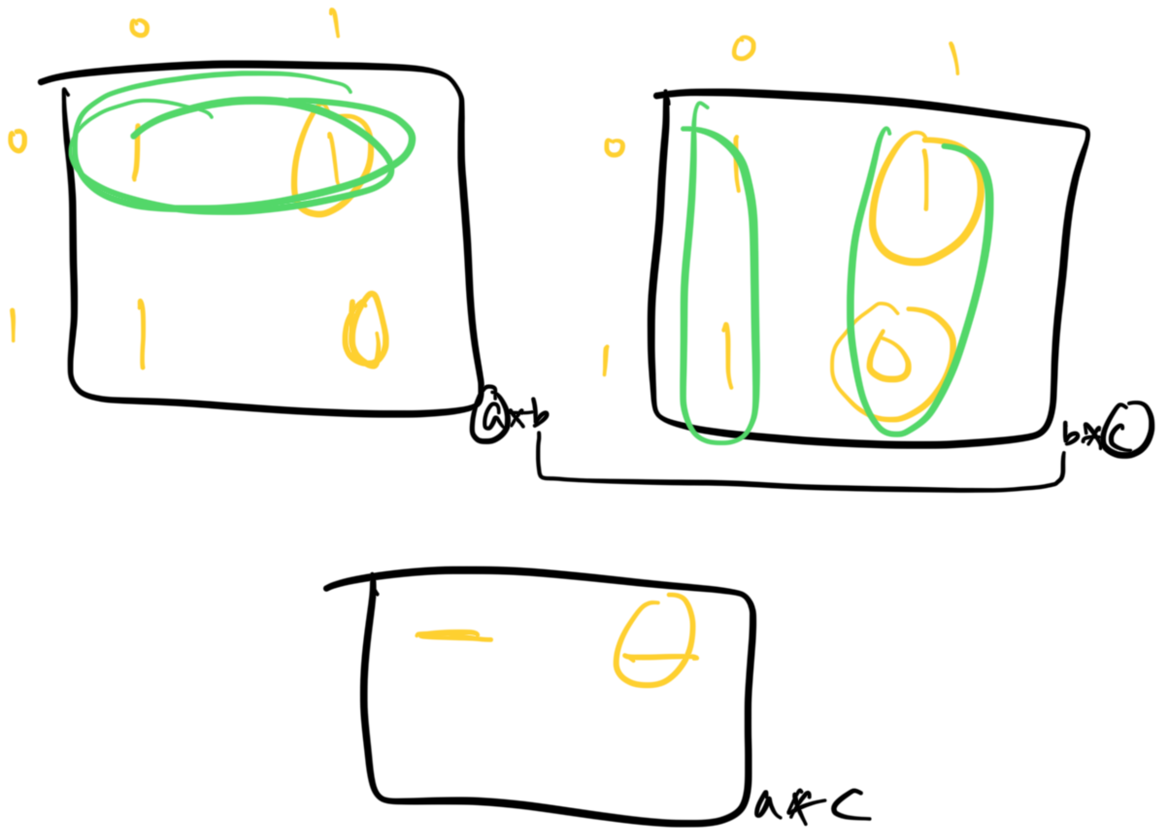
T^5

$\log(n-1) \simeq \log(n)$ $\quad \begin{bmatrix} & \\ & \end{bmatrix}_{2\times 2}$

```
int findPower(base, exponent){
    if (exponent ==0 ){
        return 1;
    }

    half = findPower(base, exponent/2);
    result = half * half;
    if (exponent % 2 ==1) {
        result =(result * base);
    }

    return result;
}
```

```
                                   T                5
Matrix matrixExp(Matrix base , int exponent){
    if (exponent ==0 ) {
        make an identity matrix I
        return I;
    }

    half =matrixExp(base, expon/2);
    result =matrixMultipli(half,half);
    if(exponent% 2 == 1){
        result =matrixmult(result,base);
    }
```

```
}

return result;
```




```
Matrix matrixMult ( Matrix A 2x2 , Matrix B 2x2 ) { //O(1)

    Matrix C 2*2 ;

    for (i =0; i<2; i++) {

        for (j=0; j<2; j++) {

            for(k=0; k<2; k++){
```

C[i][j] = (C[i][j] +
A[i][k] * B[k][j]
);

}

}

}

$n = 6$

$F(n)$

$F(6)$

$\cancel{T.C = (\log(n))}$

Solve(n)

$F(n)$

$$\begin{bmatrix} F(n) \\ F(n-1) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{n-1}}_{2 \times 2} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\text{matrixExp}(T,\ n-1) * \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

retn result[0][0] ✓

- * Matrix Exponentiation.
- * Matrix Multiplication.

P.O.T.D

<u>Lc</u> :- 3337

(.) recc. relation

(.) degree $\rightarrow T_{x**x}$