Linear Algebra
Assignment #2

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Question

What is matrix determinant?

Answer: Matrix determinant is a special number that can be calulated in a square matrix by multiplying the diagonal entries first and then substracting them.

→ **Properties of determinent:**

1. **Reflection Property:** The determinent remains unchanged if rows are changed into columns and columns into rows.

Example:

$$\begin{vmatrix} 1 & 4 \\ 5 & 6 \end{vmatrix} = (1)\times(6) - (4)(5) = 6-20 = -14$$

Now taking transpose,

$$\begin{vmatrix} 1 & 5 \\ 4 & 6 \end{vmatrix} = (1)(6) - (5)(4) = 6 - 20 = 14$$

2. **All-zero property**:-

If element of a row or column of a matrix is zero, then the determinant will be zero

Example:

$$\begin{vmatrix} 0 & 2 \\ 0 & 4 \end{vmatrix} = (0)(4) - (0)(2) = 0$$

3. **Factor property**:

If determinant becomes zero when we put $x = \alpha$, then $x - \alpha$ is a factor of Δ

Example:-

$$\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \Rightarrow \Delta_1 = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix}$$

where C_{ij} is a cofactor of elements a_{ij} in Δ

4. **Triangle property:**

If all the elements of a determinent above or below the main diagonal consists of zeros, then the determinent is equal to the product of diagonal elements.

Example:

$$\begin{vmatrix} 3 & 0 & 0 \\ 8 & 3 & 0 \\ 7 & 9 & 1 \end{vmatrix} = (3)(3) - 0 - 0 = 9$$

$$\text{or } (3)(3)(1) = 9$$

5. **Proportionality property:**

If all the elements of a row or column are identical to all the elements of some other row or column, then the determinent will be zero.

Example:

$$\begin{vmatrix} 1 & 1 \\ 4 & 4 \end{vmatrix} = (1)(4) - (1)(4) = 0$$

6. **Switching property** :-

If we interchange any two rows or columns of a matrix, then the determinent of a new matrix changes its signs.

Example:

$$\begin{vmatrix} 2 & 8 \\ 3 & 5 \end{vmatrix} = (2)(5)-(8)(3) = -14$$

Interchanging

$$\begin{vmatrix} 3 & 5 \\ 2 & 8 \end{vmatrix} = (3)(8)-(5)(2) = 14$$

7. **Scalar Multiple property**:

If every element of a row or column is multiplied by a non-zero constant, then the determinent also gets multiplied by the same constant.

Example:

$$\begin{vmatrix} 2 & 4 \\ 5 & 6 \end{vmatrix} = -8 \xRightarrow{R_1 \times 2} \begin{vmatrix} 4 & 8 \\ 5 & 6 \end{vmatrix} = -16$$

8. Sum property:

If elements of a row or column are expressed as the sum of two or more terms, then the determinent can be expressed as the sum of two or more determinants

Example:

$$\begin{vmatrix} 2+1 & 1 & 2 \\ 3+(-2) & 0 & 5 \\ 4+2 & 3 & 7 \end{vmatrix} = 3(0-15)-1(7-30)+2(3-0)$$
$$= -45+23+6 = -16$$

$$= \begin{vmatrix} 2 & 1 & 2 \\ 3 & 0 & 5 \\ 4 & 3 & 7 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 2 \\ -2 & 0 & 5 \\ 2 & 3 & 7 \end{vmatrix}$$

$= [2(-15)-1(21-20)+2(9)]+[1(15)-1(-14-10)$
$+2(6)]$
$= [-30-1+18]+[-15+24-12]$
$= -13-3 = -16$

9. Property of invariance:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1+\alpha b_1+\beta c_1 & b_1 & c_1 \\ a_2+\alpha b_2 \ \beta c_2 & b_2 & c_2 \\ a_3+\alpha b_3 \ \beta c_3 & b_3 & c_3 \end{vmatrix}$$

Example:

$$\begin{vmatrix} 1 & 1 & 2 \\ -2 & 0 & 5 \\ 2 & 3 & 7 \end{vmatrix} = 1(15) - 1(-14-10) + 2(6) = -3$$

$$\begin{vmatrix} 1+1+2 & 1 & 2 \\ -2+0+5 & 0 & 5 \\ 2+3+7 & 3 & 7 \end{vmatrix} = \begin{vmatrix} 4 & 1 & 2 \\ 3 & 0 & 5 \\ 2 & 3 & 7 \end{vmatrix}$$

$= 4(-15) - 1(21-60) + 2(9)$

$= -60 + 39 + 18 = -3$