Application of the

Monte-Carlo Simulation

and the

Datar-Mathews method

for real option valuation

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Research Paper and Case Study

1. Introduction

The research goal of this paper is to understand the practical use and application of Monte-Carlo simulations in the context of real investments. Real investments are used as the object of this research since they entail commonly and naturally the characteristics of an option. This makes the application of a Monte-Carlo simulation, such as in the process of applying the Datar-Mathews method for option valuation, a suitable approach.

To illustrate this, I will begin by introducing a hypothetical case study on a gold mine investment. I will then dwell on the limitations of that approach and derive from these the advantages of using a Monte-Carlo simulation. The results of the simulation cannot only be used to gain previously not apparent insights into the risk and return profile of an investment but also to put an explicit value on the real option involved by applying the Datar-Mathews method.

In the following I outline the case study:

The company *Gold Digger AG* has a new investment opportunity: It can buy the right to extract gold from a previously inaccessible territory in Nigeria. With this extraction right, the company has the claim to mine gold from the gold ore that is known to exist in the territory for a period of five years once it starts the extraction. It will take the company one year, starting from the decision to launch the project to extract the first gold, until it will gain the first cash flows from the investment. The total Project Duration is therefore 6 years.

The question the management of the *Gold Digger AG* asks and that is investigated further is whether it should purchase the extraction right and how much it should be willing to pay for it.

2. Simple DCF Method (Neo-classical approach)

In the conventional, simple discounted cash flow approach only one single scenario of how the investment will unfold is taken into account. The future net cash flows of the investment are forecasted, based on best estimates of incurring costs and revenues. The management will use its experience, other expert judgement and past data to determine a most likely scenario and then discount the resulting numbers to their present value using a single discount rate/hurdle rate. The investment is considered profitable if it results in a return larger than the cost of capital which is used as the discount rate.

Now, let's look at how the *Gold Digger AG* would apply this method to determine the cash flows of the investment.

The launch costs to start with the gold extraction, including the necessary infrastructure, machines etc., from the ore are estimated by management to be 3,350 million €.

The management takes into account the current market price of gold (1045.83€ per ounce) and assumes this price to remain constant for the 5 years of gold extraction after the initial investment (= the launch costs) is made.

The best estimate by the geologist of the *Gold Producer AG* for the amount of gold found in the ore is 10 million ounces. Management assumes that this gold volume is extracted from the ore over the 5 years in equal amounts every year.

Cost accountants know from past ores that the project most likely has a gross margin of 40%, meaning the yearly operating costs are 60% of the revenues.

Based on the company's capital structure the discount rate is calculated as the WACC and amounts to 8%.

<u>Overview of used variables and values:</u>

Variable	Best Estimate	Explanation
Gold Price in €	1,045.83	Gold Price at time t0
Gold Amount in troy ounces	10,000,000.00	Best Estimate of Geologists
Gross Margin	40.00%	Best Estimate based on former ores
WACC / Hurdle Rate	8.00%	Based on current capital structure
Launch Costs in €	3,350,000,000.00	Best Estimate based on current technology

Now the calculations can begin and the management can determine the revenue cash flows based on the gold price and gold amount mined, as well as the cost cash flows.

After discounting them at the appropriate rate the Net Present Value of the investment can be determined.

<u>Overview of the revenue and cost cash flows:</u>

Years	0	1	2	3	4	5
Revenues in €	0.00	2,091,660,000.00	2,091,660,000.00	2,091,660,000.00	2,091,660,000.00	2,091,660,000.00
Costs in €	3,350,000,000.00	1,254,996,000.00	1,254,996,000.00	1,254,996,000.00	1,254,996,000.00	1,254,996,000.00
Net Cash Flows	-3,350,000,000.00	836,664,000.00	836,664,000.00	836,664,000.00	836,664,000.00	836,664,000.00

NPV in €	**-9,443,249.54**

In the end the NPV of the investment is -9,443,249 €. The management would therefore not invest in the project: The company would not buy the extraction right for the territory in Ghana.

3. Monte-Carlo Simulation

It is obvious that in the simple DCF approach from above, no deviations from the best estimates are taken into account. This means it is assumed that no variations in the input variables are possible. Therefore, the management does only receive an impression on the profitability of the project if the scenario unfolds as it was predicted.

However, any real investment, especially a Very Large Industrial Real Investment (= VLIR), such as a gold mine, is characterized by high uncertainties[1]:

- they often have a long building time and market conditions can change in this time
- they have a long economic life and market conditions can change in this time
  - uncertainty about the accuracy of cash flow estimates increases the further in the future the cash flows take place, because the uncertainty of future investment cash flows comes from a large unspecified number of sources that grows with time; the complexity of the "system" increases.
- some VLIRI can steer their markets

The first simple NPV approach does not take into account these uncertainties and the management flexibility that refers to the different possibilities that managers have to optimise the value of an investment. Mikael Collan gives a very understandable explanation on that:

“According to Zadeh's principle of incompatibility uncertain future expectations cannot yield precise (certain) estimates of investment cash flows, and the further in the future the cash flows take place, the less precise the estimates can be. Turning this around means that using precise estimates of investment cash flows cannot give a correct picture of the uncertainty of the cash flow estimates, and the further away the cash flow takes place, the less correct the picture becomes. Precise estimates about an uncertain future can cause a credibility problem, which can also be called a false sense of accuracy (or certainty). This credibility problem is especially relevant for investments with long economic lives.”[2]

The uncertainty, which is included in the various variables that determine the profitability of an investment, must be represented mathematically in order to be able to process them in further considerations such as the calculations in a Monte-Carlo

[1] Collan, Mikael. "Giga-investments: modelling the valuation of very large industrial real investments." (2004): 1-145.
[2] Ebd.

Simulation. In practice, probability distributions of the variables are created based on past data, experts and tinkering. In a new field of research in finance the mathematics of fuzzy numbers are used to derive probability distributions for variables that are qualitatively in nature or easier described qualitatively than quantitatively. This approach (e.g. Fuzzy-Payoff Method) is however not part of this research paper but will be investigated further by the authors of this paper. For insights into this approach see the work in the Appendix.

For this Gold Digger AG case, I came up with the following sources of variabilities and uncertainties, including the probability distributions with which they can be described:

<u>Gold Price:</u>
The price of gold can change during the investment (i.e. from the extraction of the first gold to five years later), thereby influencing the revenues.
A normal distribution is fitted to the gold price of the last five years.

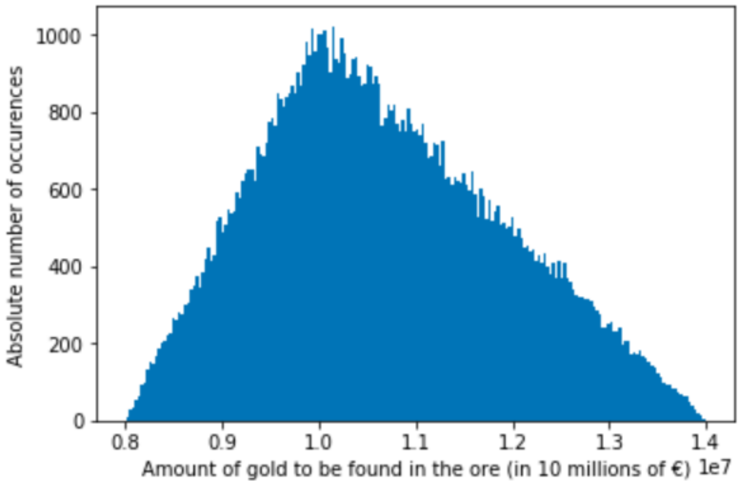
<u>Gold Amount:</u>
The amount of gold that can be extracted from the ore is uncertain and influences the revenues as well.
A triangular distribution is used with a mode of 10 million ounces, lower limit of 8 million ounces and an upper limit of 14 million ounces.

<u>Launch Costs:</u>
The initial investments necessary for the gold mine are spread out over one whole year. These costs are therefore uncertain, as personnel costs might change and the available technology for digging evolves changing the costs incurred as well.
I use a triangular distribution with a mode of 3350 Mio. € and a lower/upper limit of 290 Mio. € / 345 Mio. €.

<u>Operating Costs:</u>
The extraction costs necessary to operate the gold mine which occur in year 1 to 5 are also uncertain due to technological changes, changing material costs and changing personnel costs.
I use a triangular distribution with the costs as a percentage of revenues with a mode of 60% and a lower/upper limit of 50 % / 70%.

<u>Discount rate/WACC:</u>
The used discount rate changes with the risk-free rate, the capital structure corresponding to the financing of the project and inflation. It is therefore prone to change over the 5 year period.
I assume a triangular distribution with a best estimate of 8% and a lower/upper limit of 7% / 9%.

<u>Histogram of one of the used triangular distributions:</u>



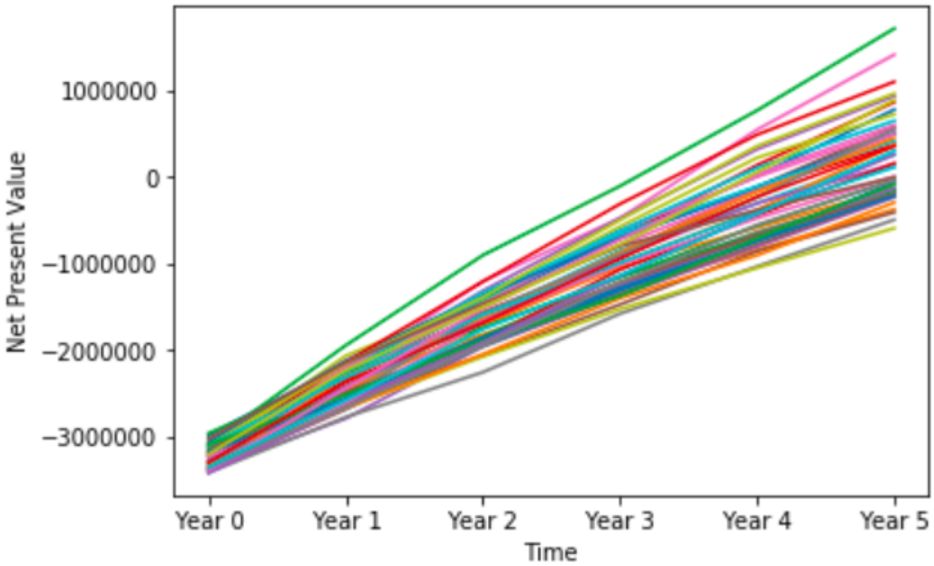
Clearly most input variables do not follow the assumed triangular distributions in reality. Nevertheless, this form of probability distribution is often used in practice due to its simplicity.: For the triangular distribution only three parameters of the distribution most be known: The “most likely value” (= the mode of the distribution), corresponding to the manifestation of the variable with the highest probability to occur, the minimum value and the maximum value of the distribution. (To ensure integrity of our case, I have used the best estimates from the application of the simple DCF method as the modes of the triangular distributions.)

In practice I often do not know more about a given variable other than theses three values and consequently the triangular distribution is used. However, if additional information is available, more sophisticated distribution functions can be applied.

After the uncertainty in the variables is declared mathematically, I can now execute a Monte-Carlo Simulation. This simulation calculates the NPV of the project while drawing the values of the input variables randomly from the corresponding distribution.

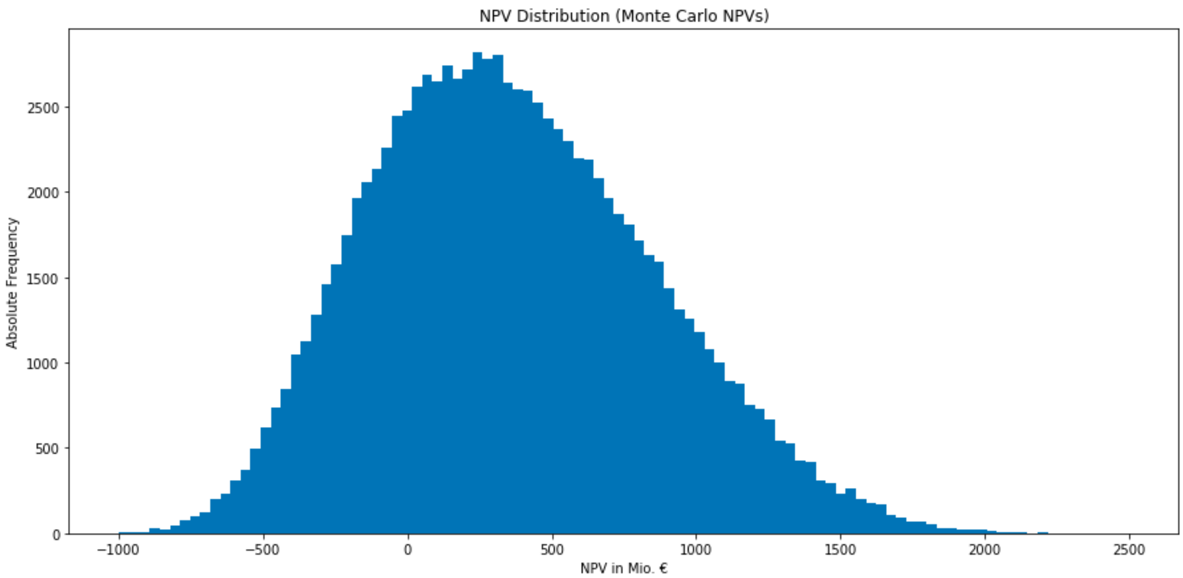
The following line plots illustrate the outcomes of 50 runs of the Monte-Carlo Simulation. The net present value of the project is calculated after each year and then plotted until I arrive at the net present in year five.

<u>50 developments of the net present value over the 5 years of the investment:</u>



The amount of gold to be found in the ore and the initial costs of the project are drawn only once at the beginning of the simulation, as they are fixed values after they materialize before the first gold can be extracted. All other variables are determined in each of the following years 1 to 5 as they can change each year.

For the following graphics and considerations, I run the simulation one million times, which translates into one million different net present values for each of the runs. These resulting NPVs (the so-called Monte-Carlo NPVs) then form a distribution again:



The information content of this distribution is highly important to the management. It shows the various possible outcomes of the investment and their relative likelihood of occurrence.

Instead of achieving one single value outcome on which the whole investment decision must be based (as shown in the previous chapter), I now take into account the variability of value-determining factors. The outcome is superior in the sense that the management can have previously unrecognizable insights into the risk-return trade-off of the investment. What a difference these insights can make can be seen quite vividly in my case study:

Gold Digger's management can now see that the majority of the possible outcomes of the investment are actually NPV positive. Keep in mind that the net present value calculated with the simple DCF approach yielded a significantly negative result. Due to the possibility of exposure to more value-creating developments in the input variables compared to value-destroying developments, the Monte-Carlo simulation shows however that there is a good chance of having a profitable investment at hand. When the management is aware of its opportunities to manage some of the input variables in the right way (e.g. in the financing decision of the project that leads to the WACC distribution) the decision to definitely not invest into the project seems not justifiable anymore.

Still, an important question is at hand: How much is the investment actually worth? How much should the *Gold Digger AG* pay? The distribution, with all its new insights, is already a big step forward from the conventional approach but the management needs a specific value that it can base its decision on and use in negotiations for the deal.

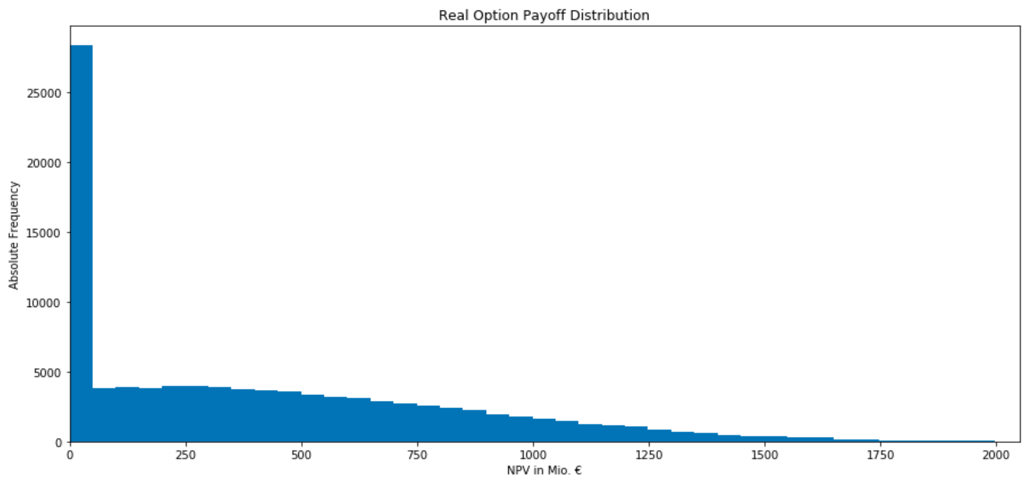
4. Real option valuation using the Datar-Mathews method

When I assume an attentive management that is aware of its influence on the value-defining variables and that terminates its investment considerations during the negotiation and due diligence process if the variables manifest themselves in a disadvantageous / value-destroying way, I can think of the investment as a real option. Then the so-called Datar-Mathews method can be applied to calculate the value of the investment. This rather new and in practice not yet often applied approach was developed in the year 2000 by professor Vinay Datar and Scott H. Mathews. I will follow this method based on the steps outlined in Scott Mathews paper “A Practical Method for Valuing Real Options: The Boeing Approach”[3] (Mathews et al. , 2007).

The term “non-linear” is often applied to real options. This simply means that the project payoff has two different outcomes: zero for the terminated cases and a positive net profit for the successful cases, reflecting the contingent decision-making. A real option valuation is always positive denoting a rational decision to invest the significant launch costs only if today I forecast a positive risk-adjusted NPV at launch time.

Applying it to my case study this means: In the time preceding the actual investment (= the purchase of the concession) the management of the *Gold Digger AG* will do anything to manage the factors containing uncertainty in an advantageous, meaning value-creating, way. Furthermore, it will stop considering the investment if the input variables manifest themselves in a value-destroying way such that the forecasted NPV is negative.

The negative NPV outcomes of the Monte-Carlo simulation can therefore be mapped to a value of zero. This leads to a new payoff distribution, the payoff distribution of the real option:



Now it is possible for us to calculate the value of this real option using the formula established by Datar and Mathews:

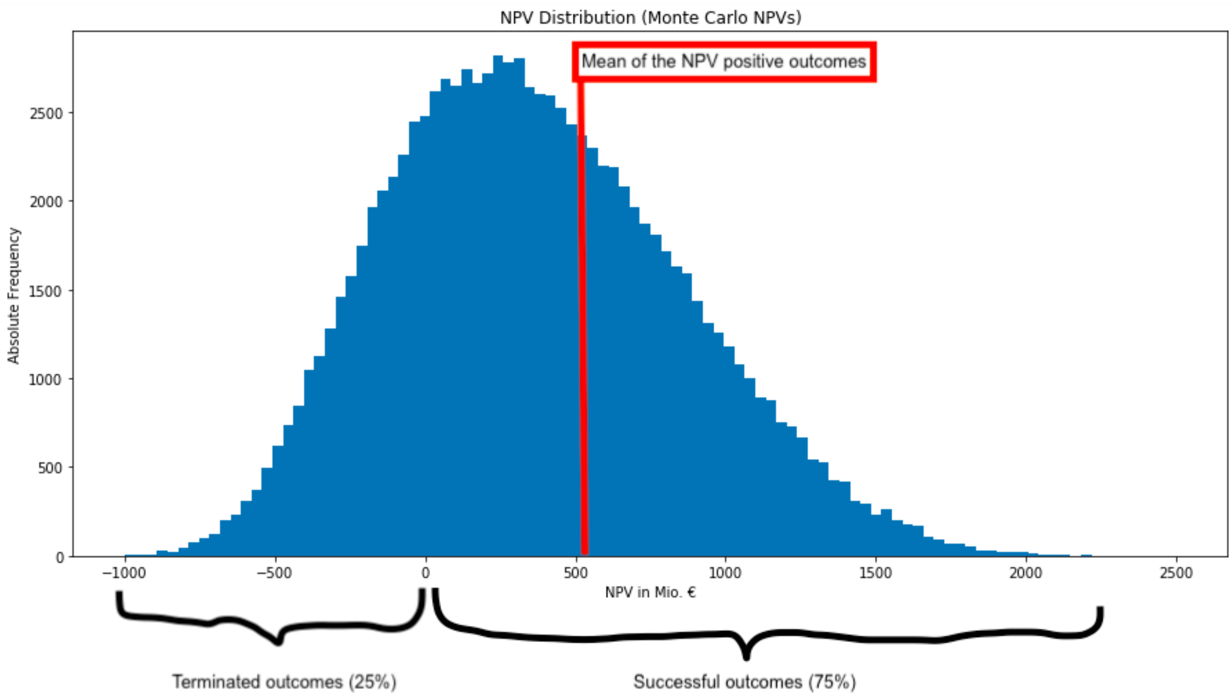
[3] Mathews, Scott, Vinay Datar, and Blake Johnson. "A practical method for valuing real options: The Boeing approach." Journal of Applied Corporate Finance 19.2 (2007): 95-104.

$$ROV = Risk\ \ adjusted\ probability\ of\ success\ \times\ Mean\ value\ of\ positive\ NPV\ outcomes$$

The risk-adjusted probability of success relates to the probability that the investment turns out to be profitable and the NPV is positive. Mathematically it is the surface under the right side of the distribution function of the Monte-Carlo NPVs.

The mean value of the positive NPV outcomes is the expected value of this right side of the probability function.

In the next graphic these explanations are made clear:



In my case study the risk-adjusted probability of success for the *Gold Digger AG* is 75.36%

The mean value of these positive outcomes is 560.94 Mio. €.
Plugging these numbers in the formula I get:

$$ROV = 0.7536\ \times\ 560{,}937{,}239€ = 422{,}722{,}303€$$

This leads to a real option value of about 422.72 Mio. €.

It is now possible to answer how much the management should be willing to pay for the extraction right. In negotiations, the calculated ROV can be used to set the walk away price.

Appendix

Fuzzy Logic Method

In order to further analyse the other methods in real option evaluation, I choose to look into the Fuzzy Logic Method and its application in the case study. The aim of this analysis is to make a comparison between the Monte-Carlo simulation and Fuzzy Logic Method and determine the best method for my case study.

By using the Monte Carlo analysis it can be statistically proved than an accurate output can be created, by realization distribution, which is representative of the entire range of possible realization outputs. However, the accuracy of the result from using the Monte Carlo simulation depends on how precisely all input parameters' distributions are defined. In many cases and projects, there are not enough data available to determine accurately input probability density functions[4].

Often the information that I have that might influence the choice of the best option is not quantitative data, uncertain information and imprecise data. The Fuzzy method is the tool that is used in these situations[5].

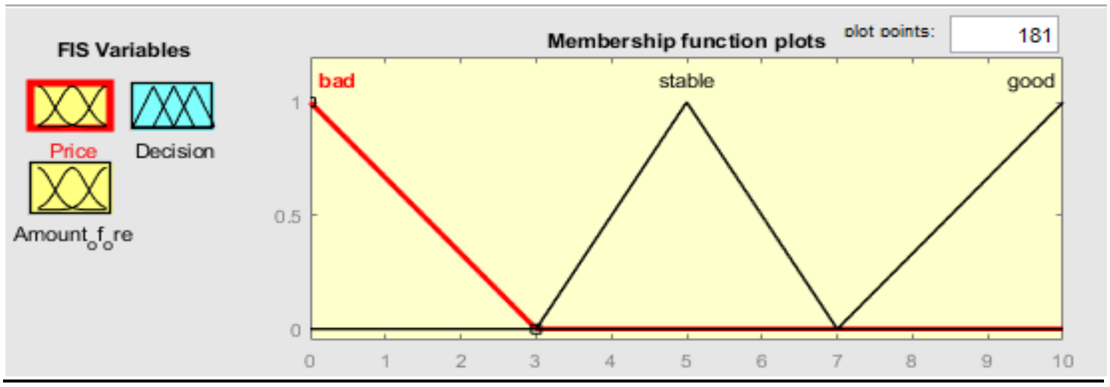
Fuzzy Logic Architecture

In building my model, I chose two variables as inputs. The two chosen variables are the gold price and the amount of gold, which has been defined by the experts. This was the first part, which is called the fuzzification module.

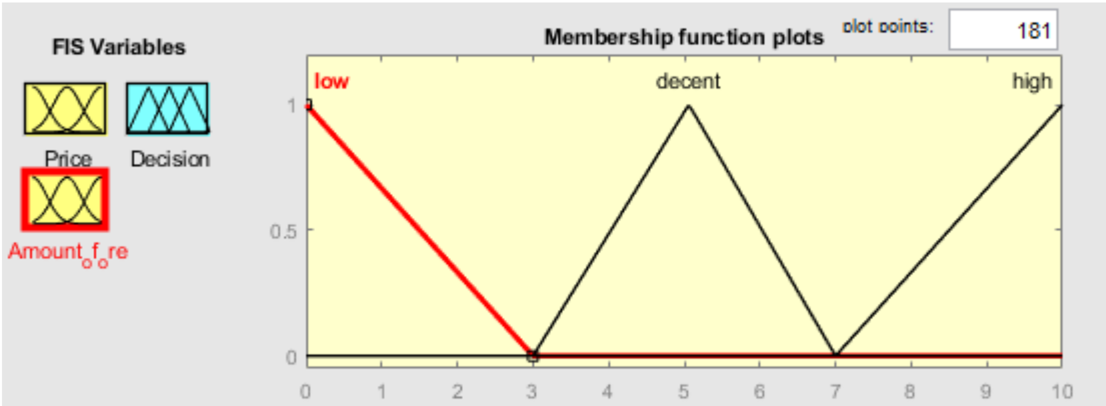
Additionally, the model consists of the IF-THEN rules. This is part of the knowledge base of the fuzzy logic architecture. By applying the IF-THEN rules I am able to come up with different scenarios of my model and analyze the real option evaluation.
The last part of the fuzzy logic architecture in my module is the defuzzification module, with which I can transform the fuzzy sets obtained into crisp values.

[4] Anna A. Loyd: "A comparison of Fuzzy indices with Monte Carlo simulations for risk assessment at the preliminary stages of transit project planning"(2006)
[5] Lambros K. Mitropoulos; Panos D. Prevedouros;Xin Yu;Eftihia G. Nathanail: " A Fuzzy and a Monte Carlo simulation approach to assess sustainability and rank vehicles in urban environment"(2016), p.297

Input variables



The first input variable that I take into consideration is the gold price. The price range from 0 to 3 is determined as Bad option, the price range from 3 to 7 is a Stable option, and 7 to 10 is a Good option.



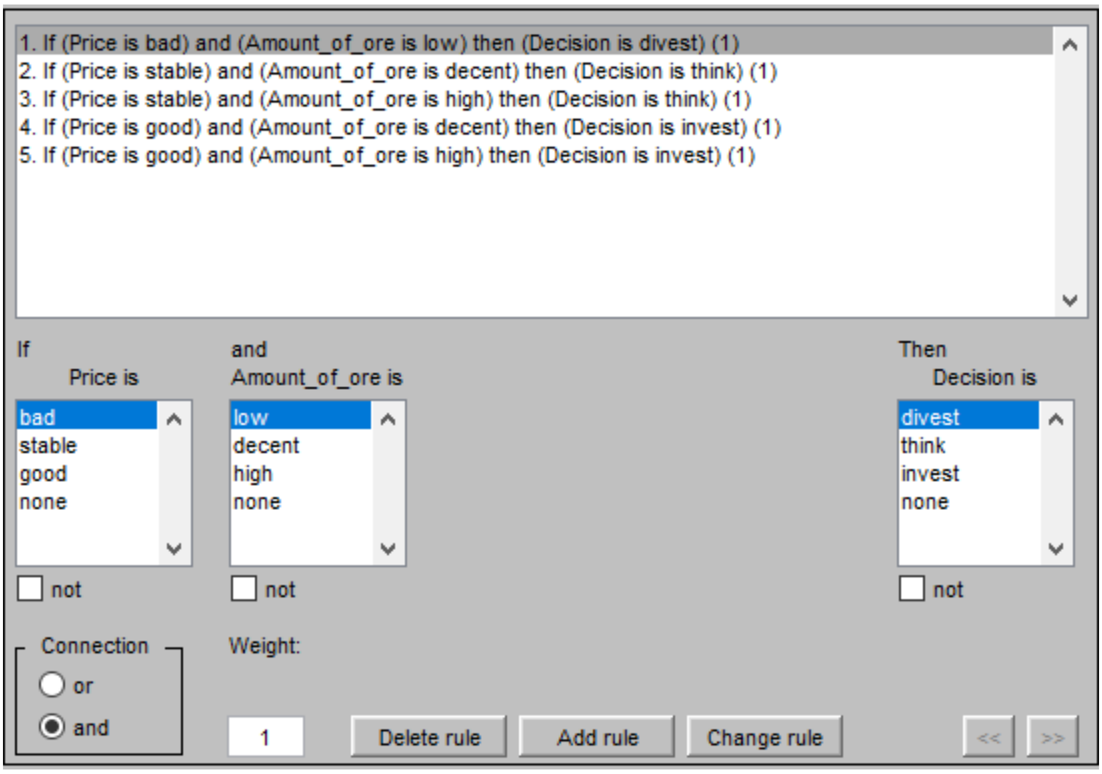
The second input variable has to do with the expert's opinion on the amount of gold ore that can be found in the ore. To determine the amount of ore, amount from 0 to 3 is considered as a Low amount, the amount range from 3 to 7 I consider as Decent, and 7 to 10 is considered as High amount of ore.

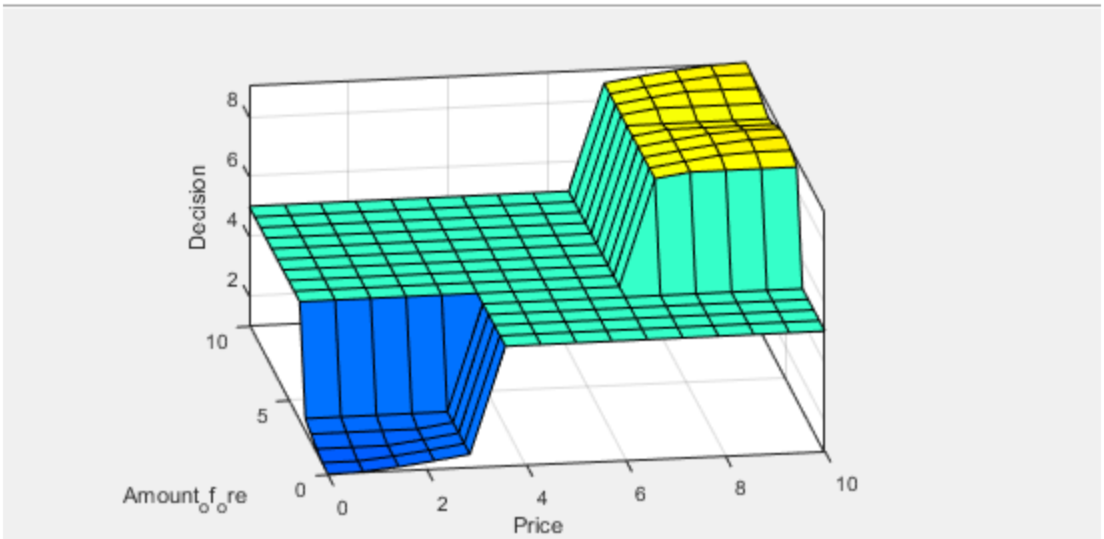
Rules

In order to determine what decision managers should make in terms of the changes in the input variables, I created several rules upon which the model works. Depending on the Price of gold and Amount of ore parameters, I can simulate three decisions, which are Divest, Think and Invest.

Firstly, in the case of Divest, if the decision range is lower than three, the model tells us that I should not invest in the option.
Secondly, by having a higher price or amount of ore, the model can suggest the decision Think in the range of 3 to 7.
Thirdly, the last decision is Invest, which means that the management should invest in the real option and that the option will be profitable.

Below I can see the rules that I have specified.





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