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Robust optimisation for vertiport location problem considering travel mode choice behaviour in urban air mobility systems

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ABSTRACT

With the rapid development of novel vehicle technologies, air taxis are becoming a new transport servicing model to achieve better urban air mobility (UAM) systems. The UAM system can mitigate severe traffic congestion problems on the ground in many metropolitan areas effectively. This research aims to design a robust UAM network to serve broader mobility needs in the future. In this study, we consider the two main competition transportation modes of air taxis (luxury ground taxis and ordinary ground taxis) and establish a travel mode choice behaviour model based on the multinomial logit choice approach to determine the aggregate air taxi demand flow. Furthermore, we account for the varying temporal preferences of different UAM passengers by incorporating different time values for heterogeneous passengers, such as leisure and business passengers, into the travel mode choice behaviour model. Additionally, to address uncertainties in user demand, we introduce a robust optimisation model for vertiport location under a budget uncertainty set. This model is then reformulated as a mixed-integer linear programming model to address computational challenges. Ultimately, we conduct a series of numerical experiments to showcase the effectiveness of our proposed mathematical model. The outcomes of this research offer valuable managerial insights and implications, aiding governments and UAM operators in scientifically designing UAM networks and making strategic decisions regarding infrastructure investments.

1. Introduction

In recent years, the aviation landscape has experienced a transformative shift, driven by the rapid advancements in electric vertical take-off and landing (eVTOL) technology (Garrow et al., 2021). This paradigm shift has propelled the urban air mobility (UAM) concept into the forefront of urban transportation discussions (Haan et al., 2021). Its significance lies in the potential to harness the underutilised low-altitude airspace efficiently and offer a sustainable solution to the perennial problem of urban traffic congestion (Willey & Salmon, 2021). A critical component of this evolving UAM system is the concept of air taxis, which have emerged as a promising and popular mode of transport (Rajendran et al., 2021). For instance, Dubai is poised to introduce air taxi services to cater to tourists, while Singapore has ambitious plans for air taxis on commercial routes connecting Marina Bay and Sentosa. In China, Shenzhen is actively implementing its UAM system in collaboration with various commercial enterprises. Air taxis represent an innovative service model with the promise of delivering faster, more efficient, and environmentally friendly transportation solutions, poised to revolutionise urban mobility when compared to traditional ground-based modes of transport (Ribeiro et al., 2023; Sun et al., 2018).

Developing an efficient UAM system encompasses a multitude of design challenges, including UAM airspace planning, flight regulations, and operational design, among others (Bauranov & Rakas, 2021; Su et al., 2022; Tang et al., 2021). The strategic decision of vertiport locations takes centre stage in the pursuit of establishing an optimally efficient UAM system. A vertiport, as noted by Hasan (2019) and Shao et al. (2021), stands as a crucial facility for UAM systems, as depicted in Fig. 1. Vertiport planning significantly impacts the performance of the UAM system, as it constitutes the foundational physical infrastructure that underpins the entire UAM system operation (Brunelli et al., 2023; Rajendran & Srinivas, 2020). Therefore, it is imperative to strategically plan the vertiport locations. In this decision-making process, user demand is a pivotal and intricate factor that will have a profound influence on this strategic decision. This intricacy emerges due to the simultaneous existence of various alternative travel modes, each with its own set of characteristics and advantages. Furthermore, the demand is also influenced by many external factors, intensifying its inherent uncertainty. For example, unpredictable weather patterns, encompassing phenomena such as rain, fog, and storms, are among the foremost factors that magnify this uncertainty. Additionally, in the realm of UAM systems, the user population can typically be segregated into two primary categories: business passengers and leisure passengers. This

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Fig. 1. Vertiport (copyright: https://www.moodiedavittreport.com/eu-aviation-safety-agency-publishes-first-vertiport-design-guidance/).

classification stems from the notably distinct temporal preferences exhibited by these user groups, thereby resulting in the manifestation of their individual time values (Wong et al., 2008). In conclusion, it is challenging to integrate the aforementioned factors into the process of vertiport location determination.

Within the context of existing literature, much attention has been dedicated to the design and location of vertiports (Kai et al., 2022; Preis, 2021). Nonetheless, there has been a relatively limited emphasis on incorporating simultaneous considerations for travel mode choice behaviour, the uncertainty of demand, and the temporal preferences of the different types of passengers (i.e. different time values of heterogeneous passengers) when making decisions regarding vertiport locations. To address this issue, this paper contributes to the literature by proposing a robust optimisation model to determine optimal vertiport locations considering the above-mentioned factors. In more detail, a travel mode choice behaviour model based on the multinomial logit choice approach is proposed to obtain the aggregate air taxi demand flow. Subsequently, a robust optimisation model considering the uncertainty of demand and the different time values of heterogeneous passengers is established to obtain robust and efficient solutions for decision-making on vertiport locations. The main contributions of this paper lie in the following aspects:

1. We consider two transportation modes as the main competition objects of air taxis, namely two kinds of ground taxis, including ordinary taxis and luxury taxis. Furthermore, a travel mode choice behaviour model is presented based on the multinomial logit choice approach to obtain the aggregate air taxi demand flow.
2. Considering the notably distinct temporal preferences of different passengers, we take into account the different time values of heterogeneous passengers (leisure passengers and business passengers) for the UAM system.
3. We establish a robust optimisation model to determine vertiport location decisions under the budget uncertainty set considering the user demand uncertainty. To make the model tractable computationally, we reformulate the model into a mixed-integer linear programming model. Additionally, we evaluate the model through both an illustrative example and a real-world case study.

The remainder of this paper is structured as follows. Section 2 discusses previous literature and proposes the research gaps of this paper. Section 3 proposes the travel mode choice behaviour model based on the multinomial logit choice approach to obtain the aggregate air taxi demand flow. Section 4 presents the robust optimisation model under the budget uncertainty set for the vertiport location problem. In Section 5, a series of numerical experiments are conducted to demonstrate the validity of the proposed model. Finally, Section 6 summarises the conclusions.

2. Literature review

As an innovative and emerging mode of transportation, scholars have conducted numerous studies to facilitate the planning and management of UAM systems (Straubinger et al., 2020). Currently, these studies mainly focus on strategic and operational levels (Kleinbekman et al., 2018; Rothfeld et al., 2018; Song & Yeo, 2021; Wang et al., 2022). Within the strategic level, most literature focuses on the determination of vertiport locations. For example, Sinha and Rajendran (2022) established a vertiport location analytics model that integrates iterative K-means clustering with a multi-criteria warm start approach. Jeong et al. (2021) determined the vertiport location according to the population of commuters and created the noise priority route to minimise the affected people. Lim and Hwang (2019) presented a K-means clustering algorithm to optimise the vertiport locations in the Seoul metro area. Fadhil (2018) proposed a GIS-based approach to determine the vertiport location for the UAM systems.

Besides, many scholars have studied the vertiport location problem by extending the hub location problem based on the optimisation technique. The hub location problem is a well-explored topic in the field of operations research. It revolves around determining the best locations for central facilities to efficiently serve a designated customer base while achieving specific objectives. The problem has many applications in logistics and transportation in recent years, such as locating distribution centres (Chao et al., 2019; Sun et al., 2021), warehouses (Wang et al., 2021), and transit transportation hubs (Auad et al., 2021; Maheo et al., 2019; Shang et al., 2021).

With the emergence and development of UAM systems, some researchers have extended this problem to this novel field to have decision-making on the vertiport locations. For example, Wu and Zhang (2021) determined the candidate locations of vertiports firstly based on Geographic Information Systems tools and established a mathematical model to obtain the optimal vertiport locations. In the literature by Chen et al. (2022), a model was presented for choosing vertiport locations, focusing on discrete demand within a grid graph. The model excludes forbidden grids, which are not eligible as candidate vertiport locations. Furthermore, the study introduced a novel variable neighbourhood search approach to solve this mathematical model. Shin et al. (2022) presented an optimisation model to determine the optimal vertiport locations. This model considered the traffic congestion with the goal of minimising the travel time, fixed cost of constructing vertiports, serving cost, and collision risk cost. Besides, a heuristic method is proposed based on a genetic algorithm to improve the calculation efficiency of the model. Rath and Chow (2022) presented a hub location problem with choice-constrained user mode choice behaviour and a case study of New York City was conducted to verify the validity of the proposed model. Furthermore, the issue of vertiport location has been approached by extending classic problems within the field of operations research. For example, Ale-Ahmad and Mahmassani (2021) introduced a capacitated location-allocation-routing problem with time windows and established an optimisation model to obtain the optimal decisions.

Overall, there is currently a lack of existing literature that focuses on the robust vertiport location problem. Therefore, this study focuses on addressing the following specific gaps. Firstly, there exists a notable research gap concerning conventional and luxury taxis as primary competitors when estimating the aggregate air taxi demand flow. Secondly, there is a scarcity of research that takes into account the concept of time values of heterogeneous passengers stemming from distinct temporal preferences. Finally, there is limited research emphasis on addressing the robust vertiport location problem while accounting for the uncertainty of user demand. To fill in these gaps, this paper proposes a travel mode choice behaviour model based on the multinomial logit choice approach considering the main competition transport modes of air taxis and different time values of heterogeneous passengers. Besides, a robust optimisation model for vertiport location is presented under the budget uncertainty set considering the uncertainty of the users in this paper.

3. Travel mode choice behaviour model

In this section, we have an estimation of the aggregate air taxi demand flow between different pairs of origins and destinations before establishing the robust optimisation model. Given the competitive relationship with ground-based transportation modes, it is essential to account for the passenger travel mode choice behaviour to determine the demand for air taxis (Fu et al., 2019). Consequently, we identify two primary competitors within the transportation sector for air taxis: ordinary taxis and luxury taxis as shown in Fig. 2. Additionally, we consider two distinct passenger categories, leisure and business passengers, each characterised by varying time values. With this framework, we evaluate the probability of passengers shifting their travel behaviour towards air taxis based on a discrete choice model. The notations used in this paper are given in Table 1.

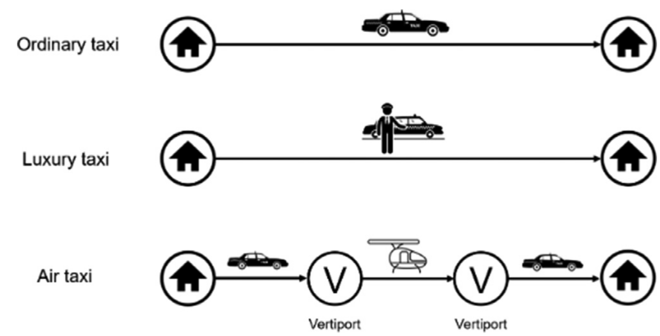

Fig. 2. Three transportation modes.

In this paper, we choose the multinomial logit choice model to estimate aggregate air taxi demand flow. The multinomial logit choice model is a significant type of discrete choice model capable of predicting an individual's selection probability among various travel modes based on the utility function (Washington et al., 2020). Furthermore, we note that the transportation mode choice for a passenger is predominantly influenced by the fare and travel time from the origin to the destination, as discussed by Ma et al. (2017). Therefore, the utility function of the choice of transportation mode m for the individual n who belongs to passenger type c can be formulated as follows:

$$U^c_{m,n} = V^c_{m,n} + \epsilon^c_{m,n}, \quad \forall m \in M, \ \forall c \in C. \tag{1}$$

where $V^c_{m,n}$ is the observable component of the utility function and $\epsilon^c_{m,n}$ is the error component that is assumed to be Gumbel distribution.

The observable components of utility functions for normal taxi $V^c_{N^T,n}$, luxury taxi $V^c_{L^T,n}$, and air taxi $V^c_{A^T,n}$ can be expressed as Eqs. (2), (3) and (4), respectively.

$$V^c_{N^T,n} = \beta^1_{N^T} \times f^{N^T}_{ij} + \beta^2_{N^T} \times t^{N^T}_{ij} \times \kappa^c, \quad \forall i \in N, \ \forall j \in N \backslash \{i\}, \ \forall c \in C, \tag{2}$$

$$V^c_{L^T,n} = \beta^1_{L^T} \times f^{L^T}_{ij} + \beta^2_{L^T} \times t^{L^T}_{ij} \times \kappa^c, \quad \forall i \in N, \ \forall j \in N \backslash \{i\}, \ \forall c \in C, \tag{3}$$

Table 1
Notations.

Notion	Explanation
Sets	
C	The set of heterogeneous passenger types of UAM system (leisure passengers/business passengers)
M	The set of travel modes (Normal taxi/luxury taxi/air taxi)
N	The set of origins and destinations (demand nodes)
K	The set of candidate nodes that are available for locating vertiports
Variables	
y_k	1,if node $k \in K$ is selected as a vertiport; 0, otherwise
x^c_{ikpj}	1, if the demand between $i \in N$ and $j \in N$ with type $c \in C$ is satisfied through vertiport $k \in K$ and $p \in K$; 0, otherwise
z_{ik}	1,if the demand node $i \in N$ is assigned to vertiport $k \in K$; 0, otherwise
Parameters	
$f^{N^T}_{ij}$	The fare of the normal taxi from node i to node j
$f^{L^T}_{ij}$	The fare of the luxury taxi from node i to node j
$f^{A^T}_{kp}$	The fare of the air taxi from node k to node p
κ^c	The time value index of passenger with type c
$t^{N^T}_{ij}$	The transportation time from node i to node j by normal taxi
$t^{L^T}_{ij}$	The transportation time from node i to node j by luxury taxi
t^{air}_{kp}	The air transportation time from vertiport k to vertiport p by air taxi
t^{gro}_{ik}	The ground transportation time from origin node i to vertiport k for the UAM passengers
t^{gro}_{jp}	The ground transportation time from vertiport p to destination node j for the UAM passengers
D^c_{ij}	The aggregate random demand for passenger type c between i and j, with air taxis, ordinary taxis, and luxury taxis
$T^{\text{gro}}_{\text{MAX}}$	The maximum acceptable time for the first-mile or last-mile ground transportation when using the UAM system
w^{TR}	The transportation cost for UAM system of the unit demand
B	The number of established vertiports
β	Dispersion coefficient of parameter
θ	The conversion factor of time value cost

$$V^c_{A^T,n} = \beta^1_{A^T} \times f^{A^T}_{kp} + \beta^2_{A^T} \times \left(t^{\text{gro}}_{ik} + t^{\text{air}}_{kp} + t^{\text{gro}}_{jp} \right) \times \kappa^c,$$
$$\forall c \in C,\ \forall i \in N,\ \forall j \in N \backslash \{i\}, \forall k \in K,\ \forall p \in K \backslash \{k\}. \tag{4}$$

Based on the above analysis, the aggregate choice probability using air taxi service for the passenger type c from the origin node i to the destination node j via vertiport k and vertiport p can be formulated as follows:

$$P^c_{ikpj} = \frac{e^{V^c_{N^T,n}}}{V^c_{N^T,n} + V^c_{L^T,n} + V^c_{A^T,n}}. \tag{5}$$

By integrating Eqs. (2), (3) and (4), we formulate the following expression to calculate the overall probability of choosing air taxi services. This enables us to derive the aggregate air taxi demand flow.

$$P^c_{ikpj} = \frac{e^{\beta^1_{A^T} \times f^{A^T}_{kp} + \beta^2_{A^T} \times \left(t^{\text{gro}}_{ik} + t^{\text{air}}_{kp} + t^{\text{gro}}_{jp}\right) \times \kappa^c}}{e^{\beta^1_{N^T} \times f^{N^T}_{ij} + \beta^2_{N^T} \times t^{N^T}_{ij} \times \kappa^c} + e^{\beta^1_{L^T} \times f^{L^T}_{ij} + \beta^2_{L^T} \times t^{L^T}_{ij} \times \kappa^c} + e^{\beta^1_{A^T} \times f^{A^T}_{kp} + \beta^2_{A^T} \times \left(t^{\text{gro}}_{ik} + t^{\text{air}}_{kp} + t^{\text{gro}}_{jp}\right) \times \kappa^c}}. \tag{6}$$

4. Mathematical programming

4.1. Problem description

In this paper, we address the robust vertiport location problem within the UAM system. Considering the multimodal nature of the UAM system, the air transportation leg between two vertiports and the first-mile and last-mile transportation between demand nodes and vertiports are included throughout the whole itinerary in this research. Note that the UAM company provides first-mile and last-mile transportation services for passengers via ground taxis.

Suppose that the set of demand nodes is known based on the historic data and is denoted by N. It follows that the number of the origin and destination pairs is $|N \times (N-1)|$. Additionally, the set of candidate vertiports is also known according to the practical geographical investigation and is expressed by K and $K \subseteq N$. The main goal of this study is to determine vertiports from the set K to satisfy the demand of $|N \times (N-1)|$ origin and destination pairs. Based on the travel mode choice behaviour model discussed earlier, we have obtained the user choice probability for the UAM system, enabling us to estimate the aggregate air taxi demand flow. Furthermore, due to the characteristic of the demand uncertainty, our vertiport location problem is modelled using the robust optimisation framework under a budget uncertainty set. In particular, we consider heterogeneous passengers that belong to the set of passenger types, which is denoted by C. To formulate a precise mathematical model, this study introduces several assumptions. Firstly, it assumes that each zone is defined by a single central point, and distances between zones are computed using these central points (Chen & Liu, 2023). Secondly, the study does not take into account factors related to capacity and congestion (Rath & Chow, 2022).

4.2. Deterministic optimisation model

In this subsection, we introduce the deterministic optimisation model for the vertiport location as follows:

$$\begin{aligned} \min \sum_{k \in K} c^{\text{CON}} y_k &+ \sum_{i \in N} \sum_{j \in N \backslash \{i\}} \sum_{k \in K} \sum_{p \in K \backslash \{k\}} \sum_{c \in C} w^{\text{TR}} \widetilde{D^c_{ij}} P^c_{ikpj} x^c_{ikpj} \\ &+ \theta \sum_{i \in N} \sum_{j \in N \backslash \{i\}} \sum_{k \in K} \sum_{p \in K \backslash \{k\}} \sum_{c \in C} \widetilde{D^c_{ij}} P^c_{ikpj} \left(t^{\text{gro}}_{ik} + t^{\text{air}}_{kp} + t^{\text{gro}}_{jp} \right) \kappa^c x^c_{ikpj} \end{aligned} \tag{7}$$

s.t.

$$\sum_{k \in K} y_k = B, \tag{8}$$

$$\sum_{k \in K} \sum_{p \in K \backslash \{k\}} x^c_{ikpj} = 1, \quad \forall i \in N,\ \forall j \in N \backslash \{i\}, \tag{9}$$

$$z_{ik} \le y_k, \quad \forall i \in N,\ \forall k \in K, \tag{10}$$

$$z_{jp} \le y_p, \quad \forall j \in N,\ \forall p \in K, \tag{11}$$

$$t^{\text{gro}}_{ik} z_{ik} \le T^{\text{gro}}_{\text{MAX}}, \quad \forall i \in N,\ \forall k \in K, \tag{12}$$

$$t^{\text{gro}}_{jp} z_{jp} \le T^{\text{gro}}_{\text{MAX}}, \quad \forall j \in N,\ \forall p \in K, \tag{13}$$

$$x^c_{ikpj} \le y_k, \quad \forall c \in C,\ \forall i \in N,\ \forall j \in N \backslash \{i\}, \forall k \in K,\ \forall p \in K \backslash \{k\}, \tag{14}$$

$$x^c_{ikpj} \le y_p, \quad \forall c \in C,\ \forall i \in N,\ \forall j \in N \backslash \{i\}, \forall k \in K,\ \forall p \in K \backslash \{k\}, \tag{15}$$

$$x^c_{ikpj} \le z_{ik}, \quad \forall c \in C,\ \forall i \in N,\ \forall j \in N \backslash \{i\}, \forall k \in K,\ \forall p \in K \backslash \{k\}, \tag{16}$$

$$x^c_{ikpj} \le z_{jp}, \quad \forall c \in C,\ \forall i \in N,\ \forall j \in N \backslash \{i\}, \forall k \in K,\ \forall p \in K \backslash \{k\}, \tag{17}$$

$$x^c_{ikpj} \ge 0, \quad \forall c \in C,\ \forall i \in N,\ \forall j \in N \backslash \{i\}, \forall k \in K,\ \forall p \in K \backslash \{k\}, \tag{18}$$

$$z_{ik} \ge 0, \quad \forall i \in N,\ \forall k \in K, \tag{19}$$

$$y_k \ge 0, \quad \forall k \in K. \tag{20}$$

The objective function (7) minimises the total cost, including vertiport establishment cost, total transportation cost of UAM system, and total time value cost of heterogenous UAM system passengers. Constraints (8) specify the number of vertiports to be established in the UAM system. Constraints (9) ensure that the demand between any origin and destination (OD) pairs needs to be satisfied. Constraints (10) and (11) indicate that the demand node cannot be linked to any node that is not selected as a vertiport. Constraints (12) and (13) show that the transportation time between a vertiport and a demand node should not exceed the maximum acceptable time for the first-mile or last-mile ground transportation when using the UAM system. Constraints (14) and (15) ensure that there is no traffic flow between two nodes that are not selected to be the vertiports. Constraints (16) and (17) ensure that there is no traffic flow between the demand node and vertiport if the link does not exist. Constraints (18)–(20) give the domain of decision variables.

4.3. Budget uncertainty set

In this paper, we consider the aggregate air taxi demand flow uncertainty of heterogeneous passenger types between different origins and destinations. We assume that $\widetilde{D^c_{ij}}$ is a bounded and symmetric random variable whose realisations fall in the interval $\left[\overline{D^c_{ij}} - \widehat{r^c_{ij}}, \overline{D^c_{ij}} + \widehat{r^c_{ij}}\right]$, where $\overline{D^c_{ij}}$ is the nominal demand value and $\widehat{r^c_{ij}}$ is the radius of corresponding interval for $\widetilde{D^c_{ij}}$. Associated with the random variable $\widetilde{D^c_{ij}}$, a scaled deviation $\xi^c_{ij} \in [-1, 1]$ is defined as follows:

$$\xi^c_{ij} = (\widetilde{D^c_{ij}} - \overline{D^c_{ij}}) / \widehat{r^c_{ij}}. \tag{21}$$

Furthermore, we can define the random variable as a function related to its nominal value and the scaled deviation. Therefore, we have $\widetilde{D^c_{ij}} = \overline{D^c_{ij}} + \widehat{r^c_{ij}} \xi^c_{ij}$, such that $\xi \in \mathscr{P}$ and

$$\mathscr{P} = \left\{ |\xi| \le 1, \sum_{j \in N \backslash \{i\}} \sum_{c \in C} \xi^c_{ij} \le \Gamma_i, \forall i \in N \right\}, \tag{22}$$

is the budget uncertainty set.

For the budget uncertainty set, we define the parameter Γ_i as the upper bound on the deviation allowed for the demand originating from

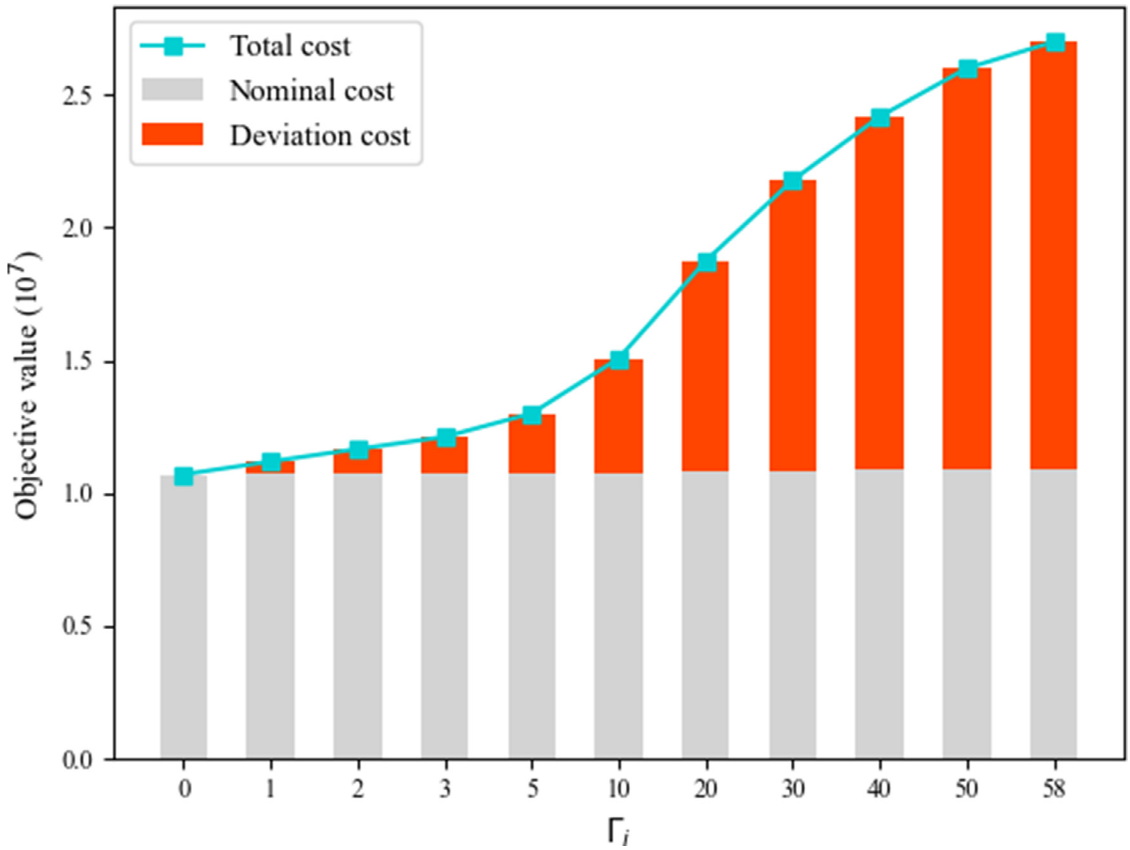

Fig. 3. The computational results under different values of parameter Γ_i.

node i, relative to its nominal value. We note that Γ_i takes values within the interval $[0, |C| \times (|N|-1)]$ (Ghaffarinasab, 2018). The parameter Γ_i can adjust the robustness of the robust optimisation model against the conservatism level of the solutions. When Γ_i is equal to 0, the robust optimisation model will be transformed into a deterministic one. Furthermore, as another special case, when the Γ_i is equal to $[0, |C| \times (|N|-1)]$, the case will correspond to the worst-case problem, which is the most conservative one. The user-specified parameter imposes an upper bound on the variation of the uncertain parameter, which can avoid the occurrence of extremely pessimistic solutions.

4.4. Robust counterpart and model reformulation

The objective of robust optimisation is to obtain the optimal solution that is feasible for any realisation of the uncertain parameter under the uncertainty set (Gorissen et al., 2015). Based on the above analysis, the robust optimisation model for vertiport location considering the uncertainty of demand can be expressed as follows:

$$\begin{aligned} min\ & \textstyle\sum_{k\in K} c^{\mathrm{CON}} y_k + \sum_{i\in N}\sum_{j\in N\setminus\{i\}}\sum_{k\in K}\sum_{p\in K\setminus\{k\}}\sum_{c\in C} w^{\mathrm{TR}}\overline{D^c_{ij}} P^c_{ikpj} x^c_{ikpj} \\ & +\theta \textstyle\sum_{i\in N}\sum_{j\in N\setminus\{i\}}\sum_{k\in K}\sum_{p\in K\setminus\{k\}}\sum_{c\in C}\overline{D^c_{ij}} P^c_{ikpj}(t^{\mathrm{gro}}_{ik}+t^{\mathrm{air}}_{kp}+t^{\mathrm{grro}}_{jp})\kappa^c x^c_{ikpj} \\ & +max \textstyle\sum_{i\in N}\sum_{j\in N\setminus\{i\}}\sum_{c\in C}\widehat{r^c_{ij}}\Big[\sum_{k\in K}\sum_{p\in K\setminus\{k\}} w^{\mathrm{TR}} P^c_{ikpj} x^c_{ikpj} \\ & + \textstyle\sum_{k\in K}\sum_{p\in K\setminus\{k\}} P^c_{ikpj}(t^{\mathrm{gro}}_{ik}+t^{\mathrm{air}}_{kp}+t^{\mathrm{grro}}_{jp})\kappa^c x^c_{ikpj}\Big] \end{aligned} \tag{23}$$

s.t. Constraints (8)–(20).

We set a variable ξ^c_{ij} that denots the deviation from the nominal demand $\overline{D^c_{ij}}$, which is in units of $\widehat{r^c_{ij}}$. Besides, we fix the variable x^c_{ikpj} in the lower level problem as X^c_{ikpj}. Therefore, the robust counterpart model is shown as follows:

$$\begin{aligned} max \sum_{i\in N}\sum_{j\in N\setminus\{i\}}\sum_{c\in C}\widehat{r^c_{ij}}\Bigg[&\sum_{k\in K}\sum_{p\in K\setminus\{k\}} w^{\mathrm{TR}} P^c_{ikpj} X^c_{ikpj} \\ &+\sum_{k\in K}\sum_{p\in K\setminus\{k\}} P^c_{ikpj}(t^{\mathrm{gro}}_{ik}+t^{\mathrm{air}}_{kp}+t^{\mathrm{grro}}_{jp})\kappa^c X^c_{ikpj}\Bigg]\xi^c_{ij} \end{aligned} \tag{24}$$

s.t.

$$\sum_{j\in N\setminus\{i\}}\sum_{c\in C}\xi^c_{ij} \le \Gamma_i, \quad \forall i\in N, \tag{25}$$

$$0 \le \xi^c_{ij} \le 1, \quad \forall c\in C,\ \forall i\in N,\ \forall j\in N\setminus\{i\}. \tag{26}$$

Based on the linear programming duality theory, we obtain the dual programming of the above robust counterpart as the following mix-integer linear programming model.

$$min \sum_{i\in N}\Gamma_i\phi_i + \sum_{i\in N}\sum_{j\in N\setminus\{i\}}\sum_{c\in C}\lambda^c_{ij} \tag{27}$$

s.t.

$$\begin{aligned} &\widehat{r^c_{ij}}\left[\sum_{k\in K}\sum_{p\in K\setminus\{k\}} w^{\mathrm{TR}} P^c_{ikpj} x^c_{ikpj} + \sum_{k\in K}\sum_{p\in K\setminus\{k\}} P^c_{ikpj}(t^{\mathrm{gro}}_{ik}+t^{\mathrm{air}}_{kp}+t^{\mathrm{grro}}_{jp})\kappa^c x^c_{ikpj}\right] \\ &\quad \le \phi_i + \lambda^c_{ij},\ \ \forall i\in N, \forall j\in N\setminus\{i\}, \forall c\in C, \end{aligned} \tag{28}$$

$$\phi_i \ge 0, \quad \forall i\in N, \tag{29}$$

$$\lambda^c_{ij} \ge 0, \quad \forall i\in N, \forall j\in N\setminus\{i\}, \forall c\in C, \tag{30}$$

where ϕ_i and λ^c_{ij} are dual variables.

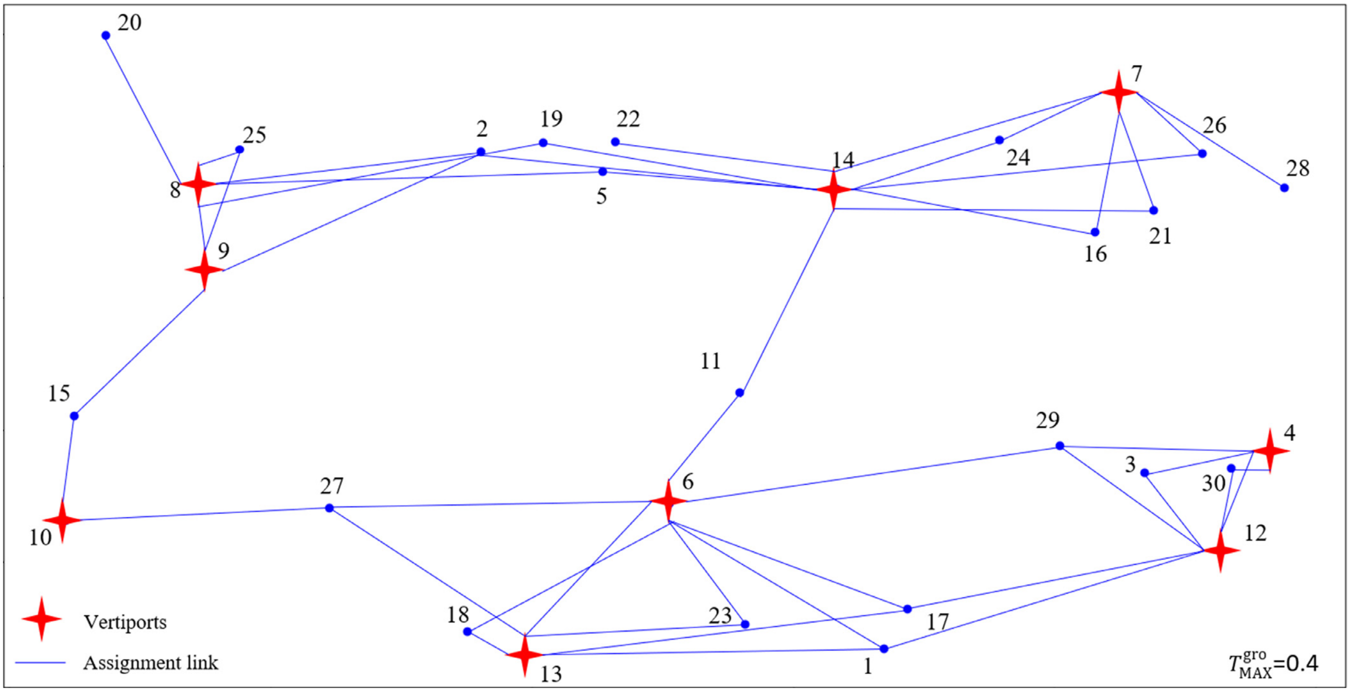

Fig. 4. The UAM network structure when the maximum acceptable time for the first-mile or last-mile ground transportation is 0.4.

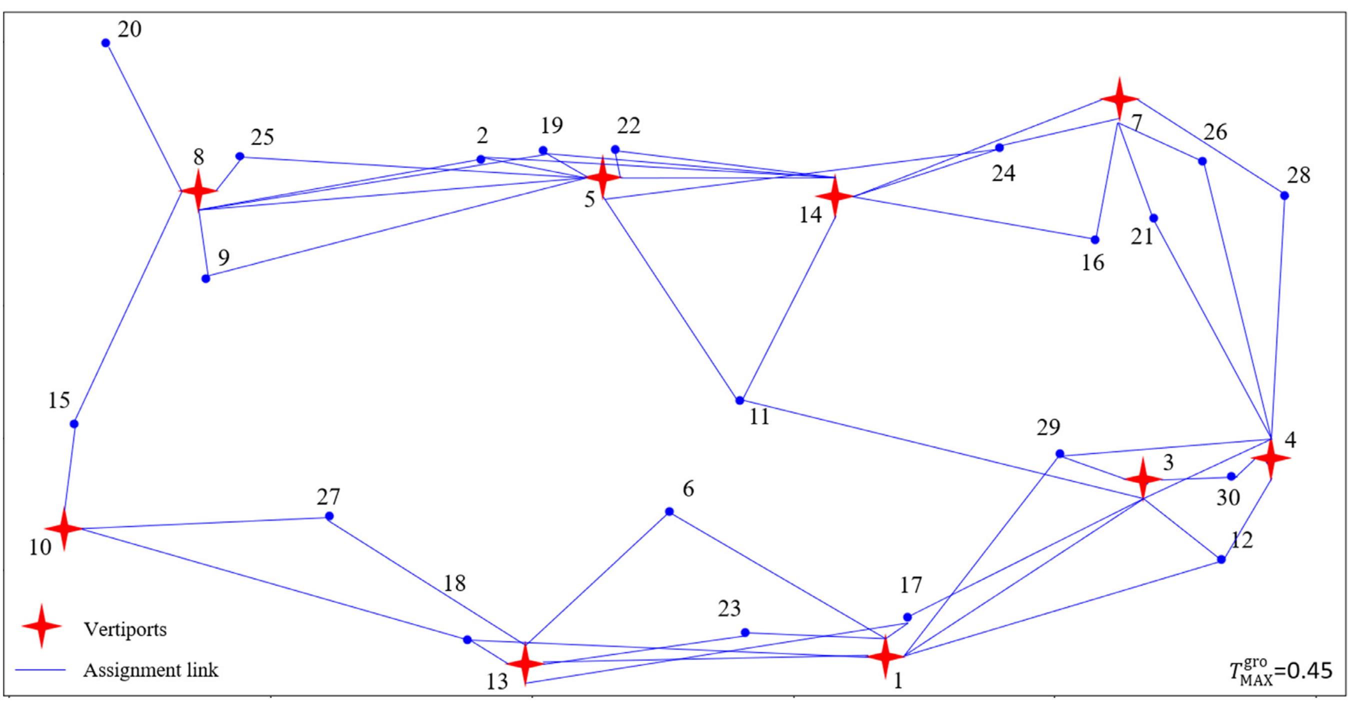

Fig. 5. The UAM network structure when the maximum acceptable time for the first-mile or last-mile ground transportation is 0.45.

Therefore, the robust optimisation model for vertiport location under the budget uncertainty set can be reformulated as the following mixed-integer linear programming model.

$$min \sum_{k\in K} c^{\mathrm{CON}} y_k + \sum_{i\in N} \sum_{j\in N\setminus\{i\}} \sum_{k\in K} \sum_{p\in K\setminus\{k\}} \sum_{c\in C} w^{\mathrm{TR}} \overline{D_{ij}^c} P_{ikpj}^c x_{ikpj}^c$$
$$+\theta \sum_{i\in N} \sum_{j\in N\setminus\{i\}} \sum_{k\in K} \sum_{p\in K\setminus\{k\}} \sum_{c\in C} \overline{D_{ij}^c} P_{ikpj}^c \left(t_{ik}^{\mathrm{gro}} + t_{kp}^{\mathrm{air}} + t_{jp}^{\mathrm{gro}}\right) \kappa^c x_{ikpj}^c$$
$$+ \sum_{i\in N} \Gamma_i \phi_i + \sum_{i\in N} \sum_{j\in N\setminus\{i\}} \sum_{c\in C} \lambda_{ij}^c \tag{31}$$

s.t. Constraints (8)–(20), (28)–(30).

5. Numerical experiments

This section conducts a set of numerical experiments to validate the proposed model. In Section 5.1, we attest to the performance of the robust optimisation model through a randomly generated numerical instance. Following this, a case study is introduced to validate the effectiveness of the proposed model. All the numerical experiments are carried out by Gurobi 10.0.0 optimisation software on an Inter(R) Core

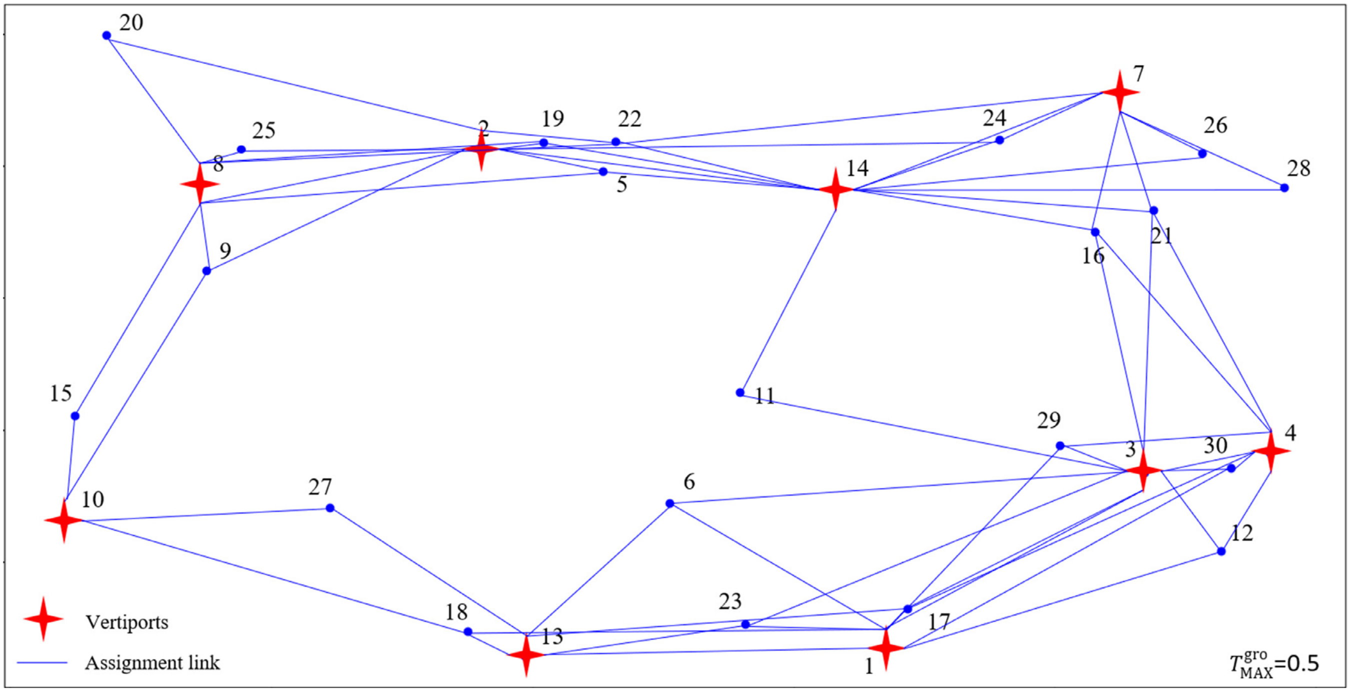

Fig. 6. The UAM network structure when the maximum acceptable time for the first-mile or last-mile ground transportation is 0.5.

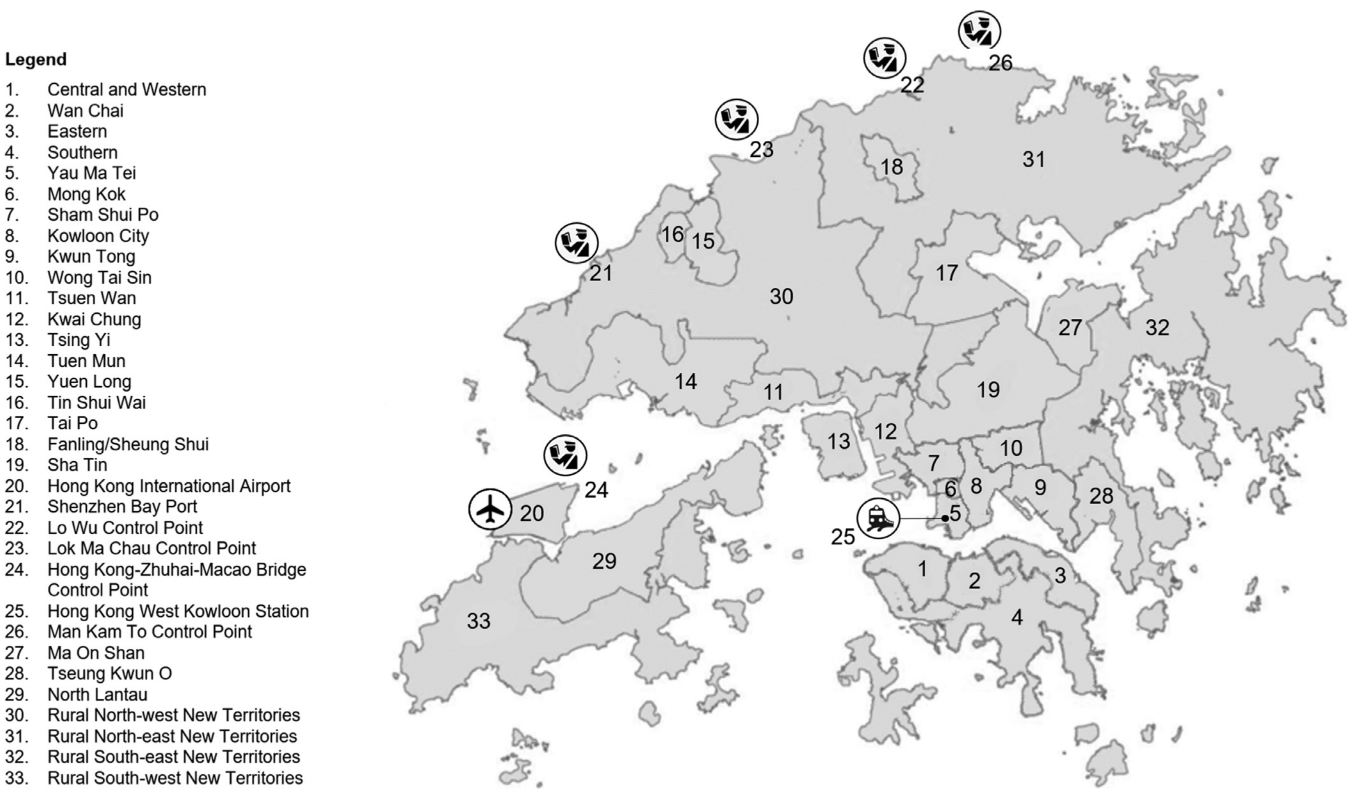

Fig. 7. Geographic region and zone division of the case study.

(TM) i7-9700 CPU 3.00GHz personal computer with 16.0GB RAM operating under Windows 10 (64bit).

5.1. An illustrative example

In this subsection, we evaluate the performance of the robust optimisation model using a randomly generated illustrative example. We generate 870 OD pairs randomly in a square area with a range of $50km \times 50km$. The number of candidate vertiports is set to be 15. Furthermore, the unit transportation cost for air travel is 2.5 USD. The fixed construction cost of a vertiport is set as 850000 USD. The maximum acceptable time for the first-mile or last-mile ground transportation when using the UAM system is set as 0.4h. For the relative dispersion coefficient, referring to the literature by Rath and Chow (2022), we set β^{1}_{NT}

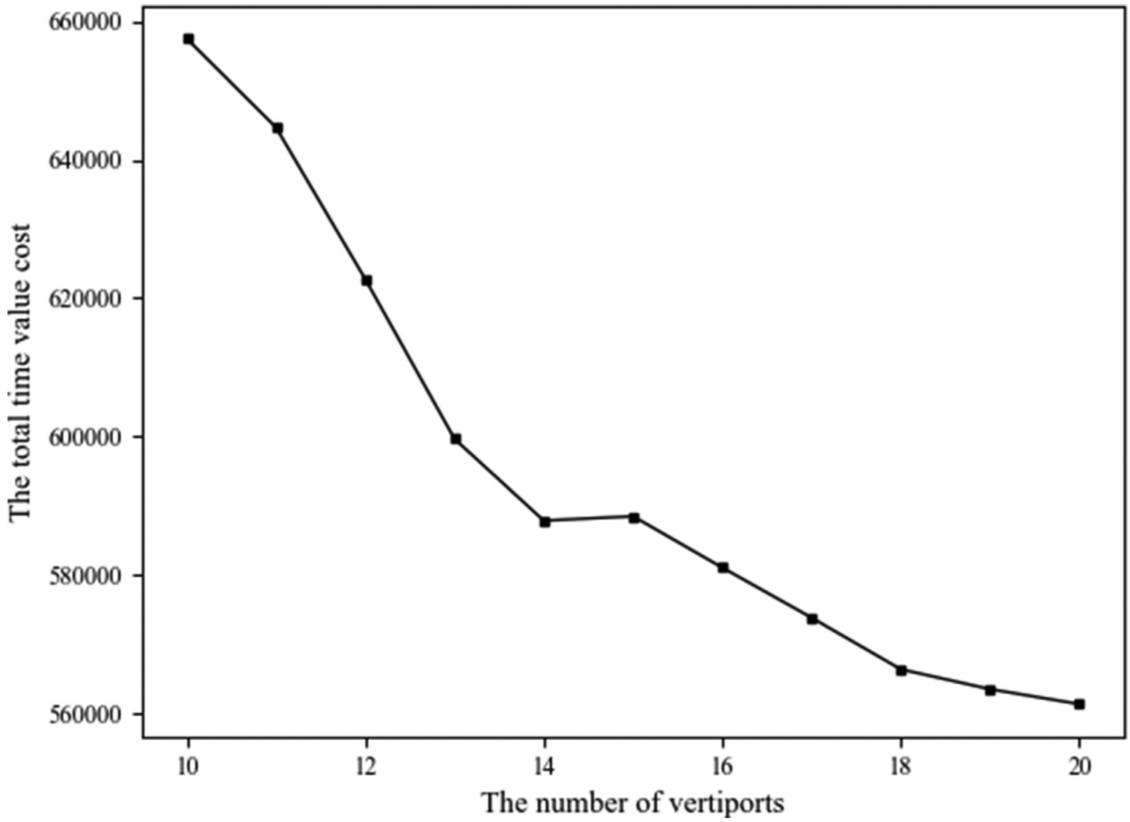

Fig. 8. The computational results of the total time value cost under different numbers of established vertiports.

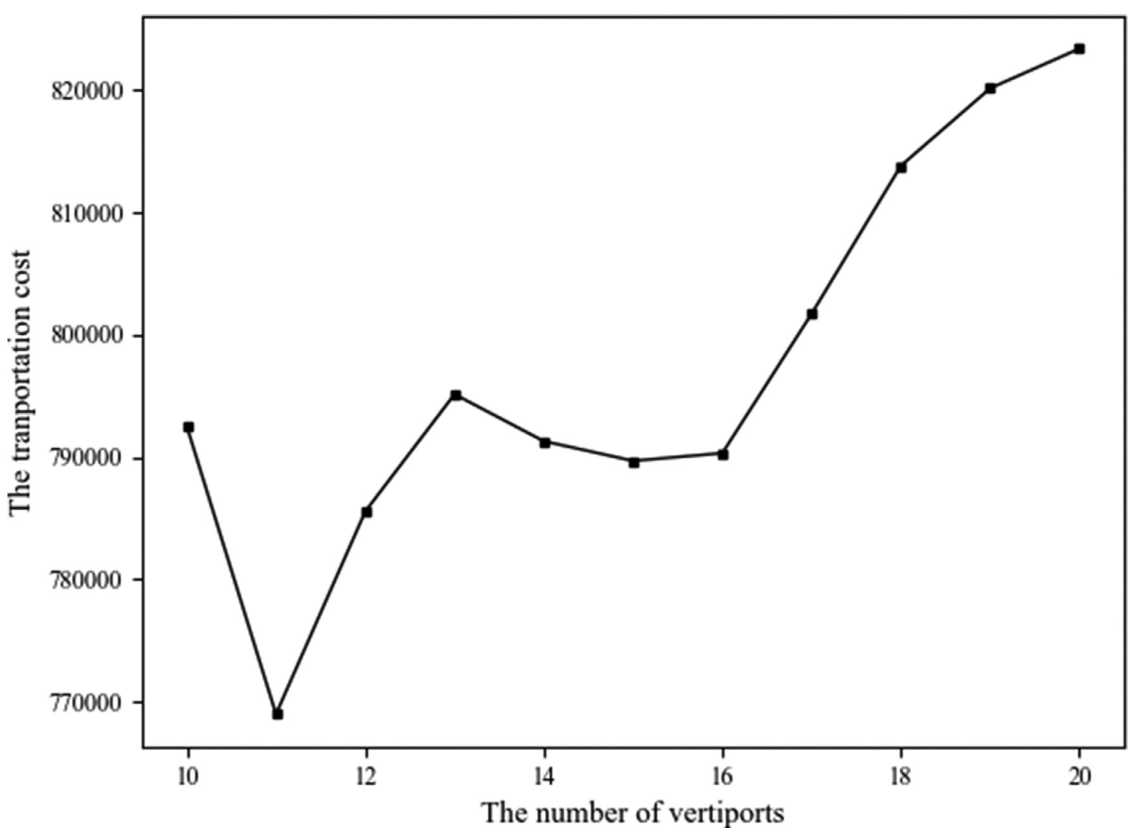

Fig. 9. The computational results of the transportation cost under different numbers of established vertiports.

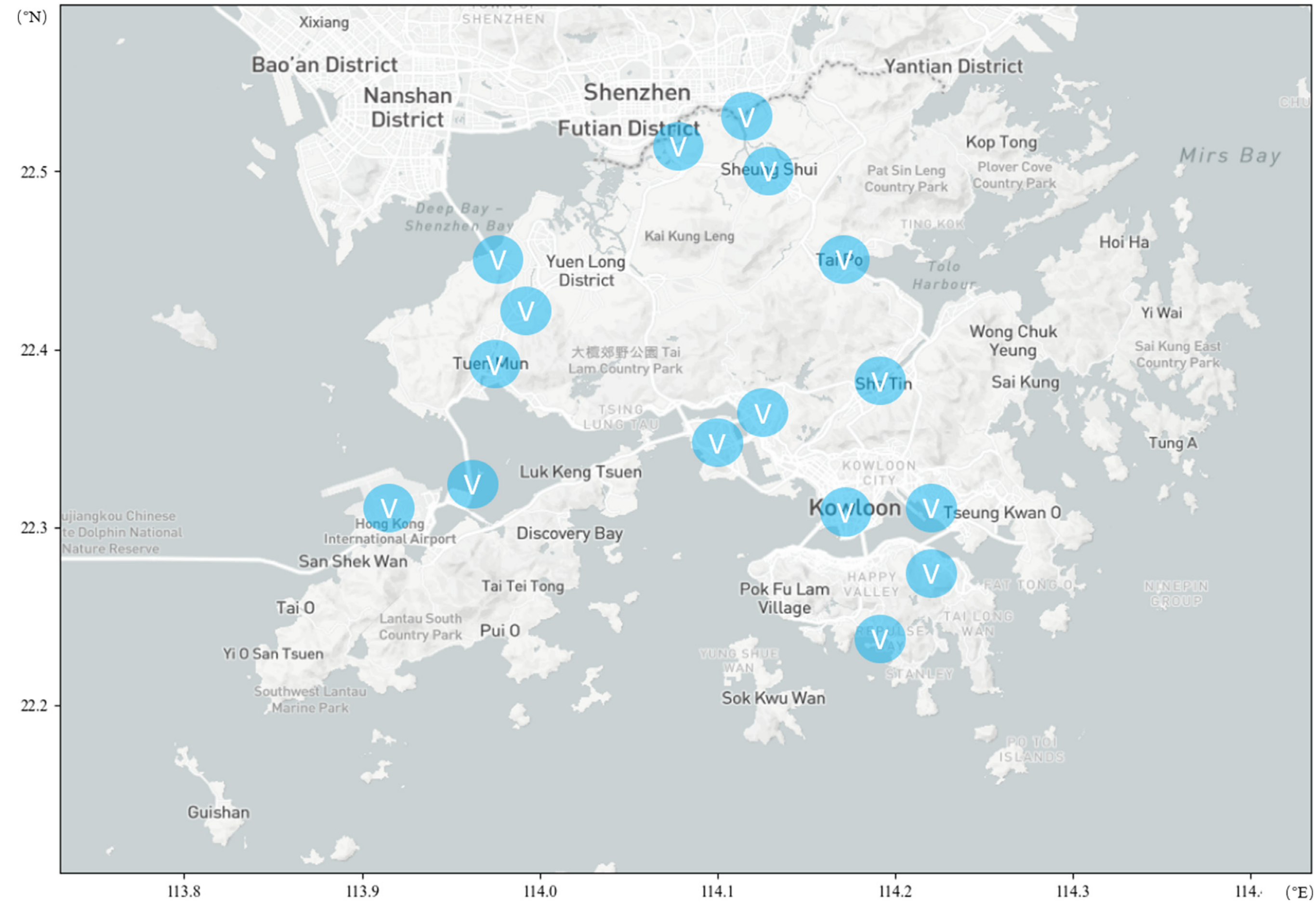

Fig. 10. The vertiport location decision results when $q = 0.5$.

$= -0.0125$, $\beta^2_{NT} = 0.0313$, $\beta^1_{LT} = -0.0155$, $\beta^2_{LT} = 0.0313$, $\beta^1_{AT} = -0.0213$, $\beta^2_{AT} = 0.018$.

To have a better visualisation of the effect of the budget of uncertainty (Γ_i) on the total cost of the network, we plot the total cost for different values of Γ_i. We select parameter Γ_i varying from 0 to 58 ($|C| \times (|N| - 1)$). Note that the instance with $\Gamma_i = 0$ corresponds to the deterministic model where the aggregate air taxi demand flow does not deviate from the nominal value. In contrast, the instance with $\Gamma_i = 58$ corresponds to the worst-case model where all the air taxi demand flow values of each node take their worst values from the corresponding intervals of uncertainty. Fig. 3 shows the computational results with different values of Γ_i. It is obvious that the total cost increases with the rise of the value of budget uncertainty. Besides, we can find that the increase rate of the objective value decreases with the rise of the parameter Γ_i.

In the context of the UAM system, it is crucial to account for the first-mile and last-mile ground transportation that connects the vertiport with the origin or destination node. This is due to the inherently multimodal nature of the system (Rimjha, 2022). The management of UAM systems should consider the integration of ground transportation into the design. Accommodating the time acceptable for first-mile and last-mile transportation of passengers can lead to more passenger-friendly and efficient UAM solutions. Therefore, we conduct the sensitivity analysis on the parameter of maximum acceptable time for the first-mile or last-mile ground transportation when using the UAM system, taking values from the set $\{0.4, 0.45, 0.5\}$. Figs. 4–6 show the location-assignment decisions under different values of maximum acceptable time for the first-mile or last-mile ground transportation. We can see that the UAM network structures under different parameters are quite different. It can be found that an extended maximum acceptable time for first-mile and last-mile ground transportation results in an increased range of route choices when air taxi companies provide multimodal transportation services. This can facilitate a more reasonable distribution of demand across various vertiports and help alleviate concerns about vertiport overcrowding. However, it is essential to note that an excessively large value can negatively impact passenger satisfaction, leading to wasted time and reduced overall system efficiency. From the perspective of management, policymakers should conduct a thorough assessment of the maximum acceptable time for first-mile and last-mile ground transportation. This assessment should be tailored to the unique characteristics of the metropolis, informed by market research, and receptive to passenger feedback. This approach serves as the key to delivering exceptional services and fostering harmonious integration of the UAM system and first-mile and last-mile ground transportation, ensuring that UAM services align with the specific needs and preferences of each metropolis.

5.2. Case study

In this subsection, we apply the proposed robust optimisation model to a practical case study. This study focuses on the vertiport location decision for the UAM system of Hong Kong. Hong Kong is a highly developed city and it is anticipated to see substantial demand for the UAM system in the near future (Chan et al., 2023).

5.2.1. Case background

We divide the study area into 33 districts according to the administrative divisions and indispensable functions. The geographic region and zone division are depicted in Fig. 7. In this case study, there are 1056 different OD pairs. The total user demand for any origin and destination

Fig. 11. The vertiport location decision results when $q = 1$.

Table 2
The computational results of different numbers of vertiports.

Number of vertiports	Vertiport location	OBJ (USD)
10	4,9,13,17,18,19,20,21,23,24	11292726
11	3,4,9,17,18,19,20,21,23,24,25	12103104
12	3,4,9,11,17,18,19,20,21,23,24,25	12914124
13	3,4,9,11,16,17,18,19,20,21,22,24,25	13731678
14	3,4,9,11,16,17,18,19,20,21,22,23,24,25	14556646
15	3,4,9,11,14,16,17,18,19,20,21,22,23,24,25	15392004
16	1,3,4,9,11,14,16,17,18,19,20,21,22,23,24,25	16231429
17	1,3,4,9,10,11,14,16,17,18,19,20,21,22,23,24,25	17072146
18	1,3,4,9,10,11,13,14,16,17,18,19,20,21,22,23,24,25	17914407
19	1,3,4,9,10,11,13,14,15,16,17,18,19,20,21,22,23,24,25	18758842
20	1,2,3,4,9,10,11,13,14,15,16,17,18,19,20,21,22,23,24,25	19603918

pair and any passenger type is randomly generated within the range of 2000 to 3000. The maximum acceptable time for the first-mile or last-mile ground transportation when using the UAM system is set as 0.3h. The remaining parameters are the same as those used in the experiment of the illustrative example discussed in the previous subsection.

5.2.2. Computational results of vertiport location

Table 2 displays vertiport location decisions and corresponding optimal objective values for various numbers of established vertiports. Furthermore, we conduct an analysis of the total time value cost and transportation cost as the number of established vertiports ranges from 10 to 20. Fig. 8 illustrates the change in total value cost as the number of vertiports varies from 10 to 20. It is evident that the total time value cost exhibits a decreasing trend as the number of vertiports increases. The analysis reveals a clear relationship between the number of established vertiports and the reduction in total time value costs. Air taxi managers should consider expanding the vertiport network to accommodate future growth. This strategy is expected to lead to cost-efficient transportation services and improve overall service quality in the long run.

Additionally, Fig. 9 shows the computational results of the transportation cost under different numbers of vertiports. When the number of vertiports varies from 10 to 16, transportation costs exhibit a fluctuating pattern. Notably, when the number of vertiports is 13, there is a slight peak. And it is evident that as the number of vertiports surpasses 16, the transportation costs consistently increase. This indicates a rising preference for air taxi services among passengers. This preference presents an opportunity for managers to expand and further develop their air taxi services. To address growing demand, consider strategies like increasing the fleet size, enhancing service quality, and improving customer experience. Besides, as the vertiport network expands, careful attention to transportation costs is crucial. A proactive cost management strategy should be employed to maintain a balance between service expansion and profitability.

5.2.3. The effect of demand patterns of heterogeneous passengers

Variations in demand patterns among heterogeneous passengers are noticeable on different days. For example, on weekdays, the demographic of passengers primarily consists of business travellers, indicating a higher need for efficient and time-sensitive travel. Conversely, the situation undergoes a perceptible transformation when the weekend arrives. During the weekends, the composition of passengers tilts in favour

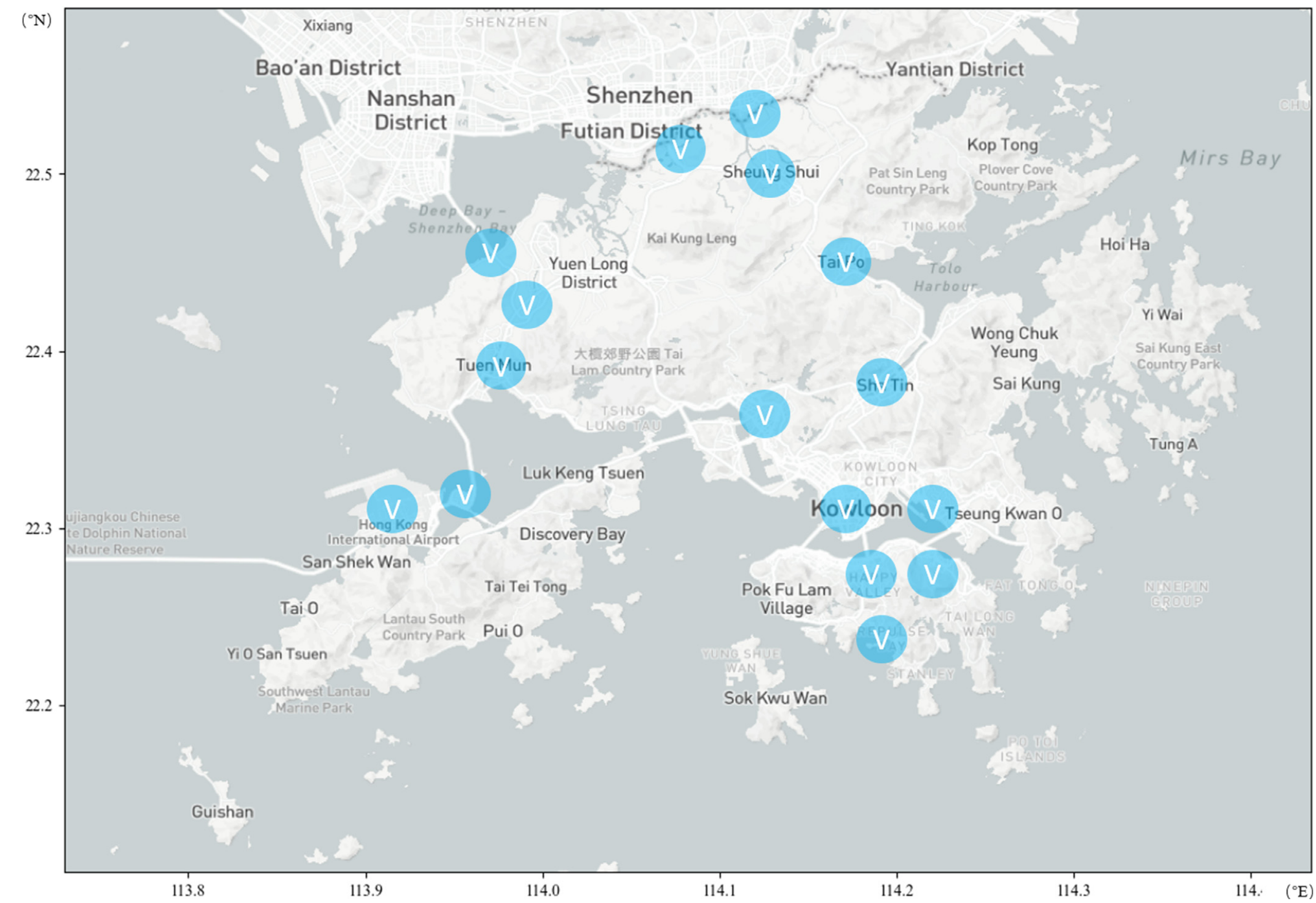

Fig. 12. The vertiport location decision results when $q = 2$.

of leisure travellers. Passengers with lower time value priorities tend to emphasize factors like comfort and relaxation over those on weekdays. In this part, we analyse the effect of demand patterns of heterogeneous passengers on the decision results of vertiport locations.

We randomly generate the number of leisure travellers ranging from 2000 to 3000 for any origin and destination pair, and subsequently, we randomly generate the number of business travellers within the range of $(2000 \times q)$ to $(3000 \times q)$, where q is a coefficient parameter. Figures 10,11,12 illustrate the vertiport location decisions under different values of q from the set $\{0.5, 1, 2\}$. These figures clearly depict the variability in vertiport locations to accommodate the distinct demand patterns of heterogeneous passengers. The result demonstrates that vertiport location decisions are highly adaptable to accommodate diverse demand patterns among leisure and business travellers. From a management perspective, it is imperative to have a flexible decision framework that can adjust vertiport locations based on the specific composition of passengers on different days. Therefore, collecting and analysing data related to passenger composition on different days is invaluable. Such data can guide decision-making processes, helping to optimise the location and capacity of vertiports to serve the needs of passengers efficiently.

6. Conclusions

This study is centred on addressing the robust vertiport location problem while considering travel mode choice behaviour. To begin with, we introduce a travel mode choice behaviour model that employs the multinomial logit choice approach. This model takes into account the two primary competitive transportation modes for air taxis: ordinary ground taxis and luxury ground taxis. Additionally, different time values of heterogeneous passengers (leisure passengers and business passengers) are also considered in this model. Based on the travel mode choice behaviour model, the aggregate air taxi demand flow is obtained. Moreover, recognising the inherent uncertainty in user demand, we construct a robust optimisation model for vertiport location decisions under a budget uncertainty set. This model aims to determine location-assignment decisions that minimise the combined costs of vertiport establishment cost, the total transportation cost of the UAM system, and the total time value costs incurred by heterogeneous UAM system passengers. To validate the effectiveness of our proposed model, we conduct an illustrative example and apply it to a real-world case study. The numerical results highlight the significant impact of the budget size on the objective value. Furthermore, our research identifies that both the maximum acceptable time for first-mile and last-mile ground transportation and the demand patterns of heterogeneous passengers influence vertiport location decisions. Leveraging the computational results, we present managerial insights that can support decision-making processes for UAM system managers.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Zhongyi Jin: Conceptualization, Data curation, Formal analysis, Methodology, Writing – original draft, Writing – review & editing.

Kam K.H. Ng: Conceptualization, Funding acquisition, Investigation, Methodology, Project administration, Resources, Supervision, Validation, Writing – review & editing. **Chenliang Zhang:** Data curation, Formal analysis, Methodology, Validation, Visualization, Writing – review & editing.

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