First to Penalty

-12

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1 Template

```
#include "bits/stdc++.h"
  //assert(x>0) si falla da RTE
  using namespace std;
  #define endl '\n'
  #define DBG(x) cerr<<\#x<< "=" << (x) << endl:
  #define RAYA cerr<<"========"<<endl:
  #define RAYAS cerr<<"...."<<endl;</pre>
  //#define DBG(x) :
   //#define RAYA ;
  //#define RAYAS ;
11
   //----SOLBEGIN-----
  int main() {
    ios_base::sync_with_stdio(false); cout.tie(NULL); cin.tie(NULL);
14
    int tC;
15
16
    cin >> tC;
17
    while (tC--) {
18
19
    }
20
21
^{22}
         -----EOSOLUTION-----
```

2 Data structures

2.1 Simplified DSU (Stolen from GGDem)

```
int uf[MAXN];
void uf_init(){memset(uf,-1,sizeof(uf));}
int uf_find(int x){return uf[x]<0?x:uf[x]=uf_find(uf[x]);}

bool uf_join(int x, int y){
    x=uf_find(x);y=uf_find(y);
    if(x==y)return false;
    if(uf[x]>uf[y])swap(x,y);
    uf[x]+=uf[y];uf[y]=x;
    return true;
}
```

2.2 Disjoint Set Union

```
class disjSet {
     int* sz;
     int* par;
   public:
     int len;
     disjSet(int tam){
           sz = new int[tam + 4]();
           par = new int[tam + 4]();
           len = 0;
           for(int i = 0; i<=tam; i++){</pre>
               par[i] = i;
11
                sz[i] = 1;
12
               len++;
13
           }
       }
15
     int finds(int el){
16
           if (el == par[el]) return el;
17
           return par[el] = finds(par[el]);
18
       }
19
     void unions(int a, int b){
           a = finds(a);
21
         b = finds(b);
22
           if (a == b) return;
23
           len--;
24
           //se hace que el gde sea padre del pequeno
25
           if (sz[a] > sz[b]) swap(a,b);
           par[a] = b;
27
           sz[b] += sz[a];
28
       }
29
      ~disjSet(){
30
           delete[] size;
31
           size = nullptr;
32
           delete[] parent;
33
           parent = nullptr;
34
35
36 };
                            2.3 Segment tree
```

```
//MAXN = 2^k, n = tam arreglo inicial
int stsize; long long int neut;int n;
long long int* st = new long long int[2*MAXN-1]();
long long int fst(long long int a, long long int b);
```

```
5 long long int build(int sti,int csize){
       if(csize == 1) return st[sti];
6
       return st[sti] = fst(build(sti*2+1,csize/2),build(sti*2+2,csize/2));
8
   void innit(){
9
       for(int i = 0; i<stsize; i++) st[i] = neut;</pre>
       /*int d = 0;
11
       for(int i = stsize-n; i<stsize && d<n; i++){</pre>
12
           st[i] = arr[d];d++;
13
       }*/
14
       build(0,n);
15
16
   void upd(int ind, long long int val){
       ind = stsize-n+ind:
18
       st[ind] = val;ind--;ind/=2;
19
       while(true){
20
           st[ind] = fst(st[ind*2+1],st[ind*2+2]);
21
           ind--;
22
           if(ind<0) break:
23
           ind/=2;
24
       }
25
26
   long long int rqu(int 1, int r,int sti, int ls, int rs){
27
       if(l<=ls && rs<= r) return st[sti];</pre>
28
       if(r<ls || l>rs) return neut;
29
       int m = (rs+ls)/2;
30
       return fst(rqu(1,r,sti*2+1,ls,m),rqu(1,r,sti*2+2,m+1,rs));
31
32
   long long int query(int 1, int r){
33
       return rqu(1,r,0,0,n-1);
34
35
   //uso, inicializa neut, determina n (asegurate que sea una potencia de
       2), define fst para determinar
37 //la opracion del segment tree
                         2.4 Segment tree Lazy
```

```
//MAXN = 2^k, n = tam arreglo inicial
#define MAXN 524288

vector<int> arr;
int stsize; long long int neut;int n;
long long int* st = new long long int[2*MAXN-1]();
long long int* pendientes = new long long int[2*MAXN-1]();
```

```
7 | long long int fst(long long int a, long long int b){return a+b;}
  long long int build(int sti,int csize){
       if(csize == 1) return st[sti];
9
       return st[sti] = fst(build(sti*2+1,csize/2),build(sti*2+2,csize/2));
10
   }
11
   bool hasChildren(int sti){sti*=2;sti++;sti++;return sti<stsize;}</pre>
   void innit(){
       for(int i = 0; i<stsize; i++) st[i] = neut;</pre>
       int d = 0:
       for(int i = stsize-n-1; i<stsize && d<n; i++) {st[i] = arr[d];d++;}
       build(0,n);
17
   }
18
   void updrec(int 1,int r, int s1, int sr,int sti, long long int val){
       if(sr<l | r< sl) return:
20
       if(1<= s1 && sr <=r){
21
           st[sti] += val*(sr-sl+1);
           if(hasChildren(sti)){pendientes[sti*2+1]+=val;pendientes[sti
23
                *2+2]+=val;}
24
           return:
       }
26
       int sm = (sl+sr)/2;
       updrec(l,r,sl,sm,sti*2+1,val);
28
       updrec(l,r,sm+1,sr,sti*2+2,val);
29
       st[sti] = fst(st[sti*2+1],st[sti*2+2]);
30
31
   void upd(int 1, int r, long long int val){updrec(1,r,0,n-1,0,val);}
33
   long long int rqu(int 1, int r, int sti, int ls, int rs){
34
       if(r<ls || l>rs) return neut:
       if(1<=1s && rs<= r){
36
           return st[sti]+pendientes[sti]*(rs-ls+1);
37
       }
38
39
       st[sti] += pendientes[sti]*(rs-ls+1);
40
       if(hasChildren(sti)){pendientes[sti*2+1]+=pendientes[sti];pendientes
41
           [sti*2+2]+=pendientes[sti];}
       pendientes[sti] = 0;
43
       int m = (rs+ls)/2;
       return fst(rqu(1,r,sti*2+1,ls,m),rqu(1,r,sti*2+2,m+1,rs));
45
46
47 long long int query(int 1, int r){
```

```
return rqu(l,r,0,0,n-1);

//uso, inicializa neut, lee n y arr, iguala n a la potencia de dos mas
cercana y mayor

//determina stsize = 2*n (asegurate que sea una potencia de 2), define
fst para determinar

//la opracion del segment tree
```

2.5 Trie

```
struct triver {
       char alphabet;
2
       bool ter;
3
       vector<triver*> child;
4
       triver(char a): alphabet(a) { child.assign(26, NULL); ter = false; }
5
6
   class trie{
   private:
       triver* root;
   public:
10
       trie() { root = new triver('!');}
11
       void insert(string s){
12
           triver* curr = root;
13
           for(char 1: s){
14
                if(curr->child[l-'A'] == NULL) curr->child[l-'A'] = new
15
                    triver(1);
                curr = curr->child[l-'A'];
16
           }
17
            curr->ter = true;
18
       }
19
       bool search(string s){
20
           triver* curr = root;
^{21}
           for(char 1: s){
^{22}
                if(curr == NULL) break;
23
                curr = curr->child[1-'A'];
24
25
           if(curr == NULL) return false;
26
           return curr->ter;
27
28
  |};
29
```

3 Graphs

3.1 Graph Transversal

3.1.1 BFS

```
#define GS 400040
  vector<int> graph[GS];
   bitset <GS> vis;
   //anchura O(V+E)
   void dfs(int curr) {
     queue<int> fringe;
     fringe.push(curr);
     while (fringe.size()) {
       curr = fringe.front(); fringe.pop();
       if (!vis[curr]) {
10
         vis[curr] = 1:
         for (int h : graph[curr]) fringe.push(h);
12
13
    }
14
15 }
                                 3.1.2 DFS
```

```
#define GS 400040
   vector<int> graph[GS];
   bitset <GS> vis;
   //profundidad O(V+E)
   void dfs(int curr) {
     stack<int> fringe;
     fringe.push(curr);
     while (fringe.size()){
       curr = fringe.top(); fringe.pop();
       if (!vis[curr]) {
10
         vis[curr] = 1;
11
         for (int h : graph[curr]) fringe.push(h);
12
13
     }
14
15 }
```

3.2 Topological Sort

```
#define GS 400040
vector<int> graph[GS];
```

for(int f = 0; f < = tam; f + +)

dist[i][f] = graph[i][f];

for(int ini = 0; ini<=tam; ini++)</pre>

//para reconstruir el camino solo basta con guardar intermedio como

el padre de ini si el cambio se hizo, -1 otherwise

for(int intermedio = 0; intermedio<=tam; intermedio++)</pre>

15

16

17

18

19

20

```
3 | bitset <GS> vis;
                                                                                                  for(int fin = 0; fin<=tam; fin++)</pre>
                                                                                  21
                                                                                                       dist[ini][fin] = min(dist[ini][fin],dist[ini][intermedio
   vector<int> topsort;
                                                                                   22
                                                                                                           ]+dist[intermedio][fin]);
   int e,n;
   //profundidad
                                                                                   23 }
   //O(N+E)
                                                                                                                    3.4 SSSP
   //Solo funciona con DAG's, no existe un top sort de un grafo Non-DAG
   void todfs(int pa) {
                                                                                                               3.4.1 Lazy Dijkstra
     vis[pa]=1;
10
     for(int h: graph[pa]){if(!vis[h]){todfs(h);}}
                                                                                   1 #define GS 1000
11
     topsort.push_back(pa);
                                                                                      #define INF 100000000
12
13
                                                                                      //destino, costo
   void topologicalSort(){
                                                                                      vector<pair<int,int>> graph[GS];
     vis.reset():
                                                                                      int dist[GS];
15
     topsort.clear();
                                                                                      void dijkstra(int origen,int tam){
     for(int i = 0; i<n; i++){if(!vis[i]){dfs(i);}}</pre>
                                                                                          for(int i = 0; i<=tam; i++){</pre>
     reverse(topsort.begin(),topsort.end());
                                                                                              dist[i] = INF;
                                                                                   8
19 }
                                                                                   9
                                                                                          priority_queue<pair<int,int>,vector<pair<int,int>>, greater<pair<int</pre>
                                                                                   10
                      3.3 APSP: Floyd Warshall
                                                                                               ,int>>> pq;
                                                                                          pair<int,int> curr;
                                                                                   11
  #define GS 1000
                                                                                   12
   #define INF 100000000
                                                                                          pq.push(make_pair(0,origen));
                                                                                   13
   //destino, costo
                                                                                   14
   int graph[GS][GS];
                                                                                          while(pq.size()){
                                                                                   15
                                                                                              curr = pq.top();pq.pop();
   //All Pairs Dist
                                                                                   16
   int dist[GS][GS]:
                                                                                              if(curr.first >= dist[curr.second]) continue:
                                                                                   17
  //Toma en cuenta nodos [0-tam] inclusivo, modificar de acuerdo a las
                                                                                   18
       necesidades
                                                                                              dist[curr.second] = curr.first;
                                                                                   19
                                                                                              for(pair<int,int> h: graph[curr.second]){
  //Ten cuidado con el valor que le pones a INF, puede provocar overflows
                                                                                   20
                                                                                                  if((h.second+curr.first)<dist[h.first]) pq.push({h.second+</pre>
       o puede no ser lo suficientemente grande.
                                                                                  21
   void Floyd_Warshall(int tam){
                                                                                                       curr.first,h.first});
       for(int i = 0; i<=tam; i++)</pre>
                                                                                              }
10
                                                                                  22
           for(int f = 0; f<=tam; f++)</pre>
                                                                                   23
11
               dist[i][f] = INF;
                                                                                      }
                                                                                   24
12
                                                                                      //Esta es la implementacion huevona
13
       for(int i = 0; i<=tam; i++)
                                                                                      //Resuelve Single Source Shortest Paths con aristas positivas
14
```

3.4.2 Bellman-Ford

//Como es la lazy implementation, si funciona con edges negativos

29 //Si no los hay puede que funcione en O((V+E)log(V)) o puede que se

siempre y cuando no hayan ciclos negativos

exponencial, si no jala prueba BellmanFord

28 //Si hay ciclos negativos se va atascar en un ciclo infinito

```
1 //esta es la implementacion huevona
   #define GS 1000
2
   //cuidado con overflows!!
   #define INF 100000000
   #define NINF -10000000
   //destino, costo
   vector<pair<int,int>> graph[GS];
   int dist[GS];
   struct edge{
       int from, to, cost;
10
   };
11
   //Corre en O(VE)
   void bellmanFord(int origen,int tam){
       for(int i = 0; i<=tam; i++){</pre>
14
           dist[i] = INF;
15
       }
16
       dist[origen] = 0;
17
       edge aux;
18
       vector<edge> aristas;
19
       bool optimal;
20
21
       for(int i = 0; i<=tam; i++){</pre>
22
           for(pair<int,int> h: graph[i]){
23
                aux.from = i; aux.to = h.first;aux.cost = h.second;
24
                aristas.push_back(aux);
25
26
       }
27
28
       //Si se relajan todos las aristas V-1 veces en un orden arbitrario
29
       //Se asegura que la distancia optima para cada vertice sera
30
           alcanzada
       for(int i = 0; i<tam && !optimal; i++){</pre>
31
           optimal = true;
32
           for(edge elem: aristas){
33
                if(dist[elem.from] + elem.cost < dist[elem.to]){</pre>
34
                    dist[elem.to] = dist[elem.from] + elem.cost;
35
                    //si algun vertice fue actualizado significa que puede
36
                    //las distancias aun no sean optimas
37
                    optimal = false;
38
39
           }
40
41
```

```
42
       //Se corre de nuevo para asegurar encontrar todos los ciclos
43
            negativos
       for(int i = 0; i<tam && !optimal; i++){</pre>
44
            optimal = true;
45
            for(edge elem: aristas){
46
                if(dist[elem.from] + elem.cost < dist[elem.to]){</pre>
47
                    //Si aun despues de correr V-1 veces se puede actualizar
48
                    //Significa que esta en un ciclo negativo
49
                    dist[elem.to] = NINF;
                    //si algun vertice fue actualizado significa que puede
51
                    //las distancias aun no sean optimas
52
                    optimal = false;
53
                }
54
            }
55
       }
56
57
58 }
```

3.5 Strongly Connected Components: Kosaraju

```
1 #define GS 2010
   vector<int> graph[GS];
   vector<int> graphI[GS];
   vector<int> orden;
   bitset<GS> vis;
6
   void invertirGrafo(int n){
       for(int p = 1; p <= n; p++)
8
           for(int h: graph[p])graphI[h].push_back(p);
9
10
   void obtOrd(int p,int n){
11
       vis[p] = 1;
12
       for(int h: graph[p]){
13
           if(!vis[h] && h<=n) obtOrd(h,n);</pre>
14
15
       orden.push_back(p);
16
   }
17
   int findSCC(int n){
18
       int res = 0;
19
       invertirGrafo(n);
20
       orden.clear();
21
```

```
for(int i = 1; i<=n; i++) vis[i] =0;
22
       for(int i = 1; i<=n; i++) if(!vis[i]) obtOrd(i,n);</pre>
23
       reverse(orden.begin(),orden.end());
^{24}
       //cuenta los connected components
25
       //vector<int> lscc;
26
       stack<int> fringe;
27
       int curr;
28
       for(int i = 1; i<=n; i++) vis[i] =0;</pre>
29
       for(int i: orden){
30
           //lscc.clear();
31
           if(!vis[i]){
32
                fringe.push(i);
33
                while (fringe.size()){
34
                    curr = fringe.top();fringe.pop();
35
                    //lscc.push_back(curr);
36
                    if (!vis[curr]) {
37
                        vis[curr] = 1:
38
                        for (int h : graphI[curr]) fringe.push(h);
39
                    }
40
                }
41
                res++;
42
43
           //hacer lo que sea con lcss
44
       }
45
       return res;
46
47
48
    //OJO esto solo jala con directed graphs
   //por definicion todas las undirected graphs tienen un solo SCC
   //NOTAR QUE LOS GRAFOS QUE USA CUMPLEN CON: O<=VERTICE<=n
```

3.6 Articulation Points and Bridges: ModTarjan

```
#define GS 50
vector<int> graph[GS];
bitset<GS> vis, isArtic;
vector<int> padre;
//id por tiempo, menor id accesible
//ya sea por descendientes o por back edges
vector<int> tId,lId;
//cantidad de hijos que tiene en el bfs spanning tree
int rootChildren;
int cnt;
```

```
int dfsRoot;
   void findAP_B(int p){
       cnt++;vis[p] = 1;tId[p] = cnt;lId[p] = tId[p];
13
14
       for(int hijo: graph[p]){
15
           if(!vis[hijo]){
16
                padre[hijo] = p;
17
                if(p == dfsRoot) rootChildren++;
18
19
                findAP_B(hijo);
20
21
                //esto significa que ni por un back edge el hijo accede al
22
                    padre
                //por lo que si el padre fuese eliminado el hijo quedaria
23
                    aislado
                if(lId[hijo] >= tId[p]) isArtic[p] = 1;
24
                if(lId[hijo] > tId[p]){
25
                    //esto significa que si se eliminase el camino de padre
26
                        ->hiio
                    //se lograria desconectar el grafo, aka bridge
27
                }
28
                lId(p) = min(lId(p),lId(hijo));
29
           }else{
30
                //si hay un ciclo indirecto, actualiza el valor para el
31
                if(hijo != padre[p]) lId[p] = min(lId[p],tId[hijo]);
32
33
       }
34
35
    //OJO esto solo jala con Undirected graphs
   /*
37
       MAIN
38
       for(int i = 0; i < n; i++){
39
           if(!vis[i]){
40
                rootChildren = 0;
41
                dfsRoot = i;
42
                findAP_B(i);
43
                //el algoritmo no puede detectar si el nodo que lo origino
44
                //es un articulation point, por lo que queda checar si
45
                //en el spanning tree que genero tiene mas de un solo hijo
46
               isArtic[i] = (rootChildren>1?1:0);
47
48
49
```

```
50 */
```

4 Math

4.1 Identities

```
C_n = \frac{2(2n-1)}{n+1}C_{n-1} C_n = \frac{1}{n+1}\binom{2n}{n} C_n \sim \frac{4^n}{n^{3/2}\sqrt{\pi}} \sigma(n) = O(\log(\log(n))) \text{ (number of divisors of } n) F_{2n+1} = F_n^2 + F_{n+1}^2 F_{2n} = F_{n+1}^2 - F_{n-1}^2 \sum_{i=1}^n F_i = F_{n+2} - 1 F_{n+i}F_{n+j} - F_nF_{n+i+j} = (-1)^n F_i F_j (Möbius Inv. Formula) Let g(n) = \sum_{d|n} f(d), then f(n) = \sum_{d} d \mid ng(d)\mu\left(\frac{n}{d}\right)).
```

4.2 Binary Exponentiation and modArith

```
long long int inf = 10000000007;
//suma (a+b)/m
//resta ((a-b)/m+m)/m
long long binpow(long long b, long long e) {
long long res = 1; b%=inf;
while (e > 0) {
    if (e & 1) res = (res * b)%inf;
    b = (b * b)%inf;
    e >>= 1;
}
return res;
}
```

4.3 Modular Inverse (dividir mod)

```
tie(a1, b1) = make_tuple(b1, a1 - q * b1);
return a1;

long long int modinverse(long long int b, long long int m){
    long long int x,y;
    long long int d = gcd(b,inf,x,y);
    if(d!=1) return -1;
    return ((x%inf)+inf)%inf;
}
```

4.4 Modular Binomial Coeficient and Permutations

```
1 long long int inf = 10000000007;
  //cat[n] = bincoef(2*n,n)/(n+1), cat[0] = 1
   class binCoef{
       long long int lim;
       long long int* fact;
   public:
6
       binCoef(long long int 1){
            lim = 1; fact = new long long int[l+1];fact[0]= 1;
            for(long long int i = 1: i <= 1: i ++ ) fact[i] = (fact[i-1]*i)%inf:
9
10
       //perm = (fact[n] * modinverse(fac[n-k],inf)%inf;
11
       long long int query(long long int n, long long int k){
12
            if(n<k) return 0;</pre>
13
            return (fact[n] * modinverse((fac[n-k]*fact[k])%inf,inf))%inf;
14
       }
15
<sub>16</sub> };
```

4.5 Non-Mod Binomial Coefficient and Permutations

```
//Solo usar con n<=20
//cat[n] = bincoef(2*n,n)/(n+1), cat[0] = 1
unsigned long long int bincoef(unsigned long long int n, unsigned long
    long int k){
    if(n<k) return 0;
    unsigned long long int num = 1, den= 1;
    for(unsigned long long int i = (n-k)+1; i<=n; i++) num*=i;
    for(unsigned long long int i = 2; i<=k; i++) den*=i;
    //perm = return num;
    return num/den;
}</pre>
```

4.6 Modular Catalan Numbers

```
long long int inf = 10000000007;
   class catalan{
       long long int* cat; long long int lim
   public:
4
       catalan(long long int 1){
5
           lim = 1; cat = new long long int[1+10];cat[0] = 1;
6
           for(long long int i = 0;i<=1; i++) cat[i+1] = ((((4LL*i+2)%inf)</pre>
7
                *cat[i])%inf) *modinverse(n+2))%inf;
8
       long long int query(long long int n){ return cat[n];}
9
10 | };
```

4.7 Ceil Fraccionario

```
1 long long int techo(long long int num, long long int den){ return (num+
      den-1)/den;}
```

Numeros de Fibonacci

```
1 //en caso de ser usados mod un m pequeno
  //recordar que los numeros de fibonacci se repiten por lo menos cada m^2
   //0(n)
3
   unsigned long long int fib(int n){
     unsigned long long int a = 1,b = 1,aux;
     if(n \le 2)
       return 1;
7
     for(int i = 3; i <= n; i++){
9
       aux = a+b;
       a = b;
       b = aux;
     return b;
14
15
  const long long int inf = 1000000007;
   unordered_map<long long int,long long int> Fib;
   //O(\log n):DD
  long long int fib(long long int n)
  {
5
       if(n<2) return 1;
6
       if(Fib.find(n) != Fib.end()) return Fib[n];
7
```

```
Page 9 of 15
       Fib[n] = (fib((n+1) / 2)*fib(n/2) + fib((n-1) / 2)*fib((n-2) / 2)) %
            inf;
       return Fib[n];
9
10 }
                             Sieve Of Eratosthenes
  #define MAXN 10e6
   class soef
   public:
       bitset<MAXN> isPrime;
       soe(){
5
           for(int i = 3; i<MAXN; i++) isPrime[i] = (i\(^2\));</pre>
6
           isPrime[2] = 1;
7
           for(int i = 3; i*i<MAXN; i+=2)</pre>
8
               if(isPrime[i])
                   for(int j = i*i; j<MAXN; j+=i)</pre>
10
                       isPrime[j] = 0;
11
12
13 };
                           Sieve-based Factorization
                    4.10
1 #define MAXN 10e6
   class soe{
   public:
       int smolf[MAXN];
       soe(){
           for(int i = 2; i < MAXN; i++) smolf[i] = (i \% 2 = = 0.2 : i);
           for(int i = 3; i*i<MAXN; i+=2)</pre>
               if(smolf[i]==i)
                    for(int j = i*i; j<MAXN; j+=i)</pre>
                        smolf[j] = min(smolf[j],smolf[i]);
13 | };
                       4.11 Berlekamp Massey
typedef long long int 11;
2 //Obtiene recurrencia lineal dados los primeros elementos en O(n^2)
   vector<ll> berlekampMassey(const vector<ll> &s) {
```

vector<ll> c;

vector<ll> oldC;

4

5

```
int f = -1;
6
       for (int i=0; i<(int)s.size(); i++) {</pre>
7
           ll delta = s[i];
           for (int j=1; j<=(int)c.size(); j++) delta -= c[j-1] * s[i-j];</pre>
9
            if (delta == 0) continue;
10
           if (f == -1) {
11
                c.resize(i + 1);
12
                mt19937 rng(chrono::steady_clock::now().time_since_epoch().
13
                    count());
                for (11 &x : c) x = rng();
14
                f = i;
15
           } else {
16
                vector<ll> d = oldC:
17
                for (11 &x : d) x = -x;
18
                d.insert(d.begin(), 1);
19
                11 df1 = 0:
20
                for (int j=1; j <= (int)d.size(); j++) df1 += d[j-1] * s[f+1-j]
21
                    1:
                assert(df1 != 0):
22
                11 coef = delta / df1;
23
                for (11 \& x : d) x *= coef;
24
                vector<ll> zeros(i - f - 1);
25
                zeros.insert(zeros.end(), d.begin(), d.end());
26
                d = zeros;
27
                vector<ll> temp = c;
28
                c.resize(max(c.size(), d.size()));
29
                for (int j=0; j<(int)d.size(); j++) c[j] += d[j];</pre>
30
                if (i - (int) temp.size() > f - (int) oldC.size()) {oldC =
31
                    temp;f = i;
           }
32
       }
33
       return c;
34
35 }
```

4.12 Modular Berlekamp Massey

```
typedef long long int ll;
long long int inf = 1000000007;
vector<ll> bermas(vector<ll> x){
    vector<ll> ls,cur;
    int lf,ld;
    for(int i = 0; i<x.size(); i++){
        long long int t = 0;
}</pre>
```

```
for(int j = 0; j < cur.size(); j++) t=(t+x[i-j-1]*(long long int)
8
                cur[j])%inf;
            if((t-x[i])%inf==0)continue;
9
            if(cur.size()==0){cur.resize(i+1);lf=i;ld=(t-x[i])%inf;continue
10
                ;}
            long long int k = (x[i]-t)*powermod(ld,inf-2)%inf;
11
            vector<ll>c(i-lf-1);c.push_back(k);
12
            for(int j = 0; j<ls.size(); j++) c.push_back(-ls[j]*k%inf);</pre>
13
            if(c.size()<cur.size()) c.resize(cur.size());</pre>
14
            for(int j = 0; j<cur.size(); j++) c[j]=(c[j]+cur[j])%inf;</pre>
15
            if(i-lf+ls.size()>=cur.size())ls=cur,lf=i,ld=(t-x[i])%inf;
16
                cur=c;
17
     }
18
       for(int i =0; i < cur.size(); i++) cur[i] = (cur[i] % inf + inf) % inf;</pre>
19
     return cur;
20
21 }
```

4.13 Matrix exponentiation

```
typedef vector<vector<long long int>> Matrix;
   long long int inf = 1000000007;
3 Matrix ones(int n) {
     Matrix r(n,vector<long long int>(n));
     for(int i= 0: i<n: i++){
           r[i][i]=1;
6
       }
7
     return r;
9
   Matrix operator*(Matrix &a, Matrix &b) {
     int n=a.size(),m=b[0].size(),z=a[0].size();
11
     Matrix r(n,vector<long long int>(m));
12
     for(int i=0; i<n; i++){
13
           for(int j=0; j<m; j++){</pre>
14
                for(int k=0; k < z; k++){
15
                   r[i][j] += ((a[i][k]\%inf)*(b[k][j]\%inf))\%inf;
16
                    r[i][i]%=inf;}}
17
     return r;
18
19
   Matrix be(Matrix b, long long int e) {
     Matrix r=ones(b.size());
21
     while(e){if(e&1LL)r=r*b;b=b*b;e/=2;}
23
     return r:
24 }
```

```
Universidad Autonoma de Ciudad Juarez - First to Penalty
25
  //Matrix mat(n,vector<long long int>(n));
                                Geometry
                                  Strings
                       6.1 Explode by token
   //#include <sstream>
2
   vector<string> explode(string const& s, char delim) {
3
     vector<string> result;
4
     istringstream iss(s);
5
     for (string token; getline(iss, token, delim); )
6
7
       result.push_back(move(token));
8
9
     return result;
10
11 }
                          Multiple Hashings DS
  struct multhash{
       unsigned long long int h1,h2;
2
      unsigned long long int alf[257];
3
```

```
bool operator < (multhash b) const {</pre>
4
       if (h1 != b.h1) return h1 < b.h1;</pre>
5
       return h2 < b.h2;
6
     bool operator == (multhash b) const { return (h1== b.h1 && h2== b.h2)
8
     bool operator != (multhash b) const { return !(h1== b.h1 && h2== b.h2)
9
          ;}
   public:
10
       string s;
11
       multhash(){
12
           h1 = 0: h2 = 0:s = "":
13
           for(char l = 'a'; l <= 'z'; l++) alf [l] = l-'a'+1;
14
       }
15
       void innit(){
16
           unsigned long long int inf,p,op;
17
18
```

```
inf = 999727999:
19
           p = 325255434; op = 325255434;
20
           for(char 1: s){
21
               h1+=(p*alf[1])%inf;
22
               p*=op;
23
               p%=inf;
24
           }
25
           inf = 1070777777;
27
           p = 10018302; op = 10018302;
           for(char 1: s){
29
               h2+=(p*alf[1])%inf;
30
               p*=op;
31
               p%=inf;
32
33
34
35
   //VALORES ALTERNATIVOS DE INF, LOG 17
   //66666555557777777
   //986143414027351997
   //974383618913296759
   //973006384792642181
   //953947941937929919
   //909090909090909091
   //VALORES PARA P, USAR PRIMOS MAYORES A |Alfabeto|
44 //31,47,53,61,79
                     6.3 Permute chars of string
void permute(string str){
     // Sort the string in lexicographically
     // ascennding order
     sort(str.begin(), str.end());
4
5
     // Keep printing next permutation while there
     // is next permutation
7
     do {
8
       cout<<str<<endl;</pre>
    } while (next_permutation(str.begin(), str.end()));
10
11 }
                 6.4 Longest common subsequence
1 //O(|te|*|pa|)
```

```
2 //cambiar score para otros problemas, str all match = +2, miss/ins/del =
   //usar char que no este en el alfabeto para denotar del/ins
   string te,pa;
   long long int ninf = -10e13;
   long long int score(char a, char b){
       if(a=='*' || b=='*') return 0;
       if(a==b) return 1;
8
       return ninf;
9
10
   long long int lcs(){
11
       long long int** dp;te = "*"+te; pa = "*"+pa;
12
       long long int res = 0;
13
14
       dp = new long long int*[te.size()];
15
       for(int i = 0; i<te.size(); i++) dp[i] = new long long int[pa.size()</pre>
16
           ]();
17
       for(int r = 1: r<te.size(): r++){</pre>
18
           for(int c = 1; c<pa.size(); c++){</pre>
19
               dp[r][c] = dp[r-1][c-1]+score(te[r],pa[c]);
20
               dp[r][c] = max(dp[r][c-1]+score(te[r],'*'),dp[r][c]);
21
               dp[r][c] = max(dp[r-1][c]+score('*',pa[c]),dp[r][c]);
22
23
       }
24
25
       return dp[te.size()-1][pa.size()-1];
26
27 }
                                 6.5 KMP
  string T,P;
  int bt[MAXN];
   //O(|Text|+|Pattern|)
   void KMPpre(){
```

```
string T,P;
int bt[MAXN];

//O(|Text|+|Pattern|)

void KMPpre(){
    int i = 0, j = -1; bt[0] = -1;
    while(i<P.size()){
        while(j>=0 && P[i]!=P[(j>=0?j:0)]) j = bt[j];
        i++;j++; bt[i] = j;
    }
}
int kmp(){
    int res =0, i = 0, j = 0;
```

```
while(i<T.size()){</pre>
13
            while(j \ge 0 \&\& T[i] != P[(j \ge 0?j:0)]) j = bt[j];
14
15
            i++; j++;
            if(j==P.size()){//match, do anything
16
                 res++; j = bt[j];
17
18
        }
19
        return res;
20
21 }
```

7 Clasicos

7.1 Job scheduling

7.1.1 One machine, linear penalty

```
//cuando se tiene que encontrar um orden optimo
//para trabajos con una funcion lineal de penalty, basta con hacer un
sort en O(n log n)

struct trabajo{
    long long int penalty,tiempo;
    int ind;
};
bool comp(const trabajo a, const trabajo b){
    if (a.tiempo * b.penalty == a.penalty * b.tiempo) return a.ind<b.ind
    ;
    return a.tiempo * b.penalty < a.penalty * b.tiempo;
}</pre>
```

7.1.2 One machine, deadlines

```
1 //calcula la maxima cantidad de jobs que se pueden hacer dados sus
       deadlines y duraciones en O(n log n)
   struct Job {
       int deadline, duration, idx;
4
       bool operator<(Job o) const {</pre>
5
           return deadline < o.deadline;</pre>
6
       }
7
   };
8
   vector<int> compute_schedule(vector<Job> jobs) {
       sort(jobs.begin(), jobs.end());
11
       set<pair<int,int>> s;
12
```

```
vector<int> schedule:
13
       for (int i = jobs.size()-1; i >= 0; i--) {
14
           int t = jobs[i].deadline - (i ? jobs[i-1].deadline : 0);
15
           s.insert(make_pair(jobs[i].duration, jobs[i].idx));
16
           while (t && !s.empty()) {
17
                auto it = s.begin();
18
                if (it->first <= t) {</pre>
19
                    t -= it->first;
20
                    schedule.push_back(it->second);
21
                } else {
22
                    s.insert(make_pair(it->first - t, it->second));
23
24
                }
25
                s.erase(it);
26
27
       }
28
       return schedule;
29
30 | }
```

7.1.3 One machine, profit

```
1 // Dado n Jobs y su profit, calcula cual es el mayor profit que se puede
        obtener en O(n^2)
  |struct Job{int start, finish, profit;};
   bool jobComparataor(Job s1, Job s2){return (s1.finish < s2.finish);}</pre>
   // Find the latest job (in sorted array) that doesn't
   // conflict with the job[i]. If there is no compatible job,
   // then it returns -1.
   vector <Job> arr;
   int* memo;
   int latestNonConflict( int i){
     for (int j = i - 1; j \ge 0; j--)
10
       if (arr[j].finish <= arr[i - 1].start)</pre>
11
         return j;
12
     return -1;
13
14
   // A recursive function that returns the maximum possible
   // profit from given array of jobs. The array of jobs must
   // be sorted according to finish time.
   int findMaxProfitRec( int n){
     // Base case
19
     if (n == 1) return arr[n - 1].profit;
20
       if (memo[n]>=0) return memo[n];
21
```

```
// Find profit when current job is included
22
     int inclProf = arr[n - 1].profit;
23
     int i = latestNonConflict(n);
24
     if (i != -1) inclProf += findMaxProfitRec( i + 1);
25
26
     // Find profit when current job is excluded
27
     int exclProf = findMaxProfitRec( n - 1);
28
29
     return memo[n]=max(inclProf, exclProf);
30
31
32
   // The main function that returns the maximum possible
   // profit from given array of jobs
   int findMaxProfit( int n){
     sort(arr.begin(),arr.end(), jobComparataor);
     return findMaxProfitRec(n);
38 }
```

7.1.4 Two machines, min time

```
1 //Obtiene el ordenamiento optimo de Jobs en dos maquinas en O(n log n)
2 struct Job {
       int a, b, idx;
       bool operator<(Job o) const {return min(a, b) < min(o.a, o.b);}</pre>
4
   };
5
   vector<Job> johnsons_rule(vector<Job> jobs) {
       sort(jobs.begin(), jobs.end());
7
       vector<Job> a, b;
8
       for (Job j : jobs) {
9
           if (i.a < i.b)
10
                a.push_back(j);
11
           else
12
                b.push_back(j);
13
14
       a.insert(a.end(), b.rbegin(), b.rend());
15
       return a;
16
17
18
   pair<int, int> finish_times(vector<Job> const& jobs) {
       int t1 = 0, t2 = 0;
20
       for (Job j : jobs) {
21
22
           t1 += j.a;
           t2 = \max(t2, t1) + j.b;
23
```

8 Flow

9 Miscellaneous

9.1 Bit Manipulation

```
#include "bits/stdc++.h"
   using namespace std;
   #define endl '\n'
5
   int main() {
     ios_base::sync_with_stdio(false); cout.tie(NULL); cin.tie(NULL);
7
     //Se representan bitmasks de 30 a 62 bits
8
     //usando signed int y signed long long int
     //para evitar problemas con el complemento de dos
     signed int a, b;
     //para multiplicar un numero por dos solo es necesario aplicar un
     // shifteo de sus bits a la izquierda
13
     a = 1:
14
     a = a << 3;
15
     cout << a << endl;</pre>
16
     //para dividir un numero entre dos es necesario aplicar un
17
     //shifteo a la derecha
18
     a = 32;
19
     a = a >> 3;
20
     cout << a << endl;</pre>
21
     //para encender el bit n de a, solo hay que igualar a = a \mid pow(2,n-1)
22
     //prende el tercer bit
23
     a = 1;
24
     b = 1 << 2;
25
     a = a \mid b;
26
     cout << a << endl;</pre>
27
     //para apagar el bit n de a, solo hay que a \&= \text{pow}(2,n-1)
28
     //prende el tercer bit
29
     a = 5;
30
     b = 1 << 2;
31
     a &= ~b;
32
     cout << a << endl;</pre>
```

```
//para revisar si el bit n de a esta encendido
     //revisa si el tercer bit esta encendido
35
     a = 5:
36
     b = 1 << 2;
37
     a = a \& b;
     cout << (a?"SI":"NO") << endl;</pre>
     //para volter el bit n de a, solo hay que igualar a = a \hat{pow}(2,n-1)
40
     //apaga el tercer bit
     a = 5;
42
     b = 1 << 2;
     a = a \hat{b};
44
     cout << a << endl;</pre>
45
     //para obtener el bit menos significativo que esta encendido a& -a
     a = 12;
47
     cout << log2(a & ((-1) * a))+1 << endl;
     //para prender todos los bits hasta n
     a = (1 << 4) - 1;
50
     cout << a << endl;</pre>
52
         -----EOSOLUTION-----
 #include "bits/stdc++.h"
   using namespace std;
   #define endl '\n'
   #pragma GCC optimize("03")
   #pragma GCC target("popcnt")
6
   //no usar con visual c++
   //solo con g++ like compilers
   int main() {
     ios_base::sync_with_stdio(false); cout.tie(NULL); cin.tie(NULL);
     signed long long int a, b, n;
11
     //Obtain the remainder (modulo) of a when it is divided by n (n is a
12
         power of 2)
     a = 15; n = 8-1;
13
     a &= n;
14
     cout << a_{n,u}=15,u=2^3 << endl;
15
     cout << a << endl:</pre>
16
     //Apaga el bit menos significativo de a
17
     a = 14:
18
     b = (a \& ((-1) * a));
19
     a &= ~b;
20
     cout << a << endl;</pre>
```

```
//enciende el ultimo cero de a
22
     a = 9;
23
     b = ~a;
^{24}
     b = (b \& ((-1) * b));
25
     a = a \mid b;
26
     cout << a<<endl;</pre>
27
     //contar bits encendidos en a
28
     cout << __builtin_popcount(a)<<endl;</pre>
29
     //checar la paridad de a
30
     cout << (__builtin_parity(a) ? "IMPAR" : "PAR") << endl;</pre>
31
     //contar leading zeroes en a
32
     cout << __builtin_clz(a)<<endl;</pre>
33
    //contar 9, trailling zeroes en a
     cout << __builtin_ctz(a)<<endl;</pre>
36
         -----EOSOLUTION-----
```

10 Testing