Project 3: Fuzzy Goal Programming

Minhajul Abedin Tajik

LU: 20521113

Most Important 20 Words

1. Fuzzy Goal Programming
2. Real-world applications
3. Multi-objective optimization
4. Imprecise
5. Deviational variables
6. Aspiration level
7. Additive fuzzy achievement
8. Eigen vector
9. Priority level
10. Membership functions
11. Objective functions
12. Non-commensurable goals
13. Simple additive model
14. Weighted additive model.
15. Preemptive priority in additive model
16. Tolerance limits
17. System constraints
18. Decision-makers (DM)
19. Weighting scheme
20. Measurable unit

Summary

Fuzzy Goal Programming (FGP) is a mathematical programming technique that uses deviational variables and membership functions to solve decision-making problems with imprecise or non-commensurable goals. FGP can be used for multi-objective optimization and has real-world applications in various fields such as environmental management, finance, healthcare, and engineering. The optimization of objective functions is a key aspect of FGP, where DMs use fuzzification to set aspiration levels and tolerance limits for system constraints. The relative importance of goals can be assessed using weighting schemes or the preemptive priority structure in additive models. Measurable units can be used to ensure consistency. The simple additive model and the weighted additive model are two common models of FGP. The objective function in the additive fuzzy achievement model is formulated by multiplying each membership of the fuzzy goal with a suitable weight and then adding them together. Eigen vectors can be used to determine priority levels for higher-level goals. Sensitivity analysis can help deal with uncertainty in real-world applications where system constraints are subject to tolerance limits set by DMs.

Model Formulation:

In this paper the mathematical model has been developed for a problem having 5 fuzzy goals with 4 variables:

Here, we need to find X to satisfy the following fuzzy goals:

$$
\begin{aligned}
&4x_1 + 2x_2 + 8x_3 + x_4 \leqslant 35,\\
&4x_1 + 7x_2 + 6x_3 + 2x_4 \geqslant 100,\\
&x_1 - 6x_2 + 5x_3 + 10x_4 \geqslant 120,\\
&5x_1 + 3x_2 + \quad + 2x_4 \geqslant 70,\\
&4x_1 + 4x_2 + 4x_3 \qquad \geqslant 40,\\
&\text{subject to}\\
&7x_1 + 5x_2 + 3x_3 + 2x_4 \leqslant 98,\\
&7x_1 + x_2 + 6x_3 + 6x_4 \leqslant 117,\\
&x_1 + x_2 + 2x_3 + 6x_4 \leqslant 130,\\
&9x_1 + x_2 + \quad + 6x_4 \leqslant 105,\\
&x_1, x_2, x_3, x_4 \geqslant 0.
\end{aligned}
$$

Here, tolerance limits of following 5 goals (55,40,70,30,10). The fuzzy goals are converted into crisp ones by using membership functions.

Case1: Simple Additive Model

The objective function and the constraint for the case is as follows:

$$
\begin{aligned}
&\text{maximize} \quad V(\mu) = \sum_{i=1}^{5} \mu_i\\
&\text{subject to} \quad \mu_1 = \frac{55 - (4x_1 + 2x_2 + 8x_3 + x_4)}{20},\\
&\mu_2 = \frac{4x_1 + 7x_2 + 6x_3 + 2x_4 - 40}{60},\\
&\mu_4 = \frac{5x_1 + 3x_2 + 2x_4 - 30}{40},\\
&\mu_5 = \frac{4x_1 + 4x_2 + 4x_3 - 10}{30},\\
&7x_1 + 5x_2 + 3x_3 + 2x_4 \leqslant 98,\\
&7x_1 + x_2 + 6x_3 + 6x_4 \leqslant 117,\\
&x_1 + x_2 + 2x_3 + 6x_4 \leqslant 130,\\
&9x_1 + x_2 + \quad + 6x_4 \leqslant 105,\\
&\mu_i \leqslant 1,\\
&x_j, \mu_i \geqslant 0, \quad i = 1, 2, \ldots, 5, \ j = 1, 2, 3, 4.
\end{aligned}
$$

Case2: Weighted Additive Model

For this case, for the following equations, the DM assigns differential weights as coefficients of the individual terms in the simple additive fuzzy achievement function to reflect their relative importance, i.e. Here, Wi is the relative weight of the I-th fuzzy goal. The considered weights are: 0.49, 0.131, 0.153, 0.114, 0.112.

$$\begin{aligned}
&\text{maximize} \quad V(\mu) = \sum_{i=1}^{m} w_i \mu_i \\
&\text{subject to} \quad \mu_i = \frac{G_i(X) - L_i}{g_i - L_i}, \\
&\qquad\qquad AX \leqslant b, \\
&\qquad\qquad \mu_i \leqslant 1, \\
&\qquad\qquad X,\ \mu_i \geqslant 0,\ \ i = 1, 2, \ldots, m.
\end{aligned}$$

Case 3: Preemptive Priority in Additive

While considering Preemptive Priority, the problem would be subdivided into k subproblems, and k is the number of priority levels. The preemptive priority structure may be stated as pi>>>pi+1 which means that the goals in the i-th priority level have higher priority than the goals in the (i+1)th priority level. The objective function is formulated by multiplying each membership of the fuzzy goal with a suitable weight and adding them together. To maximize the priority level 3. They used the u 1,2,3 function from the priority level 1 and 2.

Case 3.1 and Case 3.2: Preemptive Priority in Additive Priority 1 and 2

additional constraints. In general the i-th subproblem becomes

$$\begin{aligned}
&\text{maximize} \quad \sum_{s} (\mu_s)_{p_i} \\
&\text{subject to} \quad \mu_s = \frac{G_s - L_s}{g_s - L_s}, \\
&\qquad\qquad AX \leqslant b, \\
&\qquad\qquad (\mu)_{p_r} = (\mu^*)_{p_r},\ \ r = 1, 2, \ldots, j-1, \\
&\qquad\qquad \mu_s \leqslant 1, \\
&\qquad\qquad X,\ \mu_i \geqslant 0,\ \ i = 1, 2, \ldots, m,
\end{aligned}$$

where $(\mu_s)_{p_i}$ refers to the membership functions of the goals in the i-th priority level and $(\mu^*)_{p_r}$ is the achieved membership value in the r-th ($r \leqslant j-1$) priority level.

Case 3: Preemptive Priority in Additive Priority 1 and 2

$$
\begin{aligned}
\text{maximize} \quad & V(\mu) = \mu_4 + \mu_5 \\
\text{subject to} \quad & \mu_1 = 1, \\
& \mu_2 = 0.795, \\
& \mu_3 = 1, \\
& \mu_4 \leqslant 1, \\
& \mu_5 \leqslant 1, \\
& \text{and (5b) excluding } \mu_i \leqslant 1.
\end{aligned}
$$

Results

Case1: Simple Additive Model

Case: Simple Additive Model

Variable					Objective				
0.00	9.75	0.00	15.875		4.33				
x1	x2	x3	x4	Wi	Mfi		Results	Limits	Goals
4.00	2.00	8.00	1.00	1.00	0.981	<=	35	55.00	35.375
4.00	7.00	6.00	2.00	1.00	1.000	>=	100	40.00	100.0
1.00	-6.00	5.00	10.00	1.00	0.605	>=	120	70.00	100.25
5.00	3.00	0.00	2.00	1.00	0.775	>=	70	30.00	61.0
4.00	4.00	4.00	0.00	1.00	0.967	>=	40	10.00	39.0
7.00	5.00	3.00	2.00	<=	98.00	80.50			
7.00	1.00	6.00	6.00	<=	117.00	105.00			
1.00	1.00	2.00	6.00	<=	130.00	105.00			
9.00	1.00	0.00	6.00	<=	105.00	105.00			

Case2: Weighted Additive Model

Case: Weighted Additive Model

Variable					Objective				
0.00	9.55	0.00	15.91		0.91				
x1	x2	x3	x4	Wi	Mfi		Results	Limits	Goals
4.00	2.00	8.00	1.00	0.49	1.000	<=	35	55.00	35.000
4.00	7.00	6.00	2.00	0.13	0.977	>=	100	40.00	98.636
1.00	-6.00	5.00	10.00	0.15	0.636	>=	120	70.00	101.818
5.00	3.00	0.00	2.00	0.11	0.761	>=	70	30.00	60.455
4.00	4.00	4.00	0.00	0.11	0.939	>=	40	10.00	38.182
7.00	5.00	3.00	2.00	<=	98.00	79.55			
7.00	1.00	6.00	6.00	<=	117.00	105.00			
1.00	1.00	2.00	6.00	<=	130.00	105.00			
9.00	1.00	0.00	6.00	<=	105.00	105.00			

Case 3.1: Preemptive Priority in Additive Priority 1

Case: Preemptive Priority in Additive Priority 1

Variable					Objective				
1.70	0.00	2.18	10.74		2.00				
x1	x2	x3	x4	Wi	Mfi		Results	Limits	Goals
4.00	2.00	8.00	1.00	1.00	1.00	<=	35	55.00	35.000
4.00	7.00	6.00	2.00	1.00	0.02	>=	100	40.00	41.378
1.00	-6.00	5.00	10.00	1.00	1.00	>=	120	70.00	120.000
5.00	3.00	0.00	2.00	1.00	0.00	>=	70	30.00	30.000
4.00	4.00	4.00	0.00	1.00	0.18	>=	40	10.00	15.539
7.00	5.00	3.00	2.00	<=	98.00	39.95			
7.00	1.00	6.00	6.00	<=	117.00	89.45			
1.00	1.00	2.00	6.00	<=	130.00	70.50			
9.00	1.00	0.00	6.00	<=	105.00	79.77			

Case 3.2: Preemptive Priority in Additive Priority 2

Case: Preemptive Priority in Additive Priority 2

Variable					Objective
0.00	7.48	0.47	16.25		0.80

x1	x2	x3	x4	Wi	Mfi		Results	Limits	Goals
4.00	2.00	8.00	1.00	1.00	1.00	<=	35	55.00	35.000
4.00	7.00	6.00	2.00	1.00	0.795	>=	100	40.00	87.719
1.00	-6.00	5.00	10.00	1.00	1.00	>=	120	70.00	120.000
5.00	3.00	0.00	2.00	1.00	0.62	>=	70	30.00	54.953
4.00	4.00	4.00	0.00	1.00	0.73	>=	40	10.00	31.820
7.00	5.00	3.00	2.00	<=	98.00	71.34			
7.00	1.00	6.00	6.00	<=	117.00	107.84			
1.00	1.00	2.00	6.00	<=	130.00	105.95			
9.00	1.00	0.00	6.00	<=	105.00	105.00			

Case 3.3: Preemptive Priority in Additive Priority 3

Case: Preemptive Priority in Additive Priority 3

Variable					Objective
0.00	7.48	0.47	16.25		1.35

x1	x2	x3	x4	Wi	Mfi		Results	Limits	Goals
4.00	2.00	8.00	1.00	1.00	1.00	<=	35	55.00	35.000
4.00	7.00	6.00	2.00	1.00	0.79	>=	100	40.00	87.700
1.00	-6.00	5.00	10.00	1.00	1.00	>=	120	70.00	120.000
5.00	3.00	0.00	2.00	1.00	0.62	>=	70	30.00	54.948
4.00	4.00	4.00	0.00	1.00	0.73	>=	40	10.00	31.815
7.00	5.00	3.00	2.00	<=	98.00	71.33			
7.00	1.00	6.00	6.00	<=	117.00	107.83			
1.00	1.00	2.00	6.00	<=	130.00	105.93			
9.00	1.00	0.00	6.00	<=	105.00	105.00			