English-style RoboSoccer

Abstract: We simulate robots playing soccer on a field that they do not initially know. Each robot uses its sensors to observe fixed markings on the field to simultaneously map out the field and estimate its position on its own map. The algorithm employed is Extented Kalman Filter (EKF) Simultaneous Mapping and Localization (SLAM). Each robot performs EKF SLAM while chasing after a ball and attemping to kick the ball towards where it believes the opposite goal box is.

As the robots run around the field and make more and more observations, their uncertainties about their own positions and those of the field's markings tighten, and hence the robots' shooting accuracies improve significantly, resulting in thrilling, big-scoring soccer games!

1. The Environment

The concerned fixed environment is a rectangular Soccer Field measuring **105 meters long** by **68 meters wide**, with **two 7.32-meter-wide goals** at its two ends. The Field has a **reference coordinate system, with the center being at the origin (0, 0)**, the x-axis running along the long sides and the y-axis running along the wide sides.

In order to help robot Players orientate themselves on this Field, we place **ten markings** as follows:

- **four at the four corners**, named "***South West***", "***North West***", "***South East***" and "***North East***";
- **two at the long sides' midpoints**, named "***South***" and "***North***";
- and most importantly, **four at the four goal posts**, named "***Goal Post SW***", "***Goal Post NW***", "***Goal Post SE***" and "***Goal Post NE***".

Each **marking has a unique name / ID** that is assumed to be "loud and clear" to enable accurate data association.

2. The Ball, In- and Out-of-Play, and Goal-Scoring

The Ball is a circular object that rolls on the Field at a certain velocity and direction at a certain point in time. At each time step, absent any external impact (i.e. a kick!), the Ball's velocity slows down by a multiplicative shrinking factor (default 0.8).

The Ball is out-of-play when it rolls outside the Field's boundary. One particular out-of-play of special interest is a **scored goal**, which is when the Ball crosses between two posts of either the East-end goal box or the West-end goal box.

Whenever the Ball is out-of-play, including when a goal is scored, we throw a brand new Ball to a random location on the Field and let the game go on.

3. The Stars of the Game: (EKF SLAM) Robot Players

Each Player is a robot initially placed on the Field at a random position in either the West half (if it plays for Team West) or the East half (if it plays for Team East). The Player does not have any initial pre-conception of the Field and has to rely on sensing to understand this environment.

At each point in time, each Player performs the following capabilities:

- **chases after the Ball**: each Player has sensors that can detect the relative position of the Ball and motion actuators that enables it to run towards the Ball (at a velocity faster than the Ball's);
- **observes some of the Field's markings**: while chasing the Ball, each Player uses it sensors to observe the markings within a field of view of a certain number of degrees either side of the direction of the Ball. The Player maps out the Field's structure and locates itself on its own map, i.e. it performs *Simultaneous Localization and Mapping* (***SLAM***);

- upon **having the Ball** (by arriving close enough to it), (i) **kicks the Ball towards its target goal box** if it has seen at least one goal post, or (ii) **kicks the Ball in a random forward direction**.

(That is how soccer used to be played in England: kick the ball strong and long, and chase the ball passionately with a brave English heart… The English Premier League has grown into the world's richest soccer league, proving that there's no need to think too hard in soccer. So, let our robot Players not be too strategic or tactical about that to do with the Ball either.)

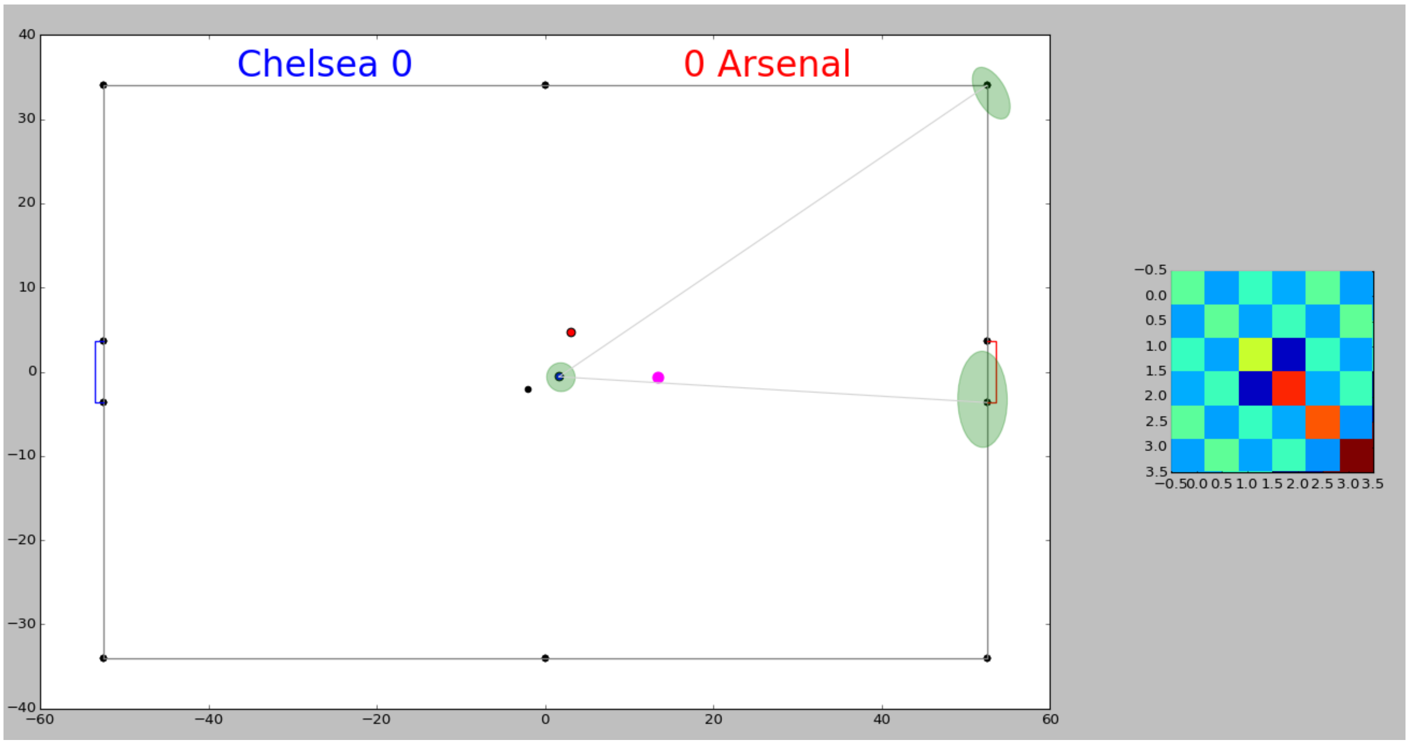

Figure 1: *Initialized state of a simple game, with two Players and a purple Ball on a Field. The transparent ellipses monitor the Blue Player's beliefs about its own position and those of the markings it has observed. The EKF covariance matrix on the right is initally sparse off the diagonal (light blue colors denoting near-zero values), meaning the beliefs about the landmarks are initially largely uncorrelated.*

4. The EKF SLAM Formulation in Detail

Let's now go into the specifics of the SLAM problem that each Player has to tackle. Each Player will use an Extended Kalman Filter (***EKF***) to do SLAM.

4.1. What's on a Map?

Each Player's map will attempt to estimate its own xy position on the Field as well as the xy position of each marking it has seen. Each xy position will be estimated by a Gaussian distribution with a mean position and a covariance matrix representing the degree of uncertainty. The **joint distribution of all concerned xy positions is hence multivariate Gaussian**:

$$\begin{bmatrix} x_t \\ y_t \\ \vdots \\ x_{\text{m } i} \\ y_{\text{m } i} \\ \vdots \end{bmatrix} \sim \mathcal{N}(\mu_t = \begin{bmatrix} \bar{x}_t \\ \bar{y}_t \\ \vdots \\ \bar{x}_{\text{m } i} \\ \bar{y}_{\text{m } i} \\ \vdots \end{bmatrix}, \Sigma_t = \begin{bmatrix} \text{var}(x_t) & \text{cov}(x_t, y_t) & \dots & \text{cov}(x_t, x_{\text{m } i}) & \text{cov}(x_t, y_{\text{m } i}) & \dots \\ \text{cov}(x_t, y_t) & \text{var}(y_t) & \dots & \text{cov}(y_t, x_{\text{m } i}) & \text{cov}(y_t, y_{\text{m } i}) & \dots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ \text{cov}(x_t, x_{\text{m } i}) & \text{cov}(y_t, x_{\text{m } i}) & \dots & \text{var}(x_{\text{m } i}) & \text{cov}(x_{\text{m } i}, y_{\text{m } i}) & \dots \\ \text{cov}(x_t, y_{\text{m } i}) & \text{cov}(y_t, y_{\text{m } i}) & \dots & \text{cov}(x_{\text{m } i}, y_{\text{m } i}) & \text{var}(y_{\text{m } i}) & \dots \\ \vdots & \vdots & \dots & \vdots & \vdots & \ddots \end{bmatrix}).$$

4.2. The Motion Model

Each Player's motion actuators attempt to move it at a **velocity v per second** at an **directional angle θ relative to "true East"**, i.e. the Field's positive x-axis. The result is a noisy motion as follows:

$$\begin{aligned}
&x_{t+1} = x_t + v_t \cos\theta_t + v_t \epsilon_{\text{motion } t}, \\
&y_{t+1} = y_t + v_t \sin\theta_t + v_t \epsilon_{\text{motion } t}, \\
&\text{where: random motion noise } \epsilon_{\text{motion } t} \sim \mathcal{N}(0, \sigma^2_{\text{motion}}).
\end{aligned}$$

That is, the Player's expected next position will shift by v_t at angle θ_t, but there is uncertainty around that expected position, representing by Gaussian distributions with standard deviations of $v_t \sigma_{\text{motion}}$ in both x- and y-directions. Note that **the motion uncertainty increases as the Player's velocity increases**.

The Jacobian matrix and the motion covariance matrix corresponding to this motion model are as follows:

$$F_{t+1} = I = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 & \dots \\ 0 & 1 & \dots & 0 & 0 & \dots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 & \dots \\ 0 & 0 & \dots & 0 & 1 & \dots \\ \vdots & \vdots & \dots & \vdots & \vdots & \ddots \end{bmatrix}, \text{ and}$$

$$R_{t+1} = \begin{bmatrix} v_t^2 \sigma^2_{\text{motion}} & 0 & \dots & 0 & 0 & \dots \\ 0 & v_t^2 \sigma^2_{\text{motion}} & \dots & 0 & 0 & \dots \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & 0 & 0 & \dots \\ 0 & 0 & \dots & 0 & 0 & \dots \\ \vdots & \vdots & \dots & \vdots & \vdots & \ddots \end{bmatrix}.$$

4.3. The Observation Model

Regarding each Player's observation of the Field's markings, the measurements taken are: (i) the **Euclidean distance from the Player to each observed marking**, and (ii) the **angle of the vector from the Player to each marking relative to "true East"**.

$$\begin{aligned}
&\hat{d}_{t,\text{ m } i} = \sqrt{(x_{\text{m } i} - x_t)^2 + (y_{\text{m } i} - y_t)^2} + \epsilon_{\text{distance } t,\text{ m } i}, \\
&\hat{\theta}_{t,\text{ m } i} = \arctan(y_{\text{m } i} - y_t, x_{\text{m } i} - x_t) + \epsilon_{\text{angle } t,\text{ m } i}, \\
&\text{where random distance measurement noise } \epsilon_{\text{distance } t,\text{ m } i} \sim \mathcal{N}(0, \sigma^2_{\text{distance}}), \\
&\text{and random angular measurement noise } \epsilon_{\text{angle } t,\text{ m } i} \sim \mathcal{N}(0, \sigma^2_{\text{angle}}).
\end{aligned}$$

That is, both distance and angular measurements are uncertain and represented by Gaussian distributions.

The Jacobian matrix and the observation covariance matrix corresponding to this observation model are as follows:

$$\begin{aligned}
&H_{t,\text{ m } i} = \begin{bmatrix} \frac{x_t - x_{\text{m } i}}{d_{t,\text{ m } i}} & \frac{y_t - y_{\text{m } i}}{d_{t,\text{ m } i}} & \dots & 0 & 0 & \frac{x_{\text{m } i} - x_t}{d_{t,\text{ m } i}} & \frac{y_{\text{m } i} - y_t}{d_{t,\text{ m } i}} & 0 & 0 & \dots \\ \frac{x_{\text{m } i} - x_t}{d^2_{t,\text{ m } i}} & \frac{y_{\text{m } i} - y_t}{d^2_{t,\text{ m } i}} & \dots & 0 & 0 & \frac{x_t - x_{\text{m } i}}{d^2_{t,\text{ m } i}} & \frac{y_t - y_{\text{m } i}}{d^2_{t,\text{ m } i}} & 0 & 0 & \dots \end{bmatrix}, \text{ and} \\
&Q_{t,\text{ m } i} = \begin{bmatrix} \sigma^2_{\text{distance}} & 0 \\ 0 & \sigma^2_{\text{angle}} \end{bmatrix}, \\
&\text{where:} \\
&d_{t,\text{ m } i} = \text{true distance } \sqrt{(x_{\text{m } i} - x_t)^2 + (y_{\text{m } i} - y_t)^2}.
\end{aligned}$$

4.4. The Augmentation of a New Marking to a Map

When each Player observes a marking it has not seen before, it augments it to its existing map as follows:

$$\mu_t^+ = \begin{bmatrix} \mu_t \\ \bar{x}_{m^+} \\ \bar{y}_{m^+} \end{bmatrix} \text{ and } \Sigma_t^+ = \begin{bmatrix} \Sigma_t & \Sigma_t G^T \\ G\Sigma_t & G\Sigma_t G^T + Q^+ \end{bmatrix},$$

where:

$\bar{x}_{m^+} = \bar{x}_t + \hat{d}_{t,\ \mathrm{m}^+} \cos\hat{\theta}_{t,\ \mathrm{m}^+}$,

$\bar{y}_{m^+} = \bar{y}_t + \hat{d}_{t,\ \mathrm{m}^+} \sin\hat{\theta}_{t,\ \mathrm{m}^+}$,

Jacobian matrix $G = \begin{bmatrix} 1 & 0 & \dots & 0 & 0 & \dots \\ 0 & 1 & \dots & 0 & 0 & \dots \end{bmatrix}$, and

Covariance matrix $Q^+ \approx \begin{bmatrix} \hat{\sigma}^2_{\bar{x}_{m^+}|\bar{x}_t} & \cos\hat{\theta}_{t,\ \mathrm{m}^+}\sin\hat{\theta}_{t,\ \mathrm{m}^+}\sigma^2_{\mathrm{distance}} \\ \cos\hat{\theta}_{t,\ \mathrm{m}^+}\sin\hat{\theta}_{t,\ \mathrm{m}^+}\sigma^2_{\mathrm{distance}} & \hat{\sigma}^2_{\bar{y}_{m^+}|\bar{y}_t} \end{bmatrix}$,

with $\hat{\sigma}_{\bar{x}_{m^+}|\bar{x}_t}$ and $\hat{\sigma}_{\bar{y}_{m^+}|\bar{y}_t}$ estimated by the largest x- and y-distances to $\bar{x}_{m^+}$ and $\bar{y}_{m^+}$ of the 8 points that are 1 s.d. away from $(\bar{x}_{m^+}, \bar{y}_{m^+})$ in distance ($\sigma_{\mathrm{distance}}$) and/or angle ($\sigma_{\mathrm{angle}}$).

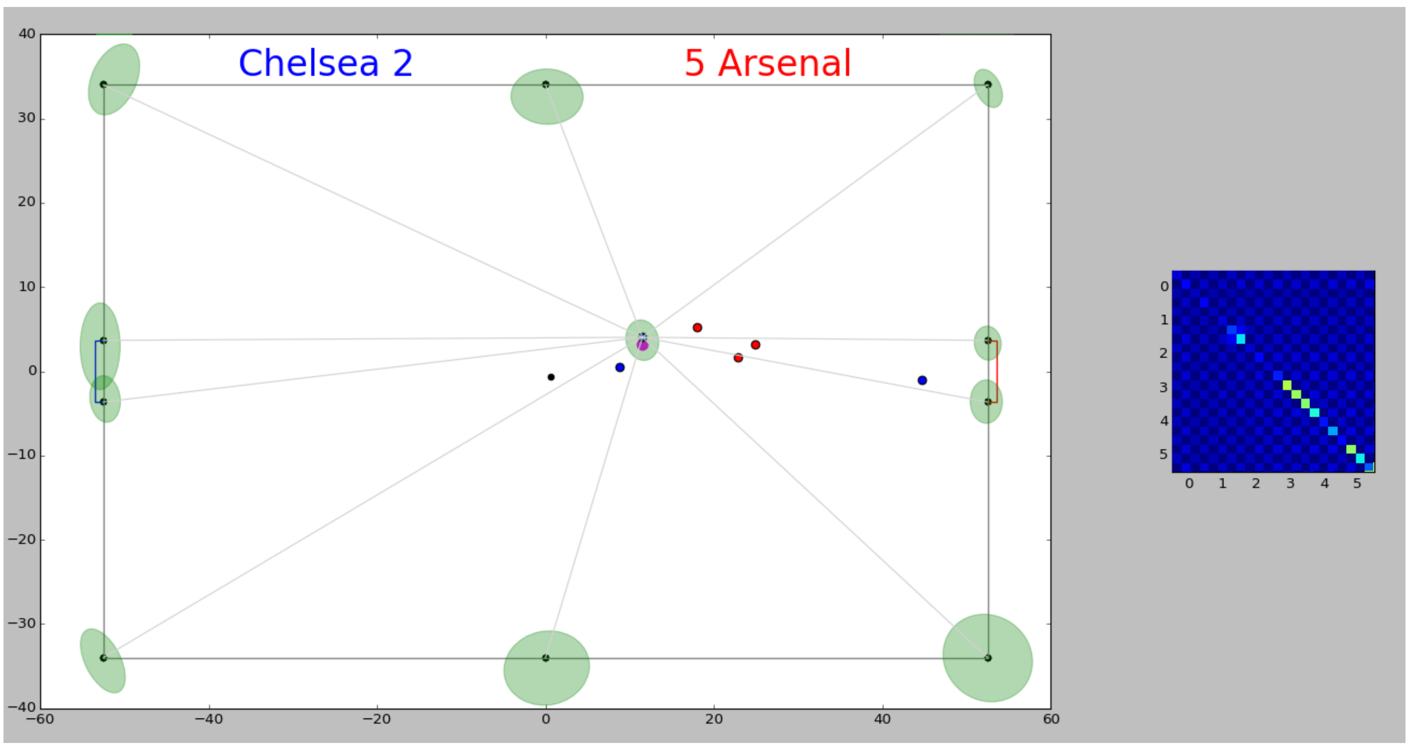

Figure 2: *The famous London (robo-)soccer derby between Chelsea (blue) and Arsenal (red) in full-blooded action. As the tracked Chelsea player observes more markings as the game progresses, it becomes more confident of its position and those of the markings, especially the target East-end goal posts that it frequently aims at. Note how all of the beliefs have now become correlated, signified by a fully dense EKF covariance matrix.*

5. Key Implementation Challenges and Their Resolutions

5.1. Distance-Related Numerical Instability

The Jacobian $H_{t,\ \mathrm{m}\ i}$ is inversely related to the distance $d_{t,\ \mathrm{m}\ i}$ and becomes numerically unstable with small distances, leading to disruptive "jumps" in the EKF algorithm. Hence we impose a restriction that each robot Player can only meaningfully "observe" a marking if the marking is not "right in its face". Specifically in this application, a Player only incoporates measurements on a marking into its EKF update step if the distance is at least six times the distance measurement noise σ_{distance}. This treatment applies to the map augmentation step as well.

5.2. EKF Inconsistency

It is known that the Extended Kalman Filter (EKF)‘s linearizations are sub-optimal and can lead to inconsistencies in the uncertainty estimates. Such inconsistencies manifest themselves in sudden “jumps” in the estimated distributions of the concerned robot’s pose and the landmarks’ positions.

In order to avoid such inconsistencies breaking down an exciting soccer game, we allow each robot Player to “calm down” and reset itself in the middle of the game if it feels “dizzy”: when the EKF update step produces a new distribution whose means are unexpectedly far from the latest estimated means, then the confused Player wipes out all of its estimates and starts observing and estimating its position all over again. This is feasible in the context of this RoboSoccer game because the Field only has ten markings and it does not take long for a Player to become confident again.

P.S.: 1) Thanks and wishes of health to 85-year-old grandpa Rudy Kálmán! :)

2) Guys, sit back and enjoy some live action!! :D