University of Moratuwa

Department of Electronic & Telecommunication Engineering

EN2074 Communication Systems Engineering

Lab Assignment

EYE DIAGRAMS AND EQUALIZATION

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THIS REPORT IS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE MODULE

EN2074 Communication Systems Engineering

6th of June 2023

Task 1

Eye Diagrams for Baseband 2-PAM

The initial step is to generate an impulse train representing BPSK symbols. To do that, a set of random numbers between 0 and 1 is compared to 0.5, and then the compared value is replaced by +1 if it is greater than 0.5, and by -1 if it is less than 0.5.

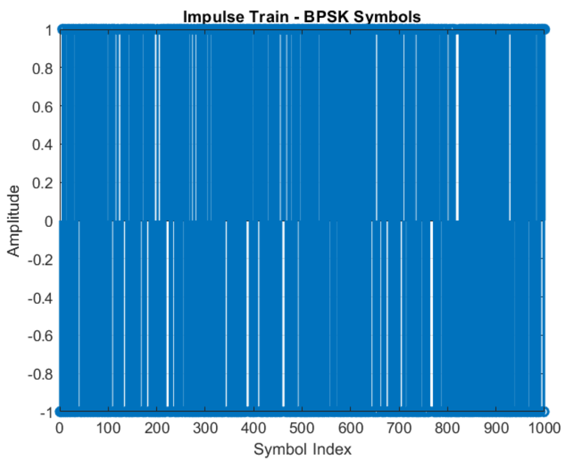

Figure 1: Impulse Train - BPSK Symbols

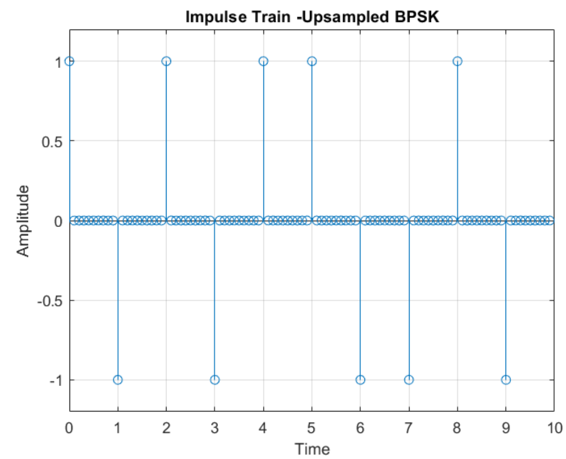

Figure 2: Impulse Train -Upsampled BPSK

Before convolving the impulse train with a pulse shaping filter, the pulse train is up sampled. In up sampling, more zeros are added to existing points. If the pulse train is not up sampled, the convoluted signal will not contain any values in between sampling points.

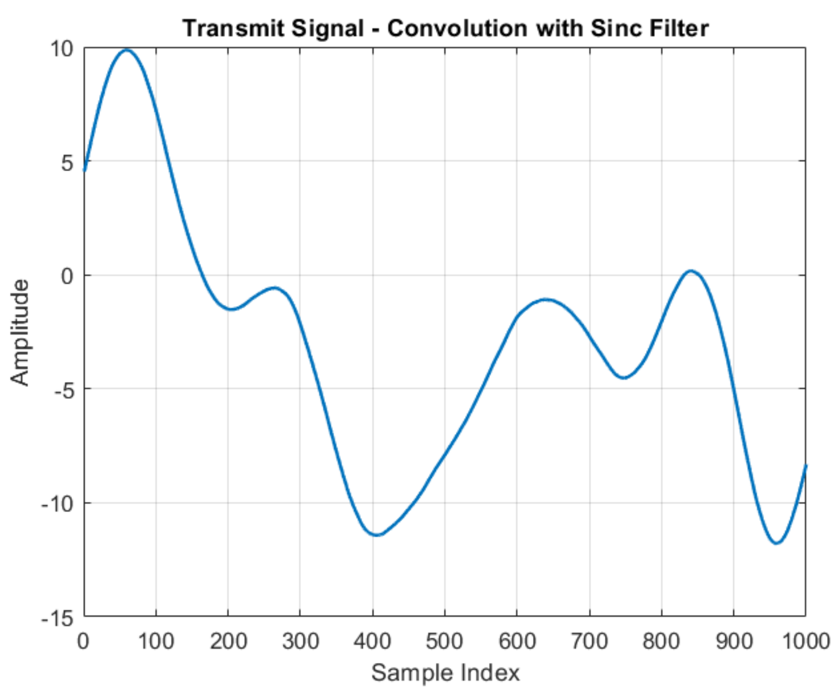

Figure 3: Transmit Signal - Convolution with Sinc Filter

The pulse shaping filter $p(t)$ is generated, where

$$p(t) = \sin(\pi R_b t) \quad \text{for} \quad 0 \leq t \leq T_b,$$

where R_b is the bit rate and T_b is the bit period.

After convolving the upsampled impulse train by the raised cosine pulse shaping filter, the received signal is reshaped.

The raised cosine pulse shaping filter is given by

$$p(t) = \sin(\pi R_b t) \times \frac{\cos(\pi r R_b t)}{1 - (2rR_b t)^2},$$

, where r is the roll-off factor.

$$r = \frac{\text{excess bandwidth}}{\text{theoretical minimum bandwidth}}.$$

, where r is the roll-off factor.

Two figures are shown below, considering $r = 0.5$ and $r = 1$.

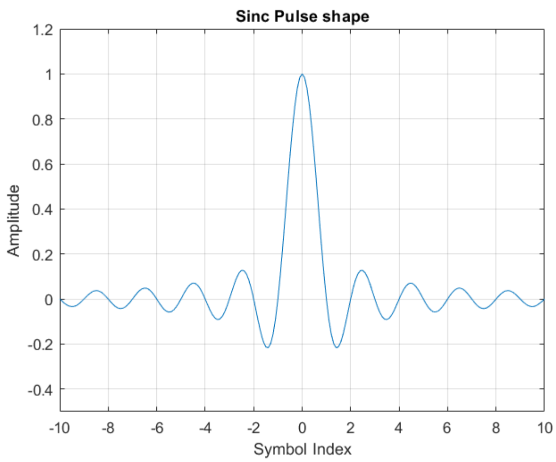

Figure 4: Sinc Pulse shape

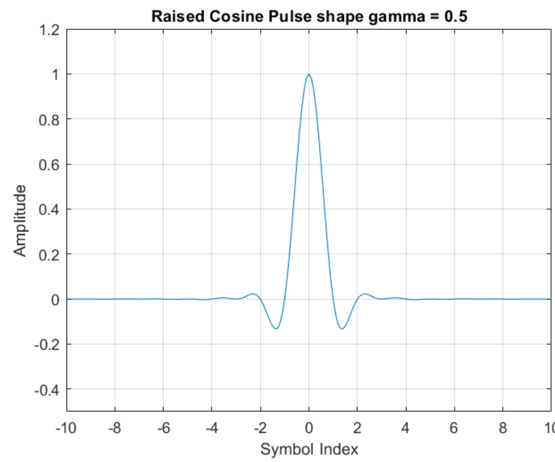

Figure 5: Raised Cosine Pulse shape gamma=0.5

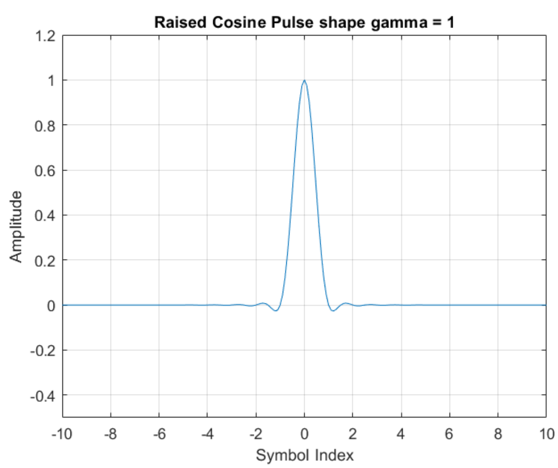

Figure 6: Raised Cosine Pulse shape gamma=1

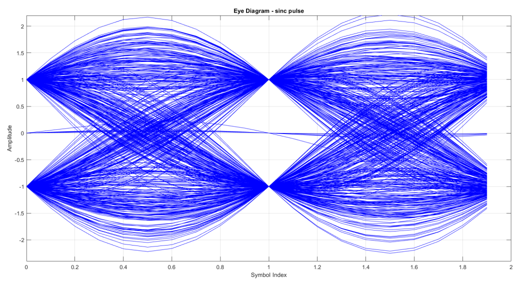

Figure 7: Eye Diagram - Sinc Pulse

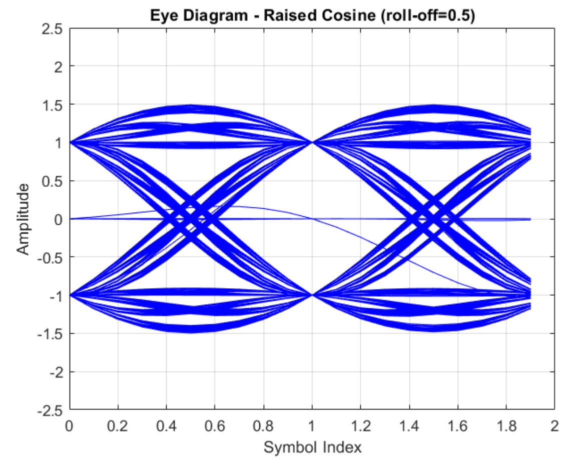

Figure 8: Eye Diagram - Raised Cosine (roll-off=0.5)

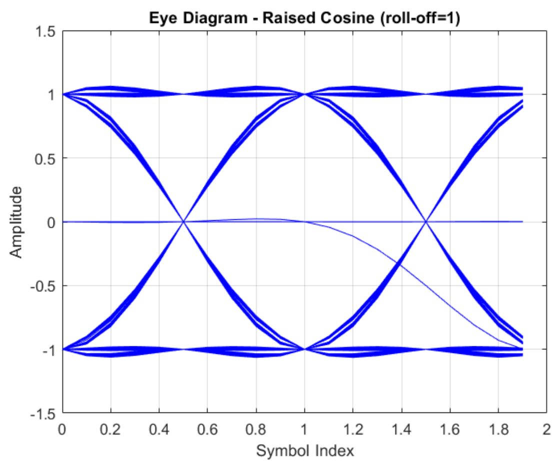

Figure 9: Eye Diagram - Raised Cosine (roll-off=1)

Comparison of the robustness of the system

1. Noise: The channel is assumed to be free of noise.
2. Sampling Time: If there are variations with high gradients near the sampling points of the eye, then small errors in the sampling time can cause large errors in the sample value. This effect is particularly noticeable when the pulse shaping filter is a Sinc function with a roll-off factor of 0. However, by increasing the roll-off factor, the gradients can be reduced, making the system more robust to small sampling time errors.
3. Synchronization Errors: Synchronization errors occur due to timing jitter. These errors are clearly visible in the eye diagram, where the amount of jitter is proportional to the amount of variation at zero-crossing locations. In the case of a Sinc function pulse shaping filter, the eye diagram exhibits the most significant amount of jitter compared to other diagrams. The timing jitter increases with the amount of variation at transition or zero-crossing locations, leading to more synchronous errors. By increasing the roll-off factor from 0 to 1, the timing jitter can be decreased.

Task 2

Repeat Task 1, in the presence of additive white Gaussian noise (AWGN)

Here the channel contains additive white Gaussian noise. It is given to set the variance of noise such that $E_b/N_0 =$ 10dB, where E_b is the average bit energy and N_0 is the noise power spectral density. Therefore, AWGN is normally distributed with zero (0) mean and 0.05 variance.

The below figures show the plots of above pulse shaping filters when noise is present.

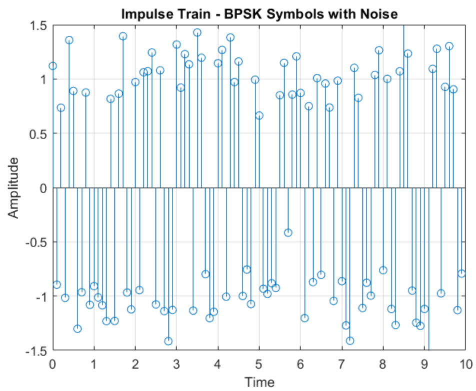

Figure 10: Impulse Response - BPSK Symbols with noise

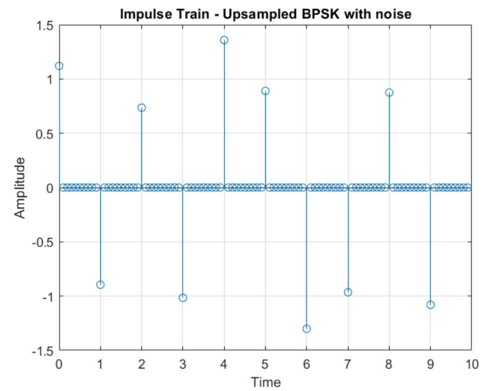

Figure 11: Impulse Response - Upsampled BPSK Symbols with noise

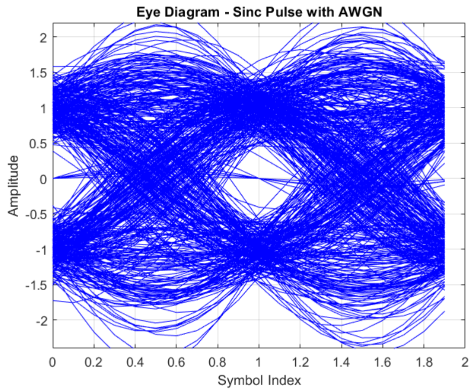

Figure 12: Eye Diagram - Sinc Pulse with AWGN

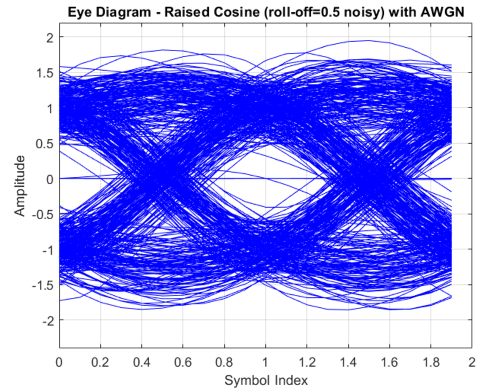

Figure 13: Eye Diagram - Raised Cosine (roll-off=0.5 noisy) with AWGN

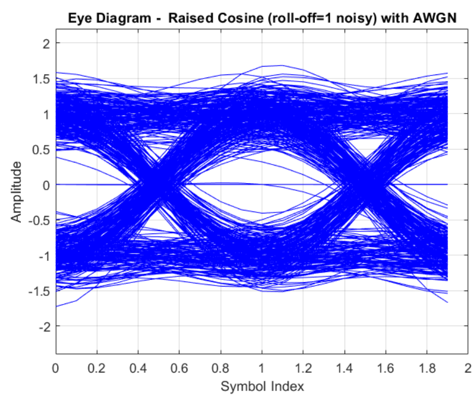

Figure 14: Eye Diagram - Raised Cosine (roll-off=1 noisy) with AWGN

Comparison of the robustness of the system

1. Noise: The AWGN present in the channel reduces the margin of error as the noise affects the value at the sampling point. Increment in the number of signal levels reduces the margin of error more and more. Therefore, the system is not reliable. From the above plots, it can be observed that for all pulse shaping filters, the effect of noise at the sampling points is present.

2. If there are variations with high gradients near the sampling points of the eye, then small errors in the sampling time can cause large error in sample value. Apart from the observations in the task 1, it is clear that the gradients near the sampling points have increased . By increasing the roll-off factor, the gradients can be reduced. Therefore, the system becomes more robust to sampling time errors

3. Due to timing jitter the synchronization errors occur. With the presence of additive white Gaussian noise, the amount of timing jitter has increased. Greater the variation at transition or zero-crossing locations, greater the timing jitter. It causes the system to occur more synchronous errors. By increasing the roll-off factor from 0 to 1, the timing jitter can be decreased.

Task 3

Zero-Forcing (ZF) equalizer for a 3-tap multipath channel

The Zero-Forcing Equalizer applies the inverse of the channel frequency response to the received signal, to restore the signal after the channel. Main goal of this is to minimize the Inter Symbol Interference (ISI) of the channel.

ZF equalizing is most useful when ISI is significant compared to noise. If noise is much larger, ZF equalizer will further increase the effect of the noise rather than removing the ISI.

If the channel acts like a M-tapped filter, N pulses are affected in each direction where M = 2N + 1.

$$P_o[k] = \sum_{n=-N}^{N} C_n P_r[k-n],$$

As for Nyquist I criteria, we need to,

$$P_o[k] = \begin{cases} 1, & \text{if } k = 0 \\ 0, & \text{if } k = \pm 1, \pm 2, \ldots, \pm N \end{cases}$$

Where:

1. P_o = filtered received signal,
2. P_r = sampled received signal, and
3. C_n = filter coefficients.

We can denote this relationship as a matrix multiplication as follows.

$$\mathbf{P_o} = \mathbf{P_r}\mathbf{C} \; Then,$$

$$\mathbf{C} = \mathbf{P_r}^{-1}\mathbf{P_o}$$

Here, $\mathbf{P_r}$ is a Toeplitz matrix. For example, a 3-tapped filter:

E.g.: 3-tapped filter

$$\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{P_r}[0] & \mathbf{P_r}[-1] & \mathbf{P_r}[-2] \\ \mathbf{P_r}[1] & \mathbf{P_r}[0] & \mathbf{P_r}[-1] \\ \mathbf{P_r}[2] & \mathbf{P_r}[1] & \mathbf{P_r}[0] \end{bmatrix} \cdot \begin{bmatrix} \mathbf{C}[-1] \\ \mathbf{C}[0] \\ \mathbf{C}[1] \end{bmatrix}$$

In this assignment, we are given channel impulse response, h = [0.3 0.9 0.4]. After obtaining the filter coefficients

Tapped Filter	Coefficients
3-tapped filter	$[-0.2899, 1.3043, -0.4348]$
5-tapped filter	$[0.0705, -0.3175, 1.3228, -0.4762, 0.1587]$
7-tapped filter	$[-0.0171, 0.0768, -0.3199, 1.3244, -0.4798, 0.1727]$
9-tapped filter	$[0.0041, -0.0186, 0.0773, -0.3201, 1.3245, -0.4802, 0.1740, -0.0626, 0.0209]$

Table 1: Coefficients of Different Tapped Filters

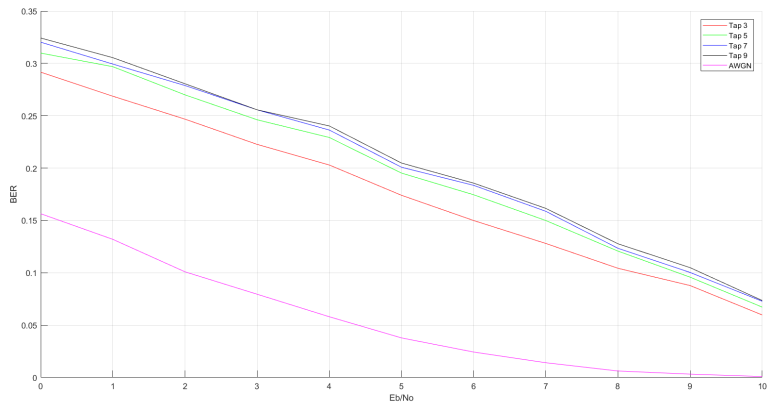

Figure 15: BER performance

In AWGN channel, there is only noise. Since there is no ISI, the BER is less. In zero equalized multipath channel, there is noise as well as ISI. We obtained zero forcing equalizers and inverted the effect of ISI of the channel. But there is an unwanted effect of that process. The effect of noise was amplified by equalizing. Therefore, the BER is higher in multipath channel than AWGN channel. When we increase the SNR, effect of the noise is reducing. Therefore, at higher SNR values, ISI is dominating. Therefore, the gap between BER values of two channels has reduced at higher SNR values.