

Linear Algebra

Assignment #2

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Question

What is matrix determinant?

Answer: Matrix determinant is a special number that can be calculated in a square matrix by multiplying the diagonal entries first and then subtracting them.

→ Properties of determinant:

1. Reflection Property:

The determinant remains unchanged if rows are changed into columns and columns into rows.

Example:

$$\begin{vmatrix} 1 & 4 \\ 5 & 6 \end{vmatrix}$$

$$= (1)(6) - (4)(5) = 6 - 20 = -14$$

Now taking transpose,

$$\begin{vmatrix} 1 & 5 \\ 4 & 6 \end{vmatrix} = (1)(6) - (5)(4) = 6 - 20 = -14$$

2. All-zero property:

If element of a row or column of a matrix is zero, then the determinant will be zero.

Example:

$$\begin{vmatrix} 0 & 2 \\ 0 & 4 \end{vmatrix} = (0)(4) - (0)(2) = 0$$

3. Factor property:

If determinant becomes zero when we put $x = \alpha$, then $x - \alpha$ is a factor of Δ .

Example:-

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \Rightarrow \Delta = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$$

where C_{ij} is a cofactor of elements a_{ij} in Δ .

4. Triangle property:

If all the elements of a determinant above or below the main diagonal consists of zeros, then the determinant is equal to the product of diagonal elements.

Example:

$$\begin{vmatrix} 3 & 0 & 0 \\ 8 & 3 & 0 \\ 7 & 9 & 1 \end{vmatrix} = (3)(3) - 0 - 0 = 9$$
$$\text{or } (3)(3)(1) = 9$$

5. Proportionality property:

If all the elements of a row or column are identical to all the elements of some other row or column, then the determinant will be zero.

Example:

$$\begin{vmatrix} 1 & 1 \\ 4 & 4 \end{vmatrix} = (1)(4) - (1)(4) = 0$$

8. Sum property.

If elements of a row or column are expressed as the sum of two or more terms, then the determinant can be expressed as the sum of two or more determinants.

Example:

$$\begin{vmatrix} 2+1 & 1 & 2 \\ 3+(-2) & 0 & 5 \\ 4+2 & 3 & 7 \end{vmatrix} = 3(0-15) - 1(7-30) + 2(3-0) = -45 + 23 + 6 = -16$$

$$\begin{aligned} &= \begin{vmatrix} 2 & 1 & 2 \\ 3 & 0 & 5 \\ 4 & 3 & 7 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 2 \\ -2 & 0 & 5 \\ 2 & 3 & 7 \end{vmatrix} \\ &= [2(-15) - 1(21 - 20) + 2(9)] + [1(15) - 1(-14 - 10) + 2(6)] \\ &= [-30 - 1 + 18] + [-15 + 24 - 12] \\ &= -13 - 3 = -16 \end{aligned}$$

9. Property of invariance:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 + \alpha b_1 + \beta c_1 & b_1 & c_1 \\ a_2 + \alpha b_2 + \beta c_2 & b_2 & c_2 \\ a_3 + \alpha b_3 + \beta c_3 & b_3 & c_3 \end{vmatrix}$$

Example:

$$\begin{vmatrix} 1 & 1 & 2 \\ -2 & 0 & 5 \\ 2 & 3 & 7 \end{vmatrix} = 1(15) - 1(-14 - 10) + 2(6) = -3$$

$$\begin{vmatrix} 1+1+2 & 1 & 2 \\ -2+0+5 & 0 & 5 \\ 2+3+7 & 3 & 7 \end{vmatrix} = \begin{vmatrix} 4 & 1 & 2 \\ 3 & 0 & 5 \\ 2 & 3 & 7 \end{vmatrix}$$

$$\begin{aligned} &= 4(-15) - 1(21 - 60) + 2(9) \\ &= -60 + 39 + 18 = -3 \end{aligned}$$