

First to Penalty



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1 Template

```

1 #include "bits/stdc++.h"
2 //assert(x>0) si falla da RTE
3 using namespace std;
4 #define endl '\n'
5 #define DBG(x) cerr<<#x<< "=" << (x) << endl;
6 #define RAYA cerr<<"===== "<<endl;
7 #define RAYAS cerr<<"..... "<<endl;
8 // #define DBG(x) ;
9 // #define RAYA ;
10 // #define RAYAS ;
11
12 //-----SOLBEGIN-----
13 int main() {
14     ios_base::sync_with_stdio(false); cout.tie(NULL); cin.tie(NULL);
15     int tC;
16
17     cin >> tC;
18     while (tC--) {
19
20     }
21 }
22
23 //-----EOSOLUTION-----

```

2 Data structures

2.1 Simplified DSU (Stolen from GGDem)

```

1 int uf[MAXN];
2 void uf_init(){memset(uf,-1,sizeof(uf));}
3 int uf_find(int x){return uf[x]<0?x:uf[x]=uf_find(uf[x]);}
4 bool uf_join(int x, int y){
5     x=uf_find(x);y=uf_find(y);
6     if(x==y)return false;
7     if(uf[x]>uf[y])swap(x,y);
8     uf[x]+=uf[y];uf[y]=x;
9     return true;
10 }

```

2.2 Disjoint Set Union

```

1 class disjSet {
2     int* sz;
3     int* par;
4 public:
5     int len;
6     disjSet(int tam){
7         sz = new int[tam + 4]();
8         par = new int[tam + 4]();
9         len = 0;
10        for(int i = 0; i<=tam; i++){
11            par[i] = i;
12            sz[i] = 1;
13            len++;
14        }
15    }
16    int finds(int el){
17        if (el == par[el]) return el;
18        return par[el] = finds(par[el]);
19    }
20    void unions(int a, int b){
21        a = finds(a);
22        b = finds(b);
23        if (a == b) return;
24        len--;
25        //se hace que el gde sea padre del pequeno
26        if (sz[a] > sz[b]) swap(a,b);
27        par[a] = b;
28        sz[b] += sz[a];
29    }
30    ~disjSet(){
31        delete[] size;
32        size = nullptr;
33        delete[] parent;
34        parent = nullptr;
35    }
36 };

```

2.3 Segment tree

```

1 //MAXN = 2^k, n = tam arreglo inicial
2 #define MAXN 262160
3 int stsize; long long int neut;int n;
4 long long int* st = new long long int[2*MAXN-1]();

```

```

5 long long int fst(long long int a, long long int b);
6 long long int build(int sti,int csize){
7     if(csize == 1) return st[sti];
8     return st[sti] = fst(build(sti*2+1,csize/2),build(sti*2+2,csize/2));
9 }
10 void innit(){
11     for(int i = 0; i<stsize; i++) st[i] = neut;
12     /*int d = 0;
13     for(int i = stsize-n; i<stsize && d<n; i++){
14         st[i] = arr[d];d++;
15     }*/
16     build(0,n);
17 }
18 void upd(int ind, long long int val){
19     ind = stsize-n+ind;
20     st[ind] = val;ind--;ind/=2;
21     while(true){
22         st[ind] = fst(st[ind*2+1],st[ind*2+2]);
23         ind--;
24         if(ind<0) break;
25         ind/=2;
26     }
27 }
28 long long int rqu(int l, int r,int sti, int ls, int rs){
29     if(l<=ls && rs<= r) return st[sti];
30     if(r<ls || l>rs) return neut;
31     int m = (rs+ls)/2;
32     return fst(rqu(l,r,sti*2+1,ls,m),rqu(l,r,sti*2+2,m+1,rs));
33 }
34 long long int query(int l, int r){
35     return rqu(l,r,0,0,n-1);
36 }
37 //uso, inicializa neut, n = primera potencia de 2 >= n del problema,
38     stsize = 2*n-1
39 //llena arr de neutros hasta que su tam sea el nuevo n
40 //DEFINE LA FUNCION fst

```

2.4 Segment tree Lazy

```

1 //MAXN = 2^k, n = tam arreglo inicial
2 #define MAXN 262160
3 vector<int> arr;
4 int stsize; long long int neut;int n;

```

```

5 long long int* st = new long long int[2*MAXN-1]();
6 long long int* pendientes = new long long int[2*MAXN-1]();
7 long long int fst(long long int a, long long int b){return a+b;}
8 long long int build(int sti,int csize){
9     if(csize == 1) return st[sti];
10    return st[sti] = fst(build(sti*2+1,csize/2),build(sti*2+2,csize/2));
11 }
12 bool hasChildren(int sti){sti*=2;sti++;sti++;return sti<stsize;}
13 void innit(){
14     for(int i = 0; i<stsize; i++) st[i] = neut;
15     int d = 0;
16     for(int i = stsize-n; i<stsize && d<n; i++) {st[i] = arr[d];d++;}
17     build(0,n);
18 }
19 void updrec(int l,int r, int sl, int sr,int sti, long long int val){
20     if(sr<l || r< sl) return;
21     if(l<= sl && sr <=r){
22         st[sti] += val*(sr-sl+1);
23         if(hasChildren(sti)){pendientes[sti*2+1]+=val;pendientes[sti
24             *2+2]+=val;}
25         return;
26     }
27     int sm = (sl+sr)/2;
28     updrec(l,r,sl,sm,sti*2+1,val);
29     updrec(l,r,sm+1,sr,sti*2+2,val);
30     st[sti] = fst(st[sti*2+1]+pendientes[sti*2+1],st[sti*2+2]+pendientes
31         [sti*2+2]);
32 }
33 void upd(int l, int r, long long int val){updrec(l,r,0,n-1,0,val);}
34 long long int rqu(int l, int r,int sti, int ls, int rs){
35     if(r<ls || l>rs) return neut;
36     if(l<=ls && rs<= r){
37         return st[sti]+pendientes[sti]*(rs-ls+1);
38     }
39
40     st[sti] += pendientes[sti]*(rs-ls+1);
41     if(hasChildren(sti)){pendientes[sti*2+1]+=pendientes[sti];pendientes
42         [sti*2+2]+=pendientes[sti];}
43     pendientes[sti] = 0;
44     int m = (rs+ls)/2;

```

```

45     return fst(rqu(l,r,sti*2+1,ls,m),rqu(l,r,sti*2+2,m+1,rs));
46 }
47 long long int query(int l, int r){
48     return rqu(l,r,0,0,n-1);
49 }
50 //uso, inicializa neut, n = primera potencia de 2 >= n del problema,
51     stsize = 2*n-1
52 //llena arr de neutros hasta que su tam sea el nuevo n
53 //DEFINE LA FUNCION fst

```

2.5 Trie

```

1 struct triver {
2     char alphabet;
3     bool ter;
4     vector<triver*> child;
5     triver(char a): alphabet(a) { child.assign(26, NULL); ter = false; }
6 };
7 class trie{
8 private:
9     triver* root;
10 public:
11     trie() { root = new triver('!');}
12     void insert(string s){
13         triver* curr = root;
14         for(char l: s){
15             if(curr->child[l-'A'] == NULL) curr->child[l-'A'] = new
16                 triver(l);
17             curr = curr->child[l-'A'];
18         }
19         curr->ter = true;
20     }
21     bool search(string s){
22         triver* curr = root;
23         for(char l: s){
24             if(curr == NULL) break;
25             curr = curr->child[l-'A'];
26         }
27         if(curr == NULL) return false;
28         return curr->ter;
29     }
30 };

```

3 Graphs

3.1 Graph Transversal

3.1.1 BFS

```

1 #define GS 400040
2 vector<int> graph[GS];
3 bitset <GS> vis;
4 //anchura O(V+E)
5 void dfs(int curr) {
6     queue<int> fringe;
7     fringe.push(curr);
8     while (fringe.size()) {
9         curr = fringe.front(); fringe.pop();
10        if (!vis[curr]) {
11            vis[curr] = 1;
12            for (int h : graph[curr]) fringe.push(h);
13        }
14    }
15 }

```

3.1.2 DFS

```

1 #define GS 400040
2 vector<int> graph[GS];
3 bitset <GS> vis;
4 //profundidad O(V+E)
5 void dfs(int curr) {
6     stack<int> fringe;
7     fringe.push(curr);
8     while (fringe.size()){
9         curr = fringe.top(); fringe.pop();
10        if (!vis[curr]) {
11            vis[curr] = 1;
12            for (int h : graph[curr]) fringe.push(h);
13        }
14    }
15 }

```

3.2 Topological Sort

```

1 #define GS 400040
2 vector<int> graph[GS];

```

```

3 bitset <GS> vis;
4 vector<int> topsort;
5 int e,n;
6 //profundidad
7 //O(N+E)
8 //Solo funciona con DAG's, no existe un top sort de un grafo Non-DAG
9 void todfs(int pa) {
10     vis[pa]=1;
11     for(int h: graph[pa]){if(!vis[h]){todfs(h);}}
12     topsort.push_back(pa);
13 }
14 void topologicalSort(){
15     vis.reset();
16     topsort.clear();
17     for(int i = 0; i<n; i++){if(!vis[i]){dfs(i);}}
18     reverse(topsort.begin(),topsort.end());
19 }
    
```

3.3 APSP: Floyd Warshall

```

1 #define GS 1000
2 #define INF 100000000
3 //destino, costo
4 int graph[GS][GS];
5 //All Pairs Dist
6 int dist[GS][GS];
7 //Toma en cuenta nodos [0-tam] inclusivo, modificar de acuerdo a las
8 //necesidades
9 //Ten cuidado con el valor que le pones a INF, puede provocar overflows
10 //o puede no ser lo suficientemente grande.
11 void Floyd_Warshall(int tam){
12     for(int i = 0; i<=tam; i++)
13         for(int f = 0; f<=tam; f++)
14             dist[i][f] = INF;
15
16     for(int i = 0; i<=tam; i++)
17         for(int f = 0; f<=tam; f++)
18             dist[i][f] = graph[i][f];
19
20     //para reconstruir el camino solo basta con guardar intermedio como
21     //el padre de ini si el cambio se hizo, -1 otherwise
22     for(int intermedio = 0; intermedio<=tam; intermedio++)
23         for(int ini = 0; ini<=tam; ini++)
    
```

```

21         for(int fin = 0; fin<=tam; fin++)
22             dist[ini][fin] = min(dist[ini][fin],dist[ini][intermedio]
23             ]+dist[intermedio][fin]);
    
```

3.4 SSSP

3.4.1 Lazy Dijkstra

```

1 #define GS 1000
2 #define INF 100000000
3 //destino, costo
4 vector<pair<int,int>> graph[GS];
5 int dist[GS];
6 void dijkstra(int origen,int tam){
7     for(int i = 0; i<=tam; i++){
8         dist[i] = INF;
9     }
10     priority_queue<pair<int,int>,vector<pair<int,int>>, greater<pair<int
11     ,int>>> pq;
12     pair<int,int> curr;
13
14     pq.push(make_pair(0,origen));
15
16     while(pq.size()){
17         curr = pq.top();pq.pop();
18         if(curr.first >= dist[curr.second]) continue;
19
20         dist[curr.second] = curr.first;
21         for(pair<int,int> h: graph[curr.second]){
22             if((h.second+curr.first)<dist[h.first]) pq.push({h.second+
23             curr.first,h.first});
24         }
25     }
26
27     //Esta es la implementacion huevona
28     //Resuelve Single Source Shortest Paths con aristas positivas
29     //Como es la lazy implementation, si funciona con edges negativos
30     //siempre y cuando no hayan ciclos negativos
31     //Si hay ciclos negativos se va atascar en un ciclo infinito
32     //Si no los hay puede que funcione en O((V+E)log(V)) o puede que se
33     //exponencial, si no jala prueba BellmanFord
    
```

3.4.2 Bellman-Ford

```

1 //esta es la implementacion huevona
2 #define GS 1000
3 //cuidado con overflows!!
4 #define INF 100000000
5 #define NINF -100000000
6 //destino, costo
7 vector<pair<int,int>> graph[GS];
8 int dist[GS];
9 struct edge{
10     int from,to,cost;
11 };
12 //Corre en O(VE)
13 void bellmanFord(int origen,int tam){
14     for(int i = 0; i<=tam; i++){
15         dist[i] = INF;
16     }
17     dist[origen] = 0;
18     edge aux;
19     vector<edge> aristas;
20     bool optimal;
21
22     for(int i = 0; i<=tam; i++){
23         for(pair<int,int> h: graph[i]){
24             aux.from = i; aux.to = h.first;aux.cost = h.second;
25             aristas.push_back(aux);
26         }
27     }
28
29     //Si se relajan todos las aristas V-1 veces en un orden arbitrario
30     //Se asegura que la distancia optima para cada vertice sera
31     //alcanzada
32     for(int i = 0; i<tam && !optimal; i++){
33         optimal = true;
34         for(edge elem: aristas){
35             if(dist[elem.from] + elem.cost < dist[elem.to]){
36                 dist[elem.to] = dist[elem.from] + elem.cost;
37                 //si algun vertice fue actualizado significa que puede
38                 //que
39                 //las distancias aun no sean optimas
40                 optimal = false;
41             }
42         }
43     }
44 }

```

```

42
43 //Se corre de nuevo para asegurar encontrar todos los ciclos
44 //negativos
45 for(int i = 0; i<tam && !optimal; i++){
46     optimal = true;
47     for(edge elem: aristas){
48         if(dist[elem.from] + elem.cost < dist[elem.to]){
49             //Si aun despues de correr V-1 veces se puede actualizar
50             //Significa que esta en un ciclo negativo
51             dist[elem.to] = NINF;
52             //si algun vertice fue actualizado significa que puede
53             //que
54             //las distancias aun no sean optimas
55             optimal = false;
56         }
57     }
58 }

```

3.5 Strongly Connected Components: Kosaraju

```

1 #define GS 2010
2 vector<int> graph[GS];
3 vector<int> graphI[GS];
4 vector<int> orden;
5 bitset<GS> vis;
6
7 void invertirGrafo(int n){
8     for(int p = 1;p<= n; p++)
9         for(int h: graph[p])graphI[h].push_back(p);
10 }
11 void obtOrd(int p,int n){
12     vis[p] = 1;
13     for(int h: graph[p]){
14         if(!vis[h] && h<=n) obtOrd(h,n);
15     }
16     orden.push_back(p);
17 }
18 int findSCC(int n){
19     int res = 0;
20     invertirGrafo(n);
21     orden.clear();

```

```

22     for(int i = 1; i<=n; i++) vis[i] =0;
23     for(int i = 1; i<=n; i++) if(!vis[i]) obtOrd(i,n);
24     reverse(orden.begin(),orden.end());
25     //cuenta los connected components
26     //vector<int> lsc;
27     stack<int> fringe;
28     int curr;
29     for(int i = 1; i<=n; i++) vis[i] =0;
30     for(int i: orden){
31         //lsc.clear();
32         if(!vis[i]){
33             fringe.push(i);
34             while (fringe.size()){
35                 curr = fringe.top();fringe.pop();
36                 //lsc.push_back(curr);
37                 if (!vis[curr]) {
38                     vis[curr] = 1;
39                     for (int h : graphI[curr]) fringe.push(h);
40                 }
41             }
42             res++;
43         }
44         //hacer lo que sea con lcsc
45     }
46     return res;
47 }
48
49 //OJO esto solo jala con directed graphs
50 //por definicion todas las undirected graphs tienen un solo SCC
51 //NOTAR QUE LOS GRAFOS QUE USA CUMPLEN CON: 0<=VERTICE<=n
    
```

3.6 Articulation Points and Bridges: ModTarjan

```

1 #define GS 50
2 vector<int> graph[GS];
3 bitset<GS> vis, isArtic;
4 vector<int> padre;
5 //id por tiempo, menor id accesible
6 //ya sea por descendientes o por back edges
7 vector<int> tId,lId;
8 //cantidad de hijos que tiene en el bfs spanning tree
9 int rootChildren;
10 int cnt;
    
```

```

11 int dfsRoot;
12 void findAP_B(int p){
13     cnt++;vis[p] = 1;tId[p] = cnt;lId[p] = tId[p];
14
15     for(int hijo: graph[p]){
16         if(!vis[hijo]){
17             padre[hijo] = p;
18             if(p == dfsRoot) rootChildren++;
19
20             findAP_B(hijo);
21
22             //esto significa que ni por un back edge el hijo accede al
23             //padre
24             //por lo que si el padre fuese eliminado el hijo quedaria
25             //aislado
26             if(lId[hijo] >= tId[p]) isArtic[p] = 1;
27             if(lId[hijo] > tId[p]){
28                 //esto significa que si se eliminase el camino de padre
29                 //se lograria desconectar el grafo, aka bridge
30             }
31             lId[p] = min(lId[p],lId[hijo]);
32         }else{
33             //si hay un ciclo indirecto, actualiza el valor para el
34             //padre
35             if(hijo != padre[p]) lId[p] = min(lId[p],tId[hijo]);
36         }
37     }
38 }
39 //OJO esto solo jala con Undirected graphs
40 /*
41 MAIN
42 for(int i = 0; i<n; i++){
43     if(!vis[i]){
44         rootChildren = 0;
45         dfsRoot = i;
46         findAP_B(i);
47         //el algoritmo no puede detectar si el nodo que lo origino
48         //es un articulation point, por lo que queda checar si
49         //en el spanning tree que genero tiene mas de un solo hijo
50         isArtic[i] = (rootChildren>1?1:0);
51     }
52 }
    
```

50 */

4 Math

4.1 Identities

Coefficientes binomiales.

$$(a+b)^n = \sum_{k=0}^n \binom{n}{k} a^{n-k} b^k$$

$$\binom{n}{k} = \binom{n}{n-k}$$

$$\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

$$k \binom{n}{k} = n \binom{n-1}{k-1}$$

$$\sum_{k=0}^n \binom{n}{k} = 2^n$$

$$\sum_{k=0}^n \binom{n}{k} (-1)^k = 0$$

$$\binom{n+m}{t} = \sum_{k=0}^t \binom{n}{k} \binom{m}{t-k}$$

$$\sum_{j=k}^n \binom{n}{j} = \binom{n+1}{k+1}$$

Numeros Catalan.

$$C_n = \frac{2(2n-1)}{n+1} C_{n-1}$$

$$C_n = \frac{1}{n+1} \binom{2n}{n}$$

$$C_n \sim \frac{4^n}{n^{3/2} \sqrt{\pi}}$$

$$\sigma(n) = O(\log(\log(n))) \text{ (number of divisors of } n)$$

$$F_{2n+1} = F_{n+1}^2 + F_n^2$$

$$F_{2n} = F_{n+1}^2 - F_{n-1}^2$$

$$\sum_{i=1}^n F_i = F_{n+2} - 1$$

$$F_{n+i} F_{n+j} - F_n F_{n+i+j} = (-1)^n F_i F_j$$

(Möbius Function)

0 if n is square-free

1 if n got even amount of distinct prime factors

0 if n got odd amount of distinct prime factors

(Möbius Inv. Formula)

Let $g(n) = \sum_{d|n} f(d)$, then $f(n) = \sum_{d|n} g(d) \mu\left(\frac{n}{d}\right)$.

Permutaciones objetos repetidos

$$P(n, k) = \frac{n!}{n_1! n_2! \dots}$$

Separadores, Ecuaciones lineales a variables = b

$$\binom{a}{b} = \binom{a+b-1}{b} = \binom{a+b-1}{a-1}$$

Teorema chino

sean $\{n_1, n_2, \dots, n_k\}$ primos relativos

$$P = n_1 \cdot n_2 \cdot \dots \cdot n_k$$

$$P_i = \frac{P}{n_i}$$

$$x \cong a_1(n_1)$$

$$x \cong a_2(n_2) \dots x \cong a_k(n_k)$$

$P_1 S_1 \cong 1(n_1)$ Donde S soluciones.

$$x = P_1 S_1 a_1 + P_2 S_2 a_2 \dots P_k S_k a_k$$

4.2 Binary Exponentiation and modArith

```

1 long long int inf = 10000000007;
2 //suma (a+b)%m
3 //resta ((a-b)%m+m)%m
4 //mult (a*b)%m
5 long long binpow(long long b, long long e) {
6     long long res = 1; b%=inf;
7     while (e > 0) {
8         if (e & 1) res = (res * b)%inf;
9         b = (b * b)%inf;
10        e >>= 1;
11    }
12    return res;
13 }
```

4.3 Modular Inverse (dividir mod)

```

1 long long int inf = 10000000007;
2 long long int gcd(long long int a, long long int b, long long int& x,
3     long long int& y) {
4     x = 1, y = 0;
5     long long int x1 = 0, y1 = 1, a1 = a, b1 = b;
6     while (b1) {
7         long long int q = a1 / b1;
8         tie(x, x1) = make_tuple(x1, x - q * x1);
9         tie(y, y1) = make_tuple(y1, y - q * y1);
10        tie(a1, b1) = make_tuple(b1, a1 - q * b1);
11    }
12    return a1;
13 }
14 long long int modinverse(long long int b, long long int m){
15     long long int x,y;
16     long long int d = gcd(b,inf,x,y);
17     if(d!=1) return -1;
18     return ((x%inf)+inf)%inf;
19 }
```

4.4 Modular Binomial Coefficient and Permutations

```

1 long long int inf = 10000000007;
2 //cat[n] = bincoef(2*n,n)/(n+1), cat[0] = 1
3 class binCoef{
4     long long int lim;
```



```

5     long long int* fact;
6 public:
7     binCoef(long long int l){
8         lim = l; fact = new long long int[l+1];fact[0]= 1;
9         for(long long int i = 1; i<=l; i++) fact[i] = (fact[i-1]*i)%inf;
10    }
11    //perm = (fact[n] * modinverse(fac[n-k],inf)%inf;
12    long long int query(long long int n, long long int k){
13        if(n<k) return 0;
14        return (fact[n] * modinverse((fac[n-k]*fact[k])%inf,inf))%inf;
15    }
16 };
    
```

4.5 Non-Mod Binomial Coefficient and Permutations

```

1 //Solo usar con n<=20
2 //cat[n] = bincoef(2*n,n)/(n+1), cat[0] = 1
3 unsigned long long int bincoef(unsigned long long int n, unsigned long
4     long int k){
5     if(n<k) return 0;
6     unsigned long long int num = 1, den= 1;
7     for(unsigned long long int i = (n-k)+1; i<=n; i++) num*=i;
8     for(unsigned long long int i = 2; i<=k; i++) den*=i;
9     //perm = return num;
10    return num/den;
11 }
    
```

4.6 Modular Catalan Numbers

```

1 long long int inf = 10000000007;
2 class catalan{
3     long long int* cat; long long int lim
4 public:
5     catalan(long long int l){
6         lim = l; cat = new long long int[l+10];cat[0] = 1;
7         for(long long int i = 0;i<=l; i++) cat[i+1] = (((((4LL*i+2)%inf)
8             *cat[i])%inf) *modinverse(n+2))%inf;
9     }
10    long long int query(long long int n){ return cat[n];}
11 };
    
```

4.7 Ceil Fraccionario

```

1 long long int techo(long long int num, long long int den){ return (num+
    den-1)/den;}
    
```

4.8 Numeros de Fibonacci

```

1 //en caso de ser usados mod un m pequeno
2 //recordar que los numeros de fibonacci se repiten por lo menos cada m^2
3 //O(n)
4 unsigned long long int fib(int n){
5     unsigned long long int a = 1,b = 1,aux;
6     if(n<=2){
7         return 1;
8     }
9     for(int i = 3; i<=n; i++){
10        aux = a+b;
11        a = b;
12        b = aux;
13    }
14    return b;
15 }

1 const long long int inf = 10000000007;
2 unordered_map<long long int,long long int> Fib;
3 //O(log n) :DD
4 long long int fib(long long int n)
5 {
6     if(n<2) return 1;
7     if(Fib.find(n) != Fib.end()) return Fib[n];
8     Fib[n] = (fib((n+1) / 2)*fib(n/2) + fib((n-1) / 2)*fib((n-2) / 2)) %
9         inf;
10    return Fib[n];
11 }
    
```

4.9 Sieve Of Eratosthenes

```

1 #define MAXN 10e6
2 class soe{
3 public:
4     bitset<MAXN> isPrime;
5     soe(){
6         for(int i = 3; i<MAXN; i++) isPrime[i] = (i%2);
7         isPrime[2] = 1;
8         for(int i = 3; i*i<MAXN; i+=2)
9             if(isPrime[i])
10                 for(int j = i*i; j<MAXN; j+=i)
11                     isPrime[j] = 0;
    
```

```

12     }
13 };
    
```

4.10 Sieve-based Factorization

```

1 #define MAXN 10e6
2 class soe{
3 public:
4     int smolf[MAXN];
5     soe(){
6         for(int i = 2; i<MAXN; i++) smolf[i] = (i%2==0?2:i);
7
8         for(int i = 3; i*i<MAXN; i+=2)
9             if(smolf[i]==i)
10                for(int j = i*i; j<MAXN; j+=i)
11                    smolf[j] = min(smolf[j],smolf[i]);
12     }
13 };
    
```

4.11 Cycle Finding

```

1 void cyclef(long long int sem){
2     long long int hare = f(sem),tort=f(sem);hare = f(hare);
3     //liebre avanza dos pasos, tortuga solo uno
4     while(hare!=tort){
5         tort = f(tort); hare = f(f(hare));
6     }
7     //Se detiene en el inicio del ciclo
8     tort = sem;
9     while(hare!=tort){
10        tort = f(tort); hare = f(hare);
11    }
12
13    int len = 1;
14    tort = f(sem);
15    while(hare!=tort){
16        tort=f(tort);
17        len++;
18    }
19 }
    
```

4.12 Berlekamp Massey

```

1 typedef long long int ll;
    
```

```

2 //Obtiene recurrencia lineal dados los primeros elementos en O(n^2)
3 vector<ll> berlekampMassey(const vector<ll> &s) {
4     vector<ll> c;
5     vector<ll> oldC;
6     int f = -1;
7     for (int i=0; i<(int)s.size(); i++) {
8         ll delta = s[i];
9         for (int j=1; j<=(int)c.size(); j++) delta -= c[j-1] * s[i-j];
10        if (delta == 0) continue;
11        if (f == -1) {
12            c.resize(i + 1);
13            mt19937 rng(chrono::steady_clock::now().time_since_epoch().
14                        count());
15            for (ll &x : c) x = rng();
16            f = i;
17        } else {
18            vector<ll> d = oldC;
19            for (ll &x : d) x = -x;
20            d.insert(d.begin(), 1);
21            ll df1 = 0;
22            for (int j=1; j<=(int)d.size(); j++) df1 += d[j-1] * s[f+1-j
23                ];
24            assert(df1 != 0);
25            ll coef = delta / df1;
26            for (ll &x : d) x *= coef;
27            vector<ll> zeros(i - f - 1);
28            zeros.insert(zeros.end(), d.begin(), d.end());
29            d = zeros;
30            vector<ll> temp = c;
31            c.resize(max(c.size(), d.size()));
32            for (int j=0; j<(int)d.size(); j++) c[j] += d[j];
33            if (i - (int) temp.size() > f - (int) oldC.size()) {oldC =
34                temp;f = i;}
35        }
36    }
37    return c;
38 }
    
```

4.13 Modular Berlekamp Massey

```

1 typedef long long int ll;
2 long long int inf = 1000000007;
3 vector<ll> bermas(vector<ll> x){
    
```

```

4     vector<ll> ls,cur;
5     int lf,ld;
6     for(int i = 0; i<x.size(); i++){
7         long long int t = 0;
8         for(int j = 0; j<cur.size(); j++) t=(t+x[i-j-1]*(long long int)
9             cur[j])%inf;
10        if((t-x[i])%inf==0)continue;
11        if(cur.size()==0){cur.resize(i+1);lf=i;ld=(t-x[i])%inf;continue
12            ;}
13        long long int k = (x[i]-t)*powermod(ld,inf-2)%inf;
14        vector<ll>c(i-lf-1);c.push_back(k);
15        for(int j = 0; j<ls.size(); j++) c.push_back(-ls[j]*k%inf);
16        if(c.size()<cur.size()) c.resize(cur.size());
17        for(int j = 0; j<cur.size();j++) c[j]=(c[j]+cur[j])%inf;
18        if(i-lf+ls.size()>=cur.size())ls=cur,lf=i,ld=(t-x[i])%inf;
19        cur=c;
20    }
21    for(int i=0; i<cur.size(); i++) cur[i]=(cur[i]%inf+inf)%inf;
22    return cur;
23 }

```

4.14 Matrix exponentiation

```

1  typedef vector<vector<long long int>> Matrix;
2  long long int inf = 1000000007;
3  Matrix ones(int n) {
4      Matrix r(n,vector<long long int>(n));
5      for(int i= 0; i<n; i++){
6          r[i][i]=1;
7      }
8      return r;
9  }
10 Matrix operator*(Matrix &a, Matrix &b) {
11     int n=a.size(),m=b[0].size(),z=a[0].size();
12     Matrix r(n,vector<long long int>(m));
13     for(int i=0; i<n; i++){
14         for(int j=0; j<m; j++){
15             for(int k=0;k<z; k++){
16                 r[i][j]+=((a[i][k]%inf)*(b[k][j]%inf))%inf;
17                 r[i][j]%=inf;}}
18     }
19     return r;
20 }
21 Matrix be(Matrix b, long long int e) {

```

```

21     Matrix r=ones(b.size());
22     while(e){if(e&1LL)r=r*b;b=b*b;e/=2;}
23     return r;
24 }
25
26 //Matrix mat(n,vector<long long int>(n));

```

4.15 Ecuaciones Diofantinas

```

1  long long int gcd(long long int a, long long int b, long long int& x,
2      long long int& y) {
3      x = 1, y = 0;
4      long long int x1 = 0, y1 = 1, a1 = a, b1 = b;
5      while (b1) {
6          int q = a1 / b1;
7          tie(x, x1) = make_tuple(x1, x - q * x1);
8          tie(y, y1) = make_tuple(y1, y - q * y1);
9          tie(a1, b1) = make_tuple(b1, a1 - q * b1);
10     }
11     return a1;
12 }
13 long long int d;
14 bool findAnySol(long long int a, long long int& x, long long int b, long
15     long int& y, long long int c) {
16     long long int g = gcd(abs(a), abs(b), x, y);
17     if (c % g != 0) return false;
18     x *= c;
19     y *= c;
20     x /= g;
21     y /= g;
22     d = c / g;
23     if (a < 0) x = -x;
24
25     if (b < 0) y = -y;
26     return true;
27 }
28 //-----SOLBEGIN-----
29 int main() {
30     ios_base::sync_with_stdio(false); cout.tie(NULL); cin.tie(NULL);
31     long long int m, a, k, n;
32     long long int f, h,res;
33     //estira en n, y despues cada m
34     //estira en k+a, y despues cada a

```

```

33 cin >> n >> m >> a >> k;
34 while (n != 0 && m != 0 && a != 0 && k != 0) {
35     m = -m;
36     if (!findAnySol(m, f, a, h, k + a - n)) {
37         cout << "Impossible" << endl;
38     }else {
39         res = f * m+n;
40         while (res > 0) res -= m * d;
41         while (res < 0) res += m * d;
42
43         cout << res << endl;
44     }
45     cin >> n >> m >> a >> k;
46 }
47 }
48 }
49 //-----EOSOLUTION-----

```

4.16 Pollard-Rho, Stolen from GGDem

```

1 long long int gcd(long long int a, long long int b){return a?gcd(b%a,a):
  b;}
2 long long int mulmod(long long int a, long long int b, long long int m)
  {
3     long long int r=a*b-(long long int)((long double)a*b/m+.5)*m;
4     return (r<0?r+m:r);
5 }
6 long long int expmod(long long int b, long long int e, long long int m){
7     if(!e)return 1;
8     long long int q=expmod(b,e/2,m);q=mulmod(q,q,m);
9     return (e&1?mulmod(b,q,m):q);
10 }
11 bool is_prime_prob(ll n, int a){
12     if(n==a)return true;
13     long long int s=0,d=n-1;
14     while(d%2==0)s++,d/=2;
15     long long int x=expmod(a,d,n);
16     if((x==1)|| (x+1==n))return true;
17     for(int i = 0; i<s-1; i++){
18         x=mulmod(x,x,n);
19         if(x==1)return false;
20         if(x+1==n)return true;
21     }

```

```

22     return false;
23 }
24 bool rabin(long long int n){ // true iff n is prime
25     if(n==1)return false;
26     int A[]={2,3,5,7,11,13,17,19,23};
27     for(int a: A) if(!is_prime_prob(n,a))return false;
28     return true;
29 }
30 long long int rho(long long int n){
31     if(!(n&1))return 2;
32     long long int x=2,y=2,d=1;
33     long long int c=rand()%n+1;
34     while(d==1){
35         x=(mulmod(x,x,n)+c)%n;
36         y=(mulmod(y,y,n)+c)%n;
37         y=(mulmod(y,y,n)+c)%n;
38         if(x>y)d=gcd(x-y,n);
39         else d=gcd(y-x,n);
40     }
41     return d==n?rho(n):d;
42 }
43 void fact(long long int n, map<long long int,int>& f){ //O (lg n)^3
44     if(n==1)return;
45     if(rabin(n)){f[n]++;return;}
46     long long int q=rho(n);
47     fact(q,f);fact(n/q,f);
48 }

```

4.17 FFT, Stolen from GGDem

```

1 // SPOJ VFMUL - AC
2 // http://www.spoj.com/problems/VFMUL/
3 #include <bits/stdc++.h>
4 #define fst first
5 #define snd second
6 #define fore(i,a,b) for(int i=a,ThxDem=b;i<ThxDem;++i)
7 #define pb push_back
8 #define ALL(s) s.begin(),s.end()
9 #define FIN ios_base::sync_with_stdio(0);cin.tie(0);cout.tie(0)
10 #define SZ(s) int(s.size())
11 using namespace std;
12 typedef long long ll;
13 typedef pair<int,int> ii;

```

```

14
15 // MAXN must be power of 2 !!
16 // MOD-1 needs to be a multiple of MAXN !!
17 // big mod and primitive root for NTT:
18 const int MOD=998244353,RT=3,MAXN=1<<20;
19 typedef vector<int> poly;
20 // FFT
21 struct CD {
22     double r,i;
23     CD(double r=0, double i=0):r(r),i(i){}
24     double real()const{return r;}
25     void operator/=(const int c){r/=c, i/=c;}
26 };
27 CD operator*(const CD& a, const CD& b){
28     return CD(a.r*b.r-a.i*b.i,a.r*b.i+a.i*b.r);}
29 CD operator+(const CD& a, const CD& b){return CD(a.r+b.r,a.i+b.i);}
30 CD operator-(const CD& a, const CD& b){return CD(a.r-b.r,a.i-b.i);}
31 const double pi=acos(-1.0);
32 // NTT
33 /*
34 struct CD {
35     int x;
36     CD(int x):x(x){}
37     CD(){}
38     int get()const{return x;}
39 };
40 CD operator*(const CD& a, const CD& b){return CD(mulmod(a.x,b.x));}
41 CD operator+(const CD& a, const CD& b){return CD(addmod(a.x,b.x));}
42 CD operator-(const CD& a, const CD& b){return CD(submod(a.x,b.x));}
43 vector<int> rts(MAXN+9,-1);
44 CD root(int n, bool inv){
45     int r=rts[n]<0?rts[n]=pm(RT,(MOD-1)/n):rts[n];
46     return CD(inv?pm(r,MOD-2):r);
47 }
48 */
49 CD cp1[MAXN+9],cp2[MAXN+9];
50 int R[MAXN+9];
51 void dft(CD* a, int n, bool inv){
52     fore(i,0,n)if(R[i]<i)swap(a[R[i]],a[i]);
53     for(int m=2;m<=n;m*=2){
54         double z=2*pi/m*(inv?-1:1); // FFT
55         CD wi=CD(cos(z),sin(z)); // FFT
56         // CD wi=root(m,inv); // NTT

```

```

57     for(int j=0;j<n;j+=m){
58         CD w(1);
59         for(int k=j,k2=j+m/2;k2<j+m;k++,k2++){
60             CD u=a[k];CD v=a[k2]*w;a[k]=u+v;a[k2]=u-v;w=w*wi;
61         }
62     }
63 }
64 if(inv)fore(i,0,n)a[i]/=n; // FFT
65 //if(inv){ // NTT
66 // CD z(pm(n,MOD-2)); // pm: modular exponentiation
67 // fore(i,0,n)a[i]=a[i]*z;
68 //}
69 }
70 poly multiply(poly& p1, poly& p2){
71     int n=p1.size()+p2.size()+1;
72     int m=1,cnt=0;
73     while(m<=n)m+=m,cnt++;
74     fore(i,0,m){R[i]=0;fore(j,0,cnt)R[i]=(R[i]<<1)|((i>>j)&1);}
75     fore(i,0,m)cp1[i]=0,cp2[i]=0;
76     fore(i,0,p1.size())cp1[i]=p1[i];
77     fore(i,0,p2.size())cp2[i]=p2[i];
78     dft(cp1,m,false);dft(cp2,m,false);
79     fore(i,0,m)cp1[i]=cp1[i]*cp2[i];
80     dft(cp1,m,true);
81     poly res;
82     n-=2;
83     fore(i,0,n)res.pb((ll)floor(cp1[i].real()+0.5)); // FFT
84     //fore(i,0,n)res.pb(cp1[i].x); // NTT
85     return res;
86 }
87
88 char s[MAXN],t[MAXN],r[MAXN];
89
90 int main(){
91     int tn;
92     scanf("%d",&tn);
93     while(tn--){
94         vector<int> a,b,c;
95         scanf("%s%s",s,t);
96         for(int i=0;s[i];++i)a.pb(s[i]-'0');reverse(a.begin(),a.end());
97         for(int i=0;t[i];++i)b.pb(t[i]-'0');reverse(b.begin(),b.end());
98         c=multiply(a,b);
99         while(!c.empty()&&!c.back())c.pop_back();

```

```

100     if(c.empty()){puts("0");continue;}
101     int n=0;
102     ll x=0;
103     fore(i,0,c.size()){
104         x+=c[i];
105         r[n++]=x%10;
106         x/=10;
107     }
108     while(x){
109         r[n++]=x%10;
110         x/=10;
111     }
112     reverse(r,r+n);
113     bool p=false;
114     fore(i,0,n){
115         putchar(r[i]+'0');
116     }
117     puts("");
118 }
119 return 0;
120 }

```

5 Geometry

6 Strings

6.1 Explode by token

```

1 // #include <sstream>
2
3 vector<string> explode(string const& s, char delim) {
4     vector<string> result;
5     istringstream iss(s);
6     for (string token; getline(iss, token, delim); )
7     {
8         result.push_back(move(token));
9     }
10    return result;
11 }

```

6.2 Multiple Hashings DS

```

1 struct multhash{

```

```

2     unsigned long long int h1,h2;
3     unsigned long long int alf[257];
4     bool operator < (multhash b) const {
5         if (h1 != b.h1) return h1 < b.h1;
6         return h2 < b.h2;
7     }
8     bool operator == (multhash b) const { return (h1== b.h1 && h2== b.h2)
9         ;}
10    bool operator != (multhash b) const { return !(h1== b.h1 && h2== b.h2)
11        ;}
12    public:
13        string s;
14        multhash(){
15            h1 = 0; h2 = 0; s = "";
16            for(char l = 'a'; l<='z'; l++) alf[l] = l-'a'+1;
17        }
18        void innit(){
19            unsigned long long int inf,p,op;
20
21            inf = 999727999;
22            p = 325255434; op = 325255434;
23            for(char l: s){
24                h1+=(p*alf[l])%inf;
25                p*=op;
26                p%=inf;
27            }
28
29            inf = 1070777777;
30            p = 10018302; op = 10018302;
31            for(char l: s){
32                h2+=(p*alf[l])%inf;
33                p*=op;
34                p%=inf;
35            }
36        }
37
38        //VALORES ALTERNATIVOS DE INF, LOG 17
39        //6666665555777777
40        //986143414027351997
41        //974383618913296759
42        //973006384792642181
43        //953947941937929919
44        //9090909090909091

```

```

43 //VALORES PARA P, USAR PRIMOS MAYORES A |Alfabeto|
44 //31,47,53,61,79
    
```

6.3 Permute chars of string

```

1 void permute(string str){
2     // Sort the string in lexicographically
3     // ascennding order
4     sort(str.begin(), str.end());
5
6     // Keep printing next permutation while there
7     // is next permutation
8     do {
9         cout<<str<<endl;
10    } while (next_permutation(str.begin(), str.end()));
11 }
    
```

6.4 Longest common subsequence

```

1 //0(|te|*|pa|)
2 //cambiar score para otros problemas, str all match = +2, miss/ins/del =
   -1
3 //usar char que no este en el alfabeto para denotar del/ins
4 string te,pa;
5 long long int ninf = -10e13;
6 long long int score(char a, char b){
7     if(a=='*' || b=='*') return 0;
8     if(a==b) return 1;
9     return ninf;
10 }
11 long long int lcs(){
12     long long int** dp;te = "*" + te; pa = "*" + pa;
13     long long int res = 0;
14
15     dp = new long long int*[te.size()];
16     for(int i = 0; i<te.size(); i++) dp[i] = new long long int[pa.size()
17         ]();
18
19     for(int r = 1; r<te.size(); r++){
20         for(int c = 1; c<pa.size(); c++){
21             dp[r][c] = dp[r-1][c-1] + score(te[r], pa[c]);
22             dp[r][c] = max(dp[r][c-1] + score(te[r], '*'), dp[r][c]);
23             dp[r][c] = max(dp[r-1][c] + score('*', pa[c]), dp[r][c]);
24         }
25     }
26 }
    
```

```

24 }
25
26     return dp[te.size()-1][pa.size()-1];
27 }
    
```

6.5 KMP

```

1 string T,P;
2 int bt[MAXN];
3 //0(|Text|+|Pattern|)
4 void KMPpre(){
5     int i = 0, j = -1; bt[0] = -1;
6     while(i<P.size()){
7         while(j>=0 && P[i] != P[(j>=0?j:0)]) j = bt[j];
8         i++;j++; bt[i] = j;
9     }
10 }
11 int kmp(){
12     int res =0, i = 0, j = 0;
13     while(i<T.size()){
14         while(j>=0 && T[i] != P[(j>=0?j:0)]) j = bt[j];
15         i++; j++;
16         if(j==P.size()){//match, do anything
17             res++;j = bt[j];
18         }
19     }
20     return res;
21 }
    
```

6.6 Suffix Array

```

1 //se asume que la longitud de la cadena sera menor a 10**6, modificar el
   ub a discrecion
2 #define ub 1000000LL
3 //pot de ub times two
4 #define ccd 12
5
6 //metodos y structs auxiliares para el suffix array
7 struct sufd{int id;long long int t;};
8 int getndigit(long long int num, int d){
9     while(d-->0) num/=10LL;
10    return (int) (num%10LL);
11 }
12 void radixSort(vector<sufd>& arr){
    
```

```

13     int count[10]; int n = arr.size();
14     vector<sufd> aux(n);
15     for(int d = 0; d<ccd; d++){
16         for(int i = 0; i<10; i++) count[i] = 0;
17         for(int i = 0; i<n; i++) count[getndigit(arr[i].t,d)]++;
18         for(int i = 1; i<10; i++) count[i]+=count[i-1];
19         for(int i = n-1; i>=0; i--){
20             count[getndigit(arr[i].t,d)]--;
21             aux[count[getndigit(arr[i].t,d)]] = arr[i];
22         }
23         for(int i = 0; i<n; i++) arr[i] = aux[i];
24     }
25 }
26 //El suffix array mismo, agregar caracter menor al alfabeto al final de
    T
27 string T,P;
28 int* sa,*lcest;
29 int stsize;
30 void makesa(){
31     int n = T.size();
32     sa = new int[n+1](); int* ra = new int[2*n+2]();
33     for(int i = 0; i<n; i++){sa[i] = i; ra[i] = T[i];}
34
35     sufd aux;vector<sufd> arr(n);
36     for(int k = 1; k<n;k*=2){
37         arr.clear();
38         for(int i = 0; i<n; i++){
39             aux.id = sa[i]; aux.t = ra[sa[i]];aux.t*=ub;aux.t += ra[sa[i]
                ]+k;
40             arr.push_back(aux);
41         }
42         //en caso de TLE calar con STL sort
43         radixSort(arr);
44         sa[0] = arr[0].id; ra[sa[0]] = 0;
45         for(int i = 1; i<n; i++){
46             sa[i] = arr[i].id;
47             ra[sa[i]] = ra[sa[i-1]]+1;
48             if(arr[i].t == arr[i-1].t) ra[sa[i]]--;
49         }
50         if(ra[sa[n-1]]==n-1) break;
51     }
52     delete[]ra;
53 }
    
```

```

54 void makelce(){
55     int n = T.size();
56     int* lce = new int[n+2]();
57     int* rank = new int[n+2]();
58     for(int i = 0; i<n; i++) rank[sa[i]] = i;
59
60     int curr = 0;
61     for(int i = 0; i<n; i++){
62         if(rank[i]==0) continue;
63         for(int j = max(curr-1,0); j+max(i,sa[rank[i]-1])<n; j++){
64             if(T[i+j] == T[sa[rank[i]-1]+j]) curr = j;
65             if(T[i+j]!=T[sa[rank[i]-1]+j]){curr = j-1; break;}
66         }
67         curr++;lce[i] = curr;
68     }
69
70     int p = 1; while(p<=n) p*=2; stsize = 2*p-1;
71     lcest = new int[stsize+2]();
72     for(int i = p-1; i-(p-1)<n; i++) lcest[i] = lce[sa[i-(p-1)]];
73     for(int i = p-2; i>=0; i--) lcest[i] = min(lcest[2*i+1],lcest[2*i +
        2]);
74     delete[] lce; delete[] rank;
75 }
76 int recque(int l, int r, int sti, int stil, int stir){
77     if(stir<l || stil>r) return ub;
78     if(l<=stil && stir<=r) return lcest[sti];
79     int stim = stil+stir; stim/=2;
80     return min(recque(l,r,sti*2+1,stil,stim),recque(l,r,sti*2+2,stim+1,
        stir));
81 }
82 int getlce(int l, int r){
83     if(l>r) return 0;
84     return recque(l,r,0,0,stsize/2);
85 }
86 int buscarRec(int l, int r,int lcp,int eas){
87     if(l>r) return -1;
88     int m = (l+r)/2;
89     //string curr = T.substr(sa[m],T.size()-sa[m]);
90     int lce = (eas>m?getlce(m+1,eas):getlce(eas+1,m));
91     if(lce>lcp){
92         if(eas<m) return buscarRec(m+1,r,lcp,eas);
93         if(eas>m) return buscarRec(l,m-1,lcp,eas);
94     }
    
```



```

95     if(lce<lcp){
96         if(eas>m) return buscarRec(m+1,r,lcp,eas);
97         if(eas<m) return buscarRec(l,m-1,lcp,eas);
98     }
99
100     for(int i = lcp,n = T.size(); sa[m]+i<n && i<P.size(); i++){if(P[i]
101         !=T[sa[m]+i]) break; lcp++;}
102     if(lcp == P.size()) return m;
103     if(l==r) return -1;
104     return (P[lcp]>T[sa[m]+lcp]?buscarRec(m+1,r,lcp,m):buscarRec(l,m-1,
105         lcp,m));
106 }
107 int buscar(){
108     int n = T.size();
109     if(P.size()>n) return -1;
110     return buscarRec(1,n-1,0,0);
111 }
112 //CODIGO DE 100 LINEAS, TE HE FALLADO MarcosK
113 //Uso: lee T, agregar signo dolar, llama makesa(); makelce(); lee P para
114     despues buscar()
115 //delete[] sa; delete[] lcest; cuando leas de nuevo T
116 //O(|T| log(|T|)) preprocesamiento, O(|P|+log**2(|T|)) cada busqueda
117 //Buscar devuelve un indice cualquiera de sa tal que el sufijo denotado
118     tenga P como prefijo
119 //Se puede hacer mas corto?

```

6.7 STL Suffix Array

```

1 //se asume que la longitud de la cadena sera menor a 10**6, modificar el
2     ub a discrecion
3 #define ub 1000000LL
4 //pot de ub times two
5 #define ccd 12
6 //metodos y structs auxiliares para el suffix array
7 struct sufd{int id;long long int t;
8     bool operator<(const sufd b) const{return t<b.t;}
9 };
10 //El suffix array mismo, agregar caracter menor al alfabeto al final de
11     T
12 string T,P;
13 int* sa,*lcest;
14 int stsize;
15 void makesa(){

```

```

14     int n = T.size();
15     sa = new int[n+1](); int* ra = new int[2*n+2]();
16     for(int i = 0; i<n; i++){sa[i] = i; ra[i] = T[i];}
17
18     sufd aux;vector<sufd> arr(n);
19     for(int k = 1; k<n;k*=2){
20         arr.clear();
21         for(int i = 0; i<n; i++){
22             aux.id = sa[i]; aux.t = ra[sa[i]];aux.t*=ub;aux.t += ra[sa[i]
23                 ]+k;
24             arr.push_back(aux);
25         }
26         //en caso de TLE calar con STL sort
27         sort(arr.begin(),arr.end());
28         sa[0] = arr[0].id; ra[sa[0]] = 0;
29         for(int i = 1; i<n; i++){
30             sa[i] = arr[i].id;
31             ra[sa[i]] = ra[sa[i-1]]+1;
32             if(arr[i].t == arr[i-1].t) ra[sa[i]]--;
33         }
34         if(ra[sa[n-1]]==n-1) break;
35     }
36     delete[]ra;
37 }
38 void makelce(){
39     int n = T.size();
40     int* lce = new int[n+2]();
41     int* rank = new int[n+2]();
42     for(int i = 0; i<n; i++) rank[sa[i]] = i;
43
44     int curr = 0;
45     for(int i= 0; i<n; i++){
46         if(rank[i]==0) continue;
47         for(int j = max(curr-1,0); j+max(i,sa[rank[i]-1])<n; j++){
48             if(T[i+j] == T[sa[rank[i]-1]+j]) curr = j;
49             if(T[i+j]!=T[sa[rank[i]-1]+j]){curr = j-1; break;}
50         }
51         curr++;lce[i] = curr;
52     }
53
54     int p = 1; while(p<=n) p*=2; stsize = 2*p-1;
55     lcest = new int[stsize+2]();
56     for(int i= p-1; i-(p-1)<n; i++) lcest[i] = lce[sa[i-(p-1)]];

```

```

56     for(int i = p-2; i>=0; i--) lcest[i] = min(lcest[2*i+1],lcest[2*i +
57         2]);
58     delete[] lce; delete[] rank;
59 }
60 int recque(int l, int r, int sti, int stil, int stir){
61     if(stir<l || stil>r) return ub;
62     if(l<=stil && stir<=r) return lcest[sti];
63     int stim = stil+stir; stim/=2;
64     return min(recque(l,r,sti*2+1,stil,stim),recque(l,r,sti*2+2,stim+1,
65         stir));
66 }
67 int getlce(int l, int r){
68     if(l>r) return 0;
69     return recque(l,r,0,0,ssize/2);
70 }
71 int buscarRec(int l, int r,int lcp,int eas){
72     if(l>r) return -1;
73     int m = (l+r)/2;
74     //string curr = T.substr(sa[m],T.size()-sa[m]);
75     int lce = (eas>m?getlce(m+1,eas):getlce(eas+1,m));
76     if(lce>lcp){
77         if(eas<m) return buscarRec(m+1,r,lcp,eas);
78         if(eas>m) return buscarRec(l,m-1,lcp,eas);
79     }
80     if(lce<lcp){
81         if(eas>m) return buscarRec(m+1,r,lcp,eas);
82         if(eas<m) return buscarRec(l,m-1,lcp,eas);
83     }
84     for(int i = lcp,n = T.size(); sa[m]+i<n && i<P.size(); i++){if(P[i
85         ]!=T[sa[m]+i]) break; lcp++;}
86     if(lcp == P.size()) return m;
87     if(l==r) return -1;
88     return (P[lcp]>T[sa[m]+lcp]?buscarRec(m+1,r,lcp,m):buscarRec(l,m-1,
89         lcp,m));
90 }
91 int buscar(){
92     int n = T.size();
93     if(P.size()>n) return -1;
94     return buscarRec(1,n-1,0,0);
95 }
96 pair<int,int> primeraYUltimaOc(){
97     int sai = buscar();

```

```

95     pair<int,int>res = {sai,sai};
96     if(sai== -1) return res;
97
98     int l,r,m;
99
100     r = sai-1; l = 0;
101     while(l<=r){
102         m = (l+r)/2;
103         if(getlce(m+1,sai)>=P.size()){
104             res.first = m; r = m-1;
105         }else{
106             l = m+1;
107         }
108     }
109     l = sai+1;r = T.size()-1;
110     while(l<=r){
111         m = (l+r)/2;
112         if(getlce(sai+1,m)>=P.size()){
113             res.second = m; l = m+1;
114         }else{
115             r = m-1;
116         }
117     }
118     return res;
119 }
120 //CODIGO DE 100 LINEAS, TE HE FALLADO MarcosK
121 //Uso: lee T, agregar signo dolar, llama makesa(); makelce(); lee P para
122     despues buscar()
123 //delete[] sa; delete[] lcest; cuando leas de nuevo T
124 //0(|T| log(|T|)) preprocesamiento, O(|P|+log**2(|T|)) cada busqueda
125 //Buscar devuelve un indice cualquiera de sa tal que el sufijo denotado
126     tenga P como prefijo
127 //Se puede hacer mas corto?

```

7 Clasicos

7.1 Job scheduling

7.1.1 One machine, linear penalty

```

1 //cuando se tiene que encontrar un orden optimo
2 //para trabajos con una funcion lineal de penalty, basta con hacer un
    sort en O(n log n)

```

```

3 struct trabajo{
4     long long int penalty,tiempo;
5     int ind;
6 };
7 bool comp(const trabajo a, const trabajo b){
8     if (a.tiempo * b.penalty == a.penalty * b.tiempo) return a.ind<b.ind
9     ;
10    return a.tiempo * b.penalty < a.penalty * b.tiempo;
11 }

```

7.1.2 One machine, deadlines

```

1 //calcula la maxima cantidad de jobs que se pueden hacer dados sus
2 //deadlines y duraciones en O(n log n)
3 struct Job {
4     int deadline, duration, idx;
5
6     bool operator<(Job o) const {
7         return deadline < o.deadline;
8     }
9 };
10 vector<int> compute_schedule(vector<Job> jobs) {
11     sort(jobs.begin(), jobs.end());
12
13     set<pair<int,int>> s;
14     vector<int> schedule;
15     for (int i = jobs.size()-1; i >= 0; i--) {
16         int t = jobs[i].deadline - (i ? jobs[i-1].deadline : 0);
17         s.insert(make_pair(jobs[i].duration, jobs[i].idx));
18         while (t && !s.empty()) {
19             auto it = s.begin();
20             if (it->first <= t) {
21                 t -= it->first;
22                 schedule.push_back(it->second);
23             } else {
24                 s.insert(make_pair(it->first - t, it->second));
25                 t = 0;
26             }
27             s.erase(it);
28         }
29     }
30     return schedule;
31 }

```

7.1.3 One machine, profit

```

1 // Dado n Jobs y su profit, calcula cual es el mayor profit que se puede
2 // obtener en O(n^2)
3 struct Job{int start, finish, profit;};
4 bool jobComparataor(Job s1, Job s2){return (s1.finish < s2.finish);}
5 // Find the latest job (in sorted array) that doesn't
6 // conflict with the job[i]. If there is no compatible job,
7 // then it returns -1.
8 vector <Job> arr;
9 int* memo;
10 int latestNonConflict( int i){
11     for (int j = i - 1; j >= 0; j--)
12         if (arr[j].finish <= arr[i - 1].start)
13             return j;
14     return -1;
15 }
16 // A recursive function that returns the maximum possible
17 // profit from given array of jobs. The array of jobs must
18 // be sorted according to finish time.
19 int findMaxProfitRec( int n){
20     // Base case
21     if (n == 1) return arr[n - 1].profit;
22     if (memo[n]>=0) return memo[n];
23     // Find profit when current job is included
24     int inclProf = arr[n - 1].profit;
25     int i = latestNonConflict(n);
26     if (i != -1) inclProf += findMaxProfitRec( i + 1);
27
28     // Find profit when current job is excluded
29     int exclProf = findMaxProfitRec( n - 1);
30
31     return memo[n]=max(inclProf, exclProf);
32 }
33 // The main function that returns the maximum possible
34 // profit from given array of jobs
35 int findMaxProfit( int n){
36     sort(arr.begin(),arr.end(), jobComparataor);
37     return findMaxProfitRec(n);
38 }

```

7.1.4 Two machines, min time

```

1 //Obtiene el ordenamiento optimo de Jobs en dos maquinas en O(n log n)
2 struct Job {
3     int a, b, idx;
4     bool operator<(Job o) const {return min(a, b) < min(o.a, o.b);}
5 };
6 vector<Job> johnsons_rule(vector<Job> jobs) {
7     sort(jobs.begin(), jobs.end());
8     vector<Job> a, b;
9     for (Job j : jobs) {
10         if (j.a < j.b)
11             a.push_back(j);
12         else
13             b.push_back(j);
14     }
15     a.insert(a.end(), b.rbegin(), b.rend());
16     return a;
17 }
18
19 pair<int, int> finish_times(vector<Job> const& jobs) {
20     int t1 = 0, t2 = 0;
21     for (Job j : jobs) {
22         t1 += j.a;
23         t2 = max(t2, t1) + j.b;
24     }
25     return make_pair(t1, t2);
26 }

```

8 Flow

8.1 Dinic, thx GGDem

```

1 #define pb push_back
2 #define mp make_pair
3 #define fst first
4 #define snd second
5 #define ALL(s) s.begin(),s.end()
6 #define SZ(x) int((x).size())
7 #define fore(i,a,b) for(int i=a,to=b;i<to;++i)
8 using namespace std;
9 typedef long long ll;
10
11 #define INF (1LL<<62)
12 // Min cut: nodes with dist>=0 vs nodes with dist<0

```

```

13 // Matching MVC: left nodes with dist<0 + right nodes with dist>0
14 struct Dinic{
15     int nodes,src,dst;
16     vector<int> dist,q,work;
17     struct edge {int to,rev;ll f,cap;};
18     vector<vector<edge>> g;
19     Dinic(int x):nodes(x),g(x),dist(x),q(x),work(x){}
20     void add_edge(int s, int t, ll cap){
21         g[s].pb((edge){t,SZ(g[t]),0,cap});
22         g[t].pb((edge){s,SZ(g[s])-1,0,0});
23     }
24     bool dinic_bfs(){
25         fill(ALL(dist),-1);dist[src]=0;
26         int qt=0;q[qt++]=src;
27         for(int qh=0;qh<qt;qh++){
28             int u=q[qh];
29             fore(i,0,SZ(g[u])){
30                 edge &e=g[u][i];int v=g[u][i].to;
31                 if(dist[v]<0&&e.f<e.cap)dist[v]=dist[u]+1,q[qt++]=v;
32             }
33         }
34         return dist[dst]>=0;
35     }
36     ll dinic_dfs(int u, ll f){
37         if(u==dst)return f;
38         for(int &i=work[u];i<SZ(g[u]);i++){
39             edge &e=g[u][i];
40             if(e.cap<=e.f)continue;
41             int v=e.to;
42             if(dist[v]==dist[u]+1){
43                 ll df=dinic_dfs(v,min(f,e.cap-e.f));
44                 if(df>0){e.f+=df;g[v][e.rev].f-=df;return df;}
45             }
46         }
47         return 0;
48     }
49     ll max_flow(int _src, int _dst){
50         src=_src;dst=_dst;
51         ll result=0;
52         while(dinic_bfs()){
53             fill(ALL(work),0);
54             while(ll delta=dinic_dfs(src,INF))result+=delta;
55         }

```

```

56     return result;
57 }
58 };
59
60 //-----SOLBEGIN-----
61 int main() {
62     ios_base::sync_with_stdio(false); cout.tie(NULL); cin.tie(NULL);
63     //l set,r set
64     int n,m;
65     cin>>n>>m;
66     m+=n;
67     Dinic d(n+m+2);
68     for(int i = 1; i<=n; i++) d.add_edge(0,i,1);
69     for(int i = n+1; i<=m; i++) d.add_edge(i,m+1,1);
70
71     int fin,q;
72     for(int i = 1; i<=n; i++){
73         cin>>q;
74         while(q--){
75             cin>>fin;
76             d.add_edge(i,n+fin,1);
77         }
78     }
79     int res =d.max_flow(0,m+1);
80     m-=n;
81     //how many were left unmatched
82     cout<<m-res<<endl;
83 }
84 //-----EOSOLUTION-----

```

9 Miscellaneous

9.1 pbds

```

1 #include "bits/stdc++.h"
2 #include <bits/extc++.h>
3 using namespace __gnu_pbds;
4 using namespace std;
5 typedef tree<pair<int,int>, null_type,less<pair<int,int>>, rb_tree_tag,
6     tree_order_statistics_node_update> ost;
7 using namespace std;
8 int main(){
9     ost arbol;

```

```

9     int n = 5;
10    for(int id = 1; id<=n; id++)
11        for(int val = 0; val<n; val++)
12            arbol.insert({val,id});
13    //te da el valor mas pequeno, en caso de empate te da el del id mas
14    //pequeno
15    cout<<(*arbol.find_by_order(0)).first<<" " <<(*arbol.find_by_order(0)
16    ).second<<endl;
17    //te da el indice (base 0) de la primera ocurrencia de .first
18    cout<<arbol.order_of_key({1,-1})<<endl;
19 }

```

9.2 Bit Manipulation

```

1 #include "bits/stdc++.h"
2 using namespace std;
3 #define endl '\n'
4
5
6 int main() {
7     ios_base::sync_with_stdio(false); cout.tie(NULL); cin.tie(NULL);
8     //Se representan bitmasks de 30 a 62 bits
9     //usando signed int y signed long long int
10    //para evitar problemas con el complemento de dos
11    signed int a, b;
12    //para multiplicar un numero por dos solo es necesario aplicar un
13    //shifteo de sus bits a la izquierda
14    a = 1;
15    a= a << 3;
16    cout << a << endl;
17    //para dividir un numero entre dos es necesario aplicar un
18    //shifteo a la derecha
19    a = 32;
20    a = a >> 3;
21    cout << a << endl;
22    //para encender el bit n de a, solo hay que igualar a = a | pow(2,n-1)
23    //prende el tercer bit
24    a = 1;
25    b = 1 << 2;
26    a = a | b;
27    cout << a << endl;
28    //para apagar el bit n de a, solo hay que a &= ~pow(2,n-1)
29    //prende el tercer bit

```

```

30 a = 5;
31 b = 1 << 2;
32 a &= ~b;
33 cout << a << endl;
34 //para revisar si el bit n de a esta encendido
35 //revisa si el tercer bit esta encendido
36 a = 5;
37 b = 1 << 2;
38 a = a & b;
39 cout << (a?"SI":"NO") << endl;
40 //para volter el bit n de a, solo hay que igualar a = a ^ pow(2,n-1)
41 //apaga el tercer bit
42 a = 5;
43 b = 1 << 2;
44 a = a ^ b;
45 cout << a << endl;
46 //para obtener el bit menos significativo que esta encendido a& -a
47 a = 12;
48 cout << log2(a & ((-1) * a))+1 << endl;
49 //para prender todos los bits hasta n
50 a = (1<<4)-1;
51 cout << a << endl;
52 }
53 //-----EOSOLUTION-----

```

```

1 #include "bits/stdc++.h"
2 using namespace std;
3 #define endl '\n'
4 #pragma GCC optimize("O3")
5 #pragma GCC target("popcnt")
6
7 //no usar con visual c++
8 //solo con g++ like compilers
9 int main() {
10 ios_base::sync_with_stdio(false); cout.tie(NULL); cin.tie(NULL);
11 signed long long int a, b, n;
12 //Obtain the remainder (modulo) of a when it is divided by n (n is a
    power of 2)
13 a = 15; n = 8-1;
14 a &= n;
15 cout << "a%n, a%u=15, n%u=2^3" << endl;
16 cout << a << endl;
17 //Apaga el bit menos significativo de a

```

```

18 a = 14;
19 b = (a & ((-1) * a));
20 a &= ~b;
21 cout << a << endl;
22 //enciende el ultimo cero de a
23 a = 9;
24 b = ~a;
25 b = (b & ((-1) * b));
26 a = a | b;
27 cout << a<<endl;
28 //contar bits encendidos en a
29 cout << __builtin_popcount(a)<<endl;
30 //chechar la paridad de a
31 cout << (__builtin_parity(a) ? "IMPAR" : "PAR") << endl;
32 //contar leading zeroes en a
33 cout << __builtin_clz(a)<<endl;
34 //contar 9,trailling zeroes en a
35 cout << __builtin_ctz(a)<<endl;
36 }
37 //-----EOSOLUTION-----

```

10 Testing

10.1 Gen and AutoRun testcases

10.1.1 Gen.cpp

```

1 #include <iostream>
2 #include <string.h>
3 #include <random>
4 #include <chrono>
5 using namespace std;
6 //args nombreDelEjecutable,seed, len
7 int main (int argc, char **argv) {
8     // argv is an array of strings
9     // atoi is a C function for converting a string into an int
10    mt19937 rng(chrono::steady_clock::now().time_since_epoch().count());
11    srand(atoi(argv[1])); // srand sets the random seed
12    int n = atoi(argv[2]);
13    int d = rng()%n; d++;
14    string test = "";
15    for (int i = 0; i < n; i++) {
16        test+= 'a'+(rng()%26);

```

```

17 }
18 cout<<test<<"□"<<d<<endl;
19 }

```

10.1.2 Stress testing

```

1 g++ -std=c++14 gen.cpp -o gen
2 g++ -std=c++14 lazy.cpp -v -o lazy
3 g++ -std=c++14 lazyn.cpp -v -o lazyn
4 for i in `seq 1 $1`; do
5     # prints the current test number
6     # I like to do this so I can see progress is being made
7     #chmod +x test.sh
8     echo $i
9     ./gen $i $((1 + i%14)) > input.txt #pasa al generador una longitud
10    entre 1 y 14, para hacer operaciones matematicas, usar $((a+b))
11    ./lazy < input.txt > output.txt
12    ./lazyn < input.txt > answer.txt
13    diff output.txt answer.txt || break
14 done

```

10.1.3 Autorun

```

1 g++ -std=c++14 gen.cpp -o gen
2 g++ -std=c++14 lazy.cpp -v -o lazy
3 for i in `seq 1 $1`; do
4     # prints the current test number
5     # I like to do this so I can see progress is being made
6     #chmod +x test.sh
7     echo $i
8
9     ./gen $i $((1 + i%14)) > input.txt
10    ./lazy < i${i}.txt > o${i}.txt
11
12    diff a${i}.txt o${i}.txt || break
13 done

```

10.2 Highly Composite Numbers

Particularly useful when testing number theoretical solutions.

1	1	
2	2	2
3	4	2 ²

4	6	4	2*3
5	12	6	2 ² *3
6	24	8	2 ³ *3
7	36	9	2 ² *3 ²
8	48	10	2 ⁴ *3
9	60	12	2 ² *3*5
10	120	16	2 ³ *3*5
11	180	18	2 ² *3 ² *5
12	240	20	2 ⁴ *3*5
13	360	24	2 ³ *3 ² *5
14	720	30	2 ⁴ *3 ² *5
15	840	32	2 ³ *3*5*7
16	1260	36	2 ² *3 ² *5*7
17	1680	40	2 ⁴ *3*5*7
18	2520	48	2 ³ *3 ² *5*7
19	5040	60	2 ⁴ *3 ² *5*7
20	7560	64	2 ³ *3 ³ *5*7
21	10080	72	2 ⁵ *3 ² *5*7
22	15120	80	2 ⁴ *3 ³ *5*7
23	20160	84	2 ⁶ *3 ² *5*7
24	25200	90	2 ⁴ *3 ² *5 ² *7
25	27720	96	2 ³ *3 ² *5*7*11
26	45360	100	2 ⁴ *3 ⁴ *5*7
27	50400	108	2 ⁵ *3 ² *5 ² *7
28	55440	120	2 ⁴ *3 ² *5*7*11
29	83160	128	2 ³ *3 ³ *5*7*11
30	110880	144	2 ⁵ *3 ² *5*7*11
31	166320	160	2 ⁴ *3 ³ *5*7*11
32	221760	168	2 ⁶ *3 ² *5*7*11
33	277200	180	2 ⁴ *3 ² *5 ² *7*11
34	332640	192	2 ⁵ *3 ³ *5*7*11
35	498960	200	2 ⁴ *3 ⁴ *5*7*11
36	554400	216	2 ⁵ *3 ² *5 ² *7*11
37	665280	224	2 ⁶ *3 ³ *5*7*11
38	720720	240	2 ⁴ *3 ² *5*7*11*13
39	1081080	256	2 ³ *3 ³ *5*7*11*13
40	1441440	288	2 ⁵ *3 ² *5*7*11*13
41	2162160	320	2 ⁴ *3 ³ *5*7*11*13
42	2882880	336	2 ⁶ *3 ² *5*7*11*13
43	3603600	360	2 ⁴ *3 ² *5 ² *7*11*13
44	4324320	384	2 ⁵ *3 ³ *5*7*11*13
45	6486480	400	2 ⁴ *3 ⁴ *5*7*11*13
46	7207200	432	2 ⁵ *3 ² *5 ² *7*11*13

47	8648640	448	$2^6 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$	90	240940299600	4800	$2^4 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
48	10810800	480	$2^4 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$	91	293318625600	5040	$2^6 \cdot 3^4 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19$
49	14414400	504	$2^6 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$	92	321253732800	5376	$2^6 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
50	17297280	512	$2^7 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$	93	481880599200	5760	$2^5 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
51	21621600	576	$2^5 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$	94	642507465600	6144	$2^7 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
52	32432400	600	$2^4 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$	95	963761198400	6720	$2^6 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
53	36756720	640	$2^4 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17$	96	1124388064800	6912	$2^5 \cdot 3^3 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
54	43243200	672	$2^6 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13$	97	1606268664000	7168	$2^6 \cdot 3^3 \cdot 5^3 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
55	61261200	720	$2^4 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17$	98	1686582097200	7200	$2^4 \cdot 3^4 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
56	73513440	768	$2^5 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17$	99	1927522396800	7680	$2^7 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
57	110270160	800	$2^4 \cdot 3^4 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17$	100	2248776129600	8064	$2^6 \cdot 3^3 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
58	122522400	864	$2^5 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17$	101	3212537328000	8192	$2^7 \cdot 3^3 \cdot 5^3 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
59	147026880	896	$2^6 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17$	102	3373164194400	8640	$2^5 \cdot 3^4 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
60	183783600	960	$2^4 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17$	103	4497552259200	9216	$2^7 \cdot 3^3 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
61	245044800	1008	$2^6 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17$	104	6746328388800	10080	$2^6 \cdot 3^4 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
62	294053760	1024	$2^7 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17$	105	8995104518400	10368	$2^8 \cdot 3^3 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
63	367567200	1152	$2^5 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17$	106	9316358251200	10752	$2^6 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29$
64	551350800	1200	$2^4 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17$	107	13492656777600	11520	$2^7 \cdot 3^4 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
65	698377680	1280	$2^4 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19$	108	18632716502400	12288	$2^7 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29$
66	735134400	1344	$2^6 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17$	109	26985313555200	12960	$2^8 \cdot 3^4 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$
67	1102701600	1440	$2^5 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17$	110	27949074753600	13440	$2^6 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29$
68	1396755360	1536	$2^5 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19$	111	32607253879200	13824	$2^5 \cdot 3^3 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29$
69	2095133040	1600	$2^4 \cdot 3^4 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19$	112	46581791256000	14336	$2^6 \cdot 3^3 \cdot 5^3 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29$
70	2205403200	1680	$2^6 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17$	113	48910880818800	14400	$2^4 \cdot 3^4 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29$
71	2327925600	1728	$2^5 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19$	114	55898149507200	15360	$2^7 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29$
72	2793510720	1792	$2^6 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19$	115	65214507758400	16128	$2^6 \cdot 3^3 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29$
73	3491888400	1920	$2^4 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19$	116	93163582512000	16384	$2^7 \cdot 3^3 \cdot 5^3 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29$
74	4655851200	2016	$2^6 \cdot 3^2 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19$	117	97821761637600	17280	$2^5 \cdot 3^4 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29$
75	5587021440	2048	$2^7 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19$	118	130429015516800	18432	$2^7 \cdot 3^3 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29$
76	6983776800	2304	$2^5 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19$	119	195643523275200	20160	$2^6 \cdot 3^4 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29$
77	10475665200	2400	$2^4 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19$	120	260858031033600	20736	$2^8 \cdot 3^3 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29$
78	13967553600	2688	$2^6 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19$	121	288807105787200	21504	$2^6 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31$
79	20951330400	2880	$2^5 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19$	122	391287046550400	23040	$2^7 \cdot 3^4 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29$
80	27935107200	3072	$2^7 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19$	123	577614211574400	24576	$2^7 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31$
81	41902660800	3360	$2^6 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19$	124	782574093100800	25920	$2^8 \cdot 3^4 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29$
82	48886437600	3456	$2^5 \cdot 3^3 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19$	125	866421317361600	26880	$2^6 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31$
83	64250746560	3584	$2^6 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$	126	1010824870255200	27648	$2^5 \cdot 3^3 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31$
84	73329656400	3600	$2^4 \cdot 3^4 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19$	127	1444035528936000	28672	$2^6 \cdot 3^3 \cdot 5^3 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31$
85	80313433200	3840	$2^4 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$	128	1516237305382800	28800	$2^4 \cdot 3^4 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31$
86	97772875200	4032	$2^6 \cdot 3^3 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19$	129	1732842634723200	30720	$2^7 \cdot 3^4 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31$
87	128501493120	4096	$2^7 \cdot 3^3 \cdot 5 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$	130	2021649740510400	32256	$2^6 \cdot 3^3 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31$
88	146659312800	4320	$2^5 \cdot 3^4 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19$	131	2888071057872000	32768	$2^7 \cdot 3^3 \cdot 5^3 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31$
89	160626866400	4608	$2^5 \cdot 3^3 \cdot 5^2 \cdot 7 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23$	132	3032474610765600	34560	$2^5 \cdot 3^4 \cdot 5^2 \cdot 7^2 \cdot 11 \cdot 13 \cdot 17 \cdot 19 \cdot 23 \cdot 29 \cdot 31$

133	4043299481020800	36864	2^7*3^3*5^2*7^2*11*13*17*19*23*29*31
134	6064949221531200	40320	2^6*3^4*5^2*7^2*11*13*17*19*23*29*31
135	8086598962041600	41472	2^8*3^3*5^2*7^2*11*13*17*19*23*29*31
136	10108248702552000	43008	2^6*3^3*5^3*7^2*11*13*17*19*23*29*31
137	12129898443062400	46080	2^7*3^4*5^2*7^2*11*13*17*19*23*29*31
138	18194847664593600	48384	2^6*3^5*5^2*7^2*11*13*17*19*23*29*31
139	20216497405104000	49152	2^7*3^3*5^3*7^2*11*13*17*19*23*29*31
140	24259796886124800	51840	2^8*3^4*5^2*7^2*11*13*17*19*23*29*31
141	30324746107656000	53760	2^6*3^4*5^3*7^2*11*13*17*19*23*29*31
142	36389695329187200	55296	2^7*3^5*5^2*7^2*11*13*17*19*23*29*31
143	48519593772249600	57600	2^9*3^4*5^2*7^2*11*13*17*19*23*29*31
144	60649492215312000	61440	2^7*3^4*5^3*7^2*11*13*17*19*23*29*31
145	72779390658374400	62208	2^8*3^5*5^2*7^2*11*13*17*19*23*29*31
146	74801040398884800	64512	2^6*3^3*5^2*7^2*11*13*17*19*23*29*31*37
147	106858629141264000	65536	2^7*3^3*5^3*7*11*13*17*19*23*29*31*37
148	112201560598327200	69120	2^5*3^4*5^2*7^2*11*13*17*19*23*29*31*37
149	149602080797769600	73728	2^7*3^3*5^2*7^2*11*13*17*19*23*29*31*37
150	224403121196654400	80640	2^6*3^4*5^2*7^2*11*13*17*19*23*29*31*37
151	299204161595539200	82944	2^8*3^3*5^2*7^2*11*13*17*19*23*29*31*37
152	374005201994424000	86016	2^6*3^3*5^3*7^2*11*13*17*19*23*29*31*37
153	448806242393308800	92160	2^7*3^4*5^2*7^2*11*13*17*19*23*29*31*37
154	673209363589963200	96768	2^6*3^5*5^2*7^2*11*13*17*19*23*29*31*37
155	748010403988848000	98304	2^7*3^3*5^3*7^2*11*13*17*19*23*29*31*37
156	897612484786617600	103680	2^8*3^4*5^2*7^2*11*13*17*19*23*29*31*37